Transfer Phenomena 2

2st
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2\ Mar \2025

fundamental transfer processes

↳ momentum : fluid flow
DF :. prusure.
- mixing
- sedimental
- filtration

↳ Heat
DF = ΔT
- ↳ convective > Newton's law of Cooling
- ↳ conductive > Fourier's law

- evaporation
- Drying
- Distillation.

↳ Mass > Fick's law.
two or more component whoes Δ concentration
↳ D.F
- Distillation
- Drying
- Liquid - Liquid extraction

$$\ast \text{ General equation } = \frac{\text{Driving force}}{\text{resistance}}$$

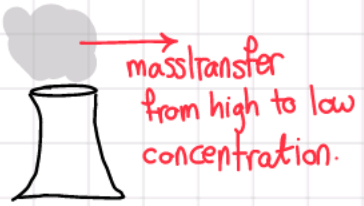
* mass transfer modes
- ↳ Diffuser
- ↳ Convection.
- ↳ Dispersion
  - ↳ مارح ندرسها ☺

حركة ال oil مع
النهر على ال Xaxis .

masstransfer Important
- ↳ need for pure materal
- ↳ to remove toxins or inactive component
- ↳ analysis of component's of the mixture DNA
- ↳ need for ultrapure materals

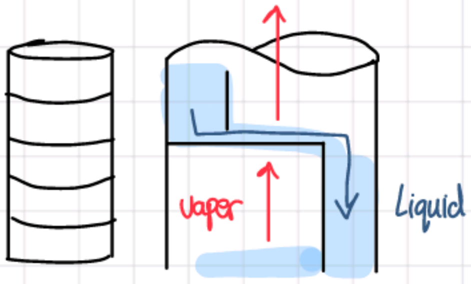
* Examples

① Dispersion of gases from stacks

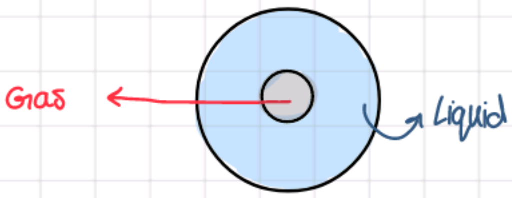


② Distillation columns

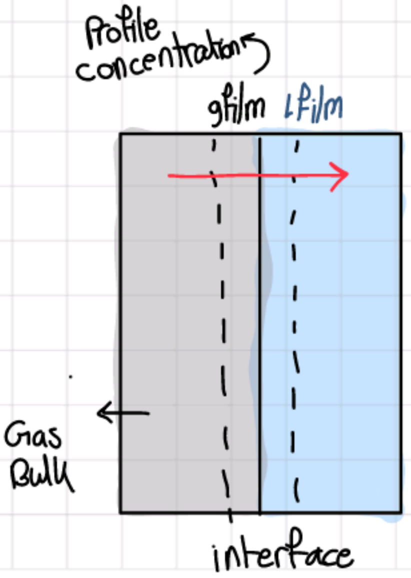
↳ Depenton V.p and B.p



③ stripping of gases from waste water.

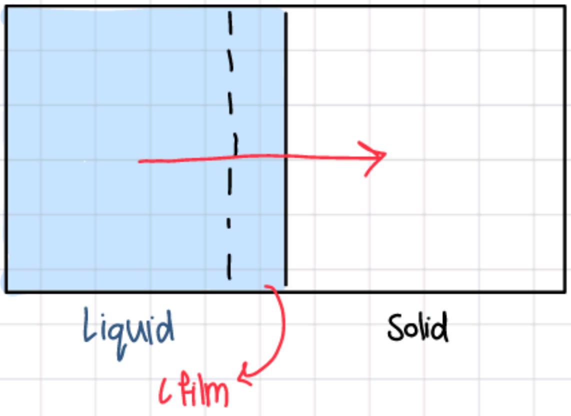


mass trans from Gas → Liquid

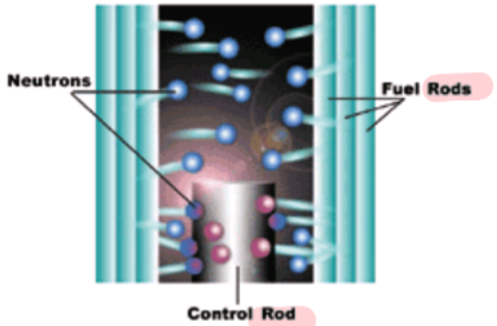


④ Adsorption of L,G to a solid

catalest.

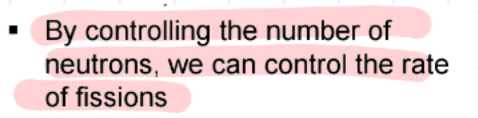


⑤ Neutron diffusion within nuclear



↳ absorbs the excess neatrons

* nuclear reaction
↳ Neutrons. بتحكم بعدد ال



- By controlling the number of neutrons, we can control the rate of fissions

⑤ Breathing

Air — Blood

$\downarrow CO_2$ ← $\uparrow CO_2$

$\uparrow O_2$ → $\downarrow O_2$

High → low

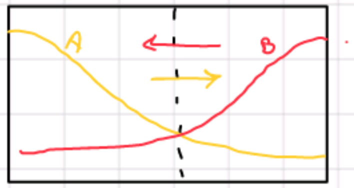
- **Diffusion**
  - random molecular motion that leads to complete mixing
  - satatioary solid, stagnant fluids or lamiar
  - 2 ≥ components

we carabout the net diffusion rate

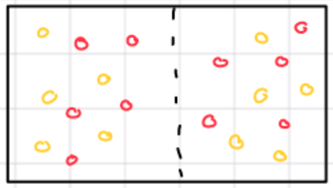
ممكن تكون in Turbulent near surface بس

ΔC = zero Single phase.
mass transfer stop

لما عنا "random motion" بس 0 = net



at time zero



net mass transfer = 0
at time ∞

- flux $\frac{amount}{Area \cdot time}$

amount → mass
amount → molar

$\frac{kg\,A}{m^2.s}$ or $\frac{mol\,A}{m^2}$

calculate J_{Az}
, can't calculate the flux in bulk flow

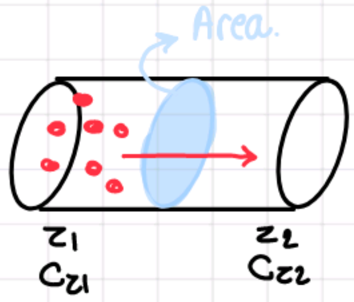
* Fick's low. empirical relation.

* isoThermal
* iso baric
* binarymixture of A and B.
* in stagnant or laminar

$$J^*_{Az} = \boxed{-}\, c_T D_{AB} \underbrace{\frac{dx_A}{dz}}$$

mass trans in Direction which the concentration drop.

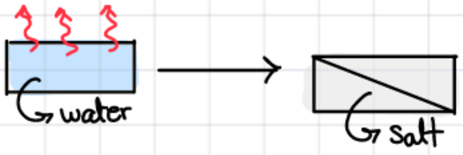
concentration greadent.

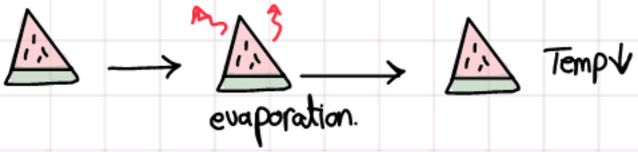


J_{Az} : molar flux of A kmol/s.m²
C_T : Total concentration A + B
D : molecular Diffusivity A in B m²/s
z : distance of diffusion
x_A : mole fraction $\frac{A}{A+B}$

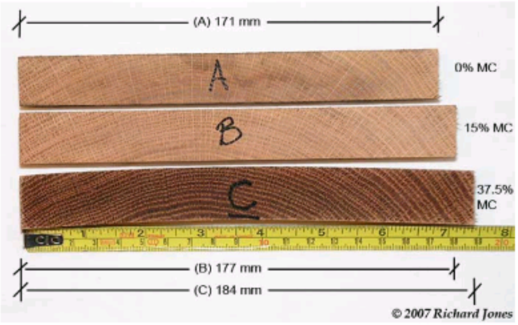
⑥ Diffusion

⑦ evaporation





- When newly cut and moist green timber is exposed to the atmosphere, the wood will dry partially when water in the timber diffuses through the wood, to the surface, and then to the atmosphere.



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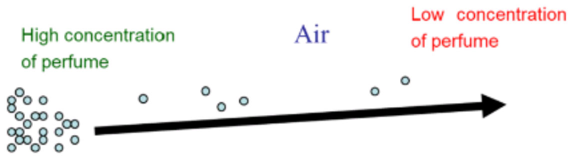
- Perfumes presents a pleasant fragrance which is imparted throughout the surrounding atmosphere.



* Diffusivity "characteristic"

↳ depent on T, P, concentration , in L or g or S $D_{in\,g} > D_{in\,L} > D_{in\,S}$

if concentration constant.

$$c\,dx_A = d\,c\,x_A = dc_A$$

$$J_{Az} = -D\,\frac{dc_A}{dz}$$

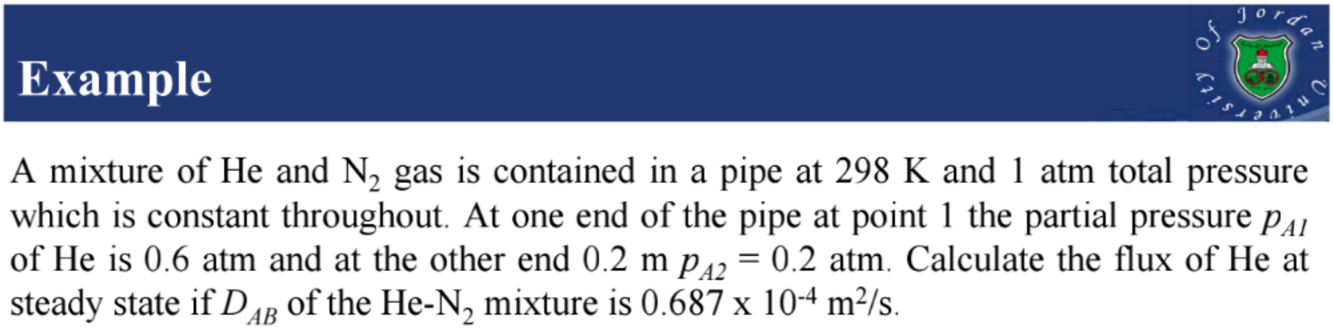
* Concentration for gas

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT}$$

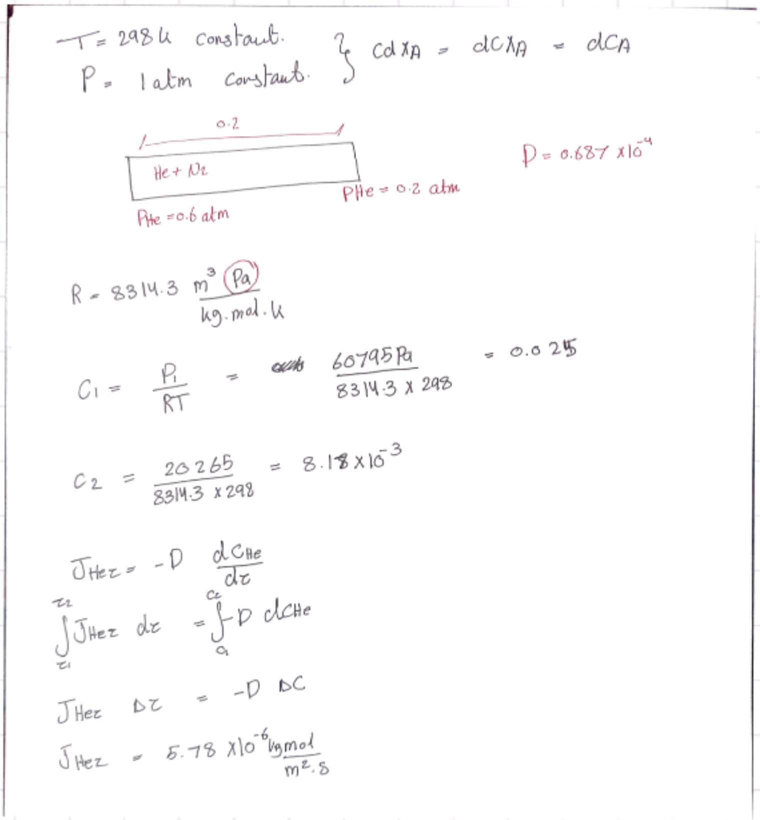
$$R: 8314.3\ \frac{m^3\,Pa}{kg\,mol.K}$$

$\cancel{Pa}$	kg mol $\cancel{K}$	
	m^3 $\cancel{Pa}$	$\cancel{K}$



Example

A mixture of He and N_2 gas is contained in a pipe at 298 K and 1 atm total pressure which is constant throughout. At one end of the pipe at point 1 the partial pressure p_{A1} of He is 0.6 atm and at the other end 0.2 m p_{A2} = 0.2 atm. Calculate the flux of He at steady state if D_{AB} of the He-N_2 mixture is 0.687 x 10^{-4} m^2/s.



T = 298 K constant.
P = 1 atm constant.
} $c\,dx_A = dc\,x_A = dc_A$

$D = 0.687 \times 10^{-4}$

$$R = 8314.3\ \frac{m^3\,(Pa)}{kg.mol.K}$$

$$C_1 = \frac{P_1}{RT} = \frac{60795\,Pa}{8314.3 \times 298} = 0.025$$

$$C_2 = \frac{20265}{8314.3 \times 298} = 8.18 \times 10^{-3}$$

$$J_{Hez} = -D\,\frac{dc_{He}}{dz}$$

$$\int_{z_1}^{z_2} J_{Hez}\,dz = \int_{c_1}^{c_2} -D\,dc_{He}$$

$$J_{Hez}\,\Delta z = -D\,\Delta C$$

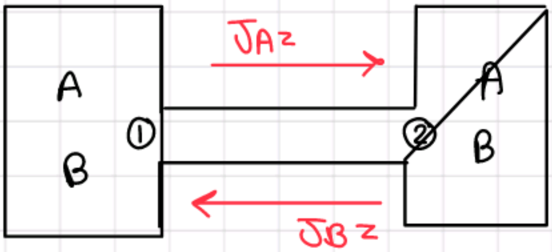
$$J_{Hez} = 5.78 \times 10^{-6}\ \frac{kg\,mol}{m^2.s}$$

* equimolar counterdiffusion of gases.

$P_T \Rightarrow$ constant

$\therefore P_{A1} + P_{A2} + P_{B1} + P_{B2} = P_T$

$P_{A1} > P_{A2} \;...\; P_{B2} > P_{B1}$



كل A بنتقل لازم يجي بداله B عشان يضل P_T ثابت

$$J_{Az} = -J_{Bz} \Rightarrow D_{-B} = D_{BA}.$$

نفس المقدار عكس الاتجاه .

$$J_{Az} = -D\frac{dc}{dz} \qquad \text{for gases} \qquad c = \frac{P}{RT}$$

$$P_A + P_B = \text{constant} \longrightarrow C_A + C_B = \text{constant}.$$

$$C = C_A + C_B$$

$$d\cancel{c} = dC_A + dC_B$$

مشتقة الصفر zero

$$dC_A = -dC_B$$

زيادة الـ A concentration نقصان بال B concentration

$$J_{Az} = -D_{AB}\frac{C_{A1} - C_{A2}}{\Delta z}$$

$$J_{Bz} = -D_{AB}\frac{C_{B1} - C_{B2}}{\Delta z}$$

$$J_{Az} = -J_{Bz}$$

$$D_{AB}\frac{(C_{A2} - C_{A1})}{\Delta z} = -D_{AB}\frac{C_{B2} - C_{B1}}{\Delta z}$$

$\Delta A = -\Delta B$ A بزيد B ع يلي بقل ن

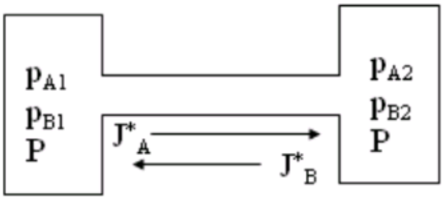
$$D_{AB}\frac{\cancel{(C_{A2} - C_{A1})}}{\cancel{\Delta z}} = -D_{AB}\frac{\cancel{(C_{A1} - C_{A2})}}{\cancel{\Delta z}} \quad (-1)$$

$$D_{AB} = D_{BA} \;\# \longrightarrow \boxed{\text{Gases}}$$

Example

Ammonia gas *(A)* is diffusing through a uniform tube 0.10 m long containing N_2 gas *(B)* at 1.0132 x 10^5 Pa press and 298 K. At point 1, $P_{A1} = 1.013\text{x}10^4$ Pa and at point 2, $P_{A2} = 0.507\ \text{x}10^4$ Pa. The diffusivity $D_{AB} = 0.230\text{x}10^{-4}\ \text{m}^2/\text{s}$.

1. Calculate the J^*_{AZ} of A at steady state.
2. Repeated for J^*_{BZ}



1) $J^*_{Az} = -D_{AB}\dfrac{dC_A}{dz}$

ideal gas. $c = \dfrac{P}{RT}$.

$J_{Az} = \dfrac{-(0.23\times10^{-3})}{8314.3\times298}\left(\dfrac{0.507\times10^{4} - 1.013\times10^{4}}{0.1}\right)$

$\boxed{J^*_{Az} = 4.69\times10^{-7}}$

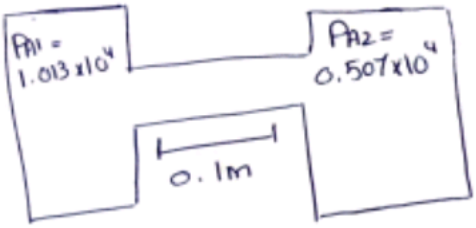
2) $J^*_{Bz} = -D_{BA}\dfrac{dC_B}{dz}$.

$J^*_{Az} = -J^*_{Bz}$

$J^*_{Bz} = -4.69\times10^{-6}$.

$J^*_{Bz} = -\dfrac{(0.23\times10^{-3})}{8314.3\times298}\left(\dfrac{96250 - 91190}{0.1}\right)$

$\boxed{J^*_{Bz} = -4.69\times10^{-7}}$ #



constant Pressure = 1.0132×10^5 Pa and Temp = 298K.

$D_{AB} = 0.23\times10^{-3}\ \text{m}^2/\text{s}$.

$P_{B1} = 1.0132\times10^5 - 1.013\times10^4$

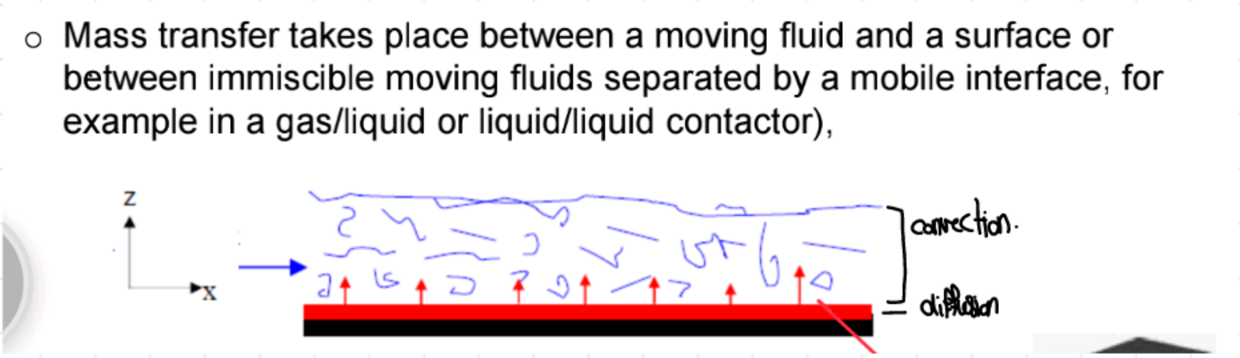
$P_{B1} = 91190$ Pa

$P_{B2} = 1.0132\times10^5 - 0.507\times10^4$.

$P_{B2} = 96250$ Pa

*Convection Bulk flow [Advection + diffusion = convection]

Advection by moving fluid
can't happen not include the simple diffusion
in solid



- Mass transfer takes place between a moving fluid and a surface or between immiscible moving fluids separated by a mobile interface, for example in a gas/liquid or liquid/liquid contactor),

*Convection mass transfer

$N_{Az} = k_c \Delta C_A$ Newton's law of cooling بيشبه قانون

N_A: molar mass transfer of A

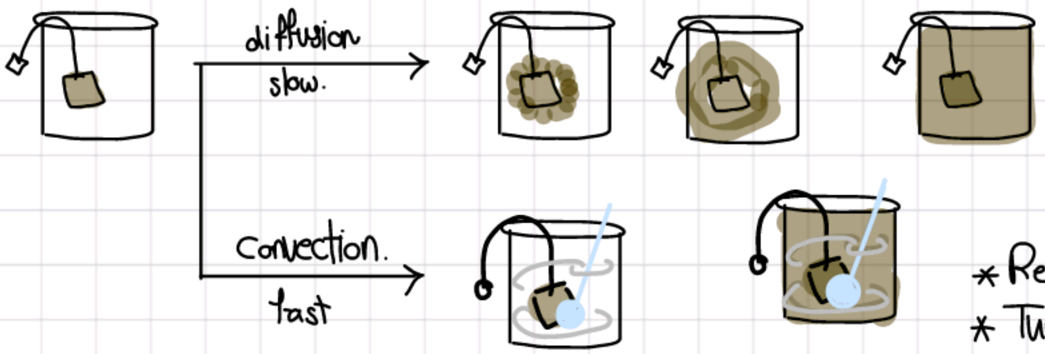
ΔC_A: concentration Δ between the boundary surface — A avg concentration of the fluid

k_c: convective mass tran coe

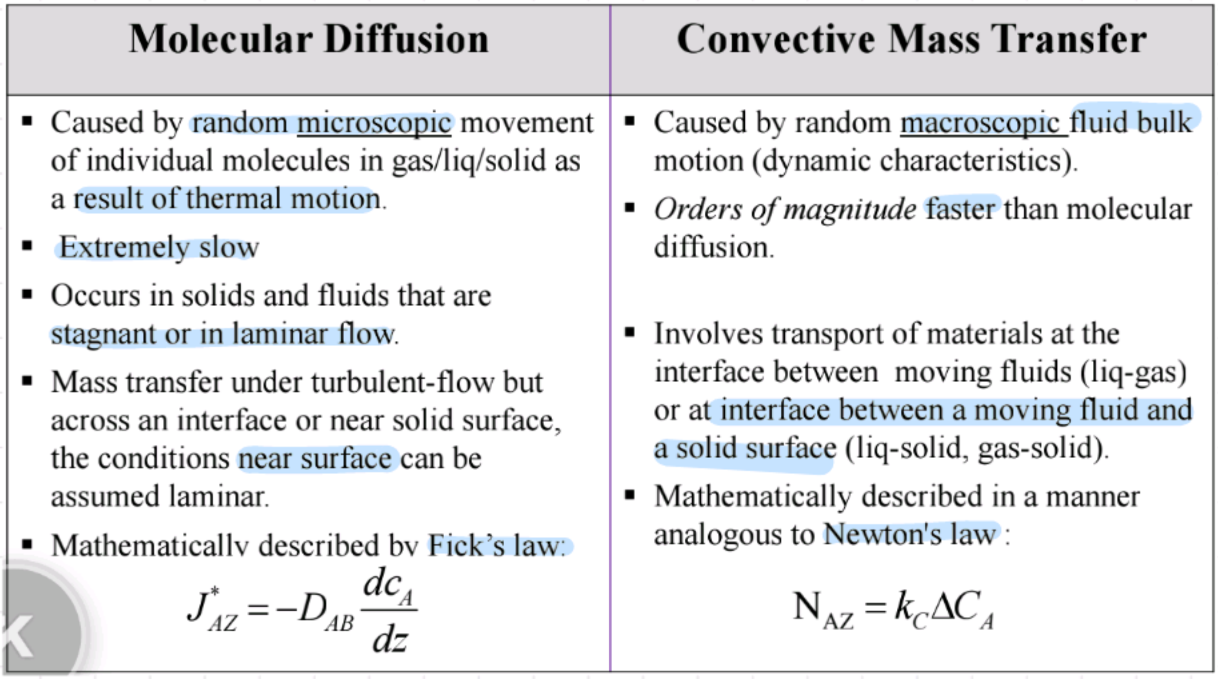
- high concentration → law concentration at our level True

- depent on → transport properti s
  → dynamic charact ristics.

Convection عـــا diffusion.



* Re ↑
* Turbulent
* ↑ masstransfer rate

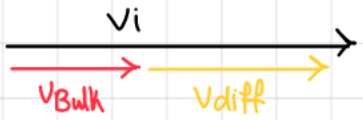


Molecular Diffusion	Convective Mass Transfer
▪ Caused by random microscopic movement of individual molecules in gas/liq/solid as a result of thermal motion. ▪ Extremely slow ▪ Occurs in solids and fluids that are stagnant or in laminar flow. ▪ Mass transfer under turbulent-flow but across an interface or near solid surface, the conditions near surface can be assumed laminar. ▪ Mathematically described by Fick's law: $$J^*_{AZ} = -D_{AB}\frac{dc_A}{dz}$$	▪ Caused by random macroscopic fluid bulk motion (dynamic characteristics). ▪ *Orders of magnitude* faster than molecular diffusion. ▪ Involves transport of materials at the interface between moving fluids (liq-gas) or at interface between a moving fluid and a solid surface (liq-solid, gas-solid). ▪ Mathematically described in a manner analogous to Newton's law : $$\mathrm{N}_{\mathrm{AZ}} = k_C \Delta C_A$$

The General Flux equation

- In Binary system of A and B
  - if ΔC of A exists there must be a ΔC of B in the opposite direction → both component of the mixture diffuse in the opposite direction
- Rate A > Rate B → mixture it self drifts in the direction of A
- total mass flux = mass trans by diffusion + mass trans bulk motion of fluid (caduiction)

↳ kinatic theory of gases.



$$V_i = V_{bulk} + V_{diff}$$

$V_i \Rightarrow V$ condiction

- $$V_{A\,diffusion} = \frac{J^*_A \ \cancel{mol}/\cancel{m^2}.s}{C_A \ \cancel{mol}/m^3 ①} \quad (m/s)$$ Diff in Binary system of A and B

- $$V_{bulk} = \frac{\sum_{i=1}^{n} C_i V_i}{C_T}$$ multicomponent mixture

$$= \frac{C_A V_A + C_B V_B}{C_T}$$ Binary system of A and B

$$N_i = \frac{w_i \text{ (molar flow rate)}}{\text{Area}} \qquad \frac{mol}{s.\,m^2}$$

$$V_{Bulk} = \frac{N_A + N_B}{C_T}$$

$$V_A = V_{diff\,A} + V_{Bulk} = C_A V_A = V_{diff}\, C_A + V_{Bulk}\, C_A$$

$$V_A = V_{diffA} + V_{Bulk} = C_A V_A = V_{diff} C_A + V_{Bulk} C_A$$

$C_A V_A = N_A$ $\quad V_{Bulk} = \frac{N_A + N_B}{C_T}$ $\quad V_{diff} = \frac{J_A}{C_A}$

$$N_A = \frac{J_A}{\cancel{C_A}} \cancel{C_A} + \frac{N_A + N_B}{C_T} C_A$$

$N_A = J_A^* + \frac{(N_A + N_B) C_A}{C_T}$ >> General flux equation #

Note: mixture very dillute in A $\quad N_A = J_A^*$

↳ why? $\frac{C_A}{C_T} \approx 0$

$$N_A = J_A^* + \cancel{\frac{(N_A + N_B) C_A}{C_T}}_{\text{zero !!}}$$

Note: $X_A = \frac{N_A}{N_B + N_A} \rightarrow \frac{C_A V_{Bulk}}{V_{Bulk} C_A + V_{Bulk} C_B}$

$$\frac{C_A \cancel{V_{Bulk}}}{(C_A + C_B)\cancel{V_{Bulk}}} \Rightarrow \frac{C_A}{C_T} \#$$

* Diffusivity D_{AB} (P, T, composition)

$$J^*_A = -D_{AB}\frac{dc}{dz}$$

$$\frac{-J^*_A\,dz}{dc} = D_{AB}$$

$$\frac{\cancel{\text{kmol}}}{\cancel{m^2}\cdot s}\Bigg|\frac{m\quad \cancel{m^3}}{\cancel{\text{kmol}}} \text{①} = \frac{m^2}{s} \equiv \frac{L^2}{t} \text{ same as } \underbrace{\text{kinatic viscocity}}_{\text{momintum transfer}} = \frac{\mu}{\rho} \text{ and } \underbrace{\text{Thermal diff}}_{\text{Heat transfer.}}\ \alpha$$

- $D_g < D_L < D_s$
  - $\hookrightarrow D_g < D_{Polymers} < D_L < D_s$.

