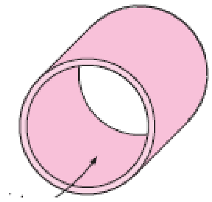
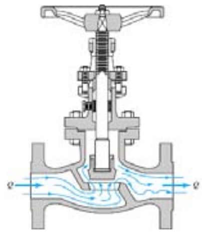
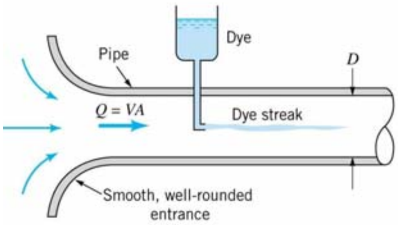
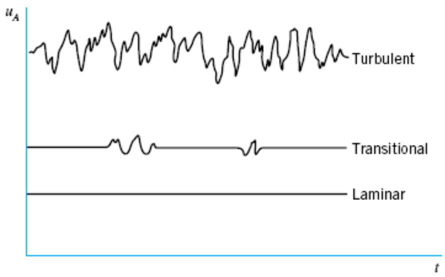


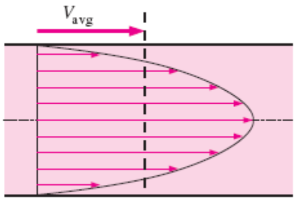
# Fluid Mechanics (0905241)

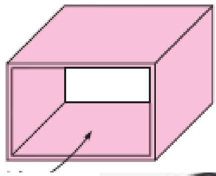


## Flow in Pipes



Prof. Zayed Al-Hamamre



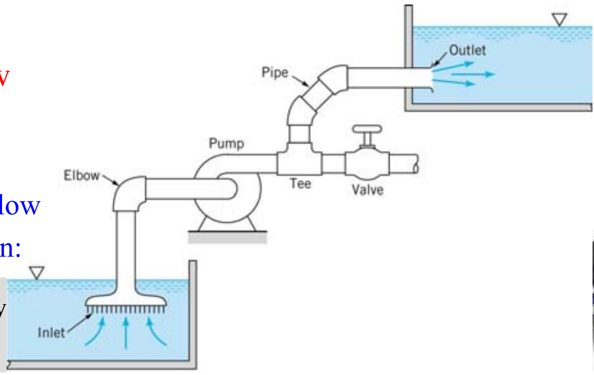

# Content

- Overview
- Laminar and turbulent flow
- Entrance and fully developed flow
- Fully developed laminar flow
- Turbulent flow
- Minor losses

# Overview

- Pipe Flow is important in daily operations and is described in general as flow in a closed conduit (pipes and ducts). It is also known as an internal flow.
- Some common examples are oil and water pipelines, flow in blood vessels, and heating and cooling applications.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- Particular attention should be paid to friction, which is directly related to the pressure drop and head loss during flow through pipes and ducts.
- When real world effects such as viscous effects are considered, it is often difficult to use only theoretical methods. **Often theoretical, experimental data, and dimensional analysis is used**.
- Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe

Some common pipe flow components are shown:

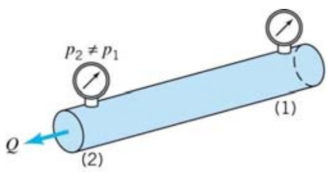




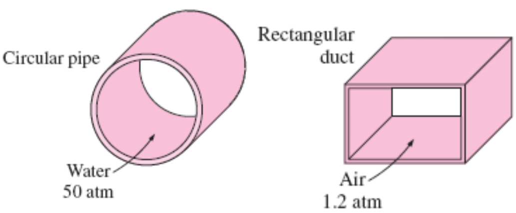

# Overview

Pipe flow versus Open-channel flow:

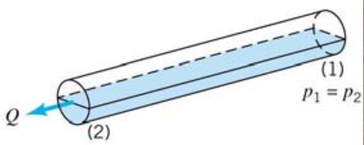
Pipe Flow:




- Pipe is completely filled with fluid
- Pressure Gradients drive the flow
- Gravity can also be important




Open-Channel Flow:



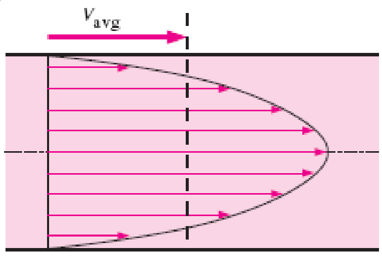

- Pipe is not full of fluids
- Pressure gradient is constant
- Gravity is the driving force

i.e., flow down a concrete spill way.

# Overview

- Pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion.
- Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small, the manufacturing and installation costs are lower,
- The fluid velocity in a pipe changes from *zero at the surface because of* the no-slip condition to a maximum at the pipe center
- $V_{avg}$, *is the* average speed through a cross section

For fully developed laminar pipe flow, $V_{avg}$ is half of maximum velocity.




*From the conservation of mass*

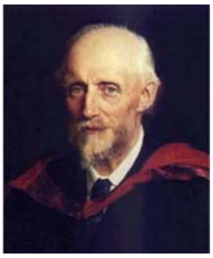
$$\dot m = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

$A_c$ is the cross-sectional area,
$u(r)$ is the velocity profile.
$\rho$ is the density
$R$ circular pipe radius

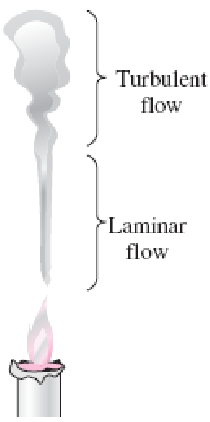


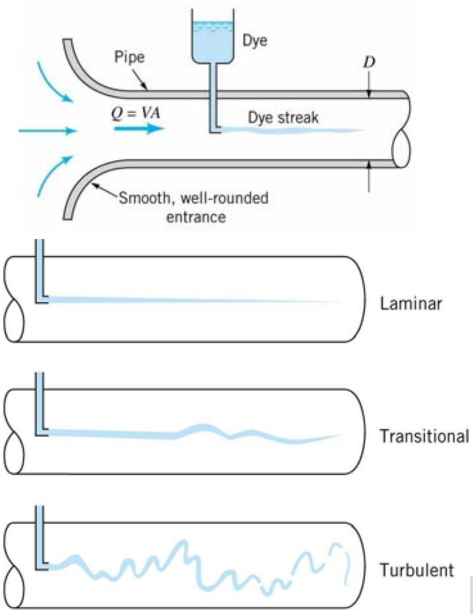
# Laminar or Turbulent Flow



Reynolds
(1842-1912)

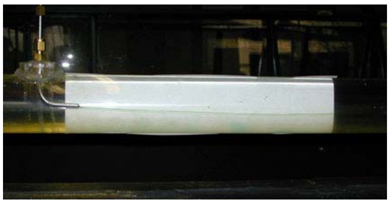
**Osborne Reynolds 1842–1912, a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus as shown**




Small flow rates

Laminar



Increasing the flow rates

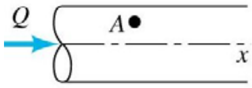
Transitional

Increasing the flow rates

Turbulent

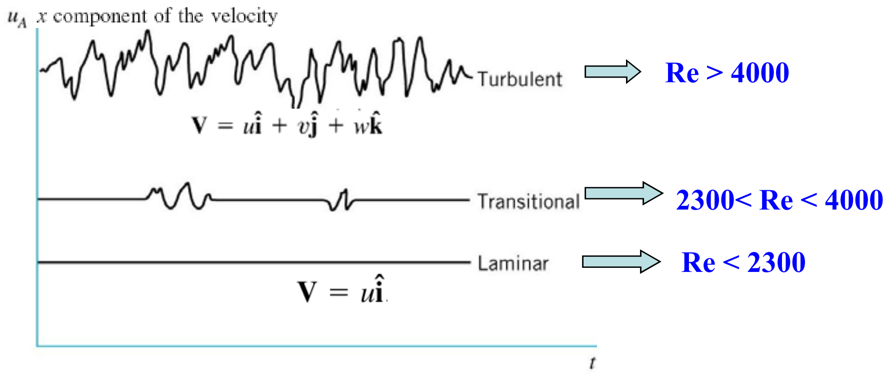
# Laminar or Turbulent Flow

If we measure the velocity at any given point with respect to time in the pipe:




**Reynolds Number Dependency:**

$$Re = \rho V D/\mu$$



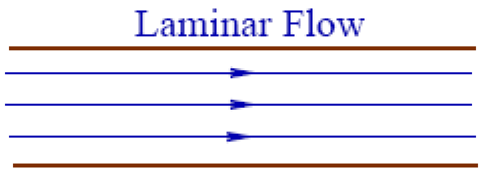

- Turbulence is characterized by random fluctuations and highly disordered motion.
- Transitional flows are relatively steady accompanied by occasional bursts.
- Laminar flow is relatively steady and characterized by smooth streamlines and highly ordered motion.
- For laminar flow there is only one flow direction: $\mathbf{V} = u\hat{\mathbf{i}}$
- For turbulent flow, there is a predominate flow direction, but there are random components normal to the flow direction: $\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$





# Laminar or Turbulent Flow


- In laminar flow, fluid particles flow in an orderly manner along path lines, and
- Momentum and energy are transferred across streamlines by molecular diffusion




- Turbulent flow is a flow regime in which the movement of fluid particles is chaotic, eddying, and unsteady, with significant movement of particles in directions transverse to the flow direction
- Turbulent flow is characterized by random and rapid fluctuations of swirling regions of fluid, called **eddies, throughout the flow.**
- **These fluctuations** provide an additional mechanism for momentum and energy transfer

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# Laminar or Turbulent Flow

## Reynolds Number

- The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature, and type of fluid, among* other things.
- The flow regime depends mainly on the ratio of *inertial forces to viscous forces in the fluid,* This ratio is called the **Reynolds number**

For internal flow in a circular pipe $$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu}$$

where $V_{avg}$ = average flow velocity (m/s), $D$ = characteristic length of the geometry (diameter in this case, in m), and $\nu = \mu/\rho$ = kinematic viscosity of the fluid ($m^2/s$).

- At large Reynolds numbers, high inertia and thus, the viscous forces cannot prevent the random and rapid fluctuations of the fluid  *turbulent*
- At *small or moderate Reynolds numbers,* the viscous forces are large enough to suppress these fluctuations and to keep the fluid in line  *laminar*



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# Laminar or Turbulent Flow

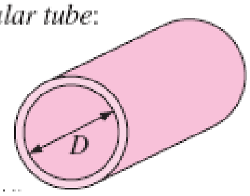
## Reynolds Number

- The Reynolds number at which the flow becomes turbulent is called the **Critical Reynolds Number, *$Re_{cr}$***
- The value of the critical Reynolds number is different for different geometries and flow conditions
- For internal flow in a circular pipe $\text{Re}_{cr} = 2300$
- For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter *$D_h$*** defined as

$$D_h = \frac{4A_c}{p}$$

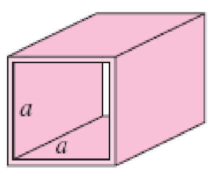
$p$ is its wetted perimeter.
$A_c$ is the cross-sectional area

*Circular tube:*



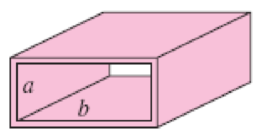

$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

*Square duct:*




$$D_h = \frac{4a^2}{4a} = a$$

*Rectangular duct:*




$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$



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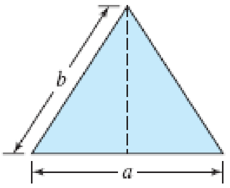
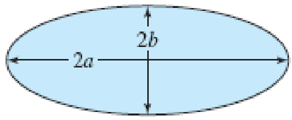
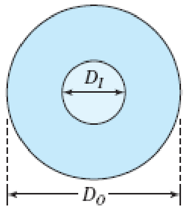
# Laminar or Turbulent Flow




$$D_H = \frac{\sqrt{4a^2b^2 - a^4}}{a + 2b}$$




$$D_H \approx \frac{2\sqrt{2}(ab)}{\sqrt{a^2 + b^2}}$$




$$D_H = D_O - D_I$$

## Example

Water at a temperature of 50 °F flows through a pipe of diameter $D = 0.73$ in. (a) Determine the minimum time taken to fill a 12-oz glass (volume $= 0.0125\ \text{ft}^3$) with water if the flow in the pipe is to be laminar. (b) Determine the maximum time taken to fill the glass if the flow is to be turbulent. Repeat the calculations if the water temperature is 140 °F.




# Example Cont.



# Example Cont.



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# Entrance and Fully Developed

The entrance region in a pipe flow is quite complex (1) to (2):

The fluid enters the pipe with nearly uniform flow.

The viscous effects create a boundary layer that merges.

When they merge the flow is fully developed.

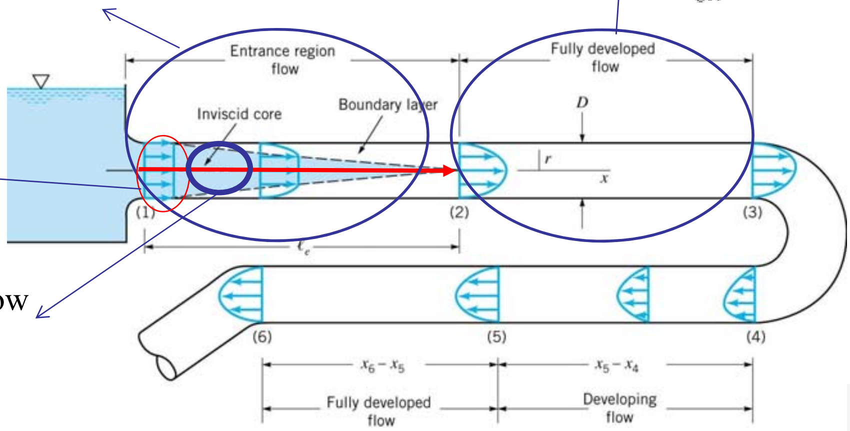
The frictional effects are negligible and the velocity profile remains essentially constant in the radial direction (Irrotational flow region)

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

The viscous effects and the velocity profile changes are significant,

Uniform velocity profile

Irrotational (core) flow region, the frictional effects are negligible



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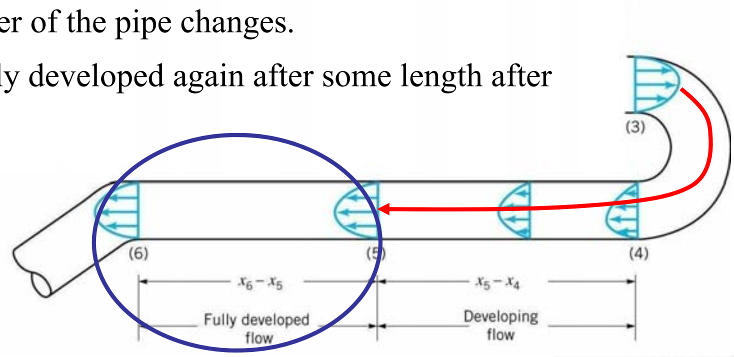
$$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow} \quad \text{and} \quad \frac{\ell_e}{D} = 4.4 (\text{Re})^{1/6} \text{ for turbulent flow}$$

- For very low Reynolds numbers (Re = 10), the entrance length is short: $\ell_e = 0.6D$
- For large Reynolds number flow the entrance length can be several pipe diameters: $\ell_e = 120D$ for Re $= 2000$
- For many practical engineering problems: $10^4 < \text{Re} < 10^5$ so that $20D < \ell_e < 30D$

Bends and T's affect Fully Developed Flow:

- Pipe is fully developed until the character of the pipe changes.
- It changes in the bend and becomes fully developed again after some length after the bend.
- Many disruptions can cause the flow to never be fully developed.
- In many flows, the fully developed region is greater than the developing region.



