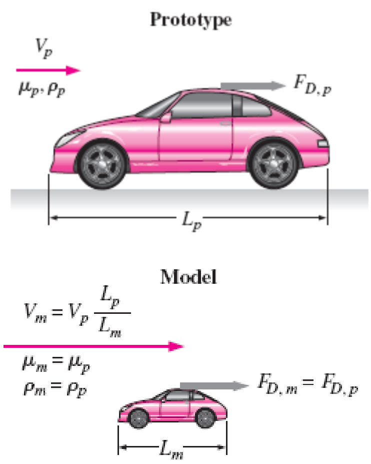
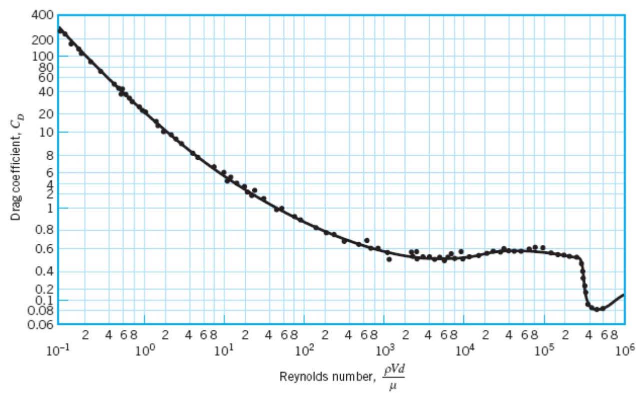
$$Re = \frac{\rho v D}{\mu}$$

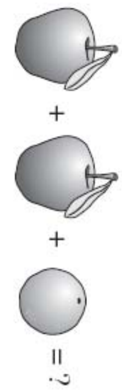
Fluid Mechanics (0905241)

$$Fr = \frac{v}{\sqrt{gD}}$$

Dimensional Analysis and Modeling







Prof. Zayed Al-Hamamre

$$Ma = \frac{v}{c}$$

Content

- Dimensional Analysis
- Buckingham Pi Theorem
- Dimensionless Groups
- Experimental Models

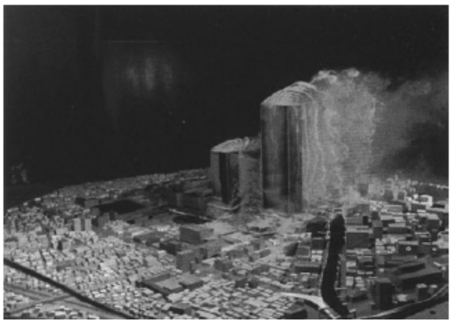
Overview

- Many flow problems can only be investigated experimentally
- Few problems in fluids can be solved by analysis alone.
- The solution to many problems is achieved through the use of a combination of analysis and experimental data
- One must know how to plan experiments.
- Correlate other experiments to a specific problem.
- Usually, the goal is to make the experiment widely applicable.
- Similitude is used to make experiments more applicable, i.e. measurements made on one system (for example, in the laboratory) can be used to describe the behavior of other similar systems (outside the laboratory2
- Laboratory flows are studied under carefully controlled conditions.

Examples

Model of an aircraft in a wind tunnel.

scale model of a section of the Missisipi River.

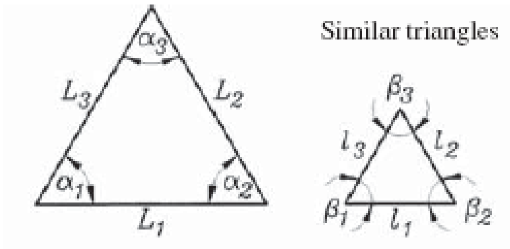


Model of San Antonio, Texas, used for determining wind patterns in this urban environment.

Full and model scale

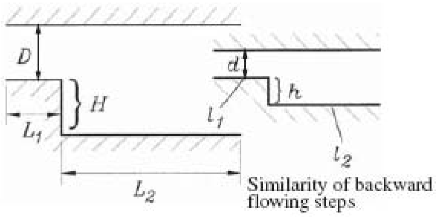
for similar triangles:

$$\frac{L_1}{l_1} = \frac{L_2}{l_2} = \frac{L_3}{l_3} = \text{constant.}$$



For a channel with a step:

$$\frac{L_1}{l_1} = \frac{L_2}{l_2} = \frac{D}{d} = \frac{H}{h} = \text{constant.}$$



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Full and model scale

- Scales: model, and full-scale
- Selection of the model scale: governed by dimensional analysis and similarity

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Dimensional Analysis

Pipe-Flow Example: Suppose we want to investigate the Pressure Drop per Unit Length in steady-state flow of water down a smooth, circular pipe of diameter D.

