Transfer Phenomena 2

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"trans 2" Lec 1

"Fundamentals of Mass transfer"

1] Momentum transfer : occurs in unit operations as fluid flow, mixing, sedimentation, filtration

2] Heat transfer : occurs in convection and conduction

Heat transfer ex) evaporation, drying, distillation

3] Mass transfer : occurs in distillation, drying, extraction adsorption, absorption, membrane processes

* General Molecular transport equation → rate = $\frac{\text{driving force}}{\text{resistance}}$

$$\left[\psi_z = -\delta \frac{d\Gamma}{dz} \right]$$

Mass transfer → when a component in a mixture transfers in the same phase or from phase to phase because of a difference in concentration between two points. higher → lower

* Mass transfer may occurs in : gas mixture, liquid solution, solid or across a phase boundary

Examples on mass transfer :-

1. Dispersion of gases from stacks "دخان المداخن"
2. Distillation columns "Separation" [ex) petroleum refinery]
3. Removal of pollutants from plant discharged streams by absorption [ex) absorption of CO_2 by water ex: packed bed]
4. Stripping of gases from waste water "تحرير الغازات الذائبة من الغازات"
5. Adsorption of liquid or gas to solid surface "يستخدم لفصل مائع من خليط"
6. Neutron diffusion within nuclear reactors
7. Breathing

Control rods → made of material that absorbs excess neutrons.

ex) Boron or cadmium

لـ مواد تستخدم لصناعة ال control rods

* By controlling the number of neutrons we can control the rate of fissions *

ال control rod بيستخدم للسيطرة على حركة التفاعل النووي داخل المفاعلات النووية

- Diffusion coefficient in gas > liquid > Solid
- in unsteady state we need initial conditions
- equilibrium means = there is no motion of molecules

- The diffusion depends on the media but the conduction depends on the material

"Modes of Mass transfer"

1 Diffusion

→ the net transport of substances in stationary Solid or Stagnant fluids or (fluid moving in laminar flow) due to concentration gradient.

↳ ex: near the solid surface

→ is caused by random molecular motion that leads to complete mixing.

* Note!, Diffusion takes place in mixtures ≥ 2 components

* Molecular Diffusion also called : Random-Walk process

at low velocity → diffusion at high velocity → convection

*The net diffusion is from high to low concentration at same T and p

Fluxes → The mass or molar vector quantity denoting the amount of the particular species that passes per given increment of time through unit area.

- molar flux $= \dfrac{(\text{mole of A transferred})}{(\text{area})\ (\text{time})} = \dfrac{\text{kgmoles}}{m^2 \cdot s}$

Fick's law :- (for diffusion)

* it's an empirical relation

* for isothermal, isobaric binary mixture of A and B

$$\boxed{J_A^* = -C_T D_{AB} \frac{dx_A}{dZ}}$$

total concentration diffusivity

$[\text{kmol (A+B)}/m^3]$ $\left[\dfrac{m^2}{s}\right]$

* Flux = rate of molecular diffusion = J^*_{Az} = mass transfer rate

* The mass transfer rate is proportional to the area normal to the direction of mass transfer

Note! Fick's law used to calculate the flux due to diffusion but not the flux due to bulk flow

* The negative sign = the diffusion occurs in the direction of drop in concentration

* Flux = rate of molecular diffusion = J^*_{AZ} = mass transfer rate

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Example 6.1-1 :- mixture of He and N_2 (T, P = 298 K, 1atm) ↳ total pressure

point 1 the partial pressure of He is 0.6 atm

point 2 0.20 atm at 0.2 m

Find the flux of He, if $D_{AB} = 0.687 \times 10^{-4}$ m²/s

① $P_{He} = 0.6$ $\quad P_{N_2} = 0.4$ $\quad$ ② $P_{He} = 0.2$ $\quad P_{N_2} = 0.8$

$$PV = nRT \rightarrow \frac{n}{V} = \frac{P}{RT} = C$$

$$J^*_{AZ} = -D_{AB}\frac{dC_A}{dZ} \longrightarrow J^*_{AZ}(Z_2 - Z_1) = -D_{AB}(C_{A_2} - C_{A_1})$$

$$J^*_{AZ} = \frac{-0.687\times10^{-4} * -0.01632}{0.2\,m} = 5.61\times10^{-6}\ \text{kgmol/s.m}^2$$

$$C_{A_1} = \frac{0.6\,\cancel{atm} \cdot kgmol\,\cancel{K}}{82.057\times10^{-3}\,m^3.\cancel{atm} * 298\,\cancel{K}} = 0.0245\ \frac{kgmol}{m3}$$

$$C_{A_2} = 8.18\times10^{-3}\ \frac{kgmol}{m3}$$

Example $\underline{6.2\text{-}1}$ 8-

ammonia gas (A) N_2 gas (B) $D_{AB} = 0.230 \times 10^{-4}\ m^2/s$

$L = 0.10\ m$ $P_{\underline{total}} = 1.0132 \times 10^5$ pa $T = 298\ K$

At point $\underline{1}$ $P_{A_1} = 1.013 \times 10^4$ Pa $J_A^* = ?$ $J_B^* = ?$

point $\underline{2}$ $P_{A_2} = 0.507 \times 10^4$ pa

$C_A = \frac{n}{v} = \frac{P_A}{RT}$ $J_A^* = -D_{AB} \frac{d\,P_A}{RT} \cdot \frac{1}{dz}$

$J_A^* (\Delta z) = -D_{AB} \frac{(P_{A_2} - P_{A_1})}{RT} \rightarrow J_A^* = \frac{-0.230 \times 10^{-4} * (-5060)}{8314 * 298 * 0.10}$

$J_A^* = 4.70 \times 10^{-7}\ kgmol/s.m^2$

$P_{B_1} = 1.0132 \times 10^5 - 1.013 \times 10^4 = 91190$

$P_{B_2} = 96250$

$J_B^* = \frac{-0.230 \times 10^{-4} * 5060}{8314 * 298 * 0.10} = -4.70 \times 10^{-7}\ kg\ mol/s.m2$

Equimolar counter diffusion in Gases :-

* if the total pressure is constant ∴ the net molecules of A diffusing must equal the net molecules of B diffusing *

$$\left(J^*_{AZ} = -J^*_{BZ} \right) \qquad \left(J^*_{BZ} = -D_{BA} \frac{dC_B}{dz} \right)$$

↳ steady state , constant C

prove that $D_{AB} = D_{BA}$:

$$J^*_{AZ} = -J^*_{BZ}$$

$$-D_{AB} \frac{dC_A}{dz} = -\left(-D_{BA} \frac{dC_B}{dz}\right) \qquad \text{Note: } dC_A = -dC_B$$

$$\cancel{-} D_{AB} \frac{\cancel{-} dC_B}{dz} = \cancel{-} \left(\cancel{-} D_{BA} \frac{dC_B}{dz}\right)$$

$$\therefore \; D_{AB} = D_{BA}$$

2 Convection

i. Advection → The net transport of substances by the moving fluid , cannot happen in solids.

ii. convection → The net transport of substances caused by both advective and diffusive transport in fluid.

* بال advection السبب يكون حركة المائع نفسه وليس بسبب فرق بكثافة او حرارة *

* بال convection السبب يكون فرق في درجات حرارة او كثافة *

- takes place between moving fluid and surface or between immiscible moving fluids separated by mobile interface

The convection rate transfer generalized in Newton's law of cooling

$$\left(\left(N_{AZ} = k_c \, \Delta C_A \right)\right)$$

General flux equation :-

* Binary mixture = equimolar diffusion

Note ! diffusion coefficient of H is high *

- Fick's law is limitted for Binary mixtures and systems without convection or laminar / stagnent fluids only, so we use the general flux equation

Mass is often transferred by two possible means :-

1) concentration difference by Fick's law (and)
2) convection differences by density differences

Stefan and Maxwell : using the kinetic theory of gases

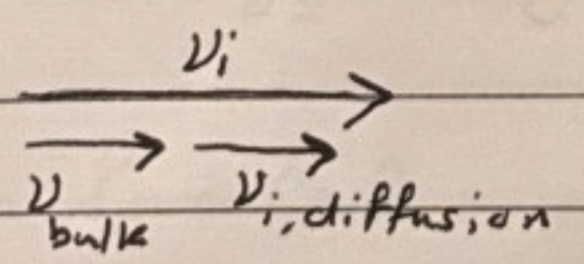
$$\Rightarrow \left[\frac{\text{total mass}}{\text{transported}}\right] = \left[\frac{\text{mass transported}}{\text{by diffusion}}\right] + \left[\frac{\text{mass transported}}{\text{by motion of fluid}}\right]$$

velocities :-

$$\left[v_{A,\text{diffusion}} = \frac{J_A^*}{C_A} \right]$$ for each species

$$\left[v_{\text{bulk}} = \frac{\sum C_i v_i}{C_{\text{Total}}} \right]$$

v_i = velocities of each type of molecules in the flow (convection velocity)



$$\left[v_i = v_{\text{bulk}} + v_{\text{diffusion}} \right]$$

$$\Big\Downarrow \quad * C_i$$

$$C_i v_i = C_i v_{\text{bulk}} + C_i v_{\text{diffusion}}$$

$$\left[N_i = C_i v_i \right]$$ v_i = convection velocity

$$\left[N_i = \frac{\bar{N}\ (\text{mol/s})}{A\ (\text{m})^2} \right]$$

the general equation ← # $N_i = J_i^* + C_i \dfrac{(N_A + N_B)}{C_T}$ #

- Note! when the mixture is diluted ∴

$$N_A = J_A^* = -D_{AB}\frac{dC_A}{dz}$$

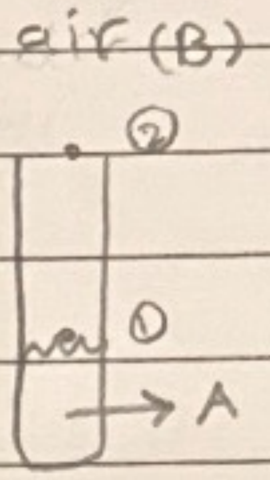
- $$N_A = -\frac{D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P}(N_A + N_B)$$

Special case 8- "A diffusing through Stagnant or Non-diffusing B" [$N_B = 0$]

In this case one boundary at the end of the diffusion path is impermeable to component B, so it cannot pass.

