Topic IV: Flow in Conduits and Energy Losses

Flow in pipes /conduits and energy losses: pressure drop; Reynolds visualization; laminar/ transition/turbulent flow in pipes; momentum boundary layer concept; Hagen-Poiseuille Equation; Darcy's Equation; Moody diagram and friction factor problems; energy losses due to fittings; hydraulic radius concept.

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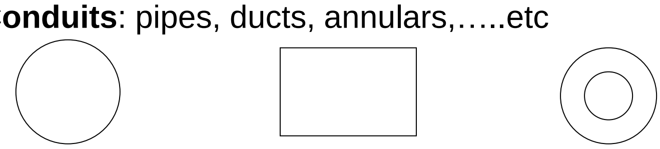
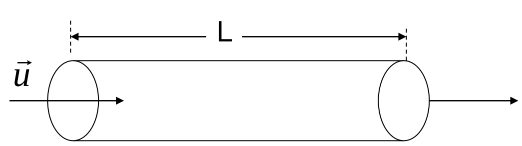
- **Conduits**: pipes, ducts, annulars,…..etc



"Pipe" "Rectangular duct" "Annular"

How to determine the energy losses due to friction for Flow in pipe of length L?



MEB: $g(z_2 - z_1) + \frac{1}{2}\left(\bar{u}_2^2 - \bar{u}_1^2\right) + \frac{P_2 - P_1}{\rho} = w_p - w_f$

$$\Rightarrow w_f = \frac{P_1 - P_2}{\rho}$$

$P_1 - P_2$: is pressure drop

$P_2 - P_1 = \Delta P$ is pressure difference

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Thus, if we know pressure drop (P_1-P_2) we can then calculate energy losses due to friction.

How to determine P_1-P_2?

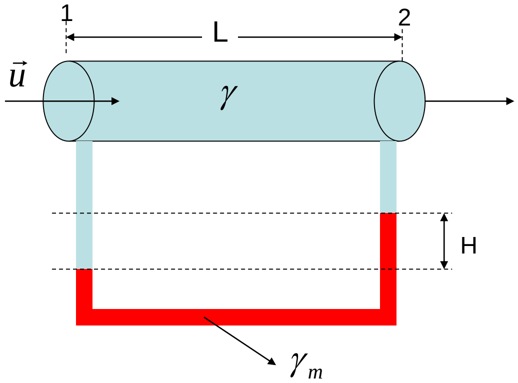
P_1-P_2 can be measured experimentally by using, for example, U-manometer:

Apply barometric Eq.:

$$P_1 + \gamma H - \gamma_m H = P_2$$

$$\Rightarrow P_1 - P_2 = H(\gamma_m - \gamma)$$

$$\therefore \quad W_f = \frac{H(\gamma_m - \gamma)}{\rho}$$



This experimental method is **impractical** for long pipes.

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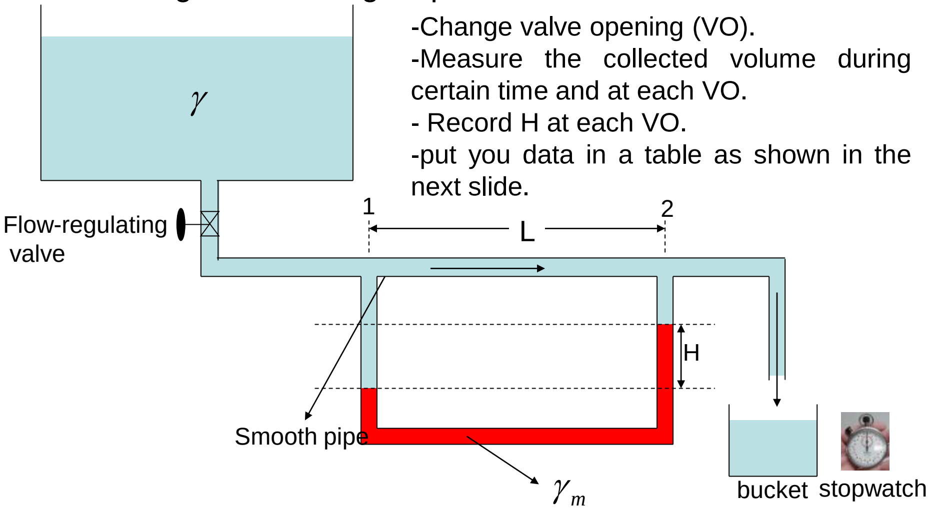
We need **an alternative way**. First let us understand the relation between pressure drop and flow rate by discussing the following experiment.

-Change valve opening (VO).

-Measure the collected volume during certain time and at each VO.

- Record H at each VO.

-put you data in a table as shown in the next slide.



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VO%	Collected volume; v	Time t	Q=V/t	H	$P_1 - P_2 = H(\gamma_m - \gamma)$
0	0	---	0	0	0
10	---	---	---	---	---
30	---	---	---	---	---
40	---	---	---	---	---
50	---	---	---	---	---
70	---	---	---	---	---
80	---	---	---	---	---
90	---	---	---	---	---
100	---	---	---	---	---

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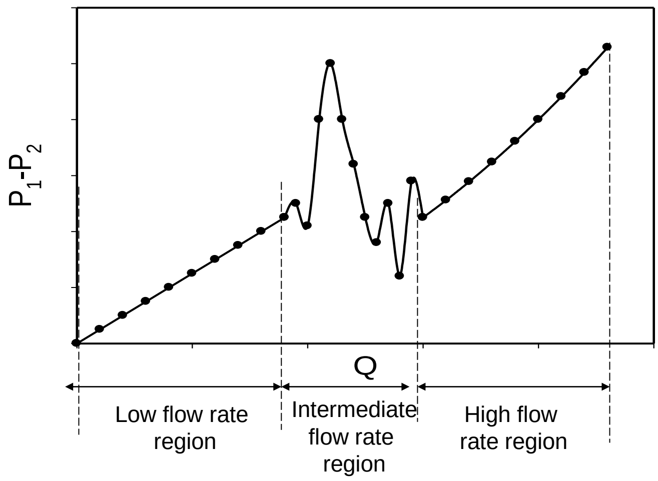
If you draw P_1-P_2 versus volumetric flow rate Q you will find a trends as shown in the figure below:



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It is clear from previous plot that:

1- **In the low flow-rate region**: Linear relationship between P_1-P_2 and Q.

2- **In the intermediate flow-rate region**: No clear trend.

3- **In the high flow-rate region**: power law relationship between P_1-P_2 and Q.

Why this strange behavior?!

In the year 1883, **Osborne Reynolds** explained this strange behavior. He performed flow visualization experiments using dye as shown in the next slide

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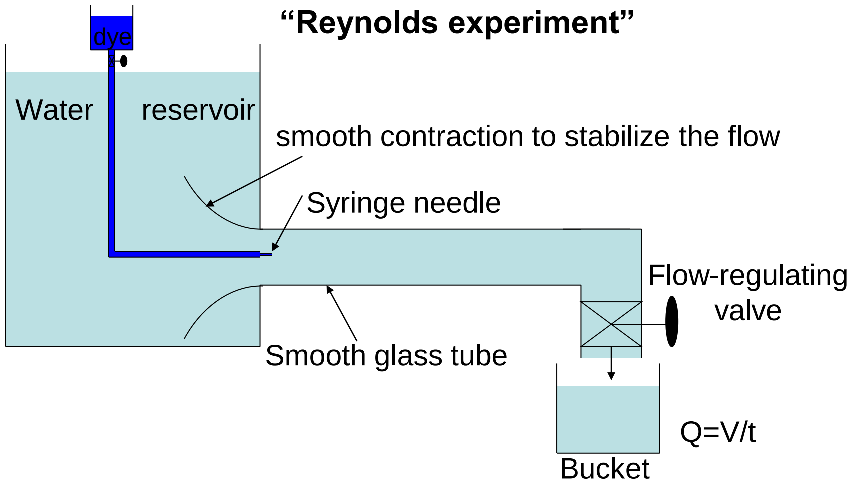
"Reynolds experiment"



Remark. Sharp edges are source of turbulence. Therefore, Reynolds used smooth contraction.

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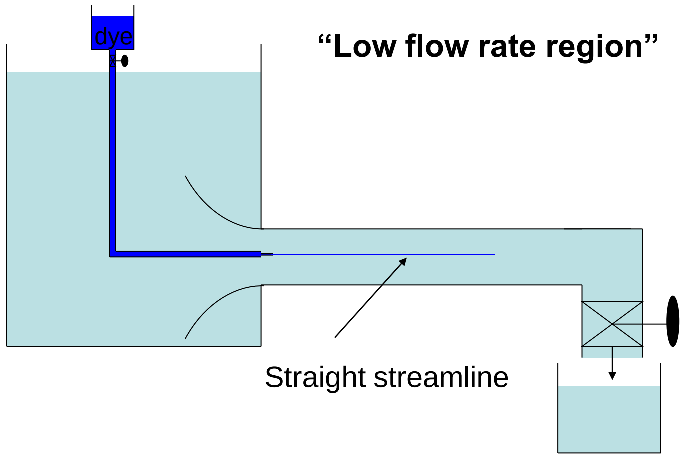

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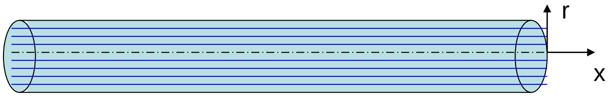
The low flow rate region is characterized by the following:

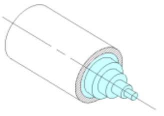
- Dye formed smooth and thin streak down the pipe.
- Streamline is straight.
- When dye is injected at different radial locations, it is found that all formed streamlines are straight and parallel to each other. This means that all motion is in the axial x-direction and thus:

$$u = u(r)$$

$$P = P(x)$$



-In this flow region, fluid moves in a thin shells or layers or **laminas**. Thus, the region is called **laminar flow region**.
-The flow is steady state.



