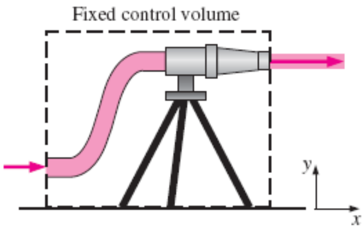
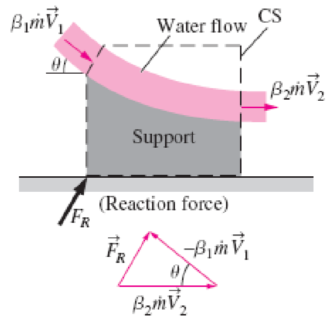
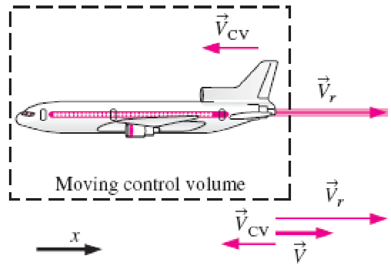


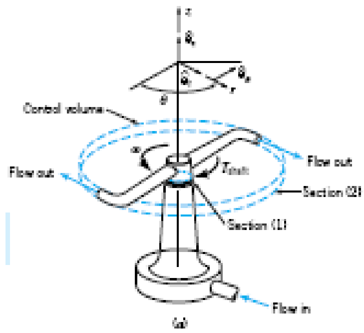
Fluid Mechanics (0905241)

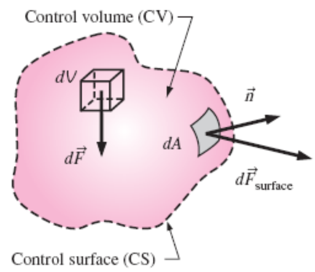
Momentum Balance



Prof. Zayed Al-Hamamre





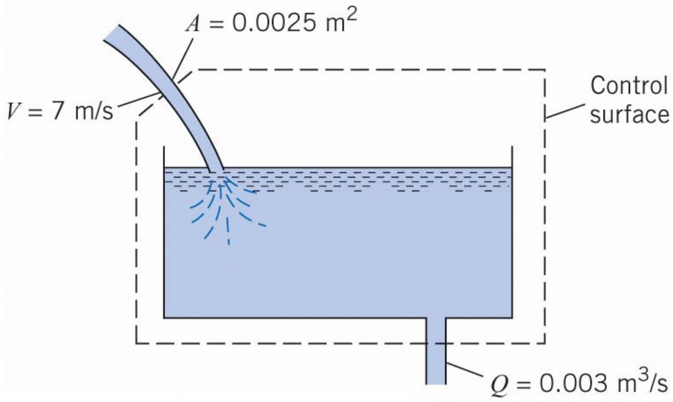


Content

- **Conservation of Mass**
- **Choosing a Control Volume**
- **Conservation of Momentum**
- **Applications of the Integral Form of the Basic Equations**

Example

A jet of water discharges into an open tank, and water leaves the tank through an orifice in the bottom at a rate of 0.003 m^3/s. If the cross-sectional area of the jet is 0.0025 m^2 where the velocity of water is 7 m/s, at what rate is water accumulating in (or evacuating from) the tank?



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Example Cont.

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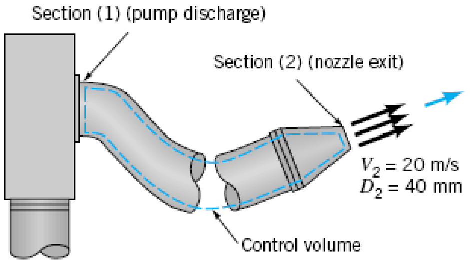
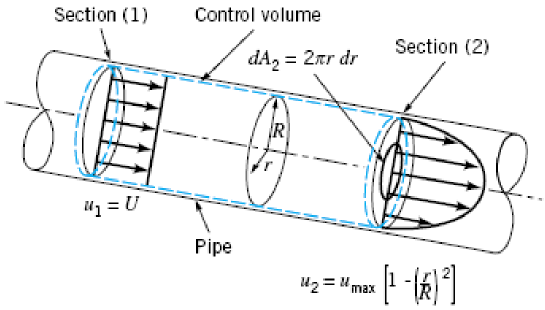
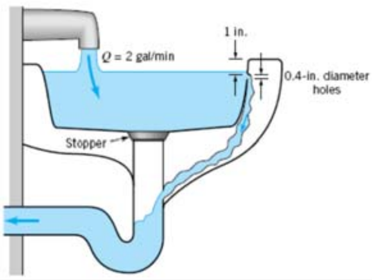
CHOOSING A CONTROL VOLUME

- A control volume can be selected as any arbitrary region in space through which fluid flows
- Its bounding control surface can be fixed, moving, and even deforming during flow

Fixed, Nondeforming Control Volume







$$\frac{\partial}{\partial t}\left(\int_{V_K} \rho \, dV\right) + \int_{A_K} \rho U_i \, dA_i = 0.$$

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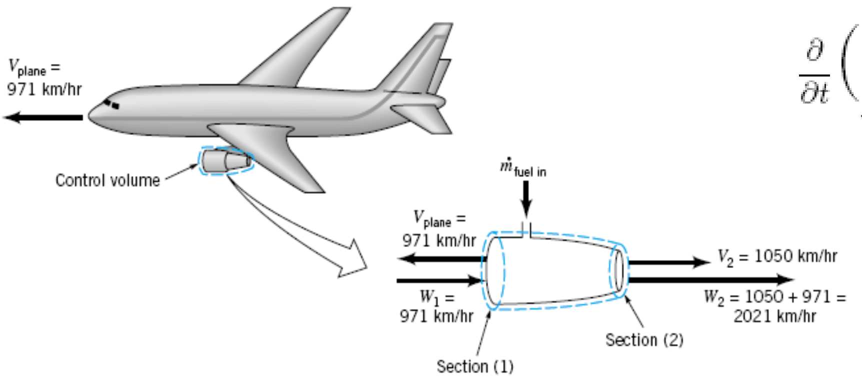
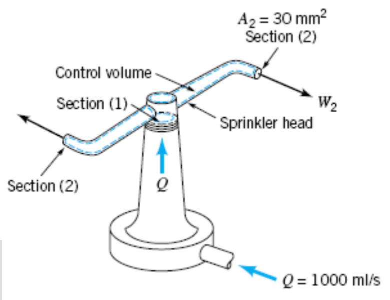
CHOOSING A CONTROL VOLUME

Moving, Nondeforming Control Volume

- when a moving control volume is used, the fluid velocity relative to the moving control volume (relative velocity) is an important flow field variable.
- The relative velocity, **W, is the fluid velocity seen by an observer moving with the control** volume.
- The control volume velocity, V_{cv}, is the velocity of the control volume as seen from a fixed coordinate system.
- The absolute velocity, **V, is the fluid velocity seen by a stationary** observer in a fixed coordinate system. These velocities are related to each other by the vector equation $\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv}$

$$\frac{\partial}{\partial t}\left(\int_{V_K} \rho \, dV\right) + \int_{A_K} \rho \mathbf{W} \, dA_i = 0.$$





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CHOOSING A CONTROL VOLUME

Deforming Control Volume

➢A deforming control volume involves changing volume size and control surface movement.