- This would be of interest to an engineer designing a pipeline

- The pressure drop per unit length that develops along a pipe as the result of friction can not be explained analytically without the use of experimental data.
- First, we determine the important factors, or variables, that will have an effect on the pressure drop per unit length :

$$\Delta p_\ell = f(D, \rho, \mu, V)$$

D is the diameter of the pipe, ρ is the density of the fluid, μ is the viscosity of the fluid, and V is the flow velocity.

So, how do we approach this problem?

- To perform the experiments in a meaningful and systematic manner, it would be necessary to change one of the variables, such as the velocity, while holding all others constant, and measure the corresponding pressure drop

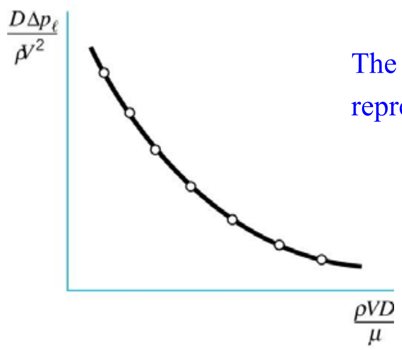
Dimensional Analysis

Fortunately, there is a simpler approach: Dimensionless Groups

The original list of variables can be collected into two dimensionless groups.

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Now instead of working with **5 variables, there are only two**.



The results of the experiment could then be represented by a single, ***universal curve***

- The experiments would consist of varying the independent variable and determining the dependent variable which is related to the pressure drop.
- Now, the curve is universal for any smooth walled, laminar pipe flow.

Dimensional Analysis

➢ The basis for this simplification lies in a consideration of the dimensions of the variables Involved.

Dimensions Associated with Common Physical Quantities

	FLT System	MLT System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

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Dimensional Analysis

➢ In pipe flow system

Dimensions are Mass (M), Length (L), Time (T), Force (F or MLT^{-2})

The basic dimensions are Mass (M), Length (L), Time (T), i.e. **3 basic dimensions**

Then, we check our dimensionless groups

$$V \doteq LT^{-1}$$
$$\mu \doteq FL^{-2}T$$
$$\Delta p_\ell \doteq FL^{-3}$$
$$D \doteq L$$
$$\rho \doteq FL^{-4}T^2$$

Substituting, we see no dimensions on our two variables:

$$\frac{D\,\Delta p_\ell}{\rho V^2} \doteq \frac{L(F/L^3)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

$$\frac{\rho VD}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

* Not only have we **reduced the number of variables** from ***five to two***, but the dimensionless plot is **independent of the system of units used**.

So, how do we know what groups of dimensionless variables to form?

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Dimensional Analysis

The purpose dimensional analysis

- Want to determine which variables to study.
- Want to determine the parameters that significantly affect the system.
- Reduce the cost/effort of experimental analysis by studying the most important groups of variables.
- The ideas can be used for any physical system.
- This will help in the design of scale test models

➢ How many dimensionless products are required to replace the original list of variables?

Buckingham Pi Theorem

Edgar Buckingham (1867-1940)

Buckingham Pi Theorem is a systematic way of forming dimensionless groups:

> If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k - r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

The dimensionless products are referred to as “pi terms”.

Requires that equation have **dimensional homogeneity**:

$u_1 = f(u_2, u_3, \ldots, u_k)$ Dimensions on the left side = dimensions on the right side

Then if pi terms are formed, they are dimensionless products one each side.

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

*The required number of pi terms is fewer than the original number of variables by r, where r is the minimum number of reference dimensions needed to describe the original set of variables (M, L, T, or F).