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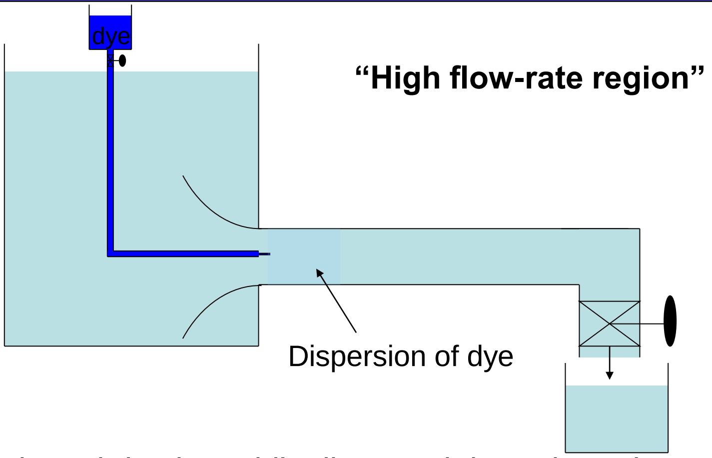

-The injected dye is rapidly dispersed throughout the entire flow filed. This means that there is rapid chaotic motion in all directions→ unsteady 3D flow → Good mixing.
- Since this region is characterized by good mixing, it is called **turbulent flow region.**

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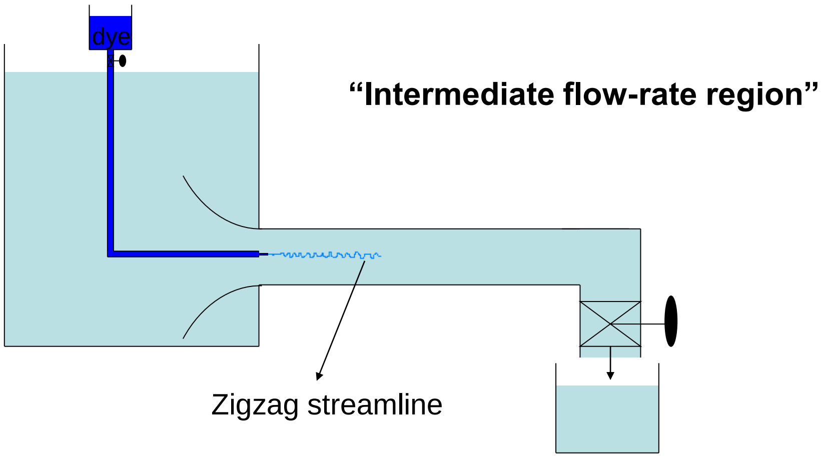

-The flow can be laminar but it is NOT stable. Any small disturbances or vibrations will switch the flow to be turbulent. Thus, this region is called **transition flow region**.
- Unsteady flow.

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-Reynolds found the most important dimensionless group in fluid mechanics which is called **Reynolds number (Re):**

$$\text{Re} = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

U : Characteristic velocity.

L : Characteristic Length.

ρ, μ, ν : Density, viscosity, and kinematic viscosity of the flowing fluid, respectively.

Exercise: Verify that Re is dimensionless quantity

- Note that Reynolds number depends on hydrodynamic conditions (U), geometry (L) and physical properties (ρ and μ).

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- For flow in pipe: $\text{Re} = \frac{\rho \bar{u} D}{\mu}$

Where D is the diameter of the pipe and $\bar{u}$ is the average velocity.

-Reynolds found that for flow of Newtonian and incompressible fluid in smooth pipe, when:

$\text{Re} = \frac{\rho \bar{u} D}{\mu} < 2000$: The flow is laminar

$2000 \leq \text{Re} < 4000$: The flow is transition

$\text{Re} \geq 4000$: The flow is turbulent

-The value of Re below which the flow switch from laminar to transition is called **critical Reynolds number** (Re_c).

For flow in pipe: $\text{Re}_c = 2000$

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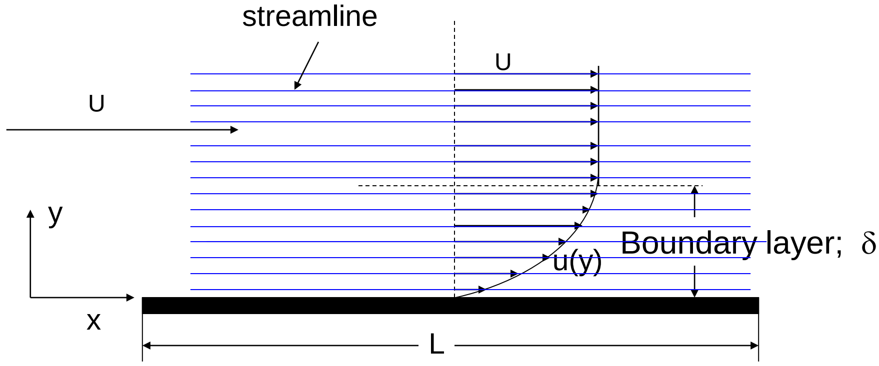

"Flow over flat plat"

For flow over flat plat, the flow is laminar If: $\mathrm{Re} = \dfrac{\rho U L}{\mu} \leq 500000$

$\Rightarrow \mathrm{Re}_c = 500000$

Momentum (velocity) Boundary layer δ: distance from the wall in which there is velocity gradient.

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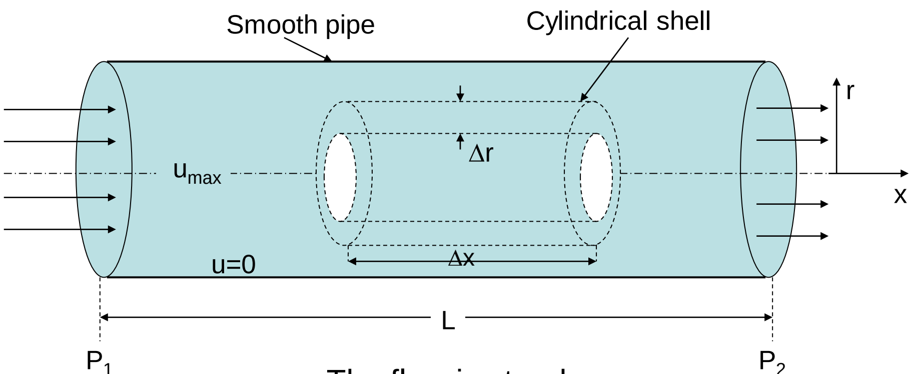
Laminar (Poiseuille) flow in pipe:



-The flow is steady
- u=u(r), p=p(x)
-Newtonian fluid
-Incompressible fluid

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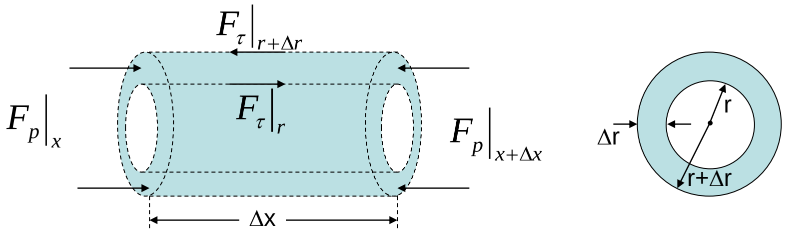
Steady state x-component force balance on the cylindrical shell:



"cylindrical shell"

x-Forces: Pressure forces F_p and shear stress forces F_τ :

$$F_p\big|_x - F_p\big|_{x+\Delta x} - F_\tau\big|_{r+\Delta r} + F_\tau\big|_r = 0$$

$$F_p\big|_x = PA\big|_x = 2\pi r\Delta r P\big|_x\ ; F_p\big|_x = PA\big|_{x+\Delta x} = 2\pi r\Delta r P\big|_{x+\Delta x}$$

$$F_\tau\big|_r = A\tau\big|_r = 2\pi r\Delta x\tau\big|_r\ ;\ F_\tau\big|_{r+\Delta r} = A\tau\big|_{r+\Delta r} = 2\pi(r+\Delta r)\Delta x\tau\big|_{r+\Delta r}$$

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$$2\pi r\Delta r P\big|_x - 2\pi r\Delta r P\big|_{x+\Delta x} - 2\pi(r+\Delta r)\Delta x\tau\big|_{r+\Delta r} + 2\pi r\Delta x\tau\big|_r = 0$$

Divide by 2π and rearrange the equation as:

$$-\left[(r+\Delta r)\Delta x\tau\big|_{r+\Delta r} - r\Delta x\tau\big|_r\right] = r\Delta r P\big|_{x+\Delta x} - r\Delta r P\big|_x$$

Divide by $\Delta r\Delta x$ and rearrange Eq. as:

$$-\left[\frac{(r+\Delta r)\tau\big|_{r+\Delta r} - r\tau\big|_r}{\Delta r}\right] = r\frac{P\big|_{x+\Delta x} - P\big|_x}{\Delta x}$$

Take limit when $\Delta x \to 0; \Delta r \to 0$: $-\dfrac{1}{r}\dfrac{d(r\tau)}{dr} = \dfrac{dP(x)}{dx}$

For Couette flow: $\tau = \mu\, du(y)/dy$ "for dy=+ve du=+ve → τ=+ve

For flow in pipe: $\tau = -\mu\, du(r)/dr$ "for dr=+ve du=-ve → τ=+ve

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$$\frac{\mu}{r}\frac{d\left(r\frac{du(r)}{dr}\right)}{dr}=\frac{dP(x)}{dx}$$

$$Or$$

$$\frac{1}{r}\frac{d\left(r\frac{du(r)}{dr}\right)}{dr}=\frac{1}{\mu}\frac{dP(x)}{dx}$$

Since the left hand side is function of r only and the right hand side is function of x only:

$$\frac{1}{\mu}\frac{dP(x)}{dx}=cons\tan t=C \Rightarrow \int_{x_1}^{x_2}\mu C dx=\int_{P_1}^{P_2}dP$$

$$\therefore \mu C(x_2-x_1)=P_2-P_1 \Rightarrow C=\frac{P_2-P_1}{\mu L}$$

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$$\frac{d\left(r\frac{du(r)}{dr}\right)}{dr}=\left(\frac{P_2-P_1}{\mu L}\right)r$$

