



Fluid Mechanics

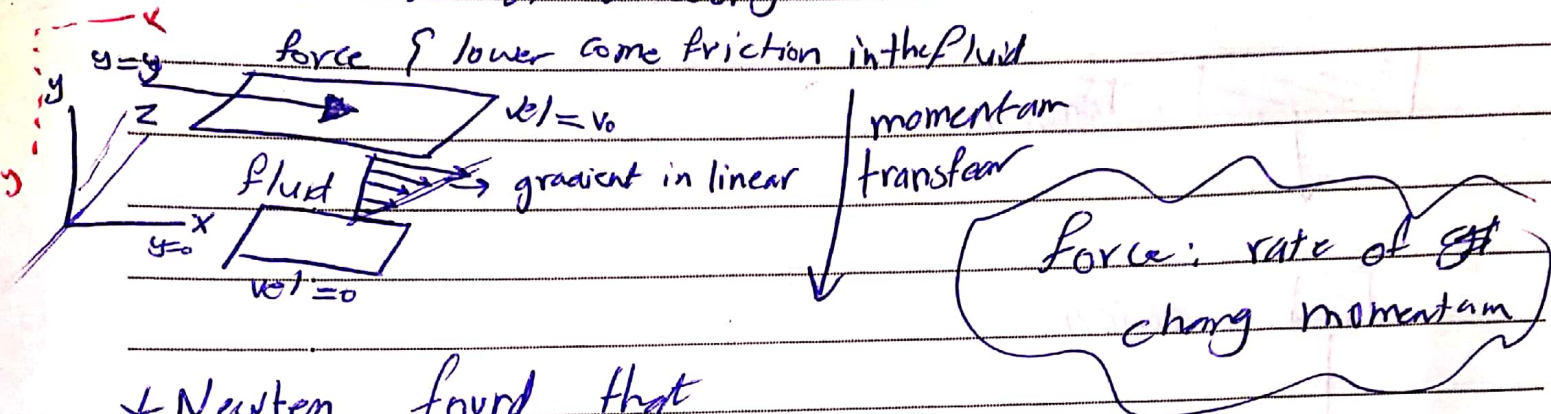
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* Viscosity: it is a measure of resistance to flow

Newton's law of viscosity

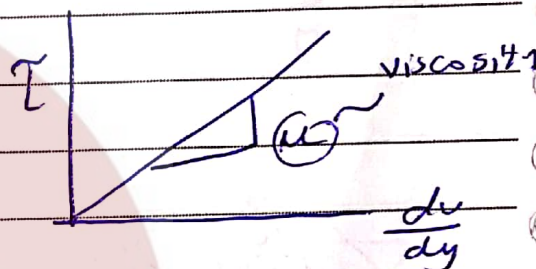


* Newton found that

Shear stress $\propto$ vel gradient

$$\tau = \frac{F}{A} \propto \frac{dv}{dy}$$

$\tau \propto \frac{dv}{dy} \rightarrow$ linear relation



$$\tau = \mu \frac{dv}{dy}$$

viscosity μ
or
coefficient of viscosity

Newton's law of viscosity

τ : shear stress $\frac{F}{A}$

$\frac{dv}{dy}$: velocity gradient (distance $\times$ velocity) (shear rate)
rate of strain
rate of shear deformation

deformation

$$\frac{dv}{dy} = \frac{d}{dy} \left(\frac{dx}{dt} \right) = \frac{d}{dt} \left(\frac{dx}{dy} \right)$$

strain

$$= \frac{d\gamma}{dt} = \dot{\gamma} \text{ rate of strain}$$

or γ (strain) y & x direction

$\dot{\gamma}$: strain

fluid is solid $\rightarrow$ deformation

* Note: in this case $\frac{dv}{dy}$ is + positive, but in

some book $\frac{dv}{dy}$ is - negative, as a result of the convention used to define the axes.

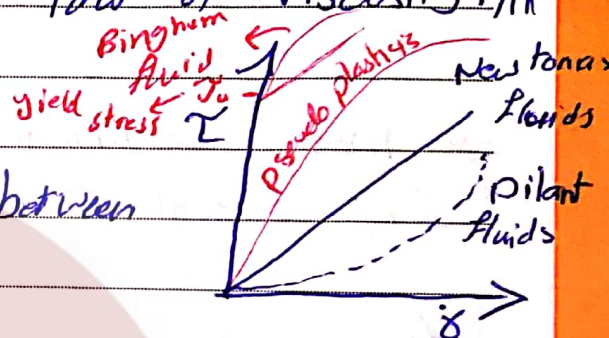
in such cases : Newtons law is written as $\tau = -\mu \frac{dv}{dz}$
 since τ is +ve quantity.

Types of Fluids.

- Newtonian Fluid : follow Newton's law of viscosity All gases and most liquids.

- Non Newtonian Fluids : relationship between τ and $\dot{\gamma}$ is not linear

Liquids with complex structure



* pseudo plastics $\mu \downarrow$ with $\dot{\gamma}$ { μ يقل مع $\dot{\gamma}$ }

* Dilant Fluid $\mu \uparrow$ with $\dot{\gamma}$ { μ يزداد مع $\dot{\gamma}$ }

independent on time

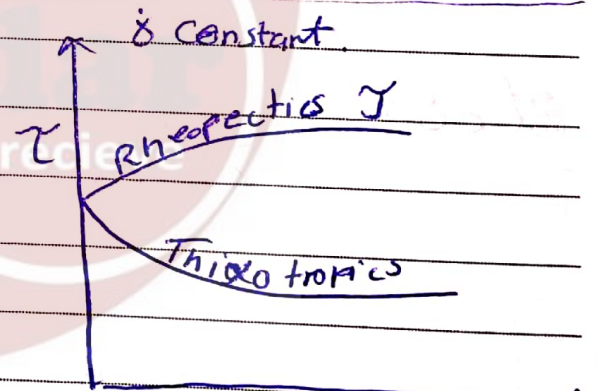
مستقلة عن الزمن

* Time dependent Fluids

*1 Rheopectics

viscosity increases with time

*2 Thixotropics $\mu \downarrow$ with time



* viscoelastic Fluids : They show elastic behavior

* Dependence of μ on T

liquids and high density gases : $\mu \downarrow$ with $T \uparrow$

but low density gases

: $\mu \uparrow$ with $T \downarrow$ { μ يزداد مع T تنخفض }
 { μ يقل مع T تزداد }

* Dependence on Pressure :

practically independent.

* Units:

$$\mu [=] \frac{\tau}{[du/dy]} = \frac{F/A}{du/dy \text{ ops}} = \frac{M L T^{-2} \cdot L^{-2}}{L^{-1} T^{-1} L^{-1}}$$

Basic units is the poise = $M L^{-1} T^{-1}$

$$1 \text{ poise} = 1 \text{ gm} / (\text{cm} \cdot \text{s})$$

$$= 6.72 \times 10^2 \text{ lbm} / (\text{ft} \cdot \text{s})$$

$$= 0.1 \text{ Pa} \cdot \text{s}$$

* most common units is the centi poise (CP)

$$1 \text{ cp} = 0.01 \text{ P}$$

$$\mu_{H_2O} \text{ at } 68^\circ \text{F} = 1 \text{ cp}$$

$$20^\circ \text{C}$$

* Kinematic viscosity

$$\nu = \frac{\mu}{\rho}$$

most common unit: Centisone.

$$1 \text{ cSt} = \frac{1 \text{ cP}}{1 \text{ g/cm}^3} = 1.08 \times 10^{-5} \text{ ft}^2/\text{s} = 10^{-6} \text{ m}^2/\text{s}$$

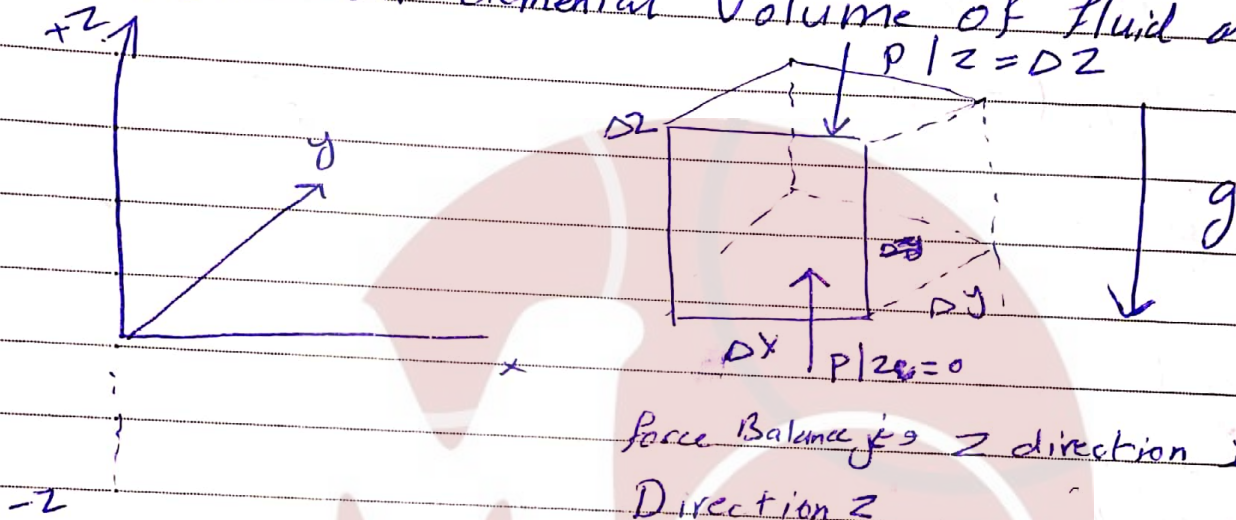
chapter 3 : Fluid statics

Basic eqn of fluid statics :

For simple fluid at rest, the pressure, at any point, in the fluid is the same in all direction
 * No shear stress *

In any direction $\sum F = 0$

consider an elemental volume of fluid at rest.



Force Balance in z direction is
 Direction z

$$\sum F_z = p|_{z=0} \times \Delta x \cdot \Delta y - p|_{z=\Delta z} \cdot \Delta x \cdot \Delta y$$

$$\Rightarrow p \Delta x \Delta y \Delta z g = 0$$

$\div \Delta x \Delta y \Delta z$ and rearranging

$$\frac{p|_{z=\Delta z} - p|_{z=0}}{\Delta z} = -g$$

Limits $\Delta z \rightarrow 0$ $\left(\frac{\Delta p}{\Delta z}\right) = \frac{dp}{dz} = -\rho g$

$\frac{dp}{dz} = -\rho g$

basic fluid statics eqn
 "Barometric equation"

P: density

Z: height

Notes:

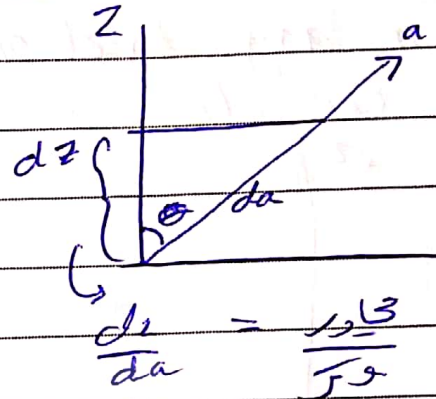
1. parametric eqn is correct for fluids at rest or fluid with no shear stresses on the vertical surface.

2. direction of change in p is not vertical

$$\frac{dp}{da} = \frac{dz}{da} \cdot \frac{dp}{dz}$$

$$= \frac{dz}{da} (-\rho g)$$

$$\therefore \frac{dp}{da} = -\rho g \cos \theta$$



3. For free surface - ^{not contain} ^{پہلو سے} of a liquid exposed to the atmosphere is ~~practically~~ practically a horizontal plane.

pressure Depth relationship

from Basic eqn: $dp = -\rho g dz$

Relation between p , g and z

- g is practically constant on the surface of the earth

- p may change with z , Appropriate expression which describes the variation must include

liquids: ρ is practically constant.

$$\int_{P_1}^{P_2} dp = -\rho g \int_{z_1}^{z_2} dz$$

$$P_2 - P_1 = -\rho g (z_2 - z_1)$$

$$P_2 - P_1 = +\rho g h$$

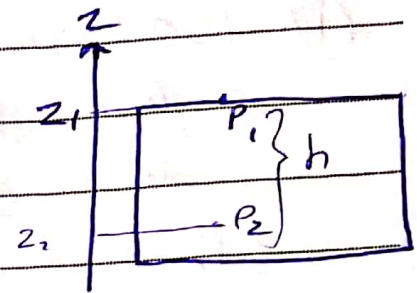
$$z_2 < z_1$$

$$(z_1 - z_2 = h)$$

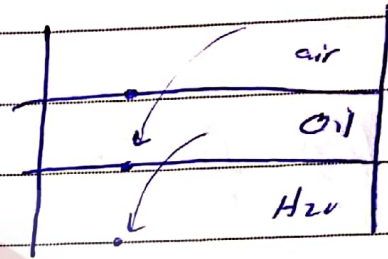
In problems involving free surface.

If $p_1 = \text{atmospheric pressure}$

$p_2 = \rho g h$ gage pressure



$$P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$$



نقطہ چینے کا اور اس کی کثافت

* Gases :

P depends on absolute temp and absolute pressure under which they exist.

For air and other real gases at conditions far from liquid phase $\Rightarrow$ super heat.

{ the ideal gas law maybe used } $\Rightarrow p \hat{V} = RT$
 absolute pressure $\nearrow$ $\hat{V}$ specific molar volume $\nwarrow$ absolute temperature $\nwarrow$ gas constant

$$p = \frac{P M_w}{R T} \quad \left. \vphantom{\frac{P M_w}{R T}} \right\} \text{EOS}$$

In general the EOS for ideal gas may be written as

$$p \hat{V}^n = \text{Constant}$$

$$K_{\text{air}} = 1.4$$

- n : +ve number
- isothermal condition $n = 1$
- $\rightarrow$ Isentropic (frictionless and adiabatic) $n = k = \frac{C_p}{C_v}$ heat capacity
- $\rightarrow$ Adiabatic with friction $n < k$ expansion
- $n > k$ Compression

① Isothermal perfect gas

$$dp = \rho g dz$$

$$dp = \frac{-\rho M_w \cdot g dz}{RT}$$

$$\int_{P_1}^{P_2} \frac{dp}{P} = \frac{-g M_w}{RT} \int_{z_1}^{z_2} dz$$

$$\ln \frac{P_2}{P_1} = - \frac{g M_w}{RT} (z_2 - z_1)$$

$$P_2 = P_1 \exp \left(- \frac{g M_w}{RT} \cdot \Delta z \right)$$

② Isotropic process

$$P \hat{V}^k = \text{Cont}$$

$$\frac{P}{e^k} = \text{Cont}$$

$$dp = -\rho g dz$$

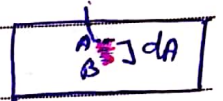
$$P_2 = P_1 \left[1 - \frac{k-1}{k} \frac{g M_w \Delta z}{RT_1} \right]^{\frac{k}{k-1}}$$

$$T_2 = T_1 \left[1 - \frac{k-1}{k} \frac{g M_w \Delta z}{RT} \right]$$

pressure forces on surfaces

static fluids exert normal forces on surfaces adjacent to them.

الضغط على السطح في السائل يعطى بالقوة $\vec{F}$ $\leftarrow$ pressure force
For an element area, the force exerted is
$$\vec{dF} = p \cdot \vec{dA}$$



النقطة A أقل عمقا من B $\leftarrow P_B > P_A$
بين الساحة يتم كسر الفرق التالي $\Delta Z \Rightarrow$ Zero
القوة عاكورة $dF = p dA$

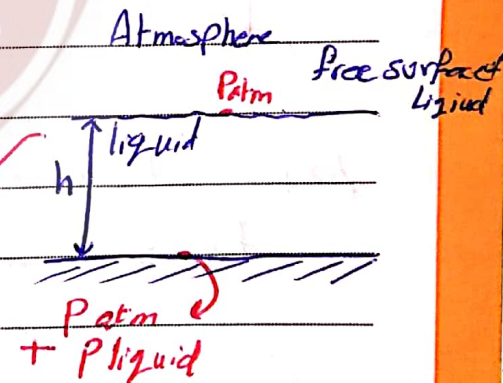
* Plane surfaces $\leftarrow$ الأسطح المسوية
All dF vectors point in the same direction
$$F = \int p dA$$

* horizontal plane surface.

* p Const

$$F = \int (P_{atm} + P_{liq}) dA$$

$$= (P_{atm} + P_{liq}) \int dA$$



$$F_{net} = F \text{ due to liquid only} = P_{liq} A = \rho g h A$$

$\leftarrow$ weight of liquid

Vertical Plane Surface

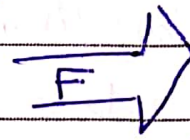
P varies on surface

$$F = \int P dA$$

$$= \int (P_{atm} + P_{liq}) dA$$

$$= P_{atm} A + \int P_{liq} dA$$

F_{sub}



$F_A < F_B < F_C$

$$dA = \sigma \cdot dA$$

$$F_{net} = \int P_{liq} dA = \rho g \int h dA$$

$$= \rho g \int_0^H \omega h dh = \rho g \omega \frac{h^2}{2} \Big|_0^H = \frac{\rho g \omega H^2}{2}$$

$$F_{net} = P_{atm} \cdot A + \rho g \int h \omega dh - P_0 A$$

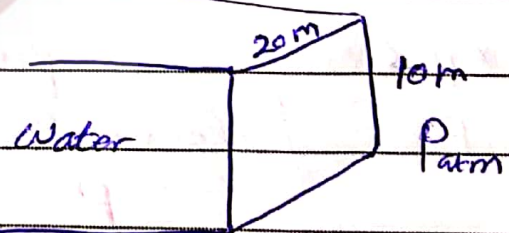
$\hookrightarrow$ atmospheric

$$P = \rho g h, \quad F = P dA$$

Ex: Find F_{net} الكبر

$$F_{net} = F_{underside} - F_{atmosphere}$$

$$P_{atm} \cdot A - \frac{\rho g H^2 \omega}{2} - P_{atm} \cdot A$$



$$= \frac{1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ m s}^{-1} \cdot 100 \text{ m}^2 \cdot 20 \text{ m}}{2}$$

$$= 9.81 \times 10^6 \text{ N}$$

$$F_{net} = \int P g h dA$$

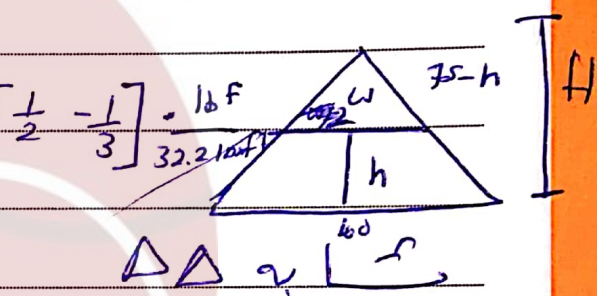
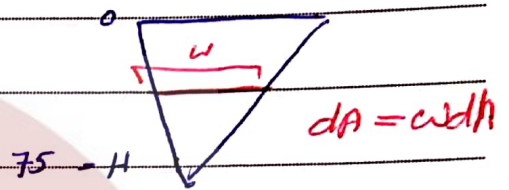
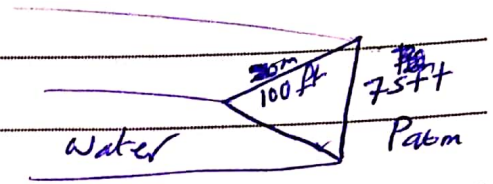
$$= P g \int_0^H h m dh$$

$$= P g \int_0^H \left(100 \left(1 - \frac{h}{H} \right) \right) h dh$$

$$100 P g \left[\frac{H^2}{2} - \frac{H^2}{3} \right]$$

$$= 100 \text{ ft} \cdot 62.3 \text{ lbm/ft}^3 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 75 \text{ ft}^2 \left[\frac{1}{2} - \frac{1}{3} \right] \cdot \frac{\text{lb f}}{32.2 \text{ lbm/ft}^3}$$

$$= 5.84 \times 10^6 \text{ lb f}$$

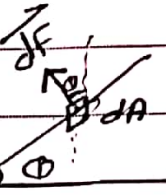


$$\frac{H-h}{H} = \frac{w}{100}$$

$$w = 100 \left(1 - \frac{h}{H} \right)$$

Inclined plane surfaces.

pressure force is $\perp$ to surface.

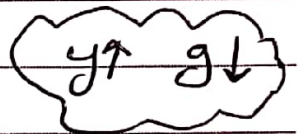


$dF = p dA$, all dF vector point in the same direction

and make angle θ with vector

Net force: $dF_x = p \sin \theta dA \rightarrow \text{to the right}$

$$dF_y = -p \cos \theta dA$$



Convention: θ measured anticlockwise

$$\theta < \frac{\pi}{2} \rightarrow \cos \theta > 0 \rightarrow F_y = -ve$$

$$\pi > \theta > \frac{\pi}{2} \rightarrow F_y = +ve$$

$$\theta = \frac{\pi}{2} \rightarrow \cos \theta = 0 \Rightarrow F_y = 0 \quad P_B = P_A$$

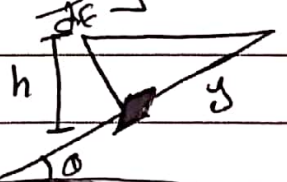
* Note: In many cases it is easier to work with Centroid [First moment of area]

$$= p g \bar{y}_c \sin \theta A$$

area $\rightarrow$ $dF_{net} = p dA$
 $= p g h dA$

$$F = \iint p g h dA$$

depth of centroid



$$h = y \sin \theta$$

$$F = p g \iint y \sin \theta dA$$

$$= p g \sin \theta \left(\iint y dA \right) \cdot \frac{A}{A}$$

Average distance

$$\left\{ \begin{aligned} h_c &= \frac{\iint y dA}{\iint dA} = \frac{\iint y dA}{A} = \text{Centroid of plane area.} \end{aligned} \right.$$

$$dF_{net} = \rho g y_c \sin \theta A$$

P at centroid.