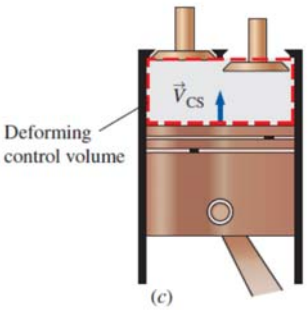
$$\frac{\partial}{\partial t}\left(\int_{V_K}\rho\, dV\right)+\int_{A_K}\rho \mathbf{W}\, dA_i=0.$$

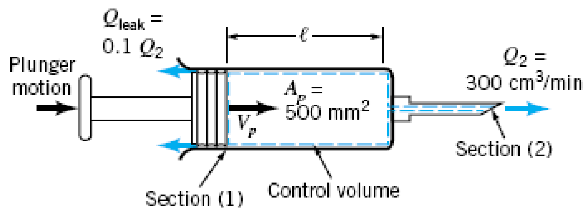
$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cs}$$

is usually nonzero and must be carefully evaluated because the extent of the control volume varies with time

W: the velocity referenced to the control surface

$\mathbf{V}_{cs}$ is the velocity of the control surface as seen by a fixed observer





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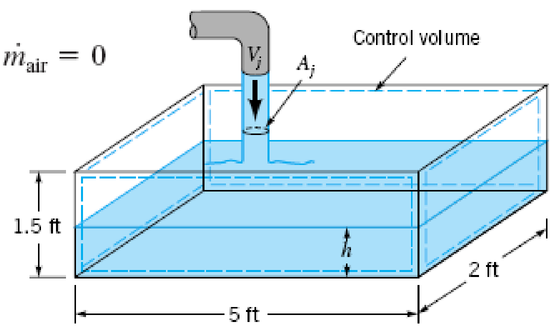
Example

Fixed, Nondeforming Control Volume

A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicated Estimate the time rate of change of the depth of water in the tub, $\partial h/\partial t$, in in./min at any instant.

$+ \dot{m}_{air} = 0$



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Example Cont.

Applications of the Integral Form of the Basic Equations

Moving, Nondeforming Control Volume

- when a moving control volume is used, the fluid velocity relative to the moving control volume (relative velocity) is an important flow field variable

The absolute velocity, $\mathbf{V}$,

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv}$$

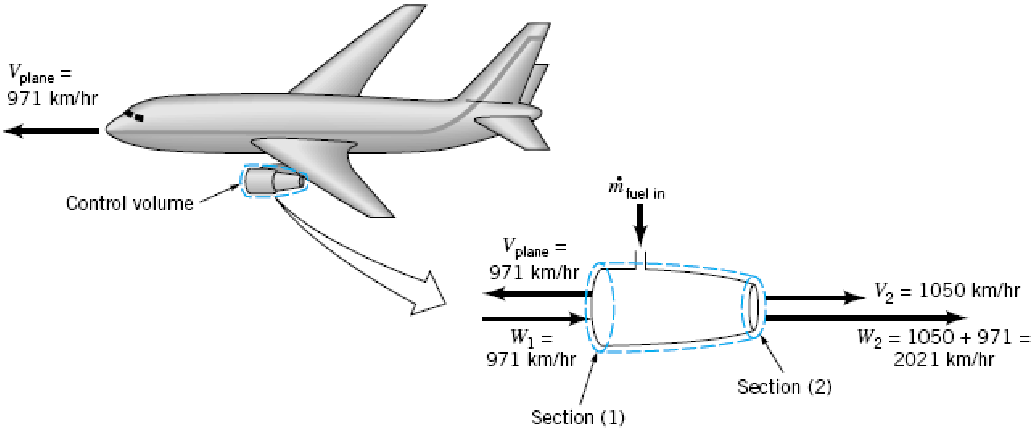
$\mathbf{W}$, The relative velocity, is the fluid velocity seen by an observer moving with the control volume

$\mathbf{V}_{cv}$, The control volume velocity is the velocity of the control volume as seen from a fixed coordinate system

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

Example

An airplane moves forward at a speed of 971 km/hr as shown The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3. Estimate the mass flowrate of fuel into the engine in kg/hr.



Assuming the intake velocity, W_1, *relative to the moving control volume, as being equal* in magnitude to the speed of the airplane 971 km/hr.

Example Cont.

Example Cont.

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Applications of the Integral Form of the Basic Equations

Deforming Control Volume

$$\frac{\partial}{\partial t}\int_{\text{CV}} \rho \, d\forall + \int_{\text{CS}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cs}$$

$\mathbf{V}_{cs}$ is the velocity of the control surface as seen by a fixed observer.

the relative velocity, $\mathbf{W}$, referenced to the control surface.

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Momentum Equation

- The product of the mass and the velocity of a body is called the *linear momentum or just the momentum of the body*

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d\left(m\vec{V}\right)}{dt}$$ *rate of change of the LINEAR momentum*

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$$

- The forces can be stated as mass forces $(\delta M_j)_{\Re}$ *caused by gravitation* forces and electromagnetic forces, as well as surface forces caused by pressure, $(\delta O_j)_{\Re}$

$$\sum \vec{F} = \underbrace{\sum \vec{F}_{gravity}}_{\text{body force}} + \underbrace{\sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} + \sum \vec{F}_{other}}_{\text{surface forces}}$$

Momentum Equation

- For a fixed control volume (no motion or deformation of the control volume), and the linear momentum equation becomes

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\,dV + \sum_{\text{out}} \beta\dot{m}\vec{V}_{\text{avg}} - \sum_{\text{in}} \beta\dot{m}\vec{V}_{\text{avg}}$$

which is stated in words as

$$\begin{pmatrix}\text{The sum of all}\\ \text{external forces}\\ \text{acting on a CV}\end{pmatrix} = \begin{pmatrix}\text{The time rate of change}\\ \text{of the linear momentum}\\ \text{of the contents of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{linear momentum out of the}\\ \text{control surface by mass flow}\end{pmatrix}$$

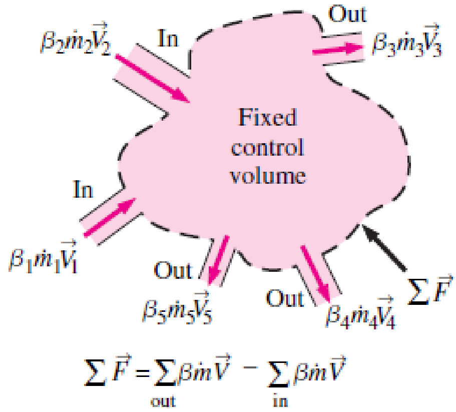
Momentum-flux correction factor: $$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{\text{avg}}}\right)^2 dA_c$$

β= 1 *for the case of uniform flow* over an inlet or outlet

Momentum Equation

Steady linear momentum equation: $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$

- For a fluid element, it is stated that the *rate of change of the momentum* in the *j direction is equal to the sum of the external forces acting in this direction* on the fluid element, plus the molecular-dependent input of momentum per unit time.



