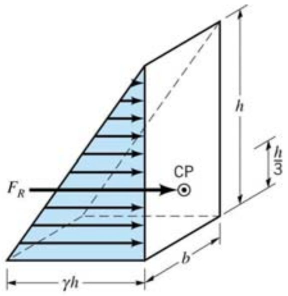
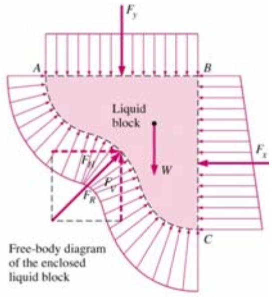
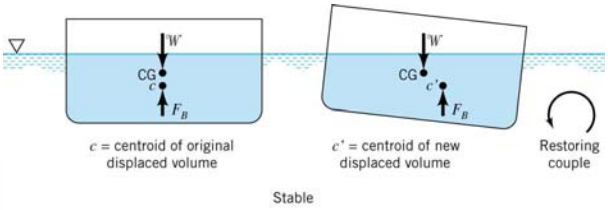
Faculty of Engineering and Technology
Department of Chemical Engineering
University Of Jordan

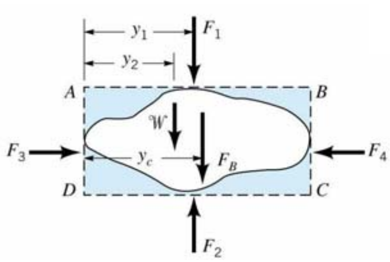


Fluid Mechanics (0905241)

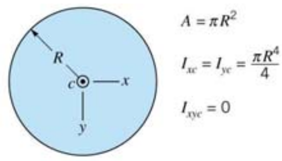
Pressure and Fluid Static (B)







Prof. Zayed Al-Hamamre



Content

- Hydrostatic Force on a Plane Surface
- Pressure Prism
- Hydrostatic Force on a Curved Surface
- Buoyancy, Flotation, and Stability
- Rigid Body Motion of a Fluid

Review

We have seen the following features of statics fluids

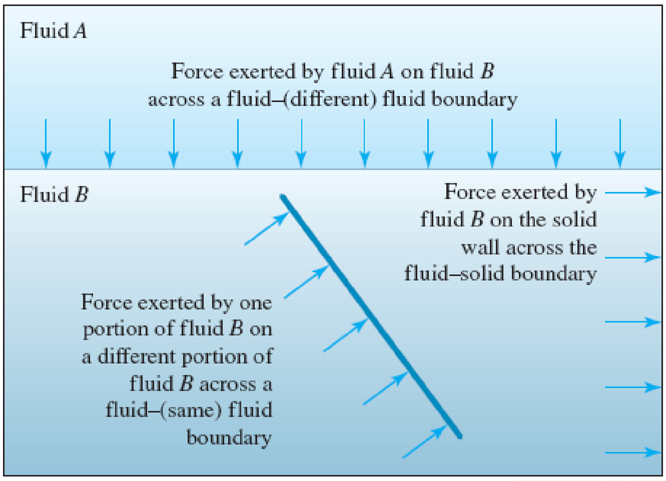
- Hydrostatic vertical pressure distribution
- Pressures at any equal depths in a continuous fluid are equal.
- Pressure at a point acts equally in all directions (Pascal. s law).
- For fluids at rest we know that the force must be *perpendicular* to the surface since there are no shearing stresses present.
- The pressure will vary linearly with depth if the fluid is incompressible.

Classification of Fluid Forces

All forces in fluid mechanics are divided into two distinctive types:

i. **Body forces** (gravity , centrifugal and electromagnetic forces), are external forces that act on a small fluid element in such a way that the magnitude of the body force is proportional to the element's mass (or volume) $(\delta M_j)_{\Re} = (\delta m)_{\Re}\, g_j = \rho \delta V_{\Re}\, g_j$

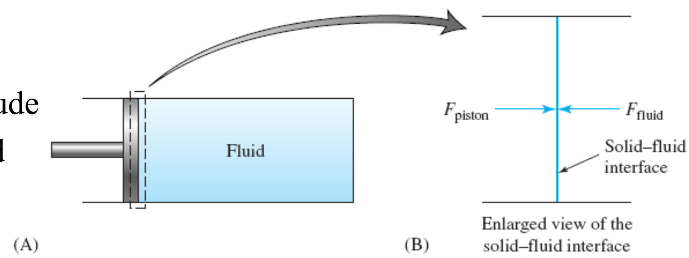
- Body forces are considered to be external forces; i.e., they are thought of as acting on a fluid, but not as forces applied by a fluid
- They exert their influence on a fluid at rest or in motion without the need for physical contact between the external source of the body force and the fluid



Classification of Fluid Forces

ii. **Surface forces,** such as those exerted by pressure or shear stress, are forces that act on a fluid element through physical contact between the element and its surroundings.

- Surface force is exerted across every boundary or interface between a fluid and another material.
- The second material may be a solid, another portion of the same fluid, or a different fluid.
- These forces exist at every interface wetted by a fluid and are present irrespective of whether the fluid is at rest (pressure only) or in motion (pressure and shear stress).
- From a macroscopic point of view, surface forces are applied to this interface by the piston and by the fluid are equal in magnitude and opposite in direction (Newton's second law).



Classification of Fluid Forces

- From a molecular perspective, the surface forces applied to this interface by the fluid are generated through molecular momentum transfer through the interaction of moving fluid molecules with the molecules of the solid piston.
- Surface forces are the macroscopic consequence of molecular momentum transfer
- Surface forces are forces exerted by a fluid on a solid is always equal and opposite to the force exerted by a solid on a fluid.
- A surface force depends on the contact area between a fluid and a second material.

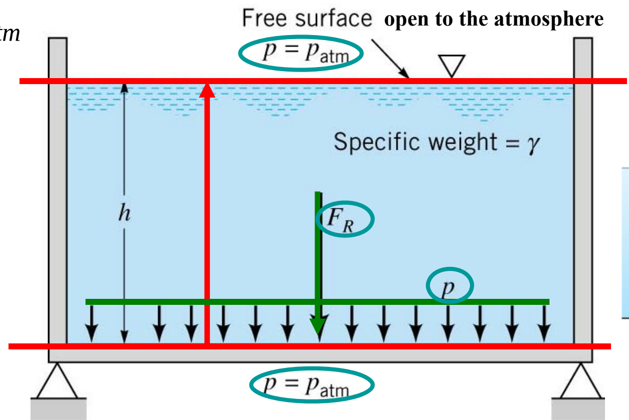
Hydrostatic Force on a Plane Surface: Tank Bottom

- When a surface is submerged in a fluid, forces develop on the surface due to the fluid.
- For fluid at rest, force must be *perpendicular* to the surface since there are no shearing stresses present.

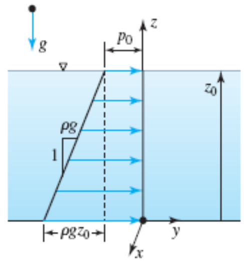
Simplest Case: Tank bottom with a uniform pressure distribution

$$p - \gamma h = p_{atm} - p_{atm}$$

$$p = \gamma h$$



For incompressible



Now, the resultant Force:

$$F_R = pA$$

Acts through the Centroid

A = area of the Tank Bottom

Hydrostatic Force on a Plane Surface: General Case

- **Determine the direction, location, and magnitude of the resultant force acting on an submerged inclined plane surface.**

The origin O is at the Free Surface.

θ is the angle the plane makes with the free surface.

y is directed along the plane surface.

A is the area of the surface.