Examples on this case:-

1] Evaporation of pure liquid benzene (A) + nondiffusing air (B)



* air is insoluble in benzene liquid *

* at point ① liquid surface is impermeable to air *

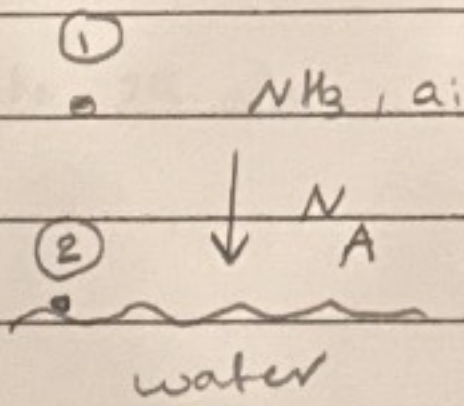
* at point ② $P_{A_2} = 0$

because I have large amount of air is passing at point ②

2] Absorption of NH_3 (A) which is in air (B) by water

* air is very slightly soluble in water

* water surface point ① is impermeable to air *



In the two examples $N_B = 0$ ∴

$$N_A = -C D_{AB} \frac{dx_A}{dz} + \frac{C_A}{C}(N_A + 0)$$

$$C = \frac{P}{RT}, \quad P_A = x_A P, \quad \frac{C_A}{C} = \frac{P_A}{P}$$

$$N_A = \frac{D_{AB} P}{RT(z_2 - z_1)} \ln \frac{P - P_{A2}}{P - P_{A1}}$$

"Diffusivity":-

D_{AB} : mass diffusivity , m^2/s

identical to other transport properties ex:

1. Kinematic viscosity $\nu = \frac{\mu}{\rho}$ → momentum transfer
2. thermal diffusivity $\alpha = \frac{k}{\rho c_p}$ → heat transfer

Diffusivity depends on : pressure / temperature / composition

Range of values of diffusivity :-

Gases → $5\times10^{-6} - 1\times10^{-5}$ m^2/s
Liquids → $10^{-6} - 10^{-9}$ m^2/s
Solids → $5\times10^{-14} - 1\times10^{-10}$ m^2/s

Diffusivity of Gases :-

* at low density are almost composition independent

$$\boxed{D_{AB} \propto \frac{1}{P}}$$ → range of pressure up to 25 atm

$$\boxed{D_{AB} \propto T^{(1.5-2)}}$$

Hirschfelder extrapolation : $$\boxed{D_{AB}\Big|_{(T_2,P_2)} = D_{AB}\Big|_{(T_1,P_1)} * \left[\frac{P_1}{P_2}\right] * \left[\frac{T_2}{T_1}\right]^{3/2}}$$

Wilke : $$\boxed{D_{1-mix} = \frac{1}{\frac{x_2}{D_{1-2}} + \frac{x_3}{D_{1-3}} + \cdots + \frac{x_n}{D_{1-n}}}}$$ → For mixture of components

10^{-7}

Fuller et al : $$D_{AB} = \frac{10^{-7} * T^{1.75} * \left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{1/2}}{P\left[(\Sigma V_A)^{1/3} + (\Sigma V_B)^{1/3}\right]^2}$$ → used when I don't have exp. data

V_i = diffusion Volume

↳ from tables

Example 6·2-5 :-

butanol (A) + air (B) $P_{abs} = 1\,atm$ Fuller et al

find D_{AB} at :

a) 0° c

$$D_{AB} = \frac{10^{-7} * T^{1.75} * (1/74.1 + 1/29)^{1/2}}{P \cdot [\Sigma \nu_A^{1/3} + \Sigma \nu_B^{1/3}]^2}$$

$\Sigma \nu_B = 20.1$

$\Sigma \nu_A = (4 * \underset{C}{16.5}) + \underset{O}{5.48} + (10 * \underset{H}{1.98}) = 91.28$ From tables !

$D_{AB} = 7.7 \times 10^{-6}\ m^2/s$

b) 0°c and 2 atm

$$\underset{(T_2,P_2)}{D_{AB}} = \underset{(T_1,P_1)}{D_{AB}} * \left[\frac{P_1}{P_2}\right] * \cancelto{0}{\left[\frac{T_2}{T_1}\right]^{3/2}}$$

$T_1 = T_2 = 0°C$

$$\underset{(T_2,P_2)}{D_{AB}} = 7.7 \times 10^{-6} * \frac{1}{2} = 3.85 \times 10^{-6}\ m^2/s$$

Diffusivity in liquids :-

* Strongly concentration dependent *

* In chemistry diffusivity limits the rate of acid-base rxns *

* In chemical industry diffusion is resonible for the rates of liquid-liquid extraction *

- Diffusion in liquids is important because it's slow

diffusion

as molecules — as ions

In certain molecules — electrolytes ionize in solutions

- each ion has a different mobility
- when : $\frac{\text{Solute}}{\text{Solvent}} > 5$ ∴ Diffusivity $\propto \frac{1}{\mu}$
- In extremely high viscosity materials, diffusion becomes independent of viscosity

$$\boxed{D_{AB} = \frac{9.96 \times 10^{-16}\, T}{\mu\, V_A^{1/3}}}$$

V : molar volume $\left(\frac{m^3}{kmol}\right)$

↳ used for : large spherical molecules of (A) of 1000 MW or more dilute in liquid solvent (B) of small molecules

\+ applicable for biological solutes such as proteins

$$\boxed{D_{AB} = 1.173 \times 10^{-16} [\varphi M_B]^{1/2} \frac{T}{\mu_B V_A^{0.6}}}$$

φ : association parameter of the solvent

↳ used for smaller (A) molecules diluted in liquid solvent (B)

\+ biological solutes

$$\boxed{D_{AB} = \frac{8.928 \times 10^{-10}\, T\,(1/n_+ + 1/n_-)}{(1/\lambda_+ + 1/\lambda_-)}}$$

↳ For electrolytes in liquid or acid-base rxn

↳ D_{AB} in cm^2/s

n : the valence الكترونات التكافؤ

λ :

Diffusivity in Solids :-

* here D is very smaller than D in liquids.

Examples :-

1) Leaching of metal ores
2) Drying of timber and foods
3) Diffusion and catalytic reaction in solid catalysts
4) separation of fluids by membranes
5) Treatment of metal at high temp by gases

* Diffusion in liquids is less than in Gases.
* The liquid is independent of pressure.
* المركبات الحلقية الها أقل D_{AB} بسبب كبر مساحتها

* pressure and concentration => Factors affect on diffusivity in gases.
* μ is a week function of temperature.
* In hydrogen attack I have corrosion.

Lec 2

Differential equation of Mass Transfer :-

$$\begin{Bmatrix}\text{net rate of mass}\\ \text{efflux of A}\end{Bmatrix} + \begin{Bmatrix}\text{net rate of}\\ \text{accumulation}\end{Bmatrix} - \begin{Bmatrix}\text{rate of chemical}\\ \text{production}\end{Bmatrix} = 0$$

net rate of mass flux $\left(\frac{\text{amount}}{\text{time}}\right)$

$\rightarrow N_{A,x}\,\Delta y\,\Delta z\Big|_{x+\Delta x} - N_{A,x}\,\Delta y\,\Delta z\Big|_{x}$

$\rightarrow N_{A,y}\,\Delta x\,\Delta z\Big|_{y+\Delta y} - N_{A,y}\,\Delta x\,\Delta z\Big|_{y}$

$\rightarrow N_{A,z}\,\Delta x\,\Delta y\Big|_{z+\Delta z} - N_{A,z}\,\Delta x\,\Delta y\Big|_{z}$

rate of accumulation $\rightarrow \frac{\partial C_A}{\partial t}\,\Delta x\,\Delta y\,\Delta z$ $\left(\frac{\text{amount}}{\text{time}}\right)$

rate of production $\rightarrow r_A\,\Delta x\,\Delta y\,\Delta z$

* r = chemical reaction rate *

* $r = \frac{\text{mole}}{\text{volume}\cdot\text{time}}$ *

$$\boxed{\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z} + \frac{\partial C_A}{\partial t} - r_A = 0}$$

↳ equation of continuity of component "A" → same equation for B !