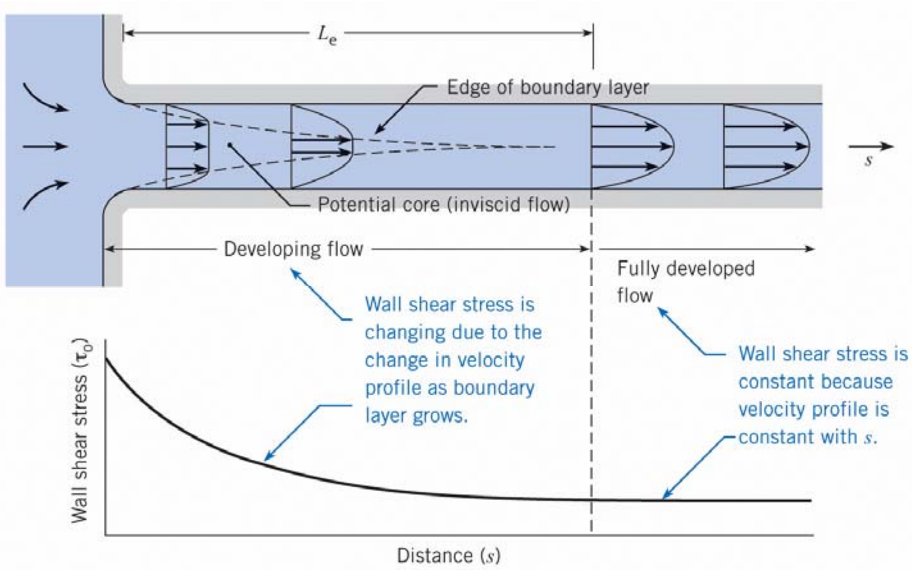







## Pressure and Shear Stress

- The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.
- In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.
- The wall shear stress is the highest at the pipe inlet where the thickness of the boundary layer is smallest, and decreases gradually to the fully developed value,

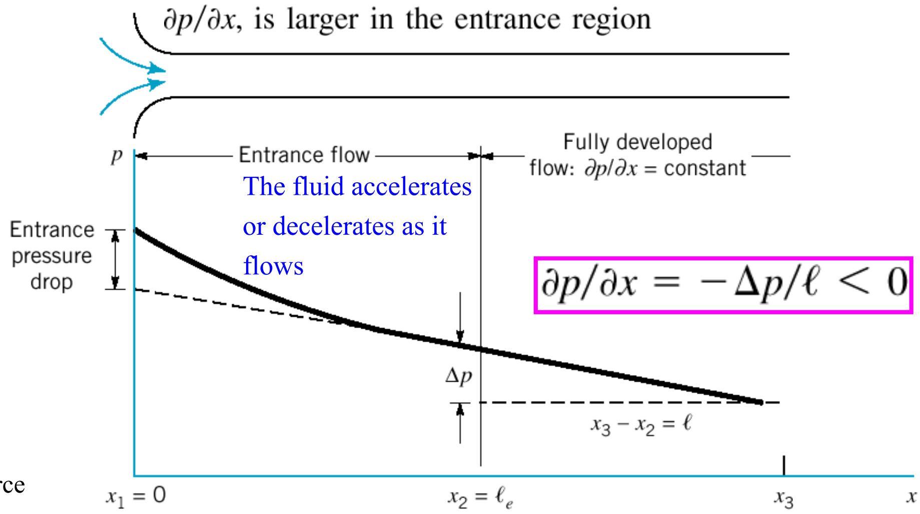





# Entrance and Fully Developed

## Pressure and Shear Stress

- In non-fully developed flow, the fluid accelerates or decelerates as it flows
- In fully developed flow, viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration




$\Delta p = p_1 - p_2$ flow driving force

- The shear stress in laminar flow is a direct result of momentum transfer along the randomly moving molecules (microscopic).
- The shear stress in turbulent flow is due to momentum transfer among the randomly moving, finite-sized bundles of fluid particles (macroscopic).
- The physical properties of shear stress are quite different between the two.



# Entrance and Fully Developed

The need for the pressure drop can be viewed from two different standpoints

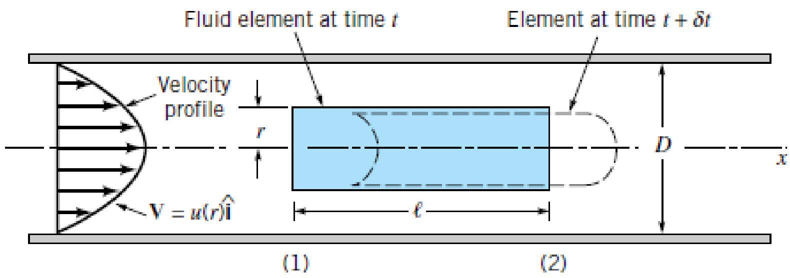
i. In terms of a force balance, the pressure force is needed to overcome the viscous forces generated.

ii. In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid.

- If the pipe is not horizontal, the pressure gradient along it is due in part to the component of weight in that direction.

# Fully Developed Flow

- If the flow is fully developed and steady, no part of the fluid experiences any acceleration as it flows.




- Every part of the fluid merely flows along its path line parallel to the pipe walls with constant velocity, although neighboring particles have slightly different velocities.
- The velocity varies from one path line to the next.
- This velocity variation, combined with the fluid viscosity, produces the shear stress

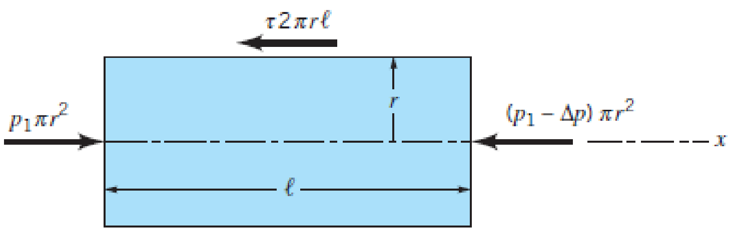


# Fully Developed Flow

- The pressure varies along the pipe from one section to the next. The pressure decreases in the direction of flow

i.e., if the pressure is $p = p_1$ at section (1), it is $p_2 = p_1 - \Delta p$ at section (2)

- A shear stress, $\tau$ acts on the surface of the cylinder of fluid.
- This viscous stress is a function of the radius of the cylinder, $\tau = \tau(r)$.




apply Newton's second law, $F_x = ma_x$.

- In this case even though the fluid is moving, it is not accelerating $a_x = 0$.
- Basic pipe flow is governed by a balance of viscous and pressure forces.

# Fully Developed Flow

- Fully developed horizontal pipe flow is merely a balance between pressure and viscous forces,
- The pressure difference acting on the end of the cylinder of area $\pi r^2$, and
- The shear stress acting on the lateral surface of the cylinder of area $2\pi r\ell$.

$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0 \quad \rightarrow \quad \frac{\Delta p}{\ell} = \frac{2\tau}{r} \qquad f(r) = g(x)$$

neither $\Delta p$ nor $\ell$ are functions of the radial coordinate, $r$,

→ $2\tau/r$ must also be independent of $r$.

→ $\tau = Cr$, where $C$ is a constant. $dP/dx$ = constant.

At $r = 0$ (the centerline of the pipe) there is no shear stress ($\tau = 0$).

At $r = D/2$
(the pipe wall) the shear stress is a maximum, denoted $\tau_w$, the *wall shear stress*.



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# Fully Developed Flow

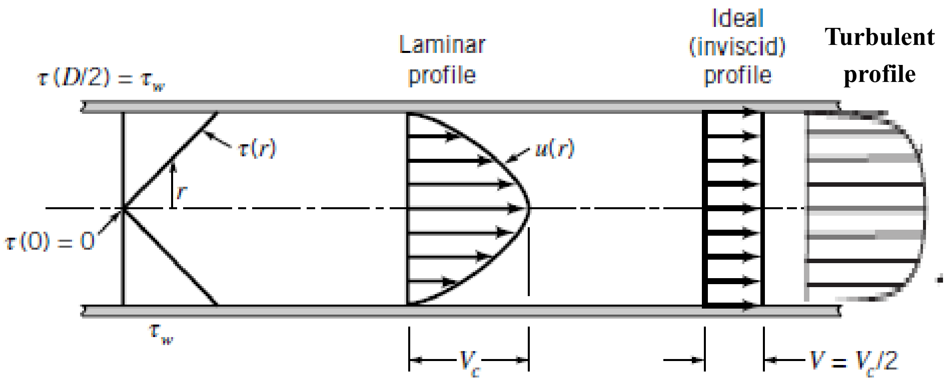
→ $C = 2\tau_w/D$

the shear stress distribution throughout the pipe is a linear function of the radial coordinate

$$\tau = \frac{2\tau_w r}{D}$$

→

$$\Delta p = \frac{4\ell\tau_w}{D}$$




A small shear stress can produce a large pressure difference if the pipe is relatively long ($\ell/D \gg 1$).

- The equation indicates that for both laminar and turbulent fully developed flows the shear stress varies linearly across the pipe, from zero at the centerline to a maximum at the pipe wall



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# Fully Developed Laminar Flow

➢ For laminar flow the stress equation is

$$\boxed{\tau = -\mu \frac{du}{dr}}$$

The negative sign is included to give $\tau > 0$ with $du/dr < 0$ (the velocity decreases from the pipe centerline to the pipe wall).

➢ The two governing laws for fully developed laminar flow of a Newtonian fluid within a horizontal pipe.

1. The one is Newton's second law of motion and the other is
2. The definition of a Newtonian fluid.

By combining these two equations

$$\longrightarrow \quad \frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell}\right) r$$



# Fully Developed Laminar Flow

$$\int du = -\frac{\Delta p}{2\mu\ell} \int r\,dr \quad \longrightarrow \quad u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

$u = 0$ at $r = D/2$. no-slip condition at the pipe surface

$$\longrightarrow \quad C_1 = (\Delta p/16\mu\ell)D^2.$$

$$\longrightarrow \quad u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

$$\boxed{\frac{u}{V_c} = 1 - \left(\frac{r}{R}\right)^2}$$

the centerline velocity. $V_c = \Delta p D^2/(16\mu\ell)$

# Fully Developed Laminar Flow

$$\rightarrow \quad u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where $R = D/2$ is the pipe radius.

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$\rightarrow \quad u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

The maximum velocity occurs at the centerline $r = 0$,

$$V_c = u_{max} = 2V_{avg}$$



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# Fully Developed Laminar Flow

The Volumetric Flow Rate:

$$Q = \int u\, dA = \int_{r=0}^{r=R} u(r) 2\pi r\, dr = 2\pi\, V_c \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr$$

$$Q = \frac{\pi R^2 V_c}{2}$$

$V_{avg} = Q/A = Q/\pi R^2$ The average velocity is $V_{avg}$

$$V_{avg} = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell} \quad \rightarrow \quad Q = \frac{\pi D^4\, \Delta p}{128\mu\ell}$$

Hagen-Poiseuille Flow

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2} \quad \rightarrow \quad \Delta P \;\alpha\; \mu$$



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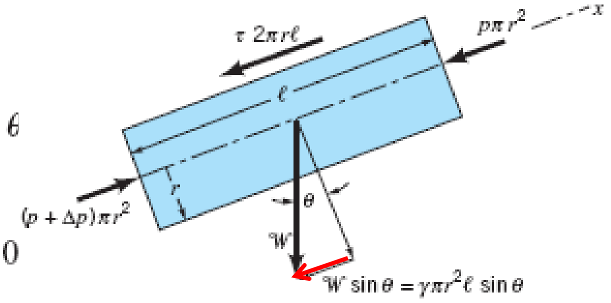
# Fully Developed Flow

## Inclined Pipes

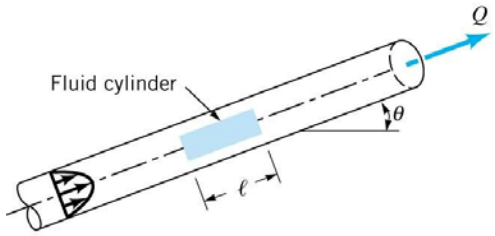
Force balance in the direction of flow

$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx)\sin\theta$$

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0$$




$$\frac{\Delta p - \gamma\ell\sin\theta}{\ell} = \frac{2\tau}{r}$$




**For laminar flow**

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g\sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$




# Fully Developed Laminar Flow

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L\sin\theta)D^2}{32\mu L} \qquad \dot{V} = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

- If the flow is downhill, gravity helps the flow (a smaller pressure drop is required; $\sin\theta < 0$)
- If the flow is uphill, gravity works against the flow (a larger pressure drop is required; $\sin\theta > 0$).

$\gamma\ell\sin\theta = \gamma\Delta z$ (where $\Delta z$ is the change in elevation) is a hydrostatic type pressure term. If there is no flow, $V = 0$ and $\Delta p = \gamma\ell\sin\theta = \gamma\Delta z$, as expected for fluid statics.

Some general remarks:

1. The flow rate is directly proportional to the pressure drop.
2. The flow rate is inversely proportional to the viscosity.
3. The flow rate is inversely proportional to the pipe length.
4. The flow rate is directly proportional to the pipe diameter to the 4th power.



# Example

Oil at 20°C ($\rho$ = 888 kg/m$^3$ and $\mu$ = 0.800 kg/m · s) is flowing steadily through a 5-cm-diameter 40-m-long pipe . The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15° upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar.



# Example Cont.

# Example Cont.



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# Example

Water at 40°F ($\rho$ = 62.42 lbm/ft$^3$ and $\mu$ = 1.038 × 10$^{-3}$ lbm/ft · s) is flowing through a 0.12-in- (= 0.010 ft) diameter 30-ft-long horizontal pipe steadily at an average velocity of 3.0 ft/s (Fig. 8–18). Determine (*a*) the head loss, (*b*) the pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.



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# Example Cont.



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# Example

An oil with a viscosity of $\mu = 0.40\ \text{N}\cdot\text{s/m}^2$ and density $\rho = 900\ \text{kg/m}^3$ flows in a pipe of diameter $D = 0.020$ m. (a) What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\ \text{m}^3/\text{s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m? (b) How steep a hill, $\theta$, must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$?

$$\text{Re} = \rho V D/\mu$$

$$V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3/\text{s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637\ \text{m/s},$$

$$\text{Re} = \rho V D/\mu = 2.87$$

which is less than 2300.