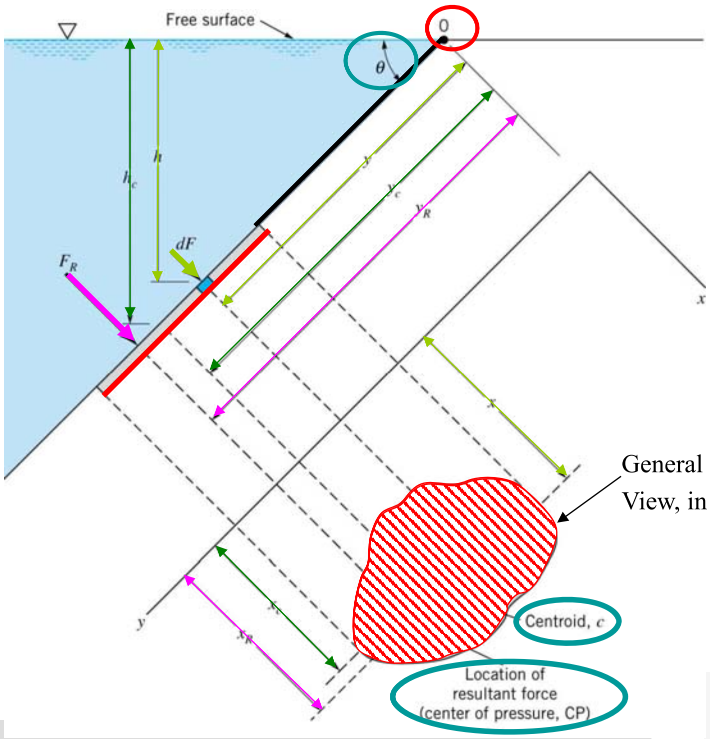
dA is a differential element of the surface.

dF is the force acting on the differential element.

C is the centroid.

CP is the center of Pressure

F_R is the resultant force acting through CP

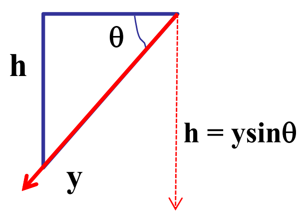


General Shape: Planar View, in the x-y plane

Hydrostatic Force on a Plane Surface: General Case

Then the force acting on the differential element:

$$dF = \gamma h \, dA$$



Then the resultant force acting on the entire surface:

$$F_R = \int_A \gamma h \, dA$$

We note h = ysinθ

$$= \int_A \gamma y \sin\theta \, dA$$

With γ and θ taken as constant:

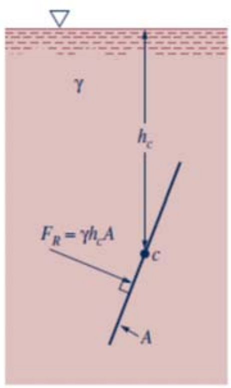
$$F_R = \gamma \sin\theta \int_A y \, dA$$

We note, the integral part is the first moment of area (first moment of inertia) about the x-axis

$$\int_A y \, dA = y_c A$$

Hydrostatic Force on a Plane Surface

Where y_c is the y coordinate to the centroid of the object.



$$F_R = \gamma A y_c \sin\theta \longrightarrow F_R = \gamma h_c A$$

(where $y_c \sin\theta = h_c$)

where h_c is the vertical distance from the fluid surface to the centroid of the area

- The magnitude of the force is independent of the angle θ and depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface
- The magnitude of the resultant force is equal to the pressure at the centroid of the area multiplied by the total area.

Hydrostatic Force on a Plane Surface

- Since all the differential forces that were summed to obtain F_R are perpendicular to the surface, the resultant F_R must also be perpendicular to the surface
- The location of the force F_R (y_R) can be determined by summation of moments around the *x axis, that is*, the moment of the resultant force must equal the moment of the distributed pressure force, or

$$F_R y_R = \int_A y\, dF = \int_A \gamma \sin\theta\, y^2\, dA \qquad \text{where} \qquad F_R = \gamma A y_c \sin\theta$$

Second moment of the area, I_{xc} → $\int_A y^2\, dA$

$$y_R = \frac{\int_A y^2\, dA}{y_c A} \quad \Rightarrow \quad y_R = \frac{I_{xc}}{y_c A} + y_c$$

Hydrostatic Force on a Plane Surface

- Where I_{xc} is the second moment of the area with respect to an axis passing through its *centroid* and parallel to the *x axis.*
- **For a submerged plane, the resultant force always acts below the centroid of the plane.**

$$I_{xc}/y_c A > 0.$$

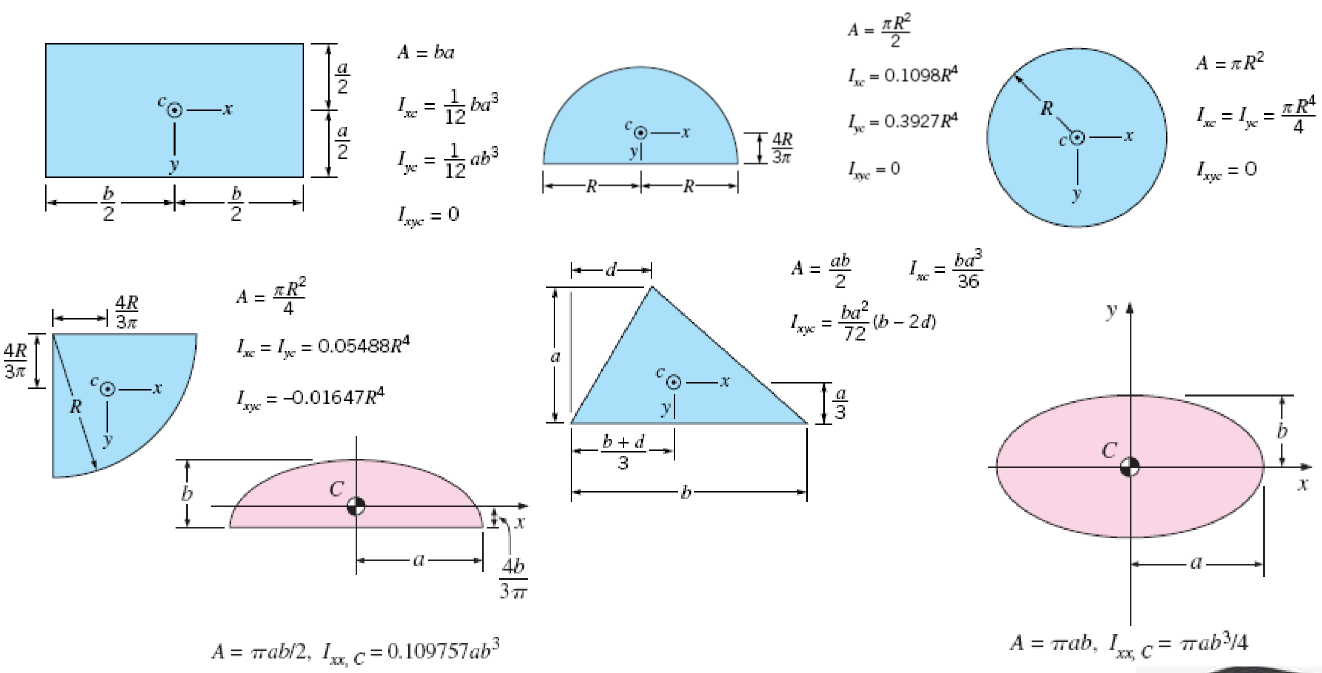
- Similarly, the x-coordinate can be found

$$F_R x_R = \int_A \gamma \sin\theta\, xy\, dA \quad \Rightarrow \quad x_R = \frac{\int_A xy\, dA}{y_c A} = \frac{I_{xy}}{y_c A} \quad \Rightarrow \quad x_R = \frac{I_{xyc}}{y_c A} + x_c$$

Keep in mind that $h_c = y_c \sin\theta$

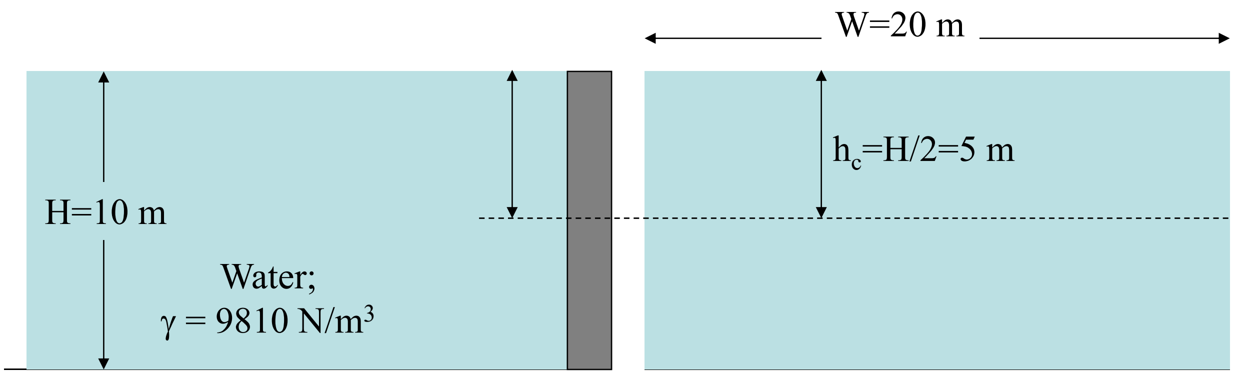
Hydrostatic Force on a Plane Surface

Geometric properties of some common shapes.



