Topic VI: Fluid Flow Measurements

Fluid flow measurement: rotameter; stagnation impact tube; Pitot-static tube; orifice meter; Venturi meter**.**

Topic VI: Fluid Flow Measurements

- The traditional method for measuring volumetric flow rate of liquid, Q, is to collect certain volume of that liquid during some period of time, t, and to calculate: Q = V/ t.

- This traditional method does NOT work for gases and it is NOT practical for continuous processes.

- This topic will explain the following fluid flow measurement devices:

A- Rotameter
B- Stagnation Impact Tube
C- Pitot-static tube
D- Orifice Meter
E- Venturi meter

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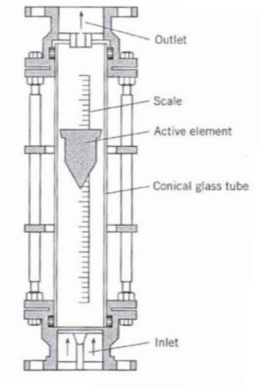
A- Rotameter:

- For low flow rates of liquids or gases.

- It measures volumetric flow rates.

- The scale is specified by traditional method for liquids and other measurement devices for gases at reference temperature and pressure.



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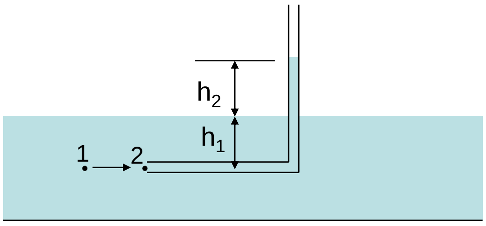
B- Stagnation Impact Tube:

-Mainly for open channel flow of liquids.

-It measures local velocity.



- Point 2 at the inlet section on the L-tube is called stagnation point since the velocity at this point will be zero(friction across small cross section).

- **Stagnation pressure**: pressure at which velocity is zero.

- Apply MEB between 1 and 2:

$$\left.\begin{array}{l}\dfrac{u_2^2-u_1^2}{2}+\dfrac{P_2-P_1}{\rho}=-w_f \\ w_f \approx 0 \; ; \; u_2=0\end{array}\right\rangle \qquad u_1=\sqrt{\frac{2(p_2-p_1)}{\rho}}$$

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From Barometric equation:

$$p_1 = \rho g h_1$$
$$p_2 = \rho g (h_1 + h_2)$$
$$\Rightarrow \boxed{u_1 = \sqrt{2 g h_2}}$$

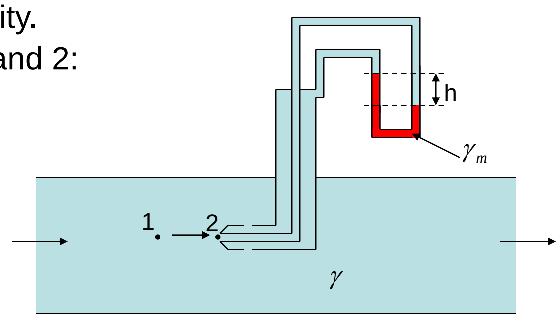
C- Pitot Static Tube:

- Mainly for closed flow of liquids or gases.
- It measures local velocity.
- Apply MEB between 1 and 2:

$$\frac{u_2^2 - u_1^2}{2} + \frac{P_2 - P_1}{\rho} = -w_f$$

$$w_f \approx 0 \;;\; u_2 = 0$$

$$\therefore \quad u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$



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From Barometric equation:

$$p_2 + \gamma h - \gamma_m h = p_1$$

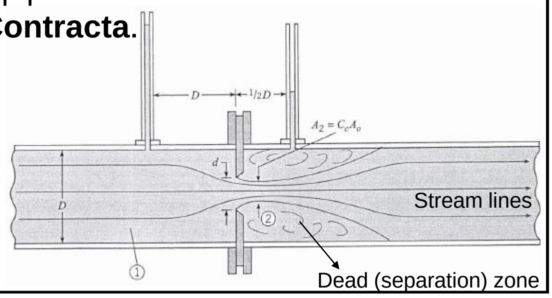
$$\Rightarrow p_2 - p_1 = h(\gamma_m - \gamma) = hg(\rho_m - \rho) \Rightarrow \boxed{u_1 = \sqrt{2gh\left(\frac{\rho_m}{\rho} - 1\right)}}$$

D- Orifice meter:

- For flow of liquids or gases.
- It measures discharge flow rate (Q).
- Orifice has cross section $A_o = \pi d^2/4$.
- Orifice is connected with a pipe of diameter D.
- Section 2 is called **Venta Contracta**.
- Contraction coefficient, C_c:

$$C_c = \frac{A_2}{A_o}$$

$$A_2 = C_c A_o = C_c \pi \frac{d^2}{4}$$



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Derivation of the Discharge Equation:

- Apply MEB between 1 and 2:

$$\frac{\bar{u}_2^2-\bar{u}_1^2}{2}+\frac{P_2-P_1}{\rho}=-w_f$$

- Let us first assume ideal flow $\Rightarrow w_f=0$

$$\frac{\bar{u}_2^2-\bar{u}_1^2}{2}+\frac{P_2-P_1}{\rho}=0$$

- Apply St. St. MB between 1 and 2: $A_2\bar{u}_2=A_1\bar{u}_1$

$$\Rightarrow \bar{u}_2=\sqrt{\frac{2(P_1-P_2)/\rho}{1-(A_2/A_1)^2}}$$

- But $A_2=C_cA_o=C_c\pi d^2/4$

$$\Rightarrow \bar{u}_2=\sqrt{\frac{2(P_1-P_2)/\rho}{1-(C_cA_o/A_1)^2}}$$

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Derivation of the Discharge Equation:

Now the volumetric flow rate is:

$$Q=A_2\bar{u}_2=C_cA_o\bar{u}_2=C_cA_o\sqrt{\frac{2(P_1-P_2)/\rho}{1-(C_cA_o/A_1)^2}}$$

$$=\frac{C_c}{\sqrt{1-(C_cA_o/A_1)^2}}A_o\sqrt{2(P_1-P_2)/\rho}$$

For real flow the energy losses to overcome friction must be considered. It will reduce the available kinetic energy and thus the volumetric flow rate will be lower than that of ideal flow. To take this effect into account, let us define another coefficient which is called velocity coefficient C_v<1.0. Then, the actual volumetric flow rate becomes:

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Derivation of the Discharge Equation:

$$Q = \frac{C_c C_v}{\sqrt{1-(C_c A_o / A_1)^2}} A_o \sqrt{2(P_1 - P_2)/\rho}$$

$$C_d = C_c C_v$$

C_d : is called discharge coefficient

$$K = \frac{C_d}{\sqrt{1-(C_c A_o / A_1)^2}}$$

K : is called flow coefficient

$$\boxed{Q = K A_o \sqrt{2(P_1 - P_2)/\rho}}$$

Discharge Equation

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$$Q = K A_o \sqrt{2(P_1 - P_2)/\rho}$$

The flow coefficient, K is function of the following:
- Volumetric flow rate Q, or average velocity $\bar{u}$.
- Diameter of the pipe D.
- Orifice diameter d.
- Density, ρ, and viscosity, μ , of the fluid.

$$\therefore K = fun\left(\mathrm{Re}_d, \frac{d}{D}\right) \Rightarrow \text{See Figure in the next slide.}$$

$$\mathrm{Re}_d = \frac{\rho \bar{u}_d d}{\mu}$$

$\bar{u}_d$: Average velocity at orifice

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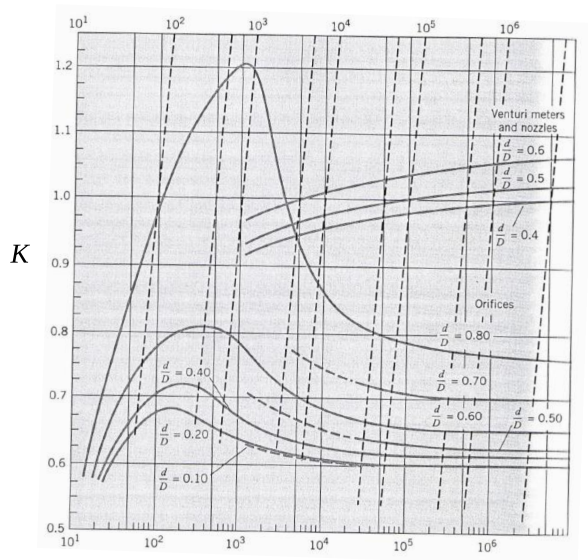

$$\mathrm{Re}_d = \rho \bar{u}_d d / \mu$$

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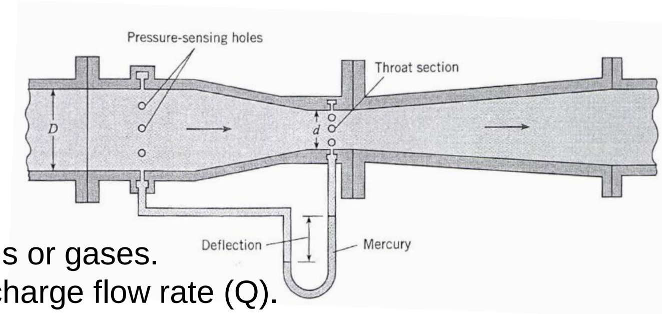
E- Venturi meter:



- For flow of liquids or gases.
- It measures discharge flow rate (Q).
- Venturi has cross section $A_o = \pi d^2/4$.
- Venturi is connected with a pipe of diameter D.

-**Venta Contracta** occurs at Venturi vent→ contraction coefficient, C_c=1

$$C_d = C_c C_v = C_v$$

$$K = \frac{C_v}{\sqrt{1-(A_o/A_1)^2}} \qquad Q = KA_o\sqrt{2(P_1 - P_2)/\rho}$$

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Example. The pressure drop across Venturi meter is 35 kPa. If Venturi diameter is 20 cm and pipe diameter is 40 cm, what is the discharge of water at 10 ^oC?