- D_g $T\uparrow\uparrow$, $P\uparrow\downarrow$, composition independent (low density)
- $D_{L,s}$ $T\uparrow\downarrow$, P indepent, composition dependent

Diffusivity in Gases:

- $D_{AB} \propto \frac{1}{P}$. bellow 25atm
- $D_{AB} \propto T^{3/2}$

$$* \quad D_{AB}(T_2,P_2) = D_{AB}(T_1,P_1)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)^{3/2}$$

عند D بعرفها عند T و P و بدي D_2 عند T_2 , P_2 .
$P\downarrow$ 25 atm.

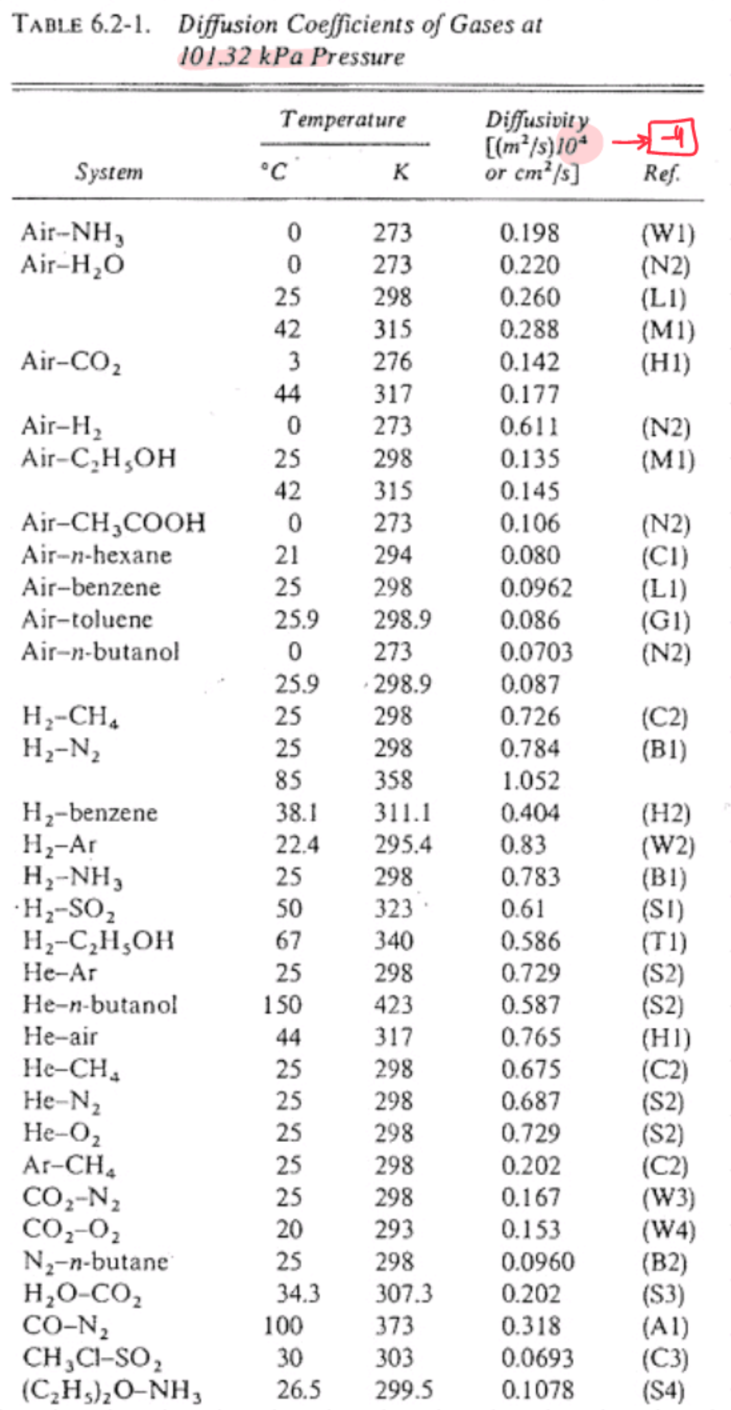


TABLE 6.2-1. *Diffusion Coefficients of Gases at 101.32 kPa Pressure*

System	Temperature °C	Temperature K	Diffusivity $[(m^2/s)10^4$ or $cm^2/s]$ $\rightarrow \boxed{-4}$	Ref.
Air–NH_3	0	273	0.198	(W1)
Air–H_2O	0	273	0.220	(N2)
	25	298	0.260	(L1)
	42	315	0.288	(M1)
Air–CO_2	3	276	0.142	(H1)
	44	317	0.177	
Air–H_2	0	273	0.611	(N2)
Air–C_2H_5OH	25	298	0.135	(M1)
	42	315	0.145	
Air–CH_3COOH	0	273	0.106	(N2)
Air–*n*-hexane	21	294	0.080	(C1)
Air–benzene	25	298	0.0962	(L1)
Air–toluene	25.9	298.9	0.086	(G1)
Air–*n*-butanol	0	273	0.0703	(N2)
	25.9	298.9	0.087	
H_2–CH_4	25	298	0.726	(C2)
H_2–N_2	25	298	0.784	(B1)
	85	358	1.052	
H_2–benzene	38.1	311.1	0.404	(H2)
H_2–Ar	22.4	295.4	0.83	(W2)
H_2–NH_3	25	298	0.783	(B1)
H_2–SO_2	50	323	0.61	(S1)
H_2–C_2H_5OH	67	340	0.586	(T1)
He–Ar	25	298	0.729	(S2)
He–*n*-butanol	150	423	0.587	(S2)
He–air	44	317	0.765	(H1)
He–CH_4	25	298	0.675	(C2)
He–N_2	25	298	0.687	(S2)
He–O_2	25	298	0.729	(S2)
Ar–CH_4	25	298	0.202	(C2)
CO_2–N_2	25	298	0.167	(W3)
CO_2–O_2	20	293	0.153	(W4)
N_2–*n*-butane	25	298	0.0960	(B2)
H_2O–CO_2	34.3	307.3	0.202	(S3)
CO–N_2	100	373	0.318	(A1)
CH_3Cl–SO_2	30	303	0.0693	(C3)
$(C_2H_5)_2O$–NH_3	26.5	299.5	0.1078	(S4)

P (395), Transport Processes and Unit Operation

D for CO_2-O_2 at $P = 1 atm$ $T = 293 K$

$= 0.153 \times 10^{-4} \ m^2/s$

D for CO_2-O_2 at $P = 1 atm$ $T = 10 C°$

عند D عال P,T وبدي D_2 عند P_2,T_2.

$$D_{AB}(T_2,P_2) = D_{AB}(T_1,P_1)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)^{3/2}$$

$$D_{CO_2-O_2}(287, 101.32) = 0.153\times10^{-4}(1)\left(\frac{283}{293}\right)^{3/2}$$

$D = 1.45 \times 10^{-5} \ m^2/s$ T↓ D↓

*
$$D_{AB} = \frac{10^{-3} T^{1.75} \left(\frac{1}{M_A} + \frac{1}{M_B} \right)^{\frac{1}{2}}}{P\left[\left(\sum V_A\right)^{\frac{1}{3}} + \left(\sum V_B\right)^{\frac{1}{3}} \right]^2}$$

$D_{AB} \propto T^{1.75}$ $D_{AB} \propto \frac{1}{P}$

used for nonpolar gases or polar – nonpolar
accuracy ↓

* M : molar mass ↑↓
* P : ⊛abs Pressure atm ↑↓
* V_A : sum of the diff volume for component A ↑↓
* T : Temp K ↑↑
* D_{AB} : cm^2/s.

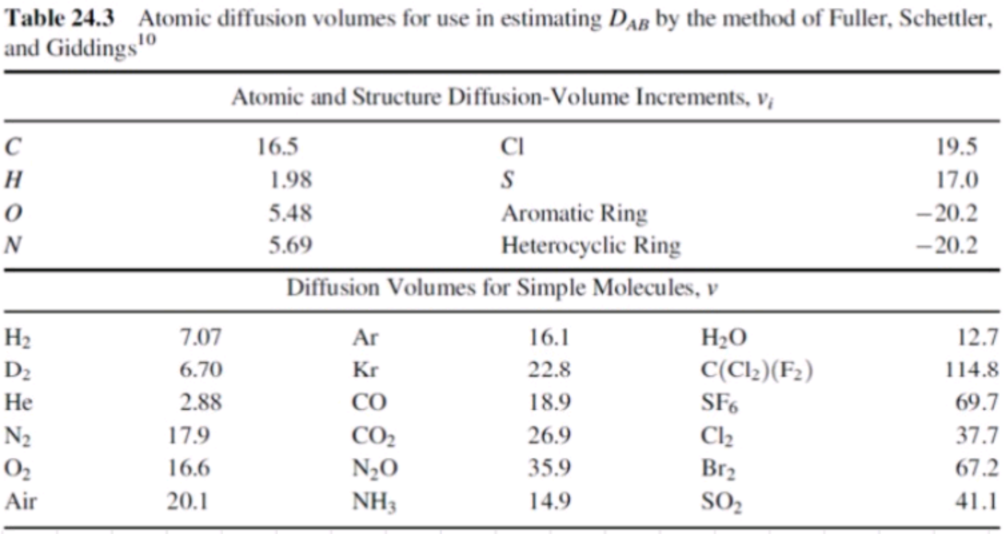


Table 24.3 Atomic diffusion volumes for use in estimating D_{AB} by the method of Fuller, Schettler, and Giddings[10]

Atomic and Structure Diffusion-Volume Increments, v_i

C	16.5	Cl	19.5
H	1.98	*S*	17.0
O	5.48	Aromatic Ring	−20.2
N	5.69	Heterocyclic Ring	−20.2

Diffusion Volumes for Simple Molecules, v

H_2	7.07	Ar	16.1	H_2O	12.7
D_2	6.70	Kr	22.8	$C(Cl_2)(F_2)$	114.8
He	2.88	CO	18.9	SF_6	69.7
N_2	17.9	CO_2	26.9	Cl_2	37.7
O_2	16.6	N_2O	35.9	Br_2	67.2
Air	20.1	NH_3	14.9	SO_2	41.1

Ex: Diffusion volume for CH_4

$V_C = 16.5$

$V_H = 1.98$

$V_{CH_4} = 16.5 + 4(1.98) = 24.42$

Normal butanol (*A*) is diffusing through air (*B*) at 1 atm abs. Using the Fuller et al. method, estimate the diffusivity D_{AB} for the following temperatures with the experimental data:

(a) For 0 °C.

(b) For 25.9 °C.

(c) For 0 °C and 2.0 atm abs.

a) T = 0C° P = 1 atm abs

T = 273.15 K.

$V_A = 4(16.5) + 10(1.98) + 5.48 = \boxed{91.28}$

$V_B = \boxed{20.1}$

$M_A = 74.1$

$M_B = 29$

$$D_{AB} = \frac{10^{-3} T^{1.75} \left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{\frac{1}{2}}}{P\left[(\Sigma V_A)^{\frac{1}{3}} + (\Sigma V_B)^{\frac{1}{3}}\right]^2}$$

$$D_{AB} = \frac{10^{-3}[273.15]^{1.75}\left[\frac{1}{74.1} + \frac{1}{29}\right]^{\frac{1}{2}}}{1\left[91.28^{1/3} + 20.1^{1/3}\right]^2} = 0.0771\ cm^2/s$$

$7.71 \times 10^{-2}\ cm^2/s$

$7.71 \times 10^{-6}\ m^2/s.$

B) T = 25.9 C

T = 25.9 + 273.15 = 299.05 K.

$$D_{AB} = \frac{10^{-3}[299.05]^{1.75}\left[\frac{1}{74.1} + \frac{1}{29}\right]^{1/2}}{1\left[91.28^{1/3} + 20.1^{1/3}\right]^2}$$

$= 0.0903\ cm^2/s$

$9.03 \times 10^{-2}\ cm^2/s$

$9.03 \times 10^{-6}\ m^2/s.$

c) T = 273.15 K P = 2 atm.

$$D_{AB} = \frac{10^{-3}[273.15]^{1.75}\left[\frac{1}{74.1} + \frac{1}{29}\right]^{1/2}}{2\left[91.28^{1/3} + 20.1^{1/3}\right]^2} = 0.0385\ cm^2/s$$

~~0.0~~

$3.85 \times 10^{-6}\ m^2/s.$

Or $D_{AB} = 7.71 \times 10^{-6}\left[\frac{1}{2}\right] = 3.85 \times 10^{-6}\ m^2/s$

Diffusivity in Liquid $\approx 10^{-6}\ 10^{-9}\ m^2/s$.

molecules in L closer than in g
↳ D_L slower D_g

$$D_{AB} = \frac{9.96\times10^{-12}\,(T)}{\mu\ V_A^{1/3}}$$

A: large spherical dilute solute of 1000 Mm or ↑
B very small.

D_{AB}: cm^2/s
T: K
μ: viscosity of soluation kg/ms ↑↓
V_A: solute molar volume at it's normal B.P $m^3/kmol$

$$* \left[\frac{D_{AB}\,\mu}{T}\right]_{T_1} = \left[\frac{D_{AB}\,\mu}{T}\right]_{T_2}$$

$$* \ D_{AB} = \frac{7.4\times10^{-8}(\phi\,M_B)^{1/2}\,T}{\mu_B\ V_A^{0.6}}$$

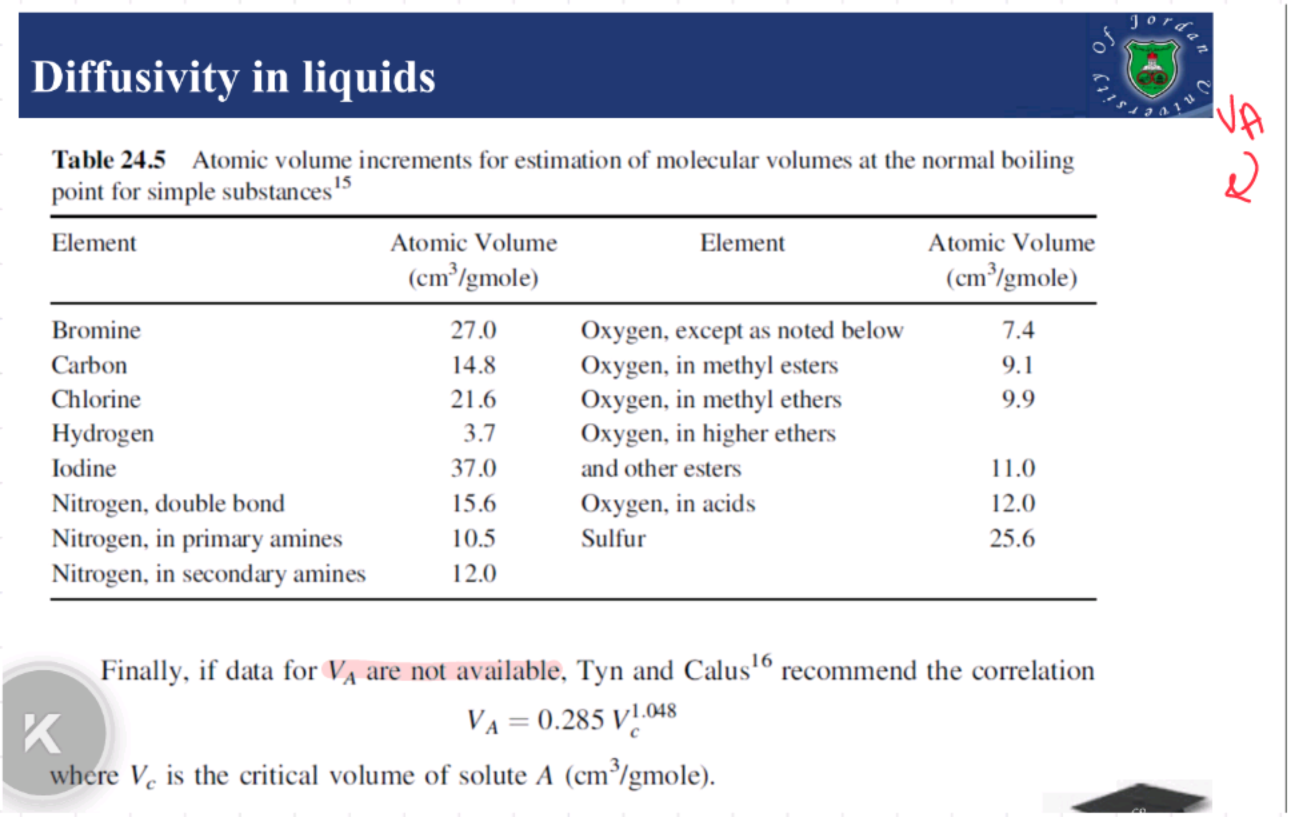
A small molecules
B dillute solvent

D_{AB}: cm^2/s
T: k
μ: cp

V_A: solute molar volume table 24.5
ϕ: association parameter of solvent

الها علاقة ↩
بال H-Bond

2.6 water
1.9 methanol
1.5 ethanol
1 non polar.



Diffusivity in liquids

VA

Table 24.5 Atomic volume increments for estimation of molecular volumes at the normal boiling point for simple substances[15]

Element	Atomic Volume (cm^3/gmole)	Element	Atomic Volume (cm^3/gmole)
Bromine	27.0	Oxygen, except as noted below	7.4
Carbon	14.8	Oxygen, in methyl esters	9.1
Chlorine	21.6	Oxygen, in methyl ethers	9.9
Hydrogen	3.7	Oxygen, in higher ethers	
Iodine	37.0	and other esters	11.0
Nitrogen, double bond	15.6	Oxygen, in acids	12.0
Nitrogen, in primary amines	10.5	Sulfur	25.6
Nitrogen, in secondary amines	12.0		

Finally, if data for V_A are not available, Tyn and Calus[16] recommend the correlation

$$V_A = 0.285\, V_c^{1.048}$$

where V_c is the critical volume of solute A (cm^3/gmole).

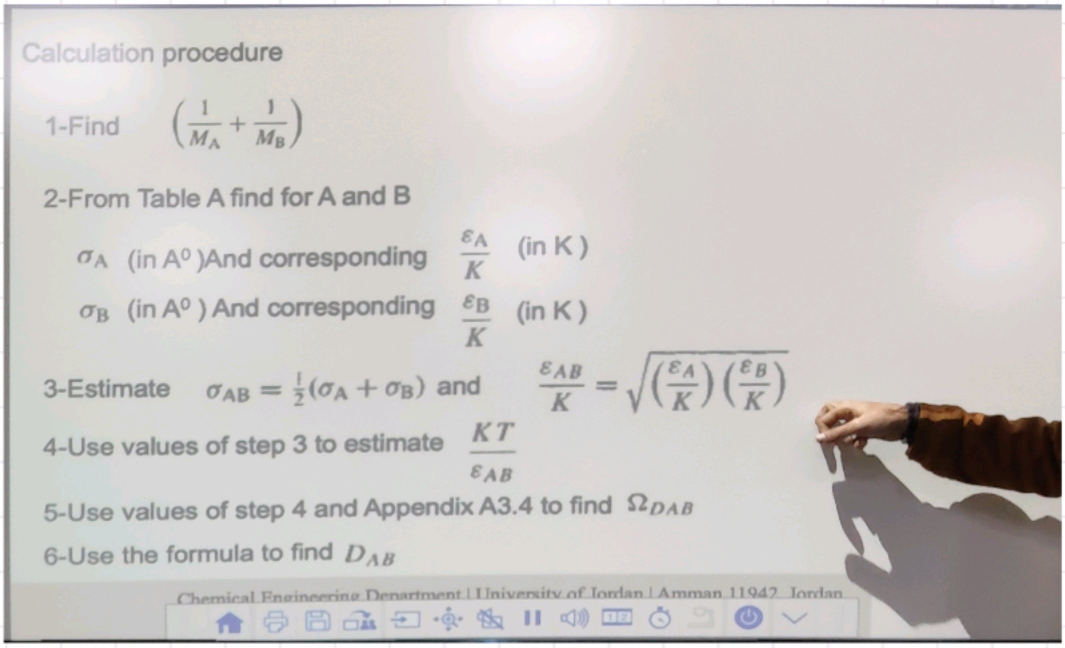
$$D_{AB} = \frac{1.8583 \times 10^{-7}\, T^{3/2}}{P\, \sigma_{AB}^{2}\, \Omega_{D.AB}} \left[\frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}$$

$$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B)$$

$$\varepsilon_{AB} = \sqrt{\varepsilon_A + \varepsilon_B}$$

$$\Omega_{DAB} = \frac{kT}{\varepsilon_{AB}}$$

D_{AB} = m^2/s
T = K
M_A = molar mass
P = abs prussure atm
σ_{AB} = avg collision Diameter
Ω_{DAB} = collision integral



Calculation procedure

1-Find $\left(\frac{1}{M_A} + \frac{1}{M_B}\right)$

2-From Table A find for A and B

σ_A (in A⁰)And corresponding $\frac{\varepsilon_A}{K}$ (in K)
σ_B (in A⁰) And corresponding $\frac{\varepsilon_B}{K}$ (in K)

3-Estimate $\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B)$ and $\frac{\varepsilon_{AB}}{K} = \sqrt{\left(\frac{\varepsilon_A}{K}\right)\left(\frac{\varepsilon_B}{K}\right)}$

4-Use values of step 3 to estimate $\frac{KT}{\varepsilon_{AB}}$

5-Use values of step 4 and Appendix A3.4 to find Ω_{DAB}

6-Use the formula to find D_{AB}

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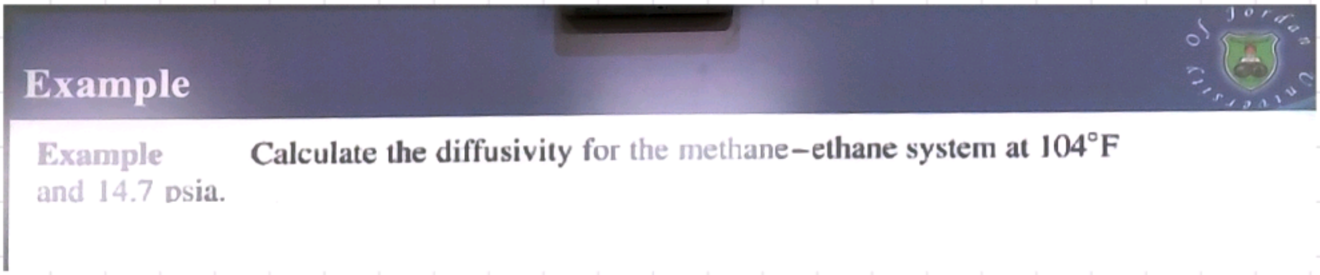


Example

Example Calculate the diffusivity for the methane–ethane system at 104°F and 14.7 psia.

$T = 313.15\,K \quad P = 1\,atm$

$\boxed{1}$ $$\frac{1}{M_A}+\frac{1}{M_B} = \frac{1}{16.04}+\frac{1}{30.07} = 0.0956$$

$\boxed{2}$ $\frac{\varepsilon_{methane}}{K} = 136.5\,K \quad \frac{\varepsilon_{ethane}}{K} = 230\,K$ from Appendix K table K.2

$\sigma_{methane} = 3.822\,A^\circ \quad \sigma_e = 4.418\,A^\circ$

$\sigma_{AB} = \frac{1}{2}(3.822+4.418) = 4.12\,A^\circ$

$$\frac{\varepsilon_{AB}}{k} = ((136.5)(230))^{0.5} = 177.19\,K.$$

$$\frac{KT}{\varepsilon_{AB}} = \frac{313.15}{177.19} = 1.767$$

$\Omega_{DAB} = 1.13$ from appendix K table K.1

$$D_{AB} = \frac{1.8583\times10^{-7}\,T^{3/2}}{P\sigma_{AB}^{2}\,\Omega_{D.AB}}\left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}$$

$$\frac{1.8583\times10^{-7}(313.15)^{3/2}\sqrt{0.0956}}{(1)(4.12)^2(1.13)}$$

$D_{AB} = 1.66\times10^{-5}\,m^2/s$

Diffusivity of Electrolytes in Liquids

- For smaller molecules (A) diffusing in a dilute liquid solution of solvent (B):

$$D^{o}_{AB} = \frac{8.928 \times 10^{-10}\, T\,(1/n_{+} + 1/n_{-})}{(1/\lambda_{+} + 1/\lambda_{-})} \qquad (14)$$

*ionic solution.

D^{o}_{AB} is diffusivity in cm²/s

n_{+} is the valence of cation

n_{-} is the valence of anion

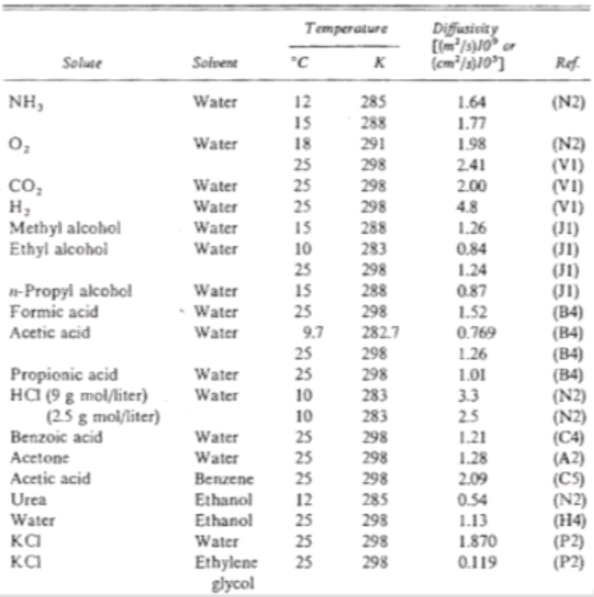
λ_{+} and λ_{-} are the limiting ionic conductance in very dilute solutions

T is 298.2 when using the above at 25°C

D_{AB} is proportional to T

Diffusion Coefficients for Dilute Liquid Solutions

TABLE 6.3-1.



Solute	Solvent	Temperature °C	Temperature K	Diffusivity [(m²/s)10⁹ or (cm²/s)10⁵]	Ref.
NH_3	Water	12	285	1.64	(N2)
		15	288	1.77	
O_2	Water	18	291	1.98	(N2)
		25	298	2.41	(V1)
CO_2	Water	25	298	2.00	(V1)
H_2	Water	25	298	4.8	(V1)
Methyl alcohol	Water	15	288	1.26	(J1)
Ethyl alcohol	Water	10	283	0.84	(J1)
		25	298	1.24	(J1)
n-Propyl alcohol	Water	15	288	0.87	(J1)
Formic acid	Water	25	298	1.52	(B4)
Acetic acid	Water	9.7	282.7	0.769	(B4)
		25	298	1.26	(B4)
Propionic acid	Water	25	298	1.01	(B4)
HCl (9 g mol/liter)	Water	10	283	3.3	(N2)
(2.5 g mol/liter)		10	283	2.5	(N2)
Benzoic acid	Water	25	298	1.21	(C4)
Acetone	Water	25	298	1.28	(A2)
Acetic acid	Benzene	25	298	2.09	(C5)
Urea	Ethanol	12	285	0.54	(N2)
Water	Ethanol	25	298	1.13	(H4)
KCl	Water	25	298	1.870	(P2)
KCl	Ethylene glycol	25	298	0.119	(P2)

Diffusivity in Solids

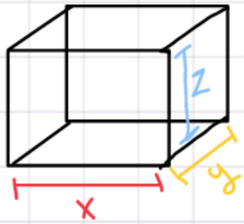
slow - slow.

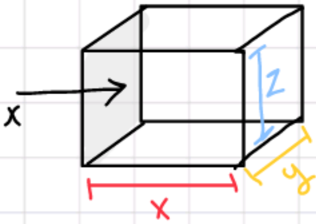
*complex

- Typical values for diffusivity in solids are ranged between $5x10^{-14}$ and $1x10^{-10}$
- One outstanding characteristic of these values is their small size, usually thousands of time less than those in a liquid, which are in turn 10,000 times less than those in a gas.
- Diffusion plays a major role in catalysis and is important to the chemical engineer. For metallurgists, diffusion of atoms within the solids is of more importance.
- Examples:
  - Leaching of metal ores
  - Drying of timber, and foods
  - Diffusion and catalytic reaction in solid catalysts
  - Separation of fluids by membranes
  - Treatment of metal at high temperature by gases.