Integrate the above Eq.: $r\frac{du(r)}{dr}=\left(\frac{P_2-P_1}{\mu L}\right)\frac{r^2}{2}+c_1$

Divide by r: $\frac{du(r)}{dr}=\left(\frac{P_2-P_1}{\mu L}\right)\frac{r}{2}+\frac{c_1}{r}$

Integrate the above Eq.: $u(r)=\left(\frac{P_2-P_1}{\mu L}\right)\frac{r^2}{4}+c_1\ln r+c_2$

Apply the following boundary conditions to find the constants c_1 and c_2: At r=0: u=u_{max} ; du/dr=0

At r=R=D/2: u=0

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At r=0 the velocity must have limited value, u_{max}, thus c_1 must be zero to avoid $\ln(0) = -\infty$

Thus, the profile becomes: $$u(r) = \left(\frac{P_2 - P_1}{\mu L}\right)\frac{r^2}{4} + c_2$$

$$\text{At } r = R = D/2 : u = 0$$

$$0 = \left(\frac{P_2 - P_1}{\mu L}\right)\frac{R^2}{4} + c_2 \Rightarrow c_2 = -\left(\frac{P_2 - P_1}{\mu L}\right)\frac{R^2}{4}$$

$$\Rightarrow u(r) = \left(\frac{P_2 - P_1}{\mu L}\right)\frac{r^2}{4} - \left(\frac{P_2 - P_1}{\mu L}\right)\frac{R^2}{4}$$

Or: $$u(r) = R^2\left(\frac{P_1 - P_2}{4\mu L}\right)\left(1 - \left(\frac{r}{R}\right)^2\right)$$

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$$\text{At } r = 0 : u = u_{max}$$

$$u = u_{max} = R^2\left(\frac{P_1 - P_2}{4\mu L}\right)\left(1 - \left(\frac{0}{R}\right)^2\right) \Rightarrow \boxed{u_{max} = R^2\left(\frac{P_1 - P_2}{4\mu L}\right)}$$

$$\boxed{u = u_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)}$$

$\Rightarrow$ Laminar flow has **parabolic velocity profile**

To find the volumetric flow rate for laminar flow:

$$Q = \int u(r)\,dA = \int_0^R u(r)2\pi r\,dr = \int_0^R 2\pi u_{max}\left(1 - \frac{r^2}{R^2}\right) r\,dr$$

$$= 2\pi u_{max}\int_0^R \left(1 - \frac{r^2}{R^2}\right) r\,dr = \pi R^2\,\frac{u_{max}}{2}$$

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$$Q = \pi R^2 \frac{u_{max}}{2}$$

But we know that: $Q = A\bar{u} = \pi R^2 \bar{u}$

Thus: $\boxed{\bar{u} = \frac{u_{max}}{2}}$

$$Q = \pi R^2 \bar{u} = \pi R^2 R^2 \left(\frac{P_1 - P_2}{4\mu L}\right) = \pi \left(\frac{D}{2}\right)^4 \left(\frac{P_1 - P_2}{4\mu L}\right)$$

$$\Rightarrow \boxed{Q = \frac{\pi D^4 (P_1 - P_2)}{128\mu L}}$$

Hagen-Poiseuille Eq.

Hagen-Poiseuille Eq. has importance in fluid mechanics. It shows that the pressure drop is linearly proportional to the volumetric flow rate as explained by Reynolds.

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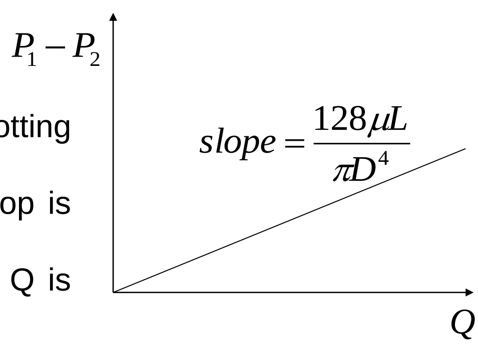
From Hagen-Poiseuille Eq.: $P_1 - P_2 = \frac{128\mu L}{\pi D^4} Q$

The equation can be used:

-To measure viscosity by plotting pressure drop versus Q.

-To determine Q if pressure drop is measured.

-To determine pressure drop if Q is measured.



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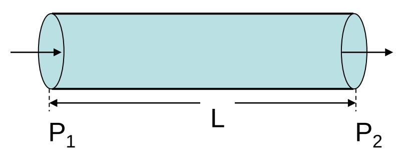
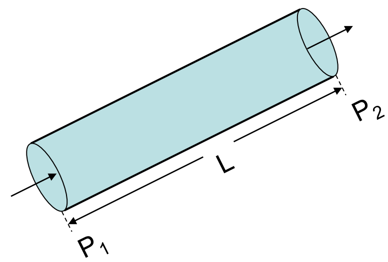
Now energy losses for **laminar flow in smooth** pipe can be determined by applying MEB:



$$g(z_2 - z_1) + \frac{1}{2\alpha}\left(\bar{u}_2^2 - \bar{u}_1^2\right) + \frac{P_2 - P_1}{\rho} = w_p - w_f$$

$$\Rightarrow w_f = \frac{P_1 - P_2}{\rho}$$

$$\therefore \boxed{w_f = \frac{128\mu L}{\pi D^4 \rho} Q}$$



- The equation is also applicable for laminar flow in vertical (or inclined) pipe. **Verify that!**

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- Turbulent flow in pipe (Re≥4000):

Random fluctuation of each velocity component in time and all directions.

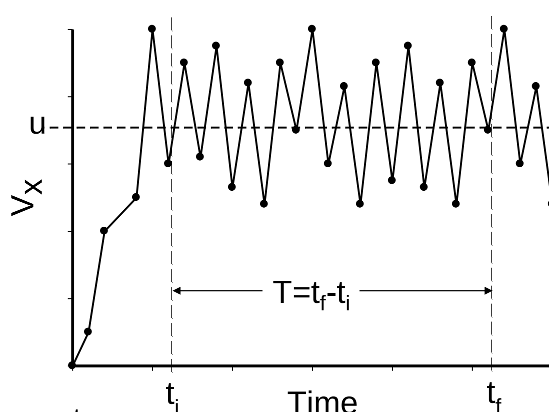
-Unsteady flow.

- Good mixing.

Velocity is averaged in time:

$$u = \frac{\int_{t_i}^{t_f} V_x(t)\,dt}{T}$$

V_x : Axial velocity component



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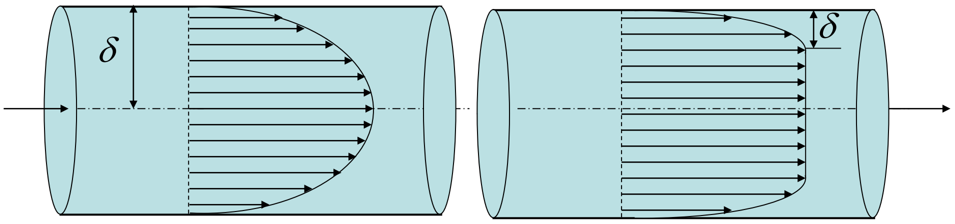
The time-averaged velocity profile for turbulent flow is:

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/7}$$

"Empirical equation obtained from experimental measurements"



Laminar flow has **parabolic** velocity profile:

$$u(r) = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Turbulent flow has **flat** velocity profile:

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/7}$$

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- Why in turbulent flow the velocity has flat velocity profile?

Turbulence means good mixing and thus a uniform values of temperature, concentration, and velocity. Therefore the velocity has a u_{max} value along the centerline and in the adjacent zone. Moreover, the velocity must drop to zero value at the wall to have a profile but with small δ .

- The thickness of momentum boundary δ for turbulent flow is smaller than that for laminar flow. Since momentum boundary layer represents a resistance to momentum transfer, turbulent flow transfers more momentum than laminar flow.
- Turbulent flows lead to high transfer rates of mass of species, heat, and momentum.

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- What is the relation between u_{max} and $\bar{u}$ for turbulent flow?

$$Q = A\bar{u} = \pi R^2\bar{u} = \int u(r)dA = \int_0^R u(r)2\pi r dr = \int_0^R 2\pi u_{max}\left(1-\frac{r}{R}\right)^{1/7} r dr$$

After integration:

$$\pi R^2\bar{u} = \pi R^2 0.8u_{max}$$

$$\therefore \quad \boxed{\bar{u} = 0.8u_{max}}$$

The value of average velocity for turbulent flow is more closed to u_{max} than that for laminar flow ($\bar{u} = 0.5u_{max}$).

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-Prove that the correction factor used in the kinetic energy term of MEB is $\alpha = 0.5$ for laminar and $\alpha = 1.0$ flow turbulent flow.