Buckingham Pi Theorem

Systematic Approach: Example Pipe Flow

Step 1. List all the **independent** variables that are involved in the problem: $\Delta p_\ell = f(D, \rho, \mu, V)$

variables (including **dimensional and non dimensional constants**) will include those that are necessary to describe the *geometry of the system (such as a pipe diameter),* to define any *fluid properties (such as a fluid viscosity), and to indicate external effects that influence the system (such as a driving pressure drop per unit length).*

Step 2. Express each of the variables in terms of basic dimensions:

$$\Delta p_\ell \doteq FL^{-3} \qquad V \doteq LT^{-1} \qquad \rho \doteq FL^{-4}T^{2} \qquad \mu \doteq FL^{-2}T \qquad D \doteq L$$

The basic dimensions are F,L,T or M,L,T, noting $F = MLT^{-2}$, 3 total

Step 3. Determine the required number of pi terms:

Then the number of pi terms are the number of variables, 5 minus the number of basic dimensions, 3. So there should be two pi terms for this case.

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Buckingham Pi Theorem

Step 4. Select a number of **repeating variables**, where the **number required is equal** to the **number** of **reference dimensions**.

We choose three independent variables as the repeating variables, there can be more than one set of repeating variables.

selecting from the original list of variables several of which can be combined with each of the remaining variables to form a pi term

Repeating variables: D, V, and ρ

- We note the these **three variables by themselves are dimensionally independent**; you can not form a dimensionless group with them alone.
- All of the required reference dimensions must be included within the group of repeating variables

Step 5. Form a pi term by multiplying one of the **nonrepeating variables by the product of repeating variables** each raised to an exponent that will make the combination dimensionless. The first group chosen usually includes the dependent variable.

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c$$

Product should be dimensionless: $(FL^{-3})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$

So, we need to solve for the exponent values.

Each π group is a function the *governing* or *repeating variables* plus one of the remaining variables.

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Buckingham Pi Theorem

Step 5 (continued).

$$1 + c = 0 \qquad (\text{for } F)$$

$$-3 + a + b - 4c = 0 \qquad (\text{for } L)$$

$$-b + 2c = 0 \qquad (\text{for } T)$$

Solving the set of algebraic equations, we obtain: a = 1, b = -2, c = -1:

$$\Rightarrow \quad \Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

μ is a remaining nonrepeating variable, so we can form another group:

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

Solving, a = -1, b = -1, and c = -1

$$\Rightarrow \quad \Pi_2 = \frac{\mu}{DV\rho}$$

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Buckingham Pi Theorem

Step 6. Repeat Step 5. for each of the remaining repeating variables.

We could have chosen D, V and μ as another repeating group (later).

Step 7. Check all the resulting pi terms to make sure they are dimensionless.

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \doteq \frac{(FL^{-3})(L)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

$$\Pi_2 = \frac{\mu}{DV\rho} \doteq \frac{(FL^{-2}T)}{(L)(LT^{-1})(FL^{-4}T^2)} \doteq F^0L^0T^0$$

or alternatively,

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \doteq \frac{(ML^{-2}T^{-2})(L)}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0$$

$$\Pi_2 = \frac{\mu}{DV\rho} \doteq \frac{(ML^{-1}T^{-1})}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0$$

Step 8. Express the final form as relationship among the pi terms and think about what it means.

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

For our case, $\frac{\Delta p_\ell D}{\rho V^2} = \tilde{\phi}\left(\frac{\mu}{DV\rho}\right)$ or $\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$

↘ Reynolds Number

Pressure drop depends on the Reynolds Number.

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Choosing variables

One of the most important aspects of dimensional analysis is choosing the variables important to the flow, however, this can also prove difficult.

We do not want to choose so many variables that the problem becomes cumbersome.

- Often we use engineering simplifications to obtain first order results sacrificing some accuracy, but making the study more tangible.
- Most variables fall in to the categories of geometry, material property, and external effects:
  - Geometry: lengths and angles, usually very important and obvious variables.
  - Material Properties: bind the relationship between external effects and the fluid response. Viscosity, and density of the fluid.
  - External Effects: Denotes a variable that produces a change in the system, pressures, velocity, or gravity.

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Choosing variables

In summary, the following points should be considered in the selection of variables:

1. Clearly define the problem. What is the main variable of interest (the dependent variable)?
2. Consider the basic laws that govern the phenomenon. Even a crude theory that describes the essential aspects of the system may be helpful.
3. Start the variable selection process by grouping the variables into three broad classes: geometry, material properties, and external effects.
4. Consider other variables that may not fall into one of the above categories. For example, time will be an important variable if any of the variables are time dependent.
5. Be sure to include all quantities that enter the problem even though some of them may be held constant (e.g., the acceleration of gravity, g). For a dimensional analysis it is the dimensions of the quantities that are important—not specific values!
6. Make sure that all variables are independent. Look for relationships among subsets of the variables.