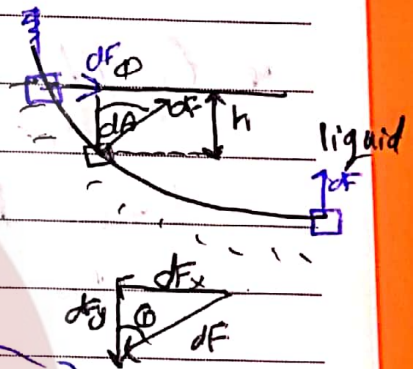
* pressure forces on curved surface.

whether curve or plane $dF = p dA$ $dF \perp dA$
 for curve surfaces. dF vectors are not all the same direction.

غير متساوية

these are correct for any surface

$$\begin{cases} F_x = \int dF_x = \iint P \sin \theta dA \\ F_y = \int dF_y = - \iint P \cos \theta dA \end{cases}$$



integral $\sin \theta$ بر ال $\sin \theta$ ← $\sin \theta$ plane $\sin \theta$
 $\cos \theta$ ← $\cos \theta$ plane $\cos \theta$
 لا يتغير

ask, believe & receive

* Notes :

① $\sin \theta dA = dA_x$

$\cos \theta dA = dA_y$

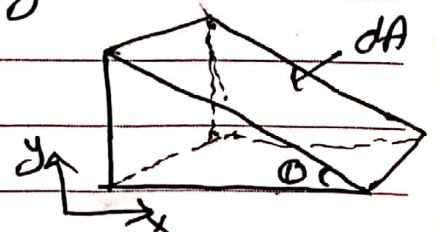
* x projection of dA *

* y " " "

② $F_x = \int P dA_x$

$= P_c \cdot A_x$

pressure at centroid.

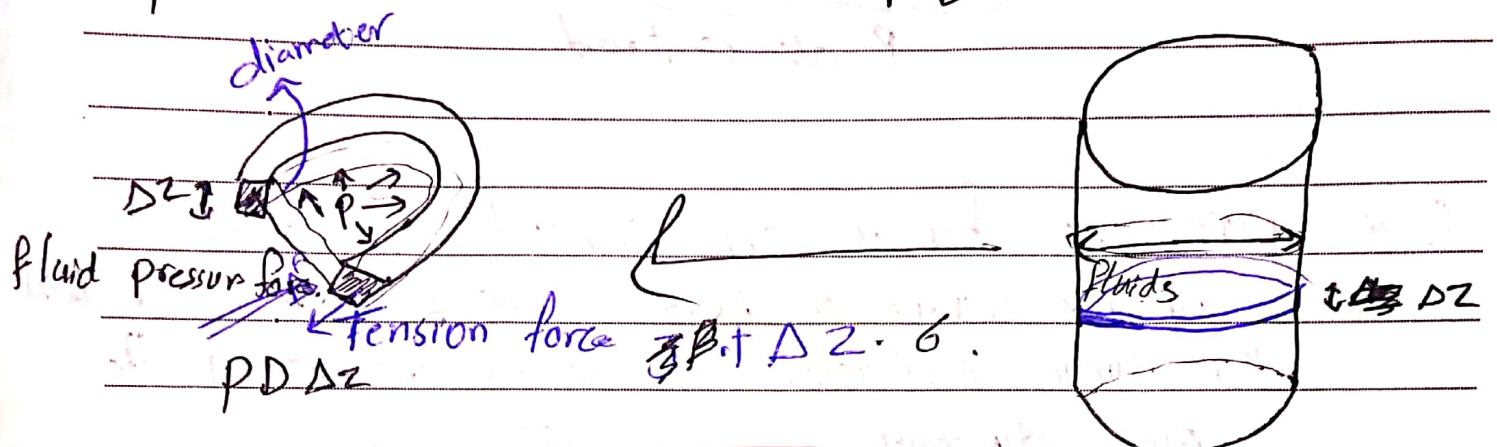


$dF_y = - P dA \cos \theta$
 projected Area

$= - \rho g h dA \cos \theta$ $du = - \rho g du$

$F_y = \rho g u$

Pressure vessels and piping:



* Liquid force is radial, but we are interested in the x direction.

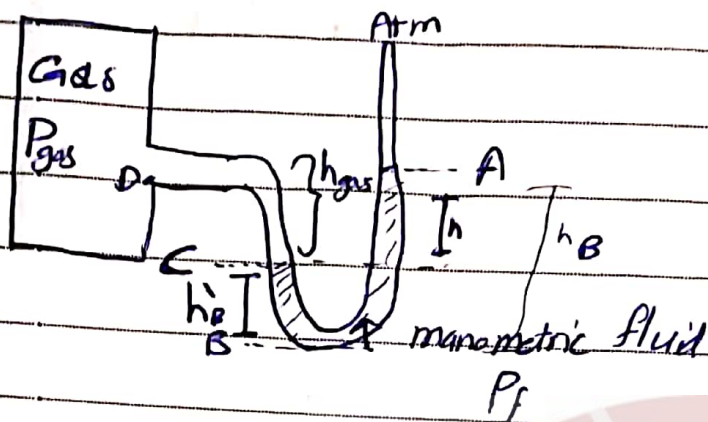
Force balance in x direction $2(6t\Delta z)pD\Delta z$

$$\text{thickness } t = \frac{pD}{2\sigma}$$

* Similar for sphere $p\frac{\pi}{6}D^2 = 6\pi Dt$

Let t_{max} be thickness of material liquid JV

Simple type : used for gases at low pressure



غير سام، ما يتغير بمرور الوقت
كأنه في حالة

* General procedure: start with known pressure and work backwards to the desired pressure

$$A: P_A = P_{atm}$$

$$B: P_B = P_{fluid} h_B + P_A$$

$$C: P_C = P_B - \rho_f g h'_B$$

$$D: P_D = P_C - \rho_{gas} g h_{gas}$$

$$P_D = P_{atm} + \rho_{fluid} (h_B - h'_B) g - \rho_{gas} h_{gas} g$$

$$P_D = P_{atm} + \rho g h - \rho_{gas} g h_{gas} \rightarrow \text{Very small (negligible)}$$

$$P_D = P_{atm} + \rho_{fluid} g h$$

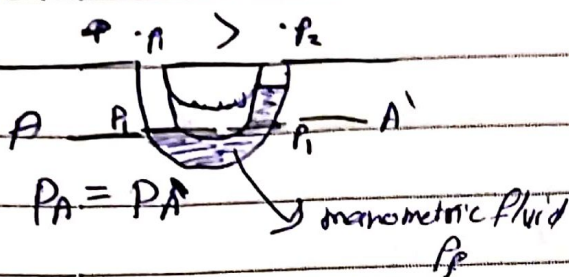
absolute pressure.

$$P_{D \text{ gage}} = \rho_{fluid} g h$$

gage pressure.

* Gas pressure difference type:

Gas →



نوع المقياس الذي يقيس الفرق في الضغط

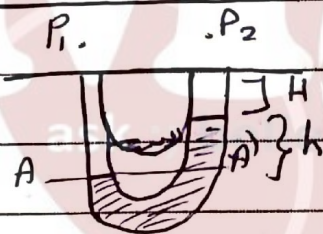
* Neglecting gas static pressure *

$$P_1 = P_2 + \rho g h$$

$$P_1 - P_2 = \Delta P = \rho g h$$

Differential type (liquid Δp)
differe in density

liquid →



صانعة بفل المقياس الذي يقيس الفرق في الضغط

$$P_A = P_{A'}$$

$$P_A = P_1 + \rho_F g (H+h)$$

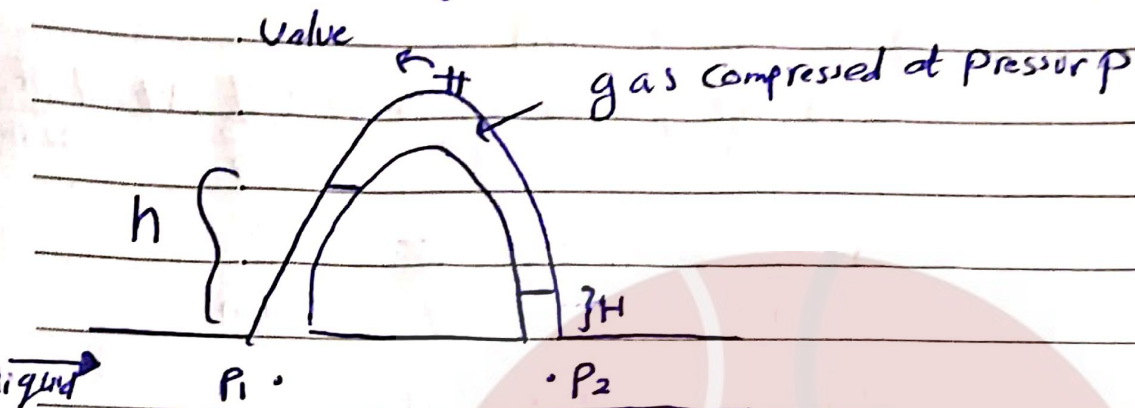
$$P_{A'} = P_2 + \rho_F g H + \rho g h$$

$$P_1 + \rho_F g (H+h) = P_2 + \rho_F g H + \rho g h$$

$$\Delta P = P_1 - P_2 = (\rho_F - \rho) g h$$

نوع المقياس الذي يقيس الفرق في الضغط

Inverted type:

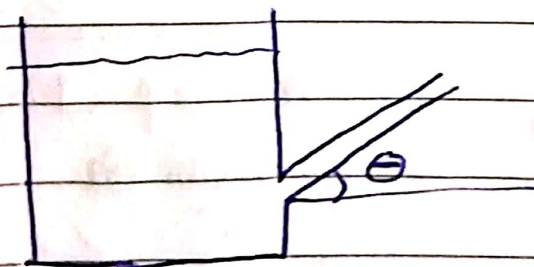


$$P_1 = P_{\text{gas}} + P_F g (H + h)$$

$$P_2 = P_{\text{gas}} + P_F g H$$

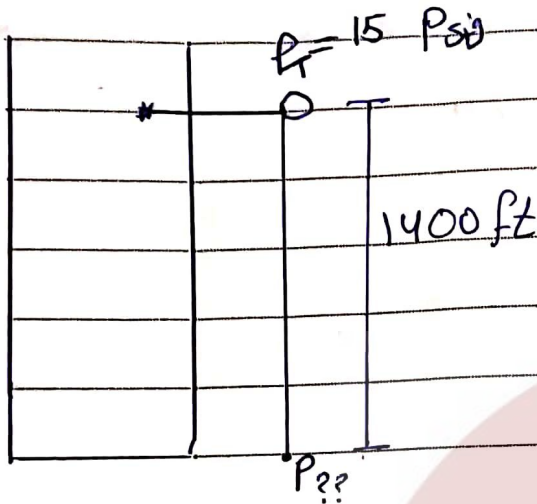
$$P_1 - P_2 = P_F g (h) \quad \text{different in height.}$$

* Inclined manometer



Problems

2 / 20



$$\begin{aligned}
 P_L &= P_T + \rho g h \\
 &= 15 \frac{\text{lb}_f}{\text{in}^2} + 62.3 \frac{\text{lb}_m}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 1400 \text{ ft} \cdot \frac{1.35}{32.2} \\
 &= 15 \frac{\text{lb}_f}{\text{in}^2} + 87220 \frac{\text{lb}_f}{\text{ft}^2} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} \\
 &= 15 \text{ Psig} + 606 \text{ Psig} \\
 &= 621 \text{ Psig}
 \end{aligned}$$

air ← vent

Air	
Gasoline	20 ft
H ₂ O	20 ft

$SG_{\text{gasoline}} = 0.72$

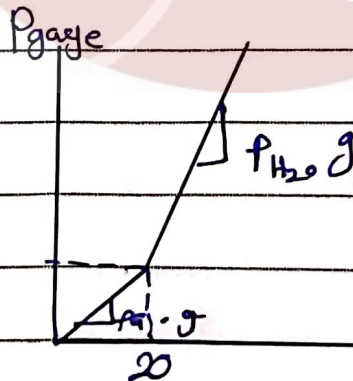
① find P_{abs} → $SG_g \cdot P_{\text{water}}$

$$P_{\text{abs}} = P_{\text{atm}} + \rho_g g h_g + \rho_{\text{H}_2\text{O}} g h_{\text{H}_2\text{O}}$$

$$= P_{\text{atm}} + (SG_g + 1) \rho_{\text{H}_2\text{O}} g h$$

$$h = h_g = h_{\text{H}_2\text{O}}$$

② plot P_{gage} & height

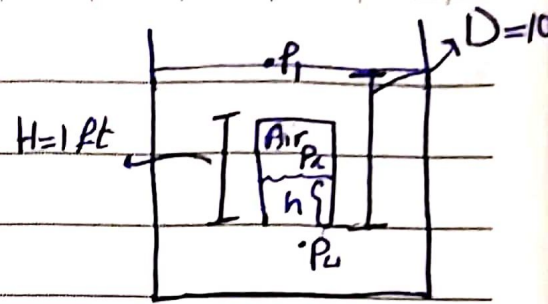


sketch of the graph
growth of the line
 $P_g < P_{\text{H}_2\text{O}}$

Originally pressure inside Can = P_{atm}

$$\frac{P_1 V_1}{T_1} = \frac{P_a V_a}{T_a}$$

$$T_1 = T_a$$



$$P_a = P_1 * \frac{V_1}{V_a}$$

$$P_1 = P_{atm}$$

$$V_1 = H * A ; V_a = (H-h) * A$$

$$P_a = P_{atm} * \frac{H A}{(H-h) A}$$

$$P_a = P_{atm} * \frac{H}{H-h}$$

$$P_w = P_{atm} + \rho g (D-h)$$

$$P_{atm} \left(\frac{H}{H-h} \right) = P_{atm} + \rho g (D-h)$$

$$h^2 - \left[\frac{P_{atm}}{\rho g} + (D+H) \right] h + DH = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = - \left[\frac{P_{atm}}{\rho g} + (D+H) \right] = \frac{-147 * 144}{62.3} \text{ ft}$$

$$c = DH = 10 \text{ ft}^2$$

* Calculate net force, F_x , F_y

$$dF = p dA$$

$$F_{net} = \int p g h dA$$

$$= \int p g h w dL$$

$$= p g w \int h dL$$

$$= p g w \int L \sin \theta dL$$

$$= p g w \sin \theta \int L dL$$

$$F_x = \int p dA \sin \theta$$

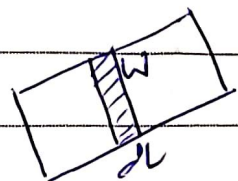
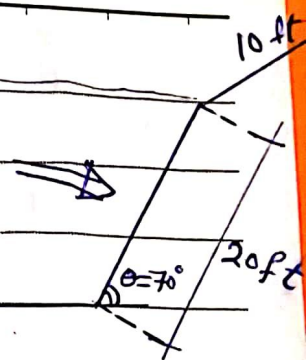
$$= p g w \int L \sin \theta \cdot \sin \theta dL$$

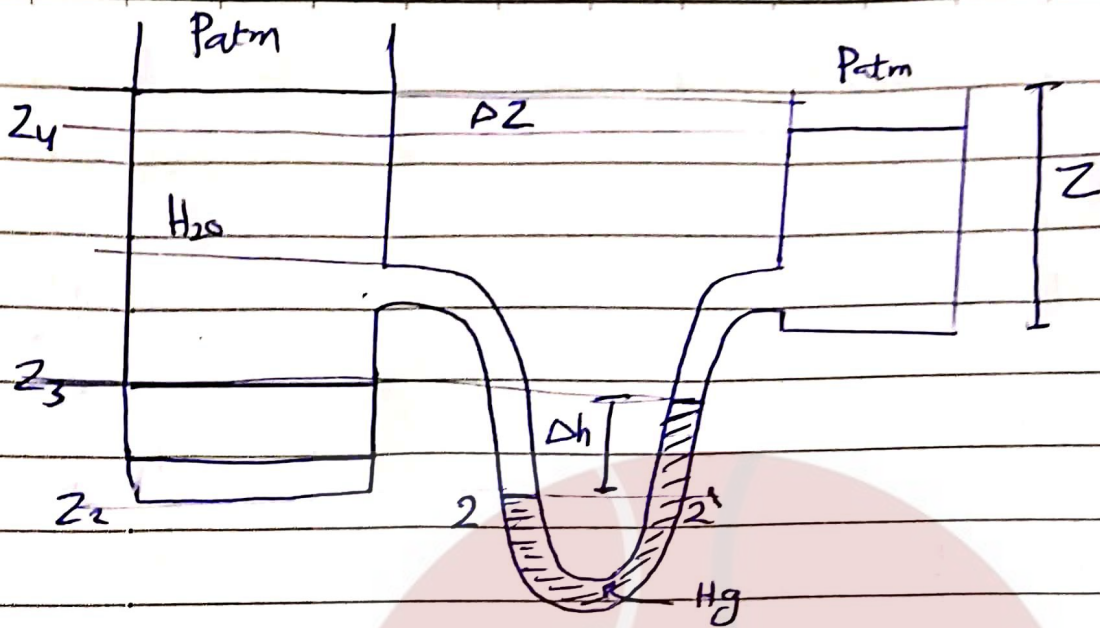
$$= F_x = F_{net} \sin \theta$$

جواب السؤال

$$F_y = \int p dA \cos \theta$$

$$= - F \cos \theta$$





$$P_2 = P_2'$$

$$P_2 = \frac{P_{atm}}{P_1} + \rho_{H_2O} g (z_1 - z_2)$$

$$P_2' = P_{atm} + \rho_{H_2O} g (z_4 - z_3) + \rho_{Hg} g (z_3 - z_2)$$

$z_2 = z_2'$

$$\rho_{H_2O} g (z_1 - z_2) = \rho_{H_2O} g (z_4 - z_3) + \rho_{Hg} g (\underbrace{z_3 - z_2}_{\Delta h})$$

$$\rho_w g \left[\underbrace{(z_1 - z_2)}_{\Delta Z} + \underbrace{(z_3 - z_2)}_{\Delta h} \right] = \rho_{Hg} g \Delta h$$

$$\Delta h = \Delta Z \frac{\rho_w}{(\rho_{Hg} - \rho_w)}$$

3.3 Mass Balance [Conservation of mass, Continuity eqn, Continuity principle]

Neglecting mass $\rightarrow$ energy conversion, the mass balance eqn's (Rate of accumulation of mass within specified boundaries)

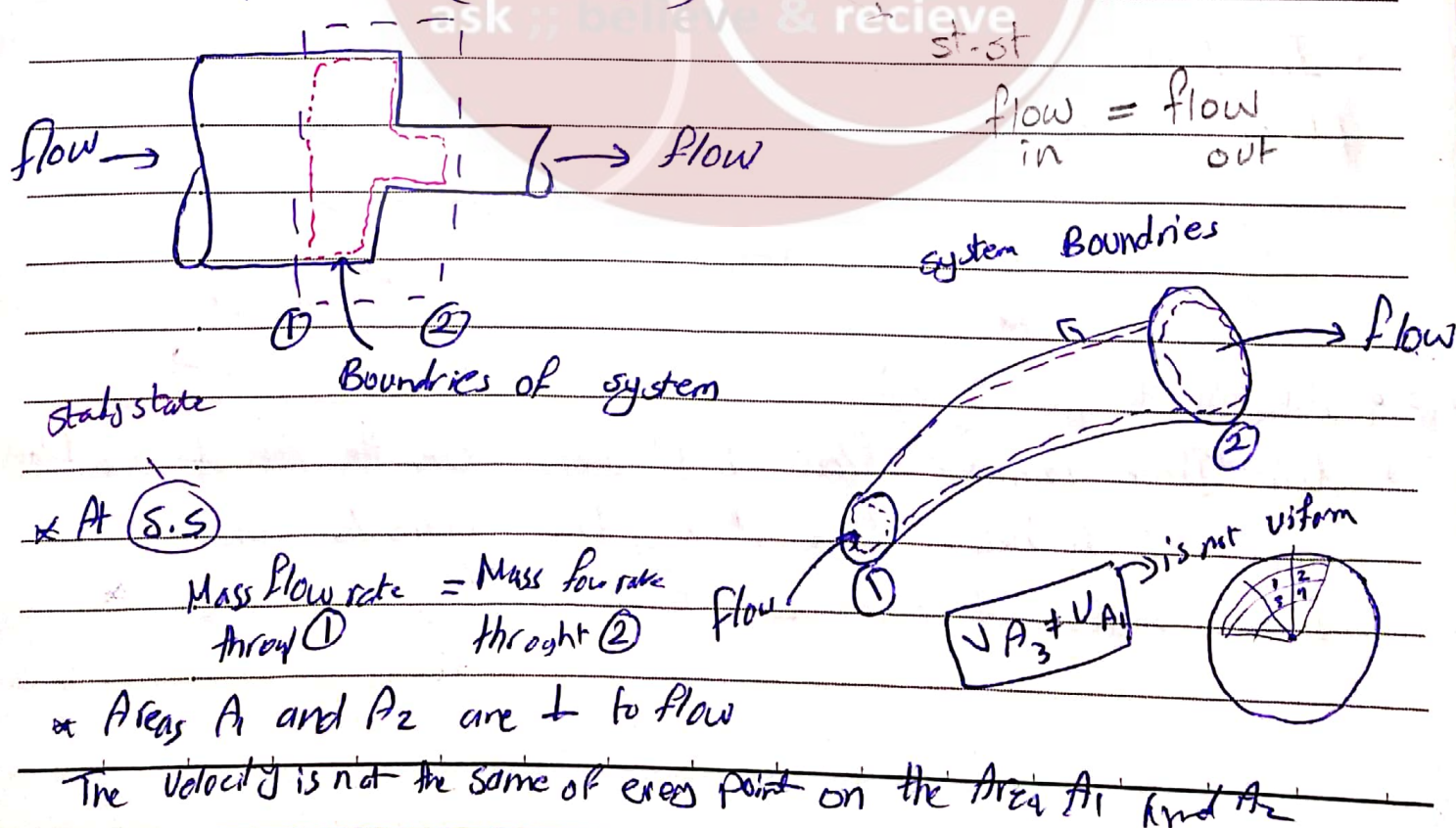
$$\text{Rate of flow of mass in} = \text{Rate of flow of mass out}$$

For mixture, component balance may contain the production or consumption terms for each component in reaction system

At steady state, the net rate of change at any point in space is zero [No Accumulation]

$$\left(\frac{\partial}{\partial t}\right)_{x,y,z} = 0$$

Macroscopic steady state Mass Balance.
~~uniform~~ Uniform: (تساوي في كل مكان)