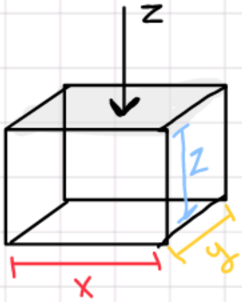
* Differential equation for mass transfer

Chapte 25
Fundamental of mometum
Heat, and mass transfer

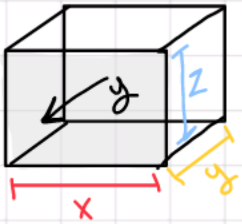




$\text{Area} = y \times Z$



$\text{Area} = X \times y$



$\text{Area} = X \times Z$

total mass = mass In − mass out >> no conc or gen

mass balanc A :-

In − Out + gen − Con = acc

mass flux In X direction

$$-n_A \Delta y \Delta z\Big|_{x+\Delta x} + n_A \Delta y \Delta z\Big|_{x} + r_A \Delta y \Delta z \Delta x = \frac{\partial \rho_A}{\partial t} \Delta x \Delta y \Delta z$$

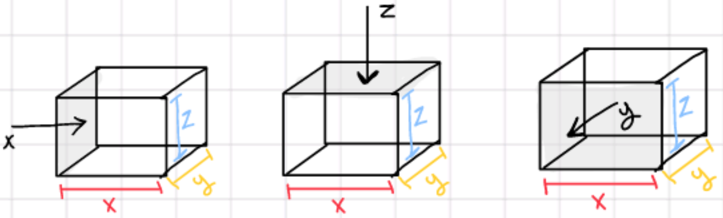
لازم كل اشي يكون . $\frac{\text{mass}}{\text{time}}$ او $\frac{\text{mole}}{\text{time}}$

net rate of mass efflux of A from control volume − net mass accumulation of A within control volume + rate chem production of A within the control volume = 0

$$n_A \Delta y \Delta z\Big|_{x+\Delta x} - n_A \Delta y \Delta z\Big|_{x} - \underbrace{r_A \Delta y \Delta z \Delta x}_{\text{no rxn } =0} + \underbrace{\frac{\partial C_A}{\partial t} \Delta x \Delta y \Delta z}_{\text{steady state } =0} = 0$$

↳ if mass flux was only in X direction.

* masstransfer all direction x, y z



$$n_A \Delta z \Delta y\Big|_{x+\Delta x} - n_A \Delta z \Delta y\Big|_{x} + n_A \Delta y \Delta x\Big|_{z+\Delta z} - n_A \Delta y \Delta x\Big|_{z} + n_A \Delta x \Delta z\Big|_{y+\Delta y}$$

$$- n_A \Delta z \Delta x\Big|_{y} - r_A \Delta z \Delta x \Delta y + \frac{\partial C_A}{\partial t} \Delta y \Delta z \Delta x = 0$$

$$\left[n_A \Delta z \Delta y\Big|_{x+\Delta x} - n_A \Delta z \Delta y\Big|_{x} + n_A \Delta y \Delta x\Big|_{z+\Delta z} - n_A \Delta y \Delta x\Big|_{z} + n_A \Delta x \Delta z\Big|_{y+\Delta y} - n_A \Delta z \Delta x\Big|_{y} - r_A \Delta z \Delta x \Delta y + \frac{\partial C_A}{\partial t} \Delta y \Delta z \Delta x = 0 \right] \div \Delta y \Delta x \Delta z$$

$$\frac{n_A}{\Delta x}\Big|_{x+\Delta x} - \frac{n_A}{\Delta x}\Big|_{x} + \frac{n_A}{\Delta y}\Big|_{\Delta y+y} - \frac{n_A}{\Delta y}\Big|_{y} + \frac{n_A}{\Delta z}\Big|_{z+\Delta z} - \frac{n_A}{\Delta z}\Big|_{z} + \frac{\partial C_A}{\partial t} - r_A = 0$$

$$\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} - r_A = 0$$

$$\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} - r_A = 0$$

$$\frac{\partial N_{Bx}}{\partial x} + \frac{\partial N_{By}}{\partial y} + \frac{\partial N_{Bz}}{\partial z} + \frac{\partial C_B}{\partial t} - r_B = 0$$

$$\nabla N_A + \frac{\partial C_A}{\partial t} - r_A = 0$$

$$+ \quad \nabla N_B + \frac{\partial C_B}{\partial t} - r_B = 0$$

$$\nabla \cdot (N_A + N_B) + \frac{\partial (C_A + C_B)}{\partial t} - (r_A + r_B) = 0$$

r_A = معدل التغير A بسبب التفاعل

r_B = معدل التغير B بسبب التفاعل

* Binary system

$A \longrightarrow B \qquad r_A \downarrow \; r_B \uparrow$

$B \longrightarrow A \qquad r_A \uparrow \; r_B \downarrow$

$$\therefore r_A = -r_B$$

$$\nabla \cdot (N_A + N_B) + \frac{\partial (C_A + C_B)}{\partial t} = 0$$

ازا بحكي عن ال mass

* when velocity of the mixture = 0, no rxn, one Dimensional :-

$$\frac{\partial \cancel{N_A}}{\partial \cancel{x}} + \frac{\partial \cancel{N_{Ay}}}{\partial y} + \frac{\partial N_A}{\partial z} + \frac{\partial C_A}{\partial t} - \underset{\text{no rxn.}}{\cancel{r_A}} = 0$$

$$\frac{\partial N_A}{\partial z} + \frac{\partial C_A}{\partial t}$$

$$N_A = J_A^* + \frac{C_A}{C_T}\underset{\text{no VBulk.}}{\cancel{[N_A + N_B]}}$$

$$N_A = -D_{AB}\frac{\partial C_A}{\partial z}$$

$$D_{AB}\frac{\partial^2 C_A}{\partial z^2} = \frac{\partial C_A}{\partial t} \quad \#$$

* when velocity of the mixture = 0, no rxn, one Dimensional steady state

$$\frac{\partial \cancel{N_A}}{\partial \cancel{x}} + \frac{\partial \cancel{N_{Ay}}}{\partial y} + \frac{\partial N_A}{\partial z} + \cancel{\frac{\partial C_A}{\partial t}} - \underset{\text{no rxn.}}{\cancel{r_A}} = 0$$

$$\frac{\partial N_A}{\partial z} = 0$$

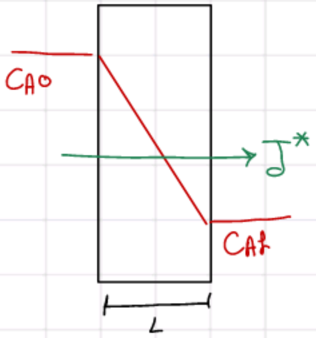
$N_A = \text{Constant}$

B.L ⇒

$C_A = C_A^\circ$ @ $z = 0$

$C_A = C_{AL}$ @ $z = L$

$$N_A = -D_{AB}\frac{dC_A}{dz}$$



$$N_a = -D_{AB}\frac{dC_A}{dz}$$

$$\int_0^L N_A\,dz = -\int_{C_{A0}}^{C_{AL}} D_{AB}\,dC_A$$

$$N_A(L) = -D_{AB}[C_{AL} - C_{A0}]$$

$$N_A = D_{AB}\frac{[C_A^\circ - C_{AL}]}{L} \quad \#$$

Concentration profile :-

$$* \frac{\partial^2 C_A}{\partial z^2} = 0$$

$$\frac{\partial C_A}{\partial z} = a \text{ [constant]}$$

$$\int \partial C_A = a \int \partial z$$

$$C_A = az + b$$

C_A° @ $z = 0$

$$C_A^\circ = b$$

C_{AL} @ $z = L$

$$C_{AL} = aL + C_A^\circ$$

$$a = \frac{[C_{AL} - C_A^\circ]}{L}$$

$$\therefore \; C_A = \frac{[C_{AL} - C_A^\circ]}{L} z + C_A^\circ \; \#$$

* for cylindrical geometry

$$\frac{\partial c_A}{\partial t} = \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{A,r}) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

* In r Direction, no rxn, no VBulk

$$\frac{\partial c_A}{\partial t} = \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{A,r}) + \frac{1}{r}\cancel{\frac{\partial N_{A,\theta}}{\partial \theta}} + \cancel{\frac{\partial N_{A,z}}{\partial z}}\right] = \cancel{R_A}$$

$$\frac{\partial C_A}{\partial t} = \frac{-1}{r}\frac{\partial}{\partial r}\left[r N_{Ar}\right]$$

$$N_A = J^* + \cancel{\frac{C_A}{C_T}\left[\cancel{N_A} + N_B\right]}$$

$$\frac{\partial C_A}{\partial t} = \frac{-1}{r}\frac{\partial}{\partial r}\left[r - D_{AB}\frac{\partial C_A}{\partial z}\right]$$

$$\frac{\partial C_A}{\partial t} = \frac{D_{AB}}{r}\frac{\partial}{\partial r}\left[r\frac{\partial C_A}{\partial z}\right] \#$$

* In spherical coordinates :-

$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\partial\theta}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial\phi}\right] = R_A$$

* In r Direction, no rxn, no VBulk

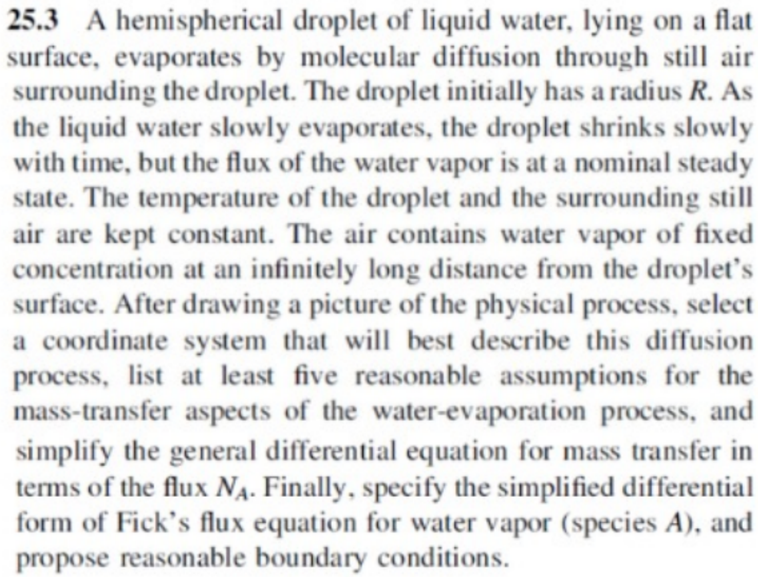
$$\frac{\partial c_A}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\cancel{\partial\theta}}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\cancel{\partial N_{A,\phi}}}{\partial\phi}\right] = \cancel{R_A}$$

$$\frac{\partial C_A}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left[r^2 N_{Ar}\right]$$

$$N_A = J^* + \frac{C_A}{C_T}\cancel{\left[N_A + N_B\right]}$$

$$\frac{\partial C_A}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r^2 - D_{AB}\frac{\partial C_A}{\partial z}\right]$$

$$\frac{\partial C_A}{\partial t} = \frac{D_{AB}}{r}\frac{\partial}{\partial r}\left[r^2\frac{\partial C_A}{\partial z}\right] \quad \#$$



25.3 A hemispherical droplet of liquid water, lying on a flat surface, evaporates by molecular diffusion through still air surrounding the droplet. The droplet initially has a radius *R*. As the liquid water slowly evaporates, the droplet shrinks slowly with time, but the flux of the water vapor is at a nominal steady state. The temperature of the droplet and the surrounding still air are kept constant. The air contains water vapor of fixed concentration at an infinitely long distance from the droplet's surface. After drawing a picture of the physical process, select a coordinate system that will best describe this diffusion process, list at least five reasonable assumptions for the mass-transfer aspects of the water-evaporation process, and simplify the general differential equation for mass transfer in terms of the flux N_A. Finally, specify the simplified differential form of Fick's flux equation for water vapor (species *A*), and propose reasonable boundary conditions.

1] steady state
2] geometric sphere ical
3] Temp, prussure constant, D constant
4] no chemical rxn
5] 1-D
6] $N_{Air} = 0$
5] concentration Constant @ r=R

Still air.
S.S → water Vaper Fixed

↗ evaporates by Diff

* stady state

$$\cancel{\frac{\partial c_A}{\partial t}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial \theta}}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\cancel{\theta}}\cancel{\frac{\partial N_{A,\phi}}{\partial \phi}}\right] = \cancel{R_A}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^2 N_{Ar}\right] = 0$$

$$r^2 N_{Ar} = \text{constant}$$

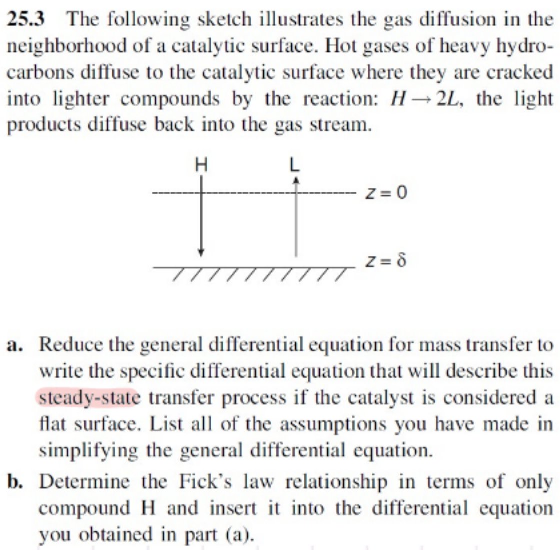
$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + \underset{N_{Air}=0}{\cancel{N_{Br}}})$$

$$N_{Ar} - y_A N_{Ar} = -cD_{AB}\frac{dy_A}{dr}$$

$$N_{Ar}[1 - y_A] = -cD_{AB}\frac{dy_A}{dr}$$

$$N_{Ar} = -\frac{cD_{AB}}{[1-y_A]}\frac{dy_A}{dr} \quad \#$$

25.3>>


25.3 The following sketch illustrates the gas diffusion in the neighborhood of a catalytic surface. Hot gases of heavy hydrocarbons diffuse to the catalytic surface where they are cracked into lighter compounds by the reaction: $H \rightarrow 2L$, the light products diffuse back into the gas stream.

a. Reduce the general differential equation for mass transfer to write the specific differential equation that will describe this steady-state transfer process if the catalyst is considered a flat surface. List all of the assumptions you have made in simplifying the general differential equation.

b. Determine the Fick's law relationship in terms of only compound H and insert it into the differential equation you obtained in part (a).

من rxn بطلع علاقة بين NA و NB.

mass transfer z Direction

Diffuation

steady state →

$$\cancel{\frac{\partial N_{Ax}}{\partial x}} + \cancel{\frac{\partial N_{Ay}}{\partial y}} + \frac{\partial N_{Az}}{\partial z} + \cancel{\frac{\partial C_A}{\partial t}} - r_A = 0$$

no mass transfer in X Direction

no mass transfer in y Direction

S.S

$$\frac{\partial N_{Az}}{\partial z} - \cancel{r_A} = 0$$

assuming no rxn

$$N_A = J_A^* + \frac{(N_A + N_B)\, C_A}{C_T} \quad >$$

$$N_{Az} = J_A^* + \frac{C_A}{C_T}(N_A + N_B)$$

$$-2N_A = N_B. \longrightarrow$$

$$H \rightarrow 2L$$

$$-2N_H = N_L$$

↳ H بختفي مع التفاعل عابزيد

$$N_A = J_A + \frac{C_A}{C_T}(N_A - 2N_A)$$

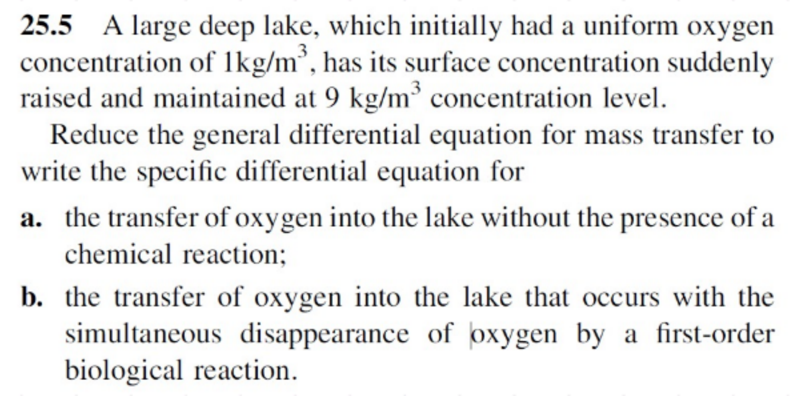
$$N_A = -D_{AB}\frac{dc_A}{dz} + \frac{C_A}{C_T}(-N_A)$$

$$N_A + N_A\frac{C_A}{C_T}$$

$$N_A\left(1 + \frac{C_A}{C_T}\right) = -\frac{D_{AB}\, dC_A}{dz}$$

$$N_A = \frac{-D_{AB}}{1 + y_A}\frac{dC_A}{dz} \quad \#$$

25.5≫



25.5 A large deep lake, which initially had a uniform oxygen concentration of 1kg/m^3, has its surface concentration suddenly raised and maintained at 9 kg/m^3 concentration level.

Reduce the general differential equation for mass transfer to write the specific differential equation for

a. the transfer of oxygen into the lake without the presence of a chemical reaction;

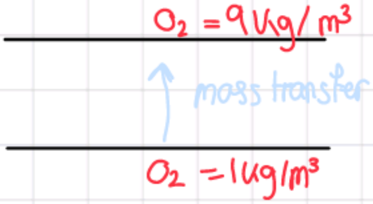
b. the transfer of oxygen into the lake that occurs with the simultaneous disappearance of oxygen by a first-order biological reaction.

اهمال $\frac{C_A}{C_T} \approx 0$.

A)

no rxn

mass trans in z Direction only



$$\cancel{\frac{\partial N_{Ax}}{\partial x}} + \cancel{\frac{\partial N_{Ay}}{\partial y}} + \frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} - \cancel{r_A} = 0$$

no mass transfer in X Direction

no mass transfer in y Direction

no rxn.

$$\frac{\partial N_{Az}}{\partial z} = 0$$

$$N_{Az} = J_A^* + \frac{C_A}{C_T}(N_A + N_B)$$

$$\frac{C_A}{C_T} \approx 0$$

نسبة ال O_2 بال very large lake قليلة

$$N_{Az} = -D_{AB}\frac{dC_A}{dz}$$

$$-D_{AB}\frac{d^2C_A}{dz^2} + \frac{dC_A}{dt} = 0$$

$$dC_A/dt = D_{AB}\frac{d^2C_A}{dz^2} \quad \#$$

b) first order rxn

$$\cancel{\frac{\partial N_{Ax}}{\partial x}} + \cancel{\frac{\partial N_{Ay}}{\partial y}} + \frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} - r_A = 0$$

no mass transfer in X Direction

no mass transfer in y Direction

$$\frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} = r_A$$

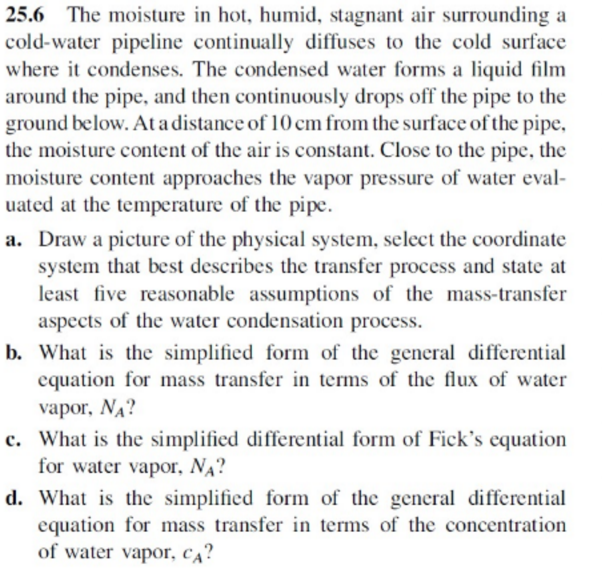
first order $r_A = -kC_A$

$$N_{Az} = -D_{AB}\frac{dC_A}{dz}$$

$$-D_{AB}\frac{\partial^2 C_A}{\partial z^2} + \frac{dC_A}{dt} = -kC_A$$

$$\frac{dC_A}{dt} = -kC_A + D_{AB}\frac{\partial^2 C_A}{\partial z^2} \quad \#$$

25.6



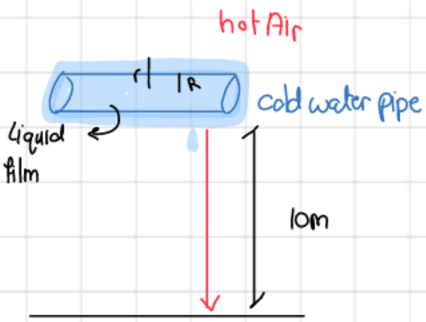
25.6 The moisture in hot, humid, stagnant air surrounding a cold-water pipeline continually diffuses to the cold surface where it condenses. The condensed water forms a liquid film around the pipe, and then continuously drops off the pipe to the ground below. At a distance of 10 cm from the surface of the pipe, the moisture content of the air is constant. Close to the pipe, the moisture content approaches the vapor pressure of water evaluated at the temperature of the pipe.

a. Draw a picture of the physical system, select the coordinate system that best describes the transfer process and state at least five reasonable assumptions of the mass-transfer aspects of the water condensation process.

b. What is the simplified form of the general differential equation for mass transfer in terms of the flux of water vapor, N_A?

c. What is the simplified differential form of Fick's equation for water vapor, N_A?

d. What is the simplified form of the general differential equation for mass transfer in terms of the concentration of water vapor, c_A?



no rxn

Diffusion in R Direction only

constant T and P.

C_A @ R+10 = Constant

C_A @ R = constant $y_A = \frac{P_A}{P_T}$

steady state

$$\cancel{\frac{\partial C_A}{\partial t}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r N_{A,r}) + \frac{1}{r}\cancel{\frac{\partial N_{A\theta}}{\partial \theta}} + \cancel{\frac{\partial N_{Az}}{\partial z}} \right] = \cancel{R_A}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r N_{Ar}) = 0$$

$$\frac{\partial}{\partial r}(r N_{Ar}) = 0$$

$$r N_{Ar} = \text{constant.}$$

$$N_{Ar} = J_A + \frac{C_A}{C_T}\left[N_A + \cancel{N_B}\right]$$

no flow Air

$$N_{Ar} - \frac{C_A}{C_T}(N_A) = -D_{AB}\frac{dC_A}{dr}$$

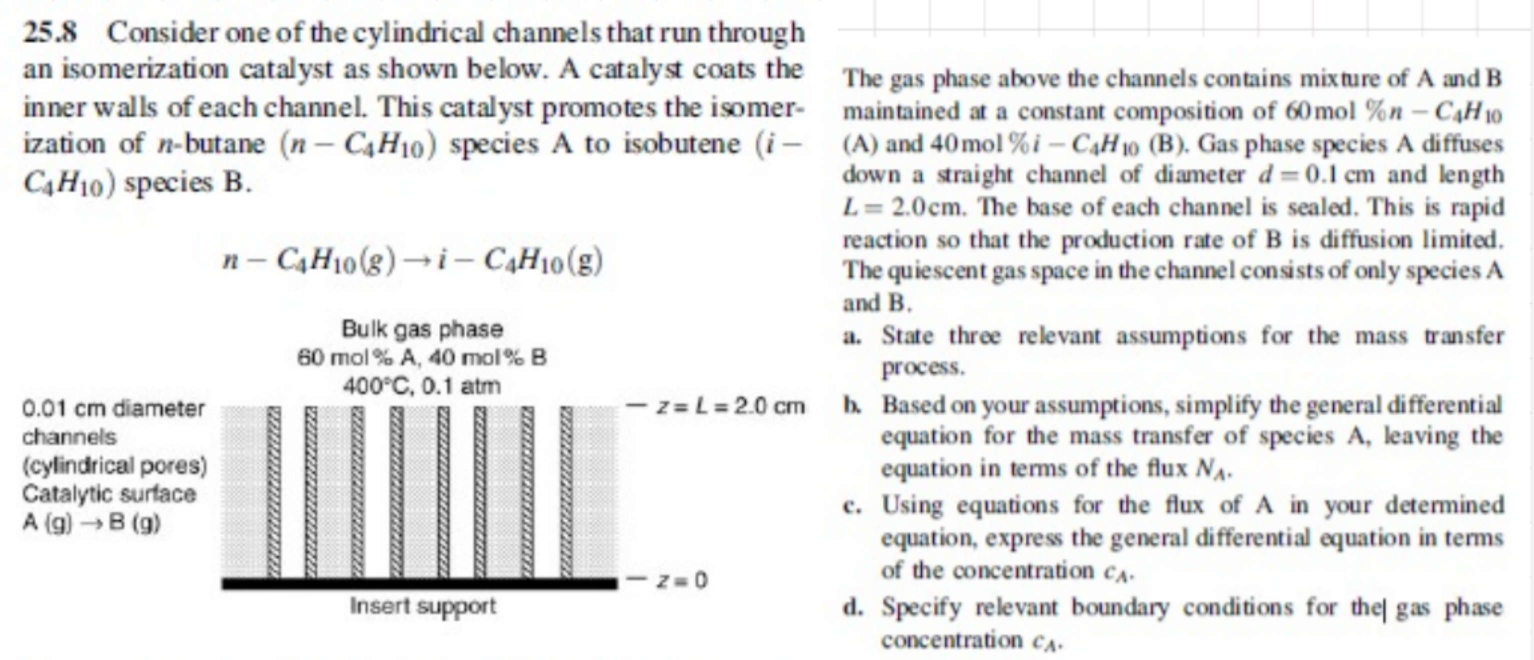
$$N_A\left(1 - \frac{C_A}{C_T}\right) = -D_{AB}\frac{dC_A}{dr}$$

$$N_A = \frac{-D_{AB}}{(1-y_A)}\frac{dC_A}{dr} \quad *$$

for Dilute concentration $y_A \approx 0$

$$N_A = -D_{AB}\frac{dC_A}{dr}$$

25.8 >>



25.8 Consider one of the cylindrical channels that run through an isomerization catalyst as shown below. A catalyst coats the inner walls of each channel. This catalyst promotes the isomerization of *n*-butane $(n - C_4H_{10})$ species A to isobutene $(i - C_4H_{10})$ species B.

$$n - C_4H_{10}(g) \rightarrow i - C_4H_{10}(g)$$

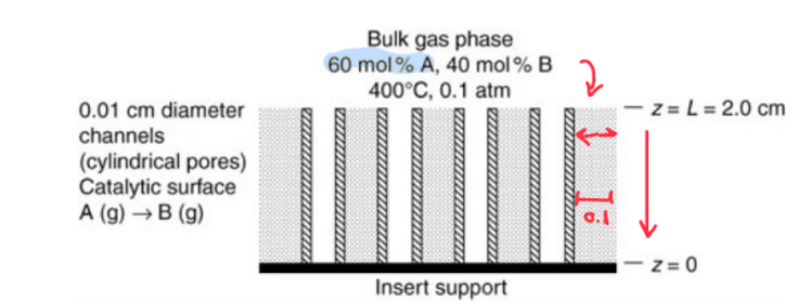
The gas phase above the channels contains mixture of A and B maintained at a constant composition of 60 mol $\% n - C_4H_{10}$ (A) and 40 mol $\% i - C_4H_{10}$ (B). Gas phase species A diffuses down a straight channel of diameter $d = 0.1$ cm and length $L = 2.0$ cm. The base of each channel is sealed. This is rapid reaction so that the production rate of B is diffusion limited. The quiescent gas space in the channel consists of only species A and B.

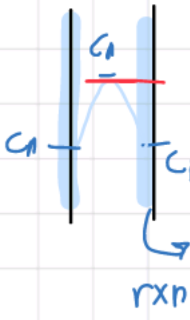
a. State three relevant assumptions for the mass transfer process.

b. Based on your assumptions, simplify the general differential equation for the mass transfer of species A, leaving the equation in terms of the flux N_A.

c. Using equations for the flux of A in your determined equation, express the general differential equation in terms of the concentration c_A.

d. Specify relevant boundary conditions for the| gas phase concentration c_A.





$$\left.\frac{dC_A}{dr}\right|_{r=0} = 0$$ B.C ①

* mass transfer ocurr in z Direction and r

* Constant P and T

* steady state

* no homogenous rxn → التفاعل بس عل الsurface

BC ②
fast rxn
= C_A at $(z, R) = 0$

BC 3:
C_A at $(0.2, r) = 0.6$

$$\cancel{\frac{\partial C_A}{\partial t}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r N_{A,r}) + \frac{1}{r}\cancel{\frac{\partial N_{A\theta}}{\partial \theta}} + \frac{\partial N_{Az}}{\partial z} \right] = \cancel{r_A}$$

no rxn

$$\frac{1}{r}\frac{\partial}{\partial r}(r N_{Ar}) + \frac{\partial N_{Az}}{\partial z} = 0$$

$$N_{Ar} = J_A + \frac{C_A}{C_T}\left[N_A + N_B\right]$$

$A \rightarrow B$

$-N_A = N_B.$

$$N_{Ar} = J_A + \frac{C_A}{C_T}\cancel{[N_A - N_A]}$$

$$N_{Ar} = -D_{AB}\frac{dC_r}{dr}$$

Same for $N_{Az} = -D_{AB}\frac{dC_z}{dz}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(-D_{AB} r \frac{\partial C_A}{\partial r}\right) \frac{-\partial}{\partial z}\left(D_{AB}\frac{dC_A}{dz}\right) = 0$$

$$-D_{AB}\left(\frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial C_A}{\partial r} + \frac{\partial^2 C_A}{\partial z^2}\right) = 0$$

* Initial condition (uniform, non uniform)

Prupose : Define concentration profile at the beging of the time interval

*unsteady state.