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In Summary

1st π theorem:

A relationship between **m** variables (physical properties such as velocity, density etc.) can be expressed as a relationship between **m-n** *non-dimensional* groups of variables (called π groups), where **n** is the number of fundamental dimensions (such as mass, length and time) required to express the variables.

2nd π theorem

Each π group is a function of **n** *governing* or *repeating variables* plus one of the remaining variables.

Repeating variables are those which we think will appear in all or most of the π groups, and are a influence in the problem.

Some rules which should be followed are

i. From the 2nd theorem there can be **n** (= 3) repeating variables.

ii. When combined, these repeating variables variable must contain all of dimensions (M, L, T) (That is not to say that each must contain M,L and T).

iii. A combination of the repeating variables must <u>not</u> form a dimensionless group.

iv. The repeating variables <u>do not</u> have to appear in all π groups.

v. The repeating variables should be chosen to be measurable in an experimental investigation. They should be of major interest to the designer. For example, pipe diameter (dimension L) is more useful and measurable than roughness height (also dimension L).

Uniqueness of Pi Terms

Now, back to our example of pressure drop, but choose a different repeating group (D, V, μ).

If we evaluate, we find $\frac{\Delta p_\ell D^2}{V\mu}$ The other pi term remains the same.

$$\frac{\Delta p_\ell D^2}{V\mu} = \phi_1\left(\frac{\rho V D}{\mu}\right)$$

But, we note that the L.H.S, is simply what we had before multiplied by the Reynolds Number.

$$\left(\frac{\Delta p_\ell D}{\rho V^2}\right)\left(\frac{\rho V D}{\mu}\right) = \frac{\Delta p_\ell D^2}{V\mu}$$

There is not a unique set of pi terms, but rather a set number of pi terms. In this case there are always two.

If we have three pi terms, we can form another by multiplying $\Pi_1 = \phi(\Pi_2, \Pi_3)$

$$\Pi_2' = \Pi_2^a \Pi_3^b \Rightarrow \Pi_1 = \phi_1(\Pi_2', \Pi_3) \text{ or } \Pi_1 = \phi_2(\Pi_2, \Pi_2')$$

*Often the set of pi terms chosen is based on previous flow analysis.

Examples

A thin rectangular plate having a width w and a height h is located so that it is normal to a moving stream of fluid. Assume the drag, $\mathfrak{D}$, that the fluid exerts on the plate is a function of w and h, the fluid viscosity and density, μ and ρ, respectively, and the velocity V of the fluid approaching the plate. Determine a suitable set of pi terms to study this problem experimentally.

Examples Cont.

Examples Cont.

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Examples Cont.

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Examples Cont.

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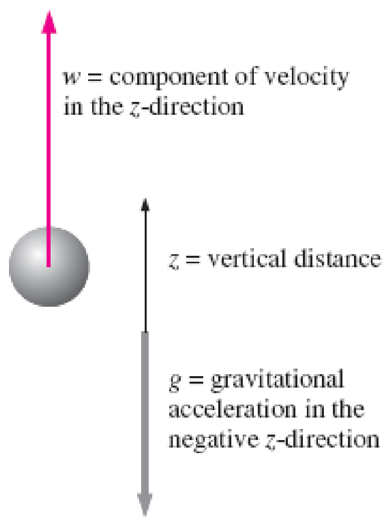
Examples

- Consider the equation of motion describing the elevation *z of an object falling by gravity through a vacuum (no air drag),*

The initial location of the object is z_0 and its initial velocity is w_0 in the z-direction.

Equation of motion: $$\frac{d^2z}{dt^2} = -g$$

Dimensional result: $$z = z_0 + w_0t - \frac{1}{2}gt^2$$



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Examples Cont.

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Examples Cont.

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Examples Cont.

Examples Cont.

Dimensionless Groups

A useful physical interpretation can often be given to dimensionless groups:

Some Common Variables and Dimensionless Groups in Fluid Mechanics

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid in important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

[a]The Cauchy number and the Mach number are related and either can be used as an index of the relative effects of inertia and compressibility. See accompanying discussion.

Dimensionless Groups

Reynolds Number: $\mathrm{Re} = \frac{\rho V \ell}{\mu}$

Osborne Reynolds
(1842 – 1912)

Osborne Reynolds, a British Engineer demonstrated that the Reynolds Number could be used as a criterion to distinguish laminar and turbulent flow.