Let U : uniform velocity over a small elemental area dA

* the fluid moves in distance Δ through dA over ~~area~~ unit time
and therefore: volume in or out is $U/\text{time} \rightarrow \text{distance } U$
 $U dA$ per unit time

the mass flowrate is $\rho U dA$.
 $\hookrightarrow$ velocity

At : S.S

$$0 = \underbrace{\int \rho U dA}_{\text{mass flow rate in}} - \underbrace{\int \rho U dA}_{\text{mass flow rate out}}$$

So $\int \rho U dA = \text{Constant}$
area $\perp$ flow

cross sections 1 2 3
 $m_{out} = m_{in}$

* In terms of average velocity

$$\rho U_1 A_1 = \rho U_2 A_2 = \text{in mass flow rate} = \text{Constant}$$

$$\frac{Q}{A} = \int \frac{U dA}{\int dA} = U_{avg}$$

$P \text{ Constant}$

(Flow rate) (مقدار جريان) (Flow rate)

* Note: ① For laminar flow : U varies over the Area $\perp$ to flow
while for turbulent flow, it may be considered constant
[most industrial flows are turbulent]

$$\textcircled{2} \quad P_1 U_1 A_1 = P_2 U_2 A_2$$

For constant density $\rho_1 = \rho_2 = \rho \rightarrow$ (for liquid in cont T Just

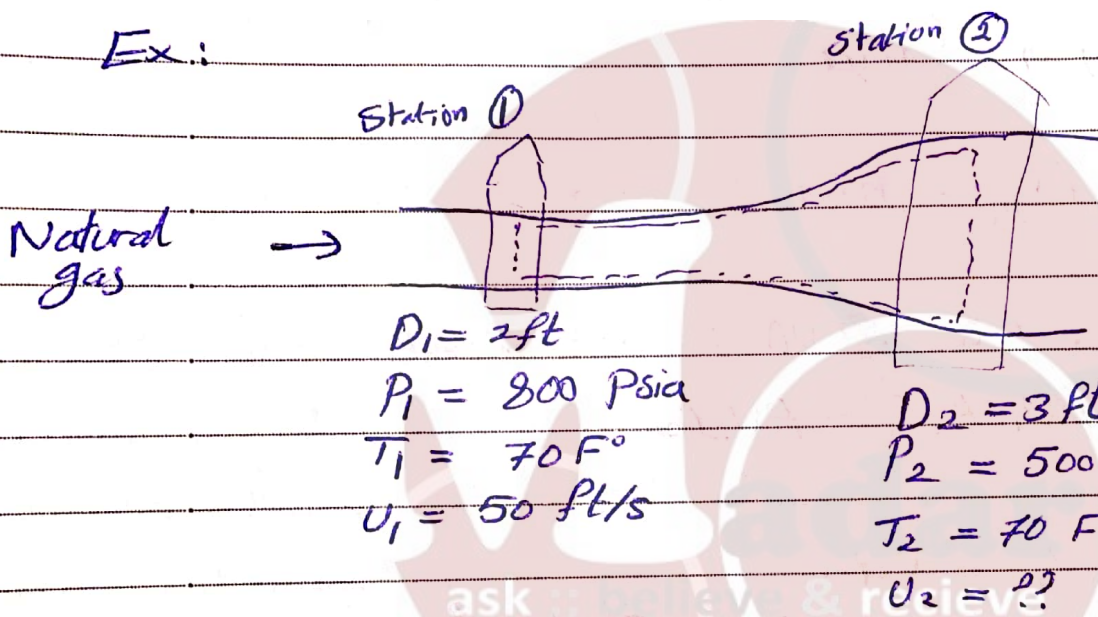
$$U_1 A_1 = U_2 A_2 \quad (\text{Continuity equation})$$

~~Units~~ Units:

$$m = \frac{\text{mass}}{\text{time}} = \frac{\text{kg}}{\text{s}} = \frac{\text{lbm}}{\text{s}}$$

$$Q : \text{Volumetric flow rate} \quad \frac{U_0 l}{\text{time}} = \frac{\text{m}^3}{\text{s}} = \frac{\text{ft}^3}{\text{s}}$$

Ex.:



$$P_1 U_1 A_1 = P_2 U_2 A_2$$

$$U_2 = U_1 \left(\frac{P_1}{P_2} \right) \left(\frac{A_1}{A_2} \right)$$

$$U_2 = U_1 \left(\frac{P_1}{P_2} \right) \left(\frac{A_1}{A_2} \right)$$

From ideal gas law

$$P = \frac{P M_w}{R T} \Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

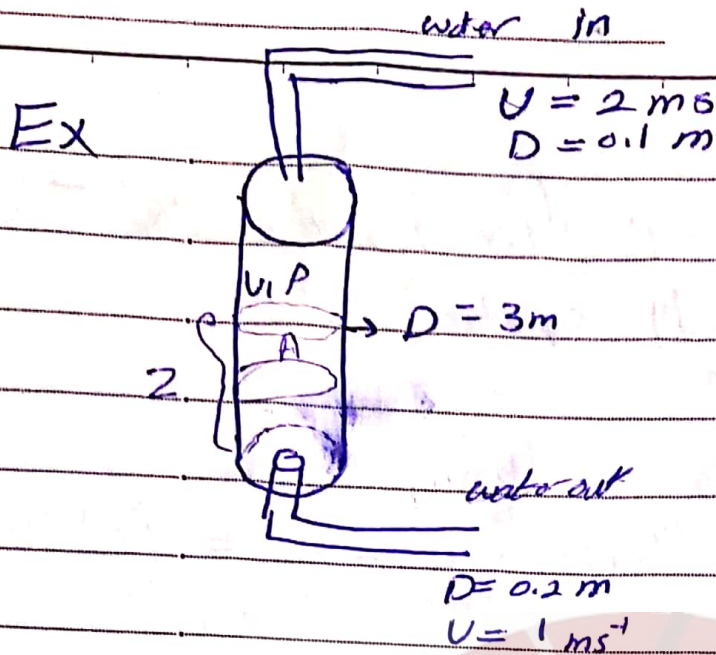
$$A = \frac{\pi D^2}{4} \quad \text{Cross section}$$

$$\frac{A_1}{A_2} = \left(\frac{D_1}{D_2} \right)^2$$

$$\text{تقوینص} = 50 \frac{\text{ft}}{\text{s}} \cdot \left(\frac{800}{500} \right) \left(\frac{2}{3} \right)^2$$

=

$$P_{\text{abs}} \text{ و } P_g \text{ و } P_{\text{atm}}$$



① Is the level rising or falling

② at what rate

Soln: $\left(\frac{dm}{dt}\right)_{\text{sys}} = m_{\text{in}} - m_{\text{out}}$

$m = \rho V$ $m = \rho Q = \rho V A$ $\rho: \text{Constant}$

$\left(\frac{d(\rho V)}{dt}\right)_{\text{sys}} = \rho V_1 A_1 - \rho V_2 A_2$

$\left(\frac{dV}{dt}\right)_{\text{sys}} = V_1 A_1 - V_2 A_2$

$= 2 \cdot \pi (0.1)^2 - 1 \cdot \pi (0.2)^2$

$= -0.0157 \text{ m}^3/\text{s} \rightarrow \text{is decreasing}$

$\left(\frac{dV}{dt}\right)_{\text{sys}} = \frac{d(Az)}{dt} = A \left(\frac{dz}{dt}\right)$ الارتفاع
بسرعة

$\frac{dz}{dt} = \frac{1}{A} \left(\frac{dV}{dt}\right)_{\text{sys}} = \frac{1}{\frac{\pi \cdot 3^2 \cdot \text{m}^2}{4}} \left(-0.0157 \frac{\text{m}^3}{\text{s}}\right)$

$= -0.0022 \text{ m/s}$

Chapter 4

* First law of thermodynamics "law of conservation of energy"

Energy - Ability to do work [Abstract term]

- Relative quantity [all calculations are based on change in energy]

- Additive property بنقصر زباجا

⇒ Forms of energy restrictions
subject to some ~~restrictions~~, all forms of energy
are

⇒ Internal energy U [u : specific internal energy]
unit (J)

⇒ measure hotness phase ability to cause chemical

* Kinetic energy KE [KE : specific KE]

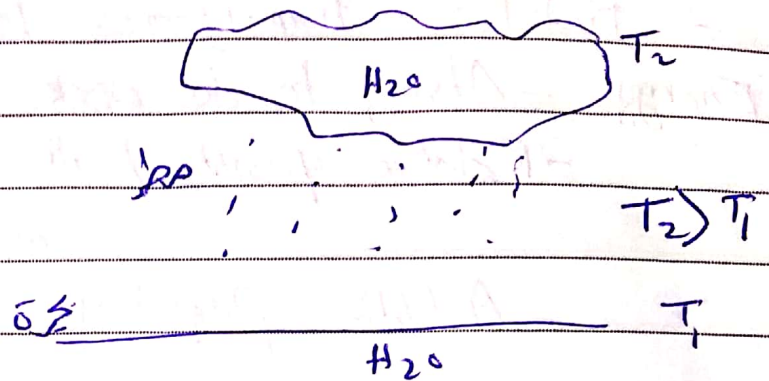
* potential energy PE [PE : specific PE]

* Fluid $\uparrow$ انواع بنساقه $\uparrow$ فزاد $\uparrow$ Courses

- من
مطلوب
موا
- * Conventional Fluid Mechanics
 - * Electrostatics energy
 - * Magnetic energy
 - * surface energy
 - * Nuclear energy

Energy transfer

* Heat: is energy in transity transfer is due to ΔT



* Work: energy transfer to or from object.

$$\text{work} = \int F dx \quad x: \text{is distance}$$

For fluid $F = pA$

$$\text{work} = \int p \cdot A dx = \int p dv \quad \begin{cases} \text{injection work} \\ -ve \text{ compression} \\ +ve \text{ expansion} \end{cases}$$

* Radiation

Electromagnetic waves

General Balance Equation

$$\text{Accumulation} = \text{Flow in} - \text{Flow out} + \text{production} - \text{Consumption}$$

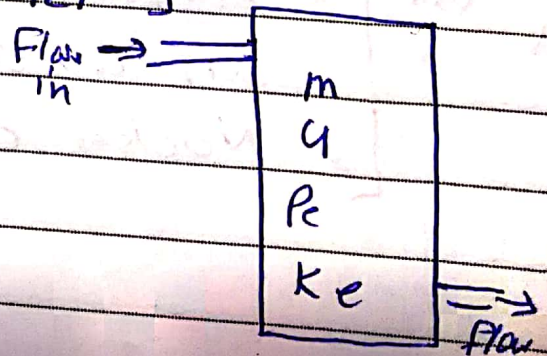
1st law of T.D. { * Energy Can neither be created or destroyed

Energy balance equation

Excludin, electrostatic, the genetik, surface, and Nuclear energy.

$$\begin{aligned} \text{Accumulation} &= \text{Flow in} - \text{Flow out} \\ &= d[(u + p_e + k_e) m] \end{aligned}$$

m: mass in system



InFlow and out Flow:

By matters:

$$In = (u + p_e + k_e) dm|_{in}$$

$$out = (u + p_e + k_e) dm|_{out}$$

By Heating or Cooling

$$dQ_{in} \rightarrow in + u$$

$$dQ_{out} \rightarrow out - u_e$$

* Mechanical work

in -ve dW_{in}

out +ve dW_{out}

-ve: surrounding on system

+ve: system on surrounding

$$d[m(u + p_e + k_e)] = \left[\begin{array}{c} (u + p_e + k_e) dm|_{in} \\ u + p_e + k_e dm|_{out} \end{array} \right] +$$

By matter

$$+ \left[\begin{array}{c} dQ_{in} \\ - dQ_{out} \end{array} \right] - \left[\begin{array}{c} dW_{in} \\ + dW_{out} \end{array} \right]$$

Heating or Cooling

measurement work

$$dQ = [dQ_{in} - dQ_{out}]$$

$$dW = -[dW_{in} - dW_{out}]$$

$$dW = dQ$$

For S.S closed system

$$0 = dQ - dW = \text{Zero}$$

[Signature]

4/3

$$d[m(u + p_e + k_e)]_{\text{syst}} = (u + p_e + k_e) \frac{dm}{dt} \Big|_{\text{in}} - (u + p_e + k_e) \frac{dm}{dt} \Big|_{\text{out}} + dQ - dW$$

$$dQ = dQ_{\text{in}} - dQ_{\text{out}} \quad \text{net going in the system}$$

$$dW = - (dW_{\text{in}} - dW_{\text{out}}) \quad \text{out}$$

$$dW = dW_{\text{inj}} + dW_{\text{ao}}$$

injection work all other work (other form of mechanical work)

$$dW_{\text{inj}} = P \left(\frac{1}{\rho} \right) dm = \frac{P}{\rho} dm$$

specific volume $\left(\frac{1}{\rho} \right)$ all other work $\left(\frac{1}{\rho} \right)$

$$dW = - \frac{P}{\rho} dm \Big|_{\text{in}} + \frac{P}{\rho} dm \Big|_{\text{out}} + dW_{\text{ao}}$$

$$-dW = \frac{P}{\rho} dm \Big|_{\text{in}} - \frac{P}{\rho} dm \Big|_{\text{out}} - dW_{\text{ao}}$$

$$p_e = \rho g z$$

$$k_e = \frac{u^2}{2}$$

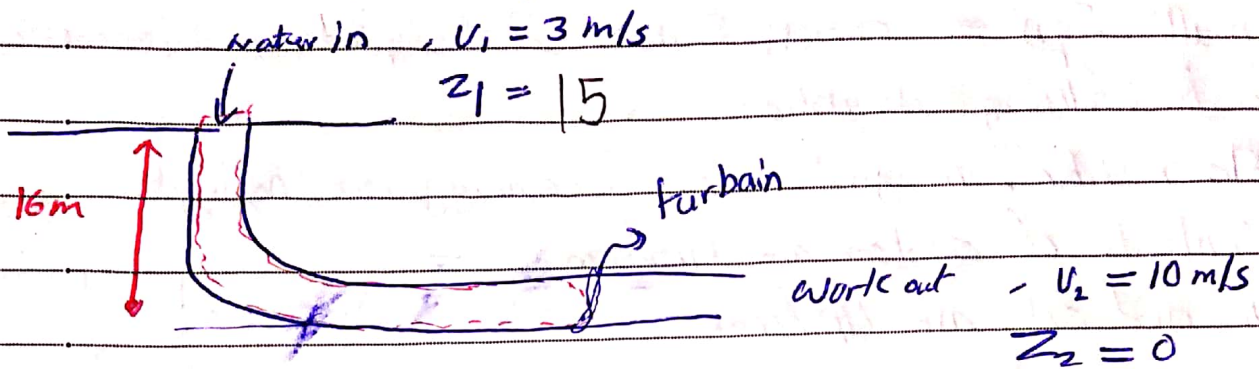
$$\left\{ d \left[m \left(u + g z + \frac{u^2}{2} \right) \right]_{\text{syst}} \right\} = \left\{ \left(u + g z + \frac{u^2}{2} \right) dm \Big|_{\text{in}} - \left(u + g z + \frac{u^2}{2} \right) dm \Big|_{\text{out}} + dQ - dW_{\text{ao}} \right\}$$

Enthalpy $h = u + p/\rho$ Energy Q W_{ao}

Zero if it's steady state.

* water power plant

* Water power plant



work done by plant

1. S.S

2. Homogenous

3. No chemical Rxn

4. $dm_{in} = dm_{out} = dm$

7. $T_{in} - T_{out} \rightarrow h_{in} = h_{out}$

6. No heat exchange ($dQ=0$)

5. No friction losses

internal energy

enthalpy

$$0 = \left(h + gz + \frac{v^2}{2} \right)_{in} dm - \left(h + gz + \frac{v^2}{2} \right)_{out} dm + dQ - dW_{ao}$$

$$\frac{dW_{ao}}{dm} = g(z_2 - z_1) + \frac{1}{2}(v_2^2 - v_1^2)$$

$$= 9.81 \frac{\text{m}}{\text{s}^2} (15 - 0) \text{ m} + \frac{1}{2} (3^2 - 10^2) \frac{\text{m}^2}{\text{s}^2}$$

$$= 101.65 \text{ m}^2/\text{s}^2 = \text{J/kg}$$

$$N = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$$

$$J = N \cdot m$$

$$\text{Power} = \frac{J}{s} = \frac{N \cdot m}{s}$$

* Application of EB:

procedure 1. write down the EB equation in any convenient form.

2. select a system of specified boundaries

3. Cancel the term, which are not relevant or which are negligible [Note: each Cancellation represent an assumption]

- Friction work per unit mass is less in fluid
- Heat capacity of liquid is higher than that for a solid

$$1^\circ \text{F} / \text{lbm H}_2\text{O} \rightarrow 8^\circ \text{F} / \text{lbm steel}$$

What is friction heating?

Involves conversion into internal energy of some other kind of energy (P_E , $\dot{K}_E$) or external work (injection, shaft work).

Increases in internal energy is industrially useless.

$\therefore$ Friction heating = Friction loss [note: energy doesn't disappear]
 Energy is valuable form (P_E , $\dot{K}_E$) $\Rightarrow$ Energy is useless form (internal energy)

* For constant density materials

$$\Delta u = \underbrace{\text{Friction heating}}_{\text{unit mass}} + \frac{d\phi}{dm} \rightarrow f = \Delta u - \frac{d\phi}{dm}$$

} cooling or heating.

* Notes: $f = 0 \rightarrow$ frictionless flow

f for real fluid flow problems is always +ve

$f = -ve \Rightarrow$ assume direction of flow is incorrect.

Q. 10.1 $\rightarrow$ For real flows, friction losses signifies decrease in P , $\dot{Z}$, U . Work extracted by a turbine or increase in work done by a pump or combination.

$$\Delta \left(\frac{P}{\rho} + gz + \frac{U^2}{2} \right) = - \frac{dw_{uc}}{dm} - \mathcal{F} \quad \text{energy distribution term.}$$

* Mechanical Energy Balance: $\Delta \left(\frac{P}{\rho} + gz + \frac{U^2}{2} \right) = -\dot{W} - \mathcal{F}$
Bernoulli's equation with terms, that can be measured mechanically.

Note: Incompressible fluid = Change in density negligible $\rightarrow$ all steady flow of liquid.
 $\rightarrow$ most steady flows of gases at low velocity.

$\rightarrow$ static flow

* zero flow: $\Delta \left(\frac{P}{\rho} + gz \right) = 0$

$$\frac{1}{\rho} \Delta P = -g \Delta Z$$

$$\frac{\Delta P}{\Delta Z} = -\rho g$$

$$\left| \frac{dP}{dz} = -\rho g \right| \quad \text{Barometric eqn}$$

* Head Form of Bernoulli's Eqn

It is convenient to divide BE eqn

$$\Delta \left(\frac{P}{\rho g} + z + \frac{U^2}{2g} \right) = - \frac{\dot{W}}{g} - \frac{\mathcal{F}}{g}$$

Pressure head, Potential head (gravity), Velocity head, Frictionless head, Work head

omit length of all terms.

* heads: height above certain datum
reference

which form of Bernoulli's eqn
specific energy
mechanical energy or head form

} Depends on the nature of the problem at hand.
If correctly applied, all give the same result

* Application of BE. diffusers and sudden expansion
Involve slowing down a fluid (diffusers) or bringing a fluid to a halt (sudden expansion)

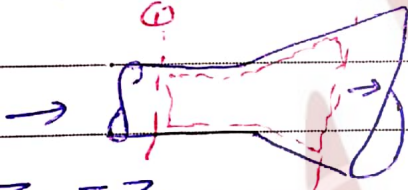
$$\cancel{P_1} U_1 A_1 = \cancel{P_2} U_2 A_2$$

$$U_1 A_1 = U_2 A_2$$

We want to apply BE

$$\Delta \left(\frac{P}{\rho} + gz + \frac{U^2}{2} \right) = -\omega - \mathcal{F}$$

Diffuser



$$z_1 = z_2$$

(mass balance)

$$BE: \frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} = -\mathcal{F}$$

$$CE: U_2 = U_1 \left(\frac{A_1}{A_2} \right)$$

توقف + ترتيب

$$P_2 - P_1 = \rho \frac{U_1^2}{2} \left[1 - \left(\frac{A_1}{A_2} \right)^2 \right] - \rho \mathcal{F}$$

Analysis: $U_1 > U_2 \rightarrow$ decrease in KE

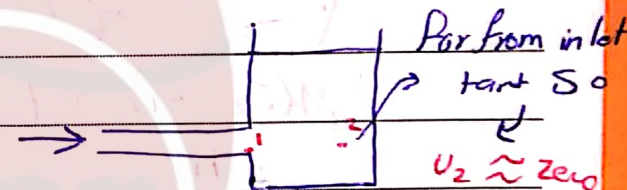
$K_E \rightarrow$ injection + friction loss

$P_2 > P_1 \Rightarrow$ pressure recovery

usually pressure recovery $\approx 90\%$ of

frictionless situation. $\mathcal{F} \approx \frac{1}{10}$ decrease in KE

sudden expansion



الكل يوقف

$$z_1 = z_2$$

$$\frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} = -\mathcal{F}$$

$$\frac{P_2 - P_1}{\rho} = \frac{U_1^2}{2} - \mathcal{F}$$

$$* CE: \rho \frac{U_1^2}{2} - \rho \mathcal{F} = P_2 - P_1$$

Analysis: similar to diffuser

$$\left[\left(\frac{A_1}{A_2} \right)^2 \approx 0 \right]$$

$$P_1 \approx P_2$$

All KE $\Rightarrow$ internal energy

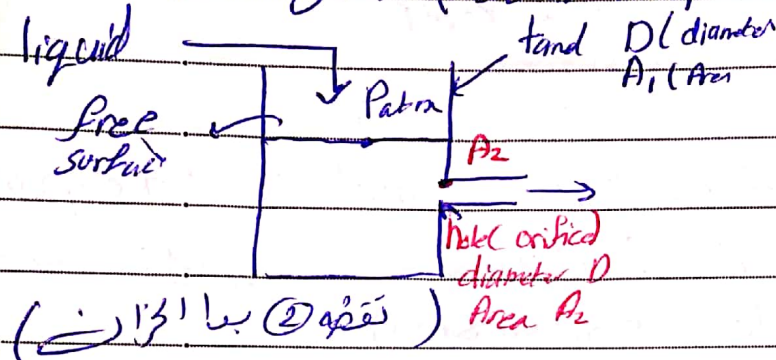
From experiment $\mathcal{F} =$ decrease in KE per unit mass
 $P_1 = P_2 \Rightarrow$ no pressure recovery.