When : * velocity of mixture is zero The motion term is zero

* no chemical reaction ∴ R or r = zero

So ↓

$$J_A^* = N_A = -D_{AB}\left[\frac{\partial C_A}{\partial x} + \frac{\partial C_A}{\partial y} + \frac{\partial C_A}{\partial z}\right]$$

↓

$$\boxed{\frac{\partial C_A}{\partial t} = D_{AB}\left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right]}$$

↳ " Fick's second law "

* continuity equations used to descripe the mass transfer *

* in membrane → we use one-dimensional equation ↓

ليش ؟ لأنو ثابت و solid

* partitionaly coefficient = solubility coefficient *

* at steady state $\frac{\partial^2 C_A}{\partial z^2}$ or $\frac{-dJ_A^*}{dz} = 0$ → means that we have constant flux !!

$$\frac{\partial^2 C_A}{\partial z^2} = 0 \longrightarrow \frac{dC_A}{dz} = a$$

$$\int dC_A = a \int dz \longrightarrow C_A = az + b$$

Boundary conditions : $z=0,\ C = C_{A_0}$ and $z = L,\ C = C_{AL}$

$$C_{A_0} = a * 0 + b \longrightarrow b = C_{A_0}$$

$$C_{AL} = aL + C_{A_0} \longrightarrow C_{AL} - C_{A_0} = aL$$

$$a = \left(\frac{C_{AL} - C_{A_0}}{L}\right)$$

$$\#\# \quad \boxed{C_A = \frac{(C_{AL} - C_{A_0})}{L} z + C_{A_0}}$$

$$J_A^* = -D_{AB} \frac{dC_A}{dz} \longrightarrow J_A^* \int_0^L dz = -D_{AB} \int_{C_{A_0}}^{C_{AL}} dC_A$$

$$J_A^* L = -D_{AB} (C_{AL} - C_{A_0})$$

$$\#\# \quad \boxed{J_A^* = -D_{AB} \frac{(C_{AL} - C_{A_0})}{L}}$$

For cylinder :-

Continuity equation ←

$$\frac{\partial C_A}{\partial t} = \left[\frac{1}{r}\frac{\partial}{\partial r}(r N_{A,r}) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

For sphere :-

Continuity equation ←

$$\frac{\partial C_A}{\partial t} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\theta}{\partial \theta}(N_{A,\theta}) + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}\right] = R_A$$

Equimolar counter diffusion :-

* اشهر مثال عليه هو distillation

When : molar latent heats of vaporization are equal for the two constituents.

$$\longrightarrow N_A$$

For binary system : $T, P \rightarrow$ constant $\qquad N_B \longleftarrow$

$$N_A = -D_{AB}\frac{\partial C_A}{\partial z} + x_A(N_A + N_B)$$

$$[\,N_A = -N_B\,]$$

$$\therefore\ N_A = -D_{AB}\frac{\partial C_A}{\partial z}$$

doing $\int \longrightarrow$ $\boxed{N_A = \frac{D_{AB}}{z_2 - z_1}(C_{A_1} - C_{A_2})}$ $\qquad \left[\, C = \frac{n}{V} = \frac{P_A}{RT} \,\right]$

$$\boxed{N_A = \frac{D_{AB}}{RT\Delta z}(P_{A_1} - P_{A_2})}$$

Concentration profile in equimolar counter diffusion :-

* N_A is constant $\quad \therefore \frac{d(N_A)}{dz} = 0$

$$\frac{d}{dz}\left(-D_{AB}\frac{\partial C_A}{\partial z}\right) = 0 \longrightarrow -D_{AB}\frac{\partial^2 C_A}{\partial z^2} = 0 \longrightarrow \frac{\partial^2 C_A}{\partial z^2} = 0$$

$$\frac{\partial C_A}{\partial z} = C_1 \longrightarrow \text{"}\, C_A = C_1 z + C_2 \,\text{"}$$

@ C_{A_1}, z_1 : $\quad C_{A_1} = C_1 z_1 + C_2$ ..... $\boxed{\frac{C_A - C_{A_1}}{C_{A_1} - C_{A_2}} = \frac{z - z_1}{z_1 - z_2}}$

@ C_{A_2}, z_2 : $\quad C_{A_2} = C_1 z_2 + C_2$

Note!

if P_{B_1} and P_{B_2} are close to each other I can take the average not the log mean

Other Relations Between N_A and N_B :- ① + ②

The relations between N_A, N_B :- Fixed by reaction Stoichiometry ①

$$mA \rightarrow nB \qquad \boxed{\frac{N_A}{m} = -\frac{N_B}{n}}$$

* The relation between N_A and N_B is Given by the Latent ② heat of Vaporization *

* mass transfer in binary liquid Solution takes place in both liquid and vapor phase *

* The more volatile component transfer from L → V
The less volatile component transfer from V → L

$$\boxed{\lambda_A N_A = -\lambda_B N_B}$$

pseudo state [unsteady state]

- بدمج بين $N_A \rightarrow$ steady state وال balance بال unsteady state
- هون بكون عندي تغيّر ب ΔZ

$$\frac{dM}{dt} = M_{in} - M_{out} \Rightarrow \text{unsteady state}$$

M = mass $= \rho V$ but I want to use it with molar Not mass

$$\left[N_A = \frac{\rho}{M_A}\frac{dz}{dt}\right] \quad ②$$

use molar density

M_A → molar mass

$$\left[N_A = \frac{C_T D_{AB}}{\Delta z}\frac{(x_{A_1} - x_{A_2})}{x_{BLm}}\right] \quad ①$$

1 + 2 ⟹

$$\frac{\rho}{M_A}\frac{dz}{dt} = \frac{C_T D_{AB}}{\Delta z}\frac{(x_{A_1} - x_{A_2})}{x_{BLm}}$$

Example Lec 3 :- Constant $T = 293\,K$ steady state

air is insoluble in water

$P_{total} = 1\,atm$

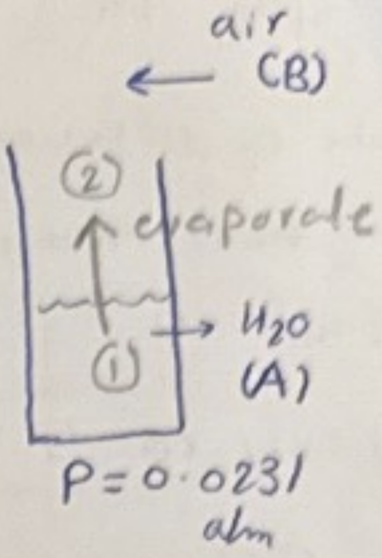
rate of evaporation = ?

dry air ($P_{water} = 0$)

$D_{AB} = 0.250 \times 10^{-4}\ \frac{m^2}{s}$

$\therefore N_B = 0$

$\Delta Z = 0.1524\ m$



$$N_A = \frac{P D_{AB}}{RT(z_2 - z_1)} \ln\left[\frac{P - P_{A_2}}{P - P_{A_1}}\right]$$

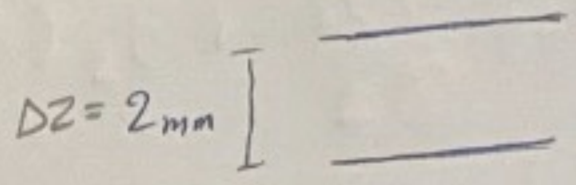
$$= \frac{1\,\cancel{atm}}{} \cdot \frac{1.01325\times10^{5}\,Pa}{1\,atm} \cdot \frac{}{293\,\cancel{K}} \cdot \frac{0.250\times10^{-4}\,\cancel{m^2}}{s} \cdot \frac{mol.\cancel{K}}{8.314\,m^3.\cancel{Pa}} \cdot \frac{}{0.1524\,m}$$

$$* \ln\left[\frac{1-0}{1-0.0231}\right]$$

$$N_A = \boxed{1.595 \times 10^{-4}\ mol/m^2.s}$$

Example Lec 3 :- O_2 is diffusing

$P = 1\,atm$ $T = 25°C$



N_2 is non-diffusing

10% and 20% volume

$N_{O_2} = ?$

$D_{AB} = 1.89 \times 10^{-5}\ m^2/s$

$P_{A_1} = 0.1 * 1 = 0.1\ atm$

$P_{A_2} = 0.2 * 1 = 0.2\ atm$

$$N_A = \frac{P D_{AB}}{RT \Delta Z} \ln\left[\frac{P - P_{A_2}}{P - P_{A_1}}\right]$$

$$= \frac{1\,\cancel{atm}}{} \cdot \frac{1.01325\times10^{5}\,pa}{1\,\cancel{atm}} \cdot \frac{}{298\,K} \cdot \frac{1.89\times10^{-5}\,m^2}{s} \cdot \frac{mol.\cancel{K}}{8.314\,m^3.\cancel{pa}} \cdot \frac{}{2\times10^{-3}\,m}$$

$$* \ln\left[\frac{1-0.2}{1-0.1}\right] = -0.0455\ mol/m^2.s$$

يعني الانتقال من الأعلى للأقل

Example Lec $\underline{\underline{3}}$:- O_2 diffuses through air to carbon surface [$R=0$]

rate O_2 diffusing per hour through $1\,m^2$?

