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Process Dynamics & Control

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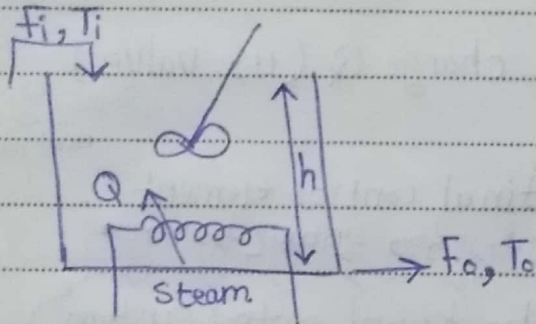


► Subject :

* Process :- Series of operation to utilize the resources to Produce useful Product in an economic way.

* Why need Process control :-

- Safety - Quality - Quantity - economically - Optimization



Process variables :-

T_i, T_o, F_i, F_o, Q, h

input
(T_i, F_i, Q)

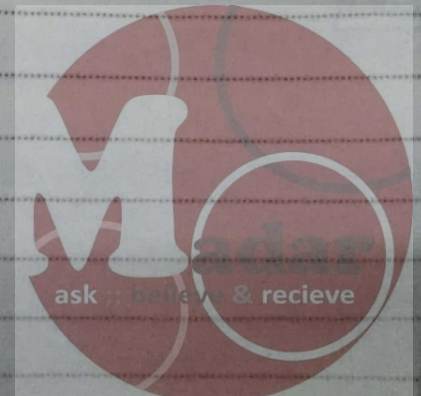
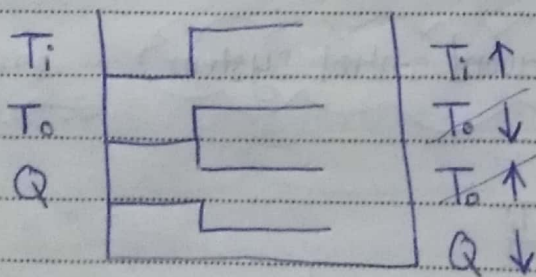
output
(T_o, F_o, h)

* input and output :- any variables effect the system (no physical direction)

* if no change Process (st.st) no need control, if we have variables we need control.

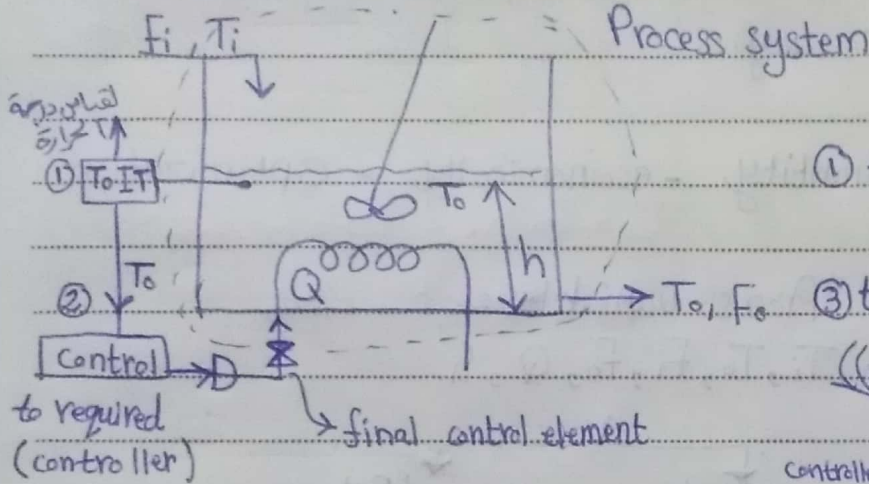
* control objective :-

$T_o, h, F_o \Rightarrow$ ~~control~~ we would like to control (output) variables (control variables)



► Subject : _____

* to control T_o :-



① temp indicator transmitter (TIT)

③ to change Q (use valve)

