THERMODYNAMICS, FLUID AND PLANT PROCESSES

The tutorials are drawn from other subjects so the solutions are identified by the appropriate tutorial.

FLUID MECHANICS – HYDROSTATIC FORCES
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

1. A mercury barometer gives a pressure head of 758 mm. The density is 13 600 kg/m3. Calculate the atmospheric pressure in bar.

   $p = \rho g h = 13\,600 \text{ X } 9.81 \text{ X } 0.758 = 10113$ Pas or 1.0113 bar

2. A manometer (fig.1.21) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains water of density 1000 kg/m^3 and the head is 250 mm. Calculate the gauge pressure of the gas in the container.

   $p = \rho g \Delta h = 1000 \times 9.81 \times 0.25 = 2452.5$ Pa

3. Calculate the pressure and force on a horizontal submarine hatch 1.2 m diameter when it is at a depth of 800 m in seawater of density 1030 kg/m^3. (8.083 MPa and 9.142 MN)

   $p = \rho g h = 1030 \times 9.81 \times 800 = 8.083 \times 10^6$ Pa

   $F = pA = 8.083 \times 10^6 \times \pi \times 1.2^2/4 = 9.142 \times 10^6$ N

SELF ASSESSMENT EXERCISE No.2

1. A vertical retaining wall contains water to a depth of 20 metres. Calculate the turning moment about the bottom for a unit width. Take the density as 1000 kg/m^3.
   (13.08 MNm)

   $\bar{y} = 10$ m
   $R = \rho g A \bar{y} = 1000 \times 9.81 \times (20 \times 1) \times 10 = 1.962 \times 10^6$ N
   Centre of pressure is 2/3 the depth and acts at 20/3 from the bottom.
   Turning moment about the bottom = $1.962 \times 10^6 \times 20/3 = 13.08 \times 10^6$ Nm=

2. A vertical wall separates seawater on one side from fresh water on the other side. The seawater is 3.5 m deep and has a density of 1030 kg/m^3. The fresh water is 2 m deep and has a density of 1000 kg/m^3. Calculate the turning moment produced about the bottom for a unit width.

   The solution is basically the same as the previous problem.

   On the sea water side
   $M = \rho g A \bar{y} \times D/3 = 1030 \times 9.81 \times (3.5 \times 1) \times (3.5/2) \times (3.5/3) = 72204$ Nm
   On the fresh water side
   $M = \rho g A \bar{y} \times D/3 = 1000 \times 9.81 \times (2 \times 1) \times (2/2) \times (2/3) = 1308$ Nm
   Resultant Moment = 72204 - 1308 = 59124 Nm

SELF ASSESSMENT EXERCISE No.3

1. A circular hatch is vertical and hinged at the bottom. It is 2 m diameter and the top edge is 2m below the free surface. Find the total force, the position of the centre of pressure and the force required at the top to keep it closed. The density of the water is 1000 kg/m^3.

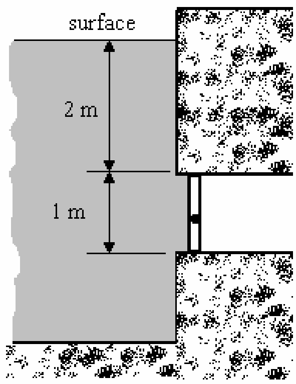
$\bar{y}$ = 3 m $R = \rho g A \bar{y}$ = 1000x 9.81 x π x $2^2/4$ x 3 = 92469 N
$I_{ss} = I_{gg} + A\bar{y}^2 = \pi D^4/64 + (\pi D^2/4)\,\bar{y}^2 = \pi 2^4/64 + (\pi 2^2/4)\,3^2 = 29.06\ m^4$
$M = \rho g\ I_{ss}$ = 1000 x 9.81 x 29.06 = 285113 N m
$\bar{h}$ = M/R = 285113/92469 = 3.08 m

Alternative solution $\bar{h} = \bar{y} + k^2/\bar{y}$ k = D/4 = 0.5 m
$\bar{h} = 3 + 0.5^2/3$ = 3.08 m

2. A large tank of sea water has a door in the side 1 m square. The top of the door is 5 m below the free surface. The door is hinged on the bottom edge. Calculate the force required at the top to keep it closed. The density of the sea water is 1036 kg/m^3.

$\bar{y}$ = 5.5 m   k = D/√12 = 1/√12 = 0.288 m
$R = \rho g A \bar{y}$ = 1036 x 9.81 x (1 x 1) x 5.5 = 55897 N
$\bar{h} = \bar{y} + k^2/\bar{y} = 5.5 + 2.88^2/5.5$ = 5.515 m This is 0.485 m from the hinge.
Moment about the hinge = 55897 x 0.485 = 27.11 kN

3. A culvert in the side of a reservoir is closed by a vertical rectangular gate 2m wide and 1 m deep as shown in fig. 4. The gate is hinged about a horizontal axis which passes through the centre of the gate. The free surface of water in the reservoir is 2.5 m above the axis of the hinge. The density of water is 1000 kg/m^3. Assuming that the hinges are frictionless and that the culvert is open to atmosphere, determine
(i) the force acting on the gate when closed due to the pressure of water.
(ii) the moment to be applied about the hinge axis to open the gate.



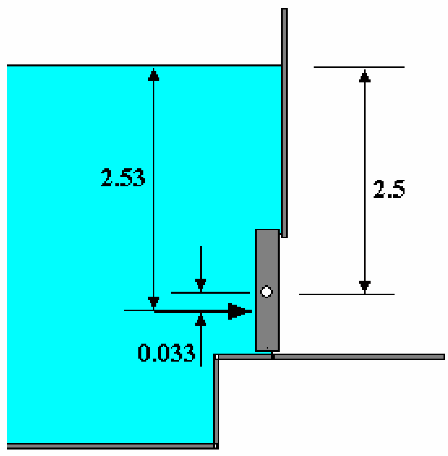
$R = \rho g A \bar{y}$ ρ= 1000   g = 9.81   A = 2x1=2
$\bar{y}$=(3 +2)/2 = 2.5 m
R=1000 x 9.81 x 2 x 2.5 =49 050 N

Centre of pressure = 2nd mom of area/1st mom of area

2nd mom of area = I = Igg + $A\bar{y}^2$ =$BD^3/12 + A\bar{y}^2$
=(2 x 1) + (2 x $(2.5)^2$)
=12.667 m^4

1st mom of area = $A\bar{y}$ =2 x 2.5 = 5 m^3

$\bar{h}$= 12.667/5= 2.53 m

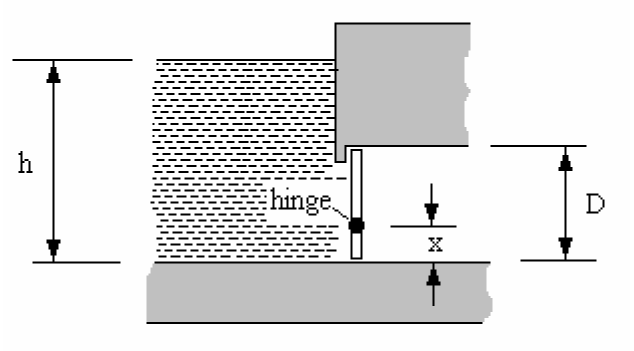


TM about the hinge is 49050 x 0.033 = 1635 Nm

4. The diagram shows a rectangular vertical hatch breadth B and depth D. The hatch flips open when the water level outside reaches a critical depth h. Show that for this to happen the hinge must be located at a position x from the bottom given by the formula

$$x = \frac{D}{2}\left\{\frac{6h - 4D}{6h - 3D}\right\}$$

Given that the hatch is 1 m deep, calculate the position of the hinge such that the hatch flips open when the depth reaches 3 metres. (0.466 m)



The hatch will flip open as soon as the centre of pressure rises above the hinge creating a clockwise turning moment. When the centre of pressure is below the hinge, the turning moment is anticlockwise and the hatch is prevented from turning in that direction. We must make the centre of pressure at position x.

$$\bar{y} = h - \frac{D}{2}$$

$$\bar{h} = h - x$$

$$\bar{h} = \frac{\text{second moment of area}}{\text{first moment of area}} \text{ about the surface}$$

$$\bar{h} = \frac{I_{gg} + A\bar{y}^2}{A\bar{y}} = \frac{\frac{BD^3}{12} + BD\,\bar{y}^2}{BD\,\bar{y}} = \frac{D^2}{12\bar{y}} + \bar{y}$$