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# Example Cont.

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4}$$

$\ell = x_2 - x_1 = 10$ m,

$$\Delta p = \frac{128(0.40 \text{ N}\cdot\text{s/m}^2)(10.0 \text{ m})(2.0 \times 10^{-5} \text{ m}^3/\text{s})}{\pi(0.020 \text{ m})^4} = 20{,}400 \text{ N/m}^2 = 20.4 \text{ kPa}$$

work done by the pressure forces that overcomes the viscous dissipation.

(b) If the pipe is on a hill of angle $\theta$ such that $\Delta p = p_1 - p_2 = 0$,

$$\sin\theta = -\frac{128\mu Q}{\pi\rho g D^4}$$

$$\sin\theta = \frac{-128(0.40 \text{ N}\cdot\text{s/m}^2)(2.0 \times 10^{-5} \text{ m}^3/\text{s})}{\pi(900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.020 \text{ m})^4}$$

$$\theta = -13.34°.$$



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# Example Cont.

- The change in potential energy of the fluid "falling" down the hill that is converted to the energy lost by viscous dissipation



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# Losses of Head due to Friction

- Energy loss through friction in the length of pipeline is commonly termed the major loss $h_f$
- This is the loss of head due to pipe friction and to the viscous dissipation in flowing water.
- Several studies have been found the resistance to flow in a pipe is:

**- Independent of pressure under which the water flows**

**- Linearly proportional to the pipe length, L**

**- Inversely proportional to some water power of the pipe diameter D**

**- Proportional to some power of the mean velocity, V**

**- Related to the roughness of the pipe, if the flow is turbulent**




# Losses of Head due to Friction

The resistance to flow in a pipe is a function of:

- The pipe length, *L*
- The pipe diameter, *D*
- The mean velocity, *V*
- The properties of the fluid ()
- The roughness of the pipe, (the flow is turbulent).

# Losses of Head due to Friction

- Pressure drop $\Delta P$ is directly related to the power requirements of the fan or pump to maintain flow
- In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as

$$\text{Pressure loss:} \quad \Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$

where $\rho V_{avg}^2/2$ is the *dynamic pressure* and $f$ is the **Darcy friction factor,**

$$f = \frac{8\tau_w}{\rho V_{avg}^2} \quad \textbf{Darcy–Weisbach friction factor,}$$

$$f = \frac{(4 \cdot \tau_0)}{(\rho V^2 / 2)} \approx \frac{\text{shear stress acting at the wall}}{\text{kinetic pressure}}$$





# Losses of Head due to Friction

- Then, the friction factor for fully developed laminar flow in a circular pipe,

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2} = \frac{32\mu L V_{avg}}{D^2}$$

$$\text{Circular pipe, laminar:} \quad f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

- In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

# Losses of Head due to Friction

- In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height,*

$$\text{Head loss:} \quad h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g}$$

***Represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe***

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

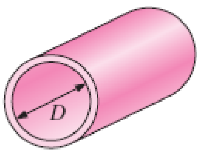
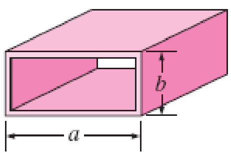
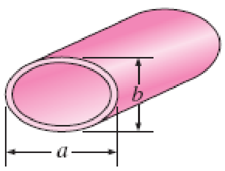
$$\dot{W}_{\text{pump},\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.



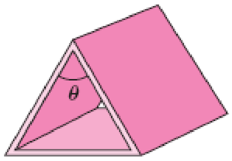

# Laminar Flow in Noncircular Pipes

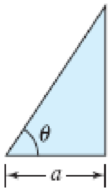
Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{\text{avg}} D_h/\nu$)

| Tube Geometry | a/b or θ° | Friction Factor f |
|---|---|---|
| Circle  | — | 64.00/Re |
| Rectangle  | a/b | |
| | 1 | 56.92/Re |
| | 2 | 62.20/Re |
| | 3 | 68.36/Re |
| | 4 | 72.92/Re |
| | 6 | 78.80/Re |
| | 8 | 82.32/Re |
| | ∞ | 96.00/Re |
| Ellipse  | a/b | |
| | 1 | 64.00/Re |
| | 2 | 67.28/Re |
| | 4 | 72.96/Re |
| | 8 | 76.60/Re |
| | 16 | 78.16/Re |

# Laminar Flow in Noncircular Pipes

| Tube Geometry | a/b or θ° | Friction Factor f |
|---|---|---|
| Isosceles triangle | θ | |
| | 10° | 50.80/Re |
| | 30° | 52.28/Re |
| | 60° | 53.32/Re |
| | 90° | 52.60/Re |
| | 120° | 50.96/Re |
| Right triangle $D_H = \dfrac{2a\sin\theta}{1+\sin\theta+\cos\theta}$ | θ (degrees) | |
| | 0 | 48.0 /Re |
| | 10 | 49.9 /Re |
| | 20 | 51.2 /Re |
| | 30 | 52.0 /Re |
| | 40 | 52.3 /Re |
| | 45 | 52.5 /Re |
| Equilateral triangle $D_H = \dfrac{a\sqrt{3}}{3}$ | | 53.3 /Re |

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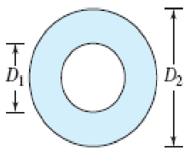
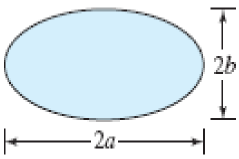
# Laminar Flow in Noncircular Pipes

| Tube Geometry | $D_1/D_2$ | Friction Factor f |
|---|---|---|
| Concentric annulus $D_H = D_2 - D_1$ | 0.0001 | 71.8 /Re |
| | 0.01 | 80.1 /Re |
| | 0.1 | 89.4 /Re |
| | 0.5 | 95.3 /Re |
| | 1.0 | 96.0 /Re |
| Ellipse $D_H \approx \dfrac{2\sqrt{2}(ab)}{\sqrt{a^2+b^2}}$ | | 64.00/Re |

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# Turbulent Flow in Pipes

**- Turbulent flow in pipe (Re≥4000):**

Random fluctuation of each velocity component in time and all directions.

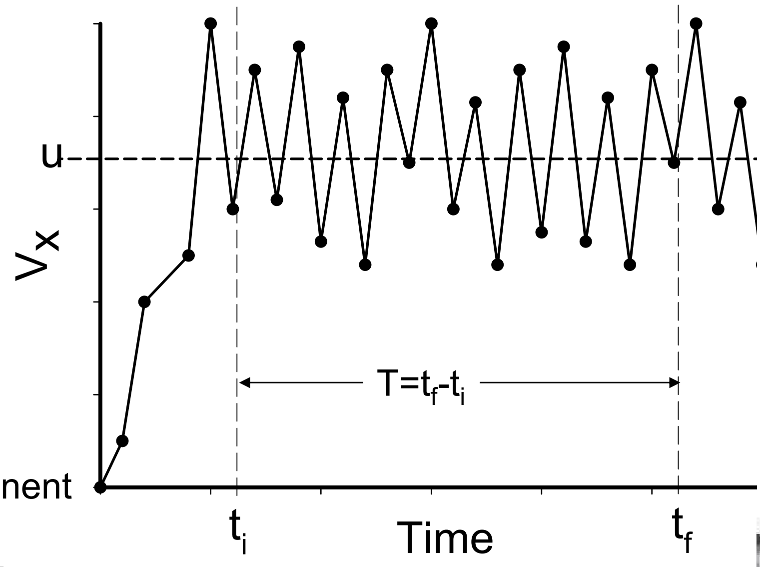
-Unsteady flow.

- Good mixing.

Velocity is averaged in time:

$$u = \frac{\int_{t_i}^{t_f} V_x(t)dt}{T}$$

$V_x$ : Axial velocity component



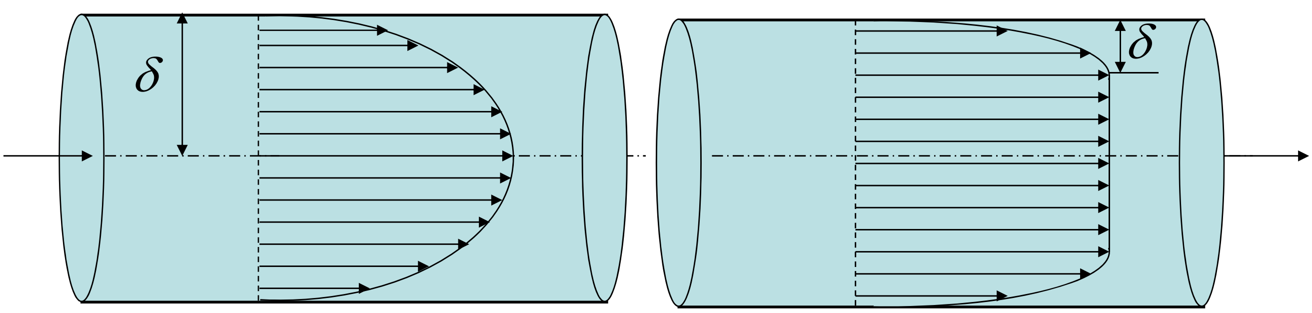



# Turbulent Flow in Pipes

The time-averaged velocity profile for turbulent flow is:

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/7}$$

"Empirical equation obtained from experimental measurements"




Laminar flow has **parabolic** velocity profile:

$$u(r) = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Turbulent flow has **flat** velocity profile:

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/7}$$

# Fully Developed Turbulent Flow

## The importance of turbulence

- Just imagine some chemical impurity being emitted from the bottom of the pipe. This chemical would only diffuse very, very slowly to the top of the pipe.
- In a turbulent flow regime, the impurity would be quickly carried to the top of the pipe.
- Turbulence is very important for the mixing of dissolved substances in fluids. Why do you stir your tea or coffee after you place the milk in?
- Turbulence flows also greatly promote heat transfer.
- In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.
- Turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients



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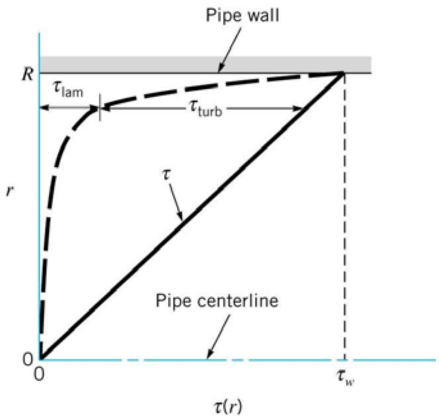
# Fully Developed Turbulent Flow

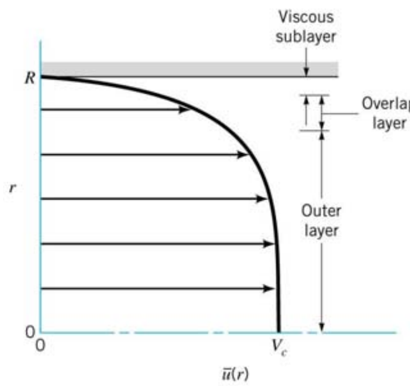
## Turbulent shear stress:

The turbulent shear components are known as Reynolds Stresses.

Shear Stress in Turbulent Flows:



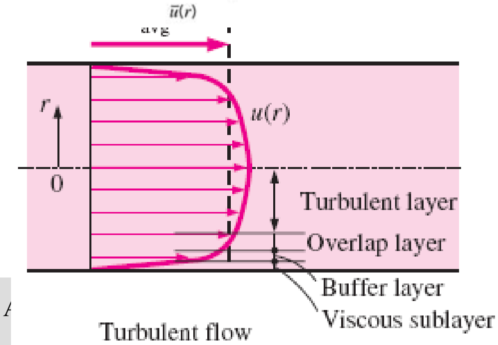

Turbulent Velocity Profile:




In viscous sublayer: $\tau_{laminar} > \tau_{turb}$ 100 to 1000 times greater.

In the outer layer: $\tau_{truib} > \tau_{laminar}$ 100 to 1000 time greater.

The viscous sublayer is extremely small.

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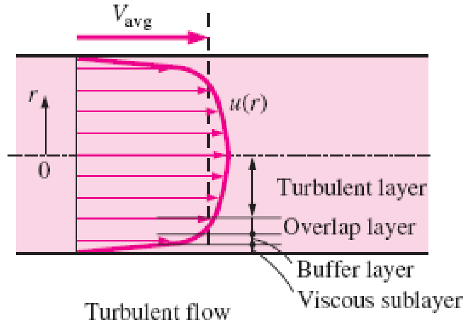
# Fully Developed Turbulent Flow

## Velocity Profile

➢ Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall

1. **Viscous (or laminar** or **linear or wall) sublayer:** The very thin layer next to the wall where viscous effects are dominant (thickness < 1% of pipe diameter)
2. **Buffer layer,** in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.
3. **Overlap (or transition) layer,** also called the inertial sublayer, in which the turbulent effects are much more significant, but still not dominant
4. **Outer (or turbulent) layer in the remaining part of the flow in** which turbulent effects dominate over molecular diffusion (viscous) effects.



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# The time-average velocity profile

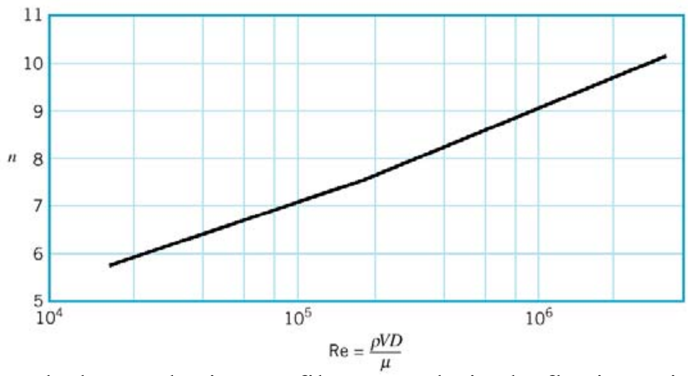
## Velocity Profile

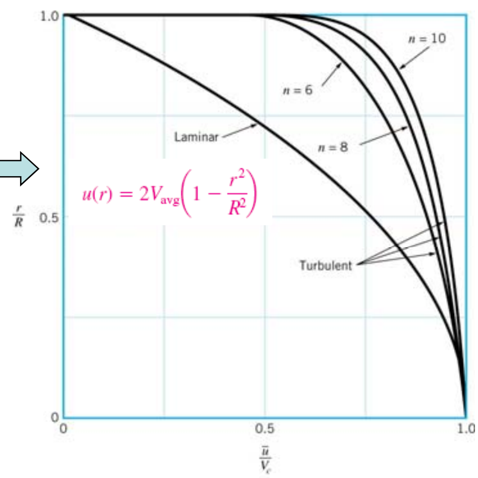
Some alternative. approach include the Power-Law equation:

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

n = 7 for many practical flows.

n, chosen based on the Reynolds number.




Turbulent velocity profiles are relatively flat in a pipe flow.