Example

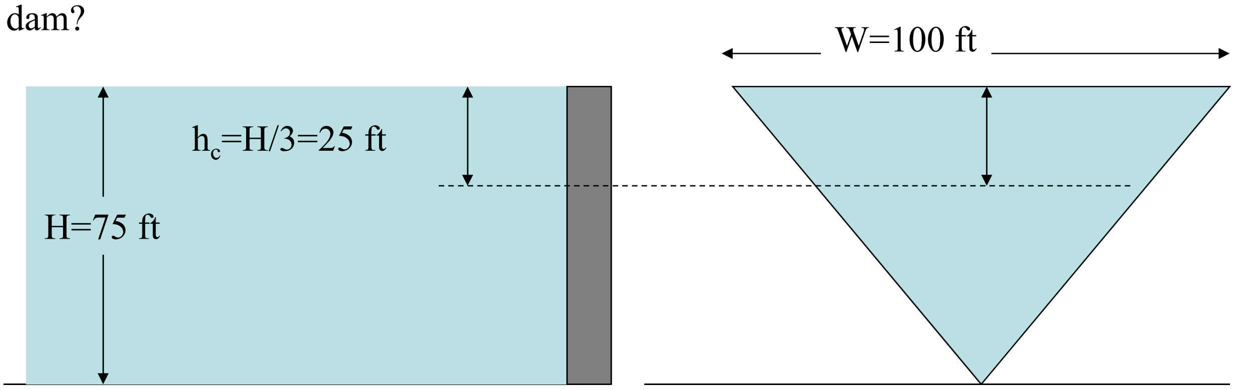
A rectangular dam (20 m wide and 10 m high is shown in the figure below. One side of the dam is exposed to atmosphere, the other side to water whose top surface is leveled with the top of the dam. What is the net force on the dam?



"Front view" "Side view"

Example

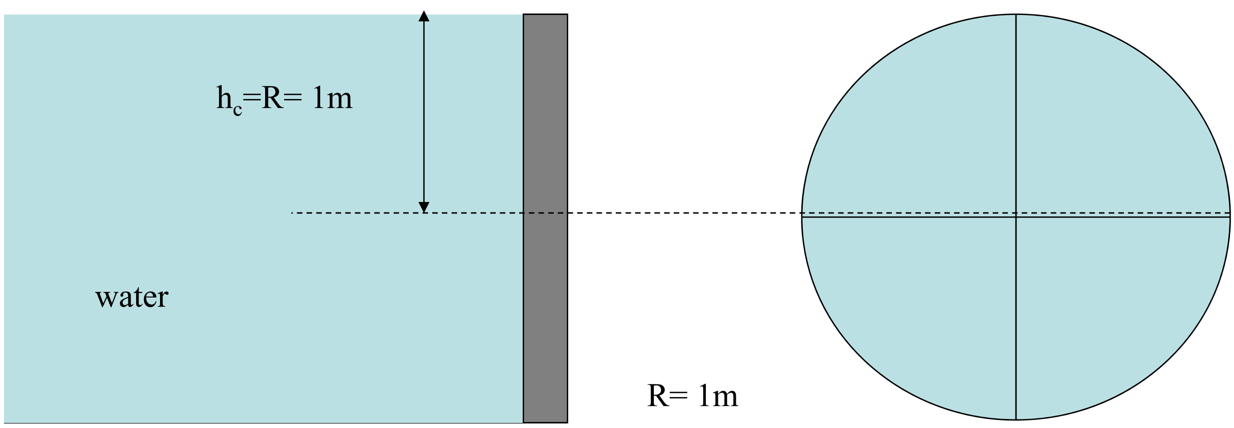
A triangular dam (100 ft across the top and 75 ft deep). Liquid water is up to the top on one side and the other side is exposed to atmosphere. What is the net force on the dam?



"Front view" "Side view"

Example

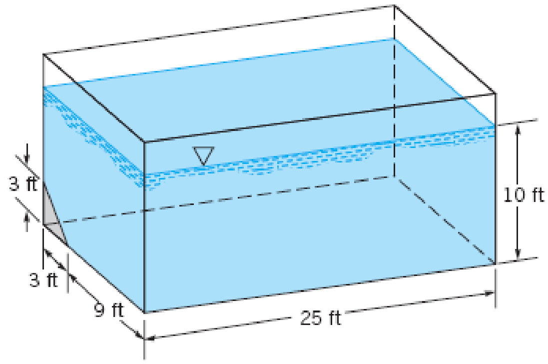
A circular gate of a dam has 2 m in diameter. Calculate the net force on the dam.



"Front view" "Side view"

Example

- A large fish-holding tank contains seawater $(\gamma = 64.0 \text{ lb/ft}^3)$ to a depth of 10 ft as shown. To repair some damage to one corner of the tank, a triangular section is replaced with a new section as illustrated. Determine the magnitude and location of the force of the seawater on this triangular area.

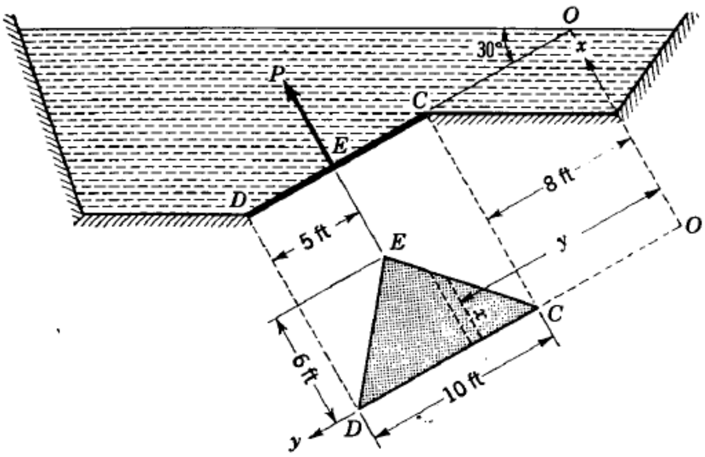


Example Cont.

Example

Triangular gate CDE is hinged along CD and is opened by a normal force F applied at E. It holds oil, s.g 0.80, above it and is open to the atmosphere on its lower side. Neglecting the weight of the gate determine (a) the magnitude of force exerted on the gate, (b) the location of pressure center; (c) the force P necessary to open the gate

$y_c = 5+8$



Example Cont.

$$F \doteq p_G A = \gamma y_c \sin\theta \, A = 62.4 \times 0.80 \times 0.50 \times 30 \times 13 = 9734.4 \text{ lb}$$

(62.4 = γ; 0.80 = s.g; 0.50 = $\sin\theta$; 30 = A; 13 = y_c)

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad = 0.0$$

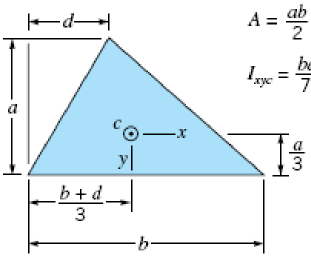
$$\frac{a}{3} = 2$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$\frac{I_{xc}}{y_c A} = 0.32$ ft

$y_c = 13$ ft

$y_R = 13.32$ ft



$A = \frac{ab}{2}$ $\quad I_{xc} = \frac{ba^3}{36}$

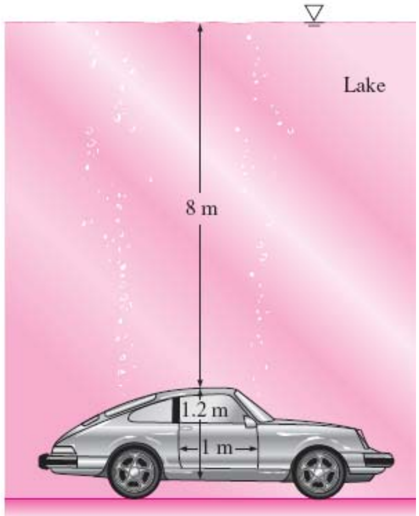
$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

When moments about CD are taken and the action of the oil is replaced by the resultant,

$$P \times 6 = 9734.4 \times 2 \qquad P = 3244.8 \text{ lb}$$

Example

- A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.