$P_{at inlet} = P$ in fluid without flow

* Toricell's equation.

Based on assumption that flow is frictionless

Tank Drainage: (constant level)



Target: $U_2 = ??$
 $Q = U_2 A_2$

Assumption: (1) s.s which height is constant.

(2) $P_1 = P_2 = P_{atm}$

(3) $f = 0$

BE: $\left(\rho z_2 + \frac{\rho U_2^2}{2} \right) - \left(\rho z_1 + \frac{\rho U_1^2}{2} \right) = 0$

$\rho \underbrace{(z_2 - z_1)}_{-h} + \left(\frac{\rho U_2^2}{2} - \frac{\rho U_1^2}{2} \right) = 0$

CE: $U_1 A_1 = U_2 A_2$ P is constant.

$U_1 = U_2 \frac{A_2}{A_1}$

Substituting $\rightarrow$ so $-gh + \frac{U_2^2}{2} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] = 0$

$\therefore U_2 = \left[\frac{2gh}{1 - \left(\frac{A_2}{A_1} \right)^2} \right]^{1/2}$ general case

Notes - $A_1 \gg A_2 \therefore U_2 = \sqrt{2gh}$ Toricellis equation

Note that in this case is less than in v. in general

since the fluid there is moving case

with measurable h in the tank

2] $A_2 = A_1$ (vertical pipe open at both ends)

Based on general eqn $U_2 = \infty$

this means that γ cannot be neglected.

$$P_E \Rightarrow \gamma$$

$$\text{b.e.: } g(z_2 - z_1) = -\gamma = -gh$$

ask ; believe & recieve

* Flow rate measured is usually less than theoretically calculated. To correct for this, a coefficient of called discharged coefficient is used

$$C_D = \frac{\text{Actual discharged}}{\text{Theoretical discharged}}$$

theoretical

$$Q_{\text{actual}} = C_D \sqrt{2gh} A_0$$

$$Q_{\text{theoretical}} = V \times A_0$$

* This difference is due to

- Actual discharged velocity (is less than theoretical)
(friction + eddying)

لـ تكوين الدوامات

very sharp

(eddies)
(دوامات)

(كل ما كانت الزاوية أصغر)
يعني الاحتكاك أقل (شكل الكواض)

تكوين الدوامات يحتاج
طاقة

Friction
very small

* C_v = Coefficient of velocity < 1

$$V_{\text{actual}} = V_{\text{theoretical}} \times C_v$$

* $Q = V_{\text{actual}} \cdot \text{Area}$

True flow area is less than orifice area.

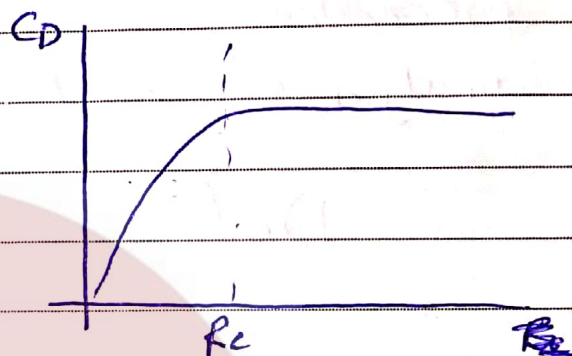
due to the contraction of flow area to a minimum

$$A_{\text{actual}} = A_0 \times C_c \quad \text{Coefficient of contraction} < 1$$

$$\therefore C_D = C_v \cdot C_c$$

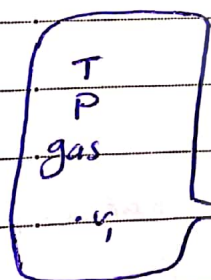
$$Q_{\text{actual}} = C_v \cdot C_c \cdot \sqrt{2gh} A_0 \quad C_D \approx 0.6$$

Vena contracta



$$Re = \frac{\rho V D}{\mu}$$

* Flow of gas system.



P_{atm}

$P > P_{\text{atm}}$

(سورجیہ طور پر)

what is the u_2

$u_1 \approx 0$ (Cross section $z=1$)

$P_2 = P_{\text{atm}}$

assumption: P constant.

BE: between ① and ②

$$\frac{P_1 - P_2}{\rho} + \frac{u_1^2 - u_2^2}{2} = 0$$

$$u_2 = \left[\frac{2(P_1 - P_{\text{atm}})}{\rho} \right]^{\frac{1}{2}}$$

perfect gas

$$u_2 = \left[\frac{2RT_1}{P_{1M}} (P_1 - P_{\text{atm}}) \right]^{\frac{1}{2}}$$

P const

→ reasonable for gas velocity

→ up to approx 200 m/s

→ Choice of T arbitrary

→ BE can be applied to a gas when $\frac{\Delta P}{P} < 0.1$

project

Imp

m

Design an experiment to measure C_u and C_c separate

- Full description of apparatus
- Theory
- procedure
- analysis of result

Deadline : 15/4

* Fluid flow measurements:

Involve devices which are based on the following principles

- BE assumption : $\bar{V}_{avg} \propto \sqrt{\Delta P}$
- negligible friction heating
- Direct measurement of pressure difference between 2 points in the flow stream. (in most cases a constriction is introduced to create a relatively large pressure difference.)

* Pitot Tube (impact tube or stagnation tube)

Apply B.E between ① & ②

$$z_1 = z_2$$

$$u_2 = 0$$

$$\frac{P_2 - P_1}{\rho} - \frac{u_1^2}{2} = 0$$

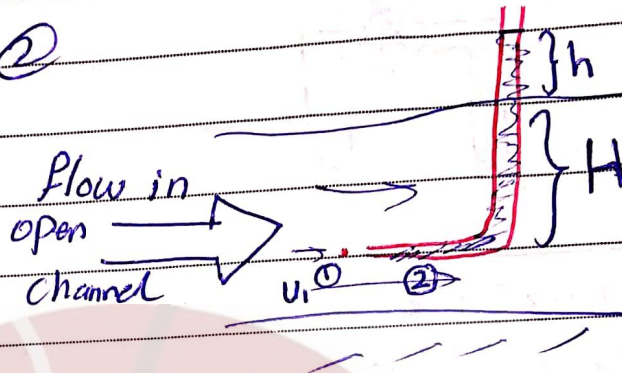
$$u_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

$$P_2 = P_{atm} + \rho g (H + h)$$

$$P_1 = P_{atm} + \rho g H$$

$$P_2 - P_1 = \rho g h$$

$$u_1 = \sqrt{\frac{2 \rho g h}{\rho}} = \sqrt{2gh}$$



$$P_2 > P_1$$

h : is measured $\rightarrow u_1$ is calculated

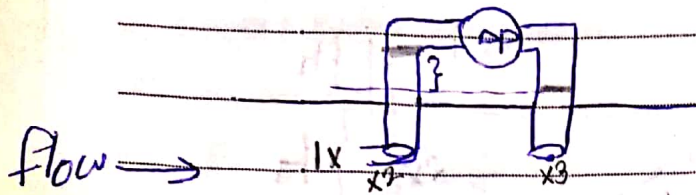
$$u \propto \sqrt{h}$$

This device is used to measure local point velocities

- Suitable for flow measurement in open channel or measurement of velocity of boats.

Pitot static tube

- Suitable for flow measurement in pipes and air speed



$$z_1 = z_2 = z_3, \quad u_2 = 0, \quad u_1 = u_3$$

BE between ① & ②

$$\frac{P_2 - P_1}{\rho} - \frac{u_1^2}{2} = 0$$

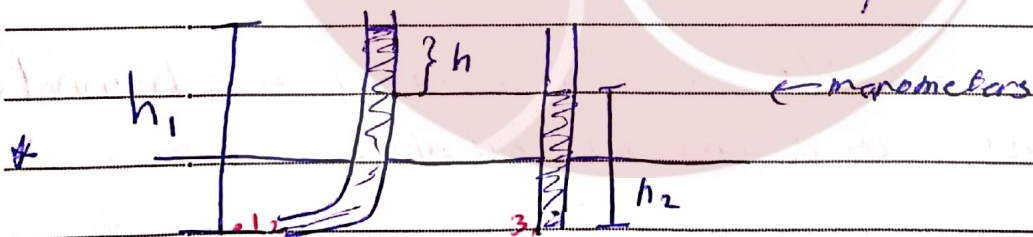
$$u_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

BE between ① & ③

$$\frac{P_3 - P_1}{\rho} + \frac{u_3^2 - u_1^2}{2} = 0$$

$$P_1 = P_3$$

$$\therefore P_2 - P_1 = P_2 - P_3 \Rightarrow u_1 = \sqrt{\frac{2(P_2 - P_3)}{\rho}}$$



$$P_2 = P_{atm} + \rho g h$$

$$P_3 = P_{atm} + \rho g h$$

$$P_2 - P_3 = \rho g h$$

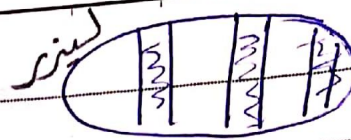
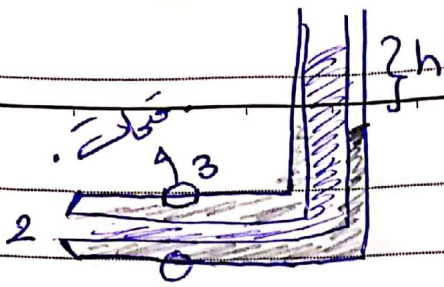
$$u_{21} = \sqrt{2gh}$$

h can be measured.

$$\frac{P}{\rho} + g z + \frac{u^2}{2} = -\omega - \mathcal{F}$$

Static + dynamic $[P_2]$

→ Static head P_1, P_3 OR Piezometric head.



Hot wire anemometer

* Volumetric flow rate Q

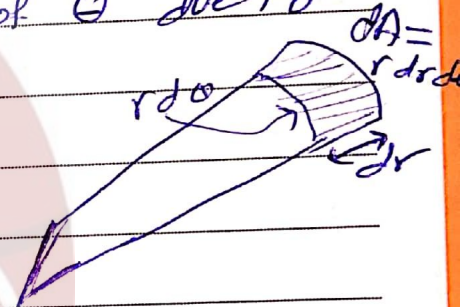
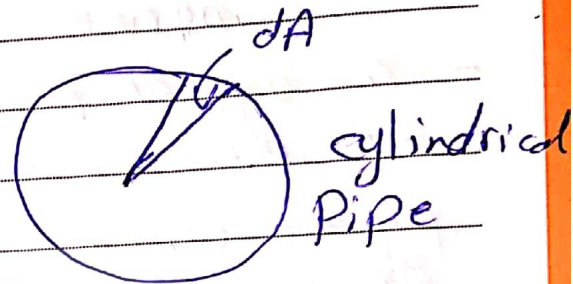
Volume flow rate through dA

$$dQ = U \cdot dA$$

U is local point velocity at distance r from center

If is function of r and independent of θ due to symmetry.

$$\therefore Q = \int U \, dA$$

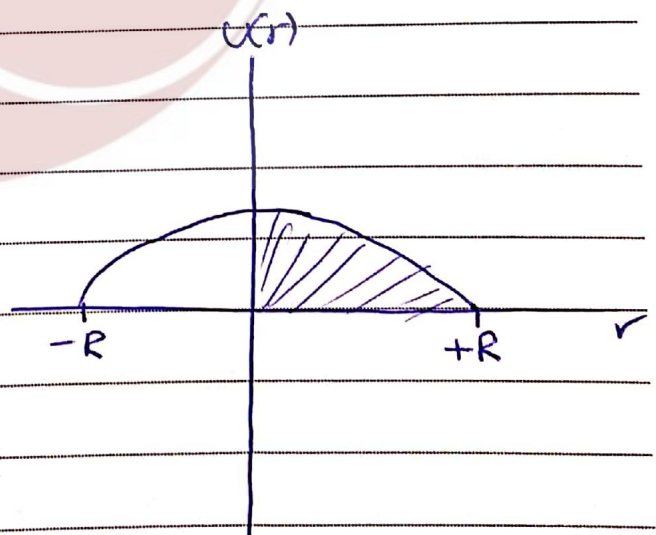


* for flow in pipe

$$Q = \int_0^{2\pi} \int_0^R U \cdot r \, dr \, d\theta$$

$$Q = 2\pi \int_0^R U r \, dr$$

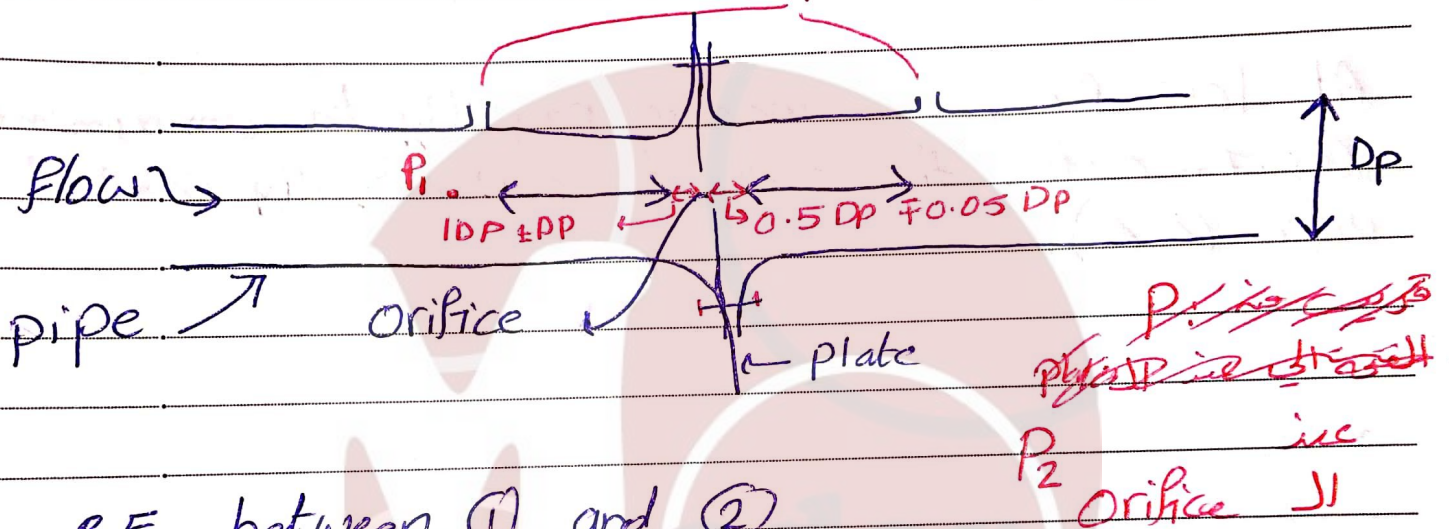
$$Q \approx 2\pi \sum U r \Delta r$$



ORIFICE METER

- simple device
- easy to construct
- relatively cheap
- suitable for small flow rates

pressure tapping.



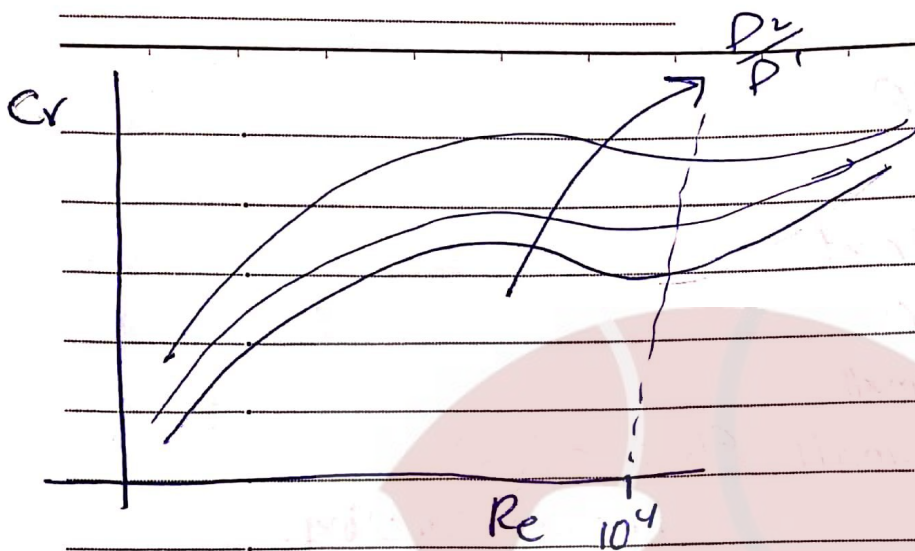
BE between ① and ②

$$\frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} = 0$$

CE $U_1 = U_2 \left(\frac{A_2}{A_1} \right)$

$$U_2 = C_r \sqrt{\frac{2(P_2 - P_1)}{\rho \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right]}}$$

$C_r = f \left(\frac{A_2}{A_1} \right)$, Re at orifice $= \frac{\rho U_2 D_o}{\mu}$



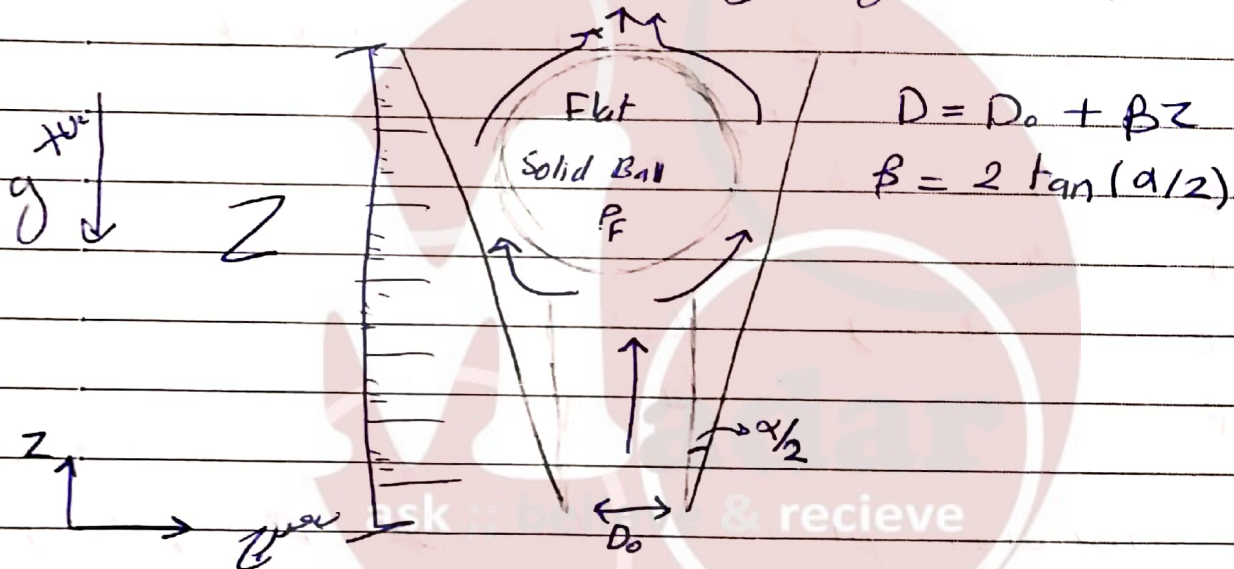
At low Re , C_r increase rapidly to a maximum (0.7-0.9), then falls to a virtually constant value (0.6).

Variable Area Flowmeter : Rotameter

- widely used to measure low flow rates
- it uses a fixed pressure difference
- and variable geometry (Area) which is function of flow rate
- Flow is linearly proportional to scale reading.

principle and mode of operation

- Fluid flows up a tapered transparent (glass or plastic) tube and supports a floating weight (Float)



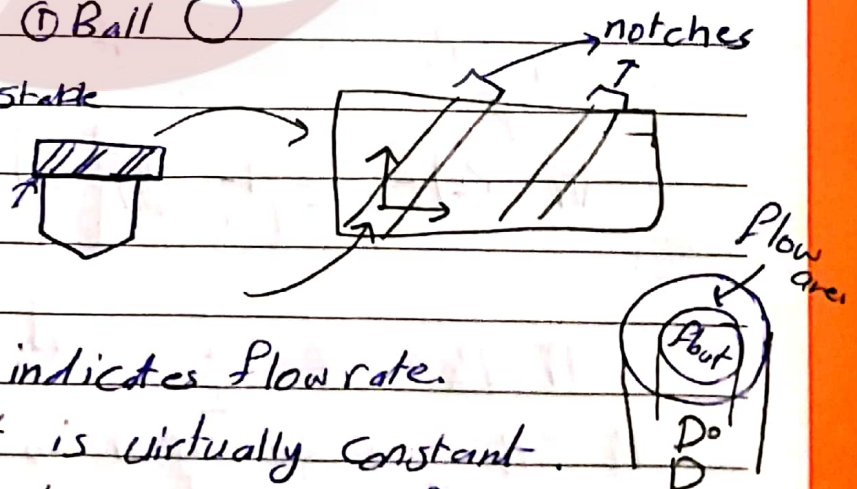
* Shape of the Float: ① Ball ○

② Cone ▽ more stable

③ Cone with notches

Float spins / ω is $\propto$

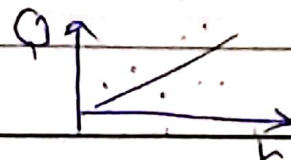
- more stability.



Height of ~~float~~ float indicates flow rate.

ΔP across the float is virtually constant.

Area between float and tube varies with flow



Theory: At S.S condition, the force balance around the float is $\Sigma F = 0$

$$0 = F_{\text{gravity}} + \underset{\text{on top}}{\text{Pressure force}} - \underset{\text{on bottom}}{F_{\text{buoyancy}} - \text{Pressure force}}$$

assume pressure to be uniform on top and bottom surface

$$0 = \rho_f V_f g + P_3 A_F - \rho_f V_f g - P_1 A_F$$

$$V_F (P_F - \rho_f) \cdot g = A_F (P_3 - P_1)$$

$$\textcircled{A} \quad \boxed{P - P_3 = \frac{V_F (\rho_f - \rho_f) g}{A_F}} \leftarrow \text{Constant}$$

$V_F = \text{Vol of float}$

$A_F = \text{projected area of float}$

ρ_f : density of float.

ρ_f : " " Fluid.