- The momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved.

This is known as the *conservation of momentum principle*

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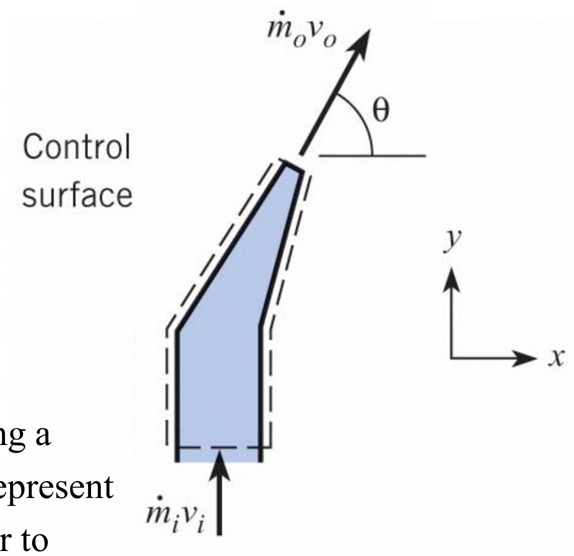
Momentum Diagram

- ✓ Consider steady flow through the control volume surrounding the nozzle
  - The fluid inside the control volume has momentum because it is moving.
  - However, the velocity and density at each point do not change with time, so the total momentum in the control volume is constant, and the momentum accumulation term is zero.



- ✓ The momentum diagram is created by sketching a control volume and then drawing a vector to represent the momentum accumulation term and a vector to represent momentum flow at each section where mass crosses the control surface.

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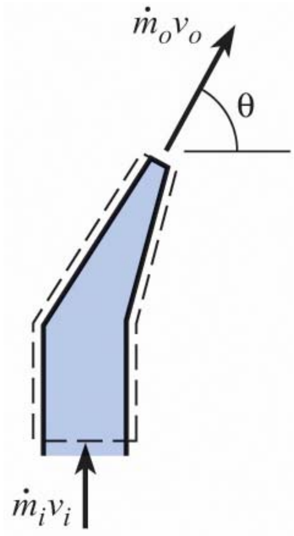
Momentum Diagram

The outlet momentum flow is $\dot{m}_o v_o$

The Inlet momentum flow is $\dot{m}_i v_i$



the momentum flow,

$$\sum_{cs} \dot{m}_o \mathbf{v}_o = [\dot{m}_o v_o \cos\theta]\mathbf{i} + [\dot{m}_o v_o \sin\theta]\mathbf{j}$$

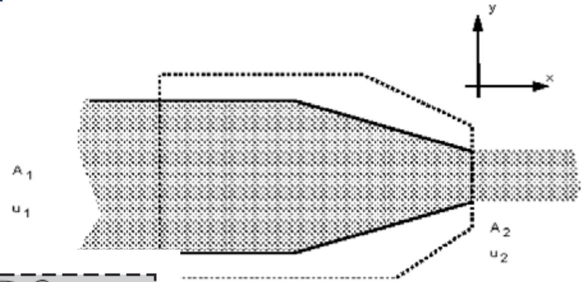
$$\sum_{cs} \dot{m}_i \mathbf{v}_i = [\dot{m}_i v_i]\mathbf{j}$$

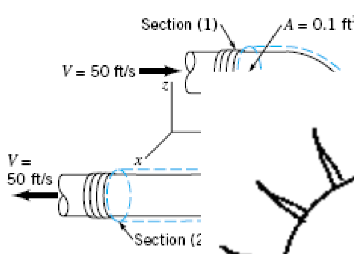
Recognizing that $\dot{m}_i = \dot{m}_o = \dot{m}$,

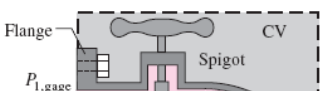
$$\sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i = [\dot{m} v_o \cos\theta]\mathbf{i} + [\dot{m} v_o \sin\theta - \dot{m} v_i]\mathbf{j}$$

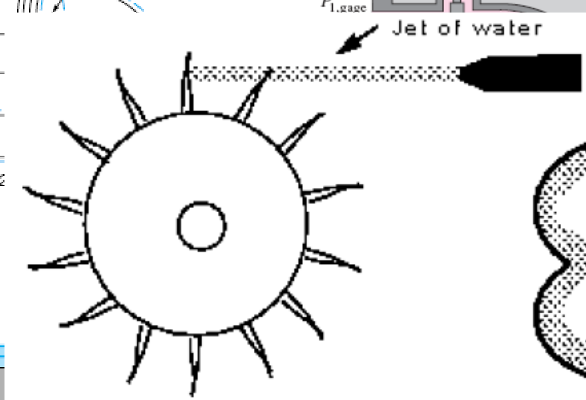
Applications of the Integral Form of the Basic Equations

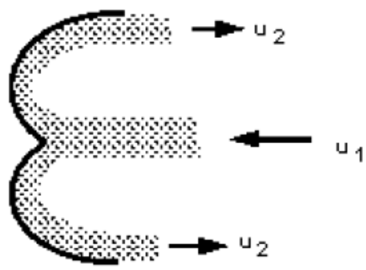
- Force on the nozzle at the outlet of a pipe. Because the fluid is contracted at the nozzle forces are induced in the nozzle. Anything holding the nozzle (e.g. a fireman) must be strong enough to withstand these forces

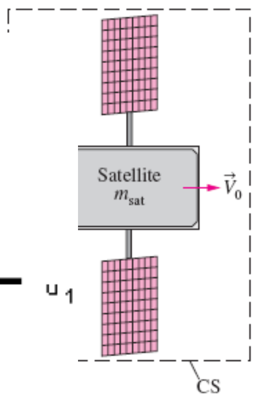


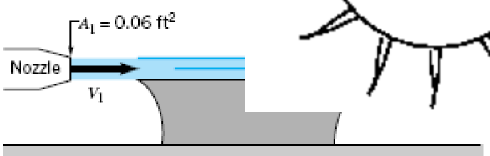


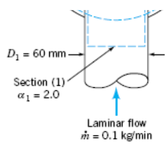












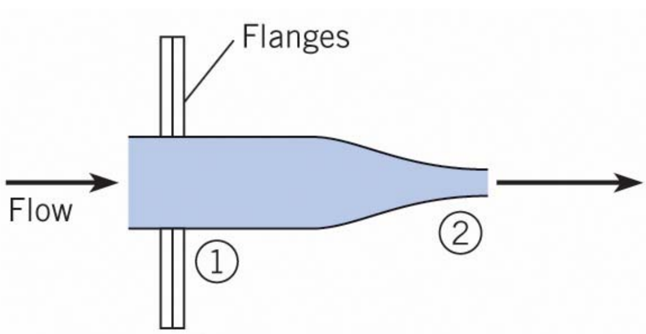
Procedure for solving a problem

1. Choose a control volume such that the force acting on the control surface is the force on the beam and where information is available for the momentum flux crossing the control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Evaluate the sum of the forces from the force diagram.
5. Evaluate momentum terms.
6. Calculate the force.