The power-law equation **is not valid at the wall**, since that would give an infinite velocity gradient.

Also, cannot be precisely valid near the centerline because the shear does not go to zero at the center-line. $d\bar{u}/dr \neq 0$ at $r = 0$.

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# The time-average velocity profile

➢ The variation of power-Jaw exponent *n with Reynolds* number (based on pipe diameter, D, and centerline velocity, U) for fully developed flow in smooth pipes is given by

$$n = -1.7 + 1.8 \log Re_U$$ J. O. Hinze

for $Re_U > 2 \times 10^4$

The value of $n$ is related empirically to $f$ by

$$n = f^{-1/2}$$

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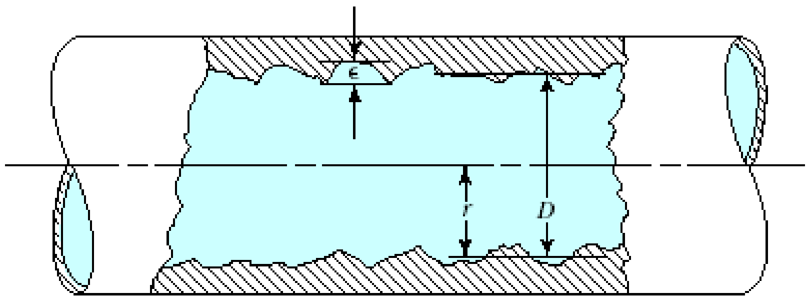
# Friction Loss In Turbulent Flow




- Using Darcy equations we can calculate the friction losses in turbulent flow. It depends on the **surface roughness** of the pipe as well as Reynolds number (IN LAMINAR, LOSSES ONLY DEPEND ON THE REYNOLD NUMBER)
- The ϵ , the average wall roughness can be obtained from tables (experiment has been conducted to determine the value). The average value is for new and clean pipe.

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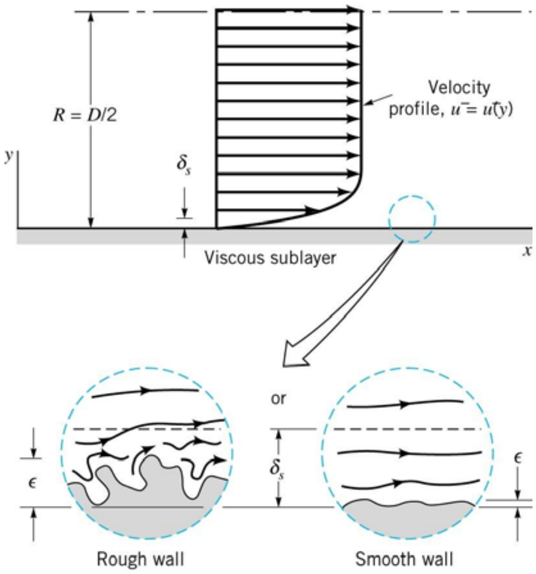
# Friction Loss In Turbulent Flow

- Most turbulent pipe flow data is based on experiments. In turbulent flow, in order to do dimensional analysis we consider the roughness of the pipe, as well as density which relates to momentum.

Variables: $\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$

$\varepsilon$ → roughness




Roughness is important in the viscous sub-layer in turbulent flows, if it protrudes sufficiently in this layer.

The viscous layer in laminar flow is so large, that small roughness does not play a role.

Range of roughness for validity of this analysis is for: $0 \leq \varepsilon/D \lesssim 0.05$

Then, the dimensionless groups are the following:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

$\varepsilon/D$: Represent the ratio of the mean height of roughness of the pipe to the pipe diameter



# Moody Chart

- As for laminar flow, the pressure drop must be proportional to the pipe length:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

Recalling the definition of the friction factor: $\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$

Then the friction factor is one of our dimensionless groups: $f = \phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$

Then using experiments, we can find the above relationship with various manufactured pipe roughness values:

| Pipe | Equivalent Roughness, $\varepsilon$ Feet | Millimeters |
|---|---|---|
| Riveted steel | 0.003–0.03 | 0.9–9.0 |
| Concrete | 0.001–0.01 | 0.3–3.0 |
| Wood stave | 0.0006–0.003 | 0.18–0.9 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Commercial steel or wrought iron | 0.00015 | 0.045 |
| Drawn tubing | 0.000005 | 0.0015 |
| Plastic, glass | 0.0 (smooth) | 0.0 (smooth) |

⇒ "Moody Chart"

Colebrook Relation for **Non-Laminar** part of the Moody Chart (curve fit):

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

- Implicit relation required iteration to find $f$

# Friction Loss In Turbulent Flow

**Roughness value, ϵ for new and clean pipe**

| Material | Roughness ϵ (m) | Roughness ϵ (ft) |
|---|---|---|
| Glass | Smooth | Smooth |
| Plastic | $3.0 \times 10^{-7}$ | $1.0 \times 10^{-6}$ |
| Drawn tubing; copper, brass, steel | $1.5 \times 10^{-6}$ | $5.0 \times 10^{-6}$ |
| Steel, commercial or welded | $4.6 \times 10^{-5}$ | $1.5 \times 10^{-4}$ |
| Galvanized iron | $1.5 \times 10^{-4}$ | $5.0 \times 10^{-4}$ |
| Ductile iron—coated | $1.2 \times 10^{-4}$ | $4.0 \times 10^{-4}$ |
| Ductile iron—uncoated | $2.4 \times 10^{-4}$ | $8.0 \times 10^{-4}$ |
| Concrete, well made | $1.2 \times 10^{-4}$ | $4.0 \times 10^{-4}$ |
| Riveted steel | $1.8 \times 10^{-3}$ | $6.0 \times 10^{-3}$ |



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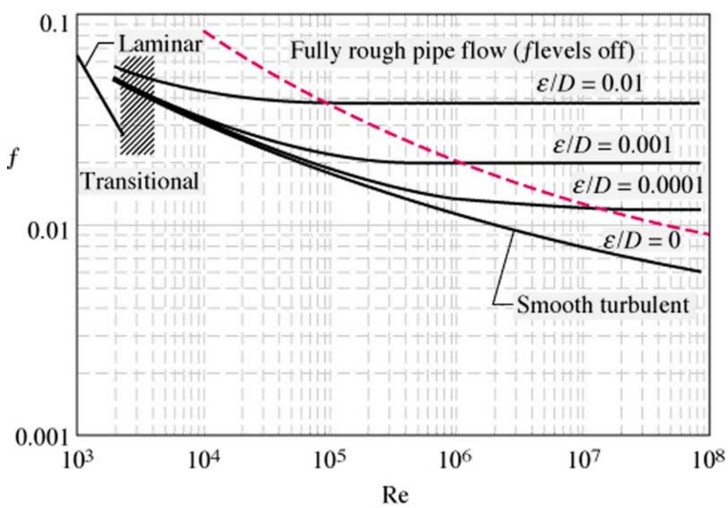
# Moody Diagram –Important Observation

- For a given Reynolds number of flow, as the relative roughness is increased, the friction factor *f* decreases.
- For a given relative roughness , the friction factor *f* decreases with increasing Reynolds number until the zone of complete turbulence is reached.



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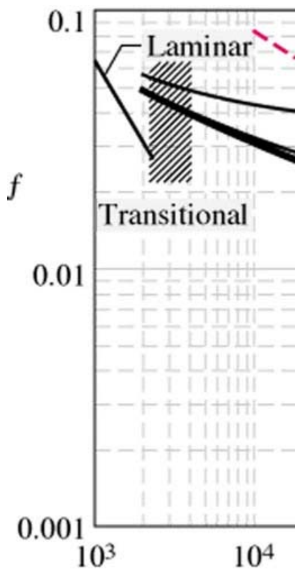
## Moody Chart




- The **transition region** is shown in the shaded area **( 2300<Re<4000)**
- The friction factors alternate between laminar and turbulent flow.



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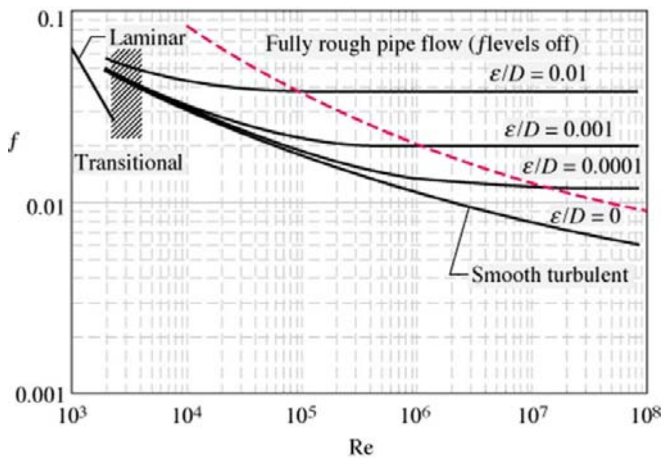
## Moody Chart

- Within the zone of complete turbulence, **the Reynolds number has no effect on the friction factor.**
- As the relative roughness increases, the value of the Reynolds number at which the zone of complete turbulence begins also increases.
- The friction factor is a minimum for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness.



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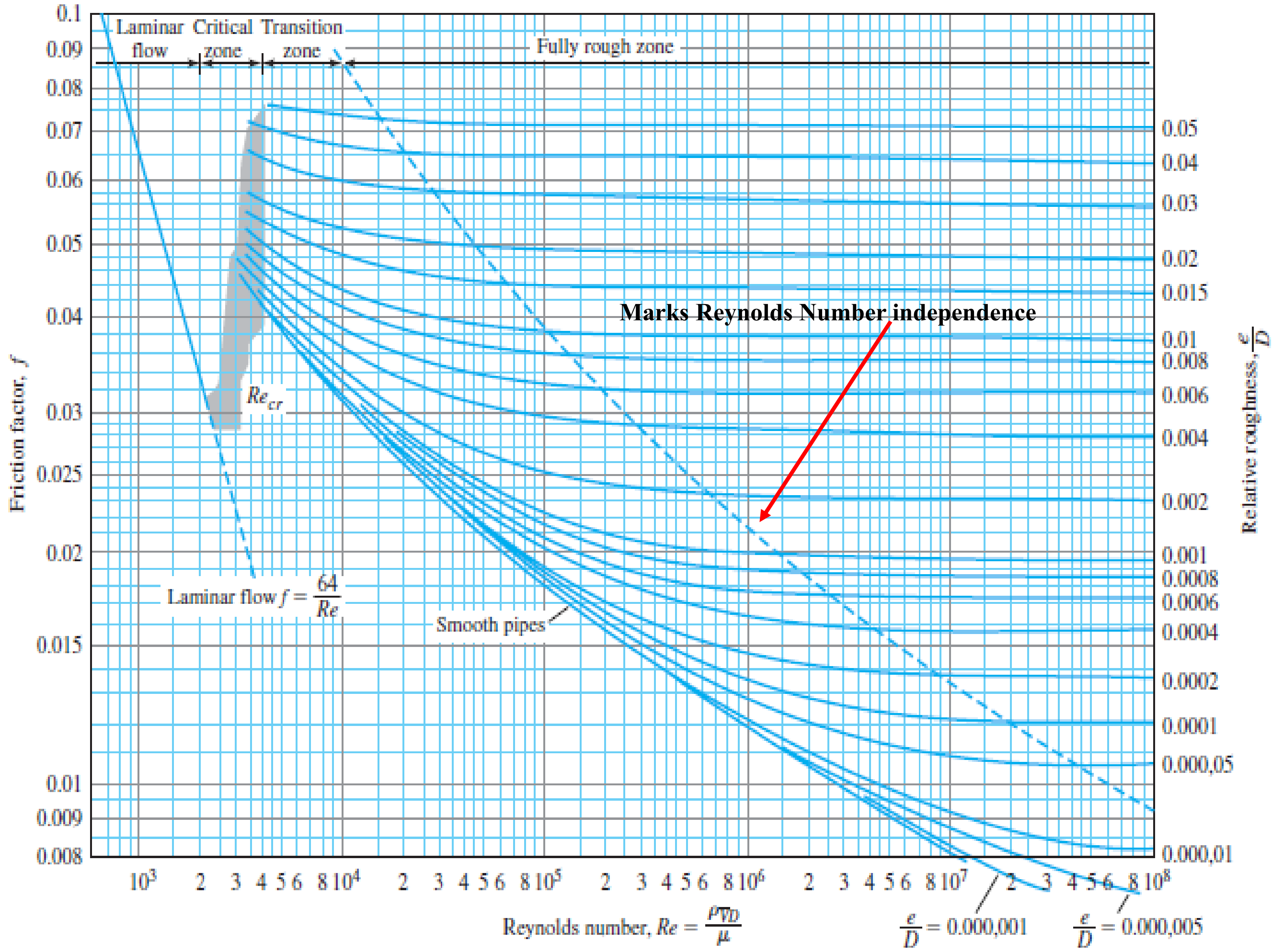


# Formula for friction factor

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness. The Colebrook equation in this case $(\varepsilon = 0)$ reduces to the **Prandtl equation expressed as**

$$1/\sqrt{f} = 2.0 \log(\text{Re}\sqrt{f}) - 0.8.$$

- The transition region from the laminar to turbulent regime (2300< Re < 4000). The flow in this region may be laminar or turbulent, depending on flow disturbances
- At very large Reynolds numbers (to the right of the dashed line on the chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number (*fully rough turbulent flow)*

*fully rough turbulent flow* $(\text{Re} \to \infty)$

The Colebrook equation is reduced to $1/\sqrt{f} = -2.0 \log[(\varepsilon/D)/3.7]$. **von Kármán equation**



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# Formula for friction factor

- An approximate explicit relation for *f was given by*

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \qquad Or \qquad f_0 = 0.25\left[\log\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}$$

*S. E. Haaland* P. K. Swamee

- The results obtained from these relation are within 2 percent and 1 percent , respectively of those obtained from the Colebrook equation.
- It can be used as a good first guess for $f$

- *For turbulent and transition region,*

$$f = 0.001375 * [1 + \left(2 * 10^4 \frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{\frac{1}{3}}]$$





# Formula for friction factor

$$f = \frac{1.325}{\{\ln[(\varepsilon/3.7D) + (5.74/\text{Re}^{0.9})]\}^2} \qquad 5000 < \text{Re} < 10^{+8} \quad \text{for } 10^{-6} < \varepsilon/D < 10^{-2}$$

**P. K. Swamee**

gave the following full range equation valid for laminar flow, turbulent flow and the transition

$$f = \left\{\left(\frac{64}{\mathbf{R}}\right)^8 + 9.5\left[\ln\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathbf{R}^{0.9}}\right) - \left(\frac{2500}{\mathbf{R}}\right)^6\right]^{-16}\right\}^{0.125}$$

- For turbulent flow in smooth pipes, the Blasius correlation, valid for $Re \leq 10^5$, is