MEB: $$g(z_2 - z_1) + \boxed{\frac{1}{2\alpha}\left(\bar{u}_2^2 - \bar{u}_1^2\right)} + \frac{P_2 - P_1}{\rho} = w_p - w_f$$

$$\boxed{\left(\frac{\dot{E}_k}{\dot{m}}\right)_2 - \left(\frac{\dot{E}_k}{\dot{m}}\right)_1}$$

$$d\dot{E}_k = 0.5u^2 d\dot{m}$$

$$d\dot{m} = \rho dQ = \rho u(r)dA = \rho 2\pi r u(r)dr$$

$$d\dot{E}_k = (0.5)(2\pi)\rho r u^3(r)dr$$

$$\dot{E}_k = \pi\rho\int_0^R r u^3(r)dr$$

<u>Laminar flow:</u>

$$\dot{E}_k = \pi\rho\int_0^R r u^3(r)dr = \pi\rho\int_0^R r\left(u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\right)^3 dr = \frac{\rho\pi R^2 u_{max}^3}{8}$$

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$$\dot{m} = \rho Q = \rho\pi R^2 \bar{u} = \rho\pi R^2 (u_{max}/2)$$

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\rho\pi R^2 u_{max}^{\ 3}/8}{\rho\pi R^2 (u_{max}/2)} = \frac{u_{max}^{\ 2}}{4} = \left(\frac{u_{max}}{2}\right)^2 = \bar{u}^2$$

$$\left(\frac{\dot{E}_k}{\dot{m}}\right)_2 - \left(\frac{\dot{E}_k}{\dot{m}}\right)_1 = \bar{u}_2^2 - \bar{u}_1^2 = \frac{1}{2\alpha}\left(\bar{u}_2^2 - \bar{u}_1^2\right) \Rightarrow \alpha = \frac{1}{2}$$

Turbulent flow:

$$\dot{E}_k = \pi\rho\int_0^R r u^3(r)\,dr = \pi\rho\int_0^R r\left(u_{max}\left[1-\left(\frac{r}{R}\right)\right]^{1/7}\right)^3 dr = \frac{\rho\pi R^2 u_{max}^{\ 3}}{4}$$

$$\dot{m} = \rho Q = \rho\pi R^2 \bar{u} = \rho\pi R^2 (0.8u_{max})$$

$$\frac{\dot{E}_k}{\dot{m}} = \frac{\rho\pi R^2 (0.8u_{max})^3/2}{\rho\pi R^2 (0.8u_{max})} = \frac{(0.8u_{max})^2}{2} = \frac{\bar{u}^2}{2}$$

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$$\left(\frac{\dot{E}_k}{\dot{m}}\right)_2 - \left(\frac{\dot{E}_k}{\dot{m}}\right)_1 = \frac{\bar{u}_2^2}{2} - \frac{\bar{u}_1^2}{2} = \frac{1}{2\alpha}\left(\bar{u}_2^2 - \bar{u}_1^2\right) \Rightarrow \alpha = 1$$

Comparison between laminar and turbulent flows:

Laminar flow	**Turbulent flow**
Steady flow	Unsteady flow
Straight and parallel streamlines	Random motion in all direction
Parabolic velocity profile	Flat velocity profile
Large boundary layer	Thin boundary layer
Low transfer rates	High transfer rates
$\alpha = 0.5$	$\alpha = 1.0$
$\bar{u} = 0.5u_{max}$	$\bar{u} = 0.8u_{max}$

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How to determine the energy losses w_f for turbulent flow?

In general, for both laminar and turbulent flows:

$$w_f = fun(\bar{u}, L, D, \mu, \rho, \varepsilon)$$

ε :**Roughness of the pipe**

Darcy found that w_f is directly proportional to the square of average velocity $\bar{u}^2$ and length of the pipe L and inversely proportional to the radius of the pipe diameter R:

$$w_f \;\alpha\; \frac{L\bar{u}^2}{R} \qquad \Rightarrow w_f = f\,\frac{L\bar{u}^2}{R}$$

$$w_f = 4f\,\frac{L}{D}\frac{\bar{u}^2}{2} \quad \text{Or} \quad h_f = 4f\,\frac{L}{D}\frac{\bar{u}^2}{2g}$$ **"Darcy's Equation"**

Where f is the proportionality constant which is **friction factor** and h_f is the head loss due to friction.

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- Effects of fluid density and viscosity and roughness of the pipe are all included through friction factor as we will see below.
- Blasius and Stanton found that as Reynolds number, Re, increases, friction factor, f, decreases. Obviously, as pipe roughness increases, friction factor increases. Hence:

$$f = fun(\mathrm{Re} = \frac{\rho\bar{u}D}{\mu}, \frac{\varepsilon}{D})$$

Where ε / D is the relative roughness

- Roughness depends on the material of construction of the pipe.

- One of the most widely used methods for evaluating the friction factor employs the Moody diagram.

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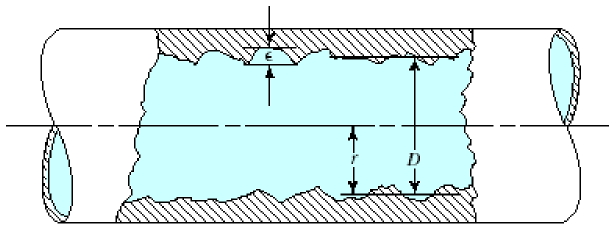
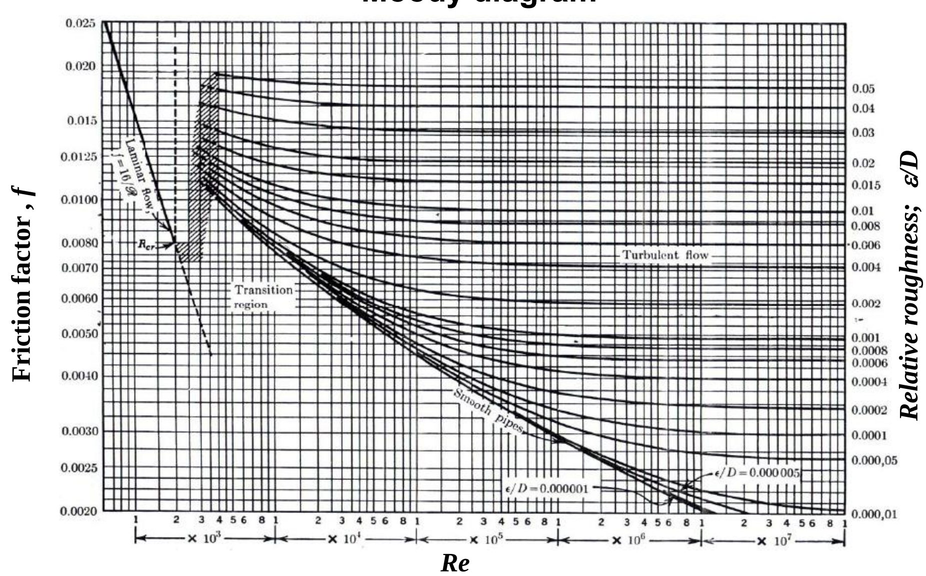


Material	Roughness ε (m)	Roughness ε (ft)
Glass	Smooth	Smooth
Plastic	3.0×10^{-7}	1.0×10^{-6}
Drawn tubing; copper, brass, steel	1.5×10^{-6}	5.0×10^{-6}
Steel, commercial or welded	4.6×10^{-5}	1.5×10^{-4}
Galvanized iron	1.5×10^{-4}	5.0×10^{-4}
Ductile iron—coated	1.2×10^{-4}	4.0×10^{-4}
Ductile iron—uncoated	2.4×10^{-4}	8.0×10^{-4}
Concrete, well made	1.2×10^{-4}	4.0×10^{-4}
Riveted steel	1.8×10^{-3}	6.0×10^{-3}

Smooth pipe: → $\varepsilon=0$

Topic IV: Flow in Conduits and Energy Losses

“Moody diagram”



Topic IV: Flow in Conduits and Energy Losses

- Friction factor can be evaluated using Moody diagram or using the following **Colebrook Eq.**:

$$\frac{1}{\sqrt{f}} = -4\log\left(\frac{\varepsilon/D}{3.7} + \frac{1.225}{\mathrm{Re}\sqrt{f}}\right)$$

Special case: laminar flow

$$w_f = \frac{128\mu L}{\pi D^4 \rho} Q = 4f\frac{L}{D}\frac{\bar{u}^2}{2} \qquad Q = A\bar{u} = \frac{\pi D^2}{4}\bar{u}$$

$$\frac{128\mu L}{\pi D^4 \rho}\frac{\pi D^2}{4}\bar{u} = 4f\frac{L}{D}\frac{\bar{u}^2}{2} \Rightarrow 16\frac{\mu}{\rho D} = f\bar{u}$$

$$f = \frac{16}{\mathrm{Re}}$$ **" For Laminar flow: Re < and ε=0"**

Topic IV: Flow in Conduits and Energy Losses

- Three common friction factor problems:

$$w_f = fun(\bar{u}\ or\ Q, L, D, \mu, \rho, \varepsilon)$$

Type	Given	To find	Solution method
1	$\bar{u}\ or\ Q, L, D, \mu, \rho, \varepsilon$	w_f	Direct
2	$w_f, L, D, \mu, \rho, \varepsilon$	$\bar{u}\ or\ Q$	Trial-and-error (It can be direct)
3	$w_f, \bar{u}\ or\ Q, L, \mu, \rho, \varepsilon$	D	Trial-and-error

How **Type 2** can be direct? Use Colebrook Eq. together with Darcy Eq. and Definition of Reynolds number to find:

$$\bar{u} = -\frac{8.8856}{\pi}\sqrt{\frac{Dw_f}{L}}\log\left(\frac{\varepsilon/D}{3.7} + \frac{1.7324\mu/\rho}{D\sqrt{Dw_f/L}}\right)$$ **Verify that!**

$$Q = \pi D^2\bar{u}/4$$

Topic IV: Flow in Conduits and Energy Losses

Example. Determine energy loss due to friction and pressure drop for a flow of a Newtonian fluid (μ= 0.09 kg/m.s and ρ=750 kg/m^3) through glass pipe of a length of 10 m and 100-mm-diameter in the following two cases:

(a)Volumetric flow rate is 14 L/s
(b) Volumetric flow rate is 80 L/s

***w_f =?*→Type 1 problem**

$\nu = \mu/\rho$ =1.2 $\times 10^{-4}$ m^2/s ; D=0.1m ; L=10 m.
Glass has smooth surface → ε=0

(a)Volumetric flow rate is 14 L/s: $Q = 0.014\,\text{m}^3/\text{s}$

$$\bar{u} = Q/A = 4Q/\pi D^2 = 1.782\,\text{m/s}$$

$\text{Re} = \bar{u}D/\nu = 1485$ Laminar flow: ε=0 and Re<2000

For laminar flow: $f = \dfrac{16}{\text{Re}} = 0.0108$

Topic IV: Flow in Conduits and Energy Losses

$$w_f = 4f\frac{L}{D}\frac{\bar{u}^2}{2} = 6.86\,\text{J/kg}$$

the head loss is: $h_f = \dfrac{w_f}{g} = 0.146\,\text{m}$

Or use: $w_f = 128\mu LQ/(\pi D^4\rho) = 6.86\,\text{J/kg}$

Apply MEB to find pressure drop across the pipe:

$$\frac{P_2 - P_1}{\rho} = -w_f \Rightarrow P_1 - P_2 = \rho w_f = 5145\,\text{Pa}$$

