University of Jordan
Chemical Engineering Department
905509 Statistical Quality Control

Control Charts for Attributes

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Outline

- What is an attribute?
- Types of control charts for attributes.
- Binomial distribution.
- Control charts for fraction nonconforming.
- Positive lower control limits.
- Variable sample size.

Attribute Data

- Data that can be classified into one of several categories or classifications is known as attribute data.
  - Classifications such as conforming and nonconforming are commonly used in quality control.
  - Another example of attributes data is the count of defects.

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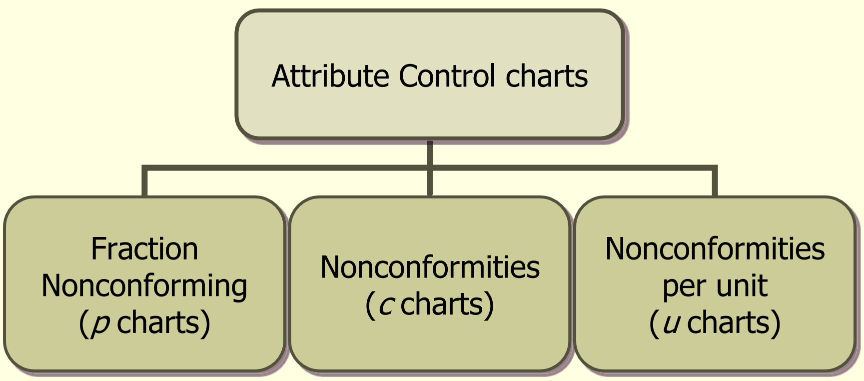
Attribute Control Charts



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Fraction Nonconforming

- Fraction nonconforming is the ratio of the number of nonconforming items in a population to the total number of items in that population.

$$p = \frac{\text{Number of nonconforming items}}{\text{Total number of items}}$$

- Control charts for fraction nonconforming are based on the binomial distribution.

Binomial Distribution

- A quality characteristic follows a binomial distribution if the following are met:
  1. All trials are independent.
  2. Each outcome is either a "success" or "failure".
  3. The probability of success on any trial is given as p, the probability of a failure is 1-p.
  4. The probability of a success is constant.

Binomial Distribution Mathematics

- The binomial distribution with parameters n>0 and 0<p<1, is given by

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

- Mean of the binomial distribution

$$\mu = np$$

- Standard deviation of the binomial distribution

$$\sigma^2 = np(1-p)$$

Notation for Fraction Nonconforming Charts

- n = number of units of product selected at random.
- D = number of nonconforming units from the sample
- p = probability of selecting a nonconforming unit from the sample.
- Then

$$P(D = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Sample Fraction Nonconforming

- The sample fraction nonconforming is given as

$$\hat{p} = \frac{D}{n}$$

- where $\hat{p}$ is a random variable with mean and variance

$$\mu = p \qquad \sigma^2 = \frac{p(1-p)}{n}$$

Control limits for Fraction Nonconforming

- Standard value of p given

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$
$$CL = p$$
$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$$

- Standard value of p not given

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$
$$CL = \bar{p}$$
$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

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Trial Control Limits

- Control limits that are based on a preliminary set of data can often be referred to as trial control limits.
- The quality characteristic is plotted against the trial limits, **if any points plot out of control**, **assignable causes** should be investigated and **points removed**.
- With removal of the points, the limits are then recalculated.

Example

- A process that produces bearing housings is investigated. Ten samples of size 100 are selected. The data are given below, is the process in statistical control?

Sample #	1	2	3	4	5	6	7	8	9	10
Number nonconforming	5	2	3	8	4	1	2	6	3	4

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Sample Size

- The sample size can be determined so that a shift of some specified amount, δ can be detected with a stated level of probability
- Duncan (1986) suggests being able to detect a process shift with a 50% chance.

$$\delta = L\sqrt{\frac{p(1-p)}{n}}$$

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

- Probability of a defect occurring is *p*=0.01. Probability of a defect occurring after a process shift is *p*=0.05. What would be an appropriate sample size to detect this shift?

$$\delta = 0.05 - 0.01 = 0.04$$

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

$$n = \left(\frac{3}{0.04}\right)^2 (0.01)(1-0.01) = 56$$

Positive LCL

- The sample size *n*, can be chosen so that the lower control limit would be nonzero:

$$LCL = p - L\sqrt{\frac{p(1-p)}{n}} > 0$$

$$n > \frac{(1-p)}{p}L^2$$

np Control Charts

- The number of nonconforming can be plotted instead of the fraction nonconforming.

$$UCL = np + 3\sqrt{np(1-p)}$$
$$CL = np$$
$$LCL = np - 3\sqrt{np(1-p)}$$

- If a standard *p* is not given use the sample estimate of *p*.