$t=0 \quad C_A = C_{A0} \quad mol/m^3$

$t=0 \quad \rho_A = \rho_{A0} \quad mass/m^3$

* Boundary condition :- How mass transfer occurs at the edges of the system.

* Known surface concentration $C_A = C_A^{\circ}$ at $z=0$

- when concentration of A is known معطيات
- Determined from equilibrum relations
  - ideal solution Raoult's law ✧ — evaporation of Volile L, $L \to V$

    $$P_A = P_A^{sat} X_A$$

  - ↓ soluble materals Henry's law ✧ — Gasabsorption in Liquid, $V \to L$

    $$P_A = H X_A$$

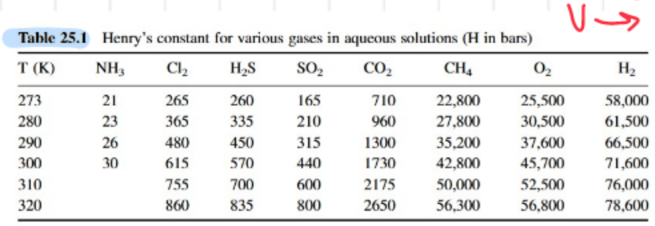


Table 25.1 Henry's constant for various gases in aqueous solutions (H in bars)

T (K)	NH_3	Cl_2	H_2S	SO_2	CO_2	CH_4	O_2	H_2
273	21	265	260	165	710	22,800	25,500	58,000
280	23	365	335	210	960	27,800	30,500	61,500
290	26	480	450	315	1300	35,200	37,600	66,500
300	30	615	570	440	1730	42,800	45,700	71,600
310		755	700	600	2175	50,000	52,500	76,000
320		860	835	800	2650	56,300	56,800	78,600

- Symmetry or Impermeable Surface $\frac{\partial C_A}{\partial z} = 0$
- Convective flux $N_A = k(C_{As} - C_{A\infty})$
  - Convective mass trans surface
- chemical rxn
  - very fast $A \to B$, $C_{As} = 0$
  - First order $N_A\big|_{z=0} = C_{As} k$

↳ for gas – solid Interphase $C_{A\,solid} = S\,P_A$

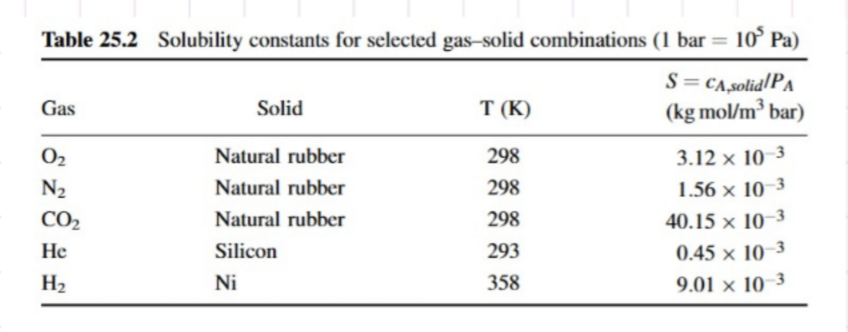


Table 25.2 Solubility constants for selected gas–solid combinations (1 bar = 10^5 Pa)

Gas	Solid	T (K)	$S = c_{A,solid}/P_A$ (kg mol/m^3 bar)
O_2	Natural rubber	298	3.12×10^{-3}
N_2	Natural rubber	298	1.56×10^{-3}
CO_2	Natural rubber	298	40.15×10^{-3}
He	Silicon	293	0.45×10^{-3}
H_2	Ni	358	9.01×10^{-3}

$$* \ \cancel{\frac{\partial N_A}{\partial x}} + \cancel{\frac{\partial N_{Ay}}{\partial y}} + \frac{\partial N_A}{\partial z} + \cancel{\frac{\partial C_A}{\partial t}} - \underset{\text{no rxn.}}{\cancel{r_A}} = 0$$

$$\frac{\partial N_A}{\partial z} = 0$$

$$N_A = \text{Constant}$$

$$N_A = -D_{AB}\frac{dC_A}{dz} + y_A\left[N_A + N_B\right]$$

$$N_A - \frac{C_A}{C_T}\left[N_A + N_B\right] = -D_{AB}\frac{dC_A}{dz}$$

$$\frac{N_A C_T - C_A\left[N_A + N_B\right]}{C_T} = -D_{AB}\frac{dC_A}{dz}$$

$$N_a \frac{[z_2]}{C_T D_{AB}} = \frac{N_A}{N_A + N_B}\ln\left[\frac{\left[\frac{N_A}{N_A+N_B}\right] - \frac{C_{A2}}{C_T}}{\left[\frac{N_A}{N_A+N_B}\right] - \frac{C_{A1}}{C_T}}\right]$$

$$N_A = = \frac{C_T D_{AB}}{z}\ \frac{N_A}{N_A + N_B}\ \ln\left[\frac{\left[\frac{N_A}{N_A+N_B}\right] - \frac{C_{A2}}{C_T}}{\left[\frac{N_A}{N_A+N_B}\right] - \frac{C_{A1}}{C_T}}\right] \quad \#$$

Unimolecular diffusion of air throw stagnant non diffusion b

$\hookrightarrow N_{B=0}$

$$N_A = J_{Az}^* + \frac{C_A}{C_T}\left[N_A + \cancel{N_B}\right]$$

$$N_A = J_{Az}^* + \frac{C_A}{C_T} N_A$$

$$N_A - \frac{C_A}{C_T} N_A = J_{Az}^*$$

$$N_A\left[1 - \frac{C_A}{C_T}\right] = J_{Az}^*$$

$$N_A = \frac{J_{Az}}{\left[1 - \frac{C_A}{C_T}\right]}$$

$$N_A = \frac{-D_{AB}\, C_T}{[1 - y_A]} \frac{dy_A}{dz}$$

$$\int N_A\, dz = -D_{AB}\, C_T \int \frac{dy_A}{1 - y_A}$$

$$N_A\, \Delta z = -D_{AB}\, C_T \ln \frac{1 - y_{A2}}{1 - y_{A1}}$$

$$N_A = \frac{-D_{AB}\, C_T}{\Delta z} \ln \frac{1 - y_{A2}}{1 - y_{A1}} \quad \#$$

* log mean avg concentration

$$X_{B,lm} = \frac{X_{B2} - X_{B1}}{\ln\left[\frac{X_{B2}}{X_{B1}}\right]}$$

$$X_B = 1 - X_A$$

$$X_{B\,lm} = \frac{X_{A1} - X_{A2}}{\ln\left[\frac{1 - X_{A2}}{1 - X_{A1}}\right]}$$

$$N_A = \frac{-C_T D_{AB}}{\Delta z} \ln\left[\frac{1-x_{A2}}{1-x_{A1}}\right]$$

$$N_A = \frac{-C_T D_{AB}}{\Delta z} \frac{x_{A1}-x_{A2}}{x_{Blm}}$$

$$x_A = \frac{C_A}{C_T} \qquad P_A = C_A RT \qquad P = C_T RT$$

▶▶ ideal gas.

$$N_A = \frac{-C_T D_{AB}}{\Delta z} \ln\left[\frac{1-x_{A2}}{1-x_{A1}}\right]$$

$$N_A = \frac{-P D_{AB}}{RT \Delta z} \ln \frac{1-\frac{P_{A2}}{P}}{1-\frac{P_{A1}}{P}}$$

$$N_A = \frac{-P D_{AB}}{RT \Delta z} \ln \frac{P - P_{A2}}{P - P_{A1}}$$

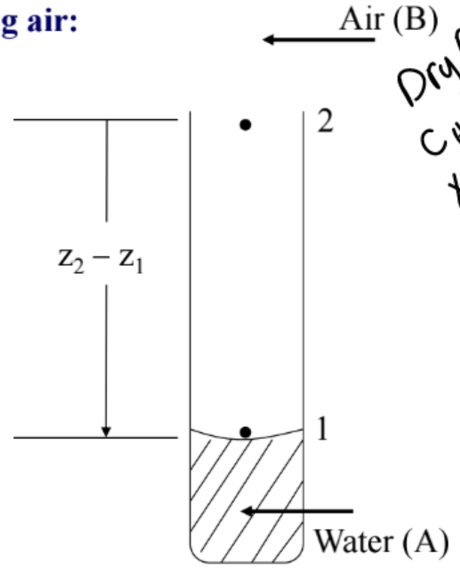
*log mean P

$$P_{Blm} = \frac{P_{B2} - P_{B1}}{\ln\left[\frac{P_{B2}}{P_{B1}}\right]}$$

$$N_A = \frac{-P D_{AB}}{RT \Delta z} \frac{(P_{A1} - P_{A2})}{P_{Blm}}$$

Diffusion of water through stagnant, non-diffusing air:

Water in the bottom of a narrow metal tube is held at a constant temperature of 293 K. The total pressure of air (assumed to be dry) is 1 atm and the temperature is 293 K. Water evaporates and diffuses through the air in the tube, and the diffusion path is 0.1524 m long. Calculate the rate of evaporation at steady state. The diffusivity of water vapour at 1 atm and 293 K is 0.250 x 10^{-4} m^2/s. Assume that the vapour pressure of water at 293 K is 0.0231 atm.



Dry Air

$C_{H_2O} = 0$

$x_{A2} = 0$

$P_A = x_A P$

$x_A = \frac{P_A}{P} = 0.0231$

* steady state

* no rxn

* constant T and P

* mass trans in one Direction.

* A = water B = Air

$C = \frac{P}{RT}$

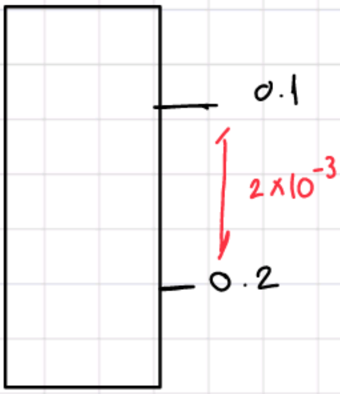
$= \frac{101325}{293\ (8.314)} = 41.63$

$$N_A = \frac{-C D_{AB}}{\Delta z} \ln\left[\frac{1 - x_{A2}}{1 - x_{A1}}\right]$$

$$\frac{41.63\,(0.25 \times 0^{-4})}{0.1524} \ln\left[\frac{1 - 0}{1 - 0.0231}\right]$$

$N_A = 1.59 \times 10^{-4}$

Oxygen is diffusing in a mixture of oxygen-nitrogen at 1 std atm, 25°C. Concentration of oxygen at planes 2 mm apart are 10 and 20 volume % respectively. Nitrogen is non-diffusing. Calculate the flux of oxygen. Diffusivity of oxygen in nitrogen = 1.89×10^{-5} m^2/sec.



$$N_A = \frac{-C_T D_{AB}}{\Delta z} \ln\left[\frac{1-X_{A2}}{1-X_{A1}}\right]$$

$$C_T = \frac{P}{RT} = \frac{101325}{8314 \times (25+273)} = 40.897$$

$$\frac{40.897 \times 1.89\times10^{-5}}{2\times10^{-3}} \ln \frac{1-0.1}{1-0.2}$$

$$= 0.045 \ \text{mol}/(m^2 \cdot s)$$

* Equimolar Counter Diffusion

$\hookrightarrow N_A = -N_B$

$$\cancel{\frac{\partial N_{Ax}}{\partial x}} + \cancel{\frac{\partial N_{Ay}}{\partial y}} + \frac{\partial N_{Az}}{\partial z} + \cancel{\frac{\partial C_A}{\partial t}} - \cancel{r_A} = 0$$

no mass trans in X

no mass trans in y

steady state

no rxn

$\therefore \frac{\partial N_{Az}}{\partial z} = 0 \rightarrow N_{Az} =$ constant

$\hookrightarrow$ concentration profile = Liner.

$$N_A = -C_T D \frac{dx_A}{dz} + x_A (N_A + N_B)$$

$$N_A = -C_T D \frac{dx_A}{dz} + \cancel{x_A (N_A - N_A)}$$

$$\int N_A \, dz = -\int C_T D \, dx_A$$

$$N_A \Delta Z = -C_T D_{AB} \Delta X_A$$

$$N_A = -C_T D_{AB} \frac{\Delta X_A}{\Delta Z}$$

for ideal gas $C = \frac{P_A}{RT}$

$$N_A = \frac{D_{AB}}{RT \Delta Z} (P_{A1} - P_{A2})$$

* The relationship between N_A and N_B by Reaction.

$$mA \rightarrow nB$$

$$\frac{N_A}{m} = -\frac{N_B}{n}$$

$$N_A = -\frac{m}{n} N_B$$

* The relationship between N_A and N_B by Latent Heats

$$\lambda_A N_A = -\lambda_B N_B$$

$$N_A = -\frac{\lambda_B}{\lambda_A} N_B$$

$$\cancel{\frac{\partial c_A}{\partial t}} = \left[\frac{1}{r}\frac{\partial}{\partial r}(rN_{A,r}) + \cancel{\frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta}} + \cancel{\frac{\partial N_{A,z}}{\partial z}}\right] = \cancel{R_A}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r N_{Ar}\right] = 0$$

$$r N_{Ar} = \text{constant.}$$

$$\cancel{\frac{\partial c_A}{\partial t}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\partial\theta}\cancel{(N_{A,\theta}\sin\theta)} + \cancel{\frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial\phi}}\right] = \cancel{R_A}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^2 N_{Ar}\right] = 0$$

$$r^2 N_{Ar} = \text{constant.}$$

* Pseudo stady steate

- change in Livel with time.

- materal Balance [unstady state] + general flux eq [steady state]

$$* \quad N_A = \frac{\rho_{AL}}{M_A}\,\frac{dz}{dt}$$

$$* \quad N_A = \frac{C_T D_{AB}}{\Delta Z}\,\frac{(x_{Ai} - x_{A2})}{x_{BlM}}$$

$$* \quad \frac{C_T D_{AB}}{\Delta Z}\,\frac{(x_{Ai} - x_{A2})}{x_{BlM}} = \frac{\rho_{AL}}{M_A}\,\frac{dz}{dt}$$

$$\int_{z_1}^{z_2} \frac{\rho_{AL}}{M_A}\,\frac{z}{C_T D_{AB}}\,\frac{x_{BlM}}{(x_{Ai} - x_{A2})}\,dz = \int_0^t dt$$

$$\frac{BlM}{C_T D_{AB}\ (x_{Ai} - x_A}\,\frac{\rho_{AL}}{M_A}\left[\frac{z_2^2 - z_1^2}{2}\right] = t \quad \#$$

A vertical glass tube 3 mm in diameter is filled with liquid toluene to a depth of 20mm from the top opened. After 275 hrs at 39.4 °C and a total pressure of 760 mm Hg the level has dropped to 80 mm from the top. Calculate the value of diffusivity.

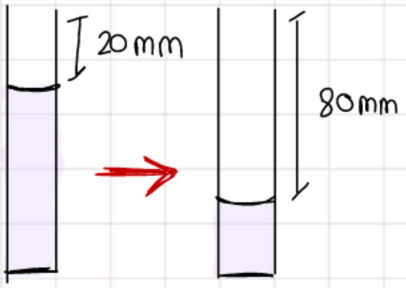
Data:

vapor pressure of toluene at 39.4°C = 7.64 kN / m²,

density of liquid toluene = 850 kg/m³

Molecular weight of toluene ($C_6H_6CH_3$)= 92

760 mm Hg = 101.3 kN/m²



$$\frac{C_T D_{AB}}{\Delta z}\frac{(x_{Ai}-x_{A2})}{x_{BlM}} = \frac{\rho_{AL}}{M_A}\frac{dz}{dt}$$

$$\int_{z_1}^{z_2}\frac{\rho_{AL}}{M_A}\frac{z \quad x_{BlM}}{C_T D_{AB} \quad (x_{Ai}-x_{A2})}dz = \int_0^t dt$$

$x_{Ai} = 7.64 \div 101.3 = 0.0754$

$x_{AB} = 0$

$$\frac{BlM}{C_T D_{AB} \quad (x_{Ai}-x_A)}\frac{\rho_{AL}}{M_A}\left[\frac{z_2^2-z_1^2}{2}\right] = t$$

$x_{Bi} = 0.925$

$x_{BB} = 1$

$$\frac{0.9618 \quad \times 850}{0.4 \quad D_{AB} \; [0.0754-0) \times 92}\left[\frac{0.08^2-0.02^2}{2}\right] = 99\times10^4$$

$x_{BM} = 0.9618$

$D_{AB} = 8.9 \ \frac{m^2}{s}$ #

$C_T = \frac{P}{RT} = 0.04$

* Diffusion into an inf standard medium:-

Large inv
D.f stay constant.

① Case 1: Vaporization of Liquid Drops :-

② Case 2: Sublimation of solid sphere :-

→ changing with time

- materal Balance [unsteady state] + general flux eq [steady state]

* assumption :- Vaper at the surface assumed to diffuse by molecular motion into the larger body of stagnant gas that surronds the droplet

$$\cancel{\frac{\partial c_A}{\partial t}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \cancel{\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(N_{A,\theta}\sin\theta)} + \cancel{\frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial\phi}}\right] = \cancel{R_A}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^2 N_{Ar}\right] = 0$$

$$r^2 N_{Ar} = \text{constant}$$

> General flux eq [S.S]

$$N_A = -C_T D_{AB}\frac{dy_A}{dr} + x_A\left[N_A + \cancel{N_B}\right] \quad \overset{}{0} \text{ stagnant}$$

↳ r Direction ○

$$N_A = -C_T D_{AB}\frac{dy_A}{dr} + x_A\left[N_A\right]$$

$$N_A = \frac{-C_T D_{AB}}{1-x_A}\frac{dx_A}{dr}$$

$$r^2 N_A\,dr = \frac{-C_T D_{AB}}{1-x_A}\,dx_A \quad \#$$

$Na \neq \text{constant}$

$r^2 Na = \text{constant} \hookrightarrow A\,N_A = \text{Constant} \hookrightarrow \underset{\hookrightarrow \text{moleRate}}{W_A} = \text{Constant}$

$$\pi r^2 4\, N_A \big|_r = \pi r^2 4\, Na \big|_{r+\Delta r}$$

$$A_0 N_{A0} = A_1 N_{A1}$$

ازا حسبت NA عند r معين حلو
بعدين طلب NA عند r تانية.

$$4\pi r^2\, N_A\big|_r - 2\pi r^2\, N_A\big|_{r+\Delta r} = 0$$

$$\lim_{\Delta \to 0} \frac{r^2 N_A\big|_r - r^2 N_A\big|_{r+\Delta r}}{\Delta r} = 0$$

$$= \frac{d r^2 N_A}{dr}$$

$\star$ $W_A \Rightarrow$ mole per time

$$\hookrightarrow N_A \times \text{Area} = \frac{\text{mol}}{\cancel{m}^2 .\, s} \times \cancel{m}^2 \Rightarrow \boxed{\frac{\text{mol}}{s}}$$

$$N_A = \frac{-C_T D_{AB}}{1 - X_A} \frac{dX_A}{dr}$$

$$\text{Area} = 4 r^2 \pi$$

$$W_A = \frac{-C_T D_{AB}}{1 - X_A} \frac{dX_A}{dr} \times 4 r^2 \pi$$

$$\int_{r_0}^{\infty} \frac{W_A}{r^2}\, dr = \int_{X_{AS}}^{X_{A\infty}} \frac{-C_T D_{AB}}{1 - X_A} \times 4\pi \, dX_A$$

$$-W_A \frac{1}{r}\Big]_{r_0}^{\infty} = -C_T D_{AB} \times 4\pi \ \ln|1 - X_A|\Big]_{X_{AS}}^{X_{A\infty}}$$

$$-W_A \Big[\underset{\text{zero}}{\cancel{\frac{1}{\infty}}} - \frac{1}{r_0}\Big] = -C_T D_{AB} \times 4\pi \ \ln\left|\frac{1 - X_{A\infty}}{1 - X_{AS}}\right|$$

$$W_A = \frac{-C_T D_{AB} \times 4\pi}{1/r_0} \ln\left|\frac{1 - X_{A\infty}}{1 - X_{AS}}\right|$$

$R = \infty$: على بعد لانهائي
conc ثابت

N_{A0} = at time = 0

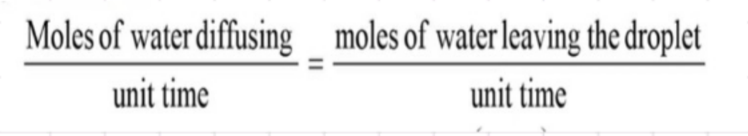
$$N_{A0} = \frac{-C_T D_{AB}}{1-X_A}\frac{dX_A}{dr}$$

$$C_T = \frac{P}{RT}$$

$P_{A1} << P$ → Dilute gas phase

$P_{BlM} = P$.

$$N_{A0} = \frac{D_{AB}P}{RT r_0}\frac{(P_{A1}-P_{A2})}{P_{BlM}}$$



* Moles of water diffusing / unit time = moles of water leaving the droplet / unit time

$$W_A = \underbrace{-}_{\text{leaving}}\frac{dn_A}{dt}$$

$$4\pi r_0^2 N_A = -\frac{d}{dt}\left[\frac{m}{M}\right]$$

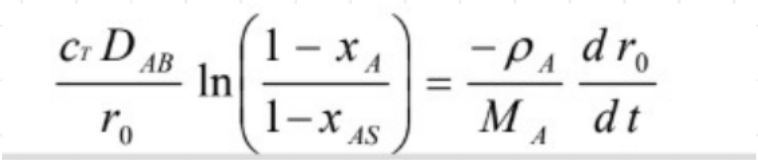
ρ x volume = m.

○ volume = $\frac{4}{3}\pi r_0^3$.

$$4\pi r_0^2 N_A = \frac{-d}{dt}\left[\frac{4}{3}\frac{\pi r_0^3\rho}{M}\right]$$

$$4\pi\frac{D_{AB}C_T}{r_0}\ln\frac{1-X_{AS}}{1-X_A} = \frac{-d}{dt}\left[\frac{4}{3}\frac{\pi r_0^3\rho}{M}\right]$$

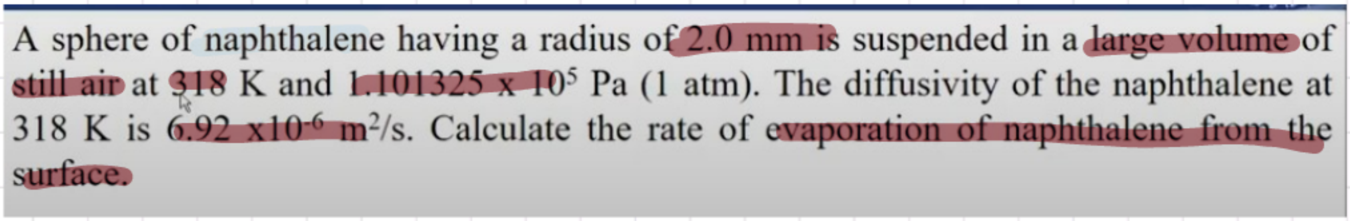
$$\cancel{4\pi}\frac{D_{AB}C_T}{r_0}\ln\frac{1-X_{AS}}{1-X_A} = -\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}\times\cancel{3}\,r^2\frac{\rho}{M}\frac{dr_0}{dt}$$



$$\frac{c_T D_{AB}}{r_0}\ln\left(\frac{1-x_A}{1-x_{AS}}\right) = \frac{-\rho_A}{M_A}\frac{dr_0}{dt}$$

$$\int_{0}^{t} dt = \frac{-\rho_A}{M_A} \frac{1}{C_T D_{AB}} \frac{1}{\ln\left|\frac{1-x_{A2}}{1-x_{A1}}\right|} \int_{r_1}^{r_2} r \cdot dr$$

$$t = \frac{-\rho A}{MA} \frac{1}{C_T D_{AB}} \frac{1}{\ln\left|\frac{1-x_{A2}}{1-x_{A1}}\right|} \frac{1}{2}\left[r_2^2 - r_1^2\right]$$



A sphere of naphthalene having a radius of 2.0 mm is suspended in a large volume of still air at 318 K and 1.101325 x 10^5 Pa (1 atm). The diffusivity of the naphthalene at 318 K is 6.92 x10^{-6} m^2/s. Calculate the rate of evaporation of naphthalene from the surface.

$$W_A = \frac{-C_T D_{AB} \times 4\pi}{1/r^\circ} \ln \left| \frac{1 - x_{A\infty}}{1 - x_{AS}} \right|$$

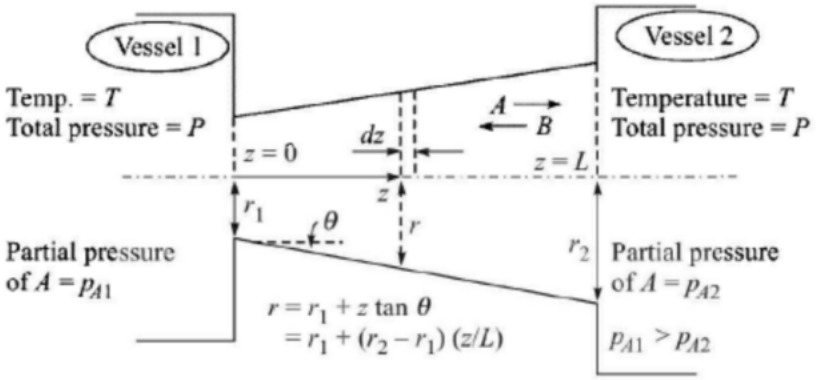
vp of napthalene = 0.555 mm Hg [illegible] → 74 pa

$p_{A2} = 0$

$$C_T = \frac{P}{RT} = \frac{10^5}{8314 \times 318} = 0.04$$

$$\frac{0.04 \times 10^{-6} \times 6.92 \times 4 \times \pi}{1/(2 \times 10^{-3})} \times \ln \frac{1 - 0}{1 - \frac{74}{10^5}}$$

$$= 5 \times 10^{-12} \frac{kmol}{s}$$



W_A = constant

$$\pi r^2 N_A\Big|_z = \pi r^2 N_A\Big|_{z+\Delta z}$$

$$\lim_{\Delta z \to 0} \frac{\pi r^2 N_A\Big|_z - \pi r^2 N_A\Big|_{r+\Delta z}}{\Delta z}$$

$$\frac{d\,\pi r^2 N_A}{dz} = 0$$

$\pi r^2 N_A$ = constant

$r_1^2 N_{A_1} = r^2 N_{A_2}$ #

Diffusion Through Varying Cross-sectional Area

- At position *z* in the conduit, for *A* diffusing through stagnant, non-diffusing *B*, Eq. (16) can be applied

$$N_A = \frac{-c_T D_{AB}}{1-x_A}\frac{d x_A}{d z} \qquad (6a)$$

Or

$$N_A = \frac{-D_{AB}}{RT(1-p_A/P)}\frac{d P_A}{d z} \qquad (6b)$$

- The variable radius *r* can be related to position z in the path as follows:

$$r = \left(\frac{r_2-r_1}{z_2-z_1}\right)z + r_1$$

Diffusion Through Varying Cross-sectional Area

- This value of *r* is then substituted into Eq. (6b) to eliminate *r* and the equation is integrated.

$$\frac{\bar{N}_A}{\pi}\int_{z_1}^{z_2}\frac{d z}{\left(\left(\frac{r_2-r_1}{z_2-z_1}\right)z+r_1\right)^2} = \frac{-D_{AB}}{RT}\int_{p_{A1}}^{p_{A2}}\frac{d P_A}{(1-p_A/P)}$$

$$\Rightarrow \quad \bar{N}_A = \frac{\pi D_{AB}P}{RT}\,\frac{r_1}{z}\left[r_1-(r_1-r_2)\left(\frac{z}{L}\right)\right]\,\ln\frac{P}{P-p_A^v}$$

* Diff in Liquid

* slower than Diff in gas.
[moleculels colser]

- S - L
- g - L
- L - L extraction

* eq molar →
$N_A = -N_B$

$$\frac{C_{av}\, D_{AB}}{\Delta z}\left[x_{A1} - x_{A2}\right]$$

← نفس الاشتقاق

$$C_{av} = \frac{\frac{\rho_1}{M_1} + \frac{\rho_2}{M_2}}{2}$$

not $C_T = \frac{P}{RT}$

* Diff of A in stagnant B

$N_B = 0$

$$N_A = \frac{D_{AB}}{[z_2 - z_1]\, x_{BlM}}\, C_{av}\, (x_{A1} - x_{A2})$$

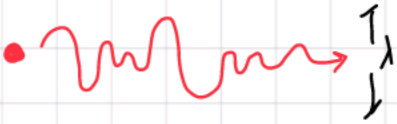
* Diffusion in solid.