Hydrostatic Force: Vertical Wall

Find the Pressure on a Vertical Wall using Hydrostatic Force Method

Pressure varies linearly with depth by the hydrostatic equation:

The magnitude of pressure at the surface $p_o = 0.0$ and at bottom is $p = \gamma h$

The depth of the fluid is "h"

The width of the wall is "b" into the board

$$F_R = p_{av} A$$

By inspection, the average pressure occurs at h/2, $p_{av} = \gamma h/2$

$$F_R = \gamma \left(\frac{h}{2}\right) A$$

The resultant force act through the center of pressure, CP:

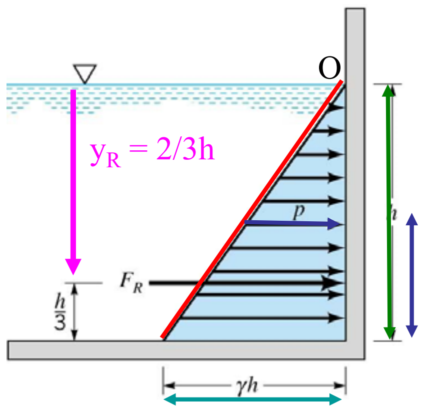
y-coordinate:

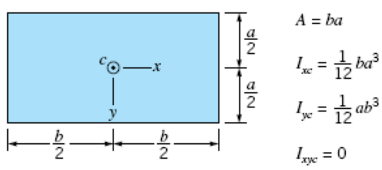
$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$I_{xc} = \frac{1}{12} b h^3 \qquad y_c = \frac{h}{2} \qquad A = bh$$

$$y_R = \frac{bh^3}{12 \frac{h}{2}(bh)} + \frac{h}{2}$$

$$y_R = \frac{h}{6} + \frac{h}{2} = \frac{2}{3} h$$





Hydrostatic Force: Vertical Wall

x-coordinate:

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

$$I_{xyc} = 0 \qquad x_c = \frac{b}{2} \qquad A = bh$$

$$\Rightarrow \quad x_R = \frac{0}{\frac{h}{2}(bh)} + \frac{b}{2}$$

$$x_R = \frac{b}{2}$$

Center of Pressure:

$$\left(\frac{b}{2}, \frac{2h}{3}\right)$$

Example

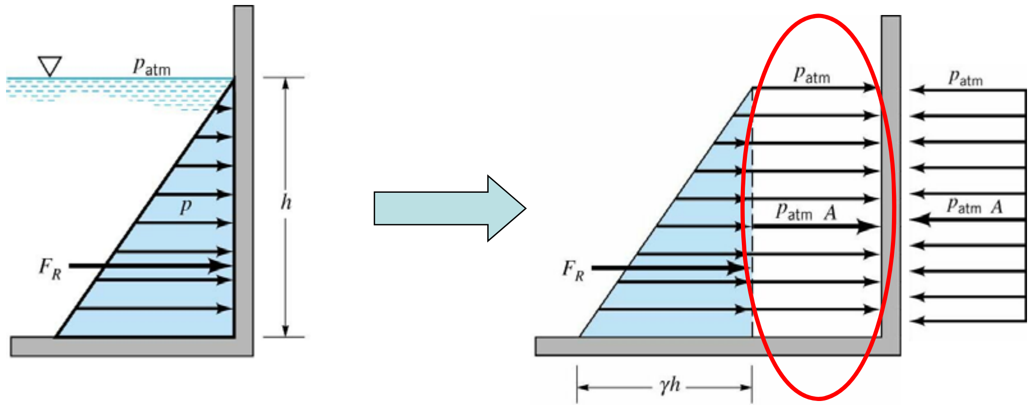
- A pressurized tank contains oil $(SG = 0.90)$ and has a square, 0.6-m by 0.6-m plate bolted to its side, as is illustrated. *When the pressure gage on the top of the tank reads* 50 kPa, what is the magnitude and location of the resultant force on the attached plate? Given that the outside of the tank is at atmospheric pressure.

Atmospheric Pressure on a Vertical Wall

Gage Pressure Analysis

Absolute Pressure Analysis

But,



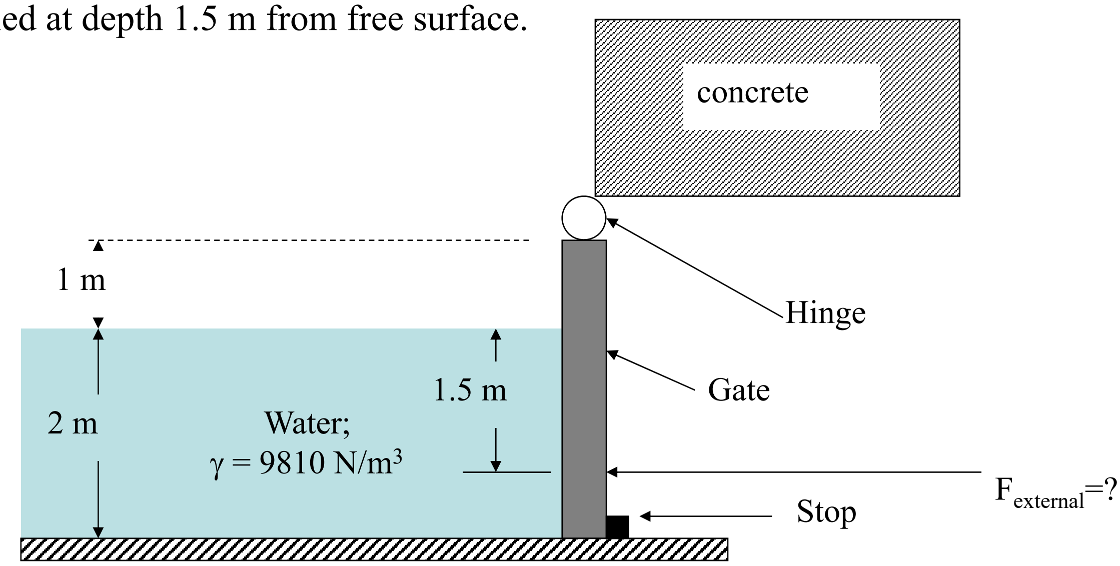
So, in this case the resultant force is the same as the gage pressure analysis.

It is not the case if the container is closed with a vapor pressure above it.

If the plane is submerged, there are multiple possibilities.

Example

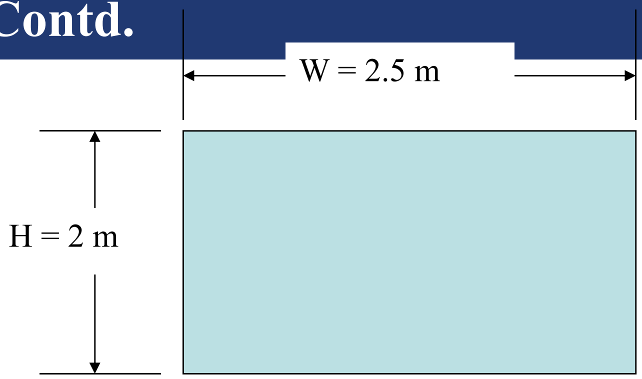
A rectangular gate (2.5 m wide and 3 m high) with a hinge and a stop as shown in the figure below. (a) What is the resultant force on the gate and locate its center of pressure? (b) Find the external force required just to open the gate if it will be applied at depth 1.5 m from free surface.



"Front view"

Example Contd.

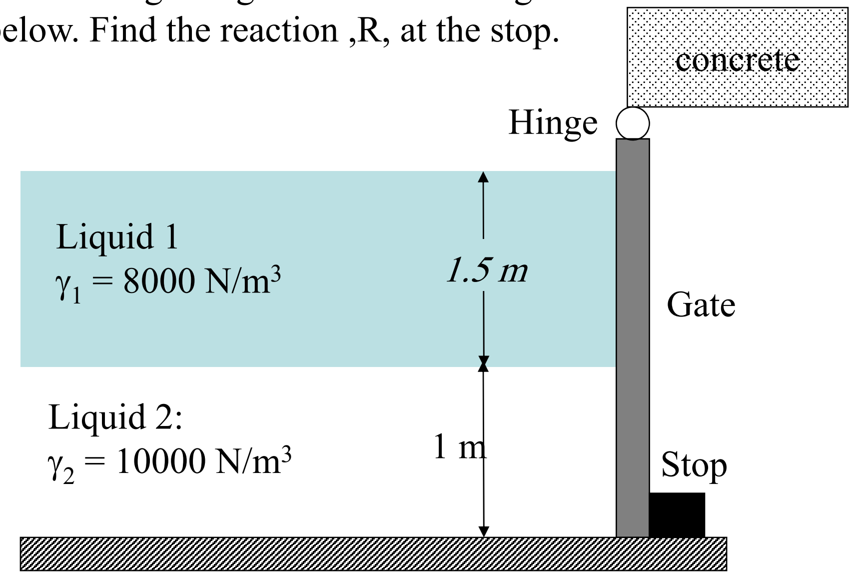
Example Contd.