final control element
جزء مربوط مع system و جزء مربوط مع controller

① Select the control variables

② measuring device

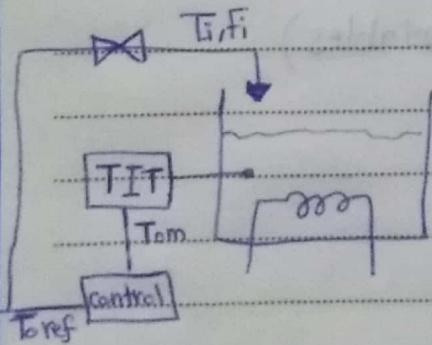
③ controller

④ Final control element

⑤ manipulated variables (I change it to achieve my control). valve

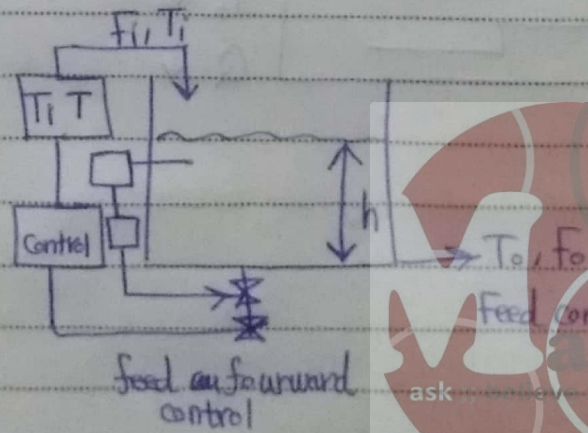
Feed backward control system

* the other control way $\Rightarrow$ Prevention and maintenance

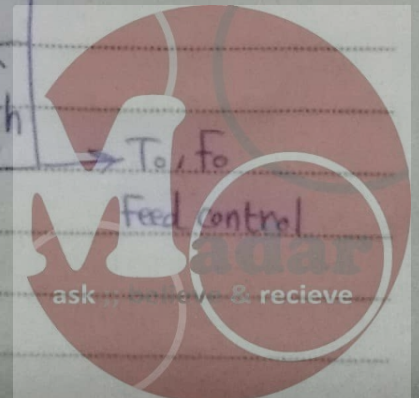


Feed forward control system

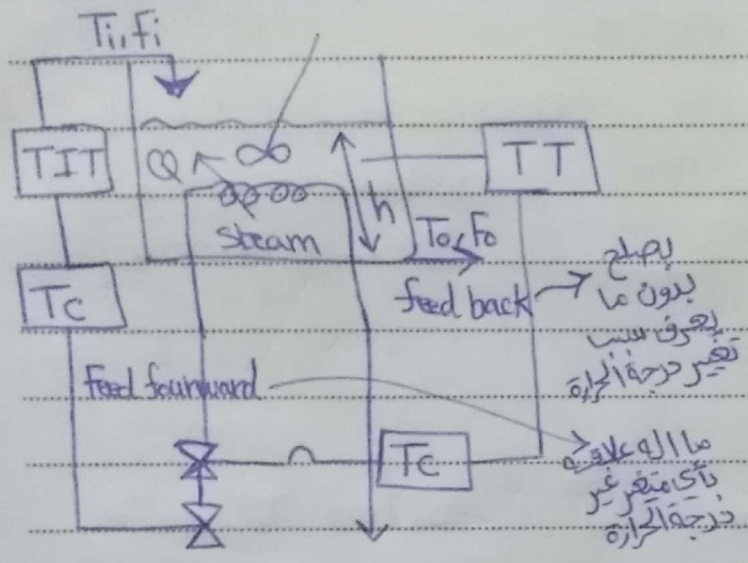
التي في
خيزر مسبق



Feed forward control



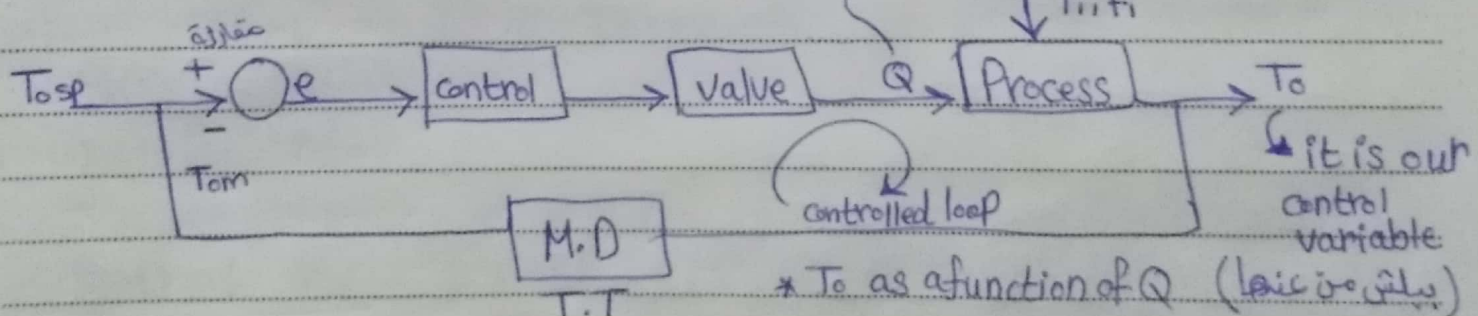
* We can combine feed back and forward



* T_o effective by T_i, F_i

بكون ما
يعرفه
تغير درجة الحرارة
ما الة على
بأنه متغير غير
درجة الحرارة

* PID → Block diagram



Disturbances (ما يتغير الحجم فيهم)

manipulated variable (أنا أغيره ونغيره هنا)

it's our control variable

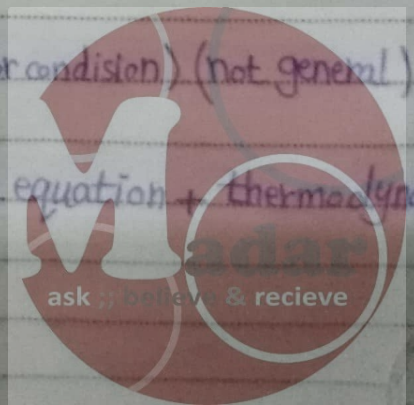
* T_o as a function of Q (بأنه من هنا)

* لازم يستر block diagram

* The dynamic modeling (unsteady state)

Theoretical (Analytical Analysis) Experimental (just is valid at this Process or condition) (not general)

* mass balance, energy balance, momentum + rate equation + thermodynamic
→ D.E or P.D.E



► Subject: _____

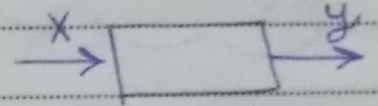
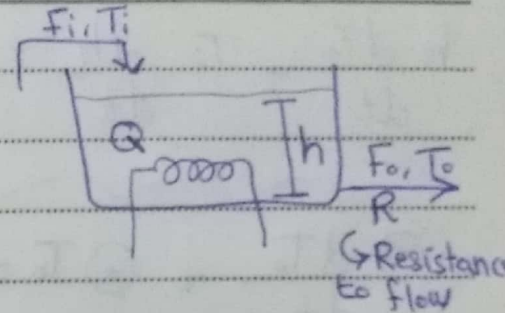
* mass balance

$$\frac{d[\rho A h]}{dt} = \rho_i f_i - \rho_o f_o$$

$$\rho A h = \rho V = m$$

f :- volumetric flowrate

assume ρ constant $\Rightarrow A$ constant



$$y = ax^2 + bx + c$$

$$A \frac{dh}{dt} = f_i - f_o$$

$$f_o = \frac{h}{R}, \quad R \text{:- resistance to flow (constant)}$$

$$AR \frac{dh}{dt} + h = R f_i \quad \rightarrow \text{I} \quad \begin{array}{l} * h \text{ only dependent on } f_i \\ * \text{ input function in output} \end{array}$$