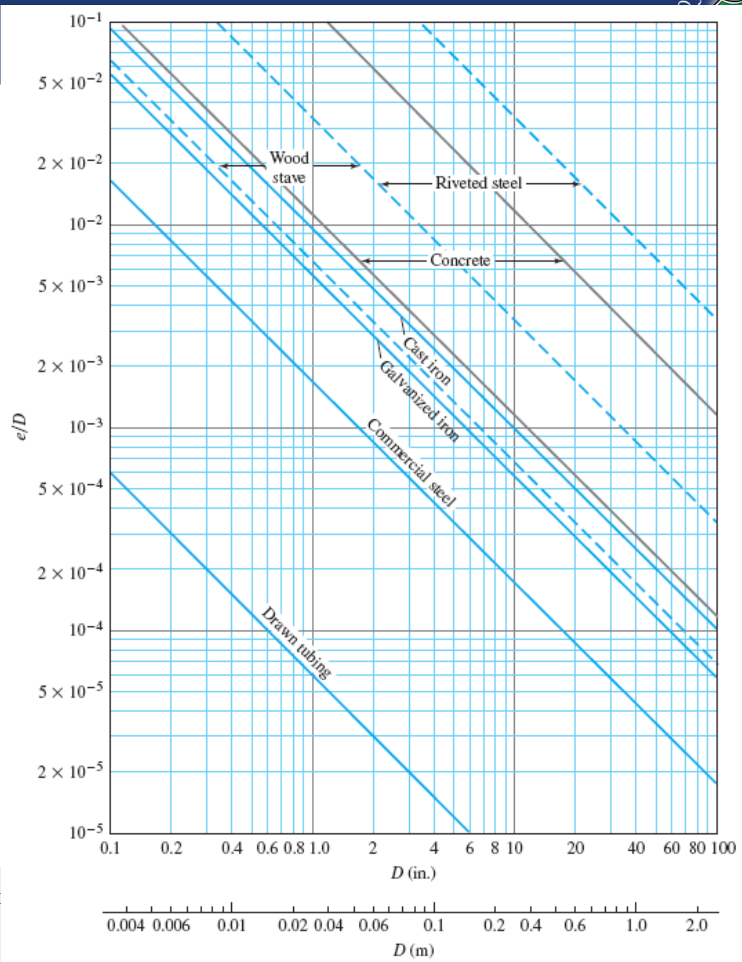
$$f = \frac{0.316}{Re^{0.25}}$$



## Relative roughness of new pipes

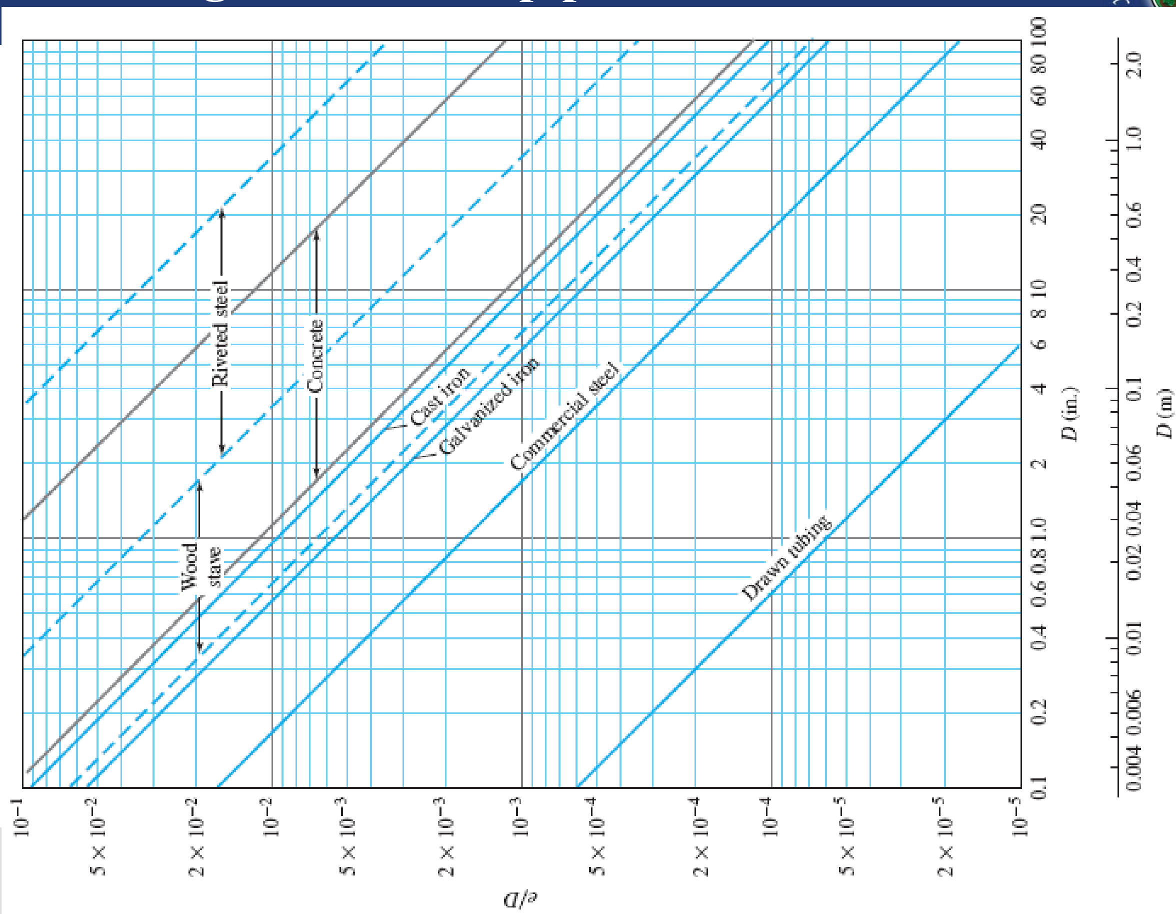
Standard sizes for Schedule 40 steel pipes

| Nominal Size, in | Actual Inside Diameter, in |
|---|---|
| $\frac{1}{8}$ | 0.269 |
| $\frac{1}{4}$ | 0.364 |
| $\frac{3}{8}$ | 0.493 |
| $\frac{1}{2}$ | 0.622 |
| $\frac{3}{4}$ | 0.824 |
| 1 | 1.049 |
| $1\frac{1}{2}$ | 1.610 |
| 2 | 2.067 |
| $2\frac{1}{2}$ | 2.469 |
| 3 | 3.068 |
| 5 | 5.047 |
| 10 | 10.02 |






# Relative roughness of new pipes.



# Relative roughness of new pipes.

Equivalent roughness values for new commercial pipes*

| | Roughness, $\varepsilon$ | |
|---|---|---|
| Material | ft | mm |
| Glass, plastic | 0 (smooth) | |
| Concrete | 0.003–0.03 | 0.9–9 |
| Wood stave | 0.0016 | 0.5 |
| Rubber, smoothed | 0.000033 | 0.01 |
| Copper or brass tubing | 0.000005 | 0.0015 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Wrought iron | 0.00015 | 0.046 |
| Stainless steel | 0.000007 | 0.002 |
| Commercial steel | 0.00015 | 0.045 |

* The uncertainty in these values can be as much as ±60 percent.

| Relative Roughness, $\varepsilon/D$ | Friction Factor, $f$ |
|---|---|
| 0.0* | 0.0119 |
| 0.00001 | 0.0119 |
| 0.0001 | 0.0134 |
| 0.0005 | 0.0172 |
| 0.001 | 0.0199 |
| 0.005 | 0.0305 |
| 0.01 | 0.0380 |
| 0.05 | 0.0716 |

* Smooth surface. All values are for Re = $10^6$ and are calculated from the Colebrook equation.

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

| | Equivalent Roughness, $\varepsilon$ | |
|---|---|---|
| Pipe | Feet | Millimeters |
| Riveted steel | 0.003–0.03 | 0.9–9.0 |
| Concrete | 0.001–0.01 | 0.3–3.0 |
| Wood stave | 0.0006–0.003 | 0.18–0.9 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Commercial steel or wrought iron | 0.00015 | 0.045 |
| Drawn tubing | 0.000005 | 0.0015 |
| Plastic, glass | 0.0 (smooth) | 0.0 (smooth) |



# Types of Fluid Flow Problems

| Problem type | Given | Find |
|---|---|---|
| 1 | $L, D, \dot{V}$ | $\Delta P$ (or $h_L$) |
| 2 | $L, D, \Delta P$ | $\dot{V}$ |
| 3 | $L, \Delta P, \dot{V}$ | $D$ |

# Types of Fluid Flow Problems

- To avoid tedious iterations in head loss, flow rate, and diameter calculations, Swamee and Jain proposed the following explicit relations that are accurate to within 2 percent of the Moody chart.

$$h_L = 1.07 \frac{\dot{V}^2 L}{g D^5} \left\{ \ln\left[ \frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9} \right] \right\}^{-2} \qquad \begin{array}{l} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3 \times 10^{8} \end{array}$$

$$\dot{V} = -0.965\left(\frac{g D^5 h_L}{L}\right)^{0.5} \ln\left[ \frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{g D^3 h_L}\right)^{0.5} \right] \qquad \text{Re} > 2000$$

$$D = 0.66\left[ \varepsilon^{1.25}\left(\frac{L\dot{V}^2}{g h_L}\right)^{4.75} + \nu \dot{V}^{9.4}\left(\frac{L}{g h_L}\right)^{5.2} \right]^{0.04} \qquad \begin{array}{l} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^{8} \end{array}$$



# Example

Air under standard conditions flows through a 4.0-mm-diameter drawn tubing with an average velocity of $V = 50$ m/s Determine the pressure drop in a 0.1-m section of the tube if the flow is laminar. (b) Repeat the calculations if the flow is turbulent.

# Example Cont.

# Example Cont.

# Example

**Determining the Head Loss in a Water Pipe**

Water at 60°F ($\rho$ = 62.36 lbm/ft$^3$ and $\mu$ = 7.536 × 10$^{-4}$ lbm/ft · s) is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft$^3$/s . Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200-ft-long section of the pipe.



# Example Cont.

# Example Cont.



# Example

Heated air at 1 atm and 35°C is to be transported a in a 300-m-long circular plastic duct with 0.267 m diameter . If the head loss in the pipe is not to exceed 20 m, determine the minimum flow rate through the duct.

# Example Cont.



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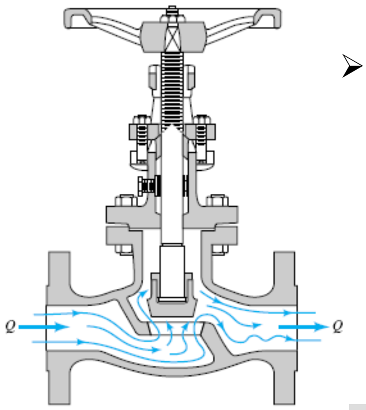
# Example Cont.

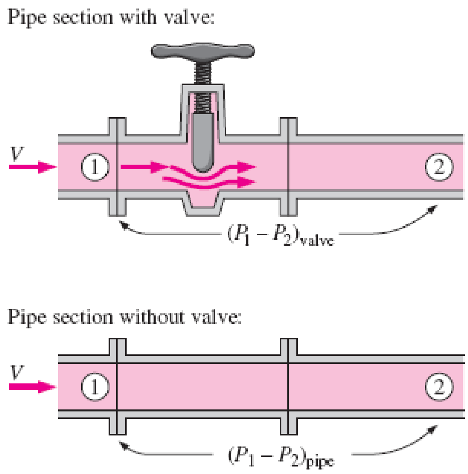
| Iteration | $f$ (guess) | $V$, m/s | Re | Corrected $f$ |
|---|---|---|---|---|
| 1 | 0.04 | 2.955 | $4.724 \times 10^4$ | 0.0212 |
| 2 | 0.0212 | 4.059 | $6.489 \times 10^4$ | 0.01973 |
| 3 | 0.01973 | 4.207 | $6.727 \times 10^4$ | 0.01957 |
| 4 | 0.01957 | 4.224 | $6.754 \times 10^4$ | 0.01956 |
| 5 | 0.01956 | 4.225 | $6.756 \times 10^4$ | 0.01956 |

# Minor Losses

- The head loss in long, straight sections of pipe can be calculated by use of the friction factor obtained from either the Moody chart or the Colebrook equation.
- The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.
- These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce



- The loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.



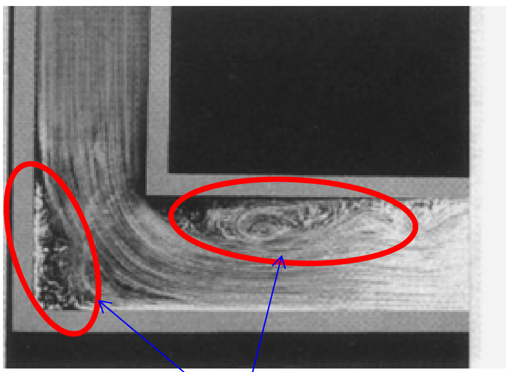

$$K_L = \frac{h_L}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

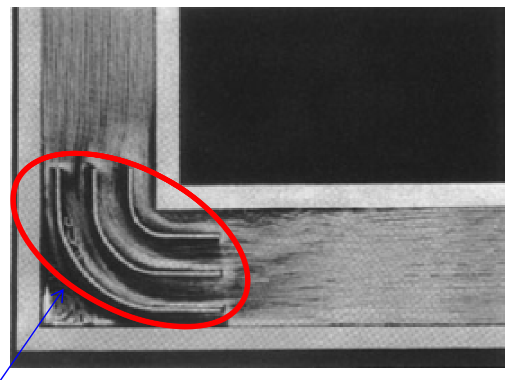
$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$



# Minor Losses

Flow ($Re = 2 \times 10^3$) through a 90° right angle bend





large eddies (flow separation)

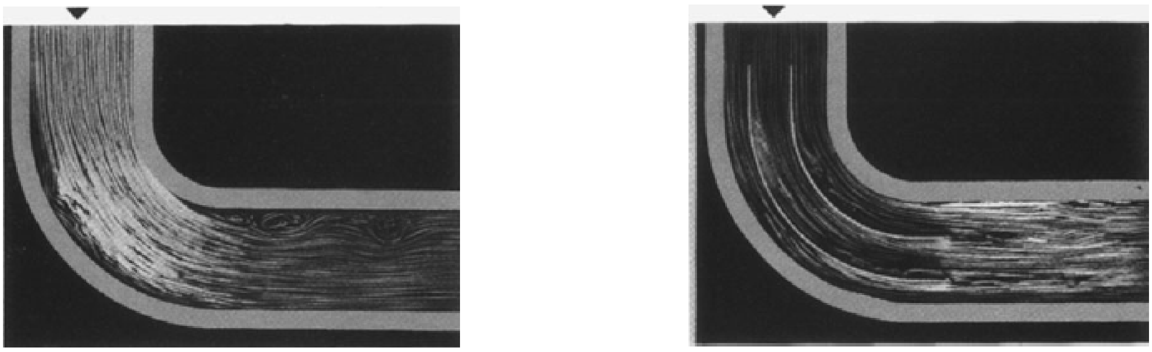
guide vanes

- Within these eddies mechanical energy is converted into heat through viscous dissipation; thus flow separation and eddy formation is a key mechanism in the minor head loss
- The eddies also decrease the effective cross-sectional area of the passage because there is no net flow downstream through them

- Eddies in the base leg and vena contracta are eliminated. The result is a smaller loss of mechanical energy than in an abrupt turn without guide vanes.
- Smaller loss of mechanical energy

# Minor Losses

Flow ($Re = 2 \times 10^3$) through a 90° gradual bend



- Reducing head loss by employing a gradual bend
- No corner in which eddies form, and the eddies in the base leg are less intense than those in the elbow
- If guide vanes are added to the bend (there is no flow separation at all and the minor head loss is further reduced



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# Minor Losses

*Loss coefficient:* $K_L = \frac{h_L}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$

so that $\Delta p = K_L \frac{1}{2}\rho V^2$ ➡ $h_L = K_L \frac{V^2}{2g}$

where $h_L$ is the *additional* irreversible head loss in the piping system caused by insertion of the component, and is defined as $h_L = \Delta P_L/\rho g$.