Equate for $\bar{h}$

$$\frac{D^2}{12\bar{y}} + \bar{y} = h - x$$

$$x = h - \frac{D^2}{12\bar{y}} - \bar{y} = h - \frac{D^2}{12\left(h - \frac{D}{2}\right)} - \left(h - \frac{D}{2}\right)$$

$$x = -\frac{D^2}{(12h - 6D)} + \frac{D}{2} = \frac{D}{2} - \frac{D^2}{(12h - 6D)}$$

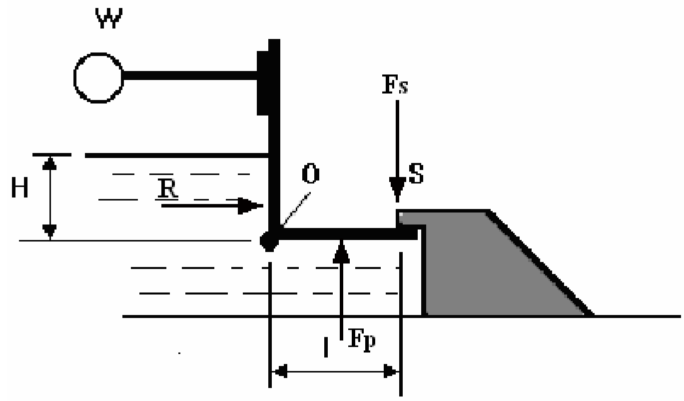
$$x = \frac{D}{2}\left\{1 - \frac{D}{6h - 3D}\right\} = \frac{D}{2}\left\{\frac{6h - 3D - D}{6h - 3D}\right\} = \frac{D}{2}\left\{\frac{6h - 4D}{6h - 3D}\right\}$$

Putting D = 1 and h = 3 we get ***x = 0.466 m***

5. An L shaped spill gate that operates by pivoting about hinge O when the water level in the channel rises to a certain height H above O. A counterweight W attached to the gate provides closure of the gate at low water levels. With the channel empty the force at sill S is 1.635 kN. The distance l is 0.5m and the gate is 2 m wide.

   Determine the magnitude of H.

   (i) when the gate begins to open due to the hydrostatic load.
   (ii) when the force acting on the sill becomes a maximum. What is the magnitude of this force.
   Assume the effects of friction are negligible.



Pressure force on the bottom = Fp = pA = ρgAH = 9810H
$\bar{y}$=H/2 A = 2H R = ρgA$\bar{y}$ = ρgAH/2 = 9810 H^2
R acts at H/3 from the bottom
Moments about O give the following.

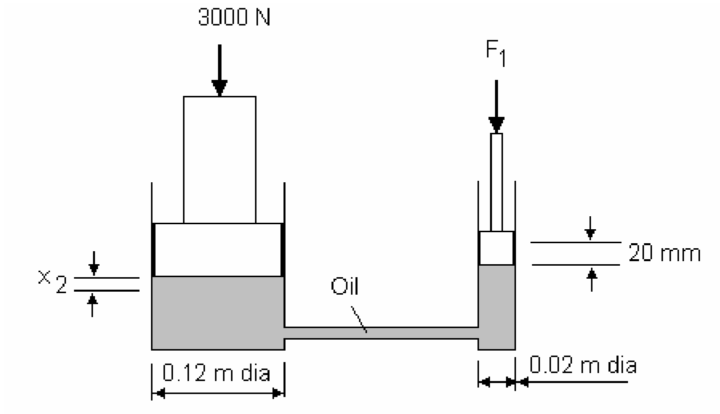
W x + Fp x 0.25 = RH/3 + Fs x 0.5

When empty, H = 0 Fp = 0 Fs = 1635 N
W x = 1635 x 0.5 = 817.5 Nm
When the gate just opens, Fs = 0
W x + Fp x 0.25 = RH/3
817.5 + 2452.5H = 3270 H^3
Solve by any suitable method and it seems that H = 1 m

When not open, Fs is unknown.
Wx + Fp x 0.25 = Fs x 0.5 + 3270 H^3
Fs = 1635 + 490 H – 6540 H^3
For max or min dFs/dH = 0
dFs/dH = 4905 – 3(6540H^2) = 0
4905 = 19620 H^2 = 0
H = 0.5 m

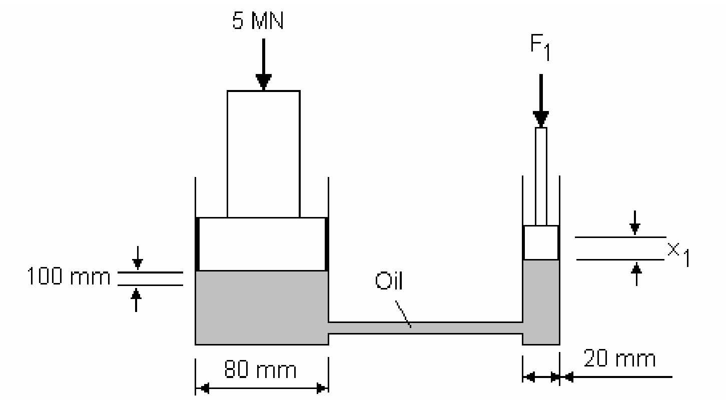
SELF ASSESSMENT EXERCISE No.4

1. Calculate F_1 and x_2 for the case shown below.



$A_2/A_1 = 0.12^2/0.02^2 = 36$
$F_1 = 3000/36 = 83.3$ N
$x_1 = 20/36 = 0.555$ mm

2. Calculate F_1 and x_1 for the case shown below.



$A_2/A_1 = 80^2/20^2 = 16$
$x_1 = 100/16 = 6.25$ mm
$F_1 = 5/16 = 0.3125$ MN

<u>SELF ASSESSMENT EXERCISE No.5</u>

1. A double acting hydraulic cylinder with a single rod must produce a thrust of 800 kN. The operating pressure is 100 bar gauge. Calculate the bore diameter required.

   $A = F/p = 800\,000/100\text{x}10^5 = 0.08\text{ m}^2$ $D = \sqrt{(4 \text{ x } 0.08/\pi)}$
   D= 0.108 m or 101.8 mm

2. The cylinder in question 1 has a rod diameter of 25 mm. If the pressure is the same on the retraction (negative) stroke, what would be the force available ?

   $A_2 = \pi \text{ x } 0.025^2/4 = 0.00049\text{ m}^2$ $A = A_1 - A_2 = 0.074509\text{ m}^2$
   $F = pA = 100 \text{ x } 10^5 \text{ x } 0.074509 = 795 \text{ x } 10^3$ N

3. A single acting hydraulic cylinder has a piston 75 mm diameter and is supplied with oil at 100 bar gauge. Calculate the thrust.

   $A = \pi \text{ x } 0.075^2/4 = 0.004418\text{ m}^2$ $F = pA = 100 \text{ x } 10^5 \text{ x } 0.004418 = 44.179 \text{ x } 10^3$ N

4. A vertical hydraulic cylinder is used to support a weight of 50 kN. The piston is 100 mm diameter. Calculate the pressure required. (6.37 MPa)

   $p = F/A = 50\,000/\{\pi \text{ x } 0.1^2/4) = 6.37 \text{ x } 10^6$ N

THERMODYNAMICS, FLUID AND PLANT PROCESSES

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FLUID MECHANICS – THE VISCOUS NATURE OF FLUIDS
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

1. Describe the principle of operation of the following types of viscometers.

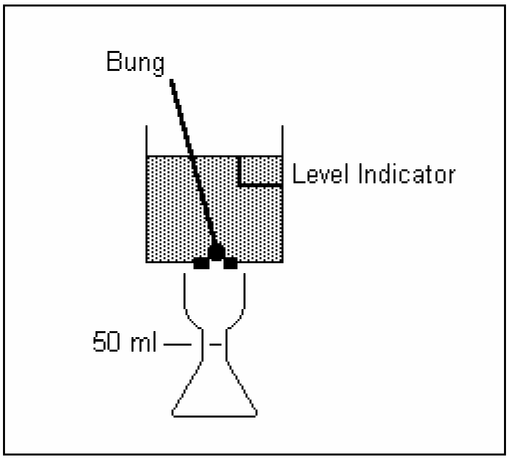
a. Redwood Viscometers.

b. British Standard 188 glass U tube viscometer.

c. British Standard 188 Falling Sphere Viscometer.

d. Any form of Rotational Viscometer

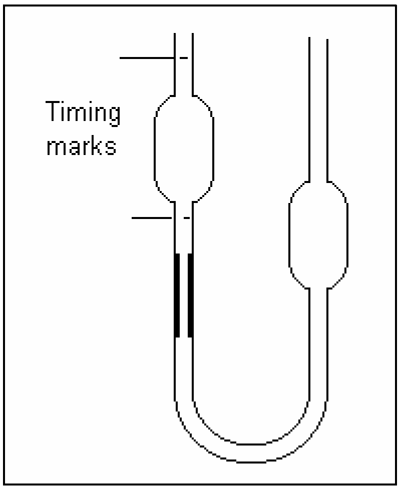
REDWOOD VISCOMETER



This works on the principle of allowing the fluid to run through an orifice of very accurate size in an agate block.