(b) Volumetric flow rate is 80 L/s: $Q = 0.08\,\text{m}^3/\text{s}$

$$\bar{u} = Q/A = 4Q/\pi D^2 = 10.186\,\text{m/s}$$

$\text{Re} = \bar{u}D/\nu = 8.5\times 10^3$ Turbulent flow: Re>4000

$$\left.\begin{array}{l}\text{Re} = 8.5\times 10^3 \\ \varepsilon/D = 0\,(smooth)\end{array}\right\rangle from\ Moody\ diagram:\ f = 0.008$$

$$w_f = 4f\frac{L}{D}\frac{\bar{u}^2}{2} = 166\,\text{J/kg} \qquad h_f = w_f/g = 16.9\,\text{m}$$

$$P_1 - P_2 = \rho w_f = 124.5\,\text{kPa}$$

Topic IV: Flow in Conduits and Energy Losses

Example. Determine energy losses and pressure drop for a flow of 140 L/s of oil (μ= 0.008 kg/m.s and ρ=800 kg/m³ through ductile iron-uncoated pipe with a length of 400 m and 200-mm-diameter. w_f =?→**Type 1 problem**

Q=140 L/s=0.14 m³/s; ν= μ/ρ =0.00001 m²/s; D=0.2m; L=400 m From a previous table ductile iron-uncoated has a roughness of $\varepsilon=2.4\times10^{-4}$ m.

$$\bar{u} = Q/A = 4Q/\pi D^2 = 4.456\,\text{m/s}$$

$$\left.\begin{array}{l} \text{Re} = \bar{u}D/\nu = 8.9\times10^4 \\ \varepsilon/D = 0.0012 \end{array}\right\rangle \textit{from Moody diagram}: f = 0.0058$$

Turbulent flow: ε≠0 and Re>4000

Topic IV: Flow in Conduits and Energy Losses

$$w_f = 4f\frac{L}{D}\frac{\bar{u}^2}{2} = 460.7\,\text{J/kg}$$

and the head loss is:

$$h_f = \frac{w_f}{g} = 47\,\text{m}$$

Apply MEB to find pressure drop across the pipe:

$$\frac{P_2 - P_1}{\rho} = -w_f \Rightarrow P_1 - P_2 = \rho w_f = 369\,\text{kPa}$$

Example. Water at 15 °C flows through 300-mm diameter riveted steel pipe with head loss of 6 m cross a length of 300 m. Find the volumetric flow rate.

Q=?→ Type 2 problem

From physical properties table of water at 15 °C: $\nu=1.13\times10^{-6}$ m²/s. From previous table riveted steel has $\varepsilon=1.8\times10^{-3}$ m.

D=0.3 m; L=300 m; h_f=6 m

Topic IV: Flow in Conduits and Energy Losses

Trial-and-error method: Use Darcy Eq. to relate $\bar{u}$ with f:

$$h_f = 4f\frac{L}{D}\frac{\bar{u}^2}{2g}$$

$$\therefore \bar{u} = \sqrt{\frac{h_f gD}{2fL}} = \frac{0.17155}{\sqrt{f}}$$

Take trial f value from moody diagram: $f = 0.01$

Find $\bar{u} = \dfrac{0.17155}{\sqrt{0.01}} = 1.716\,\text{m/s}$

Find : $\text{Re} = \dfrac{\bar{u}D}{\nu} = 4.6\times10^5$, $\varepsilon / D = 0.006$ $\Big\rangle$ $\textit{from Moody diagram}: f = 0.008$

Topic IV: Flow in Conduits and Energy Losses

Recalculate : $\bar{u} = \dfrac{0.17155}{\sqrt{0.008}} = 1.918\,\text{m/s}$

Recalculate : $\text{Re} = \dfrac{\bar{u}D}{\nu} = 5.1\times10^5$, $\varepsilon / D = 0.006$ $\Big\rangle$ $\textit{from Moody diagram}: f = 0.008$

Since this f value is similar to the previous one, stop iterations:

$$\therefore \bar{u} = 1.918 \Rightarrow Q = \bar{u}\frac{\pi D^2}{4} = 0.136\,\text{m}^3\text{/s} = 136\ \text{L/s}$$

Direct method for type 2:

$$\bar{u} = -\frac{8.8856}{\pi}\sqrt{\frac{Dw_f}{L}}\log\left(\frac{\varepsilon/D}{3.7} + \frac{1.7324\mu/\rho}{D\sqrt{Dw_f/L}}\right) = 1.910$$

$$Q = \pi D^2\bar{u}/4 = 0.135\,\text{m}^3\text{/s} = 135\,\text{L/s}$$

Topic IV: Flow in Conduits and Energy Losses

Example. Determine the diameter of commercial steel pipe required to convey 4000 gpm of oil ($\nu=1\times10^{-4}$ ft²/s) for a length of 10000 ft with head loss of 75 ft.

D=?→ Type 3 problem

From previous table, commercial steel has $\varepsilon=1.5\times10^{-4}$ ft.
L=10000 ft; h_f=75 ft

$$Q = 4000\frac{\text{gal}}{\text{min}}\frac{1\,\text{ft}^3}{7.48\,\text{gal}}\frac{1\,\text{min}}{60\,\text{s}} = 8.913\,\text{ft}^3/\text{s}$$

The strategy for solving type 3 problem is to relate the unknown D with f through Darcy Eq.

$$h_f = 4f\frac{L}{D}\frac{\bar{u}^2}{2g} = 4f\frac{L}{D}\frac{\left(4Q/\pi D^2\right)^2}{2g}$$

$$\therefore D = \sqrt[5]{\frac{32\,fLQ^2}{\pi^2 h_f g}} = 4.033\sqrt[5]{f}$$

Topic IV: Flow in Conduits and Energy Losses

Take trial f value from moody diagram: f = 0.005

Find : $D = 4.033\sqrt[5]{f} = 1.398\ ft$

Find : $\bar{u} = 4Q/\pi D^2 = 5.806\,\text{ft/s}$

Find : $\left.\begin{array}{l}\text{Re} = \dfrac{\bar{u}D}{\nu} = 8.1\times10^4 \\ \varepsilon/D = 0.00011\end{array}\right\rangle \textit{from Moody diagram}: f = 0.0049$

Recalculated : $D = 4.033\sqrt[5]{f} = 1.392\ ft$

Recalculate : $\bar{u} = 4Q/\pi D^2 = 5.857\,\text{ft/s}$

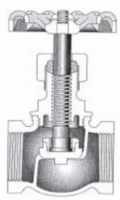
Recalculate :

$$\left.\begin{array}{l}\text{Re} = \dfrac{\bar{u}D}{\nu} = 8.2\times10^4 \\ \varepsilon/D = 0.00011\end{array}\right\rangle \begin{array}{l}\textit{from Moody diagram}: f = 0.0049 \\ \Rightarrow D = 1.392\,\text{ft}\end{array}$$

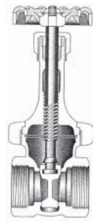
Topic IV: Flow in Conduits and Energy Losses

Energy losses due to fittings:

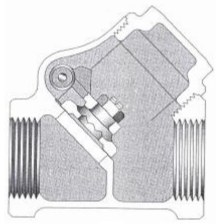
Fittings: elbow, valve, contraction, expansion, joint, bend,…etc



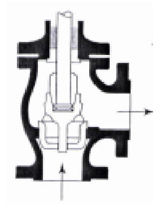
“Globe valve”



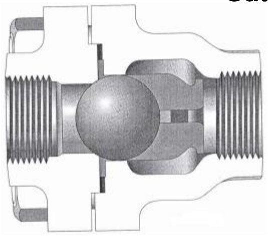
“Gate valve”



“Check valve-swing type”

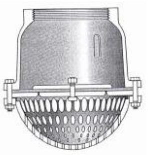


“Angle valve”



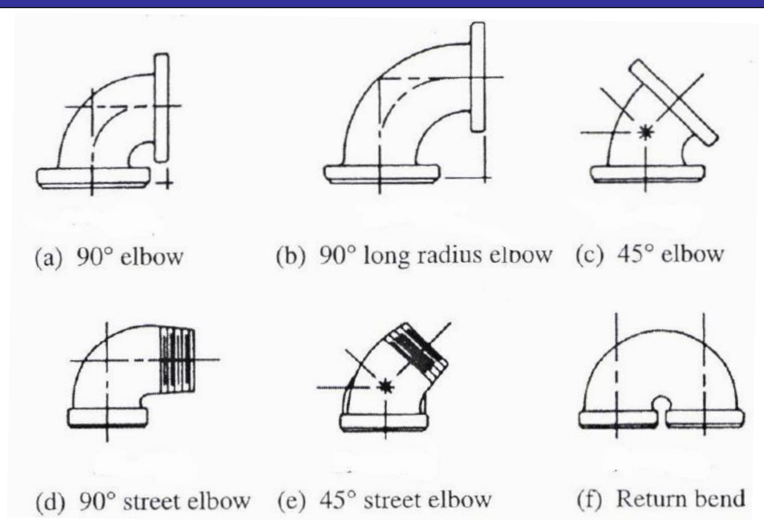
“Check valve-ball type”

“Butterfly valve”

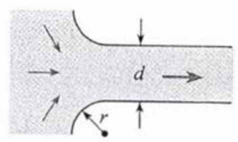


“Foot valve with strainer-hinged disc”

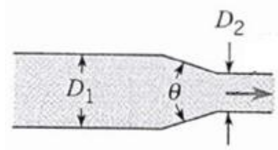
Topic IV: Flow in Conduits and Energy Losses



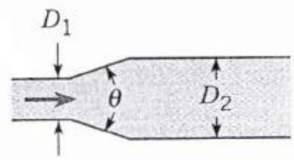
Topic IV: Flow in Conduits and Energy Losses