A O_2 $\boxed{\text{reacts}}$ to make CO / CO_2 B

at $z=0$ $y_{O_2} = 0.21$ → $z=0$ → بالنسبة لل O_2 مش يوصل عالسطح ؟

instantaneous → C_A of O_2 at surface = zero

a) only CO is produced

- steady state
- 1D diffusion
- no reaction at gas film

$$2C + O_2 \rightarrow 2CO$$ reaction at surface

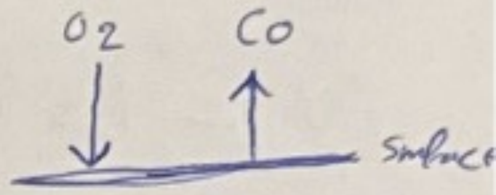
- assume that ΔZ is too small ≈ constant *

$$\frac{\partial N_{a_z}}{\partial z} = 0 \quad \dots ①$$

$$N_A = \underbrace{-D_{AB}\frac{\partial C_A}{\partial z}}_{\downarrow} + y_A(N_A + N_B)$$

$$N_A = -C D_{AB}\frac{\partial y_A}{\partial z} + y_A(N_A + N_B) \quad \dots ②$$

$$\frac{N_A}{1} = \frac{-N_B}{2} \rightarrow \boxed{N_B = -2N_A}$$



$$N_A = -C D_{AB}\frac{\partial y_A}{\partial z} + y_A(N_A - 2N_A)$$

$$N_A = -C D_{AB}\frac{\partial y_A}{\partial z} - y_A N_A \rightarrow N_A + y_A N_A = -C D_{AB}\frac{\partial y_A}{\partial z}$$

$$(1+y_A)N_A = -C D_{AB}\frac{\partial y_A}{\partial z} \rightarrow N_A \int_{z_1=0}^{z_2=\delta} \partial z = -C D_{AB}\int_{y_{A_1}=0}^{y_{A_2}=0.21} \frac{\partial y_A}{1+y_A}$$

$$N_A\,\delta = -C D_{AB}\ln(1.21)$$

$$N_A = \frac{-C D_{AB}\ln(1.21)}{\delta} \quad \#$$

b) only CO_2 is produced $C + O_2 \rightarrow CO_2$ $\boxed{N_B = -N_A}$

$$N_A = \frac{-C D_{AB}(0.21)}{\delta} \quad \#$$

c) I have CO and CO_2 as products $4C + \overset{A}{3O_2} \rightarrow \overset{B}{2CO} + \overset{C}{2CO_2}$ $\frac{N_A}{3} = \frac{N_B}{2} = \frac{N_C}{2}$

$N_B = N_C = -\frac{2}{3}$

$$N_A = -C D_{AB}\frac{\partial y_A}{\partial z} + y_A(N_A + N_B + N_C)$$

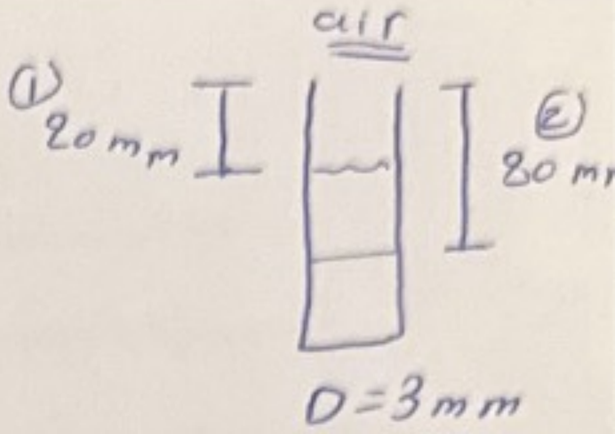
$$= -C D_{AB}\frac{\partial y_A}{\partial z} + y_A\left(N_A - \frac{2}{3}N_A - \frac{2}{3}N_A\right)$$

$$= -C D_{AB}\frac{\partial y_A}{\partial z} + y_A\left(-\frac{1}{3}N_A\right)$$

Example Lec $\underline{\underline{3}}$:- toluene$_{(L)}$ $z_1 = 20$ mm $z_2 = 80$ mm

$t = 275$ h $T = 39.4$ °C $P = 760$ mmHg

$D_{AB} = ?$



$$D_{AB} = \frac{\rho_A \, x_{B_{Lm}}}{M_A \, C_T \, (x_{A_1} - x_{A_2}) \, t} \left[\frac{z_2^2 - z_1^2}{2} \right]$$

$P_{toluene} = 7.64$ kN/m2

$\rho = 850$ kg/m^3

$M_r = 92$

$$x_{A_1} = \frac{P_{A_1}}{P_{total}} = \frac{7.64}{101.3} = 0.075$$

$$x_{A_2} = \frac{0}{P_{total}} = 0$$

$$x_{B_{Lm}} = \frac{x_{A_1} - x_{A_2}}{\ln\left[\frac{1 - x_{A_2}}{1 - x_{A_1}}\right]} = \frac{0.075}{\ln\left[\frac{1}{0.925}\right]} = 0.962$$

$$* \; C_T = \frac{P_{total}}{RT} = \frac{\text{kgmol} \cdot \cancel{\text{K}}}{8314 \, \text{m}^3 \cdot \cancel{\text{Pa}}} \left| \frac{}{312.4 \, \cancel{\text{K}}} \right| \frac{1.01325 \times 10^5 \, \cancel{\text{Pa}}}{}$$

$$= 0.039 \; \text{kgmol/m}^3$$

$$D_{AB} = \frac{850 \, \cancel{\text{kg}}}{\cancel{\text{m}^3}} \left| \frac{0.962}{92 \, \cancel{\text{kg}}} \right| \frac{\cancel{\text{m}^3} \cdot \text{mole}}{0.039 \, \cancel{\text{kgmol}}} \left| \frac{}{0.075} \right| \frac{}{275 \, \cancel{\text{h}}} \left| \frac{1 \, \cancel{\text{h}}}{3600 \, \text{s}} \right. * \; 3 \times 10^{-3} \, \text{m}^2$$

$$= 9.21 \times 10^{-6} \; \frac{\text{m}^2}{\text{s}}$$

Lec 4

Diffusion in Solids :-
Transfer of fluid or solid
in solids, examples:

- احنا بنتعامل مع pores : solid
- the only driving force is concentration gradient.

1) leaching 2) adsorption
3) drying of solids
4) separation of solution by solid membranes and chemical rxn
((taking place in pores of solid))

Diffusion in Solids

Diffusion following Fick's law [1]
doesn't depend on the structure

Diffusion that is dependent on the Nature of the Solid [2]

[1] Here, when the solute dissolve forms a homogeneous solution with the solid

in planner surface $\longrightarrow$ $N_A = D_{AB} \dfrac{C_{A_1} - C_{A_2}}{z_2 - z_1}$

N_A and D_{AB} are constant!

in cylindrical surface $\longrightarrow$ $\bar{N}_A = 2\pi L D_A \dfrac{(C_{A_i} - C_{A_o})}{\ln\left[\frac{r_o}{r_i}\right]}$

$N_A = \dfrac{\bar{N}_A}{2\pi r L}$ * here the area is not constant ∴ N_A is not constant but $\bar{N}_A$ is constant ←

$\boxed{\text{area} * \text{flux} = \text{constant}}$ total number of moles transfer is constant

in spherical cavity $\longrightarrow$ $\bar{N}_A = 4\pi r_i r_o D_A \dfrac{(C_{A_i} - C_{A_o})}{r_o - r_i}$

$N_A = \dfrac{\bar{N}_A}{4\pi r^2}$

$\boxed{2}$ Here, the porous solid and structure and Void channels are important.

a) Diffusion of liquid in solids

- The transfer path is greater than ΔZ
- The mass transfer area is smaller than total area of each face of the Solid

$$D_{Aef} = \frac{\varepsilon}{k_t^2} D_{AB}$$

$\varepsilon \rightarrow$ volume void fraction

$k_t \rightarrow$ tortuousity factor

∴

$$N_A = \frac{\varepsilon}{k_t^2} D_{AB} \frac{C_{A_1} - C_{A_2}}{z_2 - z_1}$$

b) Diffusion of gases in Solids

- depends on the relation between diameter of pores and mean free path of gas molecules

$$\lambda_A = \frac{3.2 \mu_A}{P} \left[\frac{RT}{2\pi M_A}\right]^{0.5}$$

λ_A → m

P → N/m^2

μ_A → $kg/m.s$

T → k

M_A → molecular weight

λ_A = mean free path

$R = 8134 \ J/k\text{-mole} \cdot k$

- $\lambda_A \rightarrow$ relation / Mechanisms of collision of molecules only

$$k_n = \frac{\lambda}{d_{pore}}$$

k_n = knudsen number

d_{pore} → diameter of pore

- $k_n \rightarrow$ relation between mean free path and diameter of pore

For $K_n < 0.01$ ∴ $\boxed{d_{pore} \gg \lambda}$ and $\boxed{\text{molecule} - \text{molecule collision}}$ dominant