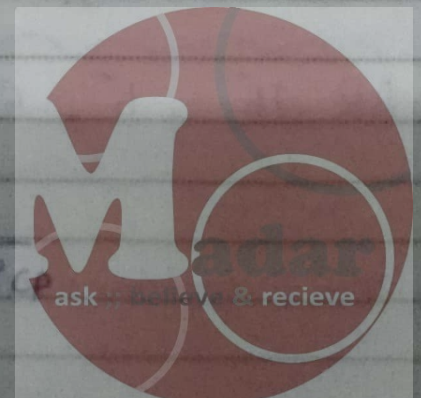
* Energy balance

$$\frac{d[\rho A h (T_o - T_{ref}) C_p]}{dt} = \rho_i C_p f_i (T_i - T_{ref}) - \rho_o C_p f_o (T_o - T_{ref}) + Q$$

assume $T_{ref} = 0$, (ρ, C_p, A) constant

$$\frac{A \rho C_p d[h T_o]}{dt} =$$

$$\frac{d[h T_o]}{dt} = \frac{1}{A} f_i T_i - \frac{1}{A} f_o T_o + \frac{Q}{A \rho C_p}$$



► Subject : _____

$$h \frac{dT_o}{dt} + T_o \frac{dh}{dt} = \frac{1}{A} f_i T_i - \frac{f_o T_o}{A} + \frac{Q}{\rho A c_p}$$

$$\left(\frac{dT_o}{dt} + T_o = T_i + f_i + Q \right) \rightarrow [2]$$

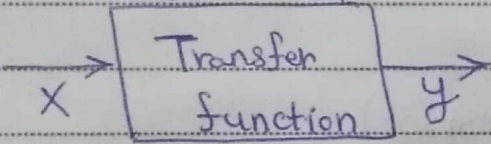
* لازم كل input يكون بالـ 1

* [1] and [2] equation
dynamic model

• the first step

* deviation form (deviation from)
steady state

• the second step



steady state من الـ 1 deviation form

$$AR \frac{dh}{dt} + h = R f_i$$

$$0 + h_{ss} = R f_{i,ss}$$

$$AR \frac{d(h - h_{ss})}{dt} + (h - h_{ss}) = R (f_i - f_{i,ss})$$

$$\text{let } h - h_{ss} = \bar{h}$$

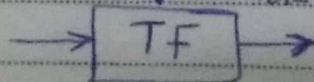
$$f_i - f_{i,ss} = \bar{f}_i$$

$$AR \frac{d\bar{h}}{dt} + \bar{h} = R \bar{f}_i$$

deviation form

* initial conditions are zero

Laplace transfer: it transfer
L.D.E to Algebraic equation



► Subject : _____

$$\mathcal{L} \{y(t)\} = Y(s)$$

$$\mathcal{L} \{y(t-a)\} = e^{-as} Y(s)$$

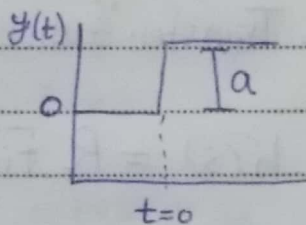
$$\mathcal{L} \left\{ \frac{dy}{dt} \right\} = s Y(s) - y(0)$$

$$\mathcal{L} \int y(t) dt = \frac{1}{s} Y(s)$$

① Step function

$$y(t) = 0 \quad t \leq 0$$

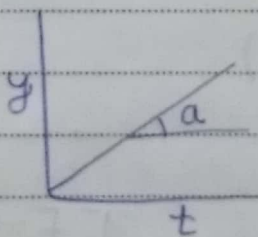
$$y(t) = a \quad t > 0$$



$$\mathcal{L} \text{ step} = \frac{a}{s}$$

② Ramp function

$$\mathcal{L} [y=at] = \frac{a}{s^2}$$



* Final value Theorem

$$y(s) = \dots$$

$$y(s) = \mathcal{L} \{y(t)\}$$

$$y(s) = \dots$$

$$y(t) = \mathcal{L}^{-1} \{y(s)\}$$

$$y(t) = \mathcal{L}^{-1} \{y(s)\}$$

$$y(t)_{\infty} = \lim_{s \rightarrow 0} s y(s)$$

→ Final value (Steady state value)



► Subject : _____

* Initial value Theorem

$$y(t_0) = \lim_{s \rightarrow \infty} s Y(s)$$

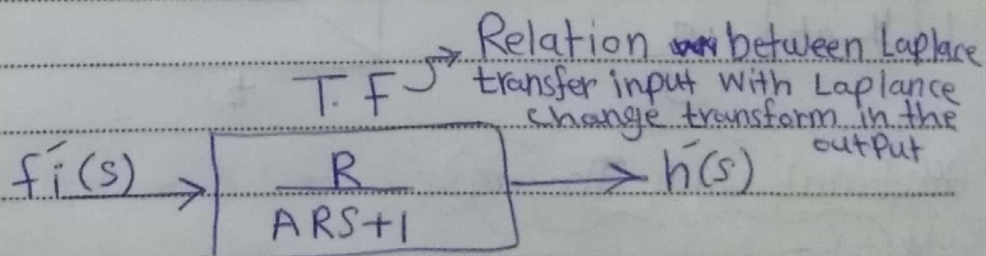
$$\mathcal{L} \left[AR \frac{dh}{dt} + h = R f_i \right] \rightarrow \text{Transfer function}$$

$$AR [s \bar{h}(s) - \bar{h}(0)] + \bar{h}(s) = R \bar{f}_i(s)$$

$$AR s \bar{h}(s) + \bar{h}(s) = R \bar{f}_i(s)$$

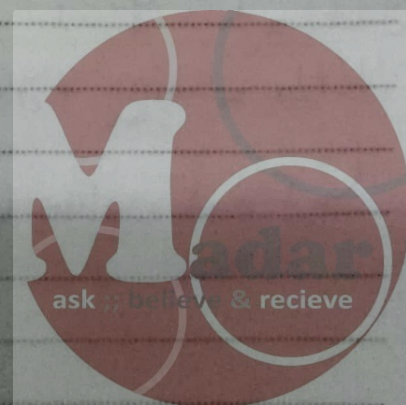
$$\bar{h}(s) [ARS + 1] = R \bar{f}_i(s)$$

$$\frac{\bar{h}(s)}{\bar{f}_i(s)} = \frac{R}{ARS + 1}$$



$$\frac{\bar{h}(s)}{\bar{f}_i(s)} \neq \frac{h(t)}{f_i(t)}$$

$$G(s) = \frac{\bar{h}(s)}{\bar{f}_i(s)} = \frac{R}{ARS + 1}$$



Subject: _____

$$\frac{h}{F_i} = \frac{R}{ARS+1}$$

$$\frac{h}{F_i} = \frac{R}{ARS+1} * f_i(s)$$

* First order system.