* Bernoulli's eqn between ① to ②, ignoring changes in potential head.

$$\frac{P_1 - P_2}{\rho_f} = \frac{U_2^2 - U_1^2}{2} \quad \left\{ \begin{array}{l} \text{using continuity eqn} \\ U_1 = U_2 \frac{A_2}{A_1} \end{array} \right.$$

$$(P_1 - P_2) = \rho_f \frac{U_2^2}{2} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right]$$

$$\frac{A_2}{A_1} \ll 1$$

$$P_1 - P_2 = \rho_f \frac{U_2^2}{2}$$

$P_2 \approx P_3$, sudden expansion

$$\textcircled{B} \quad \boxed{P_1 - P_3 = \rho_f \frac{U_2^2}{2}}$$

① = ③ So that

$$\rho \frac{V_2^2}{2} = \frac{V_F (\rho_F - \rho) g}{A_F}$$

$$V_2 = \sqrt{\frac{2 V_F g}{A_F} \cdot \frac{\rho_F - \rho}{\rho}}$$

Volumetric flowrate

$$Q = C_D \cdot A_z \cdot V_2$$

$$A_F = \frac{\pi D_o^2}{4}$$

$$D_o = D_F$$

$$A_z = \frac{\pi}{4} (D^2 - D_o^2) =$$

$$= \frac{\pi}{4} [(D_o + 2h \tan(\alpha/2))^2 - D_o^2]$$

$$= \frac{\pi}{4} [D_o^2 + 4h \tan \frac{\alpha}{2} \cdot D_o + 4h^2 \tan^2(\alpha/2) - D_o^2]$$

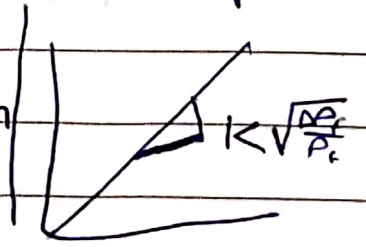
$$= \pi h D_o \tan \alpha/2$$

~~Volumetric flowrate~~

$$\therefore Q = C_D \cdot \pi h D_o \tan \alpha/2 \cdot \frac{\sqrt{2 V_F g \rho_F (\rho_F - \rho)}}{\sqrt{\frac{\pi D_o^2}{4}}} \cdot \frac{(\rho_F - \rho)}{\rho}$$

$$\therefore Q = k \sqrt{\frac{\rho_F - \rho}{\rho}} \cdot h \quad \left\{ \quad k = C_D \cdot \tan \alpha/2 \cdot \sqrt{2 \pi g V_F \rho_F} \right.$$

* mass flowrate: $m = k \sqrt{\rho_F (\rho_F - \rho)} \cdot h$



k: Can be obtained from calibration

* errors are least when $\frac{dm}{d\rho} = 0$, $\rho_F = 2\rho$

Effect of temp or P_f on reading from calibration
 h indicates a certain

$$\Delta P = P_f - P_f$$

$$Q = k \sqrt{\frac{\Delta P}{P_f}} \cdot h$$

at condition ① & ② we have for the same h

$$Q_1 = k \sqrt{\left(\frac{\Delta P}{P_{f1}}\right)} \cdot h$$

$$Q_2 = k \sqrt{\left(\frac{\Delta P}{P_{f2}}\right)} \cdot h$$

Same h يشتمل نوع الفلوز
أو د/ حرارة

$$\frac{Q_1}{Q_2} = \sqrt{\left(\frac{\Delta P_1}{\Delta P_2}\right) \cdot \left(\frac{P_{f2}}{P_{f1}}\right)}$$

$\Delta P \approx \text{constant}$

$$\frac{\Delta P_1}{\Delta P_2} = 1$$

$$Q = Q_2 \sqrt{\frac{P_{f2}}{P_{f1}}}$$

h for N_2 $100 \text{ cm}^3/\text{min}$

Q_{helium}

$$Q_{\text{helium}} = Q_{N_2} \left(\frac{P_{N_2}}{P_{H_2}} \right)^{1/2}$$

$$\therefore Q_{H_2} = 100 \frac{\text{cm}^3}{\text{min}} \left(\frac{28}{4} \right)^{1/2} = 265 \text{ cm}^3/\text{min}$$

20/3

* Negative absolute pressure : Cavitation
under certain condition RE predicts -ve pressure
$$P_2 = P_1 - \rho \left[\frac{\Delta V^2}{2} + g \Delta z \right] \Rightarrow -ve$$

* Cases :- * -ve absolute pressure has no physical meaning

* Indicate wrong assumptions. $\rightarrow$ Equations for compressible flow must be used.

* liquids in -ve absolute pressure can be predicted under certain situations

* negative absolute pressure are unstable

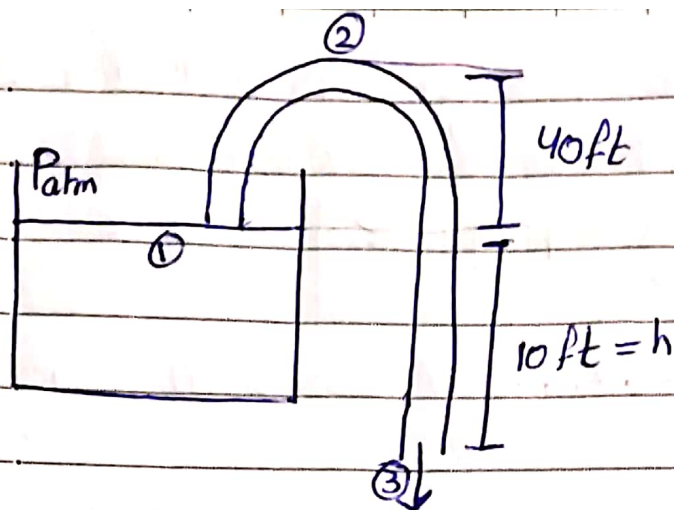
* P_L on liquid $<$ vap pressure of liquid.

* Boiling occurs $\rightarrow$ two phase flow which has higher value of f than 1 phase

when P_{calc} on liquid $<$ vap pressure

$\rightarrow$ Flow on Calculated is physically impossible $\rightarrow$ actual flow has higher friction effect and flow velocity will be less than assumed in the original calculation.

Ex:



الضغط في مخرج السيفون هو P_{atm}

BE ~~between~~ $\approx z \approx 0$, $U_1 \approx 0$, $P_1 = P_3$

BE between ① to ③

$$g(z_1 - z_3) = \frac{U_3^2}{2}$$

$$U_3 = \sqrt{2gh} = 25.3 \text{ ft/s}$$

* BE between ① & ②

zero

$$U_2 = U_3$$

$$\frac{(P_1 - P_2)}{\rho} + g(z_1 - z_2) + \frac{U_1^2 - U_2^2}{2} = 0$$

$$P_2 = P_1 - \rho \left[\frac{U_3^2}{2} + g(z_2 - z_1) \right]$$

$$= 14.7 \text{ lbf/in}^2 - 62.3 \frac{\text{lbm}}{\text{ft}^3} \left[\frac{25.3^2}{2} \left(\frac{\text{ft}^2}{\text{s}^2} \right) + \frac{32.2 \text{ ft}^2 \cdot 40 \text{ ft}}{\text{s}^2} \right]$$

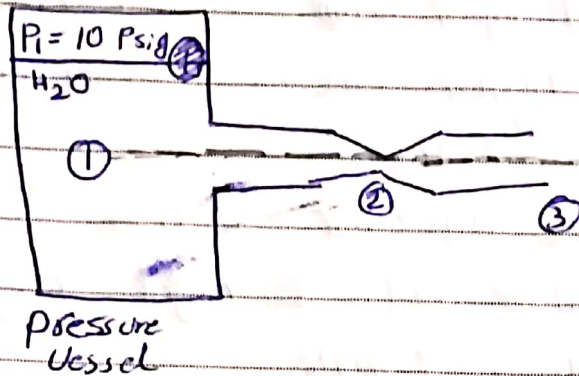
$$\times \frac{1 \text{ lbf s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 14.7 - 21.6 = -6.9 \frac{\text{lbf}}{\text{in}^2} \rightarrow \text{impossible}$$

siphon will not flow.

* Max. height for water siphon $\approx 34 \text{ ft}$

important Ex:



Be between ① & ③
 $(A_2/A_3) = 0.5$, $P_3 = P_{atm}$, $P_2 = ??$, $F \neq 0$

$$\frac{P_1 - P_3}{\rho} + \frac{1}{2}(U_1^2 - U_2^2) = 0 \quad , \quad U_1 \approx \text{Zero}$$

$$U_3 = \sqrt{\frac{2(P_1 - P_3)}{\rho}} \quad \text{R gage}$$

$$= \sqrt{\frac{2 \times 10 \text{ bfl/in}^2}{62.3 \text{ bml/ft}^3} \times \frac{32.2 \text{ bml/ft} \times 144 \text{ in}^2}{\text{bfl} \cdot \text{s}^2} \times \frac{\text{ft}^2}{\text{ft}^2}}$$

$$= 38.6 \text{ ft/s}$$

CE: $U_1 = U_3 (A_3/A_2) = 38.6 \text{ ft/s} \times \frac{1}{0.5} = 77 \text{ ft/s}$

* BE between ① & ②

$$\frac{P_1 - P_2}{\rho} + \frac{U_1^2 - U_2^2}{2} = 0$$

$$P_2 = P_1 - \rho (U^2/2) =$$

$$= (10 + 14.7) \frac{\text{bfl}}{\text{in}^2} - 62.3 \frac{\text{bml}}{\text{ft}^3} \times \frac{77^2 (\text{ft/s})^2}{2} \times \frac{\text{bfl} \cdot \text{s}^2}{32.2 \text{ bml} \cdot \text{ft}} \times \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= -15.4 \text{ psia}$$

UNREAL

Between ① & ②

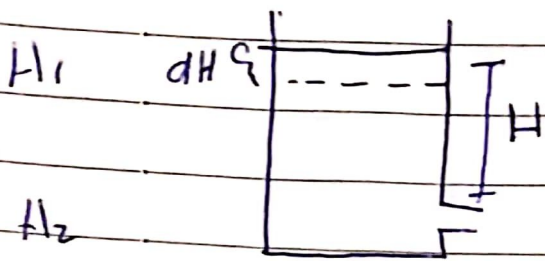
Vel increases } liq may boil if p_2 is -ve
Pressure decreases } vapor bubbles formed at ② travel to
higher pressure region and then
collapse

* → Collapse can cause a pressure pulse

In pumps pressure pulse to high frequencies
can cause damage to pump

~~phen~~ phenomena of local boiling is called
CAVITATION

Unsteady ~~flow~~ flow



time	height
t	H
$t + \Delta t$	$H + dH$

change in volume $\frac{d(\rho A H)}{dt} = \rho A \frac{dH}{dt}$

$A \{ H - dH - H \}$

Discharged rate $= -A \frac{dH}{dt}$

at any instant in time $-\frac{A dH}{dt} = C_D \cdot A_0 \sqrt{2gH}$

$\int_0^t dt = \left(\frac{-A}{C_D A_0 \sqrt{2g}} \right) \int_{H_1}^{H_2} H^{1/2} dH$

$t = + \frac{A}{C_D A_0} \frac{\sqrt{2}}{\sqrt{g}} \left[\sqrt{H_1} - \sqrt{H_2} \right]$

* Unsteady state $A_{cc} = In - out$

$\frac{dm}{dt} = 0 - out$

$\frac{\rho U}{dt} = \rho A \frac{dH}{dt} = - C_D \cdot U_0 \cdot A_0$

$\rho A \frac{dH}{dt} = - C_D \sqrt{2gh} \cdot \rho \cdot A_0$

end of chapter 5

~~X (50x) (12) X~~

Chapter 6 : Fluid Friction in steady one dimensional Flow.

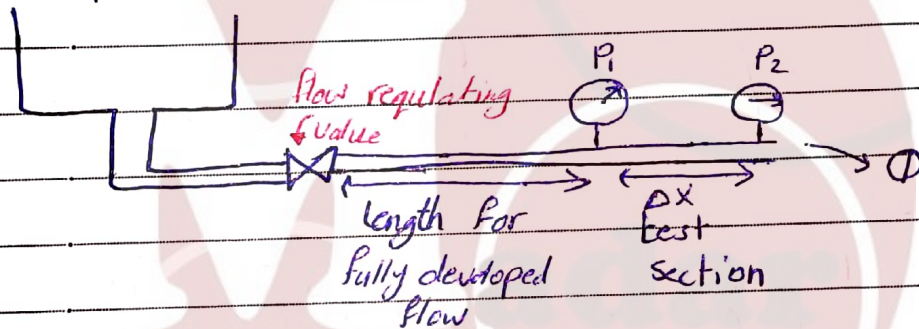
Introduction:

$$BE: \frac{\Delta P}{\rho} + gz + \frac{U^2}{2} = -\dot{w} - f$$

Q: How is f evaluate?

Ans: the formal f depends on geometry of system
 pipe, duct, channel
 - nature of flow : laminar, turbulent,

* pressure Drop:



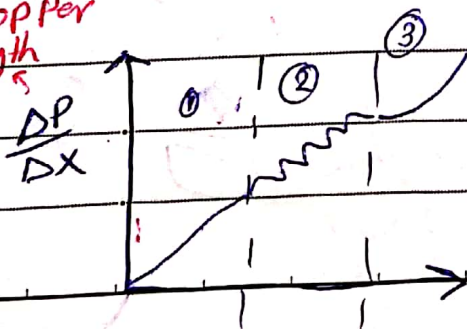
$$BE: \frac{P_2 - P_1}{\rho} = -f$$

must be +ve

$$\boxed{\frac{P_1 - P_2}{\rho} = f}$$

* For Certain Fluid Flowing in a Certain pipe

pressure drop per unit length



① linear

② wavy (not stable)

③

$$\textcircled{1} \frac{\Delta P}{\Delta x} \propto Q^{1.0}$$

low Q $\Rightarrow (1.0)$

② Result are not reproducible (Intermittent flow)

$$\textcircled{3} \frac{\Delta P}{\Delta x} \propto Q^{1.8-2.0}$$

high Q

1.8: smooth pipe

2.0: rough pipe

* Nature of Fluid Flow:

Reynolds Number: Dimensionless number

$$Re = \frac{\rho V L}{\mu} \quad ; \quad L: \text{Characteristic length for flow in pipe } L = D$$

= $\frac{\text{Inertial Force}}{\text{Viscous Force}}$

$$Re [=] \frac{ML^{-3} \cdot LT^{-1} \cdot L}{\frac{MLT^{-2} L^{-2}}{T^{-1}}} = \frac{MLT^{-2}}{MLT^{-2}} = \text{dimensionless}$$

* Experimental observations made by Reynolds revealed three type of fluid flow:

جریان لایه‌ای (laminar) ① laminar flow - low Q

- Fluid flow as laminar

$$- \frac{\Delta P}{\Delta x} \propto Q'$$

$$- Re < 2000$$

② Unstable Region

- intermediate flow

$$- 2000 < Re < 4000$$

- Flow is unstable
Laminar or turbulent

③ Turbulent Flow

- high flow rate

$$- Re > 4000$$

$$- \frac{\Delta P}{\Delta x} = \text{①}^{1.8-2.0}$$

- Motion in all direction
superimposed in overall motion.

* Laminar Flow:

Assumptions:

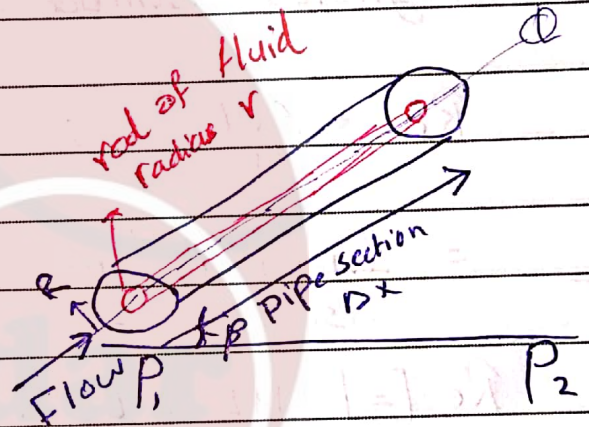
- s.s flow

- ρ, μ constant

- fluid Newtonian fluid

- location ① is far from
entrance (fully developed flow)

- Constant cross section



Force Balance on rod of fluid with radius r .

$$* \text{ Net pressure force} = \pi r^2 (P_1 - P_2)$$

$$* \text{ gravity force} = -\pi r^2 \cdot \Delta x \cdot \rho \cdot \sin \beta \cdot g$$

$$* \text{ shear force resisting flow} = -2\pi r \Delta x \cdot \tau$$

$$\sum F_x = 0$$

$$2\pi r \Delta x \tau = \pi r^2 (p_1 - p_2) - \pi r^2 \Delta x \rho g \sin \beta$$

$$\tau = \frac{r(p_1 - p_2) - r \Delta x \rho g \sin \beta}{2 \Delta x}$$

$$\tau = \frac{1}{2} r \left[\frac{(p_1 - p_2) - \rho g \Delta x \sin \beta}{\Delta x} \right]$$

$$\tau = \frac{1}{2} r \frac{\Delta \phi}{\Delta x}$$

$$\tau = -\mu \frac{du_x}{dr}$$

$$\frac{du_x}{dr} = - \frac{\Delta \phi}{2\mu \Delta x} \cdot r$$

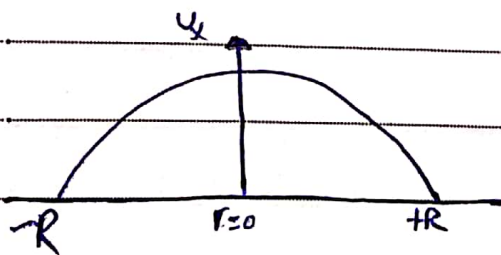
$$u_x = - \frac{\Delta \phi}{4\mu \Delta x} r^2 + C$$

$$r=R, u_x = 0$$

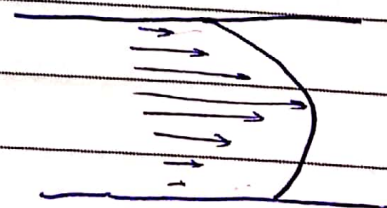
$$C = \frac{\Delta \phi R^2}{4\mu \Delta x}$$

$$u_x = - \frac{\Delta \phi}{4\mu \Delta x} r^2 + \frac{\Delta \phi R^2}{4\mu \Delta x}$$

$$u_x = \frac{\Delta \phi R^2}{4\mu \Delta x} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$



مقطع توزيع السرعة



Velocity profile

$$V_{x, \max} = \text{at } r=0$$

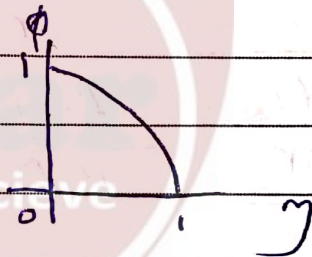
$$U_{x, \max} = \frac{\Delta \phi R^2}{4\mu \Delta x} \quad \left| \quad U_x = U_{x, \max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \right.$$

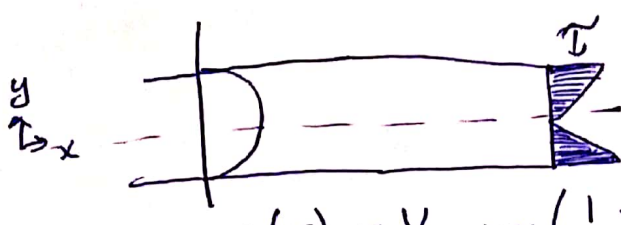
$$\frac{U_x}{U_{x, \max}} = \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$\boxed{\phi = 1 - \eta^2}$$

$$\phi \in [0, 1]$$

$$\eta \in [1, 0]$$





$$u_x(r) = V_{x, \max} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$\tau = \frac{1}{2} r \cdot \frac{\Delta p}{\Delta x}$$

$$Q = ??$$


$$dA = r dr d\theta$$

$$dQ = u_x(r) dA$$

$$Q = \iint u_x(r) dA$$

$$Q = \int_0^{2\pi} \int_0^R V_{x, \max} \left[1 - \left(\frac{r}{R} \right)^2 \right] r dr d\theta$$

$$Q = \frac{\Delta p R^2}{4 \mu \Delta x} \cdot \frac{2\pi R L}{4} = \frac{\pi \Delta p R^4}{8 \mu \Delta x}$$

$$Q = \frac{\pi \Delta p D^4}{128 \mu \Delta x} \quad \left. \vphantom{Q} \right\} \text{Hagen-Poiseuille equation}$$

$$\therefore \Delta p \propto Q^{1.0}$$

$$V_{\text{avg}} = \frac{Q}{\pi R^2} = \frac{1}{2} V_{x, \max}$$

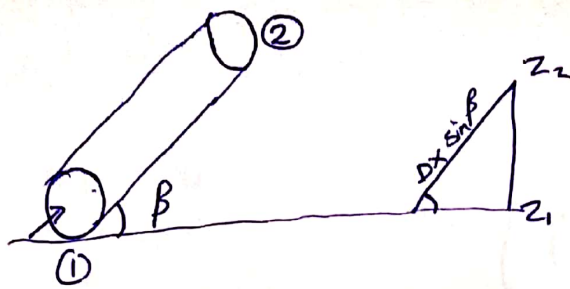
$$\frac{1}{2} V_{x, \max} = V_{x, \max} \left(1 - \left(\frac{r}{R} \right)^2 \right)$$

$$1 = 2 - 2 \left(\frac{r}{R} \right)^2$$

$$2 \left(\frac{r}{R} \right)^2 = 2 - 1$$

$$\left(\frac{r}{R} \right)^2 = \frac{1}{2}$$

only to Laminar Flow



$$BE: \frac{(P_2 - P_1)}{\rho} + g \Delta z = -\mathcal{F}$$

$$-\frac{(P_1 - P_2)}{\rho} + g (z_2 - z_1) = -\mathcal{F}$$

$$-\frac{(P_1 - P_2)}{\rho} + g \Delta x \sin(\beta) = -\mathcal{F}$$

$$\frac{-(P_1 - P_2 - \rho g \Delta x \sin \beta)}{\rho} = \mathcal{F}$$

$$-\frac{\Delta P}{\rho} = -\mathcal{F} \rightarrow \boxed{\frac{\Delta P}{\rho} = \mathcal{F}}$$

* For Hagen equation

$$\mathcal{F} = Q \Delta x \frac{\mu}{\rho} \frac{128}{\pi D^4}$$

Applies to pipe with any inclination:

Note: From Force Balance

$$\tau = \frac{1}{2} \frac{\Delta \phi}{\Delta x} \cdot r$$

$$\Delta \phi = 2 \frac{\tau}{r} \Delta x$$

$$BE: \frac{\Delta P}{\rho} = \mathcal{F}$$

$$\boxed{\mathcal{F} = 2 \frac{\tau \Delta x}{\rho} \cdot \frac{1}{r}}$$

Applies for any flow.