50 ml of fluid are allowed to empty from the level indicator into a measuring flask. The time taken is the viscosity in Redwood seconds. There are two sizes giving Redwood No.1 or No.2 seconds. These units are converted into engineering units with tables.

U TUBE VISCOMETER



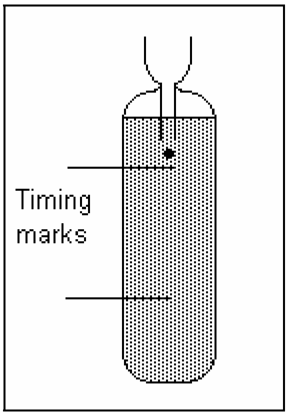
The fluid is drawn up into a reservoir and allowed to run through a capillary tube to another reservoir in the other limb of the U tube.

The time taken for the level to fall between the marks is converted into cSt by multiplying the time by the viscometer constant.

$$\nu = ct$$

The constant c should be accurately obtained by calibrating the viscometer against a master viscometer from a standards laboratory.

FALLING SPHERE VISCOMETER



This viscometer is covered in BS188 and is based on measuring the time for a small sphere to fall in a viscous fluid from one level to another. The buoyant weight of the sphere is balanced by the fluid resistance and the sphere falls with a constant velocity. The theory is based on Stoke's Law and is only valid for very slow velocities. The theory is covered later in the section on laminar flow where it is shown that the terminal velocity (u) of the sphere is related to the dynamic viscosity (μ) and the density of the fluid and sphere (ρ_f and ρ_S) by the formula

$$\mu = F\ gd^2(\rho_S - \rho_f)/18u$$

Fig.2.5

F is a correction factor called the Faxen correction factor, which takes into account a reduction in the velocity due to the effect of the fluid being constrained to flow between the wall of the tube and the sphere.

ROTATIONAL TYPES

There are many types of viscometers, which use the principle that it requires a torque to rotate or oscillate a disc or cylinder in a fluid. The torque is related to the viscosity. Modern instruments consist of a small electric motor, which spins a disc or cylinder in the fluid. The torsion of the connecting shaft is measured and processed into a digital readout of the viscosity in engineering units.

You should now find out more details about viscometers by reading BS188, suitable textbooks or literature from oil companies.

SELF ASSESSMENT EXERCISE No.2

Find examples of the following non- Newtonian fluids by searching the web.

1. Pseudo Plastic

   Gelatine, clay, milk, and liquid cement (particles suspended in liquid generally)

2. Bingham's Plastic
   Foodstuffs containing high level of fats approximate to this model (butter, margarine, chocolate and Mayonnaise).

3. Casson Plastic
   Fluids containing rod like solids and is often applied to molten chocolate and blood.

4. Dilatent Fluid

   Concentrated solutions of sugar in water and aqueous solutions of rice starch.

THERMODYNAMICS, FLUID AND PLANT PROCESSES

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FLUID MECHANICS – THE FLOW OF REAL FLUIDS
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE 3.1

1. A pipe 100 mm bore diameter carries oil of density 900 kg/m^3 at a rate of 4 kg/s. The pipe reduces to 60 mm bore diameter and rises 120 m in altitude. The pressure at this point is atmospheric (zero gauge). Assuming no frictional losses, determine:

i. The volume/s (4.44 dm^3/s)
ii. The velocity at each section (0.566 m/s and 1.57 m/s)
iii. The pressure at the lower end. (1.06 MPa)

$Q = m/\rho = 4/900 = 0.00444\ m^3/s$
$u_1 = Q/A_1 = 0.00444/(\pi \times 0.05^2) = 0.456$ m/s $\quad u_2 = Q/A_2 = 0.00444/(\pi \times 0.03^2) = 1.57$ m/s
$h_1 + z_1 + u_1^2/2g = h_2 + z_2 + u_2^2/2g \quad h_2 = 0 \quad z_1 = 0$
$h_1 + 0 + 0.566^2/2g = 0 + 120 + 1.57^2/2g$
$h_1 = 120.1$ m $\quad p = \rho g h = 900 \times 9.81 \times 120.1 = 1060$ kPa

2. A pipe 120 mm bore diameter carries water with a head of 3 m. The pipe descends 12 m in altitude and reduces to 80 mm bore diameter. The pressure head at this point is 13 m. The density is 1000 kg/m^3. Assuming no losses, determine

i. The velocity in the small pipe (7 m/s)
ii. The volume flow rate. (35 dm^3/s)

$h_1 + z_1 + u_1^2/2g = h_2 + z_2 + u_2^2/2g \quad 3 + 12 + u_1^2/2g = 13 + 0 + u_2^2/2g$
$2 = (u_2^2 - u_1^2)/2g \quad (u_2^2 - u_1^2) = 39.24$
$u_1 A_1 = Q = u_2 A_2 \quad u_1 = u_2 (80/120)^2 = 0.444\ u_2$
$39.24 = u_2^2 - (0.444\ u_2)^2 = 0.802\ u_2^2 \quad u_2 = 6.99$ m/s $\quad u_1 = 3.1$ m/s
$Q = u_2 A_2 = 6.99 \times \pi \times 0.04^2 = 0.035\ m^3/s$ or 35 dm^3/s

3. A horizontal nozzle reduces from 100 mm bore diameter at inlet to 50 mm at exit. It carries liquid of density 1000 kg/m^3 at a rate of 0.05 m^3/s. The pressure at the wide end is 500 kPa (gauge). Calculate the pressure at the narrow end neglecting friction. (196 kPa)

$A_1 = \pi D_1^2/4 = \pi(0.1)^2/4 = 7.854 \times 10^{-3}\ m^2$
$A_2 = \pi D_2^2/4 = \pi(0.05)^2/4 = 1.9635 \times 10^{-3}\ m^2$
$u_1 = Q/A_1 = 0.05/7.854 \times 10^{-3} = 6.366$ m/s
$u_2 = Q/A_2 = 0.05/1.9635 \times 10^{-3} = 25.46$ m/s
$p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
$500 \times 10^3 + 100 \times (6.366)^2/2 = p_2 + 1000 \times (25.46)^2/2 \quad p_2 = 196$ kPa

4. A pipe carries oil of density 800 kg/m^3. At a given point (1) the pipe has a bore area of 0.005 m^2 and the oil flows with a mean velocity of 4 m/s with a gauge pressure of 800 kPa. Point (2) is further along the pipe and there the bore area is 0.002 m^2 and the level is 50 m above point (1). Calculate the pressure at this point (2). Neglect friction. (374 kPa)

$800 \times 10^3 + 800 \times 4^2/2 + 0 = p_2 + 800\ 10^2/2 + 800 \times 9.81 \times 50$
$p_2 = 374$ kPa

5. A horizontal nozzle has an inlet velocity u_1 and an outlet velocity u_2 and discharges into the atmosphere. Show that the velocity at exit is given by the following formulae.

$$u_2 = \{2\Delta p/\rho + u_1^2\}^{½}$$

and $$u_2 = \{2g\Delta h + u_1^2\}^{½}$$

$p_1 + \rho u_1^2/2 + \rho g z_1 = p_2 + \rho u_2^2/2 + \rho g z_2 \qquad z_1 = z_2$

$p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$

$p_1 - p_2 = (\rho/2)(u_2^2 - u_1^2) \qquad 2(p_1 - p_2)/\rho = (u_2^2 - u_1^2)$

$u_2 = \sqrt{(2\Delta p/\rho + u_1^2)}$

Substitute $p = \rho g h$ and $u_2 = \sqrt{\{2g\Delta h + u_1^2\}}^{½}$

SELF ASSESSMENT EXERCISE 3.2

1. Oil flows in a pipe 80 mm bore diameter with a mean velocity of 0.4 m/s. The density is 890 kg/m^3 and the viscosity is 0.075 Ns/m^2.

