University of Jordan
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905509 Statistical Quality Control

Control Charts for Variables

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Outline

- What is a variable?
- Control charts for mean and range.
- Estimating process capability.
- Changing sample size.
- Moving range and unity sample size.

Variables

- A variable is a single quality characteristic that can be measured on a numerical scale such as density, viscosity, surface tension, temperature etc.
- When working with variables, we should monitor both
  - the mean value of the characteristic, and
  - the variability associated with the characteristic.
- Purpose of the control chart is to give information about the performance or capability of the process.

Control Chart For Mean and Range

- Notation
  - n - size of the sample (sometimes called a subgroup) chosen at a point in time
  - m - number of samples selected
  - $\bar{X}_i$ = average of the observations in the i^{th} sample (where $i = 1, 2, \ldots, m$)
- $\bar{\bar{X}}$ = grand average or "average of the averages" (this value is used as the center line of the control chart)

Notation

- R_i = range of the values in the i^{th} sample

$$R_i = x_{max} - x_{min}$$

- $\bar{R}$ = average range for all m samples
- μ is the true process mean
- σ is the true process standard deviation

Control limits for $\bar{X}$ Charts

$$UCL = \bar{\bar{X}} + SL\frac{\hat{\sigma}}{\sqrt{n}}$$

$$CL = \bar{\bar{X}}$$

$$LCL = \bar{\bar{X}} - SL\frac{\hat{\sigma}}{\sqrt{n}}$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Control limits for R Charts

$$UCL = \bar{R} + SL\hat{\sigma}_R$$

$$LCL = \bar{R} - SL\hat{\sigma}_R$$

$$\hat{\sigma}_R = \bar{R}\frac{d_3}{d_2}$$

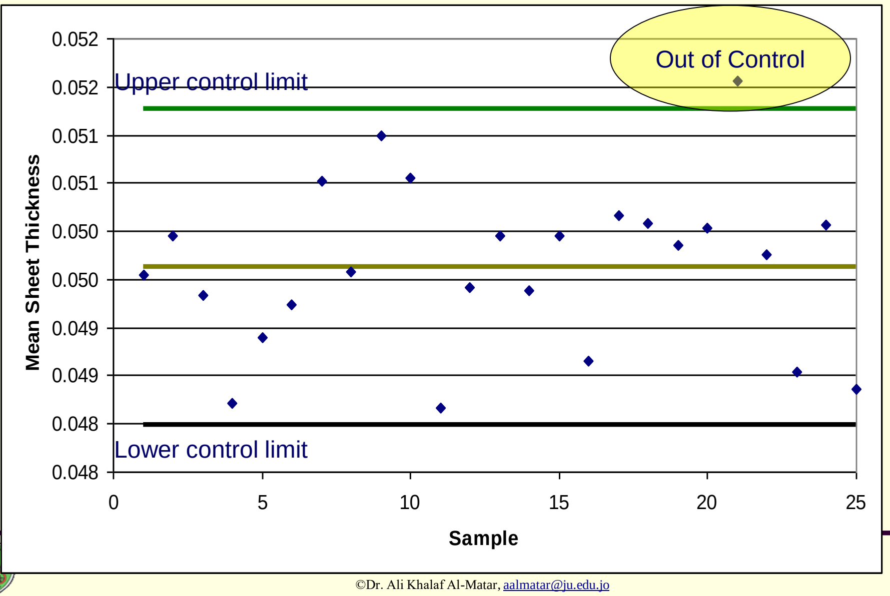
n	d_2	d_3
2	1.128	0.853
3	1.693	0.888
4	2.059	0.880
5	2.326	0.864
6	2.534	0.848
7	2.704	0.833
10	3.078	0.797
15	3.472	0.755
20	3.735	0.729
25	3.931	0.709

Example

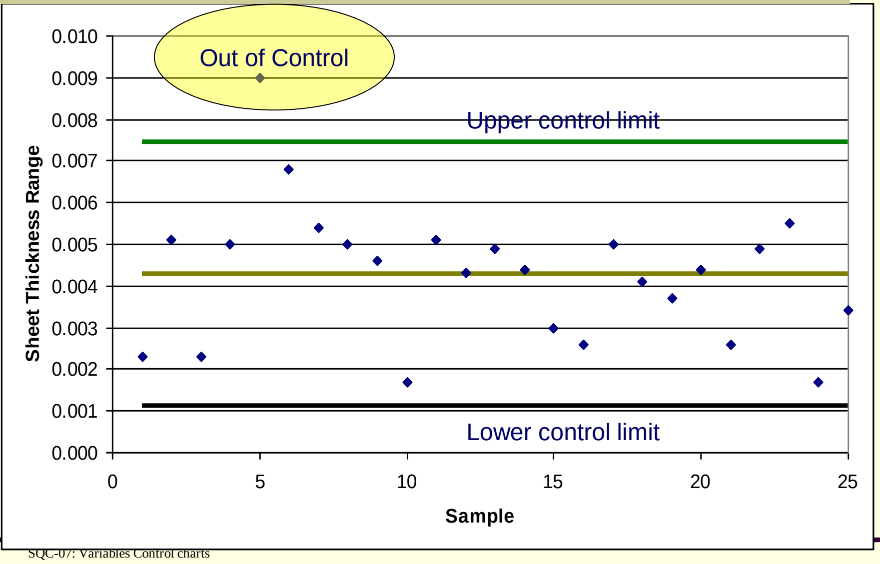
- A steel mill has implemented a procedure for controlling the quality of rolled sheet metal. A major product is sheets one-twentieth of an inch thick. Although individual thicknesses will vary within the same sheet and from sheet to sheet, no sheet should deviate much from the target dimension. The true population mean and standard deviation are unknown and may be estimated from sample data. The data given in the Excel worksheet were collected. Every hour, a sample of n=5 sheets is selected and measured for thickness. The mean, range and standard deviation are calculated for each sample. We want to construct control charts based on the mean and range, and check to see if any points were out of control.