Re << 1, Viscous forces dominate, we neglect inertial effects, creeping flows.

Re large, inertial effects dominate and we neglect viscosity (not turbulent though).

Froude Number: $\mathrm{Fr} = \frac{V}{\sqrt{g\ell}}$

William Froude
(1810 – 1879)

William Froude, a British civil engineer, mathematician, and naval architect who pioneered the use of towing tanks to study ship design.

The Froude number is the only dimensionless group that contains acceleration of gravity, thus indicating the weight of the fluid is important in these flows.

Important to flows that include waves around ships, flows through river or open conduits.

Dimensionless Groups

Leonhard Euler

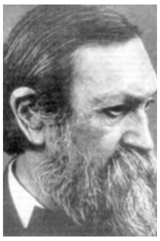
(1707 – 1783)

Euler Number: $\mathrm{Eu} = \dfrac{p}{\rho V^2}$

Leonhard Euler was a Swiss mathematician who pioneered the work between pressure and flow.

Ratio of pressure forces to inertial forces. Sometime called the pressure coefficient.

Euler number is used in flows where pressure differences may play a crucial role.



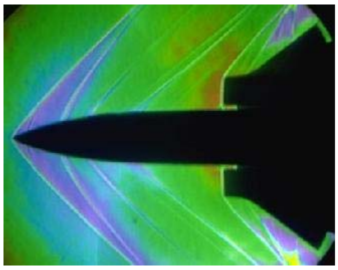
Ernst Mach

(1838 – 1916)

Mach Number: $\mathrm{Ma} = \dfrac{V}{c}$ c is the speed of sound

Ernst Mach as Austrian physicist and a philosopher.

The number is important in flows in which there is compressibility.



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Dimensionless Groups

Vincenz Strouhal

(1850 – 1922)

Theodor von Karman

(1881 – 1963)

Strouhal Number: $\mathrm{St} = \dfrac{\omega \ell}{V}$

Vincenz Strouhal studied "singing wires" which result from vortex shedding.

This dimensionless group is important in unsteady, oscillating flow problems with some frequency of oscillation ω.

Measure of unsteady inertial forces to steady inertial forces.

In certain Reynolds number ranges, a periodic flow will develop downstream from a cylinder placed in a moving fluid due to a regular pattern of vortices that are shed from the body.

This series of trailing vortices are known as Karman vortex trail named after Theodor von Karman, a famous fluid mechanician.

The oscillating flow is created a a discrete frequency such that Strouhaul numbers can closely be correlated to Reynolds numbers.

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Dimensionless Groups

1. *Similarity of molecular transport processes*

$$Pr = \nu/a = (\mu c_p/\lambda) = \text{Prandtl number}$$
$$Sc = \nu/D = \mu/(\rho D) = \text{Schmidt number}$$

2. *Similarity of flow processes*

$$Re = LU/\nu = \text{Reynolds number}$$
$$Fr = U^2 g/L = \text{Froude number}$$
$$Ma = U/c = \text{Mach number}$$
$$Eu = \Delta P/\rho U^2 = \text{Euler number}$$
$$St = Lf/U = \text{Strouhal number, } f \text{ is the frequency}$$
$$Gr = L^3 g \beta \rho^2 \Delta T/\mu^2 = \text{Grashof number}$$

3. *Similarity of heat transfer processes*

$$Pe = RePr = UL/a = \text{Peclet number}$$
$$E_c = U^2/c_p \Delta T = \text{Eckert number}$$

4. *Similarity of integral quantities of heat and mass transfer*

$$Nu = \alpha L/\lambda = \text{Nusselt number}$$
$$Sh = \beta L/D = \text{Sherwood number}$$

where α is introduced as heat transfer coefficient and β as mass transfer coefficient.

Dimensionless Groups

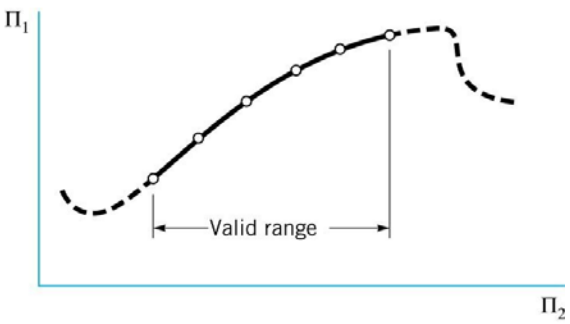
If only one pi variable exists in a fluid phenomenon, the functional relationship must be a constant.

$$\Pi_1 = C$$

The constant must be determine from experiment.