① Diffusion in Pore

* Absoption
* Catalyst rxn

some time i have diff limiting rxn this
rxn محدود diff سرعة ال

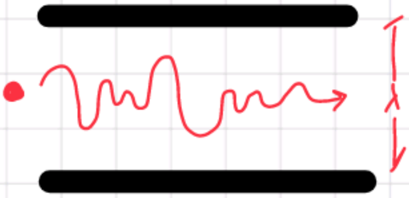
* Particle motion



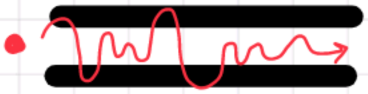
mean free bath
↑T, λ↑ bc طاقة اعلى
↑P, λ↓

طبعا صور في عرض
انو ال Pore
شكلها cylinde
يعني ideal
صار الاشي حتى Real

$\lambda_{gas} > \lambda_{Liquid}$ bc Pore in L is small.

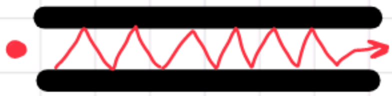


$D_{pore} > \lambda$
no effect
molec diff عادي
$Kn < 1$



$D_{pore} < \lambda$

Knudsen Diffusion



صار عندي اصطدام بالحيط.

$Kn > 1$

* Knudsen num $Kn = \frac{\lambda}{d_{pore}}$

Same as λ
↑T, ↑Kn
↓P, ↑Kn

* نعمله بس بالغازات
بالسؤال λ كتير صغيره.

The diffusion coefficient for Knudsen diffusion is obtained from the self-diffusion coefficient derived from the kinetic theory of gases:

$$D_{AA^*} = \frac{\lambda u}{3} = \frac{\lambda}{3}\sqrt{\frac{8\kappa N T}{\pi M_A}}$$

هوماشي d مو λ

For Knudsen diffusion, we replace path length λ with pore diameter d_{pore}, as species A is now more likely to collide with the pore wall as opposed to another molecule. In this instance, the Knudsen diffusivity for diffusing species A, D_{KA}, is

$$D_{KA} = \frac{d_{\text{pore}}}{3}u = \frac{d_{\text{pore}}}{3}\sqrt{\frac{8\,\kappa N\,T}{\pi M_A}} \qquad \text{(24-58a)}$$

اخلص من كل
ال constant

Or, since κ and N are physical constants, equation (24-58a) is also given by

$$D_{KA} = 4850\, d_{\text{pore}}\sqrt{\frac{T}{M_A}} \qquad \text{(24-58b)}$$

	Binary Gas D_{AB}	Knudsen Diff D_{KA}
	$D_{AB} = \frac{1.8583 \times 10^{-7}\, T^{3/2}}{P\sigma_{AB}^2\, \Omega_{D,AB}}\left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}$	$D_{KA} = 4850\, d_{pore}\sqrt{\frac{T}{M_A}}$
T	$D_{AB} \propto T^{1.5}$	$D_{KA} \propto T^{0.5}$
P	depent	In depented
other species	depent	In depented.

* efficitive Diff coe ⇒ Kn Diff + mol Diff

$$\frac{1}{D_{Ae}} = \frac{1-\alpha y_A}{D_{AB}} + \frac{1}{D_{KA}} \qquad (24\text{-}59)$$

$$\frac{1}{D_{Ae}} = \frac{1-\alpha y_A}{D_{AB}} + \frac{1}{D_{KA}} \qquad (24\text{-}59)$$

y_A : mole fraction.

$$\alpha = 1 + \frac{N_B}{N_A}$$

$$\alpha = 1 + \frac{N_B}{N_A}$$

* equimolar counter diff ⇒ $N_A = -N_B$.

* او ازا كان y_A كتير صغير

$$\alpha = 1 - \frac{N_A}{N_A} = 0.$$

$$\frac{1}{D_e} = \frac{1}{D_{AB}} + \frac{1}{D_{KA}}$$

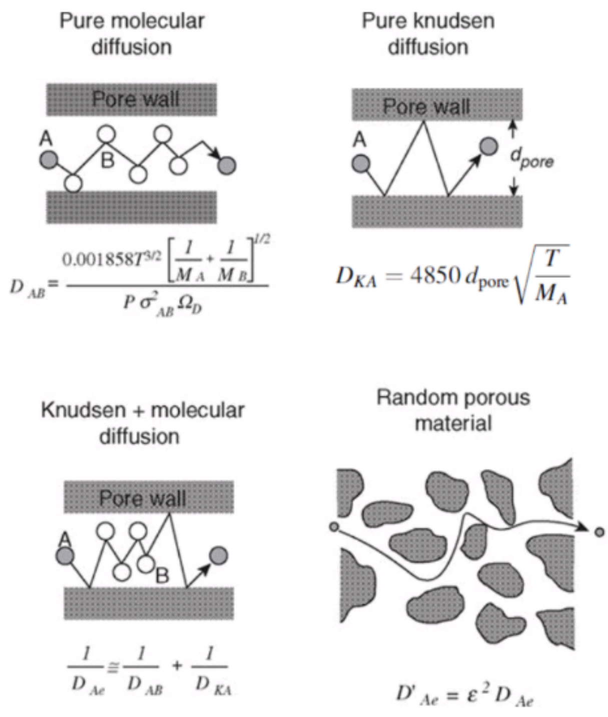
* non -Cylinder Pore *Actual case

* اكي مايدات

$$\varepsilon = \frac{P}{\underset{(s+P)}{\text{Total solid}}}$$

Void fraction

$$D'_{Ae} = \varepsilon^2 D_{Ae}$$



Convection

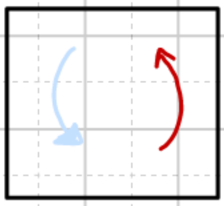
Natural convection

بسبب Δص

CΔ

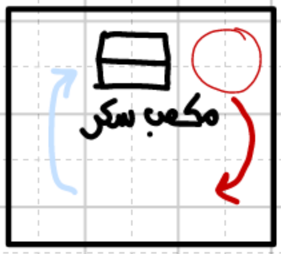
Forced convection

mixure
بكون في جهاز



■ density↓ T↑
■ density↑ T↓

صار عنى تيار
Convection



Concentration ↑
رح بنزل لتحت ص↑
تيار.

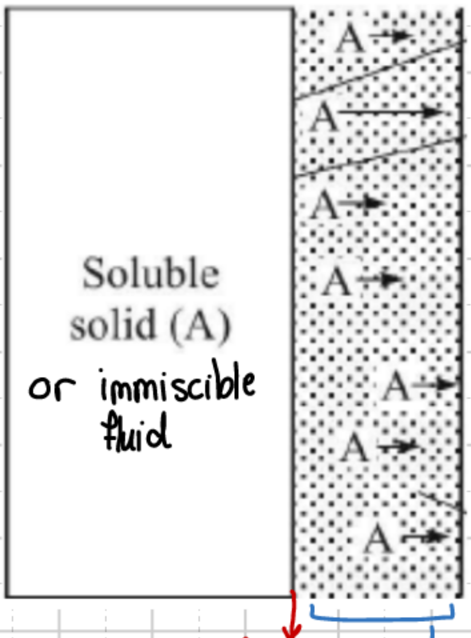
Natural Convection ✓

Concentration↑ ص↑
هو اصلا تحت فارح يعمل تيار

natural convection ✗

* Conection اسرع Diffusion
نقدر اتحكم فيه

* forced اسرع natural



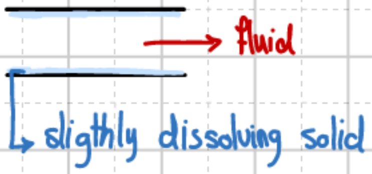
عند ال surface عندي Diff

Convection.

C_{AS} : → Solubility
Vaper prussure solid materal [volite solid]

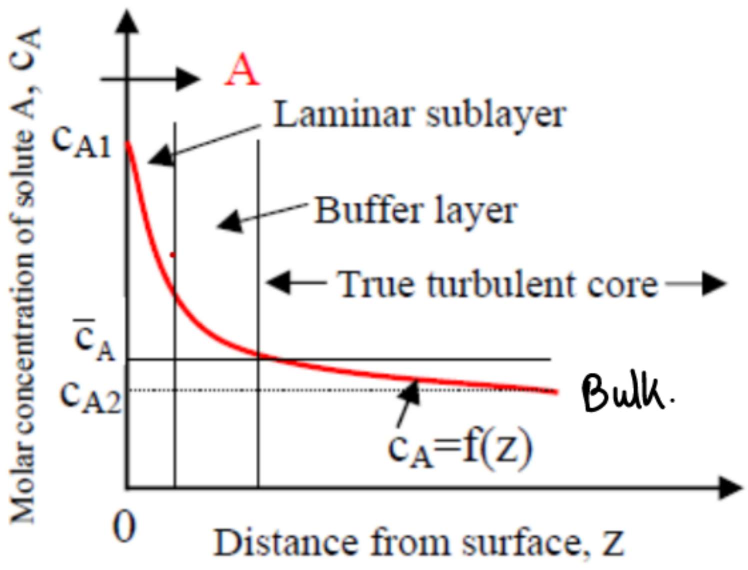
C_{AB} : → Bulk ممكن تكون ٥ ازا
كان system dilutte

* Example ①



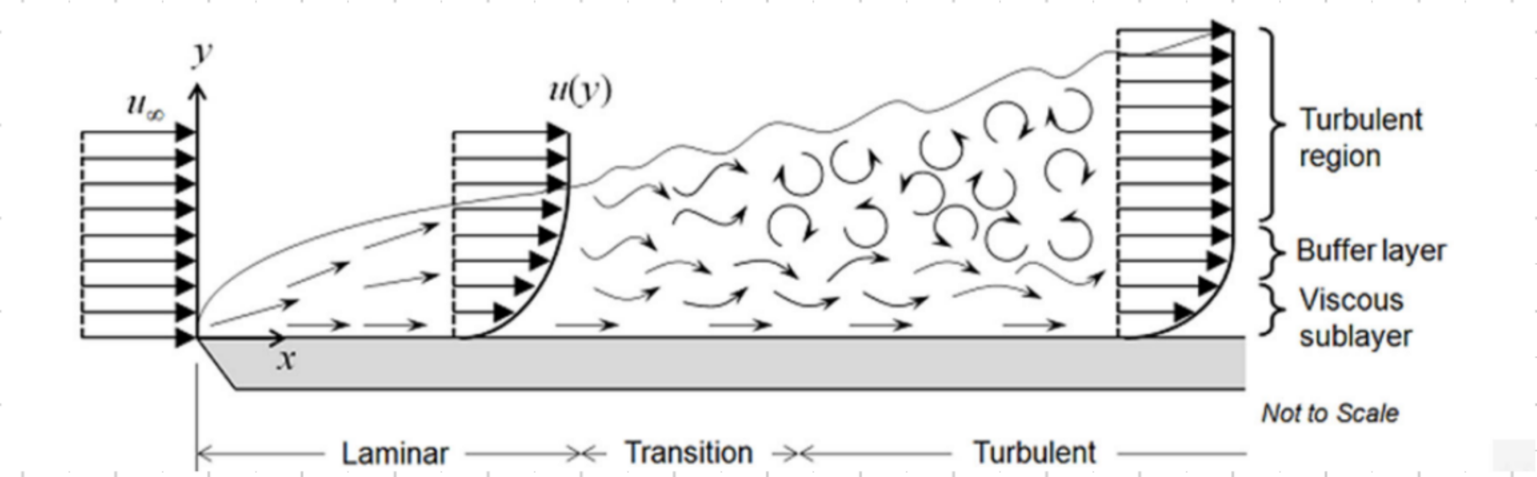
An example is a fluid flowing in a pipe; where part of the pipe wall is made by a slightly dissolving solid material such as benzoic acid.

- The benzoic acid dissolves and is transported perpendicular to the main stream from the wall.
- When a fluid is in turbulent flow and is flowing past a surface, the actual velocity of small particles of fluid cannot be described clearly as in laminar flow.
- In turbulent motion there are no streamlines, but there are large eddies or "chunks" of fluid moving rapidly in seemingly random fashion.



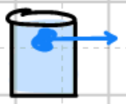
C_{A1} : عند ال surface Highest

Laminar: Diffusion
Turbulant: Convection

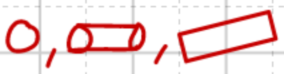


Boundary Layer

↳ Concentration Gradiant

★ Convection → Two phase
→ one phase  صبر (L-L) masstransfer.

★ mass transfer coefficient: $k_c' = \frac{\text{molar flux}}{\text{concentration D.F}}$

● function of
1. Geometry 
2. velocity
3. properites

$n_A = A k_c' (C_{AS} - C_A)$

$\frac{n_A}{A} = N_A = \text{flux}$

$N_A = k_c' (C_{AS} - C_A)$

$k_c' = \frac{N_A}{C_{AS} - C_A}$ # $\frac{mol}{m^2 s} \cdot \frac{m^3}{mol} = \boxed{\frac{m}{s}}$

unite kc لازم تكون زابطة مع DF unite

★ $1/k_c' = \text{Resistance}$

↑k_c' ↑flux ↓Resistance

★ $\text{flux} = \frac{DF}{R} = \frac{C_{AS} - C_A}{1/k_c'} = k_c' (C_{AS} - C_A)$

✩ ideal Gas

$$PV = nRT$$

$$C = \frac{n}{V} = \frac{P}{RT}$$

$$\therefore C_1 = \frac{P_1}{RT} \qquad \therefore C_2 = \frac{P_2}{RT}$$

$$N_A = k_c' \left[\frac{P_1}{RT} - \frac{P_2}{RT} \right]$$

$$N_A = \frac{k_c'}{RT} \left[P_1 - P_2 \right]$$

$$\frac{k_c'}{RT} = k_G'$$

$$\therefore N_A = k_G' \left[P_1 - P_2 \right] \#$$

$$\frac{m \quad k \quad mol}{s\,k \quad J} \qquad \frac{m \;\cancel{k}\; mol \;\; m^2 \cancel{s^2}}{\cancel{s} \cdot \cancel{k} \cdot kg} \text{①} = \boxed{\frac{m^3\,s\,mol}{kg}} \qquad J = kg \cdot m^2 \cdot s^{-2}$$

$$\frac{\cancel{m} \quad \cancel{k} \quad mol}{s\,\cancel{k} \quad Pa\,\cancel{m^3}} \text{②} \qquad \boxed{\frac{mol}{m^2\,Pa\,s}} \qquad J = Pa \cdot m^3$$

note $R \equiv \frac{J}{k \cdot mol}$

* equimolar Counter diff ⇒ $\boxed{1}$ k_G', k_c', k_y'

* diffusing throgh stagnant ⇒ ☒ k_G, k_c, k_y

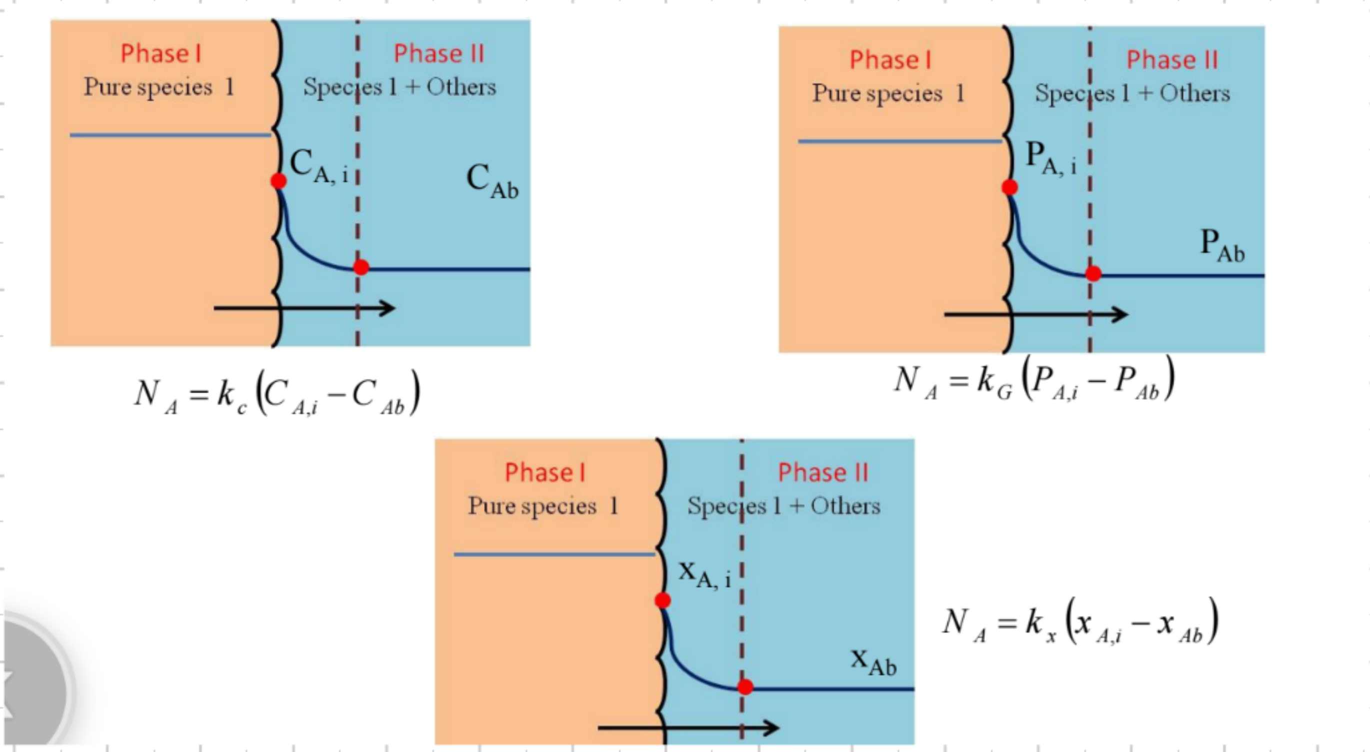
✱ Total concentration constant:

$$N_A = k_c' \, C_T \, (y_1 - y_2)$$

$$N_A = k_y (y_1 - y_2)$$

$$\frac{m}{s} \times \frac{mol}{m^3} = \boxed{\frac{mol}{m^2 \cdot s}}$$

✱ $k_c' = k_G' \times RT = k_y' \, C_T$



$$N_A = k_c (C_{A,i} - C_{Ab})$$

$$N_A = k_G (P_{A,i} - P_{Ab})$$

$$N_A = k_x (x_{A,i} - x_{Ab})$$

Diffusion throgh stagnant non diffusing B:

$$N_A = J_{AB}^{*} + x_A \left[N_A + \underset{0}{\cancel{N_B}} \right]$$

$$N_A - x_A N_A = -D_{AB}\, C_T \frac{dx}{dz}$$

$$N_A (1 - x_A) = -D_{AB}\, C_T \frac{\Delta x}{\Delta}$$

$$N_A\, dz = \frac{-D_{AB}\, C_T\, dx_A}{(1 - x_A)}$$

$$N_A\, \Delta z = D_{AB}\, C_T \ln \frac{1 - x_{A2}}{1 - x_{A1}}$$

$$N_A = \frac{D_{AB}\, C_T \ln \frac{x_{B2}}{x_{B1}}}{\Delta z}$$

$$\text{or} \quad x_{B1m} = \frac{x_{B2} - x_{B1}}{\ln \frac{x_{B2}}{x_{B1}}}$$

$$\therefore x_{B2} = 1 - x_{A2}$$

$$\therefore x_{B1} = 1 - x_{A1}$$

$$x_{B1m} \quad \frac{1 - x_{A2} - 1 + x_{A1}}{\ln x_{B2}/x_{B1}} = \frac{x_{A1} - x_{A2}}{\ln \frac{x_{B2}}{x_{B1}}}$$

$$\ln \frac{x_{B2}}{x_{B1}} = \frac{x_{A1} - x_{A2}}{x_{B1M}}$$

$$N_A = \frac{D_{AB}\, C_T \left[x_{A1} - x_{A2} \right]}{\Delta z \quad x_{B1M}.} \quad \rightarrow \quad k_x = \frac{D_{AB}\, C_T}{\Delta z\, x_{B1M}} \quad \#$$

* for Liquid

$$C_T = \left[\frac{\rho}{M}\right]_{av.}$$

$$N_A = \frac{D_{AB}\, C_T\, [x_{A1} - x_{A2}]}{\Delta z \;\; x_{BIM}.}$$

$$N_A = \frac{D_{AB}\, [x_{A1} - x_{A2}]}{\Delta z \;\; x_{BIM}.} \times \left[\frac{\rho}{M}\right]_{av} \qquad k_x = \frac{D_{AB}}{\Delta z\, x_{BIM}} \left[\frac{\rho}{M}\right]_{av}$$

Equimolar counter Diffusion

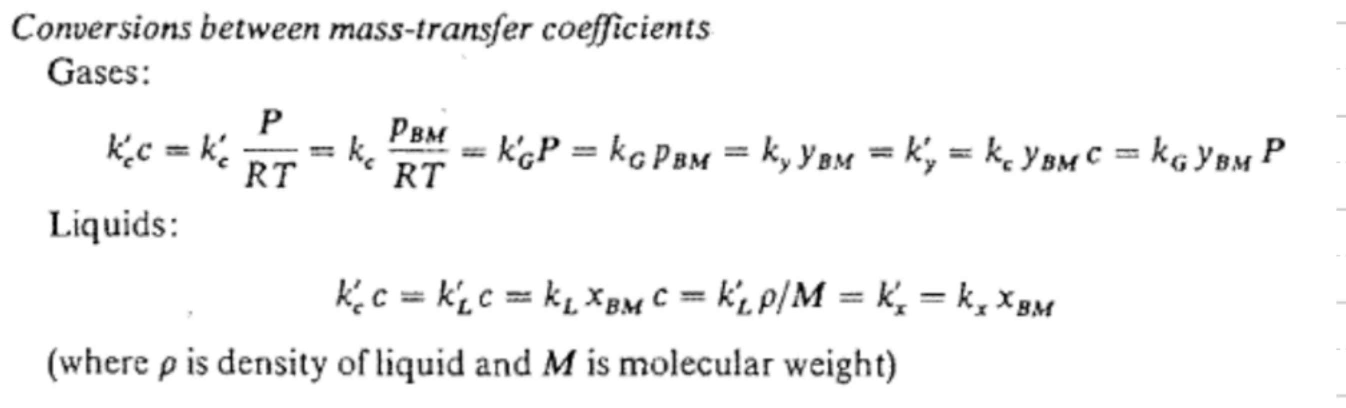
$$N_A = J^*_{AB} + x_A [N_A + N_B]$$

$$N_A = -N_B$$

$$N_A = J^*_{AB} + x_A [\cancel{N_A - N_A}]$$

$$N_A = -D_{AB}\, C_T \frac{\Delta x}{\Delta z} \rightarrow k_x' = \frac{D_{AB}\, C_T}{\Delta z} \;\#$$

$$\star \quad k_x' = k_x\, x_{BIM}$$



Conversions between mass-transfer coefficients

Gases:

$$k'_c c = k'_c \frac{P}{RT} = k_c \frac{p_{BM}}{RT} = k'_G P = k_G\, p_{BM} = k_y\, y_{BM} = k'_y = k_c\, y_{BM}\, c = k_G\, y_{BM}\, P$$

Liquids:

$$k'_c\, c = k'_L\, c = k_L\, x_{BM}\, c = k'_L\, \rho/M = k'_x = k_x\, x_{BM}$$

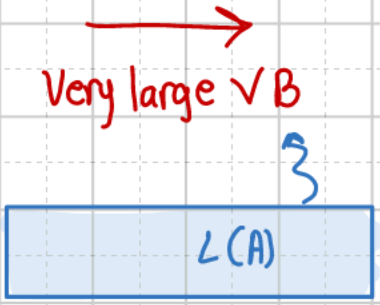
(where ρ is density of liquid and M is molecular weight)

$$k_G = \frac{k_c}{RT} \qquad k_y = k_c\, C_T \qquad k_y = k_G\, P_T \qquad k' = k_x\, y_{BIM}$$

[illegible]

Vaporizing A and Convective Mass Transfer

A large volume of pure gas B at 2 atm pressure is flowing over a surface from which pure A is vaporizing. The liquid A completely wets Pure. the surface, which is a blotting paper. Hence, the partial pressure of A at the surface is the vapor pressure of A at 298 K, which is 0.20 atm. The k'_y has been estimated to be 6.78×10^{-5} kg mol/s·m² . mol frac. Calculate N_A, the vaporization rate, and also the value of k_y and k_G.



$y_A = 0$

$y_{AS} = \frac{P_{sat}}{P_T} \quad \frac{0.2}{2} = 0.1$

$k'_y = 6.78\times10^{-5} \frac{kg\,mol}{s.\,m^2}$

$y_{A1}: 0.1 \quad y_{A2}: 0$

$y_{B1}: 0.9 \quad y_{B2}: 1$

$y_{BIM} = \frac{0.1}{\ln\frac{1}{0.9}} = 0.95$

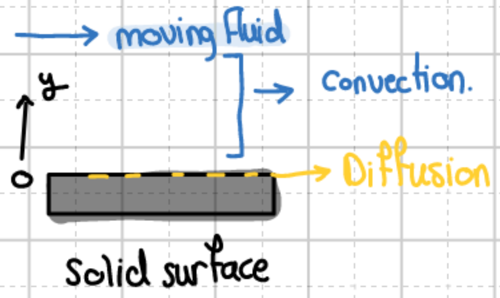
$k_y = \frac{k'_y}{x_{BIM}} = 7.14\times10^{-5} \frac{kg\,mol}{s.\,m^2}$

$k_G = \frac{k_y}{P} = 3.57\times10^{-5} \frac{kg\,mol}{s.\,m^2\,atm}$

$N_A = \frac{D_{AB}\,C_T\,[x_{A1} - x_{A2}]}{\Delta z\ x_{BIM}}$

$7.14\times10^{-5} \times 0.1 = 7.14\times10^{-6} \frac{kmol}{m^2.\,s}$

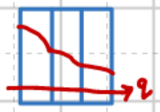
* sherwood num زي Nu بالهيت.



at steady state N_A : Constant

Diffusion flux = Convection flux

زي فكرة بالهيت وترانسى ١



$q_1 = q_2 = q_3$

- $N_A = k_c' [C_{As} - C_{A\infty}]$

at the surface : Diffusion

- $N_A = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0}$
- $-D_{AB} \left.\frac{d\,C_A - C_{As}}{dy}\right|_{y=0} = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0}$ C_{As} is constant.

$$\left[d\frac{C_A}{dy} - \underset{0}{\cancel{\frac{dC_{As}}{dy}}} \right] \rightarrow \text{مشتقة الثابت} = 0$$

- $-D_{AB} \left.\frac{d\,C_A - C_{As}}{dy}\right|_{y=0} = k_c [C_{As} - C_{A\infty}]$

$$\left[\frac{k_c'}{-D_{AB}} = \frac{\left.\frac{d\,C_A - C_{As}}{dy}\right|_{y=0}}{[C_{As} - C_{A\infty}]} \right] \times L \text{ : characteristic length.}$$

$$\frac{k_c' L}{-D_{AB}} = \frac{\left.\frac{d\,C_A - C_{As}}{dy}\right|_{y=0}}{[C_{As} - C_{A\infty}]/L} = Sh : \frac{\text{Total mass trans}}{\text{mass trans by molecular diff}} \text{ or } \frac{\text{Molecular Resicstance}}{\text{Convection Resistance.}}$$

- momentum diffusivity $\nu = \frac{\mu}{\rho}$
- Thermal diffusivity $= \frac{k}{\rho C_p}$
- mass diffusivity D_{AB}

- $Pr = \frac{\text{momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$
- $Sc = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}} = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$

☆ $k_c' = f(\underset{\text{Diameter}}{D}, D_{AB}, \rho, \mu, \underset{\text{velocity}}{\bar{U}_x})$

☆ $Sh = \alpha Re^{\beta} Sc^{\gamma}$

using Pi Theorm.

ازا بدي ادرسهم

صعب اثبت ٤ و اغير ①

وارجع اعيد و احيد عشان

صيغ بستخدم ال Dimansionless number

where

Sh = Sherwood number = kL/D_{AB}

Re$_L$ = Reynolds number = $L\upsilon\rho/\mu$

Sc = Schmidt number = $\mu / (\rho D_{AB})$

k = overall mass transfer coefficient

L = length of sheet

D_{AB} = diffusivity of A in B

υ = velocity

μ = viscosity

ρ = density, and

μ/ρ = kinematic viscosity.