Show that the flow is laminar and hence deduce the pressure loss per metre length. (150 Pa)

$$R_e = \frac{\rho u d}{\mu} = \frac{890 \text{ x } 0.4 \text{ x } 0.08}{0.075} = 379.7$$

Since this is less than 2000 flow is laminar so Poiseuille's equation applies.

$$\Delta p = \frac{32\mu 2\mu}{d^2} = \frac{32 \text{ x } 0.075 \text{ x } 1 \text{ x } 0.4}{0.08^2} = 150 \text{ Pa}$$

2. Calculate the maximum velocity of water that can flow in laminar form in a pipe 20 m long and 60 mm bore. Determine the pressure loss in this condition. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 $N s/m^2$. (0.0333 m/s and 5.92 Pa)

$$R_e = 2000 = \frac{\rho u d}{\mu} = \frac{1000 \text{ x U x } 0.06}{0.001} \qquad \text{Hence u = 0.033 m/s}$$

$$\Delta p = \frac{32 \mu L u}{d^2} = \frac{32 \text{ x } 0.001 \text{ x } 20 \text{ x } 0.033}{0.06^2} = 5.92 \text{ Pa}$$

3. Oil flow in a pipe 100 mm bore diameter with a Reynolds Number of 500. The density is 800 kg/m^3. The dynamic viscosity $\mu = 0.08$ Ns/m^2. Calculate the velocity of a streamline at a radius of 40 mm. (0.36 m/s)

$$R_e = 500 = \frac{\rho u_m d}{\mu}$$

$$u_m = \frac{500\mu}{\rho d} = \frac{500 \text{ x } 0.08}{800 \text{ x } 0.1} = 0.5 \text{ m/s}$$

Since R_e is less than 2000 flow is laminar so Poiseuille's equation applies.

$$\Delta p = \frac{32\mu 2\mu}{d^2} = \frac{32 \text{ x } 0.08 \text{ x L x } 0.5}{0.1^2} = 128L \text{ Pa}$$

$$u = \frac{\Delta p\left(R^2 - r^2\right)}{4L\mu} = \frac{128L\left(0.05^2 - 0.04^2\right)}{4L \text{ x } 0.08} = 0.36 \text{ m/s}$$

4a When a viscous fluid is subjected to an applied pressure it flows through a narrow horizontal passage as shown below. By considering the forces acting on the fluid element and assuming steady fully developed laminar flow, show that the velocity distribution is given by

$$-\frac{dp}{dx} = \mu\frac{d^2u}{dy^2}$$

b. Using the above equation show that for flow between two flat parallel horizontal surfaces distance t apart the velocity at any point is given by the following formula.

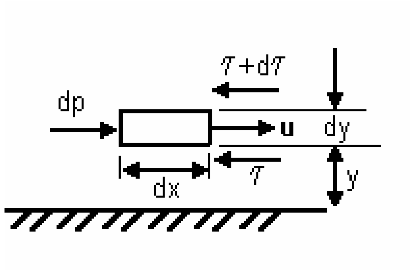
$u = (1/2\mu)(dp/dx)(y^2 - ty)$

c. Carry on the derivation and show that the volume flow rate through a gap of height 't' and width 'B' is given by $Q = -B\frac{dp}{dx}\frac{t^3}{12\mu}$.

d. Show that the mean velocity 'u_m' through the gap is given by $u_m = -\frac{1}{12\mu}\frac{dp}{dx}t^2$

Part (a)

A shear stress τ exists between each layer and this increases by $d\tau$ over each layer. The pressure difference between the downstream end and the upstream end is dp.



The pressure change is needed to overcome the shear stress. The total force on a layer must be zero so balancing forces on one layer (assumed 1 m wide) we get the following.

$$dp\,dy + d\tau\,dL = 0 \qquad \frac{d\tau}{dy} = -\frac{dp}{dL}$$

It is normally assumed that the pressure declines uniformly with distance downstream so the ***pressure gradient*** $\frac{dp}{dL}$ is assumed constant. The minus sign indicates that the pressure falls with distance. Integrating between the no slip surface (y = 0) and any height y we get

$$-\frac{dp}{dL} = \frac{d\tau}{dy} = \frac{d\left(\mu\frac{du}{dy}\right)}{dy} \qquad -\frac{dp}{dL} = \mu\frac{d^2u}{dy^2}$$

Part (b)

Starting with $-\frac{dp}{dx} = \mu\frac{d^2u}{dy^2}$

It is assumed that dp/dx does not vary with y so it may be regarded as a fixed value in the following work.

Integrating once $-y\frac{dp}{dx} = \mu\frac{du}{dy} + A$

Integrating again $-\frac{y^2}{2}\frac{dp}{dx} = \mu u + Ay + B$............(2.6A)

A and B are constants of integration. The solution of the equation now depends upon the boundary conditions that will yield A and B.

When a fluid touches a surface, it sticks to it and moves with it. The velocity at the flat plates is the same as the plates and in this case is zero. The boundary conditions are hence

u = 0 when y = 0

Substituting into equation 2.6A yields that B = 0

U = 0 when y = t

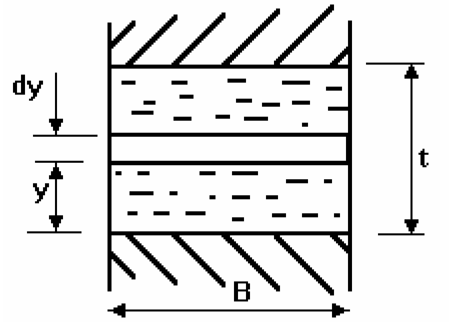
Substituting into equation 2.6A yields that A = (dp/dx)t/2

Putting this into equation 2.6A yields

$u = (dp/dx)(1/2\mu)\{y^2 - ty\}$

Part (c)

To find the flow rate we consider flow through a small rectangular slit of width B and height dy at height y.



The flow through the slit is $dQ = u\,Bdy = (dp/dx)(1/2\mu)\{y^2 - ty\}\,Bdy$

Integrating between y = 0 and y = t to find Q yields

$$\mathbf{Q = -B(dp/dx)(t^3/12\mu)}$$

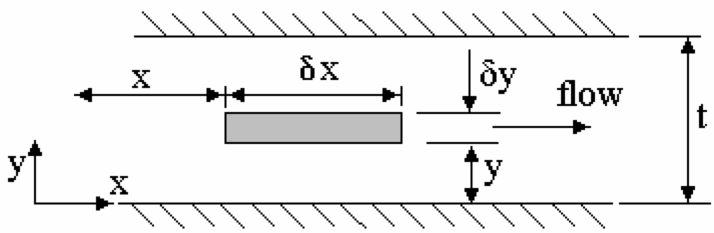
Part (d)

The mean velocity is $u_m = Q/Area = Q/Bh$

hence $\mathbf{u_m = -(dp/dx)(t^2/12\mu)}$

5. The volumetric flow rate of glycerine between two flat parallel horizontal surfaces 1 mm apart and 10 cm wide is 2 cm^3/s. Determine the following.

i. the applied pressure gradient dp/dx. (240 kPa per metre)
ii. the maximum velocity. (0.06 m/s)



For glycerine assume that μ= 1.0 Ns/m^2 and the density is 1260 kg/m^3.

$u_{mean} = Q/A = 2 \times 10^{-6}/(0.1 \times 0.001) = 0.02 m/s$
$u_{mean} = 0.02 = -(dp/dx)\{0.001^{2}/(12 \times 1)\}$

$(dp/dx) = -240$ kPa/m

$u = (dp/dx)(1/2\mu)\{y^2 - ty\}$ put y = t/2 for maximum

$u_{max} = (-240 \times 10^3)\{1/(2 \times 1)\}\{0.001^2 – 0.0005 \times 0.001\} = 0.06$ m/s

SELF ASSESSMENT EXERCISE 3.3

1. A pipe is 25 km long and 80 mm bore diameter. The mean surface roughness is 0.03 mm. It caries oil of density 825 kg/m^3 at a rate of 10 kg/s. The dynamic viscosity is 0.025 N s/m^2.

Determine the friction coefficient using the Moody Chart and calculate the friction head.
(Ans. 3075 m.)

$Q = m/\rho = 10/825 = 0.01212$ m^3/s
$u_m = Q/A = 0.01212/(\pi \times 0.04^2) = 2.411$ m/s
$Re = \rho u d/\mu = 825 \times 2.4114 \times 0.08/0.025 = 6366$
$k/D = 0.03/80 = 0.000375$
From the Moody chart $C_f = 0.0083$
$h_f = 4 C_f L u^2/2gd = 4 \times 0.0083 \times 25000 \times 2.4114^2/(2 \times 9.91 \times 0.08) = 3075$ m

2. Water flows in a pipe at 0.015 m^3/s. The pipe is 50 mm bore diameter. The pressure drop is 13 420 Pa per metre length. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2.