"Side view of area of the gate in contact with water"

Example

Two liquids are attached to a rectangular gate of 2.8 m long and 2 m width (inside the paper) as shown below. Find the reaction ,R, at the stop.



Liquid 1 affects on the gate by force F_1 at center of pressure Lp_1.
Liquid 2 affects on the gate by force F_2 at center of pressure Lp_2.
At the stop there is a reaction R.

Example Contd.

Apply the moment balance around hinge:

Example Contd.

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Example Contd.

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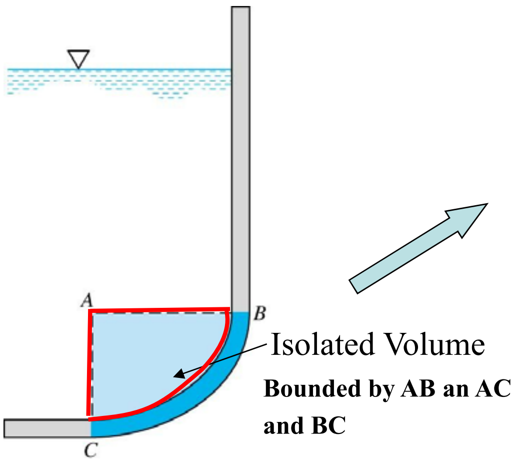
General theory of plane surfaces does not apply to curved surfaces

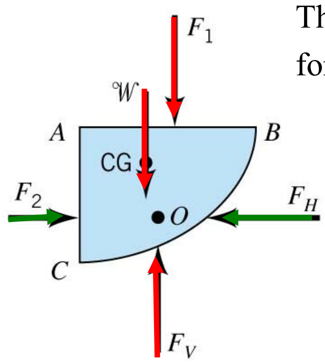
Many surfaces in dams, pumps, pipes or tanks are curved

No simple formulas by integration similar to those for plane surfaces

A new method must be used



Then we mark a F.B.D. for the volume:



F_1 and F_2 is the hydrostatic force on each planar face

F_H and F_V is the component of the resultant force on the curved surface.

W is the weight of the fluid volume.

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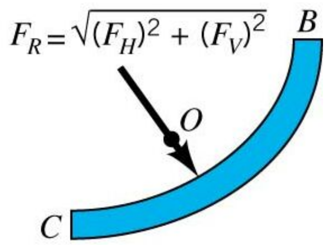
Now, balancing the forces for the Equilibrium condition:

Horizontal Force: $F_H = F_2$

Vertical Force: $F_V = F_1 + W$

Resultant Force: $F_R = \sqrt{(F_H)^2 + (F_V)^2}$



The location of the Resultant Force is through O by sum of Moments:

Y-axis: $F_1 x_1 + W x_c = F_V x_V$

X-axis: $F_2 x_2 = F_H x_H$

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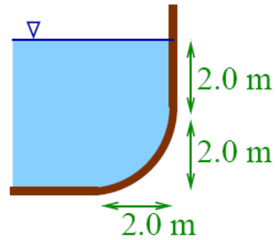
Example

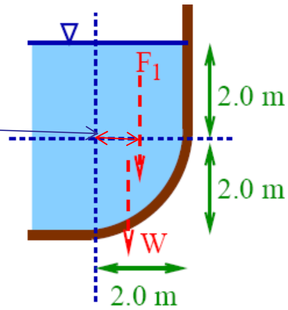
➢ Determine the resultant force on the curved part of the and also determine its line of action. The bottom corner of the tank is a circle of radius 2.0 m. The tank length (out of page) is 8.0 m



The centroid of the quarter circle wedge is

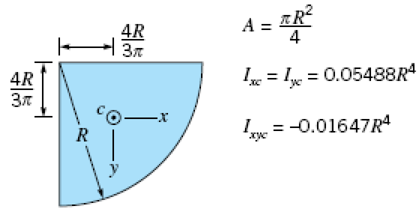
$$x_c = \frac{4R}{3\pi}$$

$$x_c = 0.84883 \text{ m}$$



$$F_1 = \gamma 2.0(2.0 \times 8.0)$$

$$F_1 = 313.6 \text{ kN}$$



$$W = \gamma \frac{1}{4}\pi(2.0)^2 \times 8.0$$

$$W = 246.3 \text{ kN}$$

The total vertical force is

$$246.3 + 313.6 = 559.9 \text{ kN}$$

F_1 line of action 1.0 m from wall.

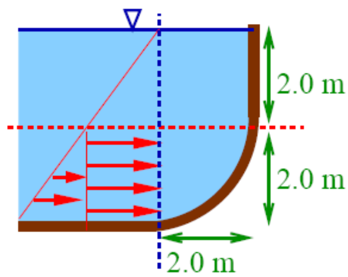
W line of action $2.0 - 0.84883 = 1.151$ m from wall.

Line of action for F_1 and W $\quad x_R = \dfrac{313.6 \times 1.0 + 246.3 \times 1.151}{559.9} = 1.066$ m

Example Cont.

$$F_{\square} = \gamma 3.0(2.0 \times 8.0)$$

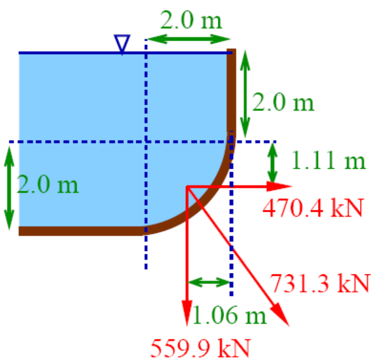
$$F_{\square} = 470.4 \text{ kN}$$



The net force is

$$F_R = \sqrt{559.9^2 + 470.4^2} = 731.3 \text{ kN}$$

$$\tan\theta = \frac{F_V}{F_H} = 1.19 \qquad \theta = = 50^{o}$$



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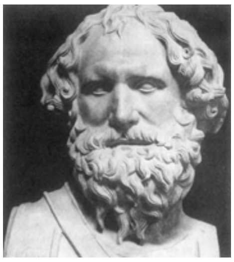
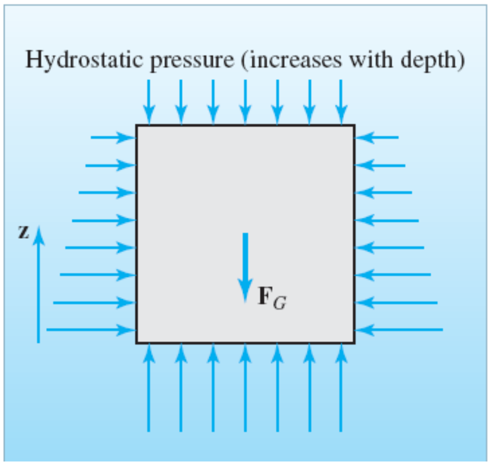
Buoyancy: Archimedes' Principle



Archimedes (287-212 BC)

Archimedes' Principle states that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward through the centroid of the displaced volume.