The actual value of $K_L$ is strongly dependent on the geometry of the component considered. It may also be dependent on the fluid properties. That is,

$$K_L = \phi(\text{geometry, Re})$$

- For many practical applications the Reynolds number is large enough so that the flow through the component is dominated by inertia effects, with viscous effects being of secondary importance

Minor losses are sometimes given in terms of an *equivalent length*, $\ell_{eq}$.

$$h_L = K_L \frac{V^2}{2g} = f\frac{\ell_{eq}}{D}\frac{V^2}{2g} \quad \Rightarrow \quad \ell_{eq} = \frac{K_L D}{f}$$



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# Minor Losses

- Where $f$ is the friction factor and $D$ is the diameter of the pipe that contains the component.
- The head loss caused by the component is equivalent to the head loss caused by a section of the pipe whose length is $L_{equiv.}$
- The contribution of a component to the head loss can be accounted for by simply adding $L_{equiv}$ to the total pipe length.

*Total head loss (general):*

$$h_L = h_{L,major} + h_{L,minor}$$

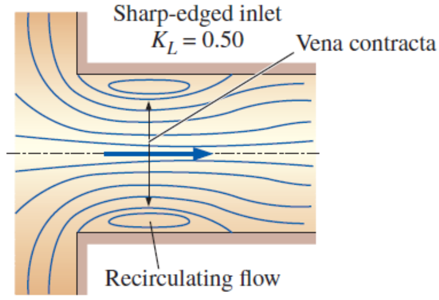
$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g}}_{\text{i pipe sections}} + \underbrace{\sum_j K_{L,j}\frac{V_j^2}{2g}}_{\text{j components}}$$

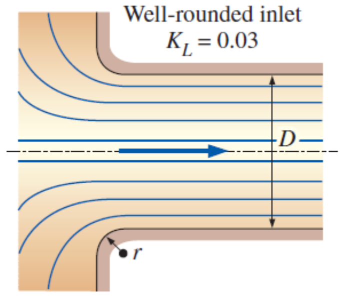
- If the piping system has constant diameter $h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$




# Minor Losses

The head loss at the inlet of a pipe is almost negligible for well-rounded inlets ($K_L$ = 0.03 for $r/D$ > 0.2) but increases to about 0.50 for sharp-edged inlets.



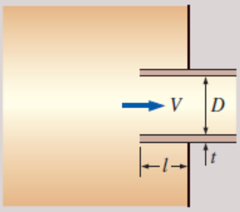




# Minor Losses

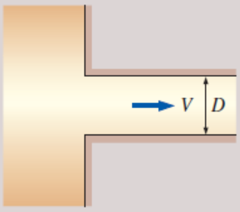
Loss coefficients $K_L$ of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where $V$ is the average velocity in the pipe that contains the component)*

*Pipe Inlet*

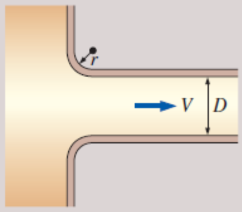
*Reentrant:* $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)




*Sharp-edged:* $K_L = 0.50$

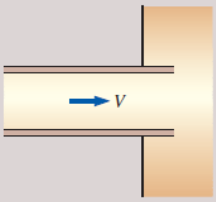



*Well-rounded* ($r/D > 0.2$): $K_L = 0.03$
*Slightly rounded* ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–39)

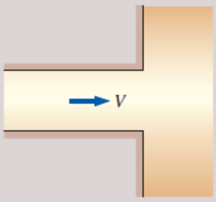



*Pipe Exit*

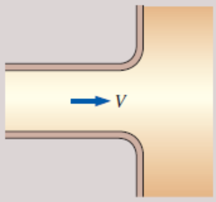
*Reentrant:* $K_L = \alpha$




*Sharp-edged:* $K_L = \alpha$




*Rounded:* $K_L = \alpha$




Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.



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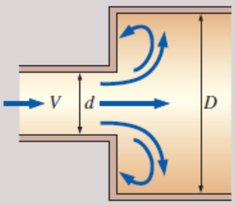
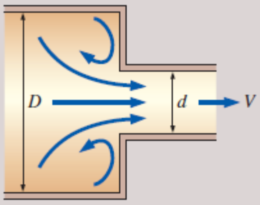
# Minor Losses

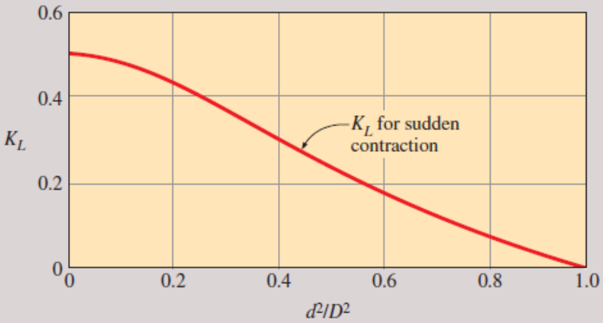
*Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)*

*Sudden expansion:* $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$



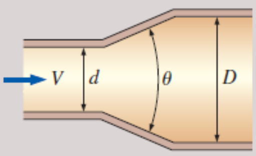

*Sudden contraction:* See chart.



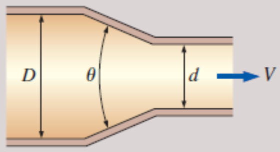




*Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)*

*Expansion* (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$




*Contraction:*
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$



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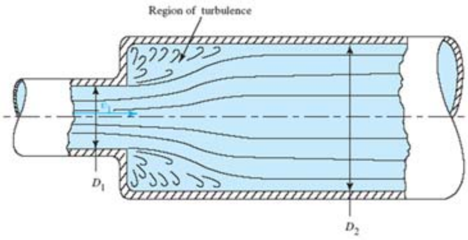
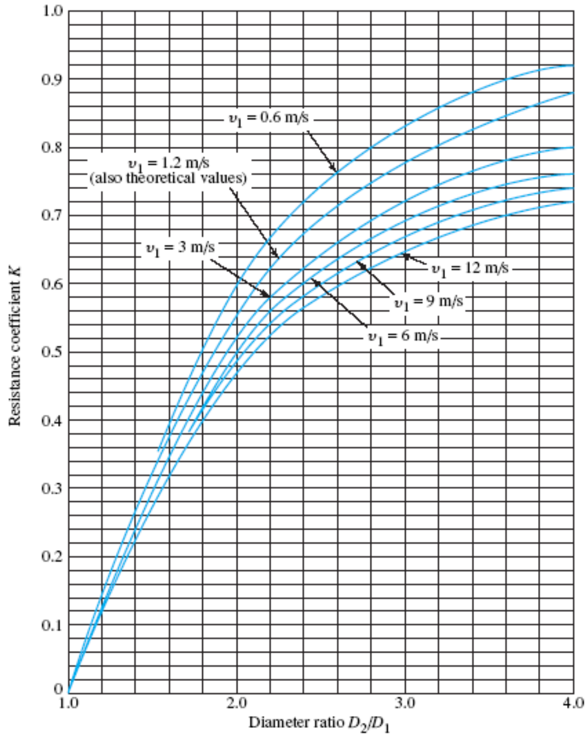
# Minor Losses

Loss coefficient for sudden enlargement.



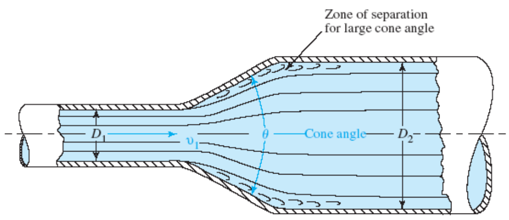



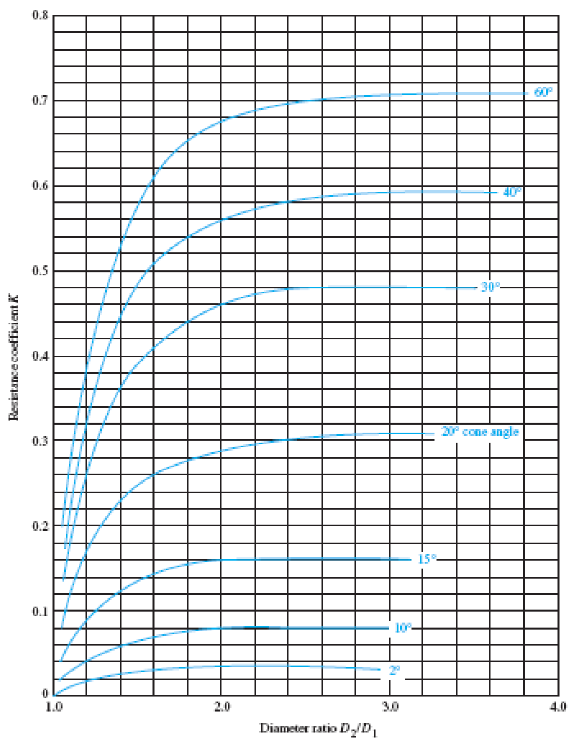




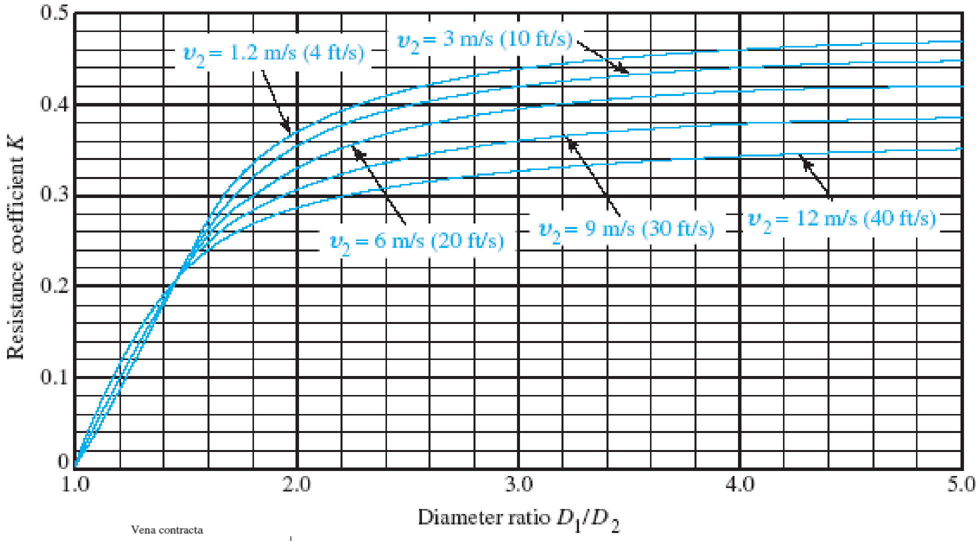

# Minor Losses

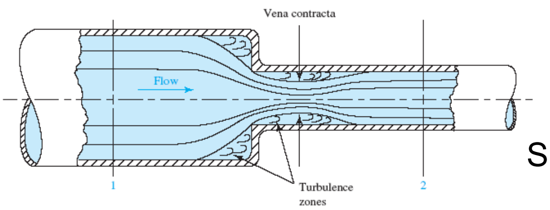
Loss coefficient for gradual enlargement.




# Minor Losses




Sudden contraction



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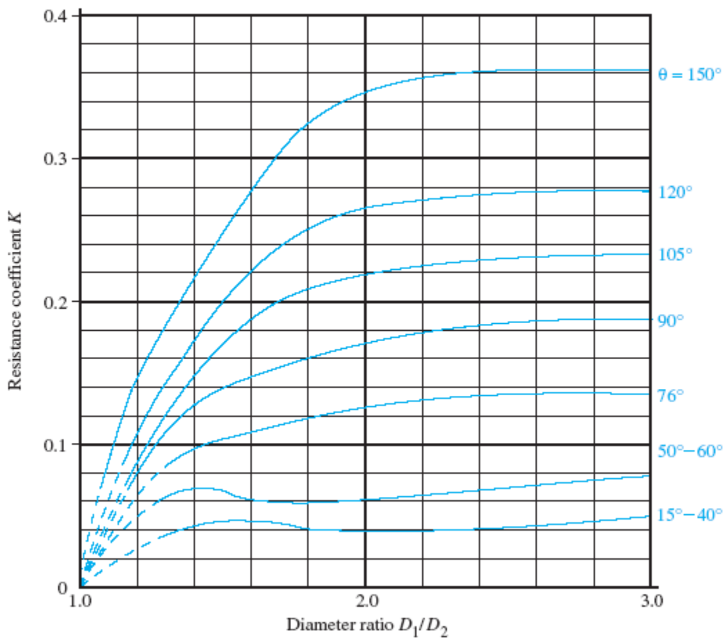
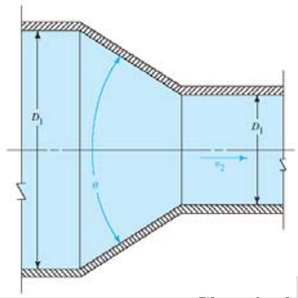
# Minor Losses

- As the cone angle of the contraction decreases, the resistance coefficient actually increases
- The reason is that the data include the effects of both the local turbulence caused by flow separation and pipe friction.
- For the smaller cone angles, the transition between the two diameters is very long, which increases the friction losses.