Determine the following.
i. The wall shear stress (167.75 Pa)
ii. The dynamic pressures (29180 Pa).
iii. The friction coefficient (0.00575)
iv. The mean surface roughness (0.0875 mm)

$u_m = Q/A = 0.015/(\pi \times 0.025^2) = 7.64$ m/s

Dynamic Pressure = $\rho u^2/2 = 1000 \times 7.64^2/2 = 29180$ Pa

$C_f = \tau_o$/Dyn Press = 167.75/29180 = 0.00575

From the Moody Chart we can deduce that $\varepsilon = 0.0017 = k/D$ $k = 0.0017 \times 50 = 0.085$ mm

3. Explain briefly what is meant by fully developed laminar flow. The velocity u at any radius r in fully developed laminar flow through a straight horizontal pipe of internal radius r_o is given by

$$\mathbf{u = (1/4\mu)(r_o^2 - r^2)dp/dx}$$

dp/dx is the pressure gradient in the direction of flow and μ is the dynamic viscosity. The wall skin friction coefficient is defined as $C_f = 2\tau_o/(\rho u_m^2)$.

Show that $C_f = 16/R_e$ where $R_e = \rho u_m D/\mu$ an ρ is the density, u_m is the mean velocity and τ_o is the wall shear stress.

3) THE BOUNDARY LAYER IS ENTIRELY LAMINAR AND CONSTANT THICKNESS

$$u = \frac{1}{4\mu}(r_0^2 - r^2)\frac{dp}{dx} \qquad \text{ASSUME } \frac{dp}{dx} = \frac{\Delta p}{L}$$

$$dQ = u \times 2\pi r\, dr \quad Q = \frac{1}{4\mu}\frac{\Delta p}{L} \times 2\pi \int_0^{r_0} (r r_0^2 - r^3)\, dr$$

$$Q = \frac{\Delta p \pi}{2\mu L}\left[\frac{r^2 r_0^2}{2} - \frac{r^4}{4}\right]_0^{r_0} = \frac{\Delta p \pi}{2\mu L}\left[\frac{r_0^4}{2} - \frac{r_0^4}{4}\right]$$

$$Q = \frac{\Delta p \pi}{2\mu L}\frac{r_0^4}{4} = \frac{\Delta p \pi\, r_0^4}{8\mu L} \qquad r_0 = D/2$$

$$Q = \frac{\Delta p \pi D^4}{128\mu L} \qquad Q = u_m \times A = u_m \frac{\pi D^2}{4}$$

$$u_m = \frac{\Delta p D^2}{32\mu L} \qquad \Delta p = \frac{32\mu L\, u_m}{D^2}$$

$$C_f = \frac{2\tau_0}{\rho u_m^2} \qquad \tau_0 \times \pi D L = \Delta p \times \frac{\pi D^2}{4}$$

$$\tau_0 = \frac{\Delta p}{L}\frac{D}{4}$$

$$C_f = \frac{2\Delta p\, D}{\rho u_m^2\, 4L}$$

$$C_f = \frac{\Delta p\, D}{2\rho u_m^2 L} = \frac{\Delta p\, D}{2\rho L u_m} \times \frac{32\mu L}{\Delta p\, D^2} = \frac{32\mu}{2\rho D u_m}$$

$$C_f = \frac{16\mu}{\rho u_m D} = \frac{16}{Re}$$

4. Oil with viscosity 2 x 10^{-2} Ns/m^2 and density 850 kg/m^3 is pumped along a straight horizontal pipe with a flow rate of 5 dm^3/s. The static pressure difference between two tapping points 10 m apart is 80 N/m^2. Assuming laminar flow determine the following.

i. The pipe diameter.
ii. The Reynolds number.

Comment on the validity of the assumption that the flow is laminar.

4.

$\mu = 2\times10^{-2}\ Ns/m^2$

$\rho = 850\ kg/m^3$

$Q = 5\ dm^3/s$

$\Delta p = 80\ N/m^2$

$L = 10\ m$

POISEUILLES EQUATION

$$\Delta p = \frac{32 \mu L u_m}{D^2}$$

$$D^2 = \frac{32 \mu L u_m}{\Delta p} = \frac{32 \times 2\times10^{-2} \times 10 \times u_m}{80}$$

$D^2 = 0.08\ u_m$

$u_m = \frac{4Q}{\pi D^2} = \frac{4 \times 5\times10^{-3}}{\pi D^2}$

$D^2 = \frac{.08 \times .006366}{D^2}$

$u_m = \frac{.006366}{D^2}$

$D^4 = 509\times10^{-6}$

$D = 0.150\ m$

$$Re = \frac{\rho u D}{\mu} = \frac{850 \times u_m \times .15}{2\times10^{-2}}$$

$u_m = .006366 / .15^2 = .2829\ m/s$

$$Re = \frac{850 \times .2829 \times .15}{2\times10^{-2}} = 1803.7$$

$Re < 2000$

JUST LAMINAR

SELF ASSESSMENT EXERCISE 3.4

1. A pipe carries oil at a mean velocity of 6 m/s. The pipe is 5 km long and 1.5 m diameter. The surface roughness is 0.8 mm. The density is 890 kg/m^3 and the dynamic viscosity is 0.014 N s/m^2. Determine the friction coefficient from the Moody chart and go on to calculate the friction head h_f.

(Ans. C_f = 0.0045 h_f = 110.1 m)

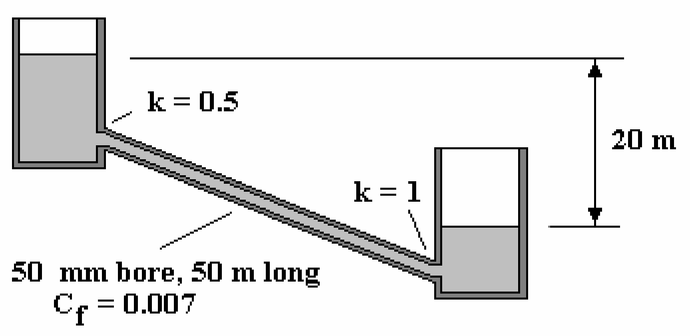
$\varepsilon = k/D = 0.8/1500 = 0.000533$

$R_e = \rho u D/\mu = 890 \times 6 \times 1.5/0.014 = 572 \times 10^3$

C_f = 0.0045 (from the Moody chart)

$h_f = 4 C_f L u^2/2Dg = 4 \times 0.0045 \times 5000 \times 6^2/(2g \times 1.5) = 110.1$ m

2. The diagram shows a tank draining into another lower tank through a pipe. Note the velocity and pressure is both zero on the surface on a large tank. Calculate the flow rate using the data given on the diagram. (Ans. 7.16 dm^3/s)



Apply Bernoulli between the free surfaces and h_L = 20 m

Losses are Inlet $0.5 u^2/2g$ Outlet $u^2/2g$ Pipe $4 C_f L u^2/2Dg$

$h_L = 0.5 u^2/2g + u^2/2g + 4 \times 0.007 \times 50 \times u^2/(2g \times 0.05)$

$20 = 0.5 + 1.0 + 28) u^2/(2g) = 29.5 u^2/(2g)$

$u^2 = 13.3$ $u = 3.647$ m/s

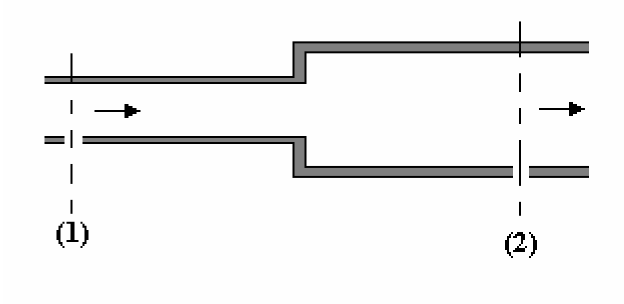
$Q = A u = (\pi \times 0.05^2/4) \times 3.647 = 7.161 \times 10^{-3}$ m^3/s

3. Water flows through the sudden pipe expansion shown below at a flow rate of 3 dm^3/s. Upstream of the expansion the pipe diameter is 25 mm and downstream the diameter is 40 mm. There are pressure tappings at section (1), about half a diameter upstream, and at section (2), about 5 diameters downstream. At section (1) the gauge pressure is 0.3 bar.

Evaluate the following.