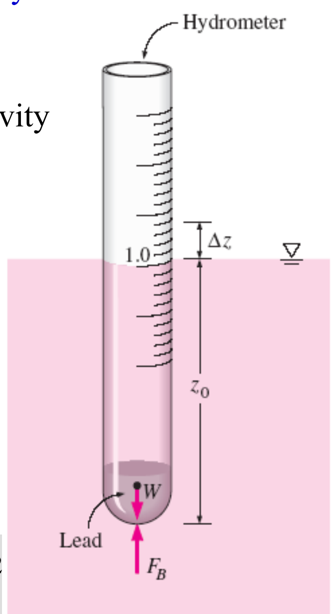
- Buoyant force is a force that results on a floating or submerged body in a fluid.
- The force results from different pressures on the top and bottom of the object
- The pressure forces acting from below are greater than those on top



Buoyancy: Archimedes' Principle

- The altitude of a hot air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air. When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

Measuring Specific Gravity by a Hydrometer



$$SG_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

Show this

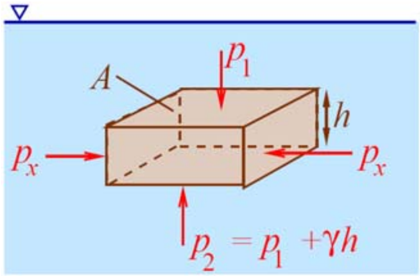
Buoyancy and Flotation: Archimedes' Principle

Balancing the Forces of the F.B.D. in the vertical Direction:

$$F_{Hydro} = p_2 A - p_1 A$$

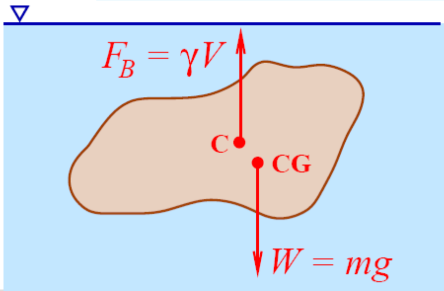
$$F_{Hydro} = (p_1 A + \gamma h) A - p_1 A$$

$$F_{Hydro} = \gamma h A = \gamma V = F_b$$



- The net hydrostatic force of the fluid on the body is vertically upward and is known as the Buoyant Force. The force is equal to the weight of the fluid it displaces.
- The Buoyancy force of a submerged body passes through a centroid called the center of buoyancy

$$F_{net} = (\rho_H - \rho_C)\forall g$$



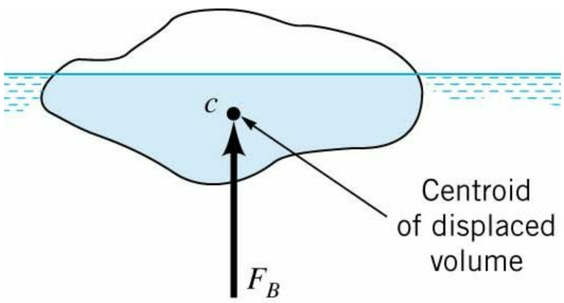
- The weight force passes through the center of gravity and does not always pass through the buoyancy centroid.

$$F_{net} = F_{gravity} - F_{buoyancy} = W_{obj} - W_{fluid}$$

$$\mathbf{F_{net} = (\rho_{obj} - \rho_F)\, Vg}$$

Buoyancy and Flotation: Archimedes' Principle

We can apply the same principles to floating objects:



- Conclude that the *buoyant force passes through the centroid of the displaced volume as shown.*

- If the specific weight varies in the fluid, the buoyant force does not pass through the centroid of the displaced volume, but through the center of gravity of the displaced volume.

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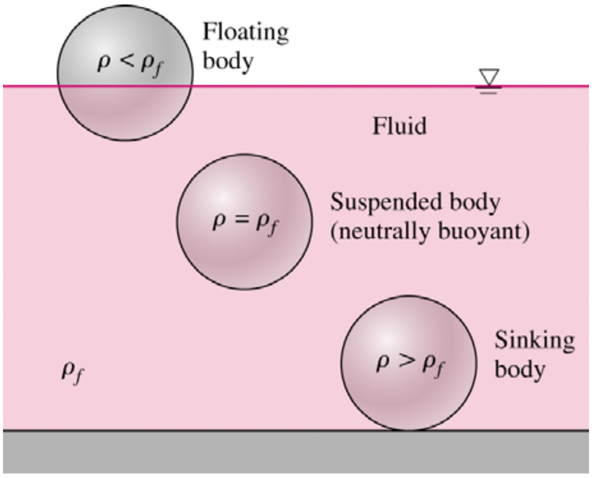
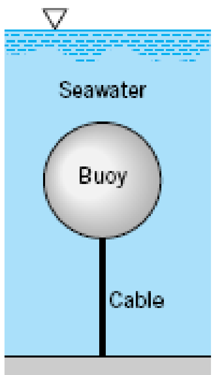
Buoyancy and Flotation: Archimedes' Principle



- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body}<\rho_{fluid}$: Floating body
  2. $\rho_{body}=\rho_{fluid}$: Neutrally buoyant
  3. $\rho_{body}>\rho_{fluid}$: Sinking body

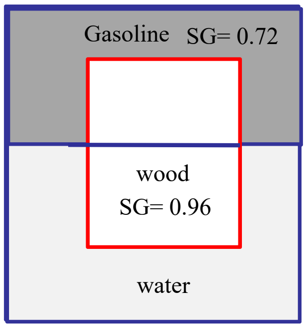
Example

➢ A spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the sea floor with a cable as is shown. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed as illustrated. For this condition what is the tension of the cable?



Example

- A block of wood floating at the interface between a layer of gasoline and a layer of water as shown. What fraction of the wood is below the interface?



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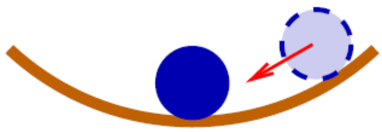
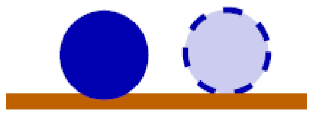
Stability

- The stability of a body depend on what happens when it is displaced from the equilibrium position
- The equilibrium is stable if the forces (as gravity)) acting on the object act to return it to its equilibrium position



- The equilibrium is unstable if the forces acting on the object act to send it away from its equilibrium position.

- The equilibrium is neutral if there are no net forces acting on the object to return it or remove it from the new equilibrium (It has no tendency to move back to its original location, nor does it continue to move away)



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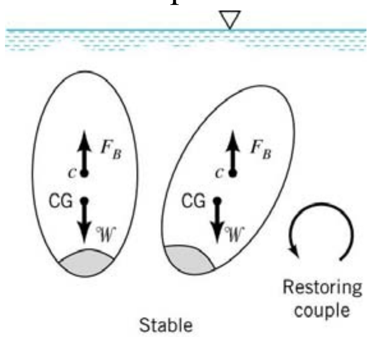
Stability of Immersed Bodies

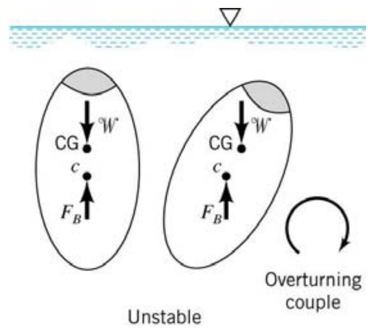
- Stable Equilibrium: if when displaced returns to equilibrium position.
- Unstable Equilibrium: if when displaced it returns to a new equilibrium position.

Stable Equilibrium:



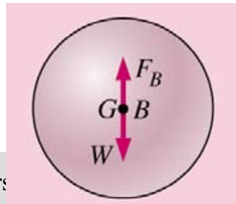
C > CG, "Higher"

Unstable Equilibrium:



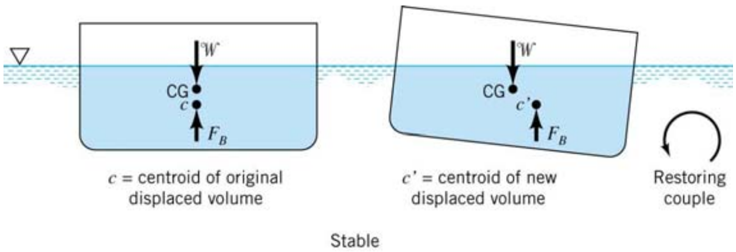
C < CG, "Lower"

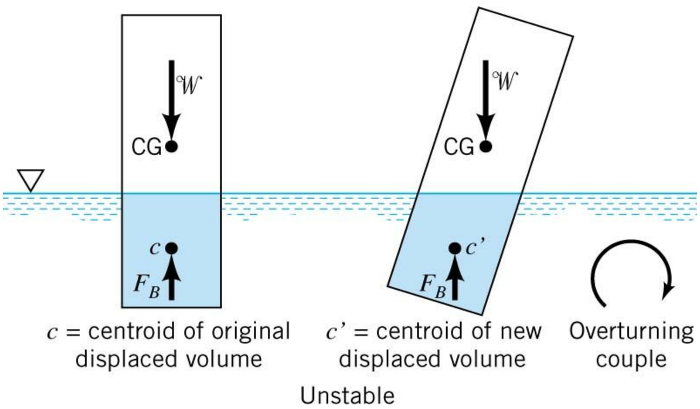
- Neutrally stable: C*G* coincides with C



Buoyancy and Stability: Floating Object

- Slightly more complicated as the location of the center buoyancy can change:



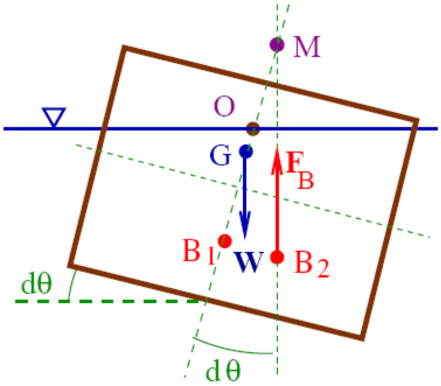


- The centroid (of the displaced volume) can shift as the body has an angular displacement. It is the movement of the centroid to the right that gives this body its stability.
- A rotational disturbance of the body in such cases produces a restoring moment to return the body to its original stable position
- The buoyancy force and gravitational force act to create an overturning torque.