Points Below LCL

- Care must be exercised in interpreting points that plot below the lower control limit.
  - They often do not indicate a real improvement in process quality.
  - They are frequently caused by errors in the inspection process or improperly calibrated test and inspection equipment.

Variable Sample Size

- Suppose you want to do100% inspection of the process output over some period of time.
- Since different numbers of units could be produced in each period, the control chart would then have a variable sample size.
- Three approaches can be used
  - Variable width control limits.
  - Control limits based on average sample Size.
  - Standardized Control Chart.

Variable Width Control Limits

- Determine control limits for each individual sample that are based on the specific sample size.
- Upper and Lower control limits are given by

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n_i}}$$

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n_i}}$$

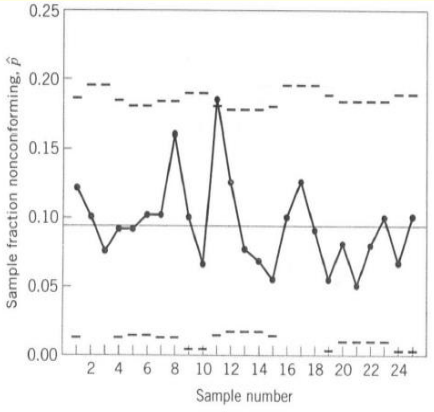

Figure 6-6 Control chart for fraction nonconforming with variable sample size.

Control Limits Based on Average Sample Size

- Control charts based on the average sample size results in an approximate set of control limits.
  - Average sample size is given by

$$\overline{n} = \frac{\sum_{i=1}^{m} n_i}{m}$$

  - Upper and lower control limits are given by

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}}$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}}$$

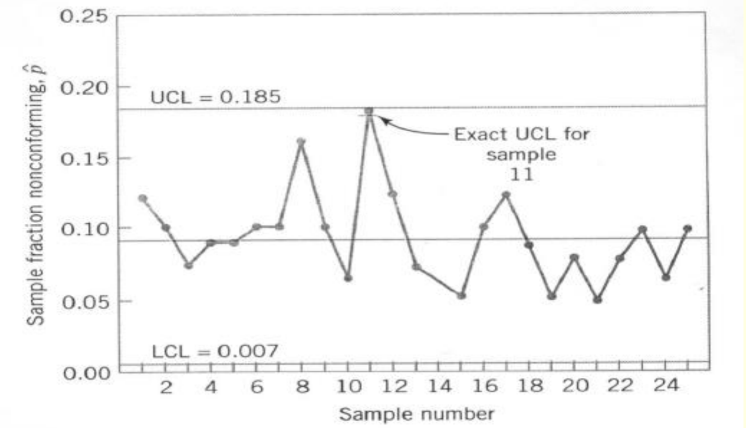

Figure 6-8 Control chart for fraction nonconforming based on average sample size.

Standardized Control Chart

- The points plotted are in terms of standard deviation units.
- The standardized control chart has the following properties:
  - Center line at 0 (zero).
  - UCL at 3.
  - LCL at -3.
- The points plotted are given by:

$$z_i = \frac{\hat{p}_i - p}{\sqrt{p(1-p)/n_i}}$$

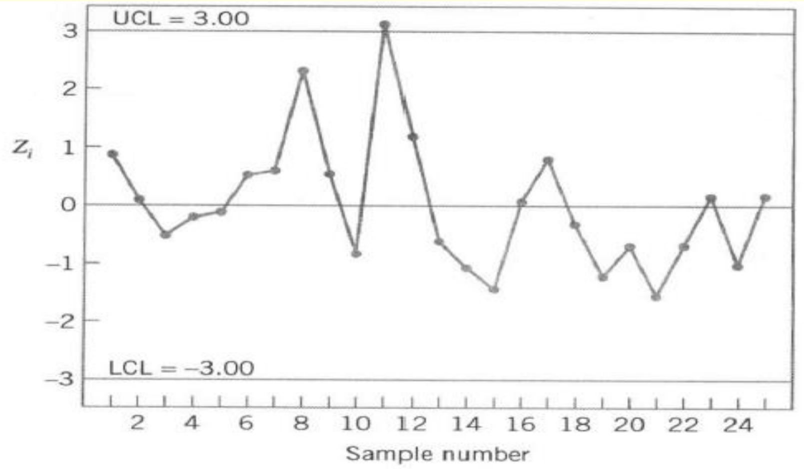

Figure 6-9 Standardized control chart for fraction nonconforming.

Control Charts for Nonconformities (Defects)

Control Charts for Nonconformities (Defects)

- There are many instances where an item will contain nonconformities but the item itself is not classified as nonconforming.
- It is often important to construct control charts for:
  - total number of nonconformities (*c*-chart) for an inspection unit, or
  - average number of nonconformities (*u*-chart) for a given "area of opportunity" per inspection unit.
- The inspection unit must be the same for each unit e.g., per unit time, per lot, per cubic meter etc.