- what is first order system? system described by a 1st order D.E

$$a_0 \frac{dy(t)}{dt} + a_1 y(t) = bX(t)$$

* لا يكون لها أكثر من (1) درجة

$$\frac{a_0}{a_1} [s y(s) - y(0)] + y(s) = \frac{b}{a_1} X(s)$$

deviation form

ويعبر عن بؤنة

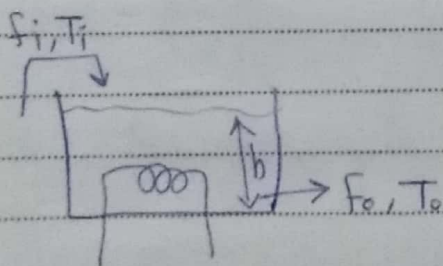
$$y(s) \left[\frac{a_0}{a_1} s + 1 \right] = \frac{b}{a_1} X(s)$$

$$G(s) = \frac{y(s)}{X(s)} = \frac{b/a_1}{\frac{a_0}{a_1} s + 1}$$

* standard form of the T.F of 1st order system.

* Polynomial (s) first degree

* two constant



$$L \left[AR \frac{dh}{dt} + h = R F_i \right]$$

$$[ARs + 1] h(s) = R F_i \Rightarrow G(s) = \frac{h(s)}{F_i(s)} = \frac{R}{ARS+1}$$

K: steady state gain

T: time constant

$$\frac{K}{Ts+1}$$

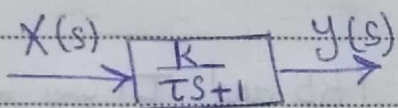
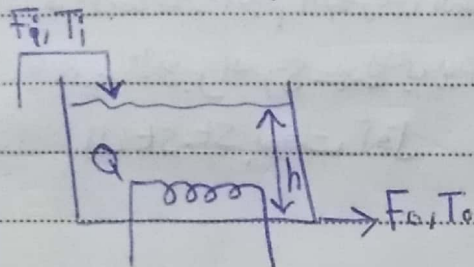
► Subject :

* first order system

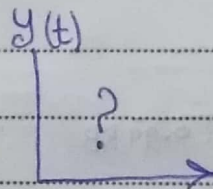
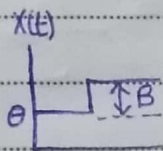
$$G(s) = \frac{K}{TS+1}$$

$$AR \frac{dh}{dt} + h = R F_i$$

$$G(s) = \frac{h(s)}{f_i(s)} = \frac{R}{ARS+1} = \frac{K}{TS+1}$$



X(t) step change



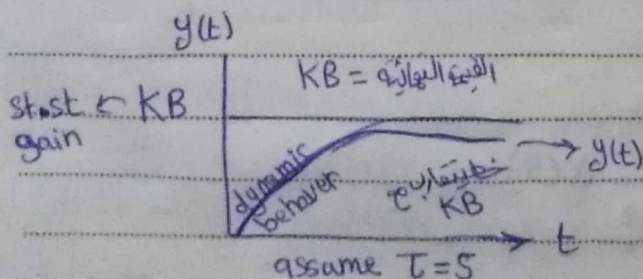
$$Y(s) = \frac{K}{TS+1} * X(s) = \frac{K}{TS+1} * \frac{B}{s}$$

$$\frac{Y(s)}{X(s)} = \frac{K}{TS+1}$$

$$Y(t) = \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \left[\frac{BK}{s(Ts+1)} \right]$$

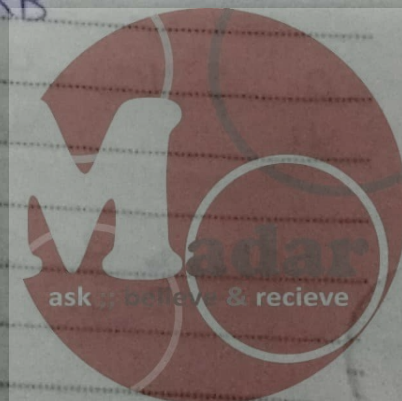
$$Y(t) = KB \left[1 - e^{-t/\tau} \right]$$

its the response of 1st order system.



at $t=0$, $y=0$

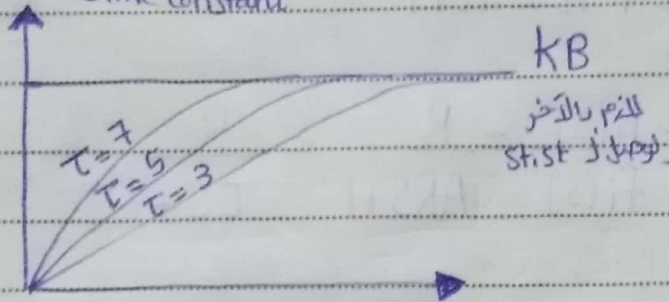
at $t=\infty$, $y=KB$



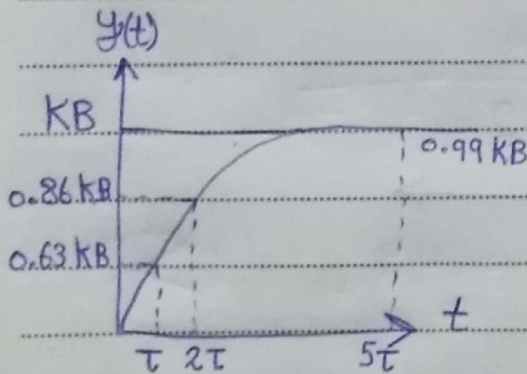
► Subject :

if $\tau = 3, 5, 7$

↳ time constant



* كلما كان الزم أخف من المرفوع
نتيجة التغير أسرع ← يعني بوصول
الـ St. St. بوقت أقل



* بغير (KB and τ)

لما أغير ← Rand A

بغير Physics of system.