"Pipe entrance"

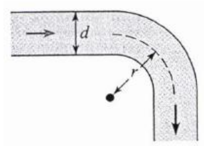


"Contraction"

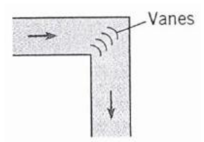


"Expansion"

Sudden expansion/contraction when θ=180°



"90° smooth bend"



"90° miter bend"

There are other fittings in the market……

Topic IV: Flow in Conduits and Energy Losses

These fittings dissipate energy. The energy loss due to fitting is expressed into two ways:

A- Loss (resistance) coefficient method:

$$\left(w_f\right)_{fitting} = K\frac{\overline{u}^2}{2} \quad or \quad \left(h_f\right)_{fitting} = K\frac{\overline{u}^2}{2g}$$

Where $\left(w_f\right)_{fitting}$: Energy loss due to fitting

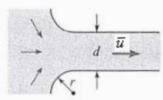
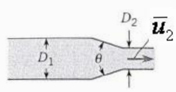
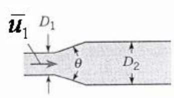
$\left(h_f\right)_{fitting}$: Head loss due to fitting

K : Loss (resistance) coefficient of fitting

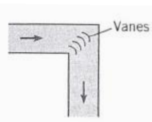
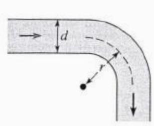
$\overline{u}$: Largest average velocity (inlet average velocity or outlet average velocity, depending on its value.

Topic IV: Flow in Conduits and Energy Losses

LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

Description	Sketch	Additional Data		K
Pipe entrance $h_f = K_e \frac{\bar{u}^2}{2g}$		r/d 0.0 0.1 >0.2		K_e 0.50 0.12 0.03
Contraction $h_f = K_C \frac{\bar{u}_2^2}{2g}$		D_2/D_1 0.0 0.20 0.40 0.60 0.80 0.90	K_C $u = 60°$ 0.08 0.08 0.07 0.06 0.06 0.06	K_C $u = 180°$ 0.50 0.49 0.42 0.27 0.20 0.10
Expansion $h_f = K_E \frac{\bar{u}_1^2}{2g}$		D_1/D_2 0.0 0.20 0.40 0.60 0.80	K_E $u = 20°$ 0.30 0.25 0.15 0.10	K_E $u = 180°$ 1.00 0.92 0.70 0.41 0.15

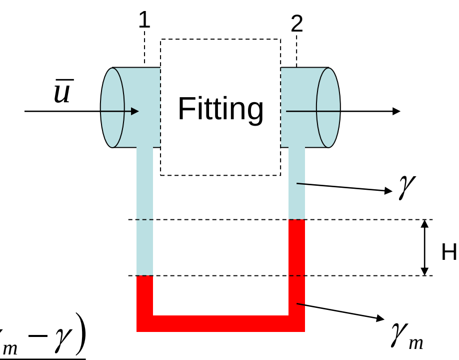
Topic IV: Flow in Conduits and Energy Losses

Description	Sketch	Additional Data	K
90° miter bend $h_f = K_b \frac{\bar{u}^2}{2g}$		Without vanes With vanes	$K_b = 1.1$ $K_b = 0.2$
90° smooth bend $h_f = K_b \frac{\bar{u}^2}{2g}$		r/d 1 2 4 6 8 10	 $K_b = 0.35$ 0.19 0.16 0.21 0.28 0.32
Threaded pipe fittings $h_f = K \frac{\bar{u}^2}{2g}$	Globe valve—wide open Angle valve—wide open Gate valve—wide open Gate valve—half open Return bend Tee straight-through flow side-outlet flow 90° elbow 45° elbow	$K_v = 10.0$ $K_v = 5.0$ $K_v = 0.2$ $K_v = 5.6$ $K_b = 2.2$ $K_t = 0.4$ $K_t = 1.8$ $K_b = 0.9$ $K_b = 0.4$	

Topic IV: Flow in Conduits and Energy Losses

How can we determine K for other fittings in the laboratory?

Barometric Eq.:



$$P_1 + \gamma H - \gamma_m H = P_2$$

$$\Rightarrow P_1 - P_2 = H(\gamma_m - \gamma)$$

$$MEB: \frac{P_2 - P_1}{\rho} = -(w_f)_{fitting}$$

$$\Rightarrow (w_f)_{fitting} = \frac{P_1 - P_2}{\rho} = \frac{H(\gamma_m - \gamma)}{\rho}$$

$$But\ (w_f)_{fitting} = K\frac{\bar{u}^2}{2} \Rightarrow \boxed{K = \frac{2H(\gamma_m - \gamma)}{\rho \bar{u}^2}}$$

Topic IV: Flow in Conduits and Energy Losses

B- Equivalent length method:

Energy loss due to fitting can be expressed in terms of the equivalent length of the pipe that has the same energy loss for the same discharge

$$(w_f)_{fitting} = 4f\frac{L_{eq}}{D}\frac{\bar{u}^2}{2} \quad or \quad (h_f)_{fitting} = 4f\frac{L_{eq}}{D}\frac{\bar{u}^2}{2g}$$

Where $(w_f)_{fitting}$: Energy loss due to fitting

$(h_f)_{fitting}$: Head loss due to fitting

L_{eq} : Equivalent length of the fitting

$\bar{u}$: average velocity in the pipe

Remark. This method does NOT used for expansion/contraction since inlet diameter differs from outlet diameter.

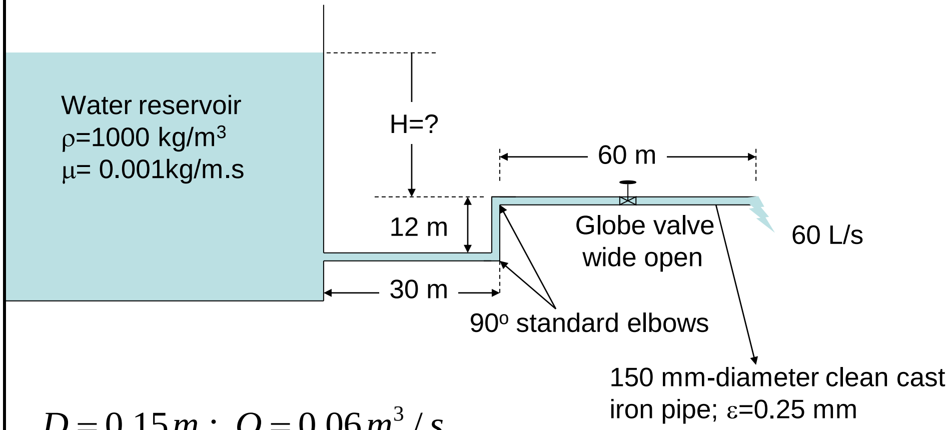
Topic IV: Flow in Conduits and Energy Losses

Equivalent length for various kinds of fittings	
Type of fitting	L_{eq}/D
Globe valve, wide open	340
Angle valve, wide open	145
Gate valve, wide open	13
Check valve (swing type)	135
90° standard elbow	30
45° standard elbow	16
90° long-radius elbow	20

Remark. The available data orient us to use either L_{eq}-method or K-method or a combination of them.

Topic IV: Flow in Conduits and Energy Losses

Example. Find the static head H shown in the figure below to have a discharge of 60 L/s



$$D = 0.15\,m\ ;\ \ Q = 0.06\,m^3/s$$

$$\therefore \bar{u} = 4Q/\pi D^2 = 3.4\,m/s$$

Topic IV: Flow in Conduits and Energy Losses

Apply MEB between reservoir free surface and pipe discharge:

$$g(z_2 - z_1) + \frac{1}{2}\left(\bar{u}_2^2 - \bar{u}_1^2\right) + \frac{P_2 - P_1}{\rho} = w_p - w_f$$

$$z_2 - z_1 = -H ; \bar{u}_1 \approx 0; w_p = 0; P_2 = P_1 = P_{atm}$$

$$-Hg + \frac{\bar{u}_2^2}{2} = -w_f \Rightarrow H = \frac{\bar{u}_2^2}{2g} + \frac{w_f}{g}$$

$$or\ H = \frac{\bar{u}_2^2}{2g} + h_f$$

Where h_f is the total head losses due to friction in pipe and fittings.

Topic IV: Flow in Conduits and Energy Losses

(a) If we neglect head losses:

$$H = \frac{\bar{u}_2^2}{2g} + 0 = \frac{\bar{u}^2}{2g} = 0.5892\, m$$

Is it safe to neglect h_f?!. Let us see

(b) If we take head losses into account:

What head losses does the problem have?
- -Head loss due to friction in pipe segments.
- -Head loss due to sudden contraction; D_2/D_1=0
- -Head loss due to 2 standard 90º elbows.
- -Head loss due Globe valve wide open.

Topic IV: Flow in Conduits and Energy Losses

-Head losses due to friction in pipe segments:

Since the pipe segments has the same diameter and the same roughness we can find total length:

L=30+12+60=102 m

$$\left.\begin{aligned} Re &= \frac{\rho \bar{u} D}{\mu} = 5.05\times 10^5 \\ \varepsilon / D &= 0.0017 \end{aligned}\right\rangle \textit{from Moody diagram}: f = 0.006$$

$$\left(h_f\right)_{pipe} = 4f\frac{L}{D}\frac{\bar{u}^2}{2g} = 9.62\,m$$

-Head losses due to contraction:

Sudden contraction (θ=180º)occurs from large reservoir diameter to pipe diameter(D_2/D_1=0). Thus from previous tables: K_c=0.5

$$\left(h_f\right)_{contraction} = K_c\frac{\bar{u}^2}{2g} = 0.29\,m$$

Topic IV: Flow in Conduits and Energy Losses

-Head loss due to 2 standard 90º elbows:

From previous tables, standard 90º elbows has K_b=0.9

$$\left(h_f\right)_{elbows} = (2)K_b\frac{\bar{u}^2}{2g} = 1.06\,m$$

- Head loss due Globe valve (wide open):

From previous tables, Globe valve (wide open) has K_v=10

$$\left(h_f\right)_{valve} = K_v\frac{\bar{u}^2}{2g} = 5.89\,m$$

Thus the total head loss is:

$$h_f = \left(h_f\right)_{pipe} + \left(h_f\right)_{contraction} + \left(h_f\right)_{elbows} + \left(h_f\right)_{valve} = 16.86\,m$$

The static head (H=0.5892 m) resulted from part (a) by neglecting head losses is NOT enough just to overcome these head losses.