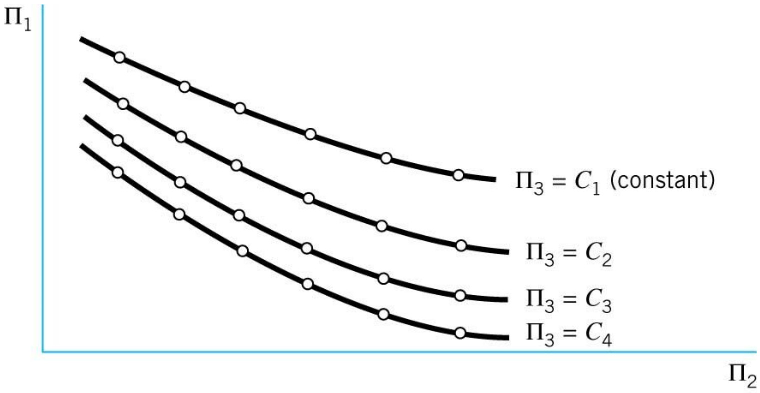
If we have two pi terms, we must be careful not to over extend the range of applicability, but the relationship can be presented pretty easily graphically:

$$\Pi_1 = \phi(\Pi_2)$$



Dimensionless Groups

If we have three pi groups, we can represent the data as a series of curves, however, as the number of pi terms increase the problem becomes less tractable, and we may resort to modeling specific characteristics.



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Similitude

Often we want to use models to predict real flow phenomenon.

We obtain similarity between a model and a prototype by equating pi terms. In these terms we must have geometric, kinematic, and dynamic similarity.

Geometric similarity: A model and a prototype are geometrically similar if and only if all body dimensions in all three coordinates have the same linear scale ratio.

$$\frac{\ell_1}{\ell_2} = \frac{\ell_{1m}}{\ell_{2m}} \Rightarrow \frac{\ell_{1m}}{\ell_1} = \frac{\ell_{2m}}{\ell_2}$$

All angles are preserved.

All flow directions are the same.

Orientations must be the same.

*Things that must be considered that are over-looked: roughness, scale of fasteners protruding.

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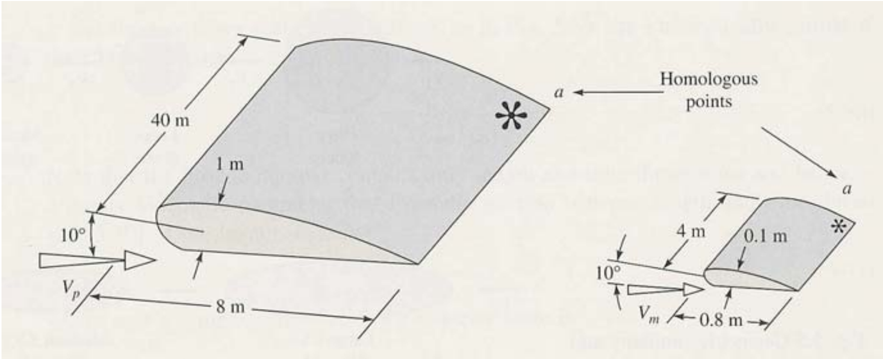
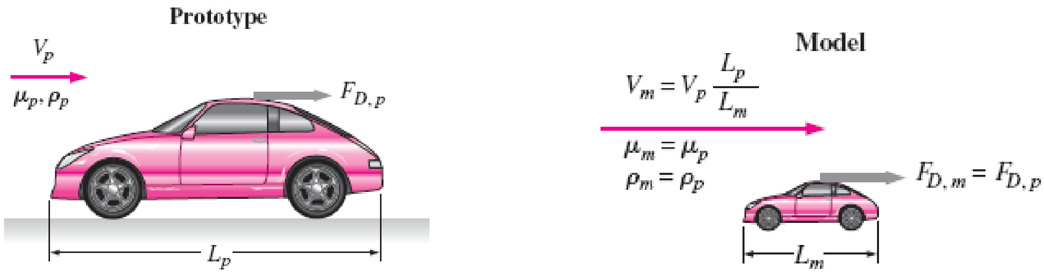
Similitude

Geometric Similarity: Scale 1/10th





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Similitude

Kinematic Similarity: Same length scale ratio and same time-scale ratio.