* Second order system :-

$$a_0 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = b X(t)$$

$$L \left[\frac{a_0}{a_2} \frac{d^2 y}{dt^2} + \frac{a_1}{a_2} \frac{dy}{dt} + y = \frac{b}{a_2} X(t) \right]$$

$$\frac{a_0}{a_2} [s^2 y(s) - y(0) - y'(0)] + \frac{a_1}{a_2} (s y(s) - y(0))$$

$$+ y(s) = \frac{b}{a_2} X(s)$$

$$\frac{a_0}{a_2} s^2 y(s) + \frac{a_1}{a_2} s y(s) + y(s) = \frac{b}{a_2} X(s)$$

$$y(s) \left[\frac{a_0}{a_2} s^2 + \frac{a_1}{a_2} s + 1 \right] = \frac{b}{a_2} X(s)$$



Subject :

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b/a_2}{\frac{a_0}{a_2} s^2 + \frac{a_1}{a_2} s + 1}$$

* standard form of
2nd order system

* Ξ constant

معامل من الدرجة الثانية
في حال كان Ξ عددًا في المقام
كل ما يوجد على Ξ نضرب به

$$G(s) = \frac{Y(s)}{X(s)} = \frac{k \rightarrow \text{st. st gain}}{s^2 T^2 + 2\zeta T s + 1}$$

time constant damping factor

$$y(t) = \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \left[\frac{k}{s^2 T^2 + 2\zeta T s + 1} \times X(s) \right]$$

علاقة تربيعية

$$Ay^2 + By + C = 0$$

$$y_{1,2} = \frac{-B \pm \sqrt{B^2 - 4ac}}{2A}$$

$$T^2 s^2 + 2\zeta T s + 1 = 0$$

$$s_{1,2} = \frac{-2\zeta T \pm \sqrt{(2\zeta T)^2 - 4T^2}}{2T^2}$$

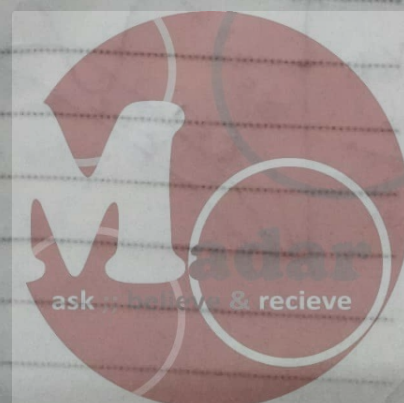
$$s_{1,2} = \frac{-2\zeta T \pm 2T \sqrt{\zeta^2 - 1}}{2T^2}$$

$$s_{1,2} = \frac{-\zeta}{T} \pm \frac{\sqrt{\zeta^2 - 1}}{T}$$

$\zeta > 1 \rightarrow 2$ Real Different Roots

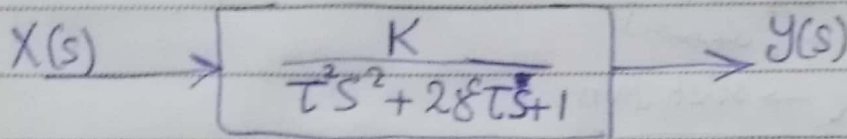
$\zeta = 1 \rightarrow$ " " equal "

$\zeta < 1 \rightarrow 2$ complex Roots $\rightarrow \mathcal{L}^{-1} y(t) = \dots \sin - \cos$



Subject :

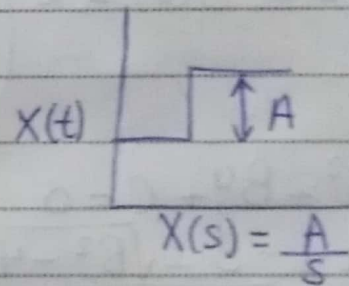
$$Y(s) = \frac{K}{(\quad)(\quad)} X(s)$$



$$Y(s) = \frac{K}{T^2 s^2 + 2\zeta T s + 1} * \frac{A}{s}$$

$\zeta \geq 1$, $y(t) = \dots \sin \dots \cos \dots$

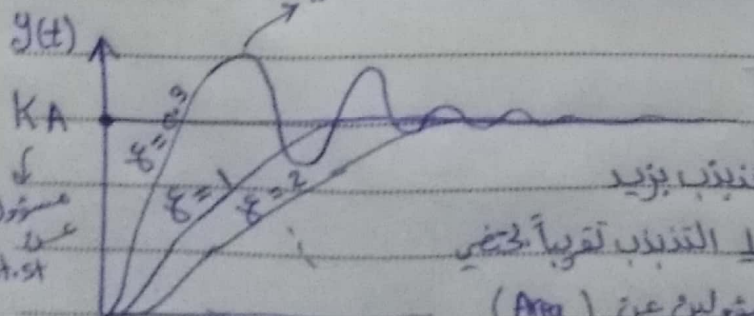
$$y(t)_{\infty} = \lim_{s \rightarrow 0} s Y(s)$$



$$y(t) = \lim_{s \rightarrow 0} s \left[\frac{K}{T^2 s^2 + 2\zeta T s + 1} * \frac{A}{s} \right] = KA$$

$$y(t)_0 = 0$$

if complex $\zeta < 1$



* كل ما قلنا ζ التذبذب يزيد
 * لما تقرب ζ من 1 التذبذب تقريباً يختفي
 * T و ζ مسؤولين عن (Area)
 KA $\downarrow$