Example : Force on a nozzle

✓ Nozzles are flow devices used to accelerate a fluid stream by reducing the cross-sectional area of the flow.

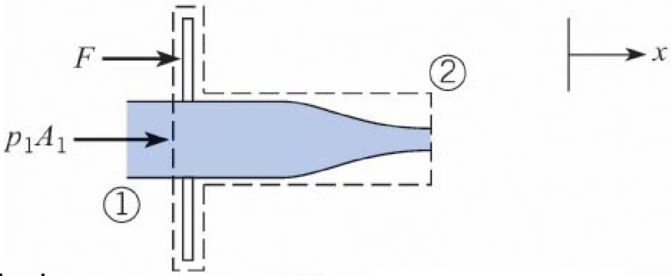
The sketch shows air flowing through a nozzle. The inlet pressure is p_1 = 105 kPa $_{abs,}$ and the air exhausts into the atmosphere, where the pressure is 101.3 kPa $_{abs}$. The nozzle has an inlet diameter of 60 mm and an exit diameter of 10 mm, and the nozzle is connected to the supply pipe by flanges. Find the air speed at the exit of the nozzle and the force required to hold the nozzle stationary. Assume the air has a constant density of 1.22 kg/m^3. Neglect the weight of the nozzle.



Example cont.

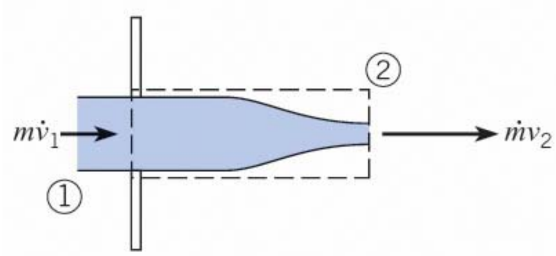
Control volume is stationary

1. Select the control volume
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the Bernoulli equation and the continuity equation to find exit (and inlet) velocity



FD the force diagram.

5. Apply the component form of the momentum equation in the *x-direction.*
6. Evaluate forces (use gage pressures for pressure force).
7. Evaluate momentum terms.
8. Calculate force on flange.



MD the momentum diagram.

Example cont.

Example cont.

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Example cont.

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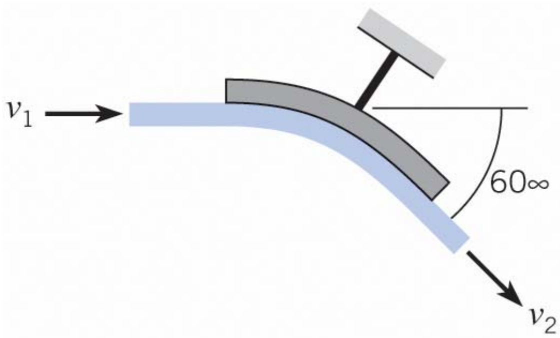
Example : Fluid jet striking a flat vane

- A *vane is a structural component, typically thin, that is used to turn a fluid jet or is turned by a fluid jet. Examples include a blade in a* turbine, a sail on a ship, and a thrust reverser on an aircraft engine.

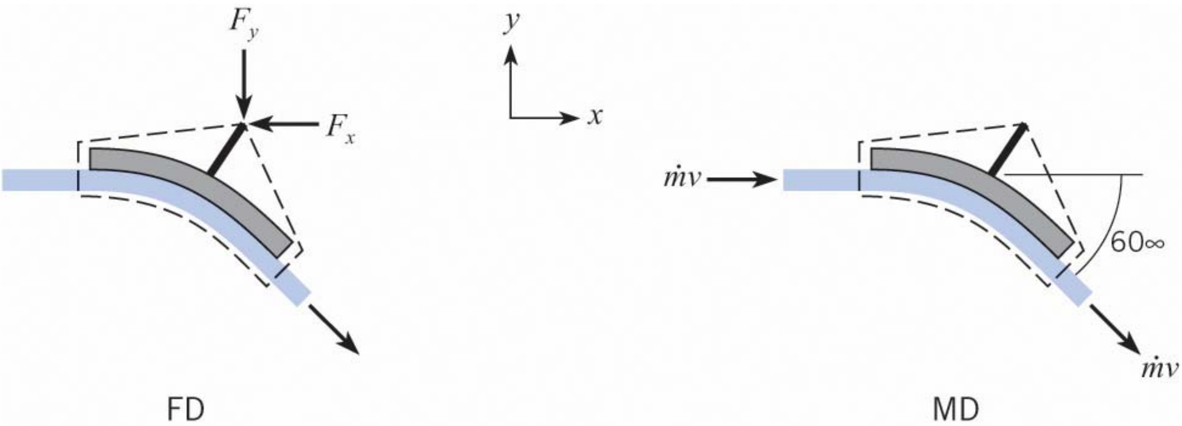
A water jet is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Find the force exerted by the jet on the vane. Neglect the influence of gravity.



Example cont.

Vector form of momentum equation.

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$



The force diagram shows only the reaction force

The momentum diagram shows an inflow and outflow

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Example cont.

Example cont.

Example cont.

l to hold the

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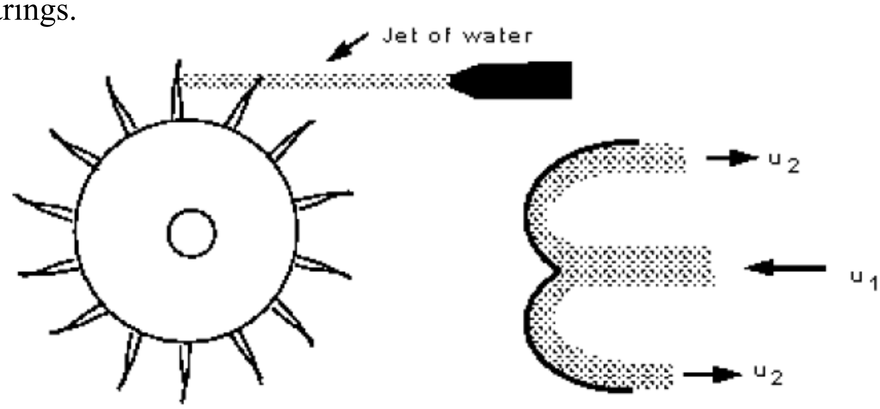
Example : Force on axisymmetric vane

Force on a Turbine Blade in a Viscosity-Free Fluid **Pelton wheel blade**

A narrow jet (usually of water) is fired at blades which stick out around the periphery of a large metal disk. The shape of each of these blade is such that as the jet hits the blade it splits in two (see figure below) with half the water diverted to one side and the other to the other. This splitting of the jet is beneficial to the turbine mounting - it causes equal and opposite forces (hence a sum of zero) on the bearings.