This requires equivalence of dimensionless groups:

Reynolds Number, Froude Number, Mach numbers, etc.

For a flow in which Froude Number and Reynolds Number is important:

Length scale:

Froude Number similarity: $$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}} \Rightarrow \frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\lambda_\ell}$$

Reynolds Number similarity: $$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu} \Rightarrow \frac{V_m}{V} = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{\ell}{\ell_m}$$

Time scale: $$\frac{t_m}{t} = \frac{l_m/V_m}{l/V} = \sqrt{\lambda_l}$$

Dynamic Similarity: the same length scale, time-scale, and force scale is required.

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Some Typical Model Studies

Flow Through Closed Conduits

- Include pipe flow and flow through valves, fittings, and metering devices. Although the conduits are often circular, they could have other shapes as well and may contain expansions or contractions.
- Since there are no fluid interfaces or free surfaces, the dominant forces are inertial and viscous.
- Geometric similarity between model and prototype must be maintained
- The Reynolds number is an important similarity parameter

$$\text{Dependent pi term} = \phi\left(\frac{\ell_i}{\ell}, \frac{\varepsilon}{\ell}, \frac{\rho V \ell}{\mu}\right)$$

the requirement of geometric similarity

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{\varepsilon_m}{\ell_m} = \frac{\varepsilon}{\ell} \qquad \text{or} \qquad \frac{\ell_{im}}{\ell_i} = \frac{\varepsilon_m}{\varepsilon} = \frac{\ell_m}{\ell} = \lambda_\ell$$

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Some Typical Model Studies

additional similarity requirement

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

- With these similarity requirements satisfied, it follows that the dependent pi term will be equal in model and prototype

$$\Pi_1 = \frac{\Delta p}{\rho V^2}$$

The prototype pressure drop would then be

$$\Delta p = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2 \Delta p_m$$

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Some Typical Model Studies

Flow Around Immersed Bodies

- Include flow around aircraft, automobiles, golf balls, and buildings.
- The dependent variable of interest for this type of problem is the drag developed on the body,
- Geometric and Reynolds number similarity is required.
- Since there are no fluid interfaces, surface tension and therefore the Weber number is **not** important.
- Also, gravity will not affect the flow patterns, so the Froude number need **not** be considered.

the dependent pi term would usually be

$$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho V^2 \ell^2}$$

C_D, *drag coefficient*,

Some Typical Model Studies

geometric similarity

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{\varepsilon_m}{\ell_m} = \frac{\varepsilon}{\ell}$$

Reynolds number similarity

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

If these conditions are met, then

$$\frac{\mathscr{D}}{\frac{1}{2}\rho V^2 \ell^2} = \frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 \ell_m^2}$$

or

$$\mathscr{D} = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{\ell}{\ell_m}\right)^2 \mathscr{D}_m$$

Some Typical Model Studies

Flow with a Free Surface

- Flows in canals, rivers, spillways, and stilling basins, as well as flow around ships, are all examples of flow phenomena involving a free surface.
- Both gravitational and inertial forces are important and, therefore, the Froude number becomes an important similarity parameter.
- Since there is a free surface with a liquid-air interface, forces due to surface tension may be significant, and
- The Weber number becomes another similarity parameter that needs to be considered along with the Reynolds number.
- Geometric variables will obviously still be important.

$$\text{Dependent pi term} = \phi\left(\frac{\ell_i}{\ell}, \frac{\varepsilon}{\ell}, \frac{\rho V \ell}{\mu}, \frac{V}{\sqrt{g\ell}}, \frac{\rho V^2 \ell}{\sigma}\right)$$

Some Typical Model Studies

Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

Reynolds number similarity

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

Weber number, We

$$\frac{\rho V^2 \ell}{\sigma} = \frac{\rho_m V_m^2 \ell_m}{\sigma_m}$$

geometric similarity

- In all previous cases, if Ma number is very high (compressible flow, then Ma similarity is necessary)

Example

The aerodynamic drag of a new sports car is to be predicted at a speed of 50.0 mi/h at an air temperature of 25°C. Automotive engineers build a one-fifth scale model of the car to test in a wind tunnel. It is winter and the wind tunnel is located in an unheated building; the temperature of the wind tunnel air is only about 5°C. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

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Examples Cont.