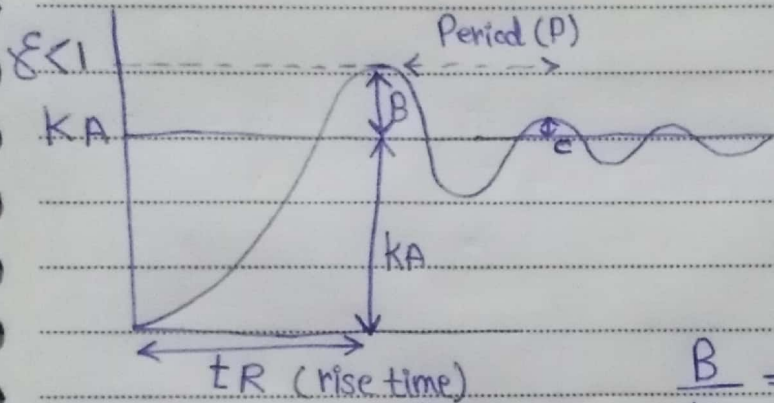
* أما ζ هي التي ممكن توحد ال Area
 باقي بعد KA



$\xi < 1$ underdamped

$\xi = 1$ critically damped

$\xi > 1$ overdamping



$$y(t) = \dots$$
$$\frac{dy}{dt} = 0$$

$$\frac{B}{KA} = \text{overshoot} = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$$

ratio between one ~~peak~~ and two Peak $\Rightarrow$ نسبة بين قمة واحدة وقمة أخرى

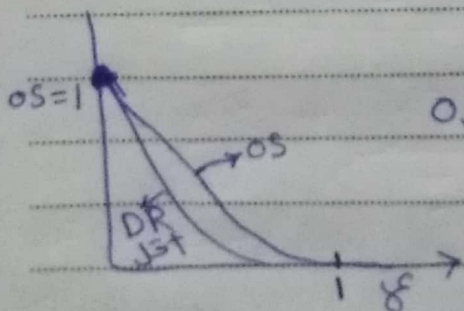
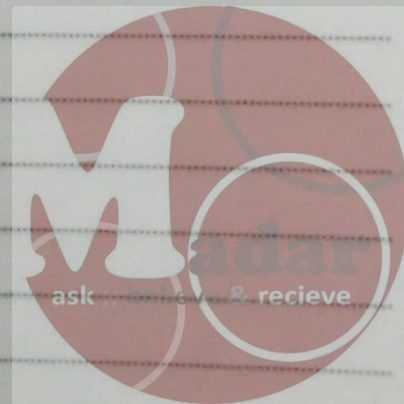
$$\frac{C}{B} = \text{Decay Ratio} = \exp\left(\frac{-2\pi\xi}{\sqrt{1-\xi^2}}\right)$$

$$DR = (OS)^2$$

Decay Ratio

overshoot

$$P = \frac{2\pi\tau}{\sqrt{1-\xi^2}}$$



$$OS = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$$

$$DR = (OS)^2$$

$$DR = (OS)^2$$
$$= (0.5)^2 = 0.25$$

* $OS \uparrow, \xi \downarrow$

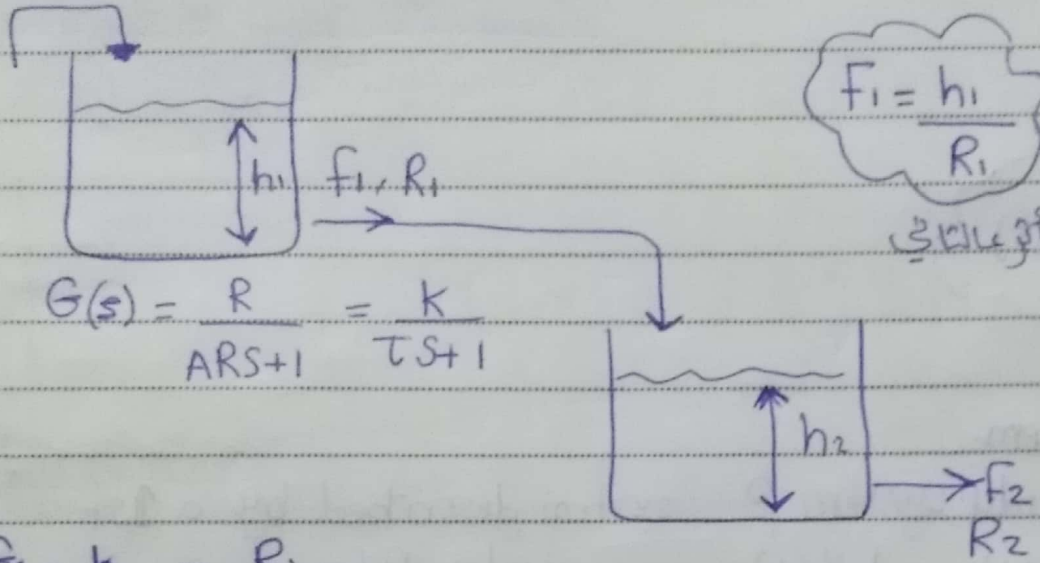
* min value $\xi = 0$

* at $\xi = 1, OS = 0$

max value $OS = 1$

min value " = 0

Subject :



تanks 1 و 2
والتي لا تتفاعل

non-interacting system

$$G_1 = \frac{h_1}{f_1} = \frac{R_1}{A_1 R_1 S + 1}$$

$$G_2 = \frac{h_2}{f_1} = \frac{R_2}{A_2 R_2 S + 1}$$

$$\frac{h_2}{f_1} = \frac{h_1}{f_1} * \frac{h_2}{f_1}, \quad f_1 = \frac{h_1}{R_1}, \quad R \text{ constant}$$

$$\begin{aligned} \frac{h_2}{f_1} &= \frac{h_1}{f_1} * \frac{h_2}{h_1} \\ &= \frac{R_1}{A_1 R_1 S + 1} * \frac{R_2}{A_2 R_2 S + 1} \\ &= \frac{R_1 R_2}{(A_1 R_1 S + 1)(A_2 R_2 S + 1)} \\ &= \frac{K_1 K_2}{(T_1 S + 1)(T_2 S + 1)} \end{aligned}$$