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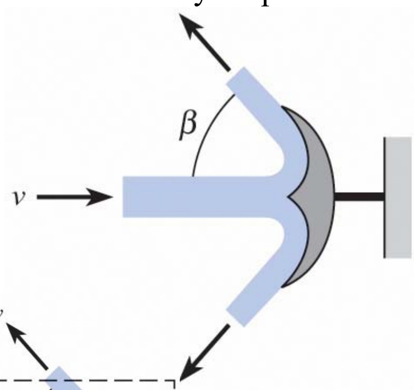
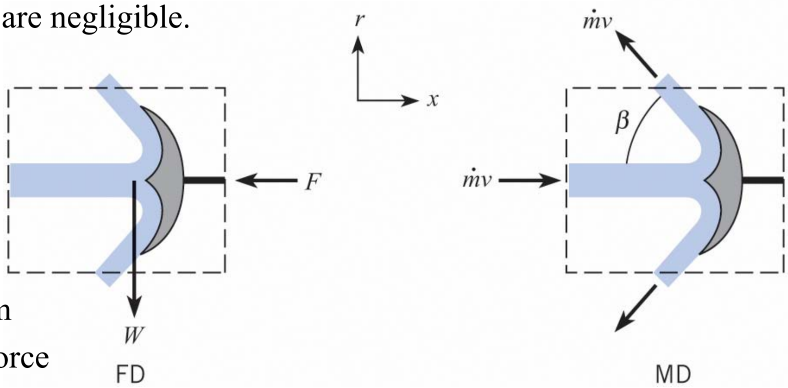
Example : Force on axisymmetric vane

An incident jet of fluid with density ρ, speed ν, and area *A is deflected through an angle β by a* stationary, axisymmetric vane. Find the force required to hold the vane stationary. Express the answer using ρ, ν, *A, and β.* Neglect the influence of gravity



Assumptions:

1. Flow is steady.
2. Fluid is incompressible.
3. Viscous effects are negligible.



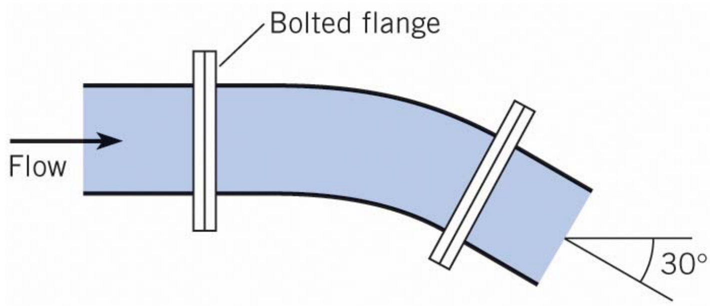
The force diagram shows only one force

Example cont.

s

Example : Pipe Bends

- Calculating the force on pipe bends is important in engineering applications using large pipes to design the support system



A 1 m–diameter pipe bend shown in the diagram is carrying crude oil ($S = 0.94$) *with a steady flow rate of 2 mm/s. The* bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is 1.2 mm, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the net force required to hold the bend in place

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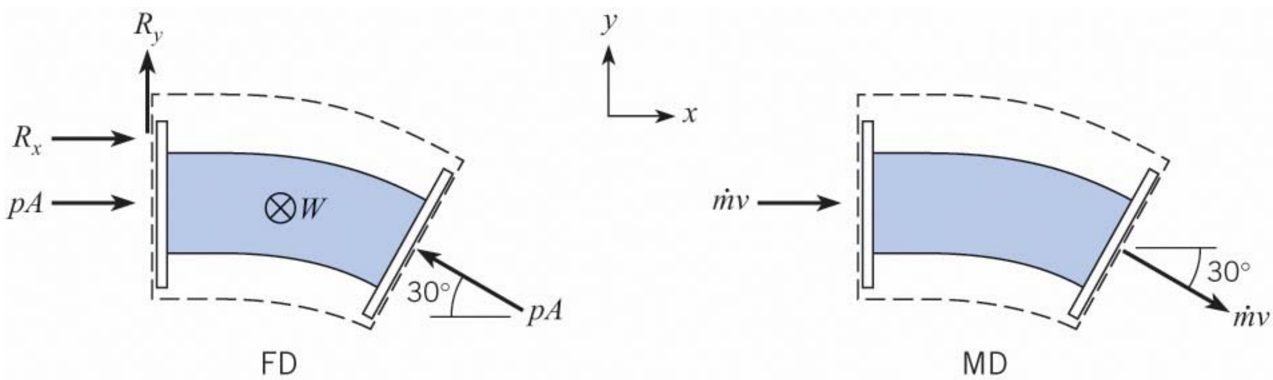
Example cont.

- Because flow in a pipe is usually turbulent, it is common practice to assume that velocity is nearly constant across each cross section of the pipe.
- ✓ The control volume is stationary



pressure forces and the
component reaction forces

The *z-direction is outward from the page*

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Example cont.

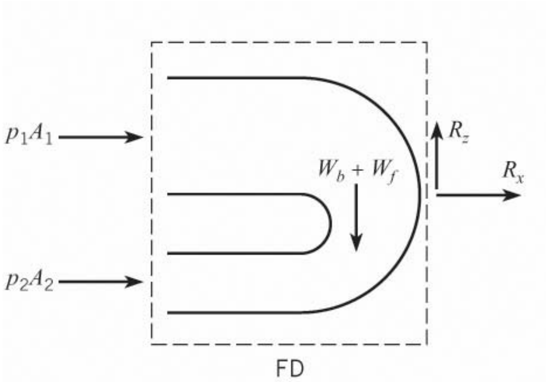
Example cont.

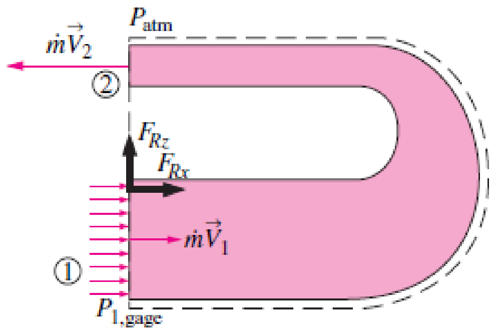
Example cont.

Example cont.

Example

Water flows through a 180° reducing bend at a rate of 14 kg/s. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. the elevation difference between the centers of the inlet and the exit sections is still 0.3 m. Determine the anchoring force needed to hold the elbow in place.





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Example : Flow through reduction bends