**Gradual contraction**



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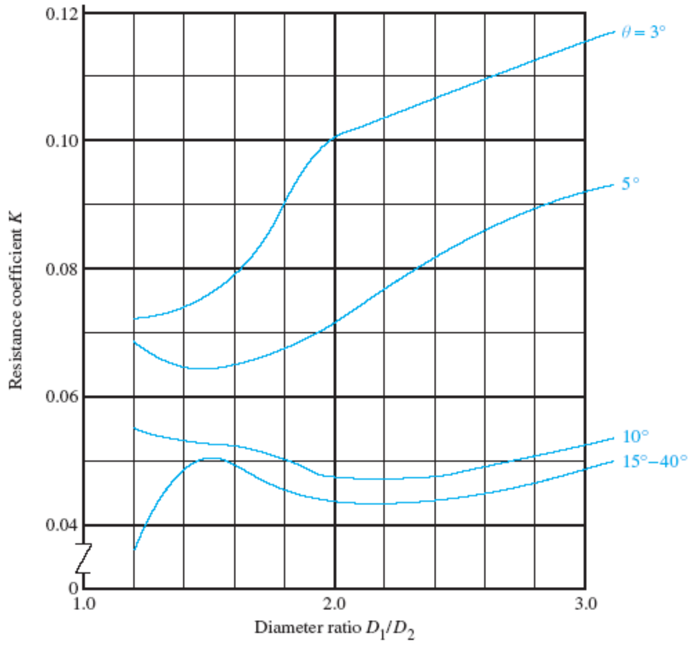
# Minor Losses

## Gradual contraction

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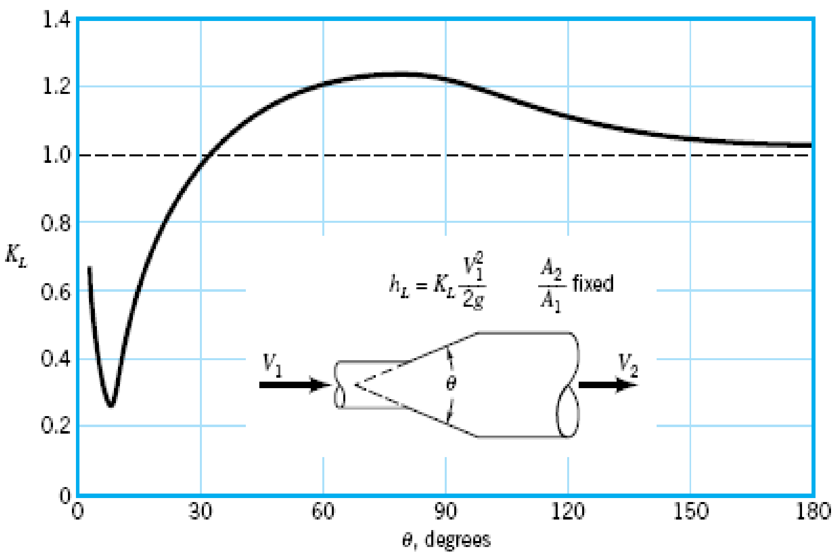
# Minor Losses

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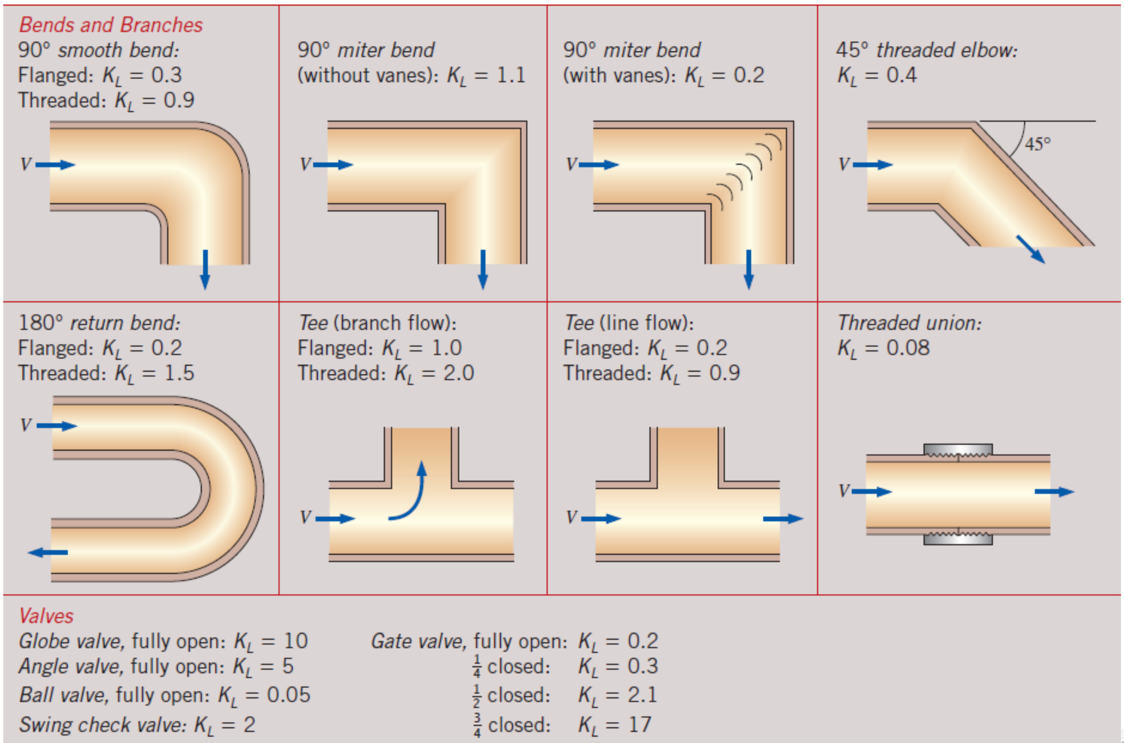
# Minor Losses



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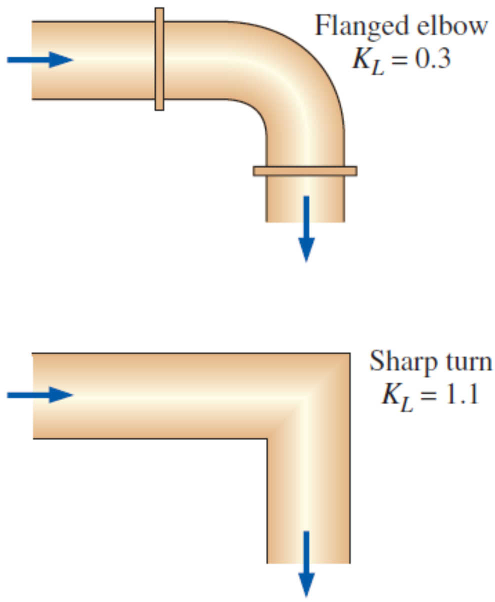
# Minor Losses

The losses during changes of direction can be minimized by making the turn “easy” on the fluid by using circular arcs instead of sharp turns.



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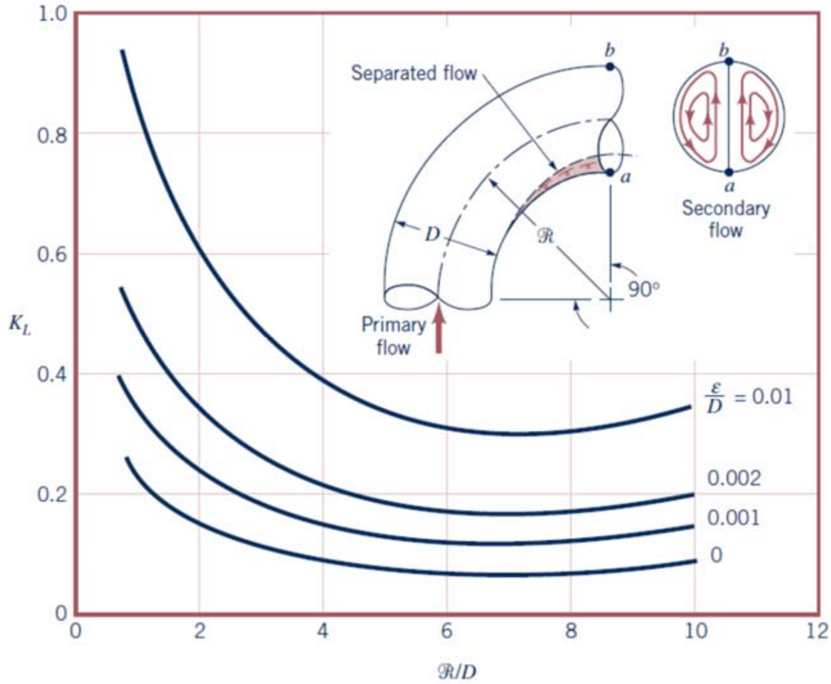
# Minor Losses

Character of the flow in a 90° bend and the associated loss coefficient

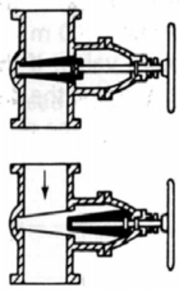





# Minor Losses

**TABLE : Values of $K_v$ for Certain Common Hydraulic Valves**

A. Gate valves

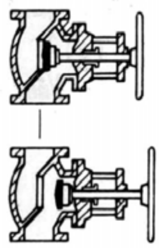


Closed

Open

$K_V$ = 0.15 (fully open)

B. Globe valves:



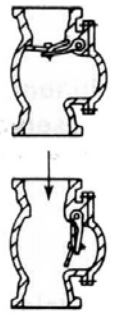
Closed

Open

$K_V$ = 10.0 (fully open)

# Minor Losses

C. Check valves:



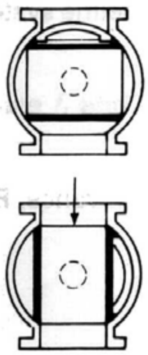
Closed

Hinge (Swing type)

Open

| | | |
|---|---|---|
| Swing type | $K_V$ = 2.5 | (fully open) |
| Ball type | $K_V$ = 70.0 | (fully open) |
| Lift type | $K_V$ = 12.0 | (fully open) |

D. Rotary valves:



Closed

$K_V$ = 10.0 (fully open)

Open



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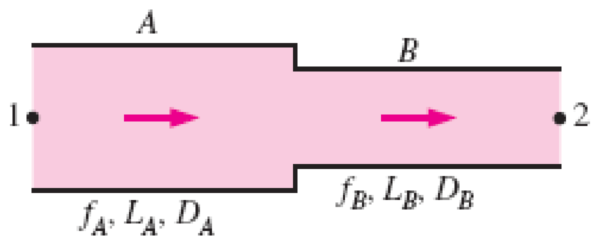
# Piping Networks

piping systems encountered in practice such as the water distribution involve

- Numerous parallel and series connections
- several sources (supply of fluid into the system) and
- loads (discharges of fluid from the system)

For pipes *in series, the flow rate is the* same in each pipe, and the total head loss is the sum of the head losses in individual pipes




$$\dot{V}_A = \dot{V}_B$$

$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$



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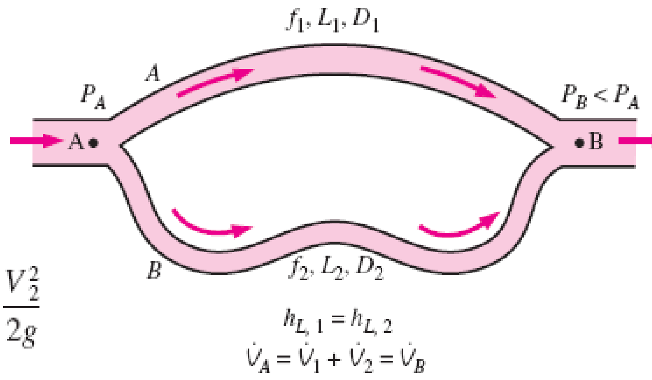
# Piping Networks and Pump Selection

For pipes *in parallel, the head loss is* the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.




$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The analysis of piping networks is based on two simple principles

Conservation of mass throughout the system must be satisfied
Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions



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# Example

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe The walls of the expansion section are angled 30° from the horizontal. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.

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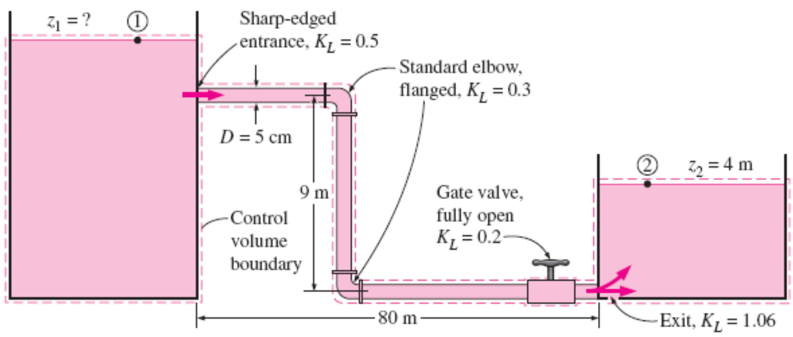
# Example Cont.



# Example

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Determine the elevation $z_1$ for a flow rate of 6 L/s.



# Example Cont.

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# Example Cont.



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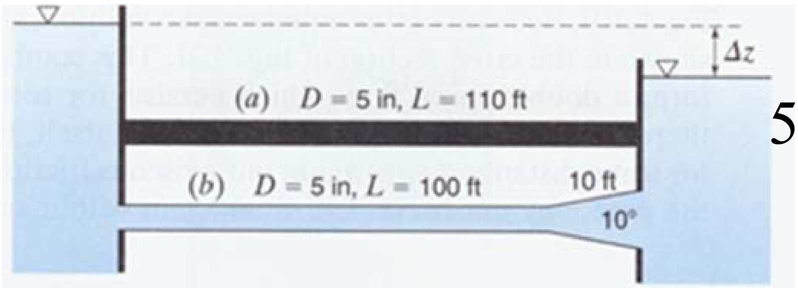
## Example

A 5-in-diameter pipe with an estimated *f* of 0.033 is 110 feet long and connects two reservoirs whose surface elevations differ by 12 feet. The pipe entrance is flushed, and the discharge is submerged.

(a) Compute the flow rate.

(b) How much would the flow rate change if the last 10 ft of the pipe were replaced with a smooth conical diffuser with a cone angle of 10°?




$$5'' = 0.417 \text{ ft}$$



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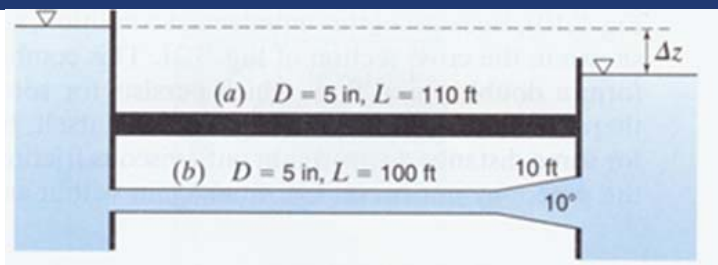
## Example Cont.

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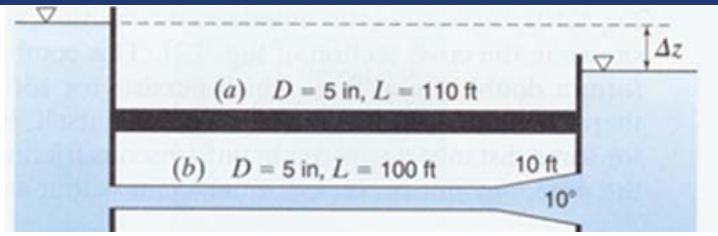
# Example Cont.



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# Example Cont.