Γ is the mass rate of liquid flow per unit of film width in the x direction.

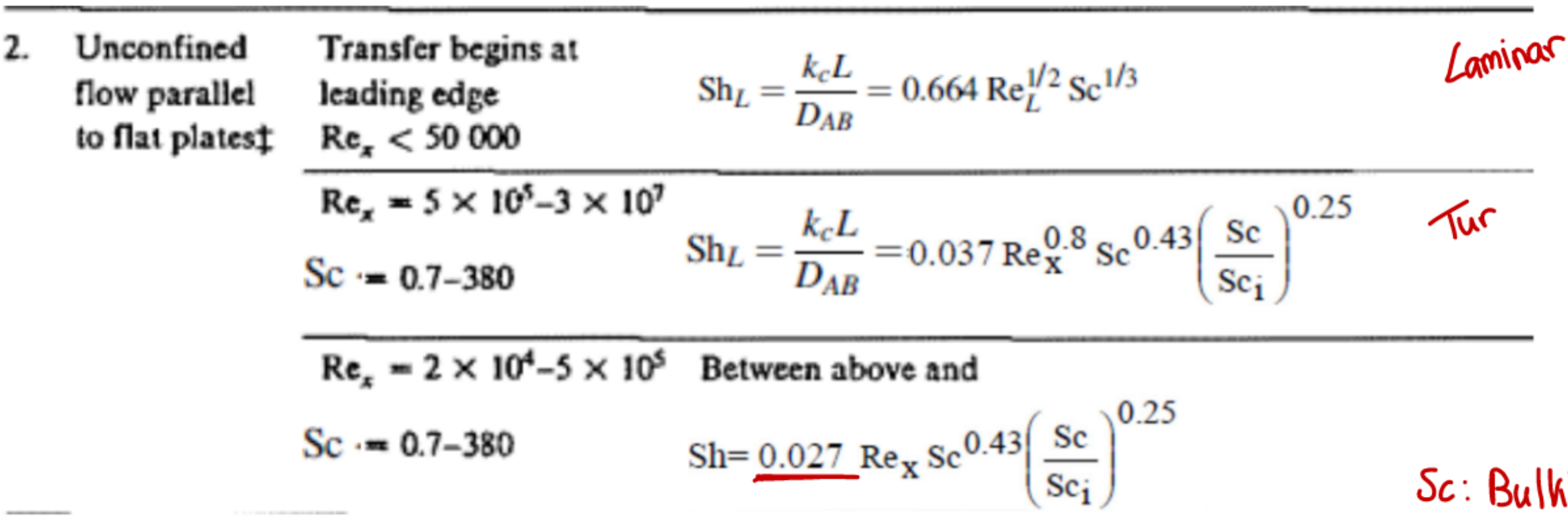
$$j_D = \frac{k_c}{v_\infty}(Sc)^{2/3}$$ *J* factor for mass transfer

$$\mathrm{St} = \mathrm{Sh}/\mathrm{Re\,Sc} = \frac{k_c}{v}$$ The Stanton number $= \frac{\text{Total mass transfer rate}}{\text{Inertia forces}}$

ci

$$\mathrm{Pe} = \mathrm{Re\,Sc} = \frac{\ell v \rho}{\mu}\frac{\mu}{\rho D} = \frac{\ell v}{D}$$ Peclet number $= \frac{\text{Inertia forces}}{\text{Mass transfer by molecular diffusion}}$

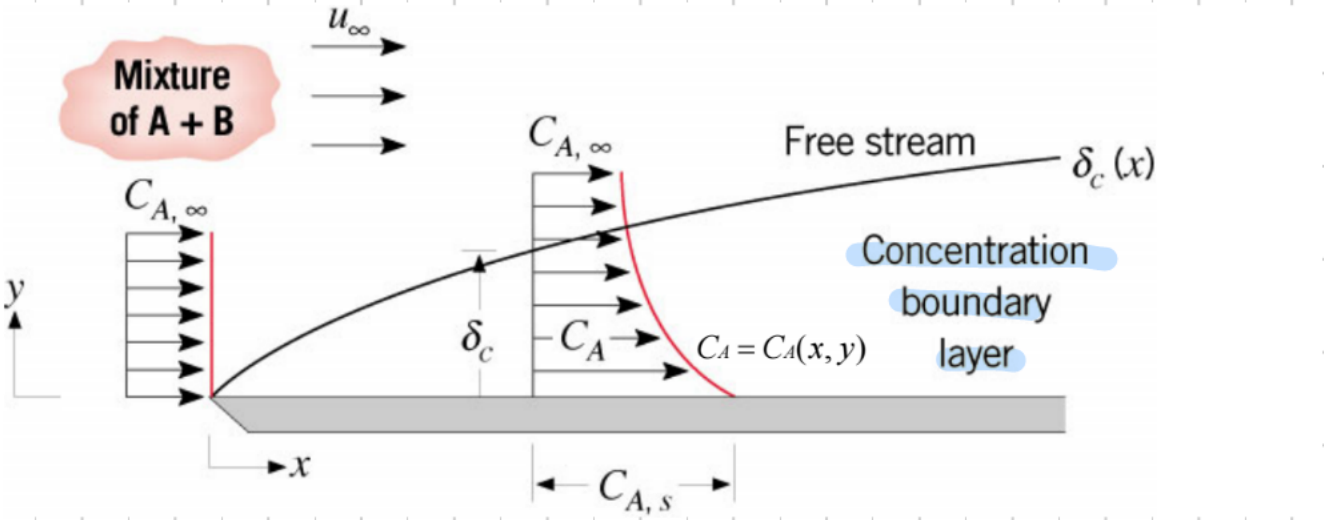
Parallel flat plate



2.	Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $Re_x < 50\,000$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.664\, Re_L^{1/2}\, Sc^{1/3}$	Laminar
		$Re_x = 5 \times 10^5 – 3 \times 10^7$ $Sc = 0.7–380$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.037\, Re_x^{0.8}\, Sc^{0.43} \left(\frac{Sc}{Sc_i}\right)^{0.25}$	Tur
		$Re_x = 2 \times 10^4 – 5 \times 10^5$ $Sc = 0.7–380$	Between above and $Sh = \underline{0.027}\, Re_x\, Sc^{0.43} \left(\frac{Sc}{Sc_i}\right)^{0.25}$	

Sc: Bulk Temp

Sci: plate temp



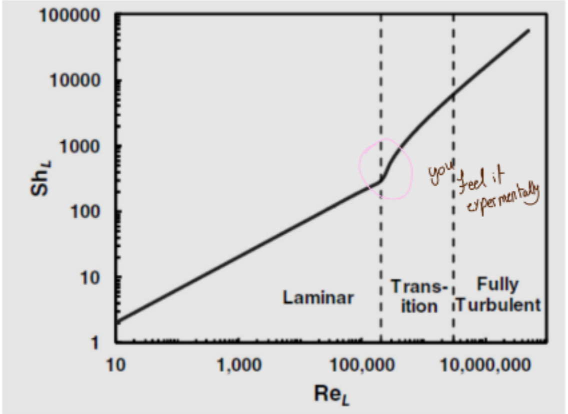
start Laminar → Turbulent [Plate is long enough]

$$\bar{k}_c = \frac{\int_0^{L_t} k_{c\,lam}\, dx + \int_{L_t}^{L} k_c\, dx}{L}$$

L_t: الطول لما بصير Turbulent

- Then The average Sherwood over the entire plate is

$$\frac{k_c L}{D_{AB}} = Sh_L = 0.664 (Re_t)^{1/2} Sc^{1/3} + 0.0365\, Sc^{1/3} \left[\underset{L}{\underline{(Re_L)}}^{4/5} - \underset{L_t}{\underline{(Re_t)}}^{4/5} \right]$$



Average Sherwood number (Sh.) at Sc =1.0 for flow over a flat plate, showing transition from laminar to fully turbulent regimes,

- Experimental data for liquids are correlated within about ± 40% by the following for

$$J_D = 0.99 N_{Re,L}^{-0.5} \qquad 600 < \text{Sc} < 50000$$

3. Confined gas flow parallel to a flat plate in a duct $\text{Re}_e = 2600\text{–}22\,000$ $\quad j_D = 0.11\,\text{Re}_e^{-0.29}$

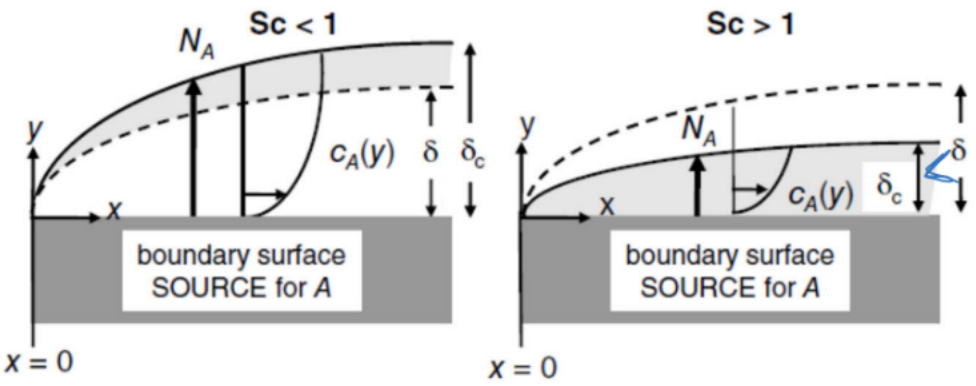
Convective Mass Transfer Correlations: Flat Plate

- For laminar flow over a flat plate, the thickness of the laminar hydrodynamic boundary layer is

$$\frac{\delta_\eta}{x} = \frac{5}{\sqrt{\text{Re}_x}}$$

- And thus, the concentration boundary layer thickness for laminar flow over a flat plate can be estimated by

$$\delta_c = 5\text{Sc}^{1/3}\sqrt{\frac{vx}{V\infty}}$$



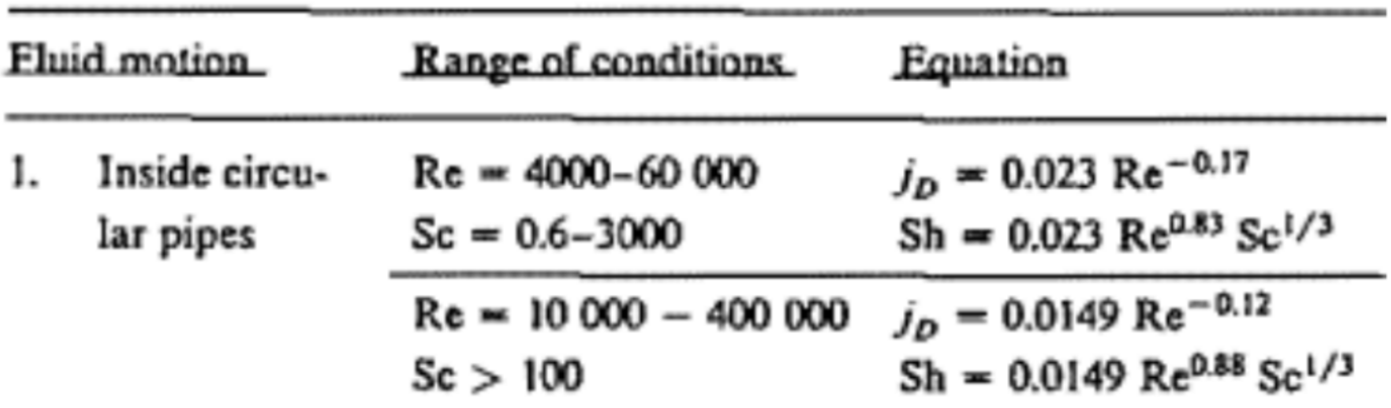
$$\frac{\delta_\eta}{\delta_c} = Sc^{1/3}$$

hydrodynamic B.L → mass transfer B.L

Sc = 1 hydrodynamic B.L ≡ mass transfer B.L

- If Sc < 1, then the concentration profile continues to develop beyond the hydrodynamic boundary layer.
- If Sc = 1, then the concentration and hydrodynamic boundary layers are of the same thickness.
- If Sc > 1, then the concentration boundary layer resides within the hydrodynamic boundary layer (as for convective mass transfer into flowing liquids).

Flow Inside Circular Pipes



Fluid motion	Range of conditions	Equation
1. Inside circular pipes	$Re = 4000–60\,000$ $Sc = 0.6–3000$	$j_D = 0.023\ Re^{-0.17}$ $Sh = 0.023\ Re^{0.83}\ Sc^{1/3}$
	$Re = 10\,000 – 400\,000$ $Sc > 100$	$j_D = 0.0149\ Re^{-0.12}$ $Sh = 0.0149\ Re^{0.88}\ Sc^{1/3}$

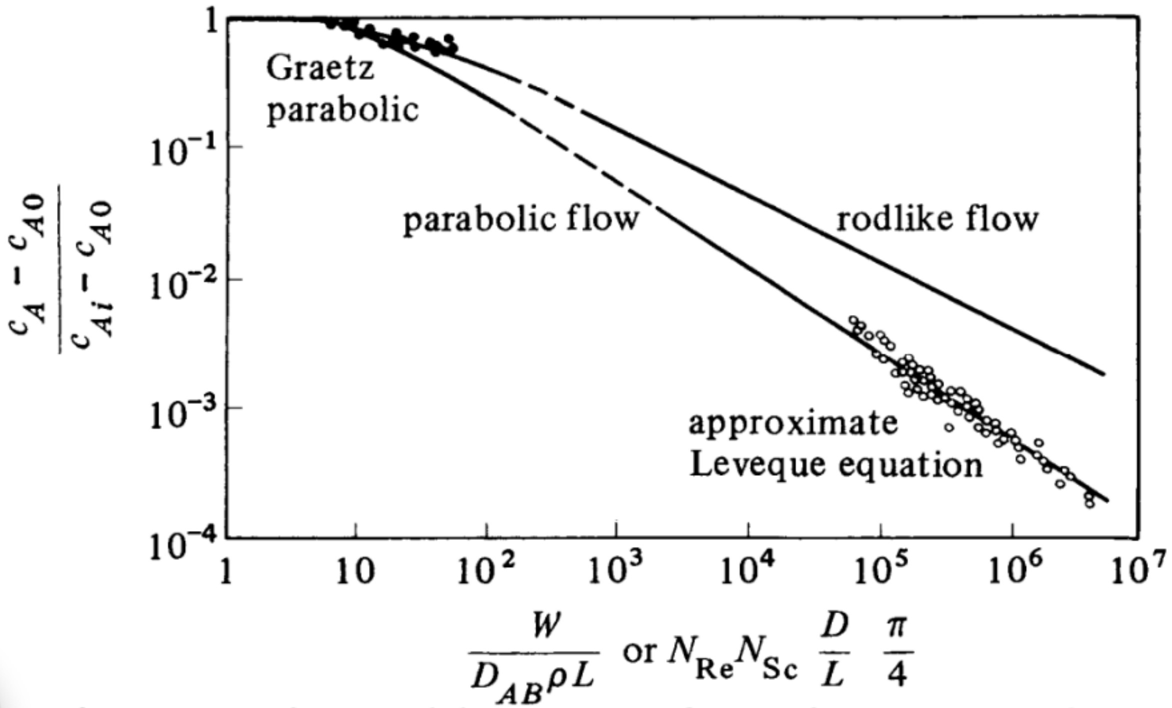
For *laminar* flow of a fluid through a tube, with a Reynolds number range of $10 < Re < 2000$, the appropriate mass-transfer correlation is

$$Sh = 1.86\left(\frac{v_\infty D^2}{LD_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}\frac{v_\infty D}{\nu}\cdot\frac{\nu}{D_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}Re\,Sc\right)^{1/3} \qquad (30\text{-}19)$$

where L the length of the pipe and v_∞ is the bulk average velocity.

Flow Inside Circular Pipes

Figure 7.3-2: Mass transfer from the inner walls of a pipe to gases **Re < 2100** and **0.5 < Sc < 3.0**



c_A is the exit concentration, c_{A0} inlet concentration, and c_{Ai} concentration at the interface between the wall and the gas. The dimensionless abscissa is $W/D_{AB}\rho L$ or $N_{Re}N_{Sc}(D/L)(\pi/4)$, where W is flow in kg/s and L is length of mass-transfer section in m.

Liquid film in Wetted Wall

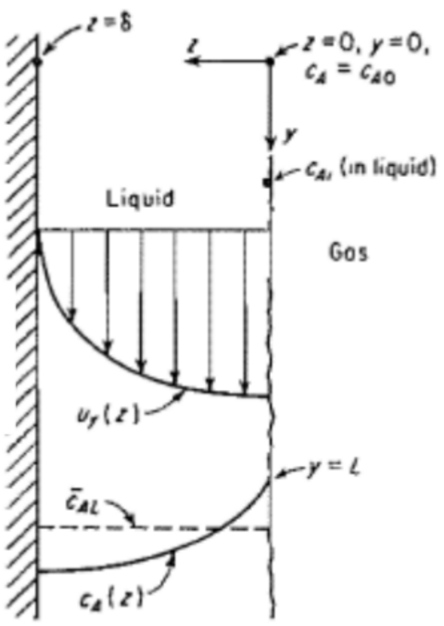
4. Liquid film in wetted-wall tower, transfer between liquid and gas

Re = 0–1200, $k_{L,\,av} = \left(\frac{6D_{AB}\Gamma}{\pi\rho\delta L}\right)^{1/2}$

$\mathrm{Re} = 4\Gamma/\mu$ less than 100 $\mathrm{Sh}_{av} \approx 3.41$

$3{,}000 < \mathrm{Re}' < 40{,}000;\ 0.5 < \mathrm{Sc} < 3$ $j_D = 0.0328(\mathrm{Re}')^{-0.23}$

$\mathrm{Re} = \frac{4\Gamma}{\mu} = 1300\text{–}8300$ $\mathrm{Sh} = (1.76 \times 10^{-5})\left(\frac{4\Gamma}{\mu}\right)^{1.506} \mathrm{Sc}^{0.5}$



The film thickness is then

$$\delta = \left(\frac{3\bar{u}_y\mu}{\rho g}\right)^{1/2} = \left(\frac{3\mu\Gamma}{\rho^2 g}\right)^{1/3}$$

The Reynolds number of the liquid flowing down the tube is defined as

$$\mathrm{Re}_L = \frac{4\Gamma}{\mu_L} = \frac{4w}{\pi D\mu_L}$$

where w is the mass flow rate of liquid, D is the inner diameter of the cylindrical column, and Γ is the mass flow rate of liquid per unit wetted perimeter of the column.

or the mass rate of liquid flow per unit of film width in the x direction.

Past Single Sphere

For mass transfer into liquid streams, the equation of Brian and Hales[4]

$$\mathrm{Sh} = \frac{k_L D}{D_{AB}} = \left(4 + 1.21\,\mathrm{Pe}_{AB}^{2/3}\right)^{1/2} \qquad Pe_{AB} = \mathrm{Re}\,Sc$$

correlates data where the mass-transfer Peclet number, Pe_{AB}, is less than 10,000. For Peclet numbers greater than 10,000, Levich[5] recommends the simpler relationship

$$\mathrm{Sh} = \frac{k_L D}{D_{AB}} = 1.01\,\mathrm{Pe}_{AB}^{1/3}$$

- Also, for liquid

$$N_{\mathrm{Sh}} = 2 + 0.95 N_{\mathrm{Re}}^{0.50} N_{\mathrm{Sc}}^{1/3} \qquad 2 < \mathrm{Re} < 2000$$

$$N_{\mathrm{Sh}} = 0.347 N_{\mathrm{Re}}^{0.62} N_{\mathrm{Sc}}^{1/3} \qquad 2000 < \mathrm{Re} < 17000$$

For mass transfer into gas streams, the Fröessling equation (evaporation and sublimation)

$$\mathrm{Sh} = \frac{k_c D}{D_{AB}} = 2 + 0.552\,\mathrm{Re}^{1/2}\,\mathrm{Sc}^{1/3} \qquad 2 < \mathrm{Re} < 48000,\ 0.6 < \mathrm{Sc} < 2.7$$

Past Single Sphere

- For drops of liquid falling through a gas, the mass transfer coefficient to and from drops may be approximately represented by

$$\mathrm{Sh} = 1.13\,\mathrm{Re}^{1/2}\,\mathrm{Sc}^{1/2} \qquad d_p > 3.0\text{ cm},\quad 500 < \mathrm{Re} < 2000$$

- Bubbles behave very much like drops, but their buoyancy and velocity of rise are much higher. Mass transfer within bubbles is relatively rapid since bubbles are filled with gas and molecular diffusion in gases is high. the following empirical equation is proposed by Johnson et al. (1969) gives a more reliable prediction

$$Sh = 1.13\,Pe\left(\frac{d_e}{0.45 + 0.2 d_e}\right)^{1/2} \qquad 0.6 < d_p < 4.0\text{ cm},\quad 500 < \mathrm{Re} < 2000$$

Past Single Sphere

- Colombet et al. (2013) proposed the following relation that is valid for a spherical bubble whatever the value of Re and Pe,

$$Sh = 1 + \left[1 + \left(\frac{4}{3\,\pi}\right)^{2/3} (2\,Pe_{max})^{2/3}\right]^{3/4},$$

where Pe_{max} is the Péclet number based on the maximal velocity U_{max} of the liquid at the interface instead of the bubble velocity V_z, which is obtained from the correlation proposed by Legendre (2007),

$$\frac{U_{max}}{V_z} = \frac{1}{2}\frac{16 + 3.315Re^{1/2} + 3Re}{16 + 3.315Re^{1/2} + Re}$$

- For very low Reynold’s number, the Sherwood number should approach a value of 2.

$$\frac{k'_c\, D_p}{D_{AB}} = N_{Sh} = 2.0$$

Past Single Sphere

The following correlation of Steinberger and Treybal[8] is recommended when the transfer occurs in the presence of natural convection

$$\mathrm{Sh} = \mathrm{Sh}_o + 0.347(\mathrm{Re\,Sc}^{1/2})^{0.62}$$

where Sh_o is dependent on Gr Sc

$$\mathrm{Sh}_o = 2 + 0.569(\mathrm{GrSc})^{0.25} \qquad \mathrm{Gr\,Sc} \le 10^8$$

$$\mathrm{Sh}_o = 2 + 0.0254(\mathrm{GrSc})^{1/3}\,(\mathrm{Sc})^{0.244} \qquad \mathrm{Gr\,Sc} \ge 10^8$$

the Grashof number is defined as

بدل Re number ←

$$\mathrm{Gr} = \frac{D^3 \rho g\, \Delta\rho}{\mu^2}$$

where density, ρ, and viscosity, μ, are taken at the bulk conditions of the flowing fluid, and $\Delta\rho$ is the positive density difference between the two phases in contact. The prediction for Sh is valid when $2 \le \mathrm{Re} \le 3 \times 10^4$ and $0.6 \le \mathrm{Sc} \le 3200$.

Past Single Sphere

- for the mass-transfer coefficient associated with the transfer of a sparingly soluble gaseous solute A into solvent B by a swarm of gas bubbles in a natural convection process, the following correlation are used:

For gas bubble diameters (d_b) less than 2.5 mm, use

$$\mathrm{Sh} = \frac{k_L d_b}{D_{AB}} = 0.31\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/3}$$

For bubble diameters greater or equal to 2.5 mm, use

$$\mathrm{Sh} = \frac{k_L d_b}{D_{AB}} = 0.42\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/2}$$

In the above correlations, the Grashof number is defined as

$$\mathrm{Gr} = \frac{d_b^3 \rho_L g\,\Delta\rho}{\mu_L^2}$$

where $\Delta\rho$ is the difference of the density of the liquid and the density of the gas inside the bubble, with density (ρ_L) and viscosity (μ_L) determined at the bulk average properties of the liquid mixture. For dilute solutions, the fluid properties of the solvent approximate the fluid properties of the liquid mixture. The diffusion coefficient D_{AB} is with respect to dissolved gaseous solute A in solvent B.

Perpendicular to Single Cylinders

5.	Perpendicular to single cylinders	$\mathrm{Re} = 400\text{–}25\,000$ $\mathrm{Sc} = 0.6\text{–}2.6$	$\dfrac{k_G P (Sc)^{0.56}}{G_M} = \dfrac{k_c (Sc)^{0.56}}{v_\infty} = 0.281(\mathrm{Re}_D)^{-0.4}$
		$\mathrm{Re}' = 0.1\text{–}10^5$ $Sc = 0.7\text{–}1500$	$Sh = (0.35 + 0.34\,Re^{0.5} + 0.15 Re^{0.58}) Sc^{0.3}$

P is the system total pressure and G_M is the superficial molar velocity of the gas flowing normal to the cylinder in units of kg mol/m$^2 \cdot$ s. The Reynolds number for flow normal to a solid cylinder, Re_D, is defined as

$$\mathrm{Re}_D = \frac{\rho v_\infty D}{\mu}$$

where D is cylinder diameter, v_∞ is the fluid velocity normal to the solid cylinder, and ρ and μ for the gas stream evaluated at the film average temperature.

Perpendicular to Single Cylinders

- Experimental data have been obtained for mass transfer from single cylinders when the flow is perpendicular to the cylinder. The cylinders are long and mass transfer to the ends of the cylinder is not considered.

$$J_D = 0.600(N_{\mathrm{Re}})^{-0.487}$$

$$\left.\begin{array}{l}\textbf{For gases } 0.6 < \mathrm{Sc} < 2.6 \\ \textbf{For liquid } 1000 < \mathrm{Sc} < 3000\end{array}\right\} \; 50 < \mathrm{Re} < 50000$$

Mass Transfer to Small Particles

Mass transfer to small particles <0.6 mm

- The following equation has been shown to hold to predict mass-transfer coefficients from small gas bubbles such as oxygen or air to the liquid phase or from the liquid phase to the surface of small catalyst particles, microorganisms, other solids, or liquid drops

$$k'_L = \underbrace{\frac{2D_{AB}}{D_p}}_{\substack{\text{the molecular}\\ \text{diffusion term}}} + \underbrace{0.31N_{Sc}^{-2/3}\left(\frac{\Delta\rho\mu_c g}{\rho_c^2}\right)^{1/3}}_{\substack{\text{free fall or rise of the sphere}\\ \text{by gravitational forces}}}$$

where D_{AB} is the diffusivity of the solute A in solution in m^2/s, D_p is the diameter of the gas bubble or the solid particle in m, μ_c is the viscosity of the solution in $kg/m \cdot s$, $g = 9.80665\ m/s^2$, $\Delta\rho = (\rho_c - \rho_p)$ or $(\rho_p - \rho_c)$, ρ_c is the density of the continuous phase in kg/m^3, and ρ_p is the density of the gas or solid particle. The value of $\Delta\rho$ is always positive.

Mass Transfer to Small Particles

Mass transfer to large gas bubbles > 2.5 mm.

- For large gas bubbles or liquid drops > 2.5 mm, the mass-transfer coefficient can be predicted by

$$k'_L = 0.42N_{Sc}^{-0.5}\left(\frac{\Delta\rho\mu_c g}{\rho_c^2}\right)^{1/3}$$

- Large gas bubbles are produced when pure liquids are aerated in mixing vessels and sieve-plate columns

Mass Transfer to and from Packed Beds

- Mass transfer to and from packed beds occurs often in processing operations, including
  - drying operations,
  - adsorption or desorption of gases or
  - liquids by solid particles such as charcoal, and
  - mass transfer of gases and liquids to catalyst particles.
- Using a packed bed a large amount of mass-transfer area can be contained in a relatively small volume.
- For packed and fluidized beds, the area of mass transfer is generally expressed in terms of specific interfacial area which is defined as the area per unit volume of packed bed. It can e expressed as: $a = \frac{\boxed{6}(1-\varepsilon)}{d}$

Area partical
volume

Mass Transfer to and from Packed Beds

Where ε is the porosity or void fraction and d_p is the particle diameter.

- For a Reynolds number range of 10-10000 for gases in a packed bed of spheres

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

where

$N_{Re} = D_p v' \rho / \mu$,

The void fraction in a bed is ε, range from 0.3 to 0.5

0.4

D_p is diameter of the spheres

v' is the superficial mass average velocity in the empty tube without packing.

inlet velocity

Mass Transfer to and from Packed Beds

7.	Through fixed beds of pellets§	$Re'' = 90–4000$ $Sc = 0.6$	$j_D = j_H = \frac{2.06}{\varepsilon} Re''^{-0.575}$
		$Re'' = 5000–10\,300$ $Sc = 0.6$	$j_D = 0.95 j_H = \frac{20.4}{\varepsilon} Re''^{-0.815}$
		$Re'' = 0.0016–55$ $Sc = 168–70\,600$	$j_D = \frac{1.09}{\varepsilon} Re''^{-2/3}$
		$Re'' = 5–1500$ $Sc = 168–70\,600$	$j_D = \frac{0.250}{\varepsilon} Re''^{-0.31}$

† Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.