Example metal sheet

X Chart



R-Chart



Process Standard Deviation

- The process standard deviation can be estimated as

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

- From which the control limits can be found as

$$UCL = \overline{\overline{X}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \overline{\overline{X}} + A_2\bar{R}$$

$$CL = \overline{\overline{X}}$$

$$LCL = \overline{\overline{X}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \overline{\overline{X}} - A_2\bar{R}$$

Estimating Process Capability

- Assuming normal distribution, the nonconforming fraction is given as

$$p = P(x < \text{LSL}) + P(x > \text{USL})$$

- Process Capability Ratio (PCR, C_p)

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

- The C_p statistic assumes that the process centered at the midpoint of the specification –it measures potential capability.

Interpretation of PCR

- PCR
  - PCR > 1 then a low number of nonconforming units will be produced.
  - PCR = 1 assuming normal distribution then 0.27% nonconforming units will be produced.
  - PCR < 1 then a large number of nonconforming units is being produced.

Specification Band

- The percentage of the specification band that the process uses up is defined as the reciprocal of PCR

$$P = \frac{1}{C_p} 100\%$$

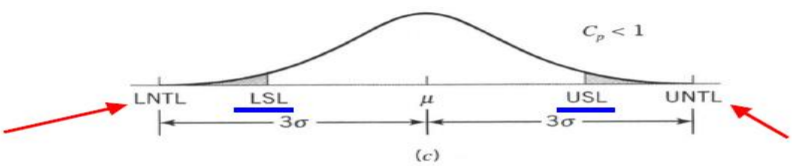


Figure 5-5 Process fallout and the process capability ratio C_p.

Limits

- Limits
  - Control limits are functions of the natural variability of the process.
  - Natural tolerance limits represent the natural variability of the process (usually set at 3σ from the mean)
  - Specification limits are determined by developers and/or designers.
- There is no mathematical relationship between control limits and specification limits.
- Typically do not plot specification limits on control charts because it causes confusion between control and capability.

Changing Sample Size

- Sometimes you may want to change the sample size for many reasons such as reduction of cost and process reevaluation.
- Compute new control limits from existing limits

$$D_3 = 1 - 3\frac{d_3}{d_2} \qquad D_4 = 1 + 3\frac{d_3}{d_2}$$

$\bar{x} - chart$

$$UCL = \bar{\bar{X}} + A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$CL = \bar{\bar{X}}$$

$$LCL = \bar{\bar{X}} - A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$R - Chart$

$$UCL = D_4\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$CL = \bar{R}_{\text{new}} = \left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$LCL = \max(0, D_3\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}})$$

Unity Sample Sizes

- What if you could not get sample sizes greater than unity ($n = 1$)?
  - Every unit manufactured must be analyzed.
  - Production rate is very slow such that it is inconvenient to allow sample size $n > 1$.
- X and MR (Moving Range) are suitable for sample sizes $n = 1$.

Moving Range (MR) Charts

- The Moving Range (MR) is defined to be the absolute difference between two successive observations

$$\text{MR}_i = \left| x_i - x_{i-1} \right|$$

- This definition helps identify shifts in the process from one observation to another.

Control Limits for MR

$$\overline{\text{MR}} = \frac{\sum_{i=1}^{m} \text{MR}_i}{m}$$

$\bar{x} - chart$

$$UCL = \bar{x} + 3\frac{\overline{\text{MR}}}{d_2}$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - 3\frac{\overline{\text{MR}}}{d_2}$$

$R - Chart$

$$UCL = D_4 \overline{\text{MR}}$$

$$CL = \overline{\text{MR}}$$

$$LCL = 0$$

Things to Remember

- X-Charts can be interpreted similar to $\bar{x}$-charts. MR charts cannot be interpreted the same as $\bar{x}$ or R charts.
- MR chart plots data that are "correlated" with one another. So, looking for patterns on the chart does not make sense.
- MR chart does not provide useful information about process variability. Should place more emphasis on interpretation of the X chart.