So the pores don't have any effect

$$\boxed{N_A = -\frac{D_{AB} P}{RT}\frac{dy_A}{dz} + y_A (N_A + N_B)}$$

For $k_n > 10$ ∴ $\boxed{\lambda \gg d_{pore}}$ and $\boxed{\text{molecules} - \text{pores wall collision}}$

$$\boxed{D_{KA} = 4850\, d_{pore} \sqrt{\frac{T}{M_A}}}$$

$$\boxed{N_A = -D_{\cancel{AB}\,KA} \frac{\partial C_A}{\partial z} = -\frac{D_{KA}}{RT}\frac{dP_A}{dz} = \frac{D_{KA} P}{RT D z}(y_{A_1} - y_{A_2})}$$

Note! Knudsen diffusion process is significant only at low pressure and small pore diameter

For $0.01 < k_n < 10$ ∴ $\boxed{\text{mixed diffusion}}$ and $\boxed{\text{resistance in series}}$

∴ I have both molecule - molecule and molecule - pores wall collisions ✓

$$\boxed{\frac{1}{D_{Ae}} = \frac{1}{D_{AB}} + \frac{1}{D_{KA}}}$$

Note! in Random pores ⇒ $\boxed{D'_{Ae} = \varepsilon^2 D_{Ae}}$

The relationship between the fluxes at the diffusion of gas in solid :-

1) closed system, constant pressure → equimolar counter diffusion

$(N_A = - N_B)$

2) open system → $(N_A \sqrt{M_A} = -N_B \sqrt{M_B})$

Concentration and Solubility :- (gas in Solid)

$$\boxed{S = \frac{m^3 \text{ solute at STP}}{m^3 \text{ solid} * \text{atm partial pressure of solute}}}$$

[STP = 0°c
1 atm]

$$\boxed{C_A = \frac{S * P_A}{22.414} \rightarrow \text{kgmol solute} / m^3 \text{ solid}}$$

$$\therefore \quad N_A = D_{AB} \frac{S\,(P_{A_1} - P_{A_2})}{22.414\,(z_2 - z_1)}$$

Concentration and permeability :- (gas in Solid)

$$\boxed{P_M = \frac{m^3 \text{ solute at STP} * 1 \text{ m thick Solid}}{\underset{\text{second}}{s} * m^2 \text{ area} * \text{atm pressure difference}}}$$

$$\boxed{P_M = D_{AB}\, S}$$

$$\left[C_B^* = \frac{P_B}{\underset{\text{henry's constant}}{H}} = \frac{P - P_A}{H} \right]$$

The relationship between the fluxes at the diffusion of gas in solid :-

1) closed system, constant pressure → equimolar counter diffusion
$(N_A = -N_B)$

2) open system → $\left(N_A \sqrt{M_A} = -N_B \sqrt{M_B}\right)$

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[STP = 0°c
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$$C_A = \frac{S * P_A}{22.414} \rightarrow \text{kgmol solute} / m^3 \text{ solid}$$

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Concentration and permeability :- (gas in Solid)

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$$\boxed{P_M = D_{AB}\, S}$$

$$\left[C_B^* = \frac{P_B}{\underset{\text{henry's constant}}{H}} = \frac{P - P_A}{H}\right]$$

membranes: separation of molecules based on size and on chemical interaction between the molecules

* diffusivity and solubility of salt less than water * (الملح بيمر ابطأ ٤ x)

* dry + الهوا اكتر من شكل : pores || *

* I have active layer or support layer on the membrane.

* Examples on diffusion in solids don't depend on structure:
Ro membrane, evaporation membrane

* T ↓ ∴ the process almost perfect

Lec 6

"Convective mass transfer"

↳ Mass transfer between a moving fluid and surface or between two immiscible moving fluids separated by a mobile interface.

Convective transfer depends on
* transport properties
* dynamic characteristics of fluid

- pump or similar device → Forced convection
- density difference → Free / natural convection

Note! Solution of higher density ∴ higher concentration

✱ in convection (natural) I have convection current ✱

$$\boxed{\text{Mass transfer Driving force} = C_s - C_b}$$

C_s → Surface concentration = saturated concentration of liquid

C_b → Bulk concentration

✱ if the componend is diluted ∴ $C_b \approx 0$

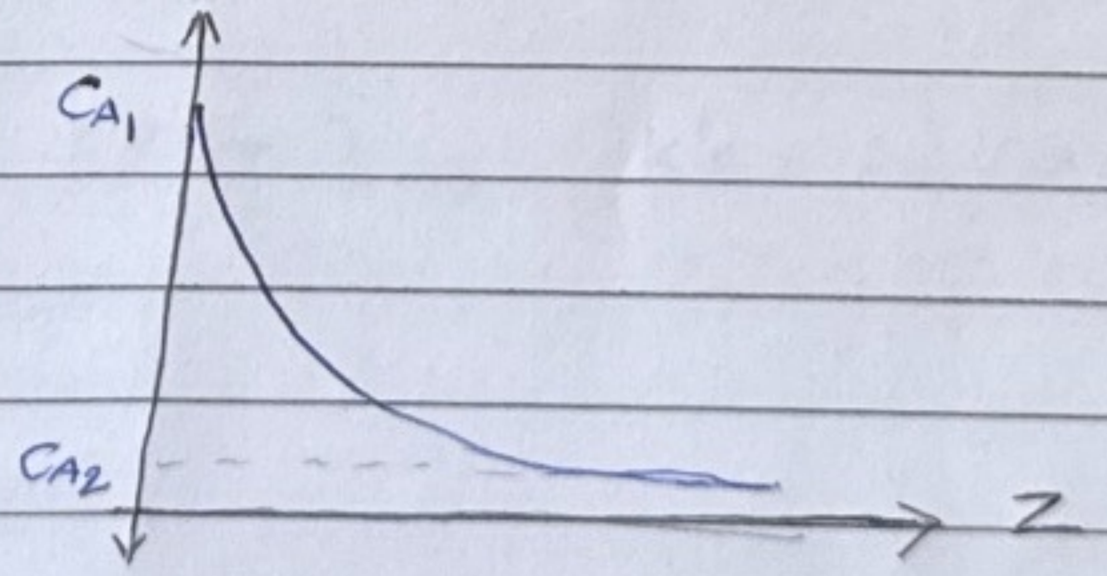
Benzoic acid → Slightly dissolving Solid material

✱ In turbulent motion there is no streamlines, but there are large eddies [chunks]

$$\text{distance} \propto \frac{1}{\text{concentration}}$$

- at surface C_{A_1} = max concentration
- at bulk C_{A_2} = constant concentration

Concentration profile



regions of mass transfer :-

1) viscous sub-layer → molecular diffusion
2) Buffer layer / transition → convection + diffusion
3) Turbulent layer → convection

Note/ (Turbulent diffusion = convection)

Cases of convection :-

[1] Only in single phase to or from phase boundary
    ex) sublimation of naphthalene (solid) into moving air
[2] in two contacting phases
    ex) extraction and absorption
    $\boxed{L+L}$ $\boxed{L+g}$

$$\boxed{n_A = k_c' A (C_{As} - C_A)} \qquad \boxed{N_A = k_c' (C_{As} - C_A)} \quad \underline{\underline{\text{flux}}}$$

- rate of mass transfer $\propto \Delta C_A$
- rate of mass transfer $\propto A$

k_c' : convective mass transfer coefficient

k_c' is a function of / depends on :-
1) geometry of the system
2) velocity
3) properties of the fluid

resistance $\propto \frac{1}{k_c'}$ $\qquad$ $\frac{1}{k_c}$ = resistance

$$\underline{\underline{k_c'}} = \frac{N_A}{\Delta C_A} = \left(\frac{m}{s}\right) \rightarrow [C = P/RT] \rightarrow k_G' = k_c'/RT$$

- using pressure : K_G / gas
- using concentration : K_L / liquid

* in equimolar we use (') prime

ex) k'_c , k'_G , k'_y , k'_L , K'_x

but in diffusing A with non-diffusing B we don't use prime

ex) k_c , k_G , k_L , k_y , k_x

Diffusing A with non-diffusing B

Example 8- large volume of gas B $P = 2$ atm flowing over surface which pure A is vaporizing [liquid A completely wets the surface] P_A at surface = 0.2 atm

so I can assume pure A

$k'_y = 6.78 \times 10^{-5}$ kg mol / s. m^2, mol frac

$N_A = ?$ $k_y = ?$ $k_G = ?$ B = non-diffusing, flow

large V is gas $\Longrightarrow$ $N_A = k_y (y_{A_1} - y_{A_2}) = 7.137 \times 10^{-5} * 0.1 = 7.137 \times 10^{-6} \frac{\text{kgmole}}{\text{s. m}^2}$