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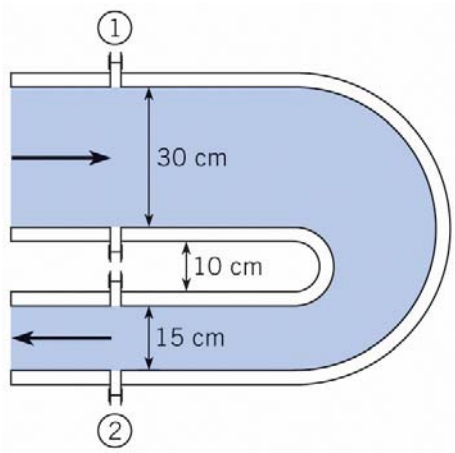
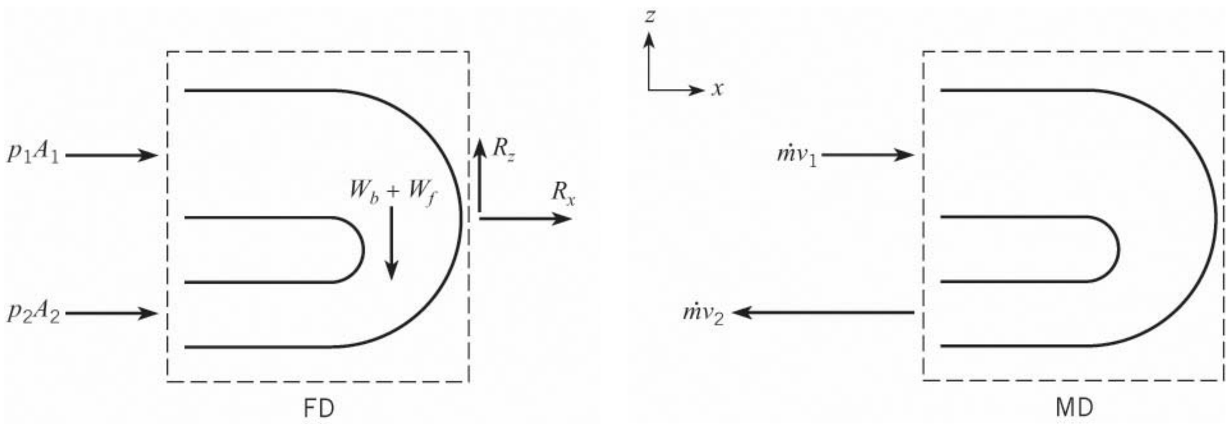
Example cont.

Example : Flow through reduction bends

Water flows through a 180° reducing bend, as shown. The discharge is 0.25 m^3/s, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is 0.10 m^3, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500 N. The water density is 1000 kg/m^3. The bend is in the vertical plane.



Example cont.



- There are two forces due to pressure and a reaction force component in the *x*-direction, and there are weight and reaction forces component in the *z*-direction.
- There is inlet and outlet momentum flux in *x-direction*

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Example cont.

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Example cont.

Example cont.

Example cont.

Example cont.

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Example cont.

- Reaction force components

$$R_x = -(11.6\ \text{kN}) - (4.42\ \text{kN})$$
$$= \boxed{-16.0\ \text{kN}}$$
$$R_z = W_b + W_f$$
$$= 500\ \text{N} + (9810\ \text{N/m}^3)(0.1\ \text{m}^3)$$
$$= \boxed{1.48\ \text{kN}}$$

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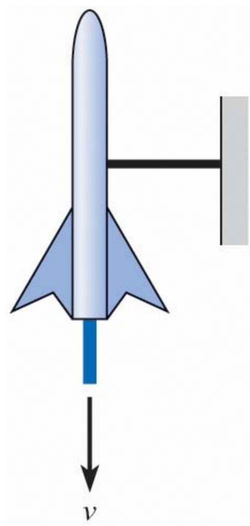
Example: Thrust of Rocket

The sketch below shows a 40 g rocket, of the type used for model rocketry, being fired on a test stand in order to evaluate thrust. The exhaust jet from the rocket motor has a diameter of *d* = *1 cm*, *a speed of* $v = 450$ *m/s, and a density of* $\rho = 0.5$ kg/m^3. Assume the pressure in the exhaust jet equals ambient pressure, and neglect any momentum changes inside the rocket motor. Find the force F_b *acting on the beam that supports the rocket.*

Plan

1. Choose a control volume such that the force acting on the control surface is the force on the beam and where information is available for the momentum flux crossing the control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Because this problem involves only one direction, the component form of the momentum equation in the *z*-direction, will be used.
5. Evaluate the sum of the forces from the force diagram.
6. Evaluate momentum terms.
7. Calculate the force.



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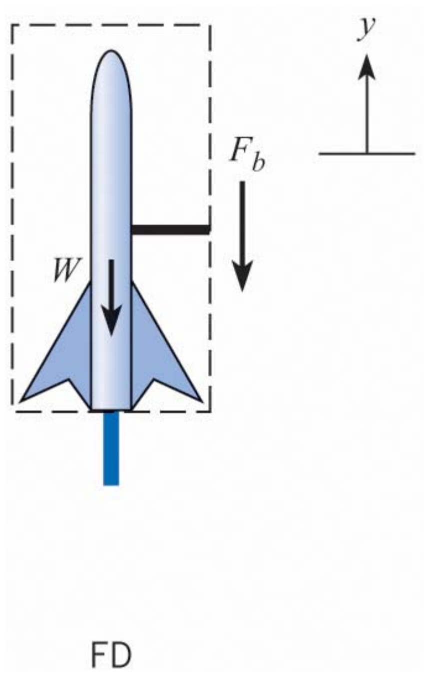
Example cont.

- The control volume is stationary
- The force on the control surface exerted by the beam is chosen as downward (negative z-direction) with magnitude F_b. (The corresponding force exerted by the rocket on the beam is upward.)
- The weight also acts downward.
- Also there is no pressure force at the nozzle exit plane because exit pressure is atmospheric



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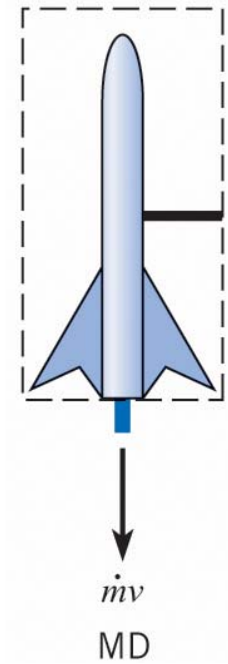
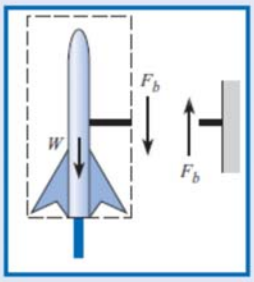
Example cont.



Example cont.

Review

1. The thrust force of the rocket motor is $\dot{m}v$ = 7.95 N (1.79 lbf); this value is typical of a small motor used for model rocketry.
2. The force F_b acts downward at the control surface, and an equal and opposite force acts upward on the support beam, as shown in the sketch below. This is an example of an action and reaction force, as described by Newton's third law of motion.



3. For solving this problem, two separate diagrams were used: the force and the momentum diagrams. The rationale for this approach is to regard forces and momentum flows as separate phenomena. This facilitates writing the equations and provides a systematic approach to more complex problems.

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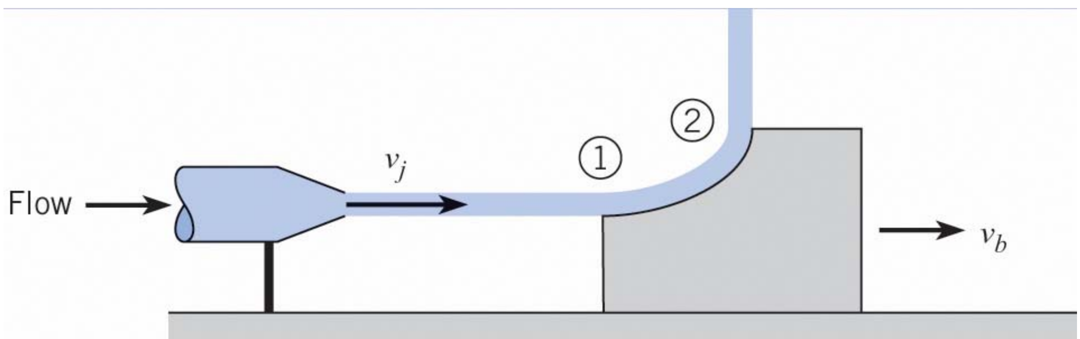
Example: Jet impinging a moving block

A stationary nozzle produces a water jet with a speed of 50 m/s and a cross-sectional area of 5 cm^2. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed of 25 m/s on a surface with friction. The density of the water is 1000 kg/m^3. Find the frictional force F acting on the block.