Turbulent flow:-

problem $\tau = ??$

$\tau_t > \tau_L$ due to movement more than one direction (specially $\perp$ are of bulk motion)

The difference is called Reynolds stress,

$$\therefore F_{\text{turb}} > F_{\text{laminar}}$$

$$\tau_i = \tau_w \quad \text{shear stress of wall}$$

$$\therefore p_1 - p_2 = \frac{2\Delta x \tau_w}{R} = \rho f$$

$$p_1 - p_2 = \frac{4\Delta x \tau_w}{D}$$

* For turbulent flow

$$\tau_w \propto \frac{\rho V^2}{2}$$

$$\frac{1}{2} \rho V^2 = \frac{1}{2} \frac{M}{Q} V^2 = \frac{k}{Q} \quad \text{volumetric flow rate}$$

$$\tau_w = f \left(\frac{1}{2} \rho V^2 \right)$$

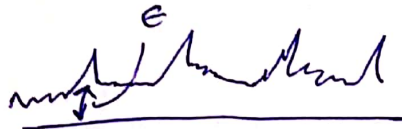
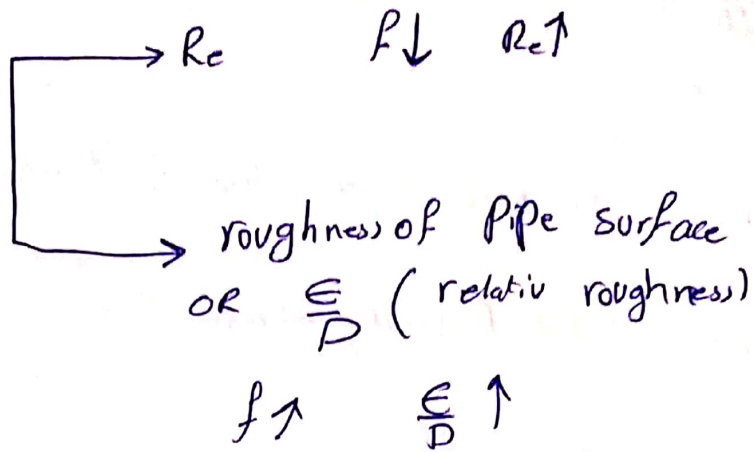
$$p_1 - p_2 = \frac{4\Delta x f \left(\frac{\rho V^2}{2} \right)}{D} = \rho f$$

f : Fanning Friction factor

$$f = \frac{F}{4 \left(\frac{\Delta x}{D} \right) \left(\frac{V^2}{2} \right)}$$

Note: f Constant $\frac{\Delta x V^2}{D}$, Constant = 24

f depends on



$$\frac{f}{2} = \frac{f'}{8} \rightarrow \text{Civ'l}$$

Chem
+
mech

Venturi meter



BE: $\frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} = 0$ / CE: $U_1 = U_2 \frac{A_2}{A_1}$

$$0 = \frac{P_2 - P_1}{\rho} + \frac{U_2^2}{2} \left(1 - \left(\frac{A_2}{A_1} \right)^2 \right)$$

$$U_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho \left(1 - \left(\frac{A_2}{A_1} \right)^2 \right)}}$$

* $Q = C_v U_2 A_2$ C_v : discharge coefficient

Different momentum $(P_2 - P_1) = \Delta P g h$

$$Q = C_v \sqrt{\frac{2g \Delta P h / \rho \cdot A_2}{1 - \left(\frac{A_2}{A_1} \right)^2}}$$

$$\frac{A_2}{A_1} = \left(\frac{P_2}{P_1} \right)^{1/2}$$

$$\frac{A_2}{A_1} = \left(\frac{D_2}{D_1} \right)^2 = \beta_2$$

$$Q = C_v \sqrt{\frac{2g \Delta P h / \rho}{1 - \beta^4}} \cdot A_2$$

C_v for well dis charged VM

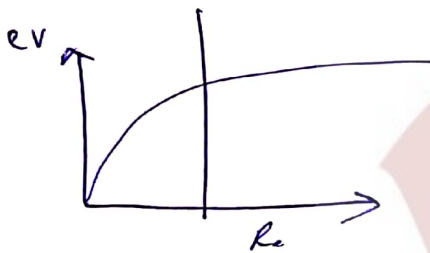
$$C_v = 0.96 - 0.98$$

- it depend on ① ratio $\frac{A_1}{A_2}$

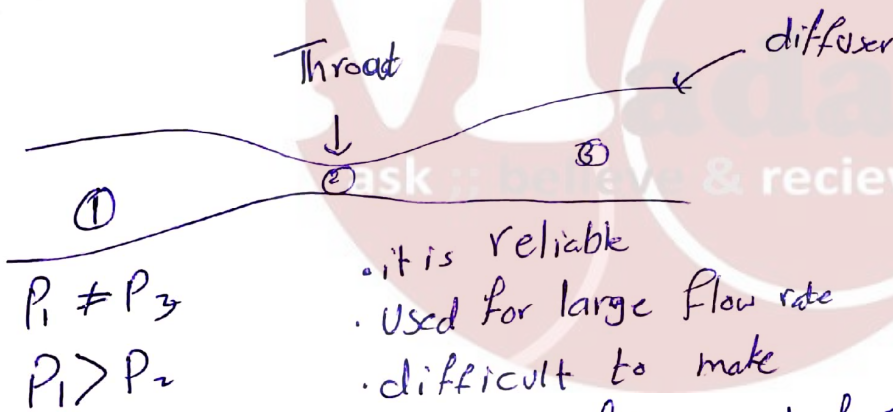
② Re no at the throat :

$$Re = \frac{\rho V_2 P_2}{\mu}$$

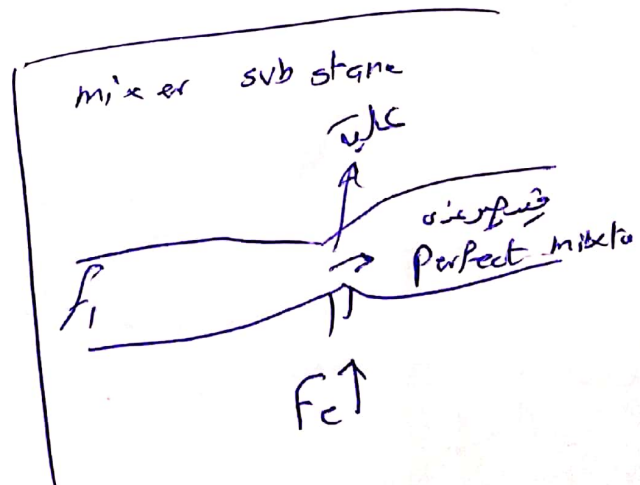
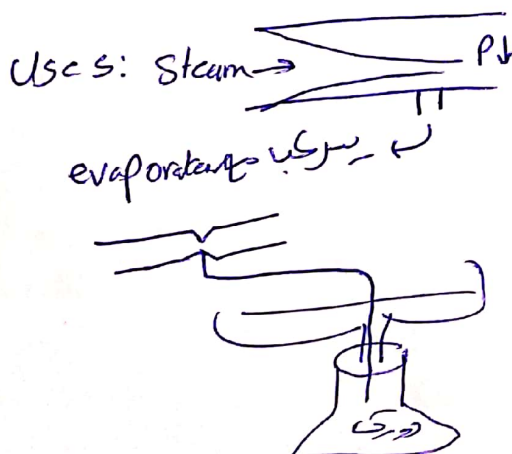
العويان ابي دور



Characteristics of VM
 - shape is important the diffuser has a narrow angle (2°) in order to slow down the recovery of pressure and minimise losses



• it is reliable
 • Used for large flow rate
 • difficult to make
 • requires a fair amount of space



Friction Factor

Laminar Flow:

$$\Delta p = \frac{Q \Delta x \mu}{\pi D^4} \quad \text{Hagen}$$

$$\Delta p = \frac{4 \Delta x f \left(\frac{1}{2} \rho v^2\right)}{D} \quad \text{Darcy}$$

$$\frac{4 \Delta x f \left(\frac{1}{2} \rho v^2\right)}{D} = \frac{Q \Delta x \mu}{\pi D^4}$$

$$f = \frac{32 Q \mu}{\rho v^2} \cdot \frac{D}{\pi D^4}$$

$$f = \frac{64 Q \mu}{\pi \rho v^2 D^3}$$

$$Q = \frac{\pi D^2 v}{4}$$

$$f = \frac{64 \pi D^2 / 4 v \mu}{\pi \rho v^2 D^3}$$

$$f = \frac{16 \mu}{\rho v D} = \frac{16}{Re}$$

* Friction Factor plots :

$$f = \frac{16}{Re}$$

$$\log f = \log 16 - \log Re$$

* ~~Can~~

Correlations for f

Laminar Flow $f = \frac{16}{Re}$

Turbulent Flow

Colebrook equation

$$\frac{1}{\sqrt{f}} = -4 \log \left(\frac{E/D}{3.7} + \frac{1.255}{Re \sqrt{f}} \right)$$

To calculate f , a numerical method is used (trial and error)

* Woods approximation:

In the range of Moody diagram

Woods

$$f = a + b Re^{-c}$$

Constant $\left[\begin{array}{l} a = 0.0235 \left(\frac{E}{D} \right)^{0.225} + 0.1325 \left(\frac{E}{D} \right) \\ b = 22 \left(\frac{E}{D} \right)^{0.44} \\ c = 1.62 \left(\frac{E}{D} \right)^{0.134} \end{array} \right]$

For given pipe roughness a, b, c are constant.

* Blasius equation

used in the range $10^4 < Re < 10^5$

$$f = 0.079 Re^{-1/4}$$

* Definition of f

Depending on how T_w is related to ρU^2
Stanton and Pannell [Colson and Richardson]

$$\phi = \frac{T_w}{\rho U^2}$$

Fanning or Darcy:

$$f = \frac{\tau_w}{\rho V^2/2}$$

civil eng Moody: $f = \frac{4 \tau_w}{\rho V^2/2}$

$$\phi = \frac{f}{2} = \frac{f'}{8}$$

Mechanical eng $\checkmark$ civil eng.

* Fluid Friction in Non-circular Conduits

Hydraulic Radius [Convenient means of expression
size and shape of Conduit]

$$R_h = \frac{A}{P}$$

A: Cross sectional area available for flow.

~~R_h~~ $\rightarrow$ p: wetted perimeter

For example for a circular pipe flowing full.

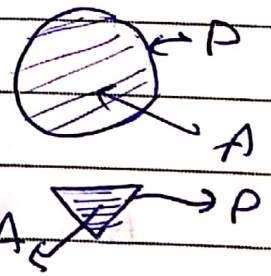
$$R_h = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

$$D = 4 R_h$$

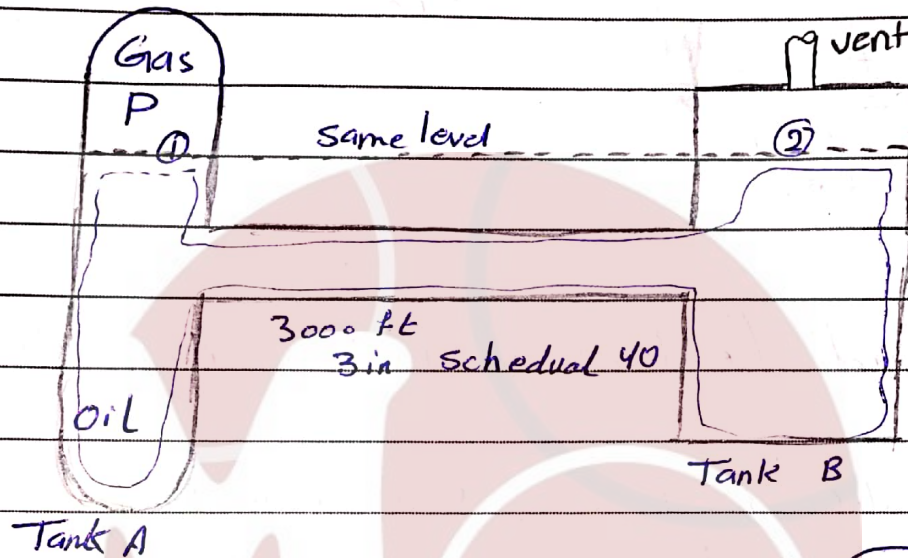
$$\frac{E}{D} \sim \frac{E}{4 R_h}$$

$$Re = \frac{\rho V (4 R_h)}{\mu}$$

$$f = \frac{\tau_w}{\frac{(4 \Delta x)}{4 R_h} \left(\frac{U^2}{2} \right)}$$



this approach gives good result for turbulent flow, Friction depends wetted perimeter
 For laminar flow result are poor since frictional phenomena depends viscous effect



$$Q = 100 \text{ gal/min}$$

$$\rho = 58 \text{ lbm/ft}^3$$

$$\mu = 100 \text{ cp}$$

$$P = ??$$



outside diameter 3 in 11.5

$$OD = ID + 2t$$

$$ID \text{ of AP} = 3.068 \text{ in}$$

BE between ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + f$$

$$\boxed{P_1 - P_2 = \rho f} \rightarrow P_1 = \rho f \text{ gage } (P_2 = P_{atm})$$

Check on Re

$$V_{avg} = \frac{Q}{A} = \frac{100 \text{ gal/min}}{\frac{\pi}{4} (3.068)^2 \text{ in}^2} \times \frac{144 \text{ in}^2}{\text{ft}^2} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ ft}^3}{7.48 \text{ gal}}$$

$$= 4.37 \text{ ft/s} \approx 1.33 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{4 \rho Q}{\pi \mu D} = \frac{4 \dot{m}}{\pi \mu D}$$

$$Re = \frac{4.58 \text{ bmft}^3}{\pi \times 100 \text{ cp} \times 3.068 \text{ in}} \times 100 \text{ gal/min} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ ft}}{7.48 \text{ gal}} \times \frac{2 \text{ in}}{1 \text{ ft}}$$

$$\times \frac{1 \text{ ft} \cdot \text{s} \cdot \text{cp}}{6.72 \times 10^{-4} \text{ bm}}$$

$$Re = 956$$

$\therefore$ Flow is laminar

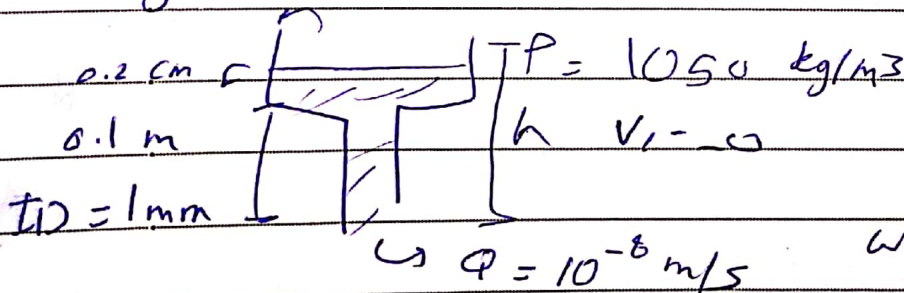
* Darcy Equation

$$P_1 - P_2 = \left(4 \Delta x f \left(\frac{1}{2} \rho V^2 \right) / D \right) = P_f$$

$$P_{1 \text{ gage}} = \dots$$

$$= 93 \frac{\text{bf}}{\text{in}^2} \text{ gage.}$$

large



What a viscosity

$$g(z_2 - z_1) + \frac{V^2}{2} = -\gamma$$

$$g \Delta h + \frac{V^2}{2} = -\gamma$$

$$V = \frac{Q}{A}$$

$$F = f \cdot u \left(\frac{\Delta x}{D} \right) \cdot \frac{u^2}{2}$$

Laminar flow $f = \frac{16}{Re}$

Turbulent $f = f(Re, \epsilon/D)$

$$f = 0.001375 \left[1 + 20000 \frac{\epsilon}{D} + \frac{10^6}{Re} \right]^{1/3}$$

* Friction factor problems:

Flow problems involve six parameters

$D, V_{ave}(Q), F, P, M, \epsilon$

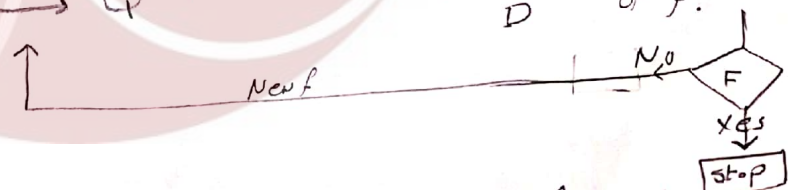
In a given flow problem P, M, ϵ are fixed.

The three most common types of problems, provided that P, M, ϵ are given:

Type	Given	Find	Solution
1	D, Q	F	Direct
2	D, F	Q	Trail and error
3	Q, F	D	Trail and error

* Type I: $P, M, D, Q \rightarrow Re$ and $\frac{\epsilon}{D} \rightarrow f \rightarrow F$ Direct solution

* Type 2: estimate of $f \xrightarrow{\text{Calc}} Q + (P, M, D) \rightarrow Re + \frac{\epsilon}{D} \rightarrow \text{New value of } f$



* Type 3: estimate $f \xrightarrow{\text{Calc}} D + (P, M, Q) \rightarrow Re + \frac{\epsilon}{D} \rightarrow \text{New value of } f$



Use ① & ②

$$\Delta \left(\frac{P}{\rho} + gz + \frac{v^2}{2} \right) = -\dot{W} - \mathcal{F}$$

2nd 2nd 4th & 2nd

3

$$-\mathcal{F} - \dot{W} = 0 \rightarrow (-\dot{W}) = \mathcal{F}$$

to the
system

* for Commercial steel $\frac{\epsilon}{D} = \frac{0.0018}{3.068} \leftarrow 3 \text{ in sch 40 (من الجدول)}$

$$Re = \frac{4PQ}{\pi MD} \quad (\text{استعمل الجدول})$$

$$= 2.05 \times 10^5 \text{ (Turbulent)} \rightarrow \text{from Chart or Equation} \rightarrow 4.8 \times 10^{-3}$$

$$\mathcal{F} = 4f \left(\frac{\Delta x}{D} \right) \frac{v^2}{2} = 176 \text{ ft} \cdot \frac{\text{bf}}{\text{bm}}$$

$$\text{work / mass} = 176 \text{ ft} \times \text{bf / bm} = \mathcal{F}$$

$$\text{Power} = \left(\frac{\text{work}}{\text{mass}} \right) \left(\frac{\text{mass}}{\text{time}} \right) = \dot{W} \times \text{in} = 176 \frac{\text{ft} \cdot \text{bf}}{\text{bm}} \times 27.8 \frac{\text{bm}}{\text{s}} \times \frac{1 \text{ hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{bf}} = 8.9 \text{ hp}$$

ask → P × Q = 27.8 bm/s recieve

$$\text{actual power} = \frac{8.9}{0.85}$$

* BE across pump $\Delta \left(\frac{P}{\rho} + gz + \frac{v^2}{2} \right) = -\dot{W} - \mathcal{F}$ friction less

$$\frac{\Delta P}{\rho} = -\dot{W} \rightarrow (P_o - P_{in}) = -\rho \dot{W} = -\rho \overset{\text{water}}{(-176 \text{ ft} \cdot \text{bf / bm})} = 76.1 \text{ bf / in}^2$$

Usually the working equation for pipe flow.

$$F = f \cdot 4 \frac{\Delta x}{D} \cdot \frac{V^2}{2}$$

$$\frac{F}{\Delta x} = f \frac{4 V^2}{2 D}$$

$$h_L = \frac{F}{\Delta x \cdot g} = \frac{2 f V^2}{D g}$$

$$V = Q/A$$

$$h_L = \left(\frac{32}{g \pi^2} \right) f \frac{Q^2}{D^5}$$

$$f = \frac{16}{Re} \text{ (laminar)} \text{ and } Re = \frac{4 \rho Q}{\pi \mu D}$$

laminar flow:

$$h_L = \left(\frac{128 \mu}{\rho g \pi} \right) \frac{Q}{D^4}$$

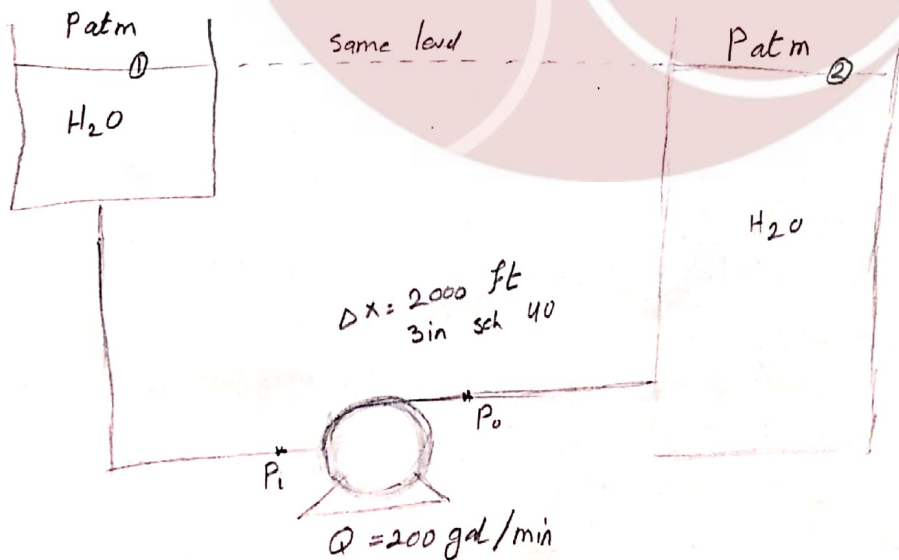
* Turbulant Flow:

$$h_L = \frac{32}{g \pi^2} f \frac{Q^2}{D^5}$$

$$f = f \left(\frac{4 \rho Q}{\pi \mu D}, \frac{E}{D} \right)$$

$\underbrace{\hspace{1cm}}_{Re} \nearrow$

Ex 6.8:



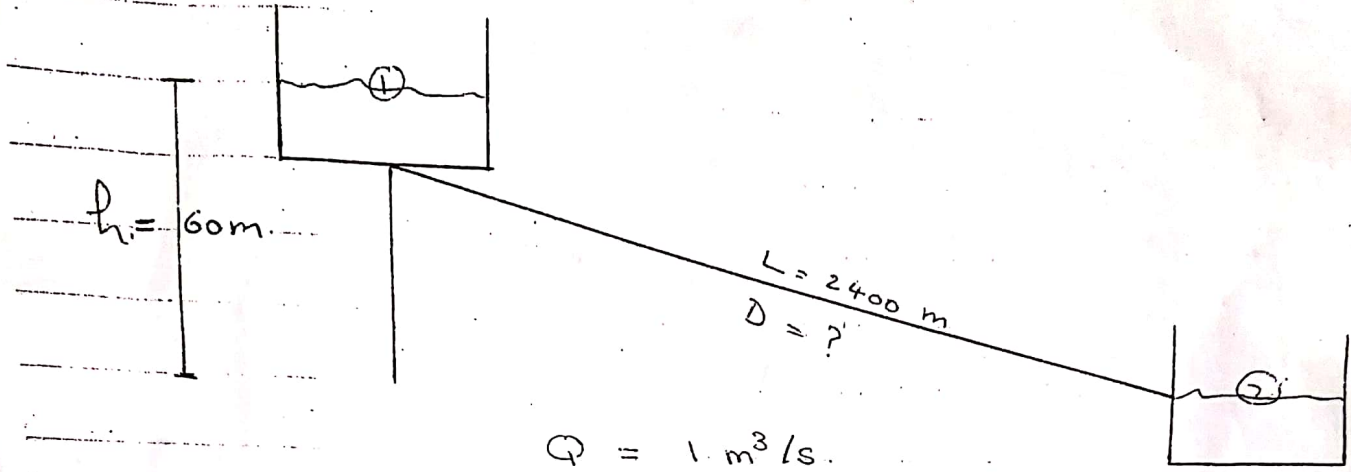
Given Q, D, ρ, μ, E
 Reqd $F \rightarrow W$
 so it type I

Reqn:

- pump work / unit mass
- pump power ($\eta = 85\%$)
- pump pressure rise

Solution $\rightarrow$

U



$Q = 1 \text{ m}^3/\text{s}$

$D = ?$

type 2

1] BE between ① & ②

$g \Delta z = -f \Rightarrow f = gh \quad (-\Delta z = h)$

2] Darcy equation.