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Example

- Suppose the engineers run the wind tunnel at 221 mi/h to achieve similarity between the model and the prototype. The aerodynamic drag force on the model car is measured with a **drag balance. Several drag readings are recorded, and the average** drag force on the model is 21.2 lbf. Predict the aerodynamic drag force on the prototype (at 50 mi/h and 25°C).

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Example

A certain spillway for a dam is 20 m wide and is designed to carry 125 m^3/s at flood stage. A 1 : 15 model is constructed to study the flow characteristics through the spillway. Determine the required model width and flowrate. What operating time for the model corresponds to a 24-hr period in the prototype? The effects of surface tension and viscosity are to be neglected.

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Example Cont.

Example Cont.

Example

EXAMPLE 7–11 Model Lock and River

In the late 1990s the U.S. Army Corps of Engineers designed an experiment to model the flow of the Tennessee River downstream of the Kentucky Lock and Dam (Fig. 7–43). Because of laboratory space restrictions, they built a scale model with a length scale factor of $L_m/L_p = 1/100$. Suggest a liquid that would be appropriate for the experiment.

Properties For water at atmospheric pressure and at $T = 20°C$, the prototype kinematic viscosity is $\nu_p = 1.002 \times 10^{-6}$ m²/s.

Example Cont.

Governing Equation Insights

Consider the 2D governing equations:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2D Navier-Stokes Equations:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \rho g + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

For a given flow, there may be characteristic or reference length, velocity, pressure, and time scales:

V, a reference pressure, p_0, a reference length, ℓ, and a reference time, τ

We can use these reference parameter to non-dimensionlize our independent, and dependent variables: u, v, p, and x, y, t.

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Governing Equation Insights

Are non-dimensional variables are starred below:

$$u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad p^* = \frac{p}{p_0}$$

$$x^* = \frac{x}{\ell} \qquad y^* = \frac{y}{\ell} \qquad t^* = \frac{t}{\tau}$$

Now, we can introduce these variables into the governing equations:

For example: $$\frac{\partial u}{\partial x} = \frac{\partial V u^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{V}{\ell}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{V}{\ell}\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

Non-Dimensional Continuity: $$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

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Governing Equation Insights

Non-Dimensional Momentum:

x-momentum: $$\left[\frac{\rho V}{\tau}\right]\frac{\partial u^*}{\partial t^*}+\left[\frac{\rho V^2}{\ell}\right]\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=-\left[\frac{p_0}{\ell}\right]\frac{\partial p^*}{\partial x^*}+\left[\frac{\mu V}{\ell^2}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

y-momentum: $$\underbrace{\left[\frac{\rho V}{\tau}\right]}_{F_{I\ell}}\frac{\partial v^*}{\partial t^*}+\underbrace{\left[\frac{\rho V^2}{\ell}\right]}_{F_{Ic}}\left(u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right)$$
$$=-\underbrace{\left[\frac{p_0}{\ell}\right]}_{F_P}\frac{\partial p^*}{\partial y^*}-\underbrace{[\rho g]}_{F_G}+\underbrace{\left[\frac{\mu V}{\ell^2}\right]}_{F_V}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$F_{I\ell}$ = inertia (local) force
F_{Ic} = inertia (convective) force
F_p = pressure force
F_G = gravitational force

F_V = viscous force

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Governing Equation Insights

Non-Dimensional Momentum: Divide by $\rho V^2/\ell$

x-momentum: $$\left[\frac{\ell}{\tau V}\right]\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\left[\frac{p_0}{\rho V^2}\right]\frac{\partial p^*}{\partial x^*}+\left[\frac{\mu}{\rho V\ell}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

y-momentum: $$\left[\frac{\ell}{\tau V}\right]\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}$$
$$=-\left[\frac{p_0}{\rho V^2}\right]\frac{\partial p^*}{\partial y^*}-\left[\frac{g\ell}{V^2}\right]+\left[\frac{\mu}{\rho V\ell}\right]\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

Then, $\ell/\tau V$ is a form of Strouhal
$p_0/\rho V^2$ the Euler number
$g\ell/V^2$ the reciprocal of the square of the Froude number
$\mu/\rho V\ell$ the reciprocal of the Reynolds number

The governing equations then can provide insight into important dimensionless parameters.

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