Poisson Distribution

- The number of nonconformities in a given area can be modeled by the Poisson distribution.
  - Number of locations for nonconformities are large.
  - Probability of occurrence of a nonconformity at any location is small.
  - In other words, can have large probability of occurring anywhere, but small probability of occurring in the same place.
- Let c ($c>0$) be the parameter for a Poisson distribution (average number of nonconformities), then the mean and variance of the Poisson distribution are equal to the value c.
- The probability of obtaining **x** nonconformities on a single inspection unit, when the average number of nonconformities is some constant, c, is found using:

$$p(x) = \frac{e^{-c} c^{x}}{x!}$$

c-Chart for Constant Sample Size

- Standard value of *c* given

$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$

- Standard value of *c* not given

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$CL = \bar{c}$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Control chart for nonconformities with sample size = 1 inspection unit

Choice of Sample Size (*u*-Chart)

- If we find *c* total nonconformities in a sample of *n* inspection units, then the average number of nonconformities per inspection unit is

$$u = \frac{c}{n}$$

- The control limits for the average number of nonconformities is

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$
$$CL = \bar{u}$$
$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

Control chart for nonconformities with sample size = *n* inspection unit

Variable Sample Size

- Three approaches can be used
  - Variable width control limits.
  - Control limits based on average sample Size.
  - Standardized Control Chart.

Variable Width Control Limits

- Determine control limits for each individual sample that are based on the specific sample size.
- Upper and Lower control limits are given by

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Control Limits Based on Average Sample Size

- Control charts based on the average sample size results in an approximate set of control limits.
  - Average sample size is given by

$$\bar{n} = \frac{\sum_{i=1}^{m} n_i}{m}$$

  - Upper and lower control limits are given by

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

Standardized Control Chart

- The points plotted are in terms of standard deviation units.
- The standardized control chart has the following properties:
  - Center line at 0 (zero).
  - UCL at 3.
  - LCL at -3.
- The points plotted are given by:

$$z_i = \frac{u_i - \bar{u}}{\sqrt{\bar{u} / n_i}}$$

Demerit System

- Demerit System
  - A system for classifying several less severe or minor defects.
  - Classify defects according to severity.
  - Provides a reasonable framework for weighting various types of defects

Demerit System Classification of Defects

Class	**D** **Minor**	**C** **Moderately serious**	**B** **Serious**	**A** **Very serious**
Weight	**1**	**10**	**50**	**100**

Number of Demerits

- Let c_{iA}, c_{iB}, c_{iC}, and c_{iD} represent the number of units in each of the four classes.
- The number of demerits in an inspection unit (d_i) is given by

$$d_i = 100c_{iA} + 50c_{iB} + 10c_{iC} + c_{iD}$$

- Number of demerits per unit (n is number of inspection units)

$$u_i = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{D}{n}$$

Control Chart Development

$$UCL = \bar{u} + 3\hat{\sigma}_u$$
$$CL = \bar{u}$$
$$LCL = \bar{u} - 3\hat{\sigma}_u$$

$$\bar{u} = 100\bar{u}_A + 50\bar{u}_B + 10\bar{u}_C + \bar{u}_D$$

$$\hat{\sigma}_u = \left[\frac{(100)^2 \bar{u}_A + (50)^2 \bar{u}_B + (10)^2 \bar{u}_C + \bar{u}_D}{n} \right]^{1/2}$$

Low Defect Levels

- When defect levels or count rates in a process become very low, say under 1000 occurrences per million, then there are long periods of time between the occurrence of a nonconforming unit.
- Zero defects occur.
- Control charts (*u* and *c*) with statistic consistently plotting at zero are uninformative.
- Alternative is to chart the time between successive occurrences of the counts – or time between events control charts.
- If defects or counts occur according to a Poisson distribution, then the time between counts occur according to an exponential distribution.

Choice Between Attributes and Variables Control Charts

- Each has its own advantages and disadvantages
- Attributes
  - Data is easy to collect and several characteristics may be collected per unit.
  - Attributes control charts will not react unless the process has already changed (more nonconforming items may be produced.
- Variables
  - Data can be more informative since specific information about the process mean and variance is obtained directly.
  - Variables control charts provide an indication of impending trouble (corrective action may be taken *before* any defectives are produced).

Guidelines for Implementing Control Charts

1. Determine *which* process characteristics to control.
2. Determine *where* the charts should be implemented in the process.
3. Choose the proper *type* of control chart.
4. Take action to *improve* processes as the result of SPC/control chart analysis.
5. Select data-collection systems and computer software.