(i) The gauge pressure at section (2) (0.387 bar)

(ii) The total force exerted by the fluid on the expansion. (-23 N)



$u_1 = Q/A_1 = 0.003/(\pi \times 0.0125^2) = 6.11$ m/s $\quad u_2 = Q/A_2 = 0.003/(\pi \times 0.02^2) = 2.387$ m/s

h_L (sudden expansion) $= (u_1^2 - u_2^2)/2g = 0.7067$ m

$u_1^2/2g + h_1 = u_2^2/2g + h_2 + h_L$

$h_1 - h_2 = 2.387^2/2g - 6.11^2/2 + 0.7067 = -0.9065$

$p_1 - p_2 = \rho g(h_1 - h_2) = 997 \times 9.81 \times (-0.9065) = -8866$ kPa

$p_1 = 0.3$ bar $\qquad p_2 = 0.3886$ bar

$p_1 A_1 - + \rho Q u_1 = p_2 A_2 - + \rho Q u_2 + F$

$0.3 \times 10^5 \times 0.491 \times 10^{-3} + 997 \times 0.003 \times 6.11 = 0.38866 \times 10^5 \times 1.257 \times 10^{-3} + 997 \times 0.003 \times 2.387 + F$

$F = -23$ N

If smooth $h_L = 0 \quad h_1 - h_2 = -1.613$ and $p_2 = 0.45778$ bar

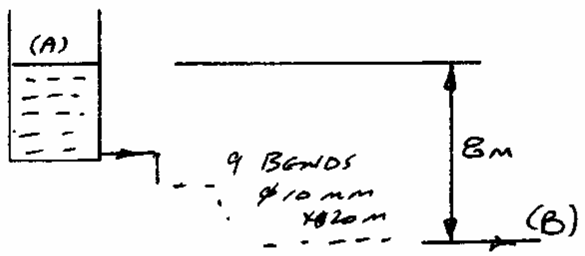
3. A domestic water supply consists of a large tank with a loss free-inlet to a 10 mm diameter pipe of length 20 m, that contains 9 right angles bends. The pipe discharges to atmosphere 8.0 m below the free surface level of the water in the tank.

Evaluate the flow rate of water assuming that there is a loss of 0.75 velocity heads in each bend and that friction in the pipe is given by the Blasius equation $C_f=0.079(Re)^{-0.25}$ (0.118 dm^3/s).

The dynamic viscosity is 0.89×10^{-3} and the density is 997 kg/m^3.

Q5 (Q5 1989)

$$C_f = 0.079\, Re^{-.25}$$
$$Re = \rho u D/\mu$$
$$\rho = 997\ kg/m^3$$
$$\mu = 0.89\times10^{-3}\ Ns/m^2$$



$$Re = 997 \times u \times 0.01 / 0.89\times10^{-3} = 11202\, u$$

$$C_f = 0.079\,(11202u)^{-.25} = 7.679\times10^{-3}\, u^{-.25}$$

$$h_f = \frac{4\, C_f\, L\, u^2}{2g\, d} = \frac{4 \times 7.679\times10^{-3}\, u^{-.25} \times 20\, u^2}{2g \times 0.01}$$

$$h_f = 3.1311\, u^{1.75}$$

$$\text{LOSS IN BENDS} = 9 \times \frac{.75 u^2}{2g} = 0.344\, u^2$$

BERNOULLI (A) → (B)

$$h_A + z_A + \frac{u_A^2}{2g} = h_B + z_B + \frac{u_B^2}{2g} + h_L$$

$$0 + 8 + 0 = 0 + 0 + \frac{u^2}{2g} + .344u^2 + 3.1311u^{1.75}$$

$$8 = .395u^2 + 3.1311u^{1.75}$$

SOLVE BY GUESSING OR NEWTON'S METHOD

$u = 1.5\ m/s$

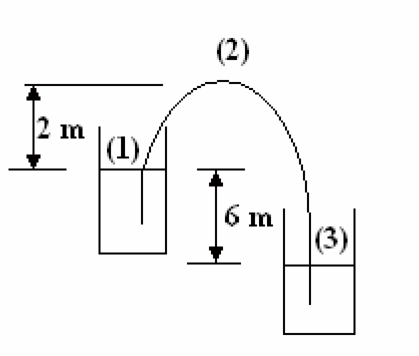
$Q = Au$

$Q = 117.8\times10^{-6}\ m^3/s$

4. A tank of water empties by gravity through a siphon into a lower tank. The difference in levels is 6 m and the highest point of the siphon is 2 m above the top surface level. The length of pipe from the inlet to the highest point is 3 m. The pipe has a bore of 30 mm and length 11 m. The friction coefficient for the pipe is 0.006.The inlet loss coefficient K is 0.6.

Calculate the volume flow rate and the pressure at the highest point in the pipe.

(Answers 2.378 dm^3/s and –4.31 m)



Total length = 11 m $C_f = 0.006$

Bernoulli between (1) and (3)

$h_1 + u_1^2/2g + z_1 = h_3 + u_3^2/2g + z_3 + h_L$

$0 + 6 + 0 = 0 + 0 + 0 + h_L$ $h_L = 6$

h_L= Inlet + Exit + pipe

$6 = 0.6\, u^2/2g + u^2/2g + (4 \times 0.006 \times 11/0.03)\, u^2/2g$

$6 = 0.6\, u^2/2g + u^2/2g + 8.8\, u^2/2g = 10.4\, u^2/2g$

u = 3.364 m/s

$Q = A\,u = (\pi \times 0.03^2/4) \times 3.364$ $Q = 0.002378\ m^3/s$

Bernoulli between (1) and (2)

$h_1 + u_1^2/2g + z_1 = h_2 + u_2^2/2g + z_2 + h_L$

$0 + 0 + 0 = h_2 + 2 + u^2/2g + h_L$

h_L= Inlet + pipe = $0.6\, u^2/2g + (3/11) \times 8.8\, u^2/2g$

$h_L = 0.6 \times 3.364^2/2g + (3/11) \times 8.8 \times 3.364/2g$

h_L= 1.73 m

$0 = h_2 + 2 + 3.364^2/2g + 1.73$

$h_2 = -4.31$ m

SELF ASSESSMENT EXERCISE 3.5

The density of water is 1000 kg/m^3 and the bulk modulus is 4 GPa throughout.

1. A pipe 50 m long carries water at 1.5 m/s. Calculate the pressure rise produced when

a) the valve is closed uniformly in 3 seconds. (25 kPa)
b) when it is shut suddenly. (3 MPa)

(a) $\Delta p = \rho Lu/t = 1000 \times 50 \times 1.5/3 = 25$ kPa

(b) $\Delta p = u(K\rho)^{0.5} = 1.5 \times (4 \times 10^9 \times 1000)^{0.5} = 3$ MPa

2. A pipe 2000 m long carries water at 0.8 m/s. A valve is closed. Calculate the pressure rise when

a) it is closed uniformly in 10 seconds. (160 kPa)
b) it is suddenly closed. (1.6 MPa)

(a) $\Delta p = \rho Lu/t = 1000 \times 2000 \times 0.8/10 = 160$ kPa

(b) $\Delta p = u(K\rho)^{0.5} = 0.8 \times (4 \times 10^9 \times 1000)^{0.5} = 1.6$ MPa

SELF ASSESSMENT EXERCISE 3.6

The density of water is 1000 kg/m^3 and the bulk modulus is 4 GPa throughout. The modulus of elasticity for steel E is 200 GPa.

1. A steel pipeline from a reservoir to a treatment works is 1 m bore diameter and has a wall 10 mm thick. It carries water with a mean velocity of 1.5 m/s. Calculate the pressure rise produced when the flow is suddenly interrupted. (1.732 MPa)

$$K' = \left\{\frac{D}{tE} + \frac{1}{K}\right\}^{-1} = \left\{\frac{1}{0.01 \text{ x } 200 \text{ x } 10^9} + \frac{1}{4 \text{ x } 10^9}\right\}^{-1} = 1.333 \text{ GPa}$$

Sudden closure $a' = (K'/\rho)^{½} = (1.333 \text{ x } 10^9/1000)^{½} = 1155$ m/s

$\Delta p = a' u \rho = 1155 \text{ x } 1.5 \text{ x } 1000 = 1.732$MPa

2. On a hydroelectric scheme, water from a high lake is brought down a vertical tunnel to a depth of 600 m and then connects to the turbine house by a horizontal high-pressure tunnel lined with concrete. The flow rate is 5 m^3/s and the tunnel is 4 m diameter.

i. Calculate the static pressure in the tunnel under normal operating conditions.(5.9 MPa)
ii. Explain the dangers to the high-pressure tunnel when the turbines are suddenly stopped.
iii. Assuming the tunnel wall is rigid, calculate the maximum pressure experienced in the high-pressure tunnel when flow is suddenly stopped. (6.7 MPa)
iv. Explain the safety features that are used in such situations to protect the tunnel.