Stability of Floating Bodies

- Consider a floating body given a small angular deflection.

➡ The magnitude of the buoyancy force will stay the same, (weight force does not change) but the location of the centre of buoyancy changes.



O = Water line point about which boat rolls.

B_1 = original buoyancy point.

B_2 = buoyancy point after displacement.

The line of action of the original buoyancy force (through the center of gravity) and new buoyancy force intersect at the metacenter.

Metacenter M: is the point of intersection between the original line of action and new line of action of the buoyancy force

G The center of gravity

d(GM) The metacentric height, the displacement of M from G.

dθ: The angular displacement should be small, e.g. less than 20^o

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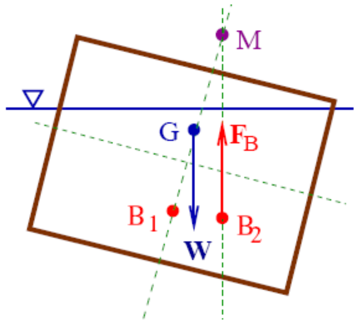
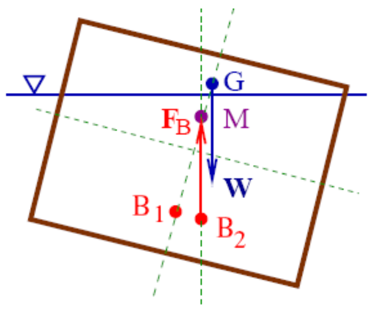
Stability of Floating Bodies

Positive Stability The ship is stable if $GM > 0$, i.e. the metacenter lies above the center of gravity.

$$GM = d(BM) - d(BG)$$



Negative Stability The ship is unstable if $GM < 0$, i.e. the metacenter lies Below the center of gravity.



Neutral stability Occurs when $GM = 0$

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Stability of Floating Bodies

1. One determines position of center of gravity
2. Determine water level
3. One determines center of buoyancy of displaced volume
4. Then d(BG) is known.
5. Finally, d(BM) is evaluated with $d(BM) = \frac{I_y}{V}$

$V =$ displaced volume

$$I_y = I_{yc} + x_c^2 A$$

Stable if $d(BM) - d(BG) > 0$

Unstable if $d(BM) - d(BG) < 0$

To improve stability

Increase width. Makes I_y larger

Lower position of G

Add ballast (near the bottom). This also tends to raise B and raises the position of M.

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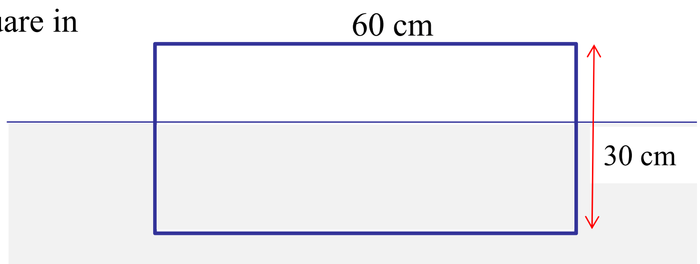
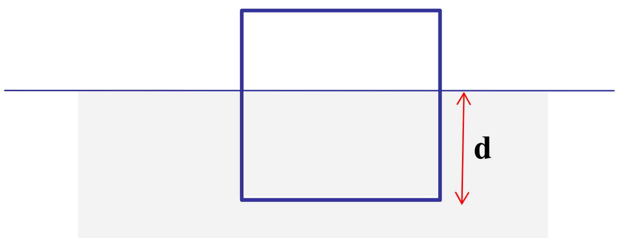
Example

- A wood block of 60 cm long and 30 cm square in





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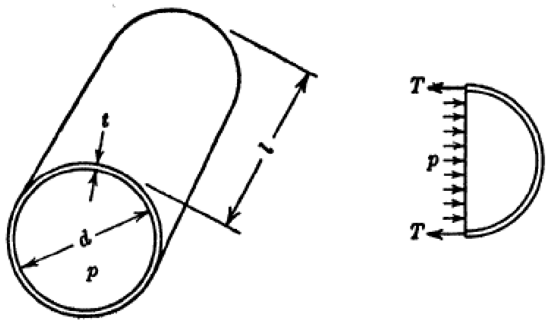
Stresses in Circular Pipes and Tanks.

- If pressure variation is neglected and if the thickness of the pipe or tank is small compared to the diameter.
- A section of pipe of length *L*, an internal diameter *d*, contains a fluid whose pressure is p; the circumferential tension stress in the walls is s_t.



$$2T - pdl = 0$$

T is the total tension force in the wall of length l and thickness t due to the stress s_t,

$$T = s_t t l$$

$$2 s_t t l = pdl$$

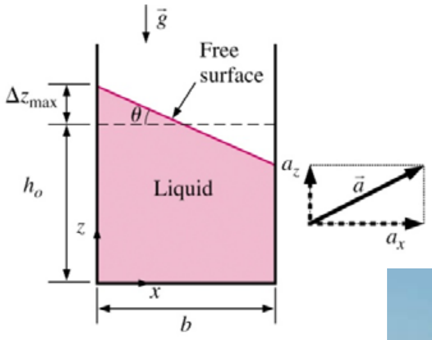
$$s_t = \frac{1}{2}\frac{pd}{t}$$

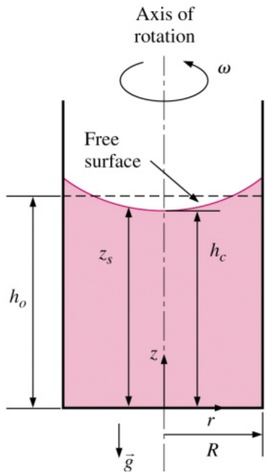
Stresses in Circular Pipes and Tanks.

- The stress caused by a given pressure may be reduced by decreasing the diameter d, or increasing the wall thickness
- Since increasing the wall thickness increases the cost, it becomes evident why small-bore tubing is in general use in high-pressure work.

Rigid-Body Motion

- There are special cases where a body of fluid can undergo rigid-body motion: linear acceleration, and rotation of a cylindrical container.





- In these cases there is no deformation, and thus there can be no shear stress,
- Newton's 2nd law of motion can be used to derive an equation of motion for a fluid that acts as a rigid body

Review

➢ Taking the pressure at the center of the element to be P

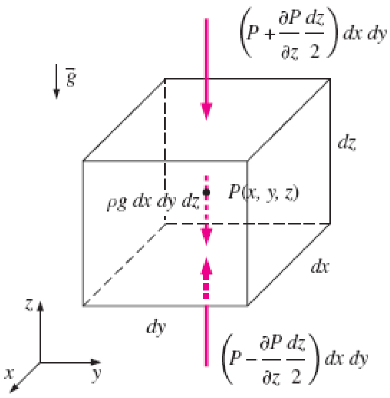
surface forces

$$\delta F_{S,z} = \left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy - \left(P + \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy = -\frac{\partial P}{\partial z} dx\, dy\, dz$$