$y_{A_1} = \frac{P_{A_1}}{P_{total}} = \frac{0.2}{2} = 0.1$ $y_{A_2} = \frac{P_{A_2}}{P_{total}} = 0$ pressure of A in gas B is ≈ zero

$$k_y = \frac{k'_y}{y_{BM}} = \frac{6.78 \times 10^{-5}}{0.95} = 7.137 \times 10^{-5}$$

$$y_{BM} = \frac{y_{A_1} - y_{A_2}}{\ln\left(\frac{1-y_{A_2}}{1-y_{A_1}}\right)} = \frac{0.1}{\ln\left(\frac{1}{0.9}\right)} = 0.95$$

Between solid surface and fluid $\Longrightarrow$ Convection $N_A = k'_c (C_{A_S} - C_{A_\infty})$

at the surface $\Longrightarrow$ diffusion $N_A = -D_{AB} \frac{d(C_A - C_{As})}{dy}$

convection = conduction (diffusion)

$$k'_c (C_{As} - C_{A\infty}) = -D_{AB} \frac{d(C_A - C_{As})}{dy}$$

$$\downarrow$$

$$k'_c = -D_{AB} \frac{d(C_A - C_{As})/dy}{(C_{As} - C_{A\infty})} \longrightarrow \frac{k'_c}{D_{AB}} = -\frac{d(C_A - C_{As})/dy}{(C_{As} - C_{A\infty})}$$

multiplying by L $\longrightarrow$ $\frac{k'_c L}{D_{AB}} = -\frac{d(C_A - C_{As})/dy}{(C_{As} - C_{A\infty})/L}$

$$\left[\frac{k'_c L}{D_{AB}} = Sh = \text{Sherwood number} = \frac{\text{total mass transfer rate}}{\text{mass transfer by diffusion}} \right]$$

or = diffusion resistance to convection resistance !

"Convective Parameters"

Three transport process by molecular diffusion :-

1) Momentum

here, Reynolds and friction factor => play major role

(momentum diffusivity $\nu = \frac{\mu}{\rho}$)

2) Heat

here, Pr and Nu => play major role

(thermal diffusivity $\alpha = \frac{k}{\rho C_p}$)

3) Mass

(Mass diffusivity D_{AB})

here, group of dimensionless number is important (next section)

* Pr = (Momentum diffusivity / thermal diffusivity) $= \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$

* Schmidt number $= Sc =$ (Momentum diffusivity / Mass diffusivity) $= \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$

* Lewis number $= Le =$ (thermal diffusivity / mass diffusivity) $= \frac{\alpha}{P_{AB}}$

ملاحظات :

- كل م نقترب على surface التركيز رح يزيد وكل م نبتعد رح يقل
- لما نوصل steady state الضغط يكون متساوي

"Dimensionless groups"

Variables that affect the mass transfer coefficient :- = 6 variables

$$K'_c = f(D \text{ or } L, D_{AB}, \rho, \mu, \bar{u}_x)$$

* The number of Ⓟ groups required to give relation is (3)

⇒ we use Buckingham pi Theorem

$$\frac{k'_c D}{D_{AB}} = \varphi\left[\frac{\rho D \bar{u}_x}{\mu}, \frac{\mu}{\rho D_{AB}}\right]$$

or $Sh = \alpha Re^{\beta} Sc^{\gamma}$ → I have $\underline{\underline{3}}$ variables ✓

Note!: (Γ (vic) = $\dfrac{\text{mass rate of liquid flow}}{\text{unit of film width}}$)

$\boxed{j_D}$ = factor of mass transfer = $\boxed{\dfrac{K_c}{v_\infty}(Sc)^{2/3}}$

$\boxed{St}$ = Stanton number = $\dfrac{Sh}{Re\,Sc}$ = $\boxed{\dfrac{K_c}{\bar{u}_x}}$ $\dfrac{\text{total mass transfer}}{\text{inertia forces}}$

$\boxed{Pe}$ = Peclet number = $Re\,Sc$ = $\boxed{\dfrac{L u_x}{D}}$ $\dfrac{\text{inertia forces}}{\text{mass trans. by diffusion}}$ = $\dfrac{\text{inertia}}{\text{viscous forces}}$

"Flat plate"

mometeum B.L = δ_η

concentration B.L = δ_c

hydrodynamic B.L = δ

$$\frac{\delta_\eta}{\delta_c} = Sc^{1/3}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

$$\delta_c = 5\,Sc^{1/3}\sqrt{\frac{ux}{u_\infty}}$$

$Sc < 1$	$Sc = 1$	$Sc > 1$
$(\delta_c > \delta)$	$(\delta_c = \delta)$	$(\delta_c < \delta)$

$Sc \propto \dfrac{1}{\delta_c}$ مهم

average sherwood over the entire plate :- (flow parallel to flat plate)

$$\Rightarrow Sh_L = 0.664 \underset{\substack{\downarrow \\ \text{for} \\ \text{transition}}}{(Re_t)}^{1/2} Sc^{1/3} + 0.0365\, Sc^{1/3} \left[Re_L^{4/5} - Re_t^{4/5} \right]$$

"inside circular pipe"

For laminar flow $(10 < Re < 2000)$ → $Sh = 1.86 \left(\frac{D}{L} Re\, Sc \right)^{1/3}$

For turbulent flow → tables !!

Note ! to find exit concentration we use figure !!

"Liquid film in Wetted wall"

$$\left(Re = \frac{4\Gamma}{\mu_L} = \frac{4 \overset{\text{rate (L)}}{w}}{\pi \underset{\substack{\Downarrow \\ \text{inner} \\ \text{diameter}}}{D} \mu_L} \right)$$

"single sphere"

$\boxed{Gr}$ = Grashof number = $\boxed{\frac{D^3 \rho g \Delta \rho}{\mu^2}}$

There is table for each geometry in the slides !!

Example 8- tube inside is napthalene, $D = 20\ mm$

$L = 1.10\ m$. Air at 318 K and 101.3 kpa flow in the pipe

$v = 0.8\ m/s$, find exist concentration. $C_{A0} = 0$

$D_{AB} = 6.92 \times 10^{-6}$

$Re = ?$ $Sc = ?$

air :-

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{0.8\ m}{s} \Big| \frac{20 \times 10^{-3}\ m \cdot s}{1.77 \times 10^{-5}\ m^2} = 903.95$$

interpolation to find ν → $$\frac{20.92 \times 10^{-6} - 15.89 \times 10^{-6}}{350 - 300} = \frac{\nu - 15.89 \times 10^{-6}}{318 - 300}$$

$$\nu = 1.77 \times 10^{-5}\ m^2/s$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.77 \times 10^{-5}}{6.92 \times 10^{-6}} = 2.56$$

$$Re * Sc * \frac{D}{L} * \frac{\pi}{4} = 33.05$$

P_{Air} at interface = 74.0 pa $\qquad C_{Ai} = \frac{P_{Ai}}{RT} = 2.80 \times 10^{-5}$

From chart $\frac{C_A}{C_{Ai}} = 0.55$ → $C_A = 1.54 \times 10^{-5}\ \frac{kgmol}{m^3}$

Example :- (large Volume) of water at 299.25 k, parallel to flat plate of solid benzoic acid, $L = 0.244$ m
$V = 0.061$ m/s, [Solubility = 0.02948 kg mol/m³] → concentration
$D_{AB} = 1.245 \times 10^{-9}$ m²/s, Find $K_L, N_A = ?$

Water :- $\rho = 997$ kg/m³ $\mu = 855 \times 10^{-6}$ N.s/m²

$$Re = \frac{\rho V L}{\mu} = \frac{997 \cancel{kg}}{m^3} \Big| \frac{0.061 \cancel{m}}{\cancel{s}} \Big| \frac{0.244 \cancel{m} \cdot \cancel{m^2}}{855 \times 10^{-6} \cancel{N} \cdot \cancel{s}} \Big| \frac{\cancel{N} \cdot \cancel{s^2}}{\cancel{kg} \cdot \cancel{m}}$$

$$= 1.6 \times 10^{4}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}} = \frac{855 \times 10^{-6}}{997 * 1.245 \times 10^{-9}} = 688.8$$

$$J_D = 0.99 * (Re)^{-0.5} = 7.84 \times 10^{-3}$$

$$J_D = \frac{k'_c}{\upsilon} * (Sc)^{2/3} \rightarrow k_c = 6.13 \times 10^{-6}$$

large Volume ∴ diluted ∴ $C_{A_2} = 0$ ∴ $k'_c \approx k_c$

$$N_A = k'_c * 0.02948 = 1.81 \times 10^{-7} \text{ kg mol/s.m}^2$$

Example : mass transfer coefficient and flux = ?

sphere , $T = 45\,^{\circ}C$, $P = 1$ atm , $v = 0.305$ m/s , $D = 25.4$ mm

$D_{AB} = 6.92 \times 10^{-6}$ m^2/s

$P_{naph} = 0.555$ mmHg

! I have sublimation of naphtalene !