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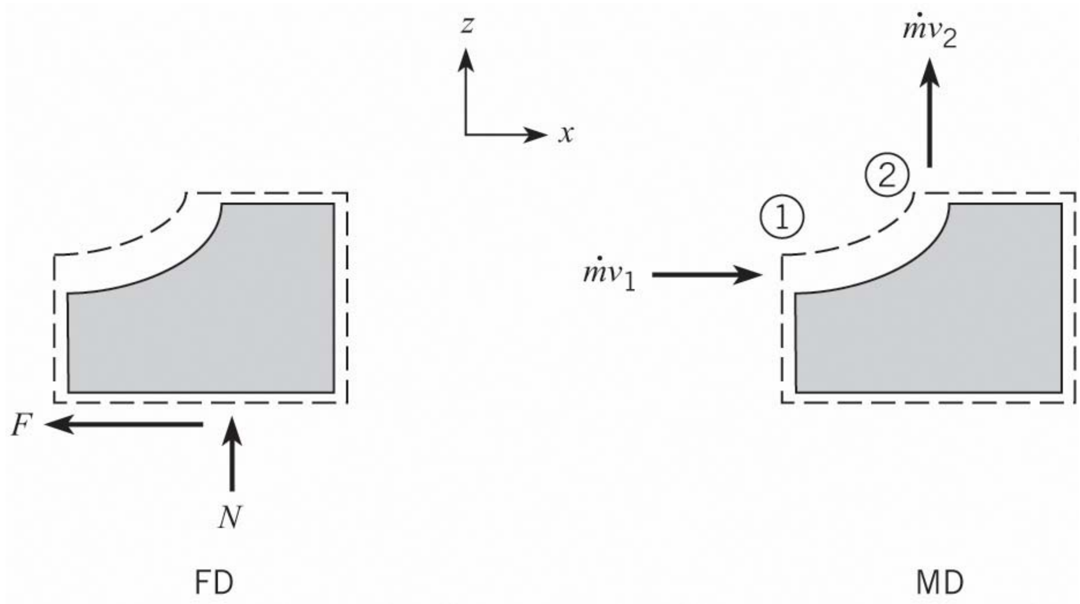
Example cont.



The force diagram shows one force in the horizontal direction

The momentum diagram shows an influx and out flux of momentum

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Example cont.

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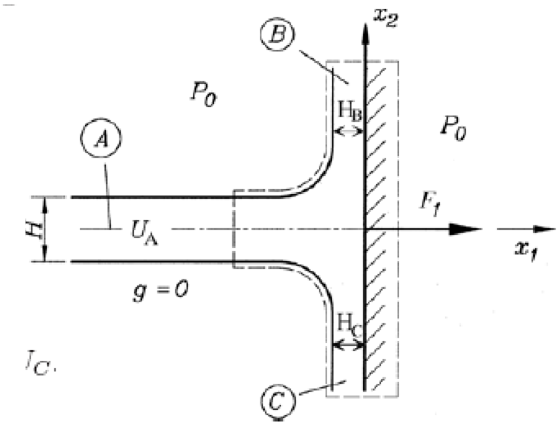
Example

Momentum on a Plane Vertical Plate

What force needs to be applied in the x_1 *direction to prevent the* deflection of the plate due to the momentum impact of the plane fluid jet.

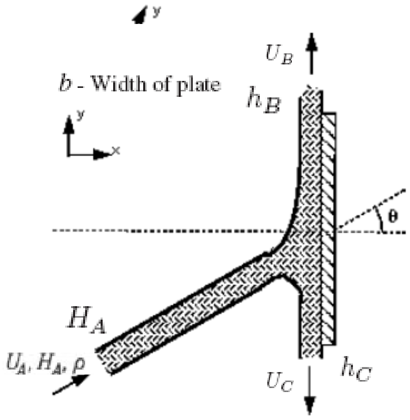


Example cont.

Example cont.

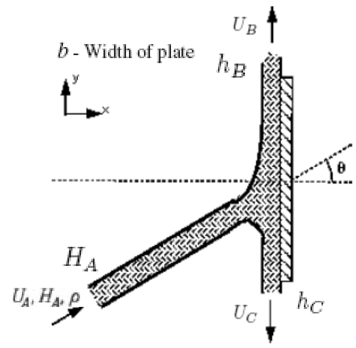
Momentum on an Inclined Plane Plate

For a fluid jet hitting an inclined plane plate, the jet behavior is shown



Example cont.

Example cont.



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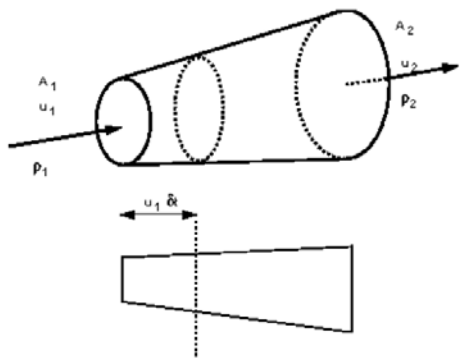
Example

determine the frictional force exerted by the pipe wall on the air flow between sections



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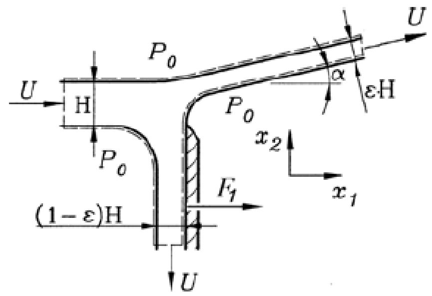
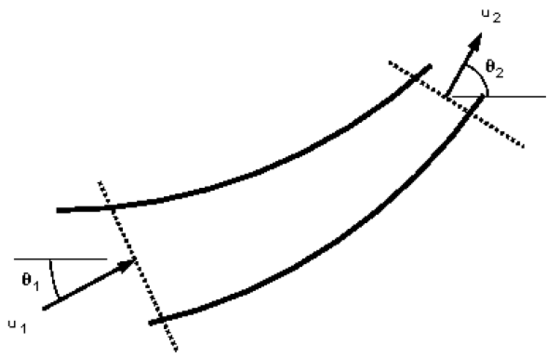
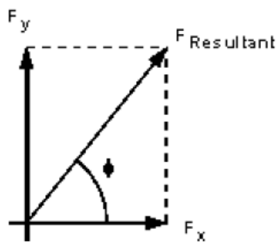
Example