Topic IV: Flow in Conduits and Energy Losses

Finally the static head required to have 60 L/s of water discharge is:

$$H = \frac{\bar{u}_2^2}{2g} + h_f = \frac{3.4^2}{2(9.81)} + 16.86 = 17.45\,m$$

Remember that you can use equivalent length method to find head losses due to some fitting. For example in this problem:

For standard 90º elbows: $\left(L_{eq}/D\right)_{elbow} = 30$

For globe valve (wide open): $\left(L_{eq}/D\right)_{valve} = 340$

$$\left(h_f\right)_{elbows} = (2)\ 4f\left(\frac{L_{eq}}{D}\right)_{elbow}\frac{\bar{u}^2}{2g} = 0.84\,m$$

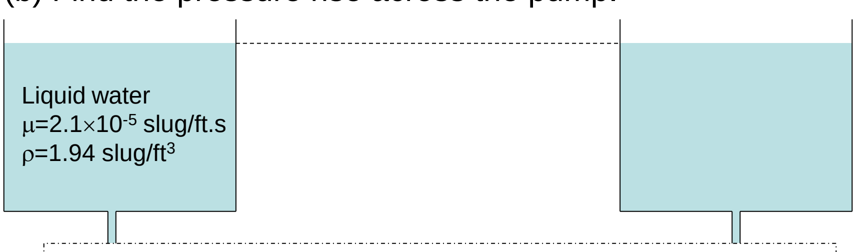
$$\left(h_f\right)_{valve} = 4f\left(\frac{L_{eq}}{D}\right)_{valve}\frac{\bar{u}^2}{2g} = 4.81\,m$$

Topic IV: Flow in Conduits and Energy Losses

Example. For flow shown in the figure below:

(a)What horsepower must be supplied to the motor of the pump, if it is 60% efficient to pump 200 gpm of water from left reservoir to the right one.

(b) Find the pressure rise across the pump.



Piping system contains: 2000 ft of 3-in-schedule 40 commercial steel pipe; 2 Globe valves (wide open), 9 90º elbows, 1 swing check valve, and pump.

Topic IV: Flow in Conduits and Energy Losses

3-in-schedule 40 riveted steel pipe: 3 in is the nominal diameter and it is **NOT** internal diameter. The dimensions of standard pipes are given in sheets. For steel:

TABLE F.1 Schedule 40

Nominal Pipe Size (in)	Outside Diameter (in)	Outside Diameter (mm)	Wall Thickness (in)	Wall Thickness (mm)	Inside Diameter (in)	Inside Diameter (ft)	Inside Diameter (mm)	Flow Area (ft^2)	Flow Area (m^2)
⅛	0.405	10.3	0.068	1.73	0.269	0.0224	6.8	0.000 394	3.660×10^{-5}
¼	0.540	13.7	0.088	2.24	0.364	0.0303	9.2	0.000 723	6.717×10^{-5}
⅜	0.675	17.1	0.091	2.31	0.493	0.0411	12.5	0.001 33	1.236×10^{-4}
½	0.840	21.3	0.109	2.77	0.622	0.0518	15.8	0.002 11	1.960×10^{-4}
¾	1.050	26.7	0.113	2.87	0.824	0.0687	20.9	0.003 70	3.437×10^{-4}
1	1.315	33.4	0.133	3.38	1.049	0.0874	26.6	0.006 00	5.574×10^{-4}
1¼	1.660	42.2	0.140	3.56	1.380	0.1150	35.1	0.010 39	9.653×10^{-4}
1½	1.900	48.3	0.145	3.68	1.610	0.1342	40.9	0.014 14	1.314×10^{-3}
2	2.375	60.3	0.154	3.91	2.067	0.1723	52.5	0.023 33	2.168×10^{-3}
2½	2.875	73.0	0.203	5.16	2.469	0.2058	62.7	0.033 26	3.090×10^{-3}
3	3.500	88.9	0.216	5.49	3.068	0.2557	77.9	0.051 32	4.768×10^{-3}
3½	4.000	101.6	0.226	5.74	3.548	0.2957	90.1	0.068 68	6.381×10^{-3}
4	4.500	114.3	0.237	6.02	4.026	0.3355	102.3	0.088 40	8.213×10^{-3}
5	5.563	141.3	0.258	6.55	5.047	0.4206	128.2	0.139 0	1.291×10^{-2}
6	6.625	168.3	0.280	7.11	6.065	0.5054	154.1	0.200 6	1.864×10^{-2}
8	8.625	219.1	0.322	8.18	7.981	0.6651	202.7	0.347 2	3.226×10^{-2}
10	10.750	273.1	0.365	9.27	10.020	0.8350	254.5	0.547 9	5.090×10^{-2}
12	12.750	323.9	0.406	10.31	11.938	0.9948	303.2	0.777 1	7.219×10^{-2}
14	14.000	355.6	0.437	11.10	13.126	1.094	333.4	0.939 6	8.729×10^{-2}
16	16.000	406.4	0.500	12.70	15.000	1.250	381.0	1.227	0.1140
18	18.000	457.2	0.562	14.27	16.876	1.406	428.7	1.553	0.1443
20	20.000	508.0	0.593	15.06	18.814	1.568	477.9	1.931	0.1794
24	24.000	609.6	0.687	17.45	22.626	1.886	574.7	2.792	0.2594

Internal diameter:

D=0.2557 ft

Topic IV: Flow in Conduits and Energy Losses

TABLE F.2 Schedule 80

Nominal Pipe Size (in)	Outside Diameter (in)	Outside Diameter (mm)	Wall Thickness (in)	Wall Thickness (mm)	Inside Diameter (in)	Inside Diameter (ft)	Inside Diameter (mm)	Flow Area (ft^2)	Flow Area (m^2)
⅛	0.405	10.3	0.095	2.41	0.215	0.017 92	5.5	0.000 253	2.350×10^{-5}
¼	0.540	13.7	0.119	3.02	0.302	0.025 17	7.7	0.000 497	4.617×10^{-5}
⅜	0.675	17.1	0.126	3.20	0.423	0.035 25	10.7	0.000 976	9.067×10^{-5}
½	0.840	21.3	0.147	3.73	0.546	0.045 50	13.9	0.001 625	1.510×10^{-4}
¾	1.050	26.7	0.154	3.91	0.742	0.061 83	18.8	0.003 00	2.787×10^{-4}
1	1.315	33.4	0.179	4.55	0.957	0.079 75	24.3	0.004 99	4.636×10^{-4}
1¼	1.660	42.2	0.191	4.85	1.278	0.106 5	32.5	0.008 91	8.278×10^{-4}
1½	1.900	48.3	0.200	5.08	1.500	0.125 0	38.1	0.012 27	1.140×10^{-3}
2	2.375	60.3	0.218	5.54	1.939	0.161 6	49.3	0.020 51	1.905×10^{-3}
2½	2.875	73.0	0.276	7.01	2.323	0.193 6	59.0	0.029 44	2.735×10^{-3}
3	3.500	88.9	0.300	7.62	2.900	0.241 7	73.7	0.045 90	4.264×10^{-3}
3½	4.000	101.6	0.318	8.08	3.364	0.280 3	85.4	0.061 74	5.736×10^{-3}
4	4.500	114.3	0.337	8.56	3.826	0.318 8	97.2	0.079 86	7.419×10^{-3}
5	5.563	141.3	0.375	9.53	4.813	0.401 1	122.3	0.126 3	1.173×10^{-2}
6	6.625	168.3	0.432	10.97	5.761	0.480 1	146.3	0.181 0	1.682×10^{-2}
8	8.625	219.1	0.500	12.70	7.625	0.635 4	193.7	0.317 4	2.949×10^{-2}
10	10.750	273.1	0.593	15.06	9.564	0.797 0	242.9	0.498 6	4.632×10^{-2}
12	12.750	323.9	0.687	17.45	11.376	0.948 0	289.0	0.705 6	6.555×10^{-2}
14	14.000	355.6	0.750	19.05	12.500	1.042	317.5	0.852 1	7.916×10^{-2}
16	16.000	406.4	0.842	21.39	14.314	1.193	363.6	1.117	0.1038
18	18.000	457.2	0.937	23.80	16.126	1.344	409.6	1.418	0.1317
20	20.000	508.0	1.031	26.19	17.938	1.495	455.6	1.755	0.1630
24	24.000	609.6	1.218	30.94	21.564	1.797	547.7	2.535	0.2344

Topic IV: Flow in Conduits and Energy Losses

$$D = 0.2557\ ft\ ;\ Q = 200\frac{\text{gal}}{\text{min}}\frac{1\text{min}}{60\text{s}}\frac{1\text{ft}^3}{7.48\,\text{gal}} = 0.4456\,\text{ft}^3/\text{s}$$

$$\therefore \bar{u} = 4Q/\pi D^2 = 8.68\,\text{m/s}$$

Apply MEB between free surfaces of reservoirs:

$$g(z_2 - z_1) + \frac{1}{2}\left(\bar{u}_2^2 - \bar{u}_1^2\right) + \frac{P_2 - P_1}{\rho} = w_p - w_f$$

$$P_1 = P_2 = P_{atm}\ ;\ z_1 = z_2\ ;\ \bar{u}_1 \approx 0\ ;\ \bar{u}_1 \approx 0$$

$$\therefore w_p = w_f$$

w_p : Specific work done by pump on water; lbf.ft/slug.

w_f : Specific energy losses due to friction in pipe and fittings; lbf.ft/slug.