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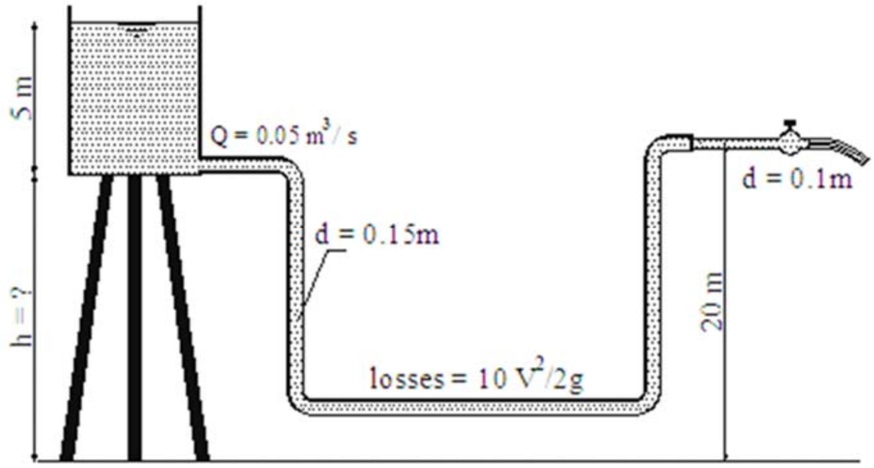
# Example

In the figure shown: Where the discharge through the system is 0.05 $m^3/s$, the total losses through the pipe is 10 $v^2/2g$ where v is the velocity of water in 0.15 m diameter pipe, the water in the final outlet exposed to atmosphere.



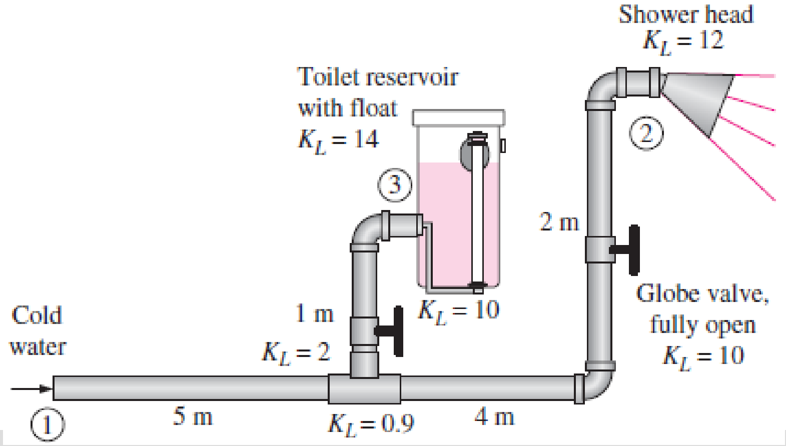

Calculate the required height **(h =?)** below the tank



# Example Cont.

# Example

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 8–49. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.






# Example Cont.

***Assumptions*** 1 The flov
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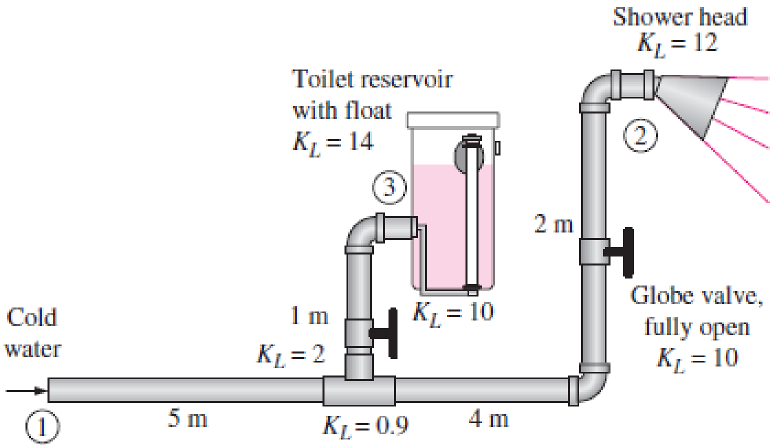
velocity heads are neglig

***Properties*** The prop
× 10⁻³ kg/m · s, a
copper pipes is ε =

1.002
ıess of




The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$).

$$\Sigma K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7.$$

the shower head is open to the atmosphere, and the velocity heads are negligible, between points 1 and 2

## Example Cont.

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

$$\rightarrow \qquad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left( f\frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$



## Example Cont.

$$18.4 = \left( f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7 \right) \frac{V^2}{2(9.81 \text{ m/s}^2)}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \qquad \rightarrow \qquad V = \frac{\dot{V}}{\pi (0.015 \text{ m})^2/4}$$

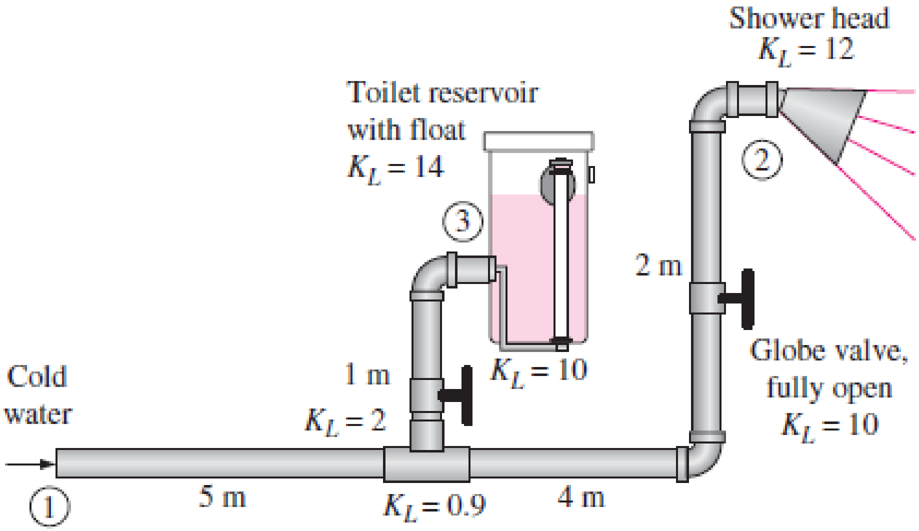
$$\text{Re} = \frac{VD}{\nu} \qquad \rightarrow \qquad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2\text{/s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left( \frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \qquad \rightarrow \qquad \frac{1}{\sqrt{f}} = -2.0 \log\left( \frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

# Example

$\dot{V} = 0.0$ ... 4,550

(*b*) When the to




$h_{L,2} = 18.4$ m and $\Sigma K_{L,2} = 24.7$,

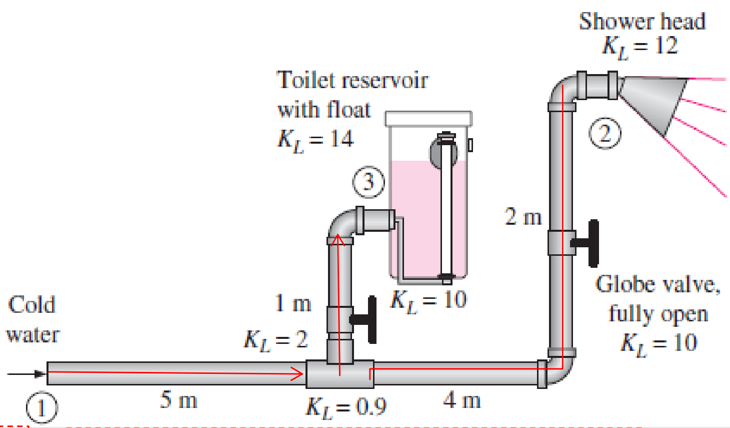
$$h_{L,3} = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 1 \text{ m} = 19.4 \text{ m}$$

$$\Sigma K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$



# Example Cont.

$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$




$$h_{L,2} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_2 \frac{6 \text{ m}}{0.015 \text{ m}} + 24.7\right) \frac{V_2^2}{2(9.81 \text{ m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_3 \frac{1 \text{ m}}{0.015 \text{ m}} + 26.9\right) \frac{V_3^2}{2(9.81 \text{ m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015 \text{ m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015 \text{ m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015 \text{ m})}{1.004 \times 10^{-6} \text{m}^2/\text{s}}$$

## Example Cont.

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\text{ m}}{3.7(0.015\text{ m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\text{ m}}{3.7(0.015\text{ m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\text{ m}}{3.7(0.015\text{ m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

$$\dot{V}_1 = 0.00090\text{ m}^3/\text{s}, \quad \dot{V}_2 = 0.00042\text{ m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048\text{ m}^3/\text{s}$$



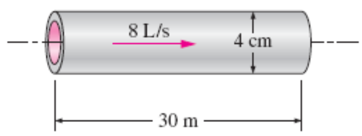

## Selected Problem

**Q1** The head loss for a certain circular pipe is given by $h_L = 0.0826fL(\dot{V}^2/D^5)$, where $f$ is the friction factor (dimensionless), $L$ is the pipe length, $\dot{V}$ is the volumetric flow rate, and $D$ is the pipe diameter. Determine if the 0.0826 is a dimensional or dimensionless constant. Is this equation dimensionally homogeneous as it stands?

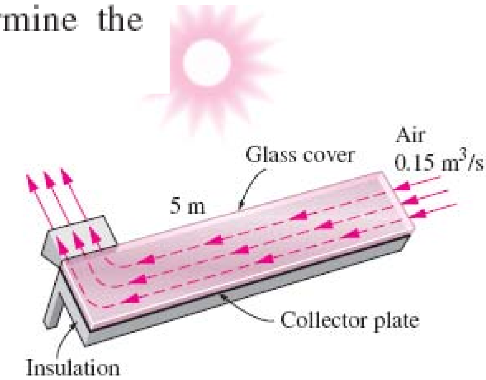
**Q2** Oil at 80°F ($\rho = 56.8$ lbm/ft$^3$ and $\mu = 0.0278$ lbm/ft · s) is flowing steadily in a 0.5-in-diameter, 120-ft-long pipe. During the flow, the pressure at the pipe inlet and exit is measured to be 120 psi and 14 psi, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 20° upward, and (*c*) inclined 20° downward.



**Q3** Water at 15°C ($\rho$ = 999.1 kg/m$^3$ and $\mu$ = 1.138 × 10$^{-3}$ kg/m · s) is flowing steadily in a 30-m-long and 4-cm-diameter horizontal pipe made of stainless steel at a rate of 8 L/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop.



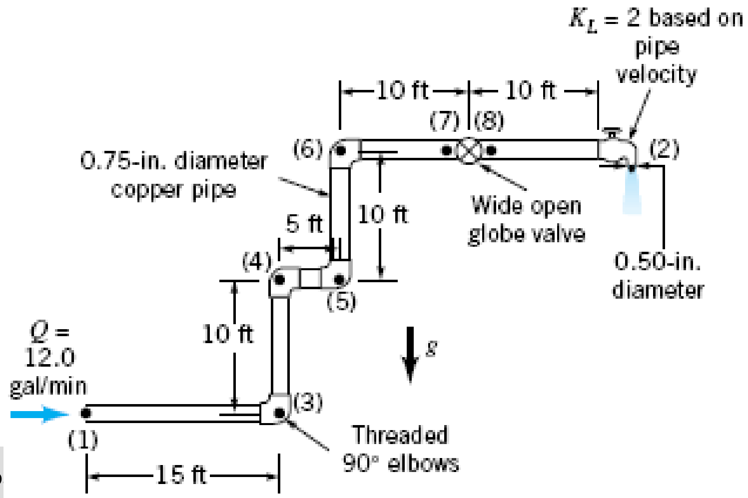

**Q4** Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45°C at a rate of 0.15 m³/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects, determine the pressure drop in the collector. *Answer: 29 Pa*



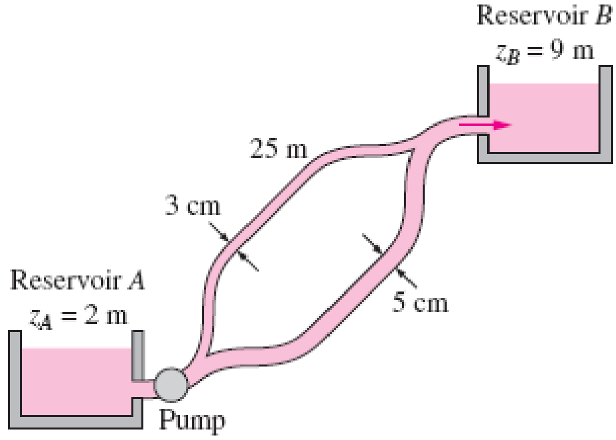



**Q5**

Water at 60 °F flows from the basement to the second floor through the 0.75-in. (0.0625-ft)-diameter copper pipe (a drawn tubing) at a rate of $Q$ = 12.0 gal/min = 0.0267 ft³/s and exits through a faucet of diameter 0.50 in. as shown . Determine the pressure at point (1) if: (a) all losses are neglected, (b) the only losses included are major losses, or (c) all losses are included.



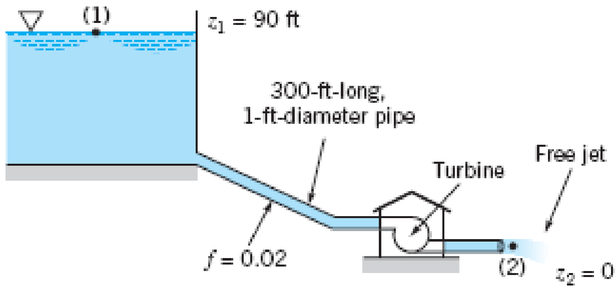
**Q5** Water at 20°C is to be pumped from a reservoir ($z_A$ = 2 m) to another reservoir at a higher elevation ($z_B$ = 9 m) through two 25-m-long plastic pipes connected in parallel. The diameters of the two pipes are 3 cm and 5 cm. Water is to be pumped by a 68 percent efficient motor–pump unit that draws 7 kW of electric power during operation. The minor losses and the head loss in the pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rates through each of the parallel pipes.






**Q6**

The turbine shown extracts 50 hp from the water flowing through it. The 1-ft-diameter, 300-ft-long pipe is assumed to have a friction factor of 0.02. Minor losses are negligible. Determine the flowrate through the pipe and turbine.




**Q7** The following equation is sometimes used in place of the Colebrook equation (Eq. 8.35):

$$f = \frac{1.325}{\{\ln[(\varepsilon/3.7D) + (5.74/\mathrm{Re}^{0.9})]\}^2}$$

for $10^{-6} < \varepsilon/D < 10^{-2}$ and $5000 < \mathrm{Re} < 10^{+8}$ (Ref. 22, pg. 220). An advantage of this equation is that given Re and $\varepsilon/D$, it does not require an iteration procedure to obtain $f$. Plot a graph of the percent difference in $f$ as given by this equation and the original Colebrook equation for Reynolds numbers in the range of validity of the above equation, with $\varepsilon/D = 10^{-4}$.

**Q8** The $\frac{1}{2}$-in.-diameter hose shown in Fig. P8.73 can withstand a maximum pressure of 200 psi without rupturing. Determine the maximum length, $\ell$, allowed if the friction factor is 0.022 and the flowrate is 0.010 cfs. Neglect minor losses.