P_1-P_2= 35000 Pa.
d=0.20 m , D= 0.40 m → **d/D=0.5**
From physical properties table for water at 10 ^oC:
ρ=1000 kg/m^3 μ=0.00131 Pa.s

Q=?

Discharge Eq.: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho}$

$$A_o = \pi\frac{d^2}{4} = \pi\frac{0.2^2}{4} = 0.03142\,m^2$$

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Look at d/D=0.5 curve for Venturi to select trial value of K:

- Trial value of K=1.0
- Find: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho} = 0.2629\,\text{m}^3/\text{s}$
- Find: $\bar{u}_d = Q/A_o = 8.367\,\text{m/s}$
- Find: $\left.\begin{array}{l}\text{Re}_d = \rho\bar{u}_d d/\mu = 1.3\times10^6 \\ d/D = 0.5\end{array}\right\rangle \text{from the chart } K = 1.02$
- Find: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho} = 0.2681\,\text{m}^3/\text{s}$
- Find: $\bar{u}_d = Q/A_o = 8.533\,\text{m/s}$
- Find: $\left.\begin{array}{l}\text{Re}_d = \rho\bar{u}_d d/\mu = 1.3\times10^6 \\ d/D = 0.5\end{array}\right\rangle \text{from the chart } K = 1.02$

$$Stop \Rightarrow Q = 0.2681\,\text{m}^3/\text{s} = 268\,\text{L/s}$$

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Example. Repeat previous example for orifice

P_1-P_2= 35000 Pa.
d=0.20 m , D= 0.40 m $\rightarrow$ **d/D=0.5**
From physical properties table for water at 10 ^oC:
ρ=1000 kg/m^3 μ=0.00131 Pa.s

Q=?

Discharge Eq.: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho}$

$$A_o = \pi\frac{d^2}{4} = \pi\frac{0.2^2}{4} = 0.03142\, m^2$$

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Look at d/D=0.5 curve for Orifice to select trial value of K:

- Trial value of K=0.7
- Find: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho} = 0.1840\,\mathrm{m^3/s}$
- Find: $\bar{u}_d = Q/A_o = 5.8566\,\mathrm{m/s}$
- Find: $\left.\begin{array}{l}\mathrm{Re}_d = \rho\bar{u}_d d/\mu = 8.9\times10^5 \\ d/D = 0.5\end{array}\right\rangle \text{from the chart } K = 0.63$
- Find: $Q = KA_o\sqrt{2(P_1 - P_2)/\rho} = 0.1656\,\mathrm{m^3/s}$
- Find $\bar{u}_d = Q/A_o = 5.2714\,\mathrm{m/s}$
- Find: $\left.\begin{array}{l}\mathrm{Re}_d = \rho\bar{u}_d d/\mu = 8\times10^5 \\ d/D = 0.5\end{array}\right\rangle \text{from the chart } K = 0.63$

$$Stop \Rightarrow Q = 0.1656\,\mathrm{m^3/s} = 165.6\,\mathrm{L/s}$$

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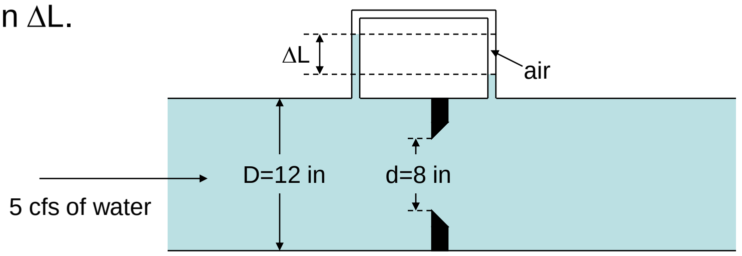
Example. Water at 60 °F flows through a pipe which has an orifice as shown in the figure below. Find the the manometer deflection ΔL.



Q= 5 ft³/s

d/D = 8/12 = 0.67

Water at 60 °F:
ν =1.22×10⁻⁵ ft²/s

$$A_o = \pi \frac{d^2}{4} = \pi \frac{0.67^2}{4} = 0.3491\ ft^2$$

$$\bar{u}_d = Q/A_o = 14.3225\,\text{ft/s} \qquad \text{Re}_d = \bar{u}_d d/\nu = 7.8\times 10^5$$

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$$\left.\begin{matrix}\text{Re}_d = 8\times 10^5 \\ d/D = 0.67\end{matrix}\right\rangle \text{from the chart (Orifice)}; K = 0.68$$

$$Q = KA_o\sqrt{2(P_1 - P_2)/\rho}$$

$$\Rightarrow (P_1 - P_2)/\rho = (Q/KA_o)^2/2 = 221.8\,\text{ft}^2/\text{s}^2$$

Apply barometric Eq.:

$$P_1 - \rho_w g\Delta L + \rho_{air} g\Delta L = P_2$$

$$\rho_{air} << \rho_w \Rightarrow \Delta L = \frac{P_1 - P_2}{\rho_w g} = \frac{221.8}{32.2} = 6.9\,\text{ft}$$

Remarks:

- Historically Venturi meter is older than Orifice meter.
- Energy losses due to orifice is greater than that due to Venturi.