Similarly,

$$\delta F_{S,x} = -\frac{\partial P}{\partial x} dx\, dy\, dz \qquad \delta F_{S,y} = -\frac{\partial P}{\partial y} dx\, dy\, dz$$

$$\delta \vec{F}_S = \delta F_{S,x}\vec{i} + \delta F_{S,y}\vec{j} + \delta F_{S,z}\vec{k} = -\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right) dx\, dy\, dz = -\vec{\nabla} P\, dx\, dy\, dz$$



body forces

$$\delta F_{B,z} = -g\delta m = -\rho g\, dx\, dy\, dz$$

➢ Then the total force acting on the element becomes

$$\delta \vec{F} = \delta \vec{F}_S + \delta \vec{F}_B \quad \Rightarrow \quad \delta \vec{F} = \delta m \cdot \vec{a} = \rho\, dx\, dy\, dz \cdot \vec{a}$$

$$= -(\vec{\nabla} P + \rho g \vec{k})\, dx\, dy\, dz$$

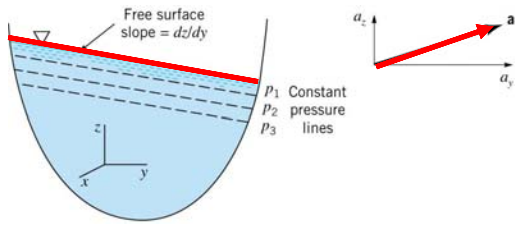
Pressure Variation, Rigid Body Motion: Linear Motion

Governing Equation with no Shear (Rigid Body Motion): $\nabla P + \rho g\vec{k} = -\rho\vec{a}$

The equation in all three directions are the following:

$$-\frac{\partial p}{\partial x} = \rho a_x \qquad -\frac{\partial p}{\partial y} = \rho a_y \qquad -\frac{\partial p}{\partial z} = \gamma + \rho a_z$$

- Consider a container partially filled with a liquid. The container is moving on a straight path with a constant acceleration
- The projection of the path of motion on the horizontal plane to be the *y-axis, and the projection* on the vertical plane to be the *z-axis*



$$\frac{\partial p}{\partial y} = -\rho a_y$$

$$\frac{\partial p}{\partial z} = -\rho(g + a_z)$$

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Pressure Variation, Rigid Body Motion: Linear Motion

Estimating the pressure between two closely spaced points apart some dy, dz the total differential of P:

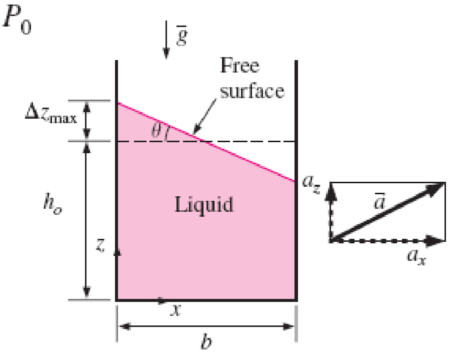
$$dp = \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$$ Substituting the partials

$$dp = -\rho a_y\,dy - \rho(g + a_z)\,dz$$

$$P_2 - P_1 = -\rho a_y(y_2 - y_1) - -\rho(g + a_z)(z_2 - z_1)$$

Taking point 1 to be the origin ($x = 0$, $z = 0$) where the pressure is P_0 point 2 to be any point in the fluid

Pressure variation: $P = P_o - \rho a_y y - \rho(g + a_z)z$



- The vertical rise (or drop) of the free surface at point 2 relative to point 1 can be determined by choosing both 1 and 2 on the free surface

$p_1 = p_2$ *isobars*

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Pressure Variation, Rigid Body Motion: Linear Motion

Along a line of constant pressure, dp = 0: $\frac{dz}{dy} = -\frac{a_y}{g + a_z}$ ⟹ Slope $= -\tan\theta$

Inclined free surface for $a_y \neq 0$

Vertical rise of surface: $\Delta z_s = z_{s2} - z_{s1} = -\frac{a_y}{g + a_z}(y_2 - y_1)$

Now consider the case where $a_y = 0$, and $a_z \neq 0$:

Recall, already: $\frac{\partial p}{\partial x} = 0$ Then, $\frac{\partial p}{\partial y} = 0, \frac{\partial p}{\partial z} = -\rho(g + a_z)$

So, $\frac{dp}{dz} = -\rho(g + a_z)$ ⟹ Non-Hydrostatic

- Pressure will vary linearly with depth, but variation is the combination of gravity and externally developed acceleration.
- A tank of water moving upward in an elevator will have slightly greater pressure at the bottom.
- If a liquid is in free-fall a_z = -g, and all pressure gradients are zero—surface tension is all that keeps the blob together.

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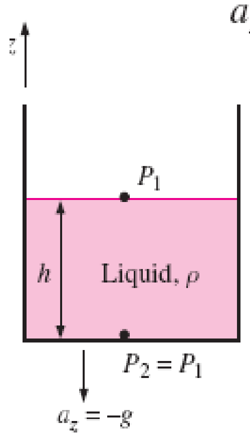
Pressure Variation, Rigid Body Motion: **freely falling body**

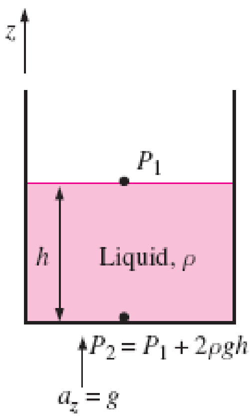
- A freely falling body accelerates under the influence of gravity.
- When the air resistance is negligible, the acceleration of the body equals the gravitational acceleration, and acceleration in any horizontal direction is zero.

$$a_x = a_y = 0 \text{ and } a_z = -g.$$

$$\frac{dp}{dz} = -\rho(g + a_z)$$



Upward acceleration of a liquid with $a_z = +g$



Free-falling fluids: $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0$

$\rightarrow \quad P = \text{constant}$

$\partial P/\partial z = -2\rho g.$

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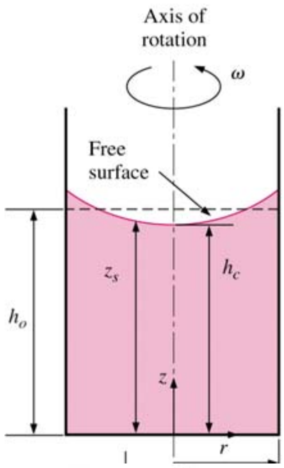
Pressure Variation, Rigid Body Motion: Rotation



Container is rotating about the z-axis with angular velocity ω

$$a_r = -r\omega^2, a_\theta = a_z = 0$$

$$\frac{\partial P}{\partial r} = \rho r \omega^2, \frac{\partial P}{\partial \theta} = 0, \frac{\partial P}{\partial z} = -\rho g$$

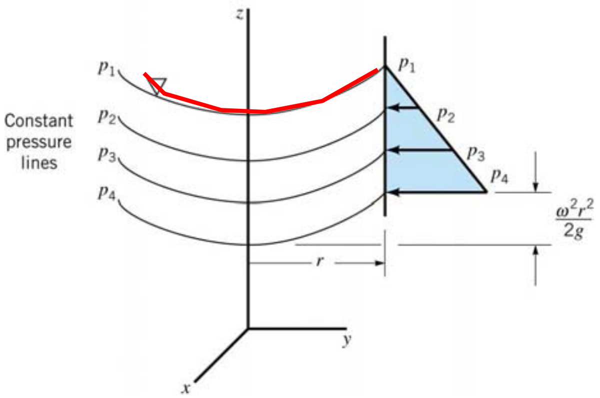
Total differential of P $\quad P = P(r, z)$

$$\Rightarrow dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz \Rightarrow dP = \rho r \omega^2 dr - \rho g dz$$

Along a line of constant pressure, dp = 0:

$$\frac{dz_{isobar}}{dr} = \frac{r\omega^2}{g} \rightarrow z_{isobar} = \frac{\omega^2}{2g}r^2 + C_1$$

The surfaces of constant pressure are parabolic



Equation of the free surface

$$z_s = h_0 - \frac{\omega^2}{4g}\left(R^2 - 2r^2\right)$$

Show this please

Pressure Variation, Rigid Body Motion: Rotation

Now, integrate to obtain the Pressure Variation:

$$\int dp = \rho\omega^2 \int r\,dr - \gamma \int dz \Rightarrow p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant}$$

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

Taking point 1 to be the origin ($r = 0$, $z = 0$) where the pressure is P_0

$$P = P_0 + \frac{\rho\omega^2}{2}r^2 - \rho g z$$

Pressure varies hydrostaticly in the vertical, and increases radialy.

- The maximum height difference between the edge and the center of the free surface the difference at at $r = R$ and also at $r = 0$,

$$\Delta z_{s,\max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g}R^2$$