$f = 4f \frac{L}{D} \cdot \frac{V^2}{2}$
 $\left\{ \begin{array}{l} V = \frac{4Q}{\pi D^2} \\ f = gh \end{array} \right.$

$D^5 = \frac{32 Q^2 L}{g h \pi^2} \cdot f$

$D = K_1 \cdot f^{0.2}$

$K_1 = \left(\frac{32 Q^2 L}{g h \pi^2} \right)^{0.2}$

$K_1 =$

Smooth Pipe سلسله

Trial Number	f	$D = K_1 f^{0.2}$	* $Re = \frac{4Q}{\pi \mu D}$	f from chart
1	0.005	0.58 m	2.2×10^6	0.0027
2	0.0027	0.5	2.5×10^6	0.00275
3	0.025			
4				

* $Re = \frac{4Q}{\pi \mu} \cdot \frac{1}{D}$

$Re = K_2 \cdot \frac{1}{D}$

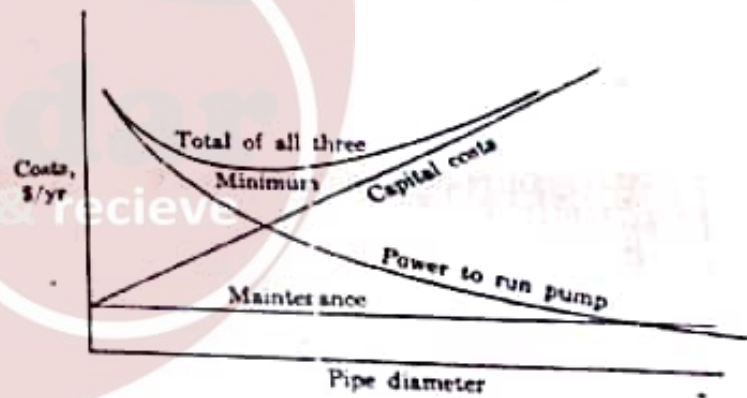
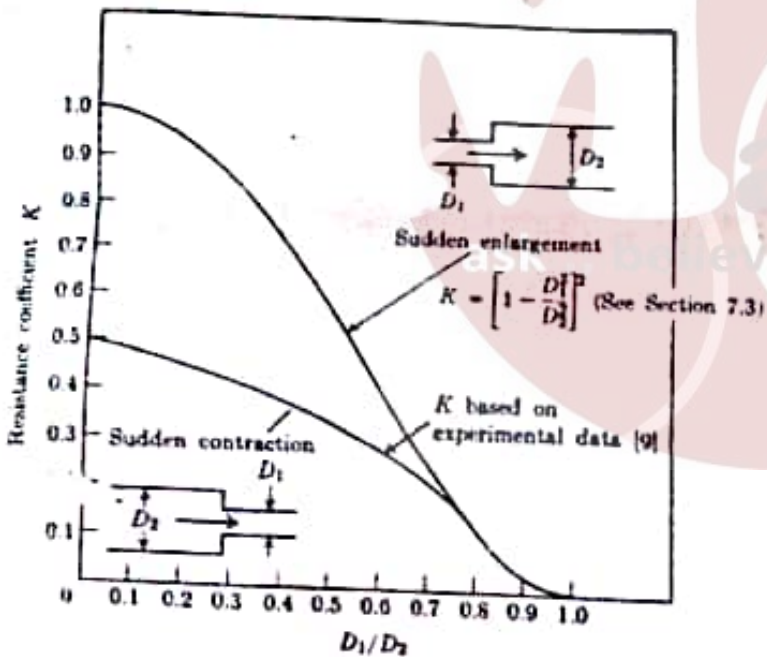
$K_2 = \frac{4Q}{\pi \mu}$
1.67



TABLE 6.2
Values of surface roughness for various materials*

Material	Surface roughness	
	ϵ , ft	ϵ , in
Drawn tubing (brass, lead, glass, etc.)	0.000005	0.00006
Commercial steel or wrought iron	0.00015	0.0018
Asphalted cast iron	0.0004	0.0048
Galvanized iron	0.0005	0.006
Cast iron	0.00085	0.010
Wood stave	0.0006-0.003	0.0072-0.036
Concrete	0.001-0.01	0.012-0.12
Riveted steel	0.003-0.03	0.036-0.36

از جدول بهوشی خارج
نمیزد



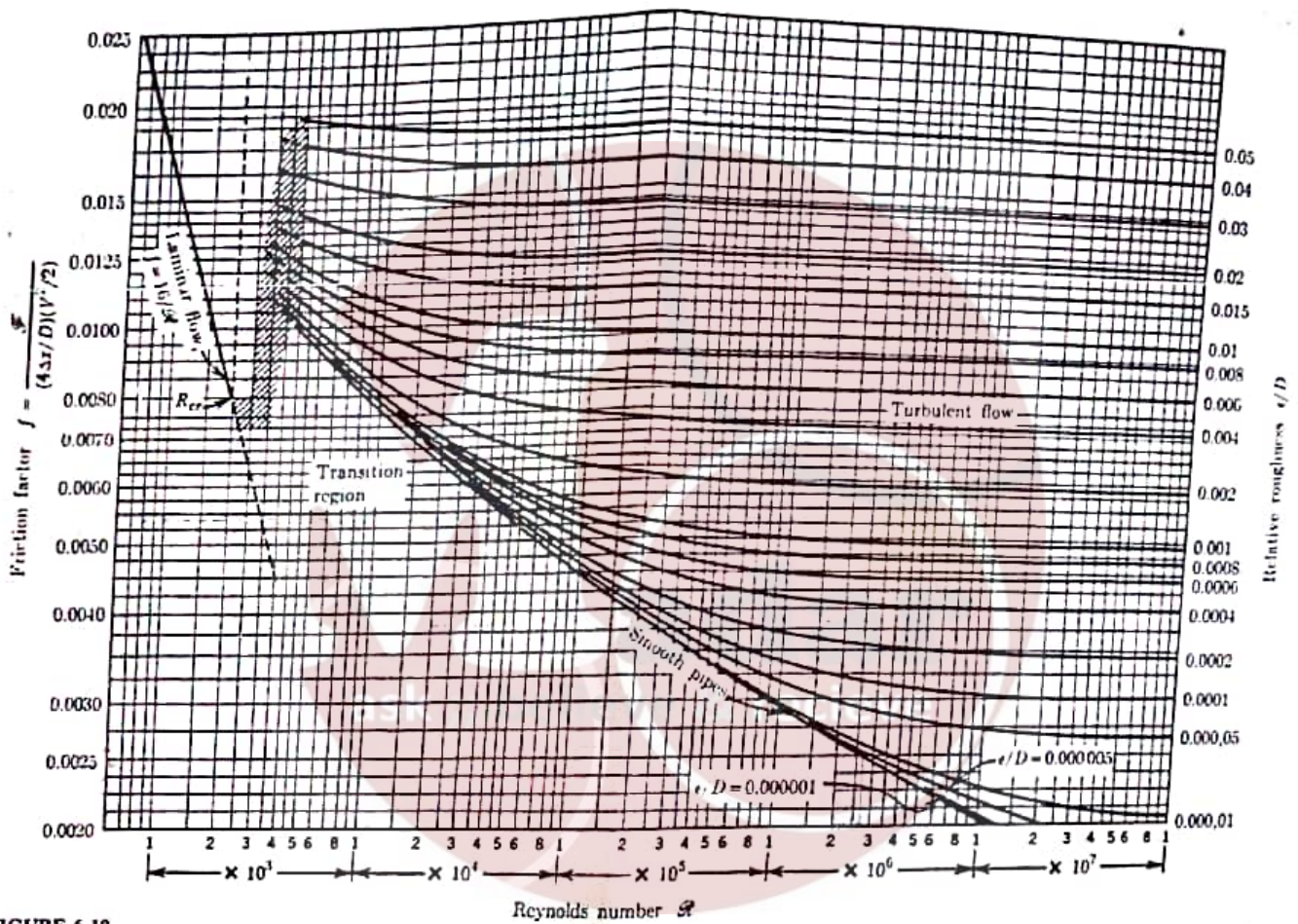
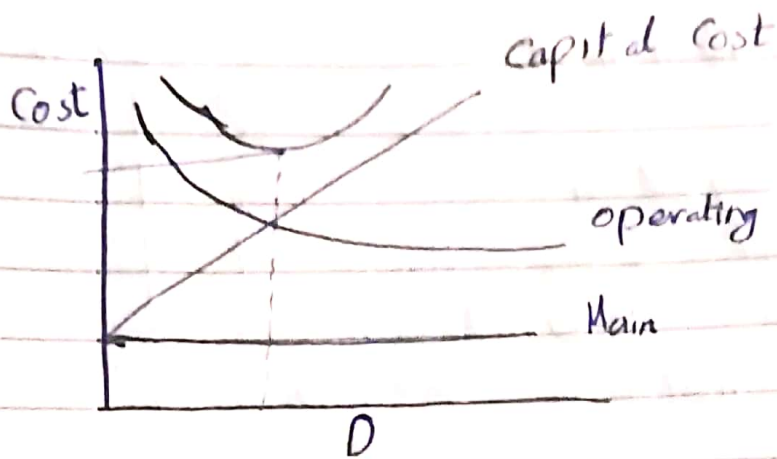


FIGURE 6.10

Friction factor plot for pipes. [From L. W. Moody, "Friction factors for pipeflow," *Trans. ASME* 66: 672 (1944). Reproduced by permission of the publisher.]



* Momentum Balance

- Extensive property $\rightarrow$ additive property.
- It is an abstract quality like energy.
- $\vec{p} = m\vec{v}$ • vector quantity.

$$\frac{\partial m\vec{v}}{\partial t} = \text{rate of change of momentum}$$

$$= m\vec{a}$$

$$\therefore F = m\vec{a} = \text{rate of change of momentum} = \frac{\partial m\vec{v}}{\partial t}$$

$$F - m\vec{a} = 0$$

$$(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) - m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) = 0$$

$$(F_x - ma_x)\hat{i} + (F_y - ma_y)\hat{j} + (F_z - ma_z)\hat{k} = 0$$

units
zero $\therefore F_x - ma_x = 0$ and $\downarrow = 0$ $\leftarrow = 0$

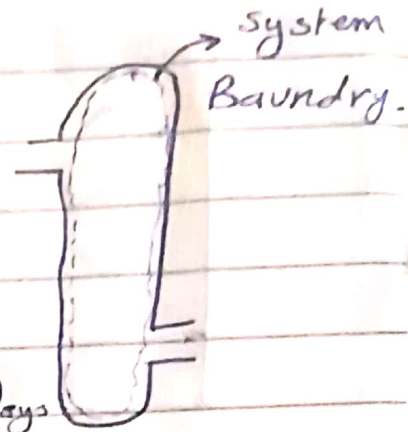
Momentum Balance

- General Balance equation

$$\text{Mom. Accumulation} = \text{Mom in} - \text{Mom out} + \text{production}$$

$$\ast \text{Momentum Accumulation} = \frac{\partial (m\vec{v})_{\text{sys}}}{\partial t}$$

$$\text{mom in} - \text{mom out} = \vec{V}_{\text{in}} \dot{m}_{\text{in}} - \vec{V}_{\text{out}} \dot{m}_{\text{out}}$$



$$\text{production} = \text{Newton's law} \quad \vec{F} = m\vec{a} = \frac{\partial (m\vec{v})_{\text{system}}}{\partial t}$$

$$\frac{\partial (m\vec{v})_{\text{system}}}{\partial t} = \vec{F} \cdot dt$$

assuming closed system: The terms of production must be forces.

$$\begin{aligned} \therefore \frac{\partial (m\vec{v})}{\partial t} &= \vec{V}_{\text{in}} \dot{m}_{\text{in}} - \vec{V}_{\text{out}} \dot{m}_{\text{out}} + \sum \vec{F} \cdot dt \\ \frac{\partial (m\vec{v})}{\partial t} &= \vec{V}_{\text{in}} \dot{m}_{\text{in}} - \vec{V}_{\text{out}} \dot{m}_{\text{out}} + \sum \vec{F} \end{aligned}$$

Newton's second law of motion for fluids
It is momentum Balance equation.

$\ast$ System Boundaries:

Chosen in such a way that one can conveniently calculate all the terms in the balance equation. It should be possible to account for all flows of matter across the boundaries and all external forces.

Steady flow application

For any direction, x For example

$$0 = \dot{V}_{x \text{ in}} \dot{m} - \dot{V}_{x \text{ out}} \dot{m} + \sum F_x$$

x direction

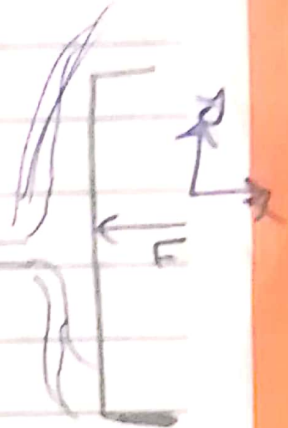
$$0 = \dot{m} (\dot{V}_{x \text{ in}} - \dot{V}_{x \text{ out}}) + \sum F_x$$

$$\sum F_x = F_x = -\rho Q \dot{V}_{x \text{ in}}$$

$$= -998.2 \frac{\text{kg}}{\text{m}^3} \times 0.01 \frac{\text{m}^3}{\text{s}} (30 \frac{\text{m}}{\text{s}} - 0)$$

$$= -299.5 \text{ N}$$

Jet of water



$$0 = \dot{m} (\dot{V}_{x \text{ in}} - \dot{V}_{x \text{ out}}) + \sum F_x$$

$$F_x = -\dot{m} (\dot{V}_{x \text{ in}} - \dot{V}_{x \text{ out}})$$

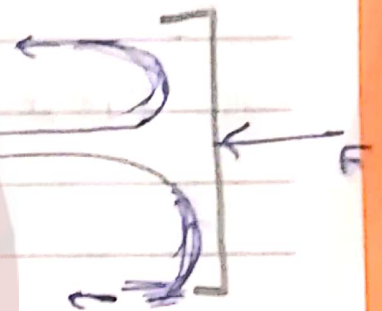
$$= 998.2 \frac{\text{kg}}{\text{m}^3} \times 0.01 \frac{\text{m}^3}{\text{s}}$$

$$\times (30 \text{ m/s} - (-15 \text{ m/s}))$$

$$= -449.3 \text{ N}$$

$$\dot{V}_x = 30 \text{ m/s} \rightarrow$$

$$\dot{V}_x = 15 \text{ m/s} \leftarrow$$



$$F = \sqrt{F_x^2 + (F_y)^2}$$

$$F_x = -0.01 \times 998.2 (30 - 30 \cos 45)$$

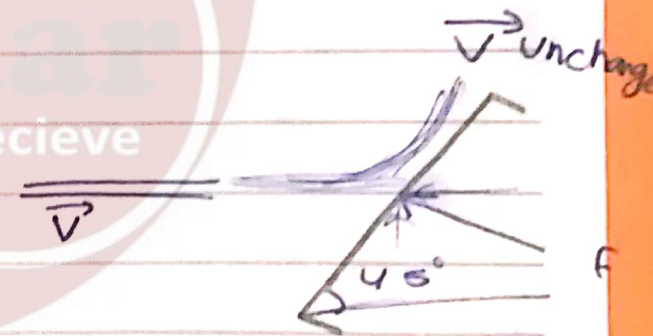
$$= -87.7 \text{ N}$$

$$F_y = -0.01 \times 998.2 (0 - 30 \sin 45)$$

$$= +211.6 \text{ N}$$

$$\therefore F = \sqrt{(-87.7)^2 + (211.6)^2}$$

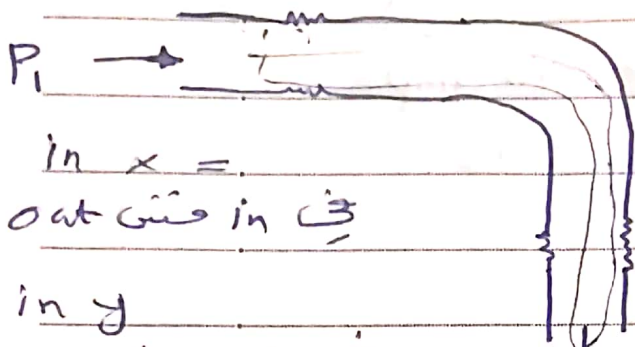
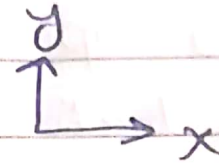
$$= 229.1 \text{ N}$$



$$\dot{V}_{x \text{ out}} = \dot{V}_{x \text{ in}} \cdot \cos 45$$

$$\dot{V}_{y \text{ out}} = |\dot{V}| \cdot \sin 45$$

Ch 7
in Books



in $x =$
out $\Rightarrow$ in $\Rightarrow$

in y
in $\Rightarrow$ out $\Rightarrow$

$$0 = m(V_{x\text{in}} - V_{x\text{out}}) + F_x \rightarrow (PA)_{\text{in}} + F_x$$

$$0 = m(V_{y\text{in}} - V_{y\text{out}}) + \sum F_y \rightarrow \begin{matrix} (+ve) \\ (PA)_{\text{out}} + F_y \end{matrix}$$

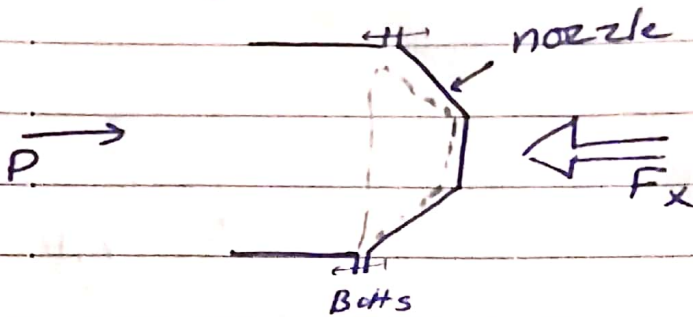
Cross section unchanged $V_{x\text{in}} = V_{x\text{out}}$