The static pressure is approximately the height less the kinetic head.
$u = Q/A = 5/(\pi \text{ x } 4^2/4) = 0.398$ m/s
$p = \rho g h - u^2/2g = 1000 \text{ x } 9.81 \text{ x } 600 - 5^2/2g = 5.88$ MPa

When the turbines are suddenly stopped the momentum of the mass in the pipe creates very high pressure and shock waves that may fracture the tunnel walls.

$\Delta p = u(K\rho)^{0.5} = 0.398(4 \text{ x } 10^9 \text{ x } 1000)^{0.5} = 0.796$ MPa

Maximum pressure is 5.88 + 0.796 = 6.67 MPa

To protect the tunnel a surge tank with a damping orifice is connected to the tunnel so that the pressure is converted into height.

THERMODYNAMICS, FLUID AND PLANT PROCESSES

The tutorials are drawn from other subjects so the solutions are identified by the appropriate tutorial.

FLUID MECHANICS – DRAG
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

1. A smooth thin plate 5 m long and 1 m wide is placed in an air stream moving at 3 m/s with its length parallel with the flow. Calculate the drag force on each side of the plate. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.6 x 10^{-5} m^2/s.

$R_{ex} = u L/\nu = 3 \times 5/1.6 \times 10^{-5} = 937.5 \times 10^3$
$C_{DF} = 0.074 R_{ex}^{-1/5} = 4.729 \times 10^{-3}$
Dynamic Pressure = $\rho u_o^2/2 = 1.2 \times 3^2/2 = 5.4$ Pa
$\tau_w = C_{DF}$ x dyn press = 0.0255 Pa
$R = \tau_w \times A = 0.0255 \times 5 = 0.128$ N

2. A pipe bore diameter D and length L has fully developed laminar flow throughout the entire length with a centre line velocity u_o. Given that the drag coefficient is given as $C_{Df} = 16/Re$ where $Re = \frac{\rho u_o D}{\mu}$, show that the drag force on the inside of the pipe is given as $R=8\pi\mu u_o L$ and hence the pressure loss in the pipe due to skin friction is $p_L = 32\mu u_o L/D^2$

$C_{DF} = 16/R_e$

$R = \tau_w \times \rho u_o^2/2 = C_{DF} \times (\rho u_o^2/2) \times A$

$R = (16/R_e)(\rho u_o^2/2) A$

$R = (16\mu/\rho u_o D)(\rho u_o^2/2) \pi DL$

$R = (16 \mu u_o \pi L/2) = 8 \pi \mu u_o L$

$p_L = R/A = 8 \pi \mu u_o L /(\pi D^2/4) = 32 \mu u_o L/D^2$

SELF ASSESSMENT EXERCISE No.2

1. Calculate the drag force for a cylindrical chimney 0.9 m diameter and 50 m tall in a wind blowing at 30 m/s given that the drag coefficient is 0.8. The density of the air is 1.2 kg/m^3.

$C_D = 0.8 = 2R/(\rho u^2 A)$ $R = 0.8 (\rho u^2/2)A = 0.8 (1.2 \times 30^2/2)(50 \times 0.9) = 19440$ N

2. Using the graph (fig.12) to find the drag coefficient, determine the drag force per metre length acting on an overhead power line 30 mm diameter when the wind blows at 8 m/s. The density of air may be taken as 1.25 kg/m^3 and the kinematic viscosity as 1.5 x 10^{-5} m^2/s. (1.8 N).

$R_e = u d/\nu = 8 \times 0.03/1.5 \times 10^{-5} = 16 \times 10^3$

From the graph

$C_D = 1.5$

$R = C_D (\rho u_o^2/2)A = 1.5 (1.25 \times 8^2/2)(0.03 \times 1) = 1.8$ N

SELF ASSESSMENT EXERCISE No.3

1. a. Explain the term Stokes flow and terminal velocity.
   b. Show that the terminal velocity of a spherical particle with Stokes flow is given by the formula $u = d^2g(\rho_s - \rho_f)/18\mu$. Go on to show that $C_D=24/R_e$

Stokes flow –for ideal fluid - no separation - Re <0.2

R = Buoyant weight = $(\pi d^3/6)g\ (\rho_s - \rho_f) = 3\ \pi\ d\ \mu\ u_t$

$u_t = d^2\ g\ (\rho_s - \rho_f)/18\ \mu$

$R = C_D(\rho\ u_t^2/2)(\pi\ d^2/4)$

$C_D = 26\mu/(\rho\ u_t\ d) = 24/R_e$

2. Calculate the largest diameter sphere that can be lifted upwards by a vertical flow of water moving at 1 m/s. The sphere is made of glass with a density of 2630 kg/m^3. The water has a density of 998 kg/m^3 and a dynamic viscosity of 1 cP.

   $C_D = (2/\rho\ u^2A)R = \{(2 \times 4)/\ (\rho\ u^2\ \pi d^2)\}\ (\pi d^3/6)g\ (\rho_s - \rho_f) = 21.38\ d$

   Try Newton Flow first
   D = 0.44/21.38 = 0.206 m

   R_e =(998 x 1 x 0.0206)/0.001 = 20530 therefore this is valid.

3. Using the same data for the sphere and water as in Q2, calculate the diameter of the largest sphere that can be lifted upwards by a vertical flow of water moving at 0.5 m/s. (5.95 mm).

   $C_D = 85.52\ d$

   Try Newton Flow

   D = 0.44/85.52 = 0.0051

   R_e =(998 x 0.5 x 0.0051)/0.001 = 2567 therefore this is valid.

4. Using the graph (fig. 12) to obtain the drag coefficient of a sphere, determine the drag on a totally immersed sphere 0.2 m diameter moving at 0.3 m/s in sea water. The density of the water is 1025 kg/m^3 and the dynamic viscosity is 1.05 x 10^{-3} Ns/m^2. (0.639 N).

   $R_e = (\rho ud/\mu) = (1025 \times 0.3 \times 0.2/1.05 \times 10^{-3}) = 58.57 \times 10^3$

   From the graph $C_D = 0.45$

   $R = C_D\ (\rho u^2/2)A = 0.45(1025 \times 0.3^2/2)(\pi \times 0.2^2/4) = 0.65$ N

5. A glass sphere of diameter 1.5 mm and density 2 500 kg/m^3 is allowed to fall through water under the action of gravity. The density of the water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.

Calculate the terminal velocity assuming the drag coefficient is
$C_D = 24\ R_e^{-1}\ (1+ 0.15R_e^{\ 0.687})$ (Ans. 0.215 m/s

C_D = F/(Area x Dynamic Pressure)

$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} \qquad C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\pi\ d^2/4)(\rho\, u^2/2)} \qquad C_D = \frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ u^2 d^2}$$

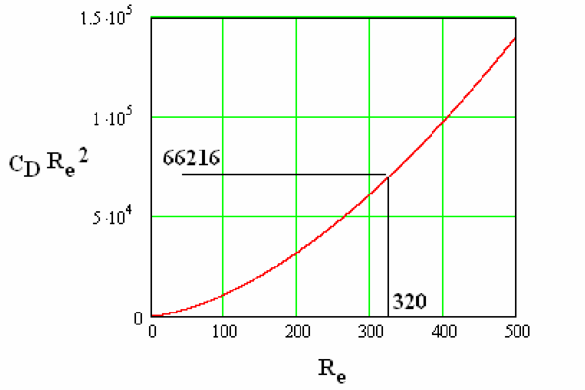
Arrange the formula into the form $C_D\ R_e^2$ as follows.

$$C_D = \frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ u^2 d^2} \text{x} \frac{\rho_f \mu^2}{\rho_f \mu^2} = \frac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2} \text{x} \frac{\mu^2}{\rho_f^2 u^2 d^2} = \frac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2} \text{x} \frac{1}{R_e^2}$$

$$C_D R_e^2 = \frac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2}$$

Evaluating this we get 66217

From $C_D = \frac{24}{R_e}\left[1 + 0.15R_e^{\ 0.687}\right]$



We may solve by plotting $C_D R_e^2$ against Re.
From the graph R_e = 320 hence u = R_e μ/ρd = 0.215 m/s