‡ Mass-transfer data for this case scatter badly but are reasonably well represented by setting $j_D = j_H$.

§ For fixed beds, the relation between ε and d_p is $a = 6(1 - \varepsilon)/d_p$, where a is the specific solid surface, surface per volume of bed. For mixed sizes [58]

Gas-phase flow through a packed bed	$10 \le Re'' \le 2500$	$j_D = 1.17(Re'')^{-0.415}$
Liquid flow through a packed bed	$Re'' < 55$	$j_D = 1.09(Re'')^{-2/3}$
	$3 < Re'' < 10{,}000$	$Sh = 2 + 1.1(Re)^{0.6}(Sc)^{0.33}$

d = tube diameter; $Re_l = l v \rho/\mu$, l = characteristic length; $Re' = d v' \rho/\mu$, v' = gas velocity relative to the surface of the falling film; $Re'' = d_p v'' \rho/\mu$, d_p = diameter of the sphere, v'' = superficial velocity of the fluid (i.e. velocity based on the bed cross-section).

Mass Transfer in Fluidized Beds of Spheres

- For fluidized beds of spheres and for gases and liquids

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069} \qquad 10 < Re < 4000$$

$$\varepsilon J_D = 1.1068\ N_{Re}^{-0.72} \qquad 1 < Re < 10$$

- For both gas and liquid packed and fluidized bed of spherical particle, اشمل.

$$St_D Sc^{2/3} = \frac{0.010}{\varepsilon} + \frac{0.863/\varepsilon}{Re^{0.58} - 0.483} \qquad \text{where } 1 < Re < 2100$$

Mass Transfer to Packed Beds

- The total flux in a packed bed,

$$N_A A = A k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln \dfrac{c_{Ai} - c_{A1} \ \text{in}}{\underset{\text{interphase}}{c_{Ai}} - c_{A2} \ \text{out}}}$$

flow بتغير
↳ in , out
لانه محصور
↳ beds
↳ Pipe

where

c_{Ai} is the concentration at the surface of the solid, in kg mol/m^3

c_{A1} is the inlet bulk fluid concentration

c_{A2} is the outlet.

A is the total external surface area *in* m^2 $A = aV_b$ → masstransferArea

a is the m^2 surface area/m^3 total volume of bed when the solids are spheres.

$$a = \frac{6(1 - \varepsilon)}{D_p}$$

V_b is total volume *in* m^3 of the bed (void plus solids),

Mass Transfer to Packed Beds

k_c is the mass transfer coefficient obtained from correlations

- Also, the material balance equation on the bulk stream is

$$N_A A = \underset{\text{↳ Volume flowrate}}{\underbrace{V}}(c_{A2} - c_{A1})$$

where V is volumetric flow rate of fluid entering in m^3/s.

Example

A tube is coated on the inside with naphthalene and has an inside diameter of 20 mm and a length of 1.10 m. Air at 318 K and of 101.3 kPa flows through this pipe at a velocity of 0.8 m/s. Assuming that the pressure remains constant. Calculate the concentration of naphthalene in the exist air

ع الوحدة.

* Air flow + velocity ⇒ Convection

* Tube.

ν at 318K → 17.7×10^{-6}

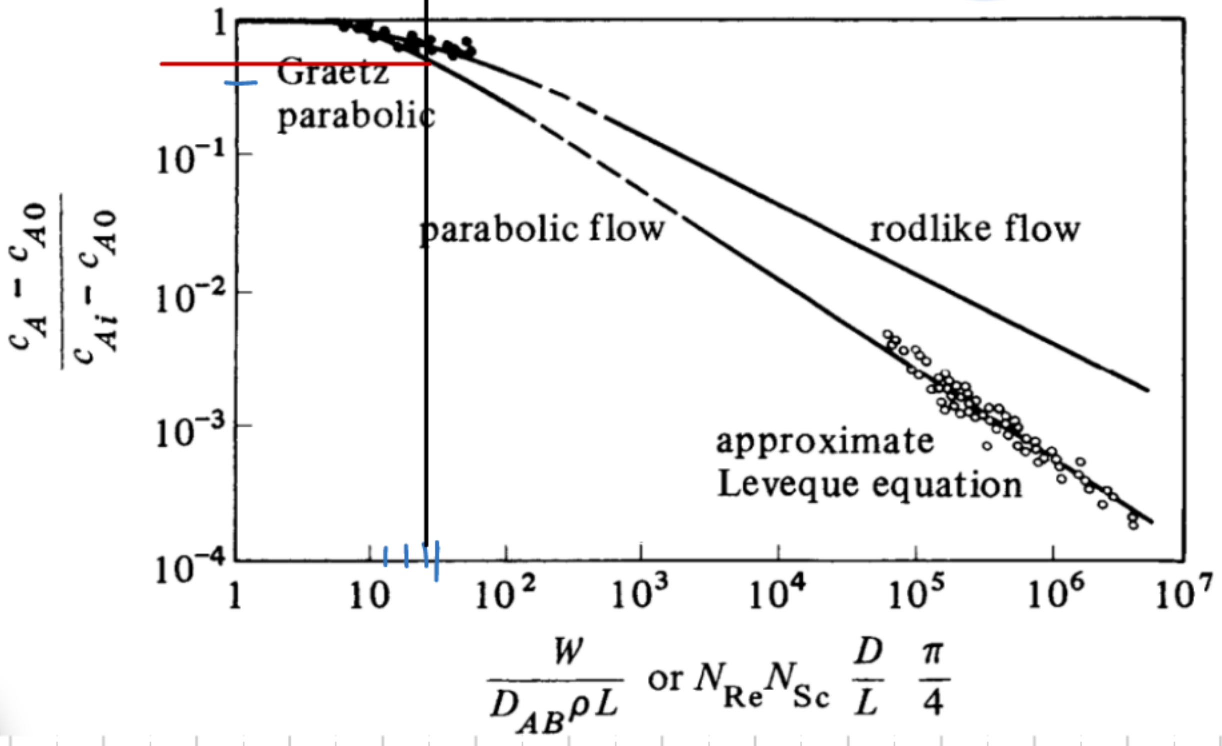
$D_{AB} = 6.92 \times 10^{-6}\ m^2/s$

$P_{Ai} = 74.0\ Pa \qquad C_{Ai} = \frac{P_{Ai}}{RT} = \frac{74}{8314.3 \times 318} = 2.8 \times 10^{-5}\ \frac{mol}{m^3}$

① $Re = \frac{u \rho D}{\mu} = \frac{0.8\ \cancel{m}}{\cancel{s}} \Big| \frac{20 \times 10^{-3}\ \cancel{m}}{17.7 \times 10^{-6}\ \cancel{m^2}} \Big| \frac{\cancel{s}}{} = 903.91$

② $Sc = \frac{\nu}{D_{AB}} \quad \frac{17.7 \times 10^{-6}\ m^2}{s} \Big| \frac{s}{6.92 \times 10^{-6}} = 2.5.$

Figure 7.3-2: Mass transfer from the inner walls of a pipe to gases **$Re < 2100$ and $0.5 < Sc < 3.0$**



* بالسؤال مو محدد نوع ال flow عادي بستخدم اي واحد.

$Re \times Sc \times \frac{D}{L}\frac{\pi}{4}$

$903.91 \times 2.5 \times \frac{20 \, 10^{-3}}{1.10} \times \frac{\pi}{4} = 33.02$

$\frac{CA - CA0}{CAi - CA0} = 0.6$

$\frac{CA - 0}{2.8 \times 10^{-5} - 0} = 0.6$

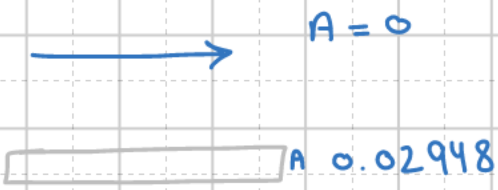
$CA = 1.68 \times 10^{-5}$ Kgmol/m^3

Example

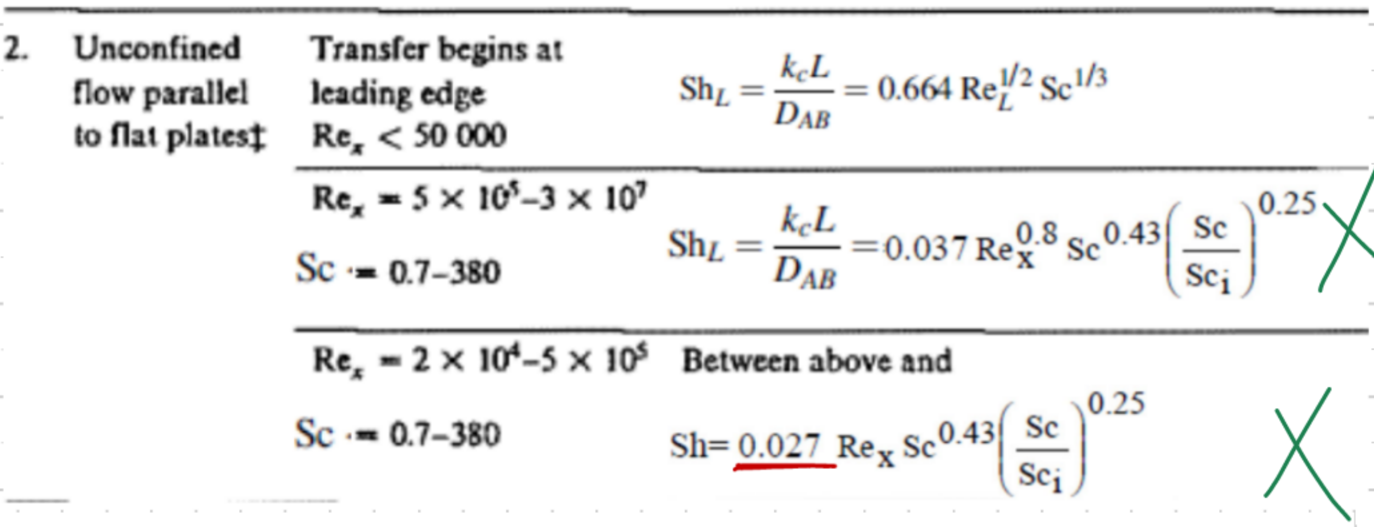
A large volume of pure water at 26.1°C is flowing parallel to a flat plate of solid benzoic acid, where $L = 0.244$ m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kg mol/m^3. The diffusivity of benzoic acid is 1.245×10^{-9} m^2/s. Calculate the mass-transfer coefficient k_L and the flux N_A.

* Flat plate

* flow + velocity → convection



① $k_L \rightarrow Sh$



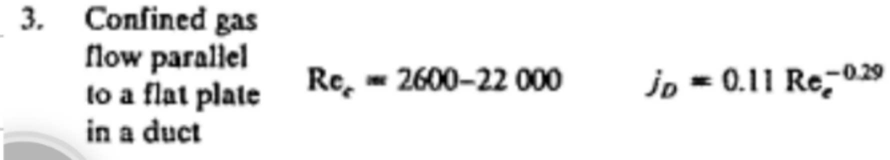
2.	Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $Re_x < 50\,000$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.664\, Re_L^{1/2}\, Sc^{1/3}$
		$Re_x = 5 \times 10^5 – 3 \times 10^7$ $Sc = 0.7–380$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.037\, Re_x^{0.8}\, Sc^{0.43}\left(\frac{Sc}{Sc_i}\right)^{0.25}$
		$Re_x = 2 \times 10^4 – 5 \times 10^5$ $Sc = 0.7–380$	Between above and $Sh = \underline{0.027}\, Re_x\, Sc^{0.43}\left(\frac{Sc}{Sc_i}\right)^{0.25}$

- Then The average Sherwood over the entire plate is

$$\frac{k_c L}{D_{AB}} = Sh_L = 0.664 (Re_t)^{1/2} Sc^{1/3} + 0.0365\, Sc^{1/3}\left[(Re_L)^{4/5} - (Re_t)^{4/5}\right]$$

- Experimental data for liquids are correlated within about ± 40% by the following for

$$J_D = 0.99 N_{Re,L}^{-0.5} \quad \boxed{600 < Sc < 50000} \checkmark$$



3.	Confined gas flow parallel to a flat plate in a duct	$Re_e = 2600–22\,000$	$j_D = 0.11\, Re_e^{-0.29}$

$$Re = \frac{\rho V L}{\mu}$$

$$\frac{996 \times 0.061 \times 0.244}{8.71 \times 10^{-4}} = 1.7 \times 10^4$$

$Sc = 702$

$J_D = 7.6 \times 10^{-3}$

$$J_D = \frac{k_c'}{V} Sc^{2/3}$$

$k_c' = 6 \times 10^{-6}$

$x_{BM} \approx 1$ very dilute

$k_c' = k_c$

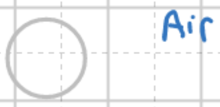
$N_A = k_c \times \Delta C$

$N_A = 6 \times 10^{-6} \times [0.0298] = 0.1788 \times 10^{-6} \ \frac{kg\,mol}{s \cdot m^2}$

Example

Calculate the value of the mass-transfer coefficient and the flux for mass transfer from a sphere of napthalene to air at 45°C and 1 atm abs flowing at a velocity of 0.305 m/s. The diameter of the sphere is 25.4 mm. The diffusivity of naphthalene in air at 45°C is 6.92×10^{-6} m²/s and the vapor pressure of solid naphthalene is 0.555 mm Hg. Use English and SI units.

* sphere



* 0.555 mmHg → 73.99 Pa

$$Re = \frac{VD}{L} = \frac{0.305 \times 0.0254 \times 1.113}{1.93 \times 10^{-5}} = 446$$

$Sc = 2.5$

For mass transfer into gas streams, the Fröessling equation (evaporation and sublimation)

$$Sh = \frac{k_c D}{D_{AB}} = 2 + 0.552\,Re^{1/2}\,Sc^{1/3} \qquad 2 < Re < 48000,\ 0.6 < Sc < 2.7$$

$Sh = 21.0$

$$Sh = \frac{k_c' D}{D_{AB}} = k_c' = 5.72 \times 10^{-3}$$

$$k_G' = \frac{k_c'}{RT} = \frac{5.72 \times 10^{-3}}{8314 \times [45 + 273]} = 2.16 \times 10^{-9}$$

$$N_A = k_G\,[P_A - P_A] = 2.16 \times 10^{-9}\,[73.99] = 1.598 \times 10^{-7}\ \frac{kmol}{m^2 \cdot s}$$

$$Area = \pi D^2 = (25.4 \times 10^{-3})^2\,\pi = 2.03 \times 10^{-3}\ m^2$$

$$\text{mass trans rate} = \text{flux} \times \text{Area} = 2.03 \times 10^{-3} \times 1.598 \times 10^{-7} = 3.24 \times 10^{-10}\ \frac{kmol}{s}$$

EXAMPLE 7.3-4. Mass Transfer of a Liquid in a Packed Bed

Pure water at 26.1°C flows at the rate of 5.514×10^{-7} m^3/s through a packed bed of benzoic acid spheres having a diameter of 6.375 mm. The total surface area of the spheres in the bed is 0.01198 m^2 and the void fraction is 0.436. The tower diameter is 0.0667 m. The solubility of benzoic acid in water is 2.948×10^{-2} kg mol/m^3.

(a) Predict the mass-transfer coefficient k_c. Compare with the experimental value of 4.665×10^{-6} m/s by Wilson and Geankoplis (W1).

(b) Using the experimental value of k_c, predict the outlet concentration of benzoic acid in the water.

✍ Packed Bed

Water 26.1c° $Q = 5.514 \times 10^{-7}\ m^3/s$

Benzoic Acid spheres $D = 6.372 \times 10^{-3}\ m$ $\quad A = 0.01198\ m^2$

$\varepsilon = 0.0667\ m$ $\qquad$ Solibility $= 2.948 \times 10^{-2}$ kmol /m^3.

Properites of water at 26.1c° → 299.1k

$\mu : 8.7 \times 10^{-4}$

$\rho : 997$ kg / m^3

$$Re = \frac{D \rho v}{\mu} \qquad D = \text{of the sphere.}$$

$$\frac{Q}{A} = \text{velocity} \rightarrow \frac{5.514 \times 10^{-7}\ m^3}{s \;\Big|\; \frac{\pi}{4}[0.0667]^2\ m^2}$$

$$= 1{,}58 \times 10^{-4}\ m/s$$

$$Re = \frac{6.375 \times 10^{-3} \times 997 \times 1.58 \times 10^{-4}}{8.7 \times 10^{-4}} = \boxed{1.15}$$

$$\frac{\cancel{m}}{\cancel{s}} \times \cancel{m} \times \frac{\cancel{kg}}{\cancel{m^3}} \times \frac{\cancel{m^2}}{\cancel{s} \cdot \cancel{kg} \cdot \cancel{m}} \times \cancel{s^2} \quad ✓$$

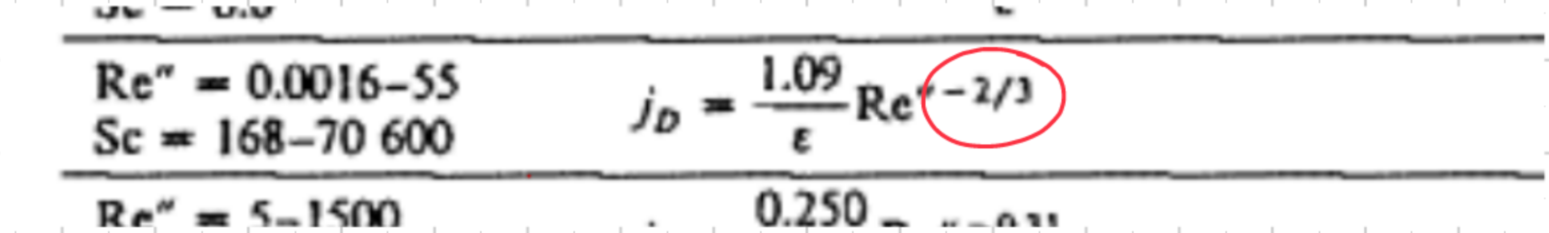
$$Sc = \frac{\mu}{\rho D_{AB}}$$

D_{AB} lec1 Table 6.31 >> $D_{AB} = 1.21 \times 10^{-9}\ m^2/s$ at 298k

$$D_{AB\,(at\ 299.1)} = 1.21 \times 10^{-9} \times \frac{299.1}{298} = 1.21 \times 10^{-9}\ m^2/s.$$

$$Sc = \frac{8.7\times10^{-4}}{997\times1.21\times10^{-9}} = 718.5$$

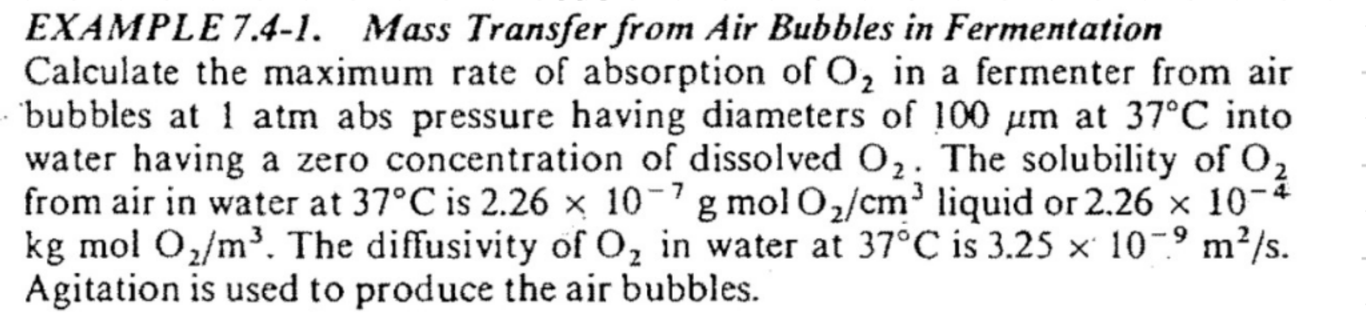
$$\cancel{Kg}\cdot\frac{\cancel{m}}{\cancel{s^2}}\cdot\frac{\cancel{s}}{\cancel{m^2}}\times\frac{\cancel{m^3}}{\cancel{Kg}}\times\frac{\cancel{s}}{\cancel{m^2}} \checkmark$$



Re″ = 0.0016–55 Sc = 168–70 600	$j_D = \frac{1.09}{\varepsilon}\text{Re}''^{-2/3}$
Re″ = 5–1500	0.250

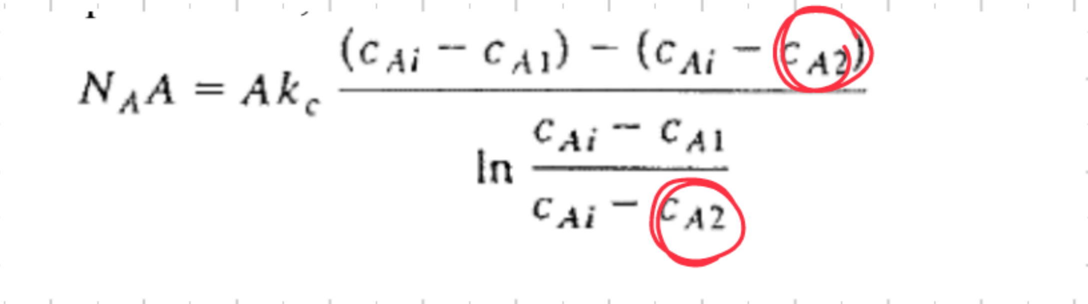
$$J_D = \frac{1.09}{0.436}\,[1.15]^{-2/3} = 2.277$$

$$J_D = \frac{k_c'}{v}\,[Sc]^{2/3}$$



EXAMPLE 7.4-1. Mass Transfer from Air Bubbles in Fermentation
Calculate the maximum rate of absorption of O_2 in a fermenter from air bubbles at 1 atm abs pressure having diameters of 100 μm at 37°C into water having a zero concentration of dissolved O_2. The solubility of O_2 from air in water at 37°C is 2.26×10^{-7} g mol O_2/cm³ liquid or 2.26×10^{-4} kg mol O_2/m³. The diffusivity of O_2 in water at 37°C is 3.25×10^{-9} m²/s. Agitation is used to produce the air bubbles.

B =



$$N_A A = Ak_c\frac{(c_{Ai}-c_{A1})-(c_{Ai}-c_{A2})}{\ln\dfrac{c_{Ai}-c_{A1}}{c_{Ai}-c_{A2}}}$$

$$N_A A = Q\,(C_{A2} - C_{A1})$$

$$N_A A = 5.514\times10^{-7}\times(C_{A2} - 2.94\times10^{-2}]$$

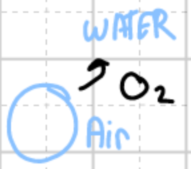
$$N_A A = 0.01198\times4.48\times10^{-6}\;\frac{(2.94\times10^{-2}-0)-(2.948\times10^{-2}-C_2)}{\ln\dfrac{2.94\times10^{-2}-0}{2.948\times10^{-2}-C_2}}$$

$$C_2 = 2.842\times10^{-3}\ \text{kmol/m}^3$$

EXAMPLE 7.4-1. ***Mass Transfer from Air Bubbles in Fermentation***

Calculate the maximum rate of absorption of O_2 in a fermenter from air bubbles at 1 atm abs pressure having diameters of 100 μm at 37°C into water having a zero concentration of dissolved O_2. The solubility of O_2 from air in water at 37°C is 2.26×10^{-7} g mol O_2/cm^3 liquid or 2.26×10^{-4} kg mol O_2/m^3. The diffusivity of O_2 in water at 37°C is 3.25×10^{-9} m^2/s. Agitation is used to produce the air bubbles.

* Bubbles



$D = 100 \times 10^{-6}$ m

$S = 2.26 \times 10^{-4}$ kg mol/ m^3

$D_{AB} = 3.25 \times 10^{-9}$ m^2/s

ρ_c water at $[37 + 273]$ = 993 kg/m^3 ρ Air = 1.13 kg/m^3

μ water at $[310]$ = 695×10^{-6} N.s/m^2

$$Sc = \frac{\mu_c}{\rho_c D_{AB}} \quad \frac{695 \times 10^{-6}}{993 \times 3.25 \times 10^{-9}} = 215$$

$$\frac{\cancel{kg}\,\cancel{m}}{\cancel{s^2}}\,\frac{\cancel{s}}{\cancel{m^2}} \times \frac{\cancel{s}}{\cancel{m^2}} \times \frac{\cancel{m^3}}{\cancel{kg}} \quad \checkmark$$

$$k_L' = \underbrace{\frac{2D_{AB}}{D_p}}_{\text{the molecular diffusion term}} + \underbrace{0.31 N_{Sc}^{-2/3}\left(\frac{\Delta\rho \mu_c g}{\rho_c^2}\right)^{1/3}}_{\text{free fall or rise of the sphere by gravitational forces}}$$

the molecular diffusion term

free fall or rise of the sphere by gravitational forces

$100 \times 10^{-6} < 0.6 \times 10^{-3}$ ✓

$\Delta\rho = \rho_c - \rho_p = 991.87$

$$k_L' = \frac{2 \times 3.25 \times 10^{-9}}{100 \times 10^{-6}} + 0.31 \times 215^{-2/3} \left[\frac{991.87 \times 695 \times 10^{-6} \times 9.8}{993^2}\right]^{1/3}$$

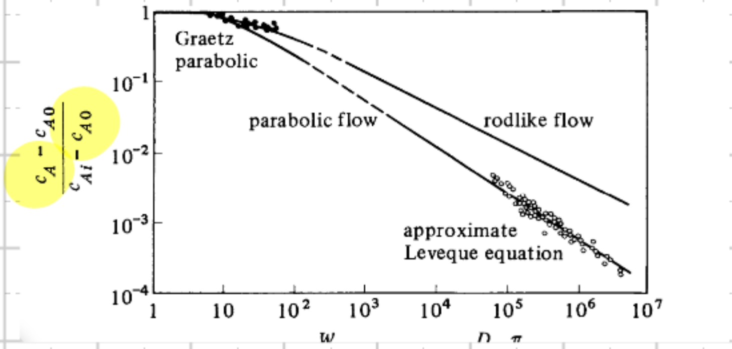
$k_L' = 2.29 \times 10^{-4}$ m/s.

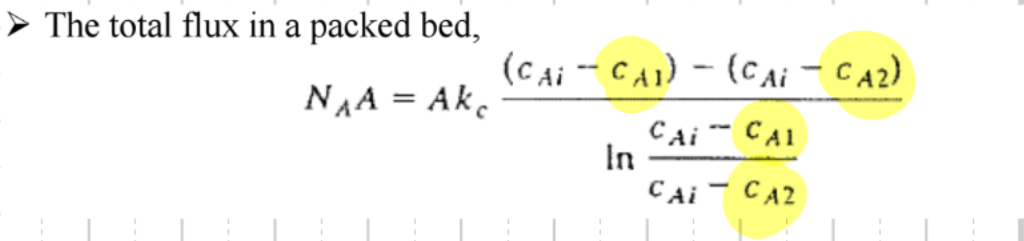
dillate soluation $k_L' = k_L$

$N_a = k_L (C_{A1} - C_{A2})$

$2.290 \times 10^{-4} \times [2.26 \times 10^{-4} - 0] = 5.18 \times 10^{-8} \frac{\text{kmol } O_2}{s \cdot m^2}$.

★ flow → Confind → Pipe
→ Packed Bed
} Cin ≠ Cout

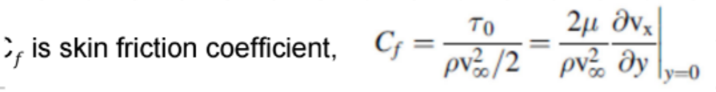



➢ The total flux in a packed bed,

$$N_A A = A k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln \frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}}$$

* Reynolds Analogy

- mechanism heat ≡ momentum ≡ mass
- $\underbrace{\frac{h}{\rho V_\infty C_p}}_{\text{heat}} = \underbrace{\frac{C_f}{2}}_{\text{momentum}} = \underbrace{\frac{k_c}{V_\infty}}_{\text{mass.}}$ → $St_H = St_m$


$_f$ is skin friction coefficient, $C_f = \frac{\tau_0}{\rho v_\infty^2/2} = \frac{2\mu}{\rho v_\infty^2}\frac{\partial v_x}{\partial y}\Big|_{y=0}$

* Prandtl and Schmidt = 1
* Turbulent eddies gases

} نقدر استخدم هاي
العلاقة بدل مااروح
للحالات بلي ورا لانه
حسابات ال Heat , momentum
اسهل

* chiton - colburn Analogy $\rightarrow J_D = J_H = \frac{f}{2}$

$St_H \neq St_M$

mass :

$J_D = St_m (Sc)^{2/3}$

$St_m: \frac{Sh}{Re\, Sc} = \frac{k_c}{V_\infty}$

Heat:

$J_H = St_H (Pr)^{2/3}$

$St_H: \frac{Nu}{Re\, Pr} = \frac{h}{\rho C_p V}$

* $0.6 < Sc < 2500$
* $0.6 < Pr < 100$

*Dimensionless Numbers:

① Sherwood number $Sh = \frac{k_c L}{D_{AB}}$

- Total mass transfer rate / mass transfer by diffusion
- R molecular diffusion / R convection

② Prandtl Number $Pr = \frac{C_p \mu}{k}$

- diffusivity of momentum / diffusivity of heat.