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{0.305 * 25.4 \times 10^{-3}}{1.77 \times 10^{-5}} = 437.68$$

$$Sc = \frac{\nu}{D_{AB}} = 2.6$$

$$Sh = \frac{k_c' D}{D_{AB}} = 2 + 0.552 * Re^{1/2} * Sc^{1/3}$$ [I have air : gas

$$\frac{k_c D}{D_{AB}} = \underbrace{2 + 15.88} \rightarrow k_c' = 4.87 \times 10^{-3} \text{ m/s}$$

→ or bubbles

Mass transfer of small particles < 0.6 mm :-

1) from $g \to L$

2) from $L \to$ surface of catalyst particle

$$\left[k'_L = \frac{2 D_{AB}}{D_p} + 0.31 * (Sc)^{-2/3} * \left(\frac{\Delta\rho \mu_c g}{\rho_c^2} \right)^{1/3} \right]$$

- ρ_c : density of the continuous phase
- ρ_p : density of gas or solid particle
- μ_c : viscosity of the solution (kg/m·s)

Note : value of $\Delta\rho$ always positive.

Mass transfer of larger particles (or bubbles) > 2.5 mm :-

$$\left[k'_L = 0.42 * (Sc)^{-0.5} * \left(\frac{\Delta\rho \mu_c g}{\rho_c^2} \right)^{1/3} \right]$$

Example :- $N_A = ?$ air bubble : $D = 100\ \mu m$ at 37°c, 1 atm

$C_{A_1} = 0$ $C_{A_i} = 2.26 \times 10^{-4}$ kg mol O_2 / m^3

$D_{AB} = 3.25 \times 10^{-9}$ m^2/s

$\rho_{Air} = 1.12812$ kg/m³ $\rho_{water} = 993.05$ kg/m³ $\mu_{water} = 695 \times 10^{-6}$

$$k'_L = 6.5 \times 10^{-5} + 0.31 * (215.3)^{-2/3} * \left(\frac{991.9 * 695 \times 10^{-6} * 9.81}{(993.05)^2} \right)^{1/3}$$

$$Sc = \frac{\mu}{\rho D_{AB}} = 215.3$$

$\therefore k'_L = 6.5 \times 10^{-5} + 0.31 * 5.2889 \times 10^{-4}$

$[k'_L = 2.29 \times 10^{-4}\ m/s]$ *

$k'_c C = k'_L C$ so $k'_c \approx k'_L$

$N_A = k_L (C_{A_1} - C_{A_2}) = 5.17 \times 10^{-8}$ kg mol O_2 / s.m² #

Transfer to and from "packed bed":-

Examples * drying operations

such as ↔ * adsorption or desorption of gas or liquid by solid particles
charcoal

* mass transfer of gas and liquid to catalyst particles

area of mass transfer → $\left[a = \frac{6(1-\varepsilon)}{D_p} \right]$

total external surface area → $\left[A = a V_b \right]$

V_b = total volume
= (void + solids)

$$N_A A = \left[A k_c \frac{(C_{Ai} - C_{A1}) - (C_{Ai} - C_{A2})}{\ln \left(\frac{(C_{Ai} - C_{A1})}{(C_{A1} - C_{A2})} \right)} \right]$$

$$\left[N_A A = \underbrace{v}_{\text{volumetric flow rate}} (C_{A2} - C_{A1}) \right]$$

$Q = VA$

Example :- pure water / $Q = V = 5.514 \times 10^{-7}\ m^3/s$ / packed bed

$D_p = 6.375\ mm$ $\varepsilon = 0.436$ $C_{Ai} = 2.948 \times 10^{-2}\ kg\,mol/m^3$

of sphere ← $A = 0.01198\ m^2$ $D_{tower} = 0.0667$ $C_{A1} = 0$

a) $k_c = ?$ $\rho = 99.7\ kg/m^3$ $\mu = 855 \times 10^{-6}$

$(D_{AB} = 1.21 \times 10^{-9}\ m^2/s)$ at $25°c$ and $\mu = 0.8940 \times 10^{-3}$

$$Re = \frac{\rho \overset{?}{v} D \xrightarrow{} \text{particle}}{\mu} = 1.173$$

$$V = \frac{Q}{A_{tower}} = \frac{5.514 \times 10^{-7}\ m^3/s}{\frac{\pi}{4} D^2 \xrightarrow{} \text{of tower}} = 1.578 \times 10^{-4}\ m/s$$

$$Sc = \frac{\mu}{\rho D_{AB}} = 714.6$$

$$\boxed{Sh = \frac{k'_c L}{D_{AB}}}$$

$$D_{AB} \text{ at } 26.1\,°c = 1.21 \times 10^{-9} * \left(\frac{299.1}{298}\right) * \left(\frac{0.8940 \times 10^{-3}}{855 \times 10^{-6}}\right)$$

$$= 1.2 \times 10^{-9}\ m^2/s$$

$$J_D = \frac{k_c}{\text{velocity}} (Sc)^{2/3} = \frac{1.09}{\varepsilon} * Re^{-2/3}$$

$k_c = 5.5 \times 10^{-6}\ m/s$

Lec $\underline{\underline{7}}$

Types of interface in separation processes :-

- Gas / liquid interface
- liquid / liquid interface
- fluid / solid interface

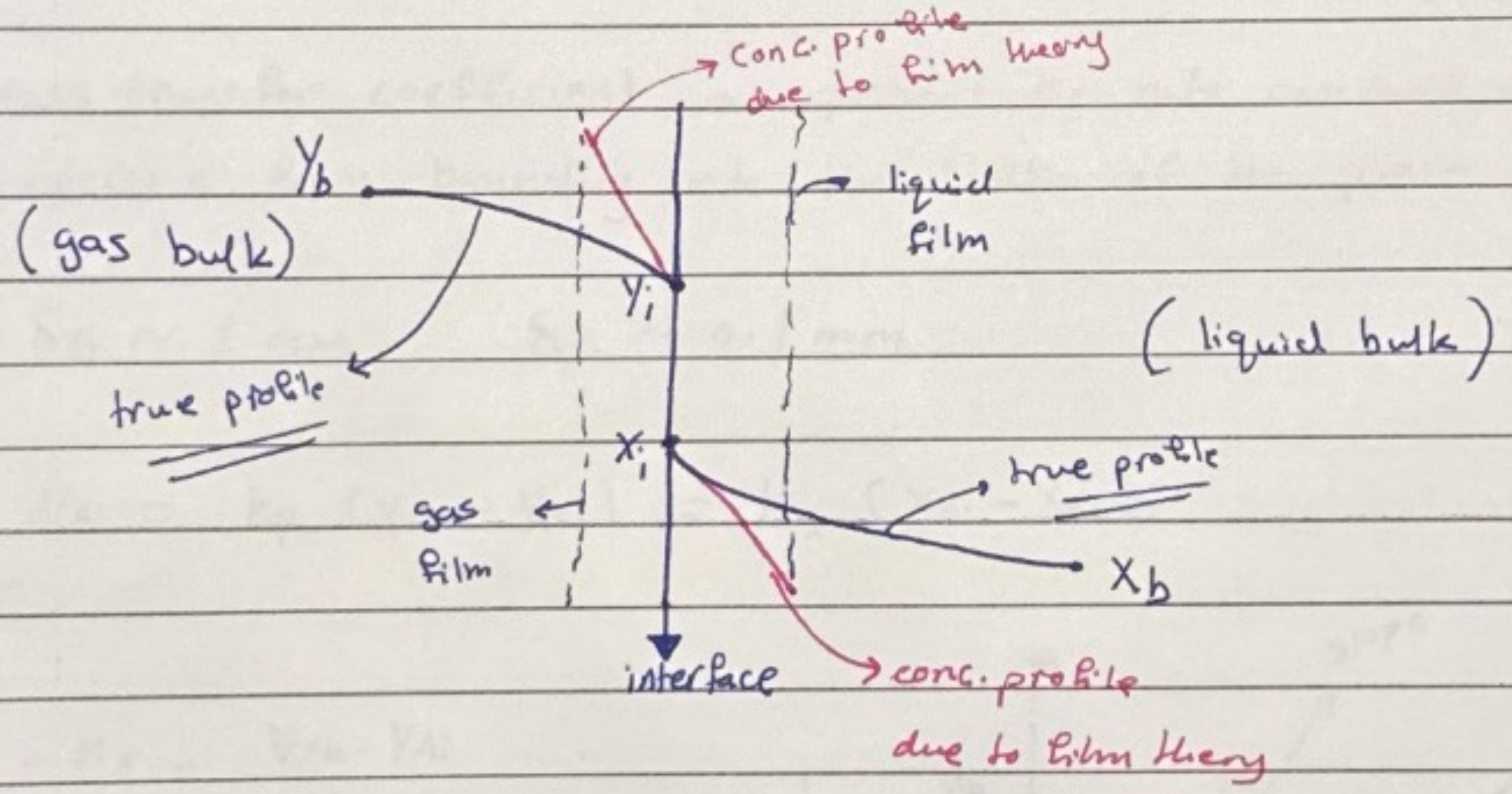
These interfaces can be found in Absorption, Distillation, Extraction / stripping

Absorption → gas / liquid

Distillation → vapor / liquid

Extraction / stripping → liquid / liquid

Note : I have real (true)^{1} concentration profile and other concentration $\underline{\underline{2}}$ profile according to film theory.