Topic IV: Flow in Conduits and Energy Losses

Energy losses:

-Energy losses due to friction in pipe segments:

L=2000 ft

For commercial steel: $\varepsilon = 1.5\times10^{-4}$ ft

$$\left.\begin{array}{l}\text{Re} = \dfrac{\rho\bar{u}D}{\mu} = 2.05\times10^5 \\ \varepsilon/D = 0.0006\end{array}\right\rangle \text{from Moody diagram}: f = 0.0048$$

$$\left(w_f\right)_{pipe} = 4f\frac{L}{D}\frac{\bar{u}^2}{2} = 5657.3\,\text{lbf.ft/slug}$$

-Energy losses due to contraction:

Sudden contraction from left reservoir to pipe: K_c=0.5

$$\left(w_f\right)_{contraction} = K_c\frac{\bar{u}^2}{2} = 18.8\ \text{lbf.ft/slug}$$

Topic IV: Flow in Conduits and Energy Losses

-Energy losses due to expansion:

Sudden expansion from pipe to the right reservoir: K_e=1.0

$$\left(w_f\right)_{expansion} = K_e \frac{\bar{u}^2}{2} = 37.6 \text{ lbf.ft/slug}$$

- Energy losses due 2 Globe valve (wide open):

For globe valve (wide open): $\left(L_{eq}/D\right)_{Globe\,valve} = 340$

$$\left(w_f\right)_{Globe\,valves} = (2)\,4f\left(\frac{L_{eq}}{D}\right)_{valve}\frac{\bar{u}^2}{2} = 491.8\,\text{lbf.ft/slug}$$

- Energy losses due 1 Swing check valve:

For swing check valve: $\left(L_{eq}/D\right)_{swing\,valve} = 135$

$$\left(w_f\right)_{swing\,valve} = (1)\,4f\left(\frac{L_{eq}}{D}\right)_{swing\,valve}\frac{\bar{u}^2}{2} = 97.6\,\text{lbf.ft/slug}$$

Topic IV: Flow in Conduits and Energy Losses

- Energy losses due 9 90º elbows:

For 90º elbows : $\left(L_{eq}/D\right)_{elbow} = 30$

$$\left(w_f\right)_{elbows} = (9)\,4f\left(\frac{L_{eq}}{D}\right)_{elbow}\frac{\bar{u}^2}{2} = 195.3\,\text{lbf.ft/slug}$$

$$w_f = \left(w_f\right)_{contraction} + \left(w_f\right)_{expansion} + \left(w_f\right)_{pipe} + \left(w_f\right)_{eswing\,valve} + \left(w_f\right)_{globe\,valves} + \left(w_f\right)_{elbows}$$

$$= 6498.7\,\text{lbf.ft/slug}$$

$$From\,MEB: w_p = w_f = 6498.7\,\text{lbf.ft/slug}$$

$$\dot{w}_p = \dot{m}w_f = \rho Q w_f = (1.94)(0.4456)(6498.7) = 5617.9\,\text{lbf.ft/s}$$

$\dot{w}_p$: Power done by pump on water.

Topic IV: Flow in Conduits and Energy Losses

To convert this power to horsepower, use the following conversion factor: $1\,hp = 550\,\text{lbf.ft/s}$

$$\dot{w}_p = 5617.9\,\text{lbf.ft/s}\,\frac{1\,\text{hp}}{550\,\text{lbf.ft/s}} = 10.2\,hp$$

But the problem asked about the power supplied to the motor of the pump and since the pump is 60% efficient:

$$Efficiency\ ;\ \eta = \frac{\dot{w}_p}{\text{Power supplied to the motor}}$$

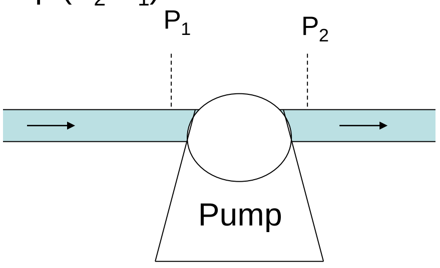
$$\Rightarrow 0.6 = \frac{10.2}{\text{Power supplied to the motor}}$$

$$\text{Power supplied to the motor} = 17\,\text{hp}$$

Thus buy a pump with at least 17 hp to have this pumping duty.

Topic IV: Flow in Conduits and Energy Losses

(b) Pressure rise across the pump (P_2-P_1):



Apply MEB across the pump:

$$\frac{P_2 - P_1}{\rho} = w_p - \left(w_f\right)_{pump}$$

$$assume\left(w_f\right)_{pump} \approx 0$$

$$P_2 - P_1 = \rho w_p = (1.94)(6498.7) = 12607.5\,\text{lbf/ft}^2$$

$$= 87.6\,\text{psi} = 5.96\,\text{atm}$$

Topic IV: Flow in Conduits and Energy Losses

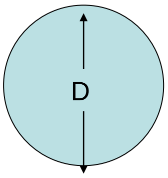
Energy losses due to friction for flow across noncircular cross-sections:

The concept of hydraulic radius (HR) permits circular and non-circular cross sections to be treated in the same manner:

$$HR = \frac{Cross-Sectional\ Area}{Wetted\ \ perimeter} = \frac{A}{W}$$

For circular cross-section:

$$HR = \frac{A}{W} = \frac{\pi D^2/4}{\pi D} \Rightarrow \boxed{D = 4HR}$$



For noncircular cross section we define the equivalent diameter D_{eq}:

$$\boxed{D_{eq} = 4HR = 4\frac{A}{W}}$$

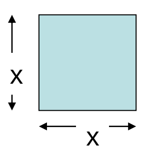
Topic IV: Flow in Conduits and Energy Losses

Equivalent diameter is used to determine energy losses due to friction for flow through non-circular cross section:

$$\mathrm{Re} = \frac{\rho \bar{u} D_{eq}}{\mu} \quad ; \quad \varepsilon / D_{eq} \quad ; \quad \left(w_f\right)_{pipe} = 4f\frac{L}{D_{eq}}\frac{\bar{u}^2}{2}$$

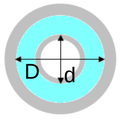
Example. Find the equivalent diameter for flow in conduit of square cross-section:

$$D_{eq} = 4\frac{A}{W} = 4\frac{x^2}{4x} = x$$



Example. Find the equivalent diameter for flow between two concentric cylinders:

$$D_{eq} = 4\frac{A}{W} = 4\frac{\frac{\pi D^2}{4} - \frac{\pi d^2}{4}}{\pi D + \pi d} = \frac{D^2 - d^2}{D + d} == D - d$$

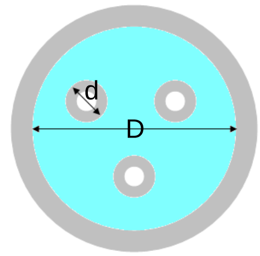


"Annular cross-section"

Topic IV: Flow in Conduits and Energy Losses

Example. Find the equivalent diameter for flow between two 3 pipes and one cylindrical shell:

$$D_{eq} = 4\frac{A}{W} = 4\frac{\frac{\pi D^2}{4} - \frac{(3)\pi d^2}{4}}{\pi D + 3\pi d} = \frac{D^2 - 3d^2}{D + 3d}$$



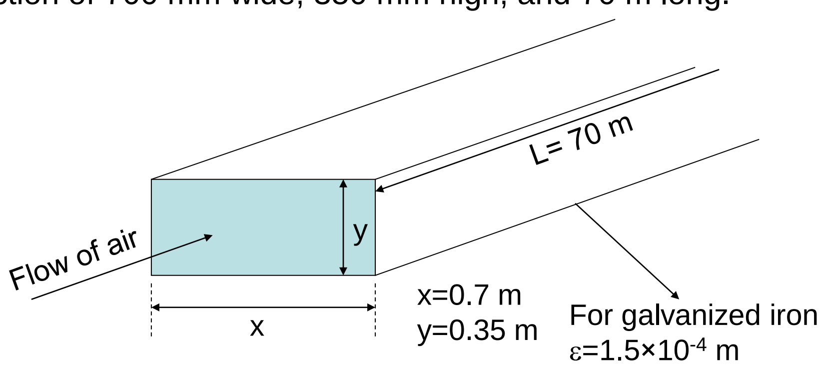
- The equivalent diameter is helpful in the design of heat/mass transfer equipments such as heat exchangers.

- The use of equivalent diameter does NOT work well for cross-sections that depart radically from circles such as flow through narrow slits.

"Narrow slits"

Topic IV: Flow in Conduits and Energy Losses

Example. Determine head loss and pressure drop for flow of 300 m^3/min of air at 20 ^oC through a rectangular galvanized iron section of 700 mm wide, 350 mm high, and 70 m long.



From physical properties table for air at 20 ^oC:
ρ = 1.204 kg/m^3
μ =1.81×10^{-5} Pa.s

$h_f = ?$ → **Type 1 problem**

Topic IV: Flow in Conduits and Energy Losses

$$Q = 300\ \mathrm{m^3/min} = 5\ \mathrm{m^3/s}$$

$$A = xy = (0.7)(0.35) = 0.245\ \mathrm{m^2}$$

$$\bar{u} = Q/A = 20.41\ \mathrm{m/s}$$

$$D_{eq} = 4\frac{A}{W} = 4\frac{xy}{2x+2y} = 4\frac{(0.7)(0.35)}{(2)(0.7)+(2)(0.35)} = 0.4667\ m$$

$$\left.\begin{aligned} Re &= \rho \bar{u} D_{eq}/\mu = 6.3\times10^5 \\ \varepsilon / D_{eq} &= 0.00032 \end{aligned}\right\rangle \textit{from Moody diagram}: f = 0.004$$

Turbulent flow: $\varepsilon \neq 0$ and Re>4000

$$h_f = 4f\frac{L}{D_{eq}}\frac{\bar{u}^2}{2g} = 51\ \mathrm{m}$$

Apply MEB to find pressure drop across the duct:

$$P_1 - P_2 = \rho w_f = \rho g h_f = 602.4\ \mathrm{Pa}$$