Subject: * 2nd order derivative
2nd system

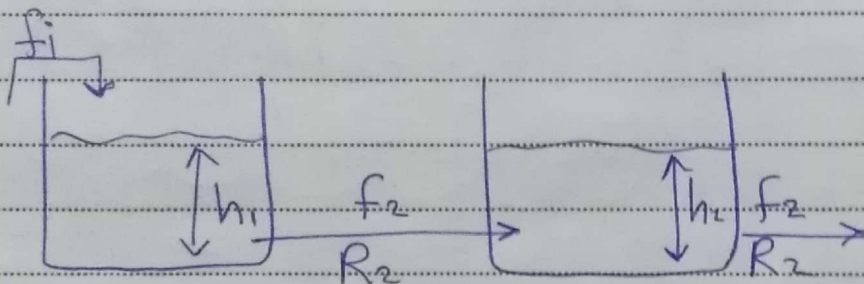
$$= \frac{K}{T^2 S^2 + 2ST + 1}$$

$$\xi = \frac{\sqrt{T_1 T_2}}{2\sqrt{T_1 T_2}} = \frac{T_1 + T_2}{2\sqrt{T_1 T_2}}$$

$$T_1 = A_1 R_1, T_2 = A_2 R_2$$

$$K_1 = R_1, K_2 = R_2$$

* how can I increase zeta



h_1 و h_2 هيا driving force
جهد h_1 و h_2

$$f_1 = \frac{h_1 - h_2}{R_1}$$

$$h_1 - h_2 = 0$$

tank 2 هيا driving force
جهد h_1 و h_2

~~mass balance of tank 1~~
mass balance of tank 1

$$A_1 \frac{dh_1}{dt} + h_1 = f_i - f_1$$

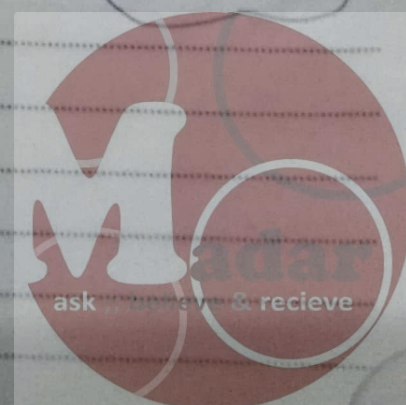
2 tank هيا interacting system

interacting system

$$* A_1 \frac{dh_1}{dt} + h_1 = f_i - \frac{h_1 - h_2}{R_1}$$

mass balance of tank 2

$$A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$



► Subject : _____

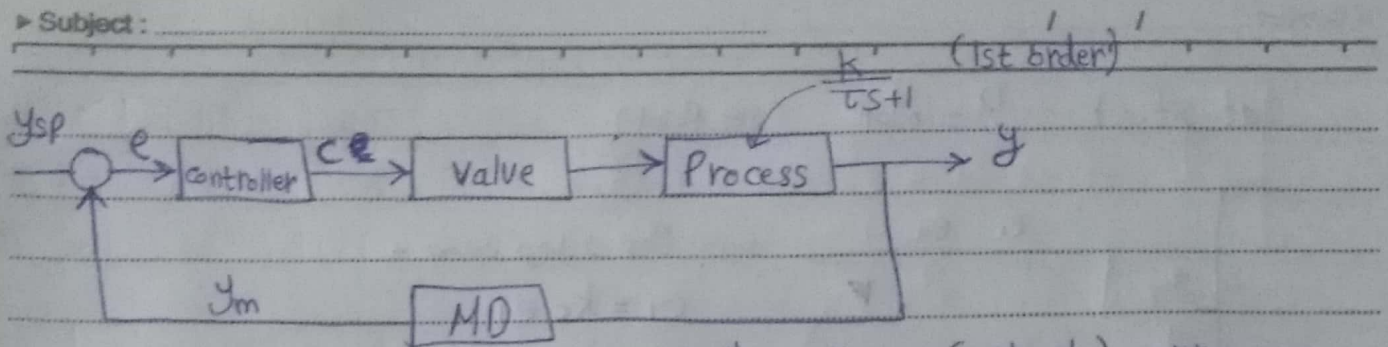
$$\frac{h_2}{f_i} = \frac{K_1 K_2}{T_1 T_2 S^2 + (T_1 + T_2) S + 1} \quad \text{for system 1}$$

$$\frac{h_2}{f_i} = \frac{?}{T_1 T_2 S^2 + (T_1 + T_2 + A_1 R_2) S + 1} \quad \text{for system 2}$$

System 1 و System 2 interaction coeff

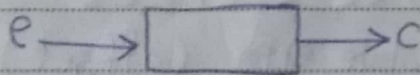


Subject:



* $y_{sp} = y_m$ (st. st) error = 0

* if Process at st. st controller doesn't work. $e(t) = 0$



$$c(t) = K_c e(t)$$

Proportional controller

ما يوصف الحلقة بالبداية * رابط علاقة

$$c(t) = K_c e(t) + \bar{c}$$

simulate the actual Process

change $\bar{c}$ ليغير Process

diviation from st. st ($\bar{c} = 0$)

$$c(t) = K_c e(t)$$

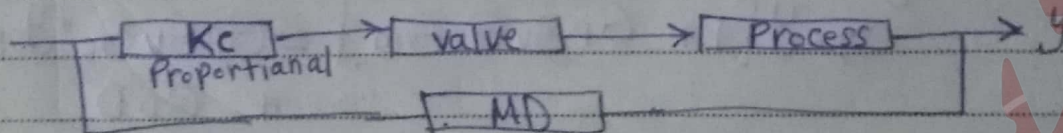
Laplace transform

$$\mathcal{L}[c(t) = K_c e(t)]$$

$$\bar{c}(s) = K_c e(s)$$

$$G_c(s) = \frac{c(s)}{e(s)} = K_c$$

Proportional controller



- dynamic Part $\frac{k}{s+1}$

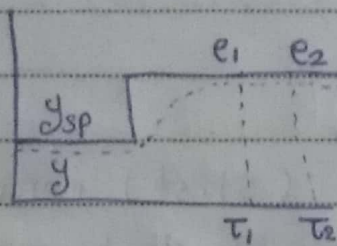
* response simultanouse (very fast controller)

* Simple and cheape

ask & believe & recieve

Subject :

at st. st $y = y_{sp}$ (set point)



بجاءت بوقت y_{sp} الجديدة -

$$c_1 = K_c e_1$$

$$c_2 = K_c e_2$$

$$\text{if } e_1 = e_2 \rightarrow c_1 = c_2$$

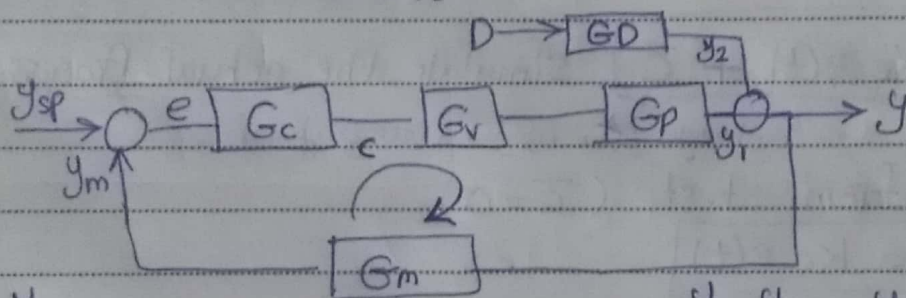
Steady state error (offset)

depend on resolution

الوقت الذي
يكتشف
خطأ
error
(offset)

Not detected

$$0.01 \rightarrow 0.015$$



$$\frac{y}{y_{sp}} = \frac{y}{D}$$

$$y = y_1 + y_2$$

$$= G_p * m + G_D * D$$

$$= G_p G_v G_c + G_D D$$

$$= G_p G_v G_c * e + G_D D$$

$$= G_p G_v G_c [y_{sp} - y_m] + G_D D$$

$$= G_p G_v G_c y_{sp} -$$

$$G_p G_v G_c G_m * y + G_D D$$

* Characteristic of the system

* Not dependent on change

$$\Rightarrow y [1 + G_p G_v G_c G_m] = G_p G_v G_c * y_{sp} + G_D D$$

closed
loop
T.F

$$y = \frac{G_c G_v G_p}{1 + G_c G_v G_p G_m} * y_{sp} + \frac{G_D}{1 + G_c G_v G_p G_m} D$$

ask & recieve

Subject: _____

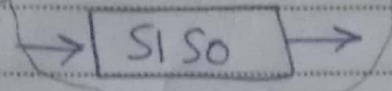
$$1 + G_c G_v G_p G_m = 0$$

characteristic equation

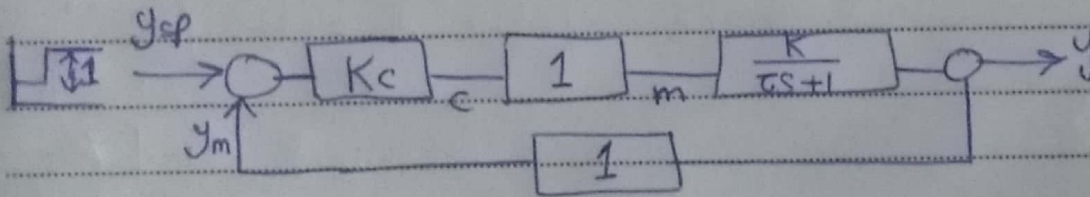
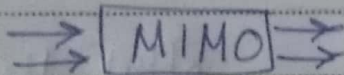
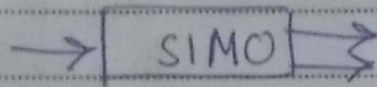
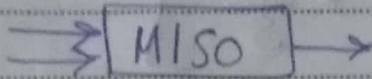
معادله مشخصه سیستم

معادله مشخصه سیستم در حلقه بسته

یک ورودی
یک خروجی



(Single input single output)



$$y = \frac{K_c \cdot 1 \left(\frac{K}{Ts+1} \right)}{1 + K_c \cdot 1 \cdot \frac{K}{Ts+1}} \cdot \frac{1}{s}$$

$$y = \frac{K_c K}{Ts+1} \cdot \frac{1}{s} \cdot \frac{Ts+1}{Ts+1 + K_c K}$$

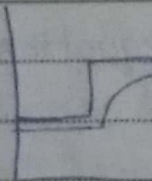
$$y = \frac{K_c K}{Ts+1}$$

$$y = \frac{K_c K (1 + K_c K)}{Ts+1 + K_c K}$$

ask & recieve

S T A R S N O T E B O O K

Subject :


$$\text{offset} = \text{new set Point} = y(t)_{\infty}$$

$$y(t)_{\infty} = \lim_{s \rightarrow 0} y(s)$$

$$y(t)_{\infty} = \lim_{s \rightarrow 0} s \left[\frac{K_c K}{1 + K_c K} \cdot \frac{1}{s} \right]$$

$$\text{offset} = \text{new } y_{sp}$$

$$y_{sp} \propto \frac{1}{K_c}$$

$$\text{offset} = \frac{1}{1 + K_c K}$$

* زيادة K_c ينقل offset
لكوننا بقدر، أنزيد K_c كثير
لأنه يأتي لا un.st.st