* y_i and x_i are realated by equilibrium distribution relation *

$$\boxed{y_{A_i} = H' x_{A_i}} \quad \boxed{P_{A_i} = H x_{A_i}} \quad \boxed{y_A^* = H x_{A_b}} \quad \boxed{y_{A_b} = H x_A^*}$$

H: Henry's constant in [atm / mole fraction]

$$H' = \frac{H}{P_{total}}$$

Two-film theory :- [Steady state]

- two fluids in contact ✓
- two stagnant films exist on either sides of interface in series ✓
- each film represent a resistance to mass. trans ✓
- two fluids at the interface assumed to be in equilibrium ✓

[Boundary layer = film] → here I have "resistance"

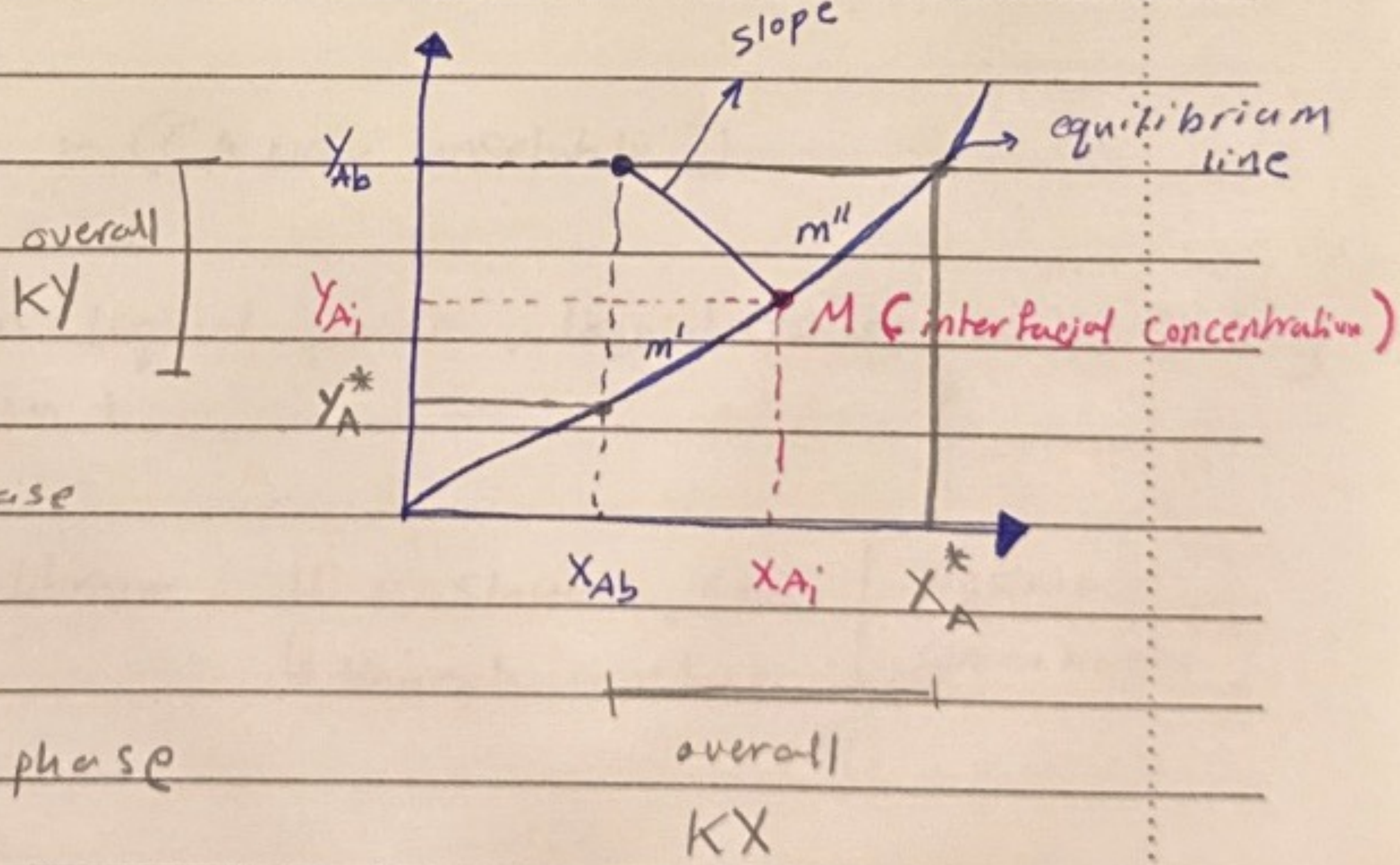
Note! if I have pure species then the concentration is uniform (constant) and I don't have concentration profile.

Mass transfer coefficient → represent the rate constant for moving species from boundary into the bulk of the phase.

($\delta_G \sim 1$ mm) ($\delta_L \sim 0.1$ mm)

$$\boxed{N_A = k_y (y_{Ab} - y_{Ai}) = k_x (x_{Ai} - x_{Ab})}$$

$$\underbrace{\frac{-k_x}{k_y}}_{\text{slope}} = \frac{y_{Ab} - y_{Ai}}{x_{Ab} - x_{Ai}}$$



$$KY = \frac{N_A}{y_{Ab} - y_A^*}$$ gas phase

$$kX = \frac{N_A}{x_A^* - x_{Ab}}$$ liquid phase

Note! interfacial concentrations are not directly measurable quantities or specified in practical problems.

$\frac{1}{KY}$ → overall mass transfer resistance on gas phase base

$\frac{1}{KX}$ → overall mass transfer resistance on liquid phase base

1 if m' is small :- ((A very soluble))

- the curve almost horizontal
- $m'/K_x \approx 0$ very small
- major resistance in gas phase, gas phase is controling

= low concentration in g

2 if m' is large :-

- major resistance in liquid phase, liquid phase is controling

= low concentration in L

3 if m'' is very large :- ((A very insoluble))

- $1/m'' k_y \approx 0$ very small
- major resistance in liquid phase, liquid phase is controling

= Low concentration in L

Note! in Linear equilibrium H = slope $[=] \left[\frac{\text{pressure}}{\text{concentration}}\right]$
↳ Henry's constant

Example 8 - Solute $\overset{\text{diffuse}}{A}$ absorbed from mixtre $A + \overset{\text{Stagnant}}{B}$

$y_{AG} = 0.380$, $x_{AL} = 0.1$ [dilute solution]

$y_{Ai} = ?$
$x_{Ai} = ?$
$N_A = ?$

$k'_y = 1.465 \times 10^{-3}$ kgmolA / s.m^2. mol fraction

$k'_x = 1.967 \times 10^{-3}$ kgmolA / s. m^2. mol fraction

for diluted solution $k'_y \approx k_y$ and $k'_x \approx k_x$

[redacted]

[redacted] [redacted]

[redacted]

- gas phase → $N_A = k_y \dfrac{(y_{AG} - y_{Ai})}{(1-y_A)_{lm}}$ diffusing
- liquid phase → $N_A = k_x \, (x_{Ai} - x_{AL})$ nondiffusing

assume steady state $\left[k_y \dfrac{(y_{AG} - y_{Ai})}{(1-y_A)_{lm}} = k_x \, (x_{Ai} - x_{AL}) \right]$

at steady state y_{Ai} and x_{Ai} at equilibrium

initial guess $x_{Ai} = 0.2$ and from the data $y_{Ai} = 0.131$

$\therefore$ find $(1-y_A)_{lm} = \dfrac{(1-y_{Ai}) - (1-y_{AG})}{\ln [(1-y_{Ai}) / (1-y_{AG})]} = \dfrac{0.869 - 0.62}{\ln 0.869 / 0.62}$

$= 0.738$

$N_A = 4.94 \times 10^{-4}$ kgmol / s. m^2 → from here I will find x_{Ai} to see if my initiay guess is true or not!

$N_A = k_x (x_{Ai} - x_{AL})$

$4.94 \times 10^{-4} = 1.967 \times 10^{-3} (x_{Ai} - 0.1)$

$x_{Ai} = 0.351$

↳ far from our inital guess

second trial $x_{Ai} \approx 0.30 \rightarrow y_{Ai} \approx 0.265$

$$(1-y_A)_{lm} = \frac{0.735 - 0.62}{\ln(0.735/0.62)} = 0.675$$

$N_A = 2.49 \times 10^{-4}$

$x_{Ai} = 0.227 \rightarrow$ قرّبنا بس طلعت أقل اذاً

، ح أخذ تجربة ثالثة نهائية

third trial $x_{Ai} \approx 0.25 \rightarrow y_{Ai} = 0.187$

$$(1-y)_{lm} = \frac{0.813 - 0.62}{\ln(0.813/0.62)} = 0.712$$

$N_A = 3.97 \times 10^{-4}$

$x_{Ai} = 0.3$

So $x_{Ai} \approx 0.25 \rightarrow y_{Ai} = 0.187$