Jet Deflection by an Edge



Example



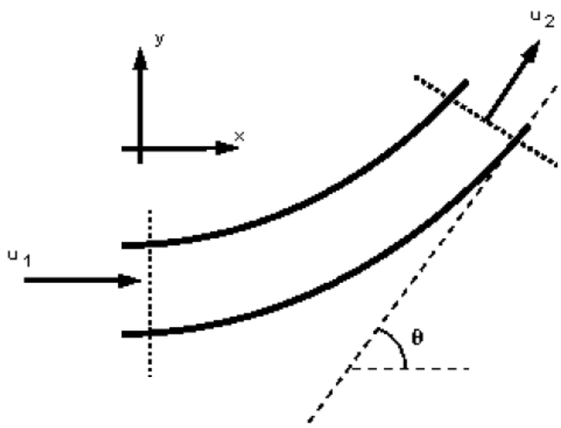


$$F_{resultant} = \sqrt{F_x^2 + F_y^2}$$

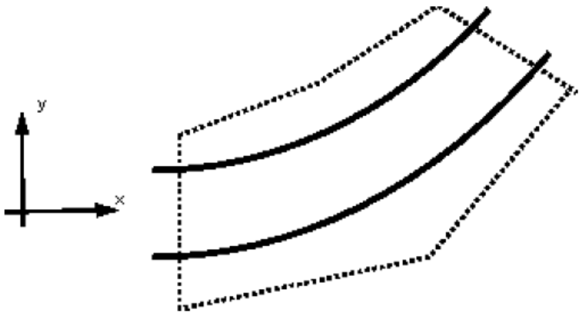
$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example cont.

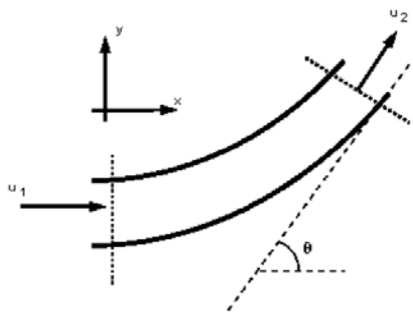
1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the **total** force



1. Control Volume



Example cont.

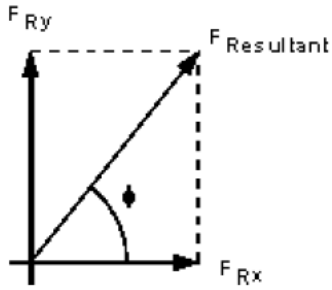


Example

In the case when the pressure is not uniform at the considered control volume

Calculate the **pressure** force

Calculate the **resultant** force



$$F_R = \sqrt{F_{R\,x}^2 - F_{R\,y}^2}$$

$$\phi = \tan^{-1}\left(\frac{F_{R\,y}}{F_{R\,x}}\right)$$

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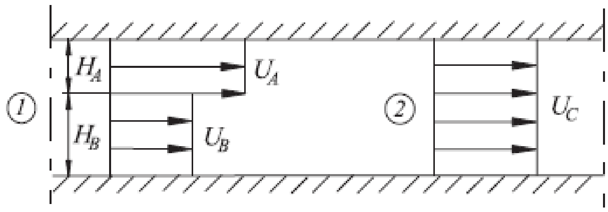
Example

Mixing Process in a Pipe of Constant Cross-Section

- In a pipe, two fluids flow at constant velocities U_A, U_B. *These fluids mix with* one another as they move downstream in a channel. The pressure at point 1 and the partial areas in which the velocities *UA, UB hold, will be* given. Sought is the pressure P_2 *at point 2 where a constant velocity* U_C *over* the pipe cross-section has been reached.



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Example

Angular Momentum Equation

- The motion of a rigid body is a combination of :
  - The translational motion of its center of mass and — linear momentum equation
  - Rotational motion about its center of mass — angular quantities

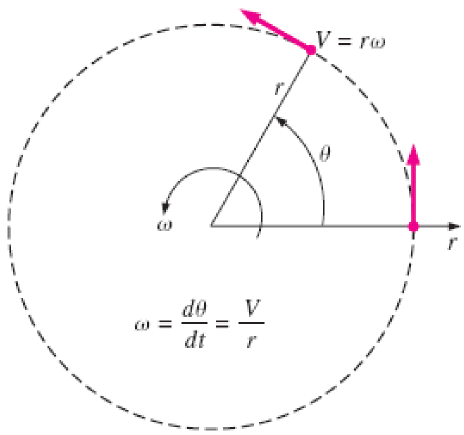
angular distance θ, angular velocity ω, angular acceleration α.

Angular velocity ω is the angular distance traveled per unit time, angular acceleration α is the rate of change of angular velocity.

circumference of a circle of radius r is $2\pi r$,

the angular distance traveled by any point in a rigid body during a complete rotation is 2π rad

$$\omega = \frac{d\theta}{dt} = \frac{d(l/r)}{dt} = \frac{1}{r}\frac{dl}{dt} = \frac{V}{r} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{dV}{dt} = \frac{a_t}{r}$$



Angular Momentum Equation

$$V = r\omega \qquad \text{and} \qquad a_t = r\alpha$$

V is the linear velocity and a_t is the linear acceleration

torque M acting on a point mass m at a normal distance r from the axis of rotation is

$$M = rF_t = rma_t = mr^2\alpha$$

The total torque acting on a rotating rigid body about an axis

$$M = \int_{\text{mass}} r^2\alpha\, dm = \left[\int_{\text{mass}} r^2\, dm\right]\alpha = I\alpha$$

where I is the *moment of inertia* of the body about the axis of rotation, which is a measure of the inertia of a body against rotation.

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Angular Momentum Equation

- The moment of a force is equal to the product of the force and the normal distance, the moment of momentum, $H = rmV = r^2m\omega,$

total angular momentum of a rotating rigid body

$$H = \int_{\text{mass}} r^2\omega\, dm = \left[\int_{\text{mass}} r^2\, dm\right]\omega = I\omega$$

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

$\vec{M}$ is the net moment or torque applied on the body,

I is the moment of inertia of the body about the axis of rotation,

and $\vec{\alpha}$ is the angular acceleration.

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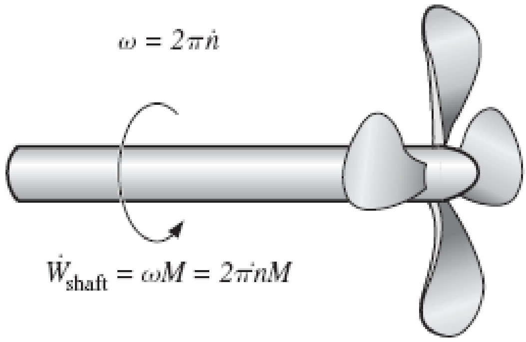
Angular Momentum Equation

➢ It can also be expressed in terms of the rate of change of angular momentum $d\vec{H}/dt$

The angular momentum equation can be stated as *the rate of change of the angular momentum of a body is equal to the net torque acting on it*



Angular velocity versus rpm: $$\omega = \frac{2\pi\dot{n}}{60} \quad \text{(rad/s)}$$

Shaft power: $$\dot{W}_{shaft} = \omega M = 2\pi\dot{n}M \quad \text{(W)}$$