α, D_{AB}, ν

③ Schmidt Number $Sc = \frac{\mu}{\rho D_{AB}}$

- diffusivity of momentum / diffusivity of mass.

④ Lewis Number $Le = \frac{k}{\rho C_p D_{AB}}$

- diffusivity of heat / diffusivity of mass

⑤ $Re = \frac{D v \rho}{\mu}$

- inertial force / viscous force

⑥ J factor for mass transfer $J_D = \frac{k_c}{V_\infty} [Sc]^{2/3}$

- empircal correlation factor combining mass trans and flow properties.

⑦ $J_H = St_H . Pr^{2/3}$

- empircal correlation factor combining Heat trans and flow properties.

⑨ Stanton number $St_m = \frac{Sh}{Re\,Sc} = \frac{k_c}{v}$

- Total mass transfer rate / Inertial force

⑩ Peclet number $Pc = Re\,Sc$

- Inertial forces / mass transfer by molecular diffusion
- Convection / Diffusion

⑪ Grashof number $Gr = \frac{D^3 \rho g \Delta\rho}{\mu^2}$

- Bouoyancy / viscosity
- Natural convection.

7.3-2. *Mass Transfer from a Pipe Wall.* Pure water at 26.1°C is flowing at a velocity of 0.0305 m/s in a tube having an inside diameter of 6.35 mm. The tube is 1.829 m long with the last 1.22 m having the walls coated with benzoic acid. Assuming that the velocity profile is fully developed, calculate the average concentration of benzoic acid at the outlet. Use the physical property data of Example 7.3-2. [*Hint*: First calculate the Reynolds number $D\upsilon\rho/\mu$. Then calculate $N_{Re}N_{Sc}(D/L)(\pi/4)$, which is the same as $W/D_{AB}\rho L$.]

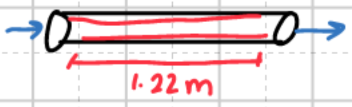
Ans. $(c_A - c_{A0})/(c_{Ai} - c_{A0}) = 0.0744$, $c_A = 2.193 \times 10^{-3}$ kg mol/m^3

$\mu = 8.71 \times 10^{-4}$ Pa.s

$\rho = 996$ kg/m^3

$D_{AB} = 1.245 \times 10^{-9}$ m^2/s

$\delta = 0.02948$ kgmol/m^3

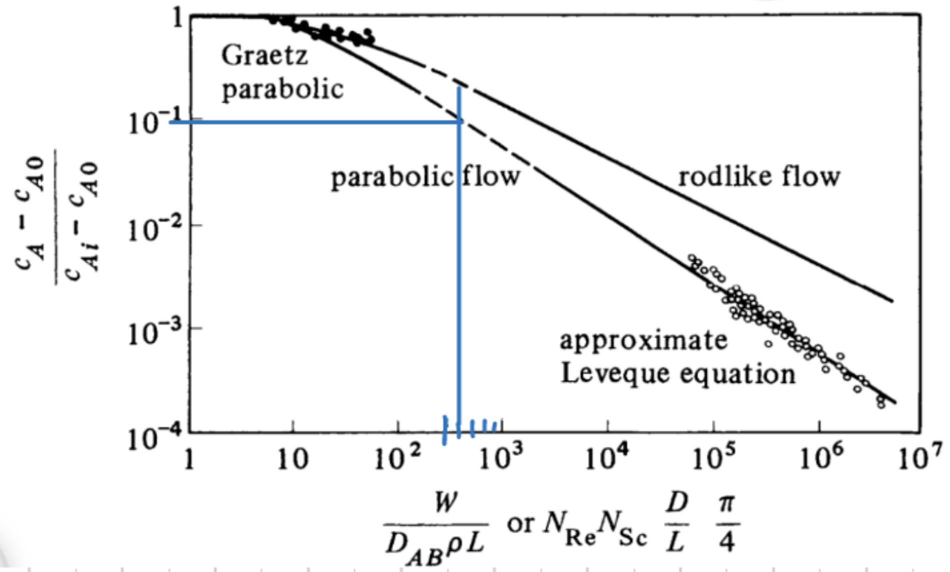


بالكتاب في معادلة
احنا عنا بس chart
والدكتور حكا اعتمدوا
السلايدات بالقوانين

① $Re = \frac{\rho V D}{\mu} = \frac{996 \times 0.0305 \times 6.35 \times 10^{-3}}{8.71 \times 10^{-4}} = 221.5$

② $Sc = \frac{\mu}{\rho D_{AB}} = \frac{8.71 \times 10^{-4}}{1.245 \times 10^{-9} \times 996} = 702.4$

Figure 7.3-2: Mass transfer from the inner walls of a pipe to gases **Re < 2100 and 0.5 < Sc < 3.0**



$= 221.5 \times 702.4 \times \frac{6.35 \times 10^{-3}}{1.22} \times \frac{\pi}{4} = 636.0$

$\frac{C_A - C_{A0}}{C_{Ai} - C_{A0}} = 9 \times 10^{-2}$

$\frac{C_A - 0}{0.02948 - 0} = 9 \times 10^{-2}$

$C_A \approx 2.7 \times 10^{-3}$ kgmol/m^3

$Q = 0.0701\ ft^3/h$

* Packed bed

* A $0.129\ ft^2$ benzoic acid

* solibility $= 0.00184$ lbmol benzoic acid$/ft^3$

* $C_{A2} = 1.80 \times 10^{-4}$

k_c ?

① velocity $= Q \div A \Rightarrow 0.0701 \frac{ft^3}{h} \times \frac{1}{0.129\ ft^2} \times \frac{h}{60 \times 60\ s} = \boxed{1.5 \times 10^{-4}\ ft/s}$

$1.5 \times 10^{-4} \frac{ft}{s} \Big| \frac{0.3048\ m}{ft}$

$= 4.6 \times 10^{-5}.$

$\rho = 997 \frac{kg}{m^3}$

$\mu = 875.8 \times 10^{-6}\ N\ s/m^2.$

$$Re = \frac{997 \times 4.6 \times 10^{-5} \times}{875.8 \times 10^{-6}} = \boxed{0.333.}$$

$$J_D = \frac{1.09}{0.4} \times 0.333^{-2/3}$$

$J_D = 5.7$

$$Sc = \frac{\mu}{\rho D_{AB}} = \boxed{725.98.}$$

$298 = 1.21 \times 10^{-9}$

$\boxed{Sh = 16.7}$

$$Sh = \frac{k_c \cancel{D}}{\cancel{D_{AB}}}$$

~~ft~~

$\frac{m}{s} \Big|$

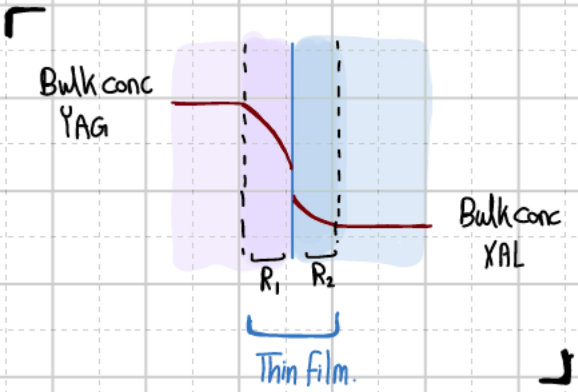
* y → Gas
* x → Liquid
* i → inter phase.

mass transfer between phases.

↳ Gas - Liquid * Absorption
* Distelation

↳ Liquid - Liquid * Extration

↳ Fluid - Liquid



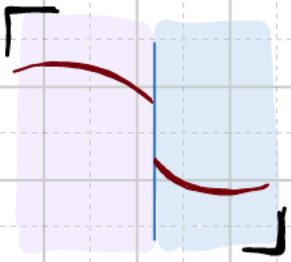
Thin film Theory ① Thin film ① Resistance
② Concentration

② Outside the thin film ① No resistance
② Constant concentration.

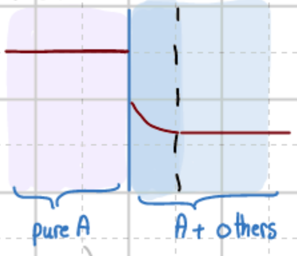
③ y_{Ai}, X_{Ai} must be in equilibrum

↲ يعني بينهم علاقة

more relastic



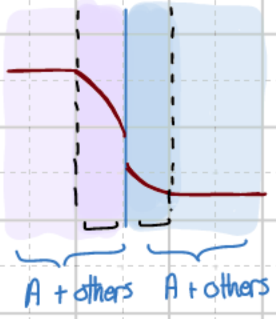
* Pure to mixture



↲ واعني B.L

$$N_A = k_c [C_{Ai} - C_{Ab}]$$

* mixture to mixture.



* ↑ Thickness for the film ↑ Resistance

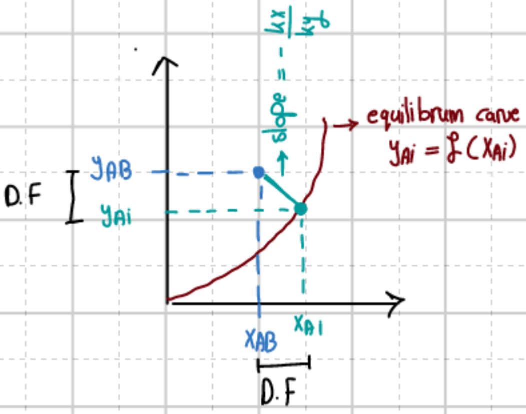
$$N_A = k_c [C_{Ai} - C_{AB}]$$

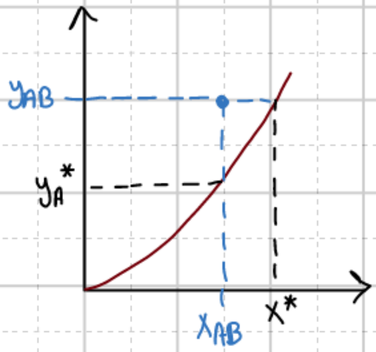
$$N_A = k_G [P_{AB} - P_{Ai}]$$

$$k_G [P_{AB} - P_{Ai}] = k_c [C_{Ai} - C_{AB}]$$ at stady state

$$* \, \frac{\ominus k_G}{k_c} = \frac{[C_{Ai} - C_{\boxed{AB}}]}{[P_{Ai} - P_{\boxed{AB}}]}$$

$$* \, \frac{-k_x}{k_y} = \frac{[y_{Ai} - y_{AB}]}{[X_{Ai} - X_{AB}]}$$





y_A^* : y equilbrum value المقابلة x_{AB}

x_A^* : X equilbrum value المقابلة y_{AB}.

- y_{Ai} — x_{Ai} in equilibrum
  y^* — x_{AB} in equilibrum
  y_{AB} — x^* in equilibrum

$y_{Ai} = M\, x_{Ai}$

$y^* = M\, x_{AB}$ M : slope for the equilibrum line

$y_{AB} = M\, x^*$

* The over all mass-transfer coefficent

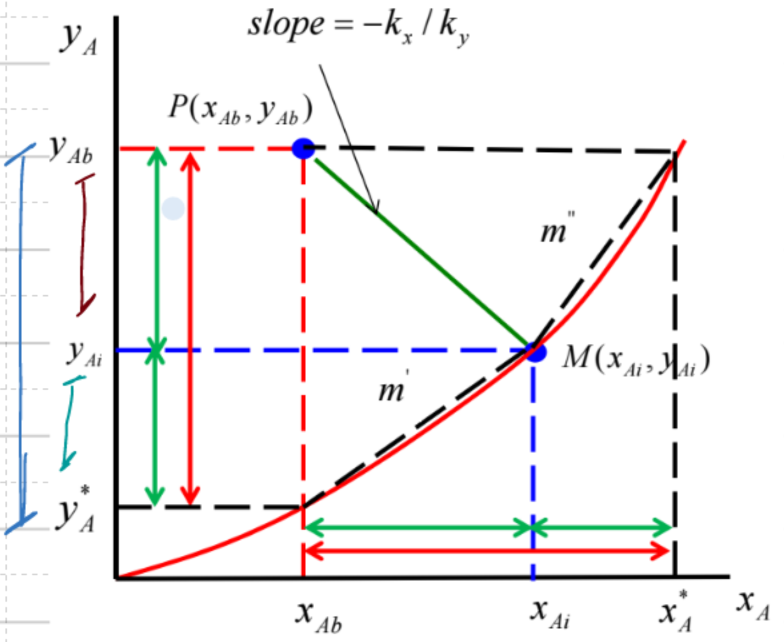
$$K_y = \frac{N_A}{y_{AB} - y_A^*} \qquad K_x = \frac{N_A}{x_A^* - x_{AB}}$$

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{AB}) = K_x(x_A^* - x_{AB}) = K_y(y_{AB} - y_A^*)$$

$$y_{Ab} - y_{Ai} = \frac{N_A}{k_y} \qquad K_x = \frac{N_A}{x_A^* - x_{AB}}$$

$$x_{Ai} - x_{AB} = \frac{N_A}{k_x} \qquad K_y = \frac{N_A}{y_{AB} - y_A^*}$$

$\therefore$ flux $= \frac{DF}{R}$ $N_A = k_y(y_{AB} - y_{Ai})$ $R = 1/k_y$



* non-Liner eq

$$''m = \frac{y_{Ai} - y_{Ab}}{x_{Ai} - x^*}$$

$$'m = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{Ab}}$$

$$y_{AB} - y_A^* = (y_{Ai} - y_A^*) + (y_{AB} - y_i)$$

$$'m = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{Ab}}$$

$$y_{AB} - y_A^* = 'm\,(x_{Ai} - x_{Ab}) + (y_{AB} - y_i)$$

$$\hookrightarrow \frac{N_A}{K_y} = \frac{N_A\,'m}{k_x} + \frac{N_A}{k_y} \rightarrow \underbrace{\frac{1}{K_y}}_{\text{overall R basis on Gas phase}} = \underbrace{\frac{m}{k_x}}_{\text{L R basis on G Phase}} + \underbrace{\frac{1}{k_y}}_{\text{G. R}}$$

$'m \rightarrow$ very small ≈ 0

$\hookrightarrow$ The gas solute A very solibile in L

$$\frac{1}{K_y} = \frac{1}{k_y}$$

major R in G phase [G → controlling]

$''m \rightarrow$ large $\rightarrow$ R in L

$$x_A^* - x_{AB} = (x_A^* - x_i) + (x_{Ai} - x_{AB})$$

$$''m = \frac{y_{Ai} - y_{Ab}}{x_{Ai} - x^*}$$

$$x_A^* - x_{AB} = \frac{y_{Ai} - y_{AB}}{''m} + (x_{Ai} - x_{AB})$$

$$\hookrightarrow \frac{N_A}{K_x} = \frac{N_A}{''m\, k_y} + \frac{N_A}{k_x} \rightarrow \frac{1}{K_x} = \frac{1}{''m\, k_y} + \frac{1}{k_x}$$

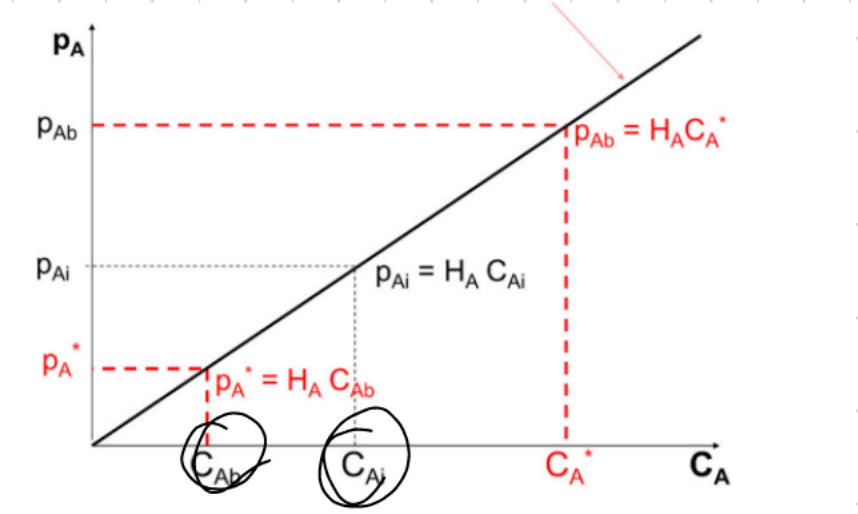
$\therefore \frac{1}{K_x}$ >> overall Resistance on the L phase basis

$\frac{1}{k_x}$ >> L - phase R

$\frac{1}{''m\, k_y}$ >> G - phase R (on L phase basis)

$''m \rightarrow$ very big

$\frac{1}{K_x} = \frac{1}{k_x}$ R in L phase [L = controlling]



* Liner eq

$$P_{Ai} = HC_{Ai}$$

$$H = \frac{P_{AB} - P_A^*}{C_A^* - C_{AB}}$$

$$H = \frac{P_i - P_A^*}{C_{Ai} - C_{AB}}$$

$$P_{AB} - P_A^* = [P_{Ai} - P_A^*] + [P_{AB} - P_{Ai}]$$

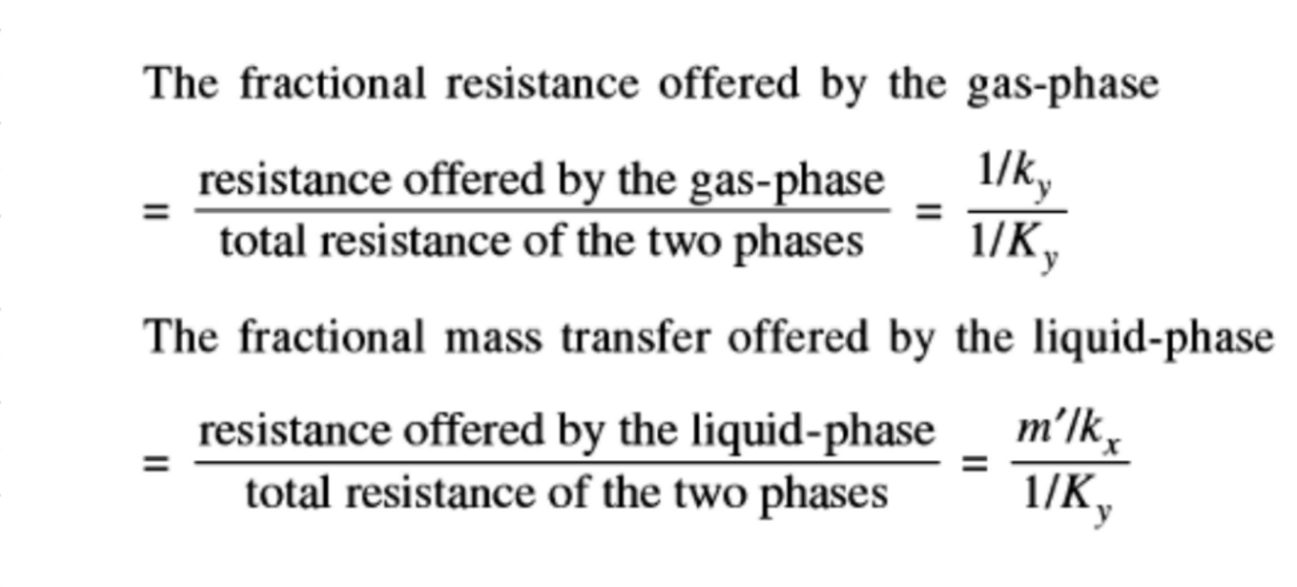
$$\frac{Na}{K_G} = H\frac{Na}{k_L} + \frac{Na}{k_G}$$

$$\frac{1}{k_G} = \frac{H}{k_L} + \frac{1}{k_G}$$

$$N_A = k_G(P_{AB} - P_A^*)$$

$$\frac{P_{AB} - P_A^*}{\left[\frac{H}{k_L} + \frac{1}{k_G}\right]}$$

$$N_A = \frac{P_{AB} - HC_{Ab}}{\left[\frac{H}{k_L} + \frac{1}{k_G}\right]} \quad \#$$



The fractional resistance offered by the gas-phase

$$= \frac{\text{resistance offered by the gas-phase}}{\text{total resistance of the two phases}} = \frac{1/k_y}{1/K_y}$$

The fractional mass transfer offered by the liquid-phase

$$= \frac{\text{resistance offered by the liquid-phase}}{\text{total resistance of the two phases}} = \frac{m'/k_x}{1/K_y}$$

In an experimental study of the absorption of ammonia by water in a wetted-wall column, the value of overall mass transfer coefficient, K_G was found to be 2.75×10^{-6} kmol/m^2-s-kPa. At one point in the column, the composition of the gas and liquid phases were 8.0 and 0.115 mole% NH_3, respectively. The temperature was 300K and the total pressure was 1 atm. Eighty five % of the total resistance to mass transfer was found to be in the gas phase. At 300 K, Ammonia –water solutions follows Henry's law upto 5 mole% ammonia in the liquid, with m = 1.64 when the total pressure is 1 atm. Calculate the individual film coefficients and the interfacial concentrations. Interfacial concentrations lie on the equilibrium line.

$Y_{AB} = 8\%$

$X_{AB} = 0.115\%$

85% over all R ⇒ GR

$m = 1.64$

$K_G = 2.75 \times 10^{-6}$

$K_y = K_G P_T$

$= 2.75 \times 10^{-6} \times 101.325$

$K_y = 2{,}8 \times 10^{-4}$ overall

$\frac{1}{K_y} \times 0.85 = \frac{1}{k_y}$

$\frac{1}{k_y} = 3035.7$

$k_y = 3.3 \times 10^{-4} \ \frac{kmol}{m^2 . s}$

$\frac{1}{K_y} = \frac{m}{k_x} + \frac{1}{k_y}$

$\frac{1}{2.786 \times 10^{-4}} = \frac{1.64}{k_x} + \frac{1}{3.3 \times 10^{-4}}$

$k_x = 2.9 \times 10^{-3}$ kmol/m^2.s

$y^* = m X_{AB}$

$y^* = 1.64 \left[0.115 \times 10^{-2}\right]$

$y^* = 0.1886 \times 10^{-3}$

$N_A = K_y \left[y_{AB} - y^*\right]$

$N_A = 2.18 \times 10^{-5}$

$N_A = k_y \left[y_{AB} - y_{Ai}\right]$

$2.18 \times 10^{-5} = 3.3 \times 10^{-4} \left[8 \times 10^{-2} - y_{Ai}\right]$

$y_{Ai} = 0.01362$

$y_{Ai} = m X_{Ai}$

$X_{Ai} = 8.3 \times 10^{-3}$

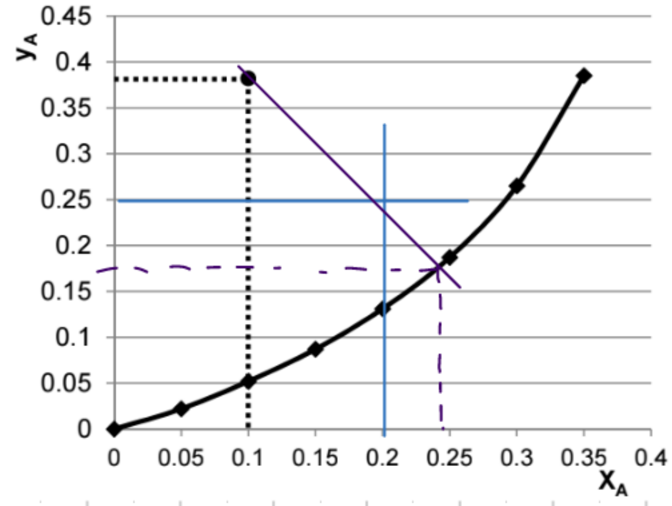
EXAMPLE 10.4-1. Interface Compositions in Interphase Mass Transfer

The solute A is being $\overset{G-L}{\text{absorbed}}$ from a gas mixture of A and B in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration $y_{AG} = 0.380$ mol fraction and the bulk liquid concentration is $x_{AL} = 0.100$. The tower is operating at 298 K and 1.013×10^5 Pa and the equilibrium data are as follows:

x_A	y_A	x_A	y_A
0	0	0.20	0.131
0.05	0.022	0.25	0.187
0.10	0.052	0.30	0.265
0.15	0.087	0.35	0.385

The solute A diffuses through stagnant B in the gas phase and then through a nondiffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for A in the gas phase is predicted as $k'_y = 1.465 \times 10^{-3}$ kg mol A/s · m² · mol frac (1.08 lb mol/h · ft² · mol frac) and for the liquid phase as $k'_x = 1.967 \times 10^{-3}$ kg mol A/s · m² · mol frac (1.45 lb mol/h · ft² · mol frac). Calculate the interface concentrations y_{Ai} and x_{Ai} and the flux N_A.



① slope $-\frac{k_x}{k_y} = \frac{k'_x/x_{BIM}}{k'_y/y_{BIM}}$

① assume Dillute soluation.
$x_{BIM} = 1$
$y_{BIM} = 1$

$$-\frac{1.967 \times 10^{-3}}{1.465 \times 10^{-3}}$$

$\boxed{\text{slope} = -1.34}$

$y - 0.380 = -1.34[x - 0.1]$

$x = 0.2 \quad y = 0.246$

$x_i = 0.247 \quad y_i = 0.183$

$y_{AB} = 0.38 \quad x_{AB} = 0.1$
$y_{BB} = 0.62 \quad x_{BB} = 0.9$

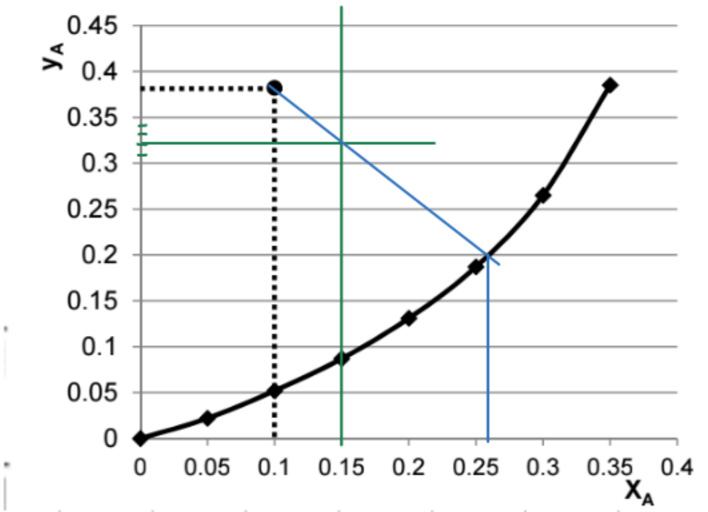
$x_{Bi} = 0.753 \quad y_{Bi} = 0.817$

$$x_{BIM} = \frac{0.753 - 0.9}{\ln\frac{0.753}{0.9}} = 0.82$$

$$y_{BM} = \frac{0.817 - 0.62}{\ln\frac{0.817}{0.62}} = 0.71.$$

② slope$_{new}$ $= \frac{k'_x/x_{BIM}}{k'_y/y_{BIM}} - \frac{1.967 \times 10^{-3} \backslash 0.82}{1.465 \times 10^{-3} \backslash 0.71} = -1.16$

error = 15%
> 10 ☹



$y - 0.380 = -1.16[x - 0.1]$

$x_{Ai} = 0.26$ $y_{Ai} = 0.2$

$x_{Bi} = 0.74$ $y_{Bi} = 0.8$

$y_{AB} = 0.38$ $x_{AB} = 0.1$

$y_{BB} = 0.62$ $x_{BB} = 0.9$

$$y_{IMB} = \frac{0.62 - 0.8}{\ln \frac{0.62}{0.8}} = 0.71$$

$$x_{IMB} = \frac{0.9 - 0.62}{\ln 0.9/0.62} = 0.75$$

③ New slope $= \frac{1.967\times10^{-3} \backslash 0.75}{1.465\times10^{-3} \backslash 0.71} = -1.27$

error = 8.7%

<10% ☺ ✓

* $N_A = k'_y [y_{AB} - y_i]$

$N_A = 3.7\times10^{-4} \frac{\text{kg mol}}{m^2\ s}$