THERMODYNAMICS, FLUID AND PROCESS ENGINEERING C106

COMBUSTION THEORY

SAE 1 COMBUSTION BY MASS

Q1
80% CO_2 18% H_2 2% S

$C + O_2 \rightarrow CO_2$
12 32 48
0.8 2.133 2.933

$2H_2 + O_2 \rightarrow 2H_2O$
4 32 36
0.18 1.44 1.62

$S + O_2 \rightarrow SO_2$
32 32 64
0.02 0.02 0.04

Total O_2 = 3.593 kg
Air needed = 3.593/23% = 15.62 kg
30% Excess air so air supplied = 1.3 x 15.62 = 20.308 kg
Contents
N_2 = 0.77x 20.308 = 15.637 kg or 79%
SO_2 = 0.04 kg or 0.2%
CO_2 = 2.933 kg or 15%
O_2 = 1.078 kg or 5.8%
Total mass of products 19.69 kg

Q2
75% CO_2 15% H_2 7% S 3% Ash

$C + O_2 \rightarrow CO_2$
12 32 48
0.75 2 2.75

$2H_2 + O_2 \rightarrow 2H_2O$
4 32 36
0.15 1.2 1.35

$S + O_2 \rightarrow SO_2$
32 32 64
0.07 0.07 0.14

Total O_2 = 3.27 kg
Air needed = 3.27/23% = 14.217 kg
20% Excess air so air supplied = 1.2 x 14.217 = 17.06 kg
Contents
N_2 = 0.77x 17.06 = 13.137kg or 78.7%
SO_2 = 0.14 kg or 0.8%
CO_2 = 2.75 kg or 16.5%
O_2 = 0.654 kg or 4%
Total mass of products 16.681 kg

Q3

80% CO_2 10% H_2 5% S

$C + O_2 \rightarrow CO_2$

12 32 48

0.8 2.133 2.933

$2\ H_2 + O_2 \rightarrow 2\ H_2O$

4 32 36

0.1 0.8 0.9

$S + O_2 \rightarrow SO_2$

32 32 64

0.05 0.05 0.1

Total O_2 = 2.983 kg

Air needed = 2.983/23% = 12.97 kg

Contents

N_2 = 0.77x 12.97 =	9.986 kg or 76.7%
SO_2 =	0.1 kg or 0.8%
CO_2 =	2.933 kg or 22.5%
O_2 =	0
Total mass of products	13.02 kg

SAE 2 COMBUSTION BY VOLUME

Q1

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$

1 m^3 3 m^3 2 m^3 2 m^3

Stoichiometric ratio =3/21% = 14.28/1

Q2

$C_4H_{10} + 6½ O_2 \rightarrow 4CO_2 + 5H_2O$

1 m^3 6½m^3 4 m^3 5 m^3

Stoichiometric ratio =6.5/21% = 30.95/1

30% excess air so air supplied = 30.95 x 1.3 = 40.238

Contents

N_2 = 0.79 x 40.238 =	31.79 m^3
CO_2 =	4 m^3
O_2 = 0.3 x 6.5 =	1.95 m^3
Total	37.74 m^3

% CO_2 = 4/37.74 = 10.6%

Q3

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

1 m^3 5 m^3 3 m^3 4 m^3

Stoichiometric ratio =5/21% = 23.8/1

20% Excess Air Air supplied = 1.2 x 23.8 = 28.571 m^3

Contents

N_2 = 0.79 x 28.571 =	22.57 m^3
CO_2 =	3 m^3
O_2 = 0.2 x 5 =	1 m^3
Total	26.57 m^3

% O_2 = 1/26.57 = 3.76%

Q4

0.4 m^3 H_2 needs 0,2 m^3 O_2 and makes .05 m^3 CO_2

0.4 m^3 CH_4 needs 0,8 m^3 O_2 and makes .4 m^3 CO_2

Total O_2 needed is 1 m^3

Stoichiometric ratio = 1/0.21 = 4.762 m^3

N_2 in air is .79 x 4.762 3.762

N_2 in gas is 0.25 m^3

Total N_2 = 3.912 m^3

Total dry gas = 0.05 + 0.4 + 3.912 = 4.362 m^3

% N_2 = 3.912/4.362 x 100 = 89.7 %

% CO_2 = 0.45/4.362 x 100 = 10.3 %

SAE 3

Q1

$C_2H_6 + 3\ ½\ O_2 \rightarrow 2CO_2 + 3H_2O$

$2C_2H_6 + 7\ O_2 \rightarrow 4CO_2 + 6H_2O$

Let the excess air be x

Stoichiometric Ratio

$O_2 \rightarrow 3.5\ m^3$

Air $\rightarrow$ 3.5/21% = 16.67 m^3

Actual Air = 16.67(1 + x)

DRY PRODUCTS

$N_2 \rightarrow$ 0.79 x 16.67(1 + x) =	13.167 +	13.167 x
$O_2 \rightarrow$		3.500 x
$CO_2 \rightarrow$	2.000	
TOTAL	15.167 +	16.667 x

% CO_2 is 8 so 8/100 = 2/(15.167 + 16.667 x)

200/8 = 15.167 + 16.667 x

25 – 15.167 = 16.667 x

9.833/16.667 = x = 0.59 or 59%

Q2

$C + O_2 \rightarrow CO_2$

12 32

0.85kg needs (32/12) x 0.85 = 11.2 kg O_2

$2H_2 + O_2 \rightarrow 2H_2O$

4 32

0.15kg needs (32/8) x 0.15 = 1.2 kg O_2

Total O_2 = 3.467 kg

Stoichiometric ratio = 3.467/0.233 = 14.88/1

DEG Consider 1 kmol

This contains 0.1 kmol of CO_2
y kmol of O_2
1 – 0.1 – y kmol of N_2

0.1 kmol of CO_2 is 0.1 x 44 = 4.4 kg of CO_2

Carbon in it is 12/44 x 4.4 = 1.2 kg Carbon

This came from the fuel.

FUEL consider 1 kmol

It contains 0.85 kmol of carbon.

0.85/12 = 0.708 kmol of Dry Exhaust Gas

NITROGEN

(0.9 – y) x 28 = 25.2 – 28y kg per kmol of DEG

OXYGEN

(25.2 – 28y)x 23.3/76.7 = 7.666 – 8.506 y per kmol of DEG (Oxygen supplied per kmol DEG)

Oxygen in CO_2 (32/44) x 4.4 = 3.2 kg per kmol DEG

Excess oxygen 32y kg per kmol DEG

Total Oxygen excluding that in the water is 32y + 3.2 kg per kmol DEG

Oxygen to burn H_2
Subtract from Oxygen supplied (7.655 – 8.506 y) – (32y + 3.2)

4.455 – 40.506y kg per kmol DEG

There are 0.708 kmol DEG

Oxygen to burn H_2 = 0.708(4.455 – 40.506y) = 3.154 – 28.678 y kg

$2H_2 + O2 \rightarrow$
4 32 ratio 8/1
1 8 for 0.1 kg H_2 0.8 kg O_2

0.8 = 3.145 – 28.678y y = 0.0818 kg

NITROGEN

25.2 – 28 x 0.0818 = 22.91 kg per kmol DEG
But there are 0.708 kmol so 25.91 x 0.708 = 16.22 kg N_2

AIR

16.22/0.767 = 21.1 kg

Excess air is 21.1 – 14.88 = 6.27 kg

% Excess = 6.27/14.88 = 42%

PLANT PROCESS PRINCIPLES

SAE 4 ENERGY OF COMBUSTION

Q1

$m = 0.4$ kg/s $\theta_2 = 70^oC$ $\theta_1 = 10^oC$

$\Phi = 0.4 \times 4.186 \times (70 - 10) = 100.464$ kW

Fuel Power $= 3.2 \times 10^{-3} \times 44\,000 = 140.8$ kW

$\eta = 100.46/140.8 = 0.713$ or 71.3 %

Q2

$m = 3$ kg/s $p_1 = 70$ bar $\theta 3 = 500^oC$ $\theta_w = 120^oC$

Specific enthalpy $h_3 = 3410$ kJ/kg

Specific enthalpy $h_w = 509$ kJ/kg

$\Phi = 3(3410 - 509) = 8703$ kW

Fuel Power $= 38 \times 10^3 \times 17/60 = 10\,767$ kW

$\eta = 8703/10\,767 = 0.81$ or 81%

Q3

$m_f = 0.98$ g

$m_w = 1.32$ kg

$Q = 1.32 \times 4.186 \times 5.7 = 31.495$ kJ/kg

Calorific value $= 31.495/0.98 \times 10^{-3} = 32\,138$ kJ/kg or 32.138 MJ/kg