OR

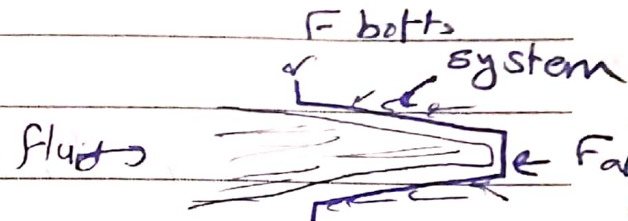


هیلر فکریہ
مکان

Example:



$$F_{bolts} = ??$$



Force exerted on fluid by nozzle = F_x

$$F_x = P \cdot A \rightarrow P_{g00} + P_{atm}$$

projected area

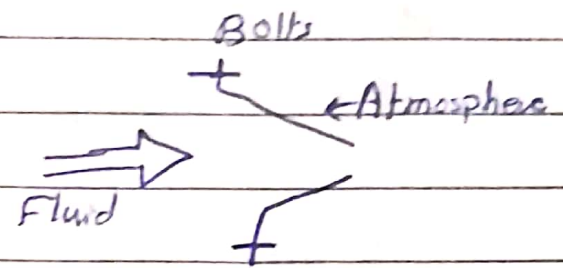
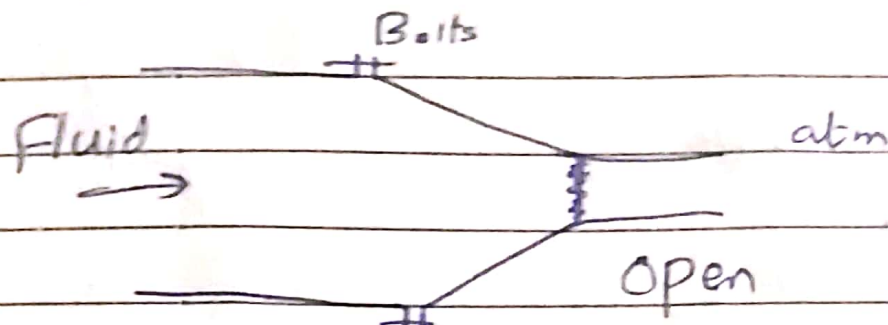
$$\sum F_x = F_{bolts} + F_{fluid} - F_{atm}$$

$$F_{bolts} = - (F_{fluid} - F_{atm})$$

$$= - (P_g + P_{atm}) A - P_{atm} A$$

القوة التي تحاول
تسبب

$$= - P_g \cdot A$$

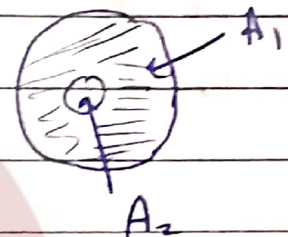


$$0 = F_{\text{fluid}} - F_{\text{bolt}} - F_{\text{atm}}$$

$$F_{\text{bolt}} = F_{\text{fluid}} - F_{\text{atm}}$$

$$F_{\text{atm}} = P_{\text{atm}} \cdot A_{\text{xn}}$$

$$P_{\text{atm}} (A_1 - A_2)$$



$$A_{\text{xn}} = A_1 - A_2$$

$$F_{\text{fluid on nozzle}} = F_{\text{nozzle on fluid}}$$

$F_{\text{nozzle on fluid}}$ can be calculated by considering the fluid in nozzle to be the system



$$0 = \text{mom}_{\text{in}} - \text{mom}_{\text{out}} + \sum F_x$$

$$\text{mom}_{\text{in}} = \dot{m} V_1 = \dot{m} V_2 \frac{A_2}{A_1}$$

$$\text{mom}_{\text{out}} = \dot{m} V_2$$

$$0 = \dot{m} V_2 \left(1 - \frac{A_2}{A_1}\right) + F_x$$

$$F_x = -\dot{m} V_2 \left(1 - \frac{A_2}{A_1}\right)$$

BUT $\Sigma F_x = P_1 A_1 - P_{atm} \cdot A_2 - F_{nozzle}$

$\therefore F_{nozzle \text{ on fluid}} = P_1 A_1 - P_{atm} A_2 + \dot{m} v_2 \left(\frac{A_2}{A_1} - 1 \right)$

so $F_{butt} = F_{fluid} - F_{atm}$

end of ch 7

ask ; believe & recieve

pumps

gas { Fan →
 blower → Q dist
 Compressor → ... factor 1/2

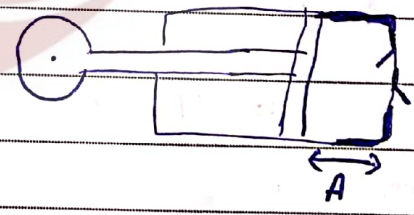
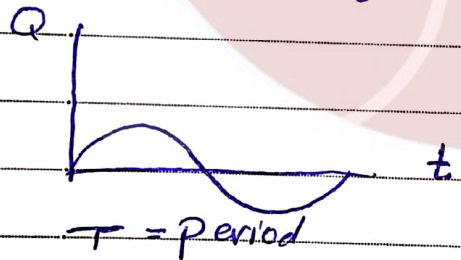
$$Z_2 + \frac{P_2}{\rho g} + \frac{u_2^2}{2} - (Z_1 + \frac{P_1}{\rho g} + \frac{u_1^2}{2}) = -\Delta h - h_f$$

Useful work head developed by pump head loss in system

total work = useful work + useless work

- * positive displacement pumps (static)
- * kinetic or dynamic (centrifugal)

Reciprocating pumps - piston and cylinder



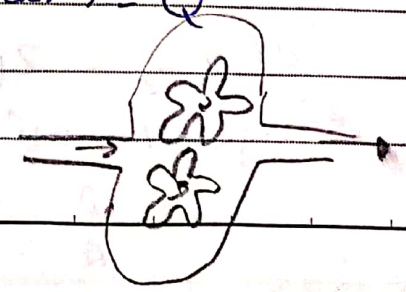
$f = \frac{1}{T}$ Frequency

A: amplitude
 $x_{\text{area}} = Q$

$Q \propto f, A$

amplitude Area $\times f = Q$

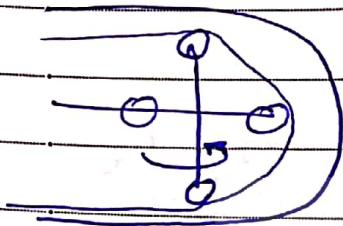
Rotary pumps: Gear pump



High pressure pumps $\rightarrow$ high pressure, high head

$Q \propto$ rotational speed.

* peristaltic pump

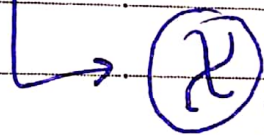
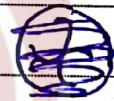


$Q \propto$ rotational speed.



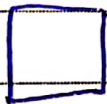
Dosing (metering pumps)

* Radial pumps }
* axial pumps }



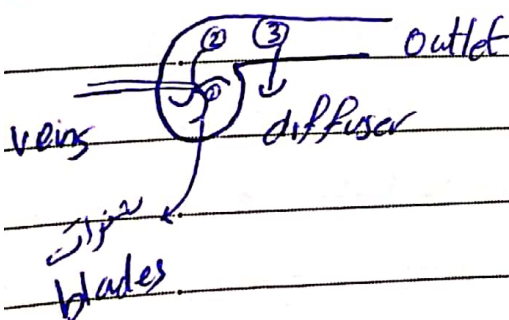
Impeller

mechanical energy.



$\rightarrow$ High, $K_E \rightarrow$ injection work $\rightarrow$ high pressure of outlet.

1 $\rightarrow$ 2



BE ① $\neq$ ②

assum friction is negligible.

$$P_2 > P_1 \rightarrow P_2 \approx P_1 / \frac{\Delta P}{\rho} \ll \frac{\Delta V^2}{2}$$

$$\frac{V_2^2}{2} \approx -\hat{w} \rightarrow \text{Mechanical work} \rightarrow K_E$$

Tangential vel = $r \times$ angular ~~speed~~ velocity

$$r \cdot 2\pi \text{ (RPM)} \rightarrow \text{radians Per minute}$$

Angular speed.



P doesn't change significantly

BE between 2 f3

$$\frac{P_3 - P_1}{P} \approx \frac{U_2^2}{2}$$

$$U_3 \ll U_2$$

$$P_2 = P_1$$

$$\frac{P_3 - P_1}{P} \approx \frac{U^2}{2}$$

for again pump

$$\left(\frac{\Delta P}{P} \right) \approx \text{Constant}$$

across
pump

$$\frac{\Delta P}{P} = \frac{U^2}{2} = \frac{P}{2} \left(\frac{D}{2} 2\pi \text{ RPM} \right)$$

$$= \frac{P}{2} (D \pi \text{ RPM})$$

Pumps

Pumps are devices for supplying energy or head to a flowing liquid in order to overcome friction and also if necessary to raise the liquid to a higher level or to flow against a higher pressure.

Pumps are associated with liquids while fans, blowers and compressors are associated with gases depending on the amount of gas to be moved and the pressure rise to be achieved.

If a pump is placed between two points in a piping system, then Bernoulli's equation is

$$\left(Z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \right) - \left(Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} \right) = -\Delta h - h_f$$

Useful work
Head developed by pump
Head loss in system

Therefore

$$\text{Total work} = \text{Useful work} + \text{useless work}$$

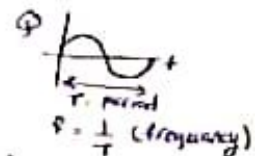
Classification of pumps:

Two basic types:

- Positive displacement pumps (static)
- Kinetic or dynamic (centrifugal)

Positive displacement pumps:

- Reciprocating pumps – Piston and cylinder $Q \propto f, A$
- Rotary pumps – Gear pumps, screw pumps $Q \propto \text{rotational speed}$



Characteristics:

- Suitable for viscosity liquids, dilatants liquids
- Difficult and expensive to maintain
- Cannot be operated against closed discharge
- Suitable for high head delivery
- High initial cost
- Can be used as metering pumps

A : Amplitude
 $x_{\text{max}} = Q$
 ampl. $\propto$ area of $f = Q$

Centrifugal pumps:

Energy or head is achieved by centrifugal action. Depending on the shape of the impeller we have:

Radial Pumps – Liquid is thrown radially (most common)

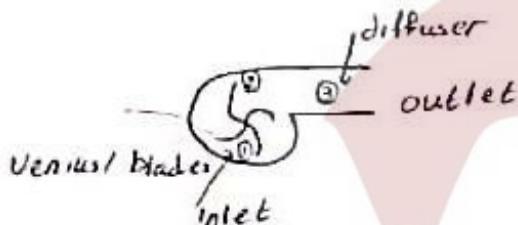
Axial Pumps – Liquid is thrown axially

Principle of operation:



Pump (Mech. Energy) → High KE → Injection work → High pressure at outlet

Theory



BE between 1 & 2 (assuming friction is negligible)

$$P_2 > P_1 \rightarrow P_2 \cong P_1 \text{ also } \frac{\Delta P}{\rho} \ll \frac{\Delta V^2}{2}$$

$$V_1 \ll V_2$$

$$\therefore \frac{\Delta V^2}{2} \cong - \frac{dW_{ao}}{dm} \approx - \hat{\omega}$$

Mech. Work → KE

Tangential velocity = $r \times$ angular velocity

$$= r (2 \pi \text{ rpm})$$

(دور في الدقيقة) ω rounds per minute

P does not change significantly

BE between 2 & 3 (diffuser section KE is diffused into P)

(Assuming friction is negligible, no work done, $V_3 \ll V_2$)

$$\frac{P_3 - P_2}{\rho} \cong \frac{V_2^2}{2} \cong \frac{P_3 - P_1}{\rho} \quad P_2 \simeq P_1$$

Centrifugal pump consists of two stages:

- Increase in KE at approximately constant P
- Conversion of KE into P in the diffuser section

For a given pump:

$$\frac{\Delta P_{\text{across pump}}}{\rho} \cong \text{Constant}$$

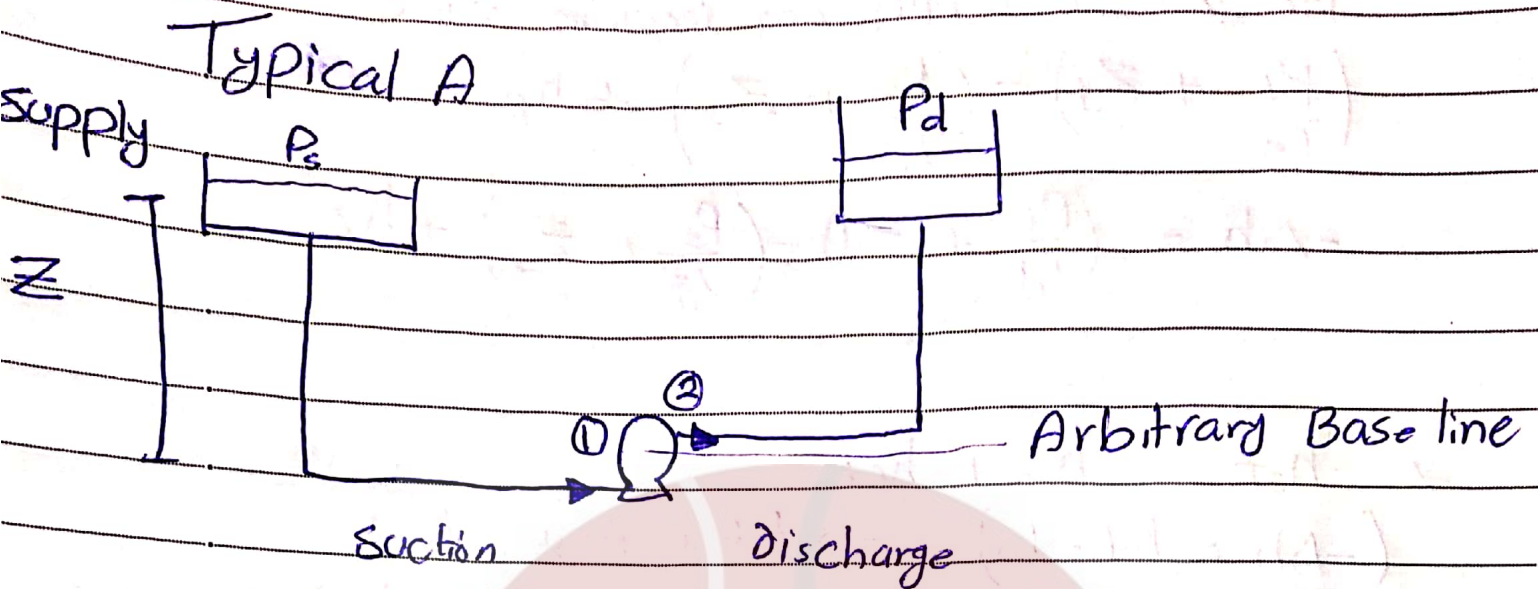
$$\Delta P = \rho \frac{V^2}{2} = \rho \frac{(r\omega)^2}{2}$$

$$= \frac{\rho}{2} \left(\frac{D}{2} 2\pi \text{rpm} \right)^2$$

$$= \frac{\rho}{2} (\pi D \text{rpm})^2$$

ask ; believe & recieve

* System Head 8



Important head to be Considered in a pumping system.

- Suction head
- Discharge head
- Total system head
- Available NPSH (Net ~~Pressure~~ Positive suction head)

Suction side

($v_s = 0$) BE between supply tank at ①

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_s}{\rho g} + Z_s - h_{fs}$$

Discharge side

BE between ② and receiving tank

$$\frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} = \frac{P_d}{\rho g} + Z_d + h_{fd}$$

* Suction head

$$h_s = \left(\frac{P_s}{\rho g} + Z_s \right) - h_{fs}$$

Note: $Z_s < 0$ if liq level is between base line

h_{fs} reduces h_s

* Discharge head

$$h_d = \frac{P_d}{\rho g} + Z_d + h_{fd}$$

Friction head

Note: h_{fd} increases h_d

System head

BE between supply and receiving tank

$$\left(\frac{P_d}{\rho g} + Z_d \right) - \left(\frac{P_s}{\rho g} + Z_s \right) + h_f$$

$$-\Delta h = \left(\frac{P_d}{\rho g} + Z_d \right) - \left(\frac{P_s}{\rho g} + Z_s \right) + h_f$$

$$* h_{ST} = h_{fs} + h_{fd}$$

$(-\Delta h = h_d - h_s) \rightarrow$ total system head

Note: $-\Delta h$ increases with Q since $(h_{fd} + h_{fs})$ increases with Q

* this is the total head which should be imparted delivered by pump to the fluid for a certain flowrate. It is called the system head. It is the work required to be done by the pump for a certain system at a certain flowrate.

* for the pump: BE between ① & ②

$$-\Delta h = \frac{P_2 - P_1}{\rho g}$$

* The system head equation can be written as:

$$-\Delta h = \underbrace{(Z_d - Z_s)}_{\text{①}} + \underbrace{\frac{P_d - P_s}{\rho g}}_{\text{②}} + \underbrace{h_{fd} - h_{fs}}_{\text{③}}$$

$$h_{fs} = 4f \frac{L_{es}}{D_s} \cdot \frac{V_s^2}{2g} + [h_{\text{entry}}]$$

$$h_{fd} = 4f \frac{L_{ed}}{D_d} \cdot \frac{V_d^2}{2g} + [h_{\text{exit}}]$$

In many cases these are negligible

- $\Delta h \propto Q$ Laminar

- $\Delta h \propto Q^n$ Turbulent

[inlet & exit]

* Available NPSH or simply NPSH

$$h_s = Z_s + \frac{P_s}{\rho g} - h_{fs} = \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

$$\frac{P_1}{\rho g} = h_s - \left(Z_1 + \frac{V_1^2}{2g} \right)$$

if Z_s is low or Q is high (high h_{fs}) $\rightarrow h_s$ will decrease

• If $P_1 \leq P_{vp} \rightarrow$ liquid will boil [Bubbles will be form at inlet]

• If P_1 is slightly higher than P_{vp} : bubbling (Cavitation) will occur inside the pump as the pressure is further decreased inside the pump as the liquid is accelerated.

$$NPSH = h_s - \left(\frac{P_{vp}}{\rho g} \right) \leftarrow \begin{array}{l} \text{head corresponding to} \\ \text{vapor pressure} \end{array}$$

$$NPSH = + \frac{(P_s - P_{vp})}{\rho g} - h_{fs}$$

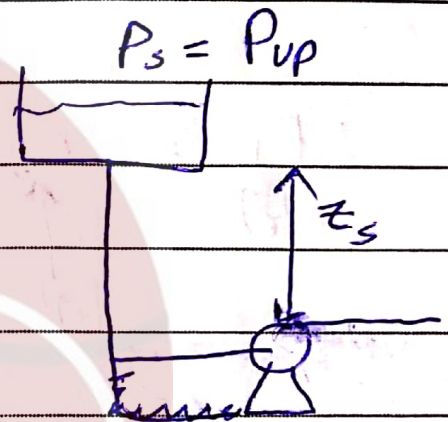
Note: • NPSH should always be +ve



- NP SH decreases with increasing Q
- At Bpt $\rightarrow$ No suction lift is possible
 $NP SH < 0$
 $Z_s < 0$

- In order to avoid Cavitation $\rightarrow$ the available NP SH must be greater than the NP SH required by the pump and specified by manufacturing

For boiling liquid $P_s = P_{vp}$
 Available NP SH = $Z_s = h_{fs}$



- $\therefore Z_s - h_{fs} > NP SH \text{ required}$
- $\therefore$ Curve must be ~~taken~~ taken in specified design
- Value of Z_s

* Centrifugal pumps:

- $\Rightarrow$ Describe performance of pump for a particular rotational speed and liquid viscosity
- $\rightarrow$ Manufacturers provide these curves for water at ambient conditions

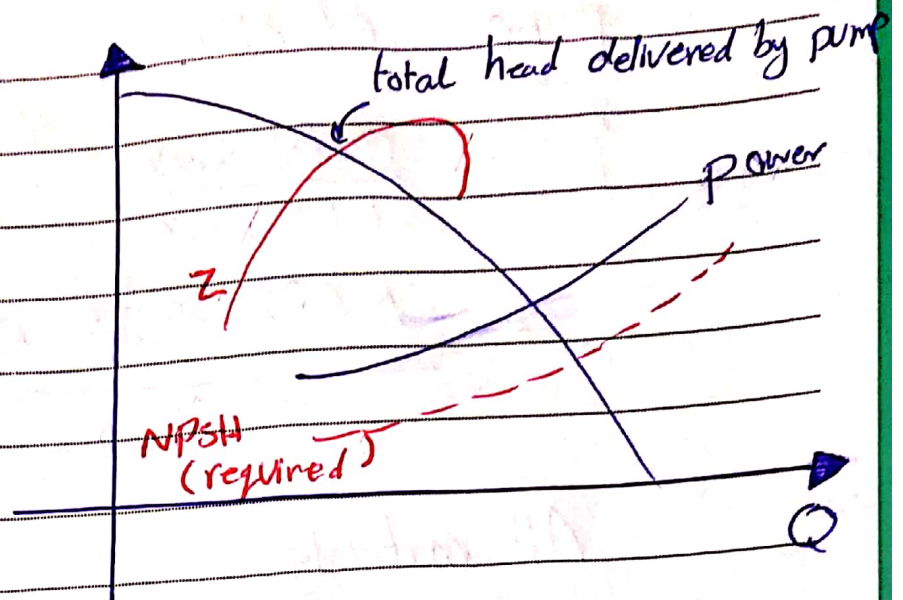
Types:

Δh Vs Q

power Vs Q

NPSH Vs Q

Z Vs Q



Δh Vs Q

is independent of P

$$\Delta P = \rho g \Delta h$$

Δh
max at
 $Q=0$

① + ②

system
 Δh system

Function
of M

ask, buy & receive

Centrifugal

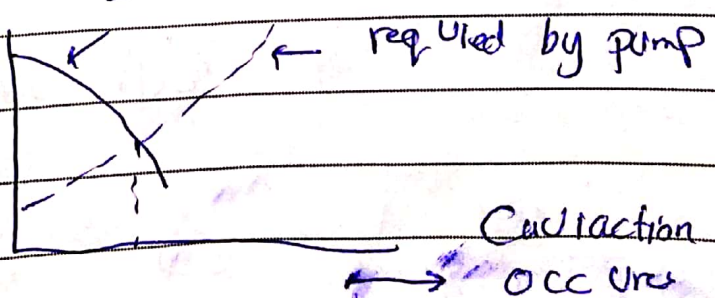
appears

→ particular system → a ~~centrifugal~~ pump ~~operator~~ only at one point of Δh Vs Q curve that is the point where the pump intersects with the system curve.

Curve

available in system

* NPSH Vs Q



pump efficiency:

$$\eta = \frac{\text{Useful work}}{\text{Total work}}$$

$$= \frac{\Delta \left(P/\rho + gz + \frac{V^2}{2} \right)}{\dot{W}}$$

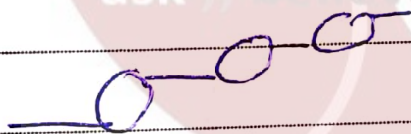
Alternatively

$$\eta = \frac{\text{hydraulic power}}{\text{power supplied}}$$

work power
↙
brake power
↘

series

(head Δh)



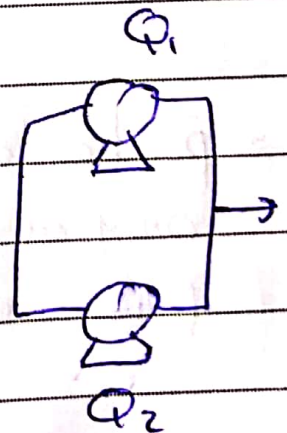
$$\Delta h_{\text{tot}} = \Delta h_1 + \Delta h_2 + \dots$$

$$Q = Q_1 + Q_2 + \dots$$

parallel (volume Δh)

$$Q_{\text{tot}} = Q_1 + Q_2$$

$$\Delta h_1 = \Delta h_2$$





* تم ثمر الله

* دفع ميكانيك الواجه 2020 / 2019

* مدار # عز منا أن كسب سحيا

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