CH.2 Cost Concepts & Design Economics

Cost Catagories:

- Fixed cost : unaffected by changes
- Variable cost : costs that vary with the quantity of output
- Incremental cost : additional cost resulting from increasing the output of a system

A contractor has a choice of two sites on which to set up the asphalt-mixing plant equipment.
It costs \$2.75 per cubic yard mile (yd3-mile) to haul the asphalt-paving material from the mixing plant to the job location.
Factors relating to the two mixing sites are as follows (production costs at each site are the same):

Cost Factor	Site A	Site B	
Average Hauling Distance	4 miles	3 miles	variable
Monthly Rental of Site	\$2,000	\$7,000	fixed
Cost to set up and remove equipment	\$15,000	\$50,000	fixed
Hauling Expenses	\$2.75/yd^3-mile	\$2.75/yd^3-mile	variable
Flagperson	Not required	\$150/day	fixed

The job requires 50,000 cubic yards of mixed-asphalt-paving material.
Four months (17 weeks of five working days per week) will be required for the job.
Required:

1. Compare the two sites in terms of their fixed, variable, and total costs.
2. Assume that the cost of the return trip is negligible. Which is the better site?
3. For the selected site, how many cubic yards of paving material does the contractor have to deliver before starting to make a profit if paid \$12 per cubic yard delivered to the job location, if the cost of material is \$1.5/yd3. At what point does he break even and begin to make a profit?

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Site A

Hauling costs $= 2.75/yd^3mile \times 4\,miles \times 50{,}000\,yd^3 = 550{,}000$

Rent $+\ 8000$

Set up & Removal $+\ 15{,}000$

$573{,}000$

Site B

Hauling Costs $= 2.75/yd^3\text{-}mile \times 3\,miles \times 50{,}000\,yd^3 = 412{,}500$

Rent $+\ 7000$

Setup & Removal $+\ 50{,}000$

Flagperson $+\ 150(17)(5)$

$482{,}250$ ↳ less costs.

c) Material $= 1.5/yd^3$

Revenue $= 12/yd^3$

total expenses = total revenue

$$1.5x + 2.75(3)(x) + 7000 + 50{,}000 + 150(17)(5) = 12x$$

$x = 40.33\ yd^3$ of Asphalt

Other Catagories of Cost

- Direct costs
- Indirect costs (overhead or burden) : difficult to allocate to a specific activity
- Standard costs (established ahead of production or service dilivery)
  - ↳ Estimating future manufacturing costs
  - ↳ Measuring operation performance by comparing (actual vs standard)
  - ↳ preparing bids on products or services
  - ↳ Establishing the value of work in process & finished inventories

Cost Terminology

- Cash cost : involves payment in cash & results in cash flow
- Book cost : doesnot involve cash transaction [noncash cost] , cost of a past transaction recorded in a book
- Sunk cost : payment occurred in the past with no relevence to the future cost & revenue estimates
- Opportunity costs ; monetary advantage foregone due to limited resources or the cost of the best rejected opprotunity
- Life cycle costs : Summation of all costs related to a product → during its lifespan
  - ↳ Investment costs
  - ↳ operation & maintenance
  - ↳ Disposal costs

Life-cycle cost example:

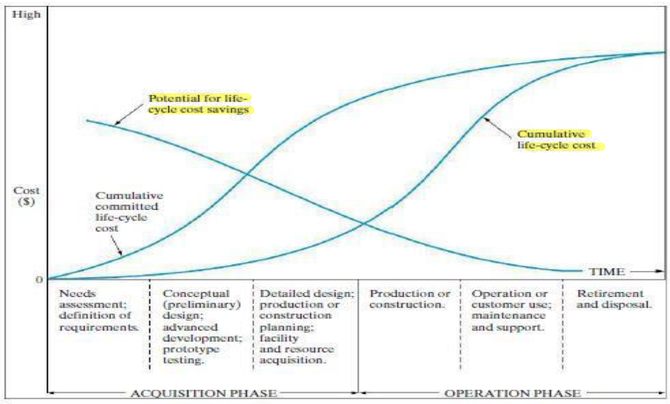

Figure 2-1 Phases of the Life Cycle and Their Relative Cost

The longer the project → less life-cycle savings

Cumulative value → S-curve

General Price - demand Relationships

constants depending on product or service

$$p = a - bD$$

Price   Demand

$a > 0$, $b > 0$

$0 < D < a/b$ $\rightarrow$ $D = (a-p)/b$

→ Total Revenue & Breakeven point

Demand is a function of price

Profit = Total revenue − Total costs

$$\text{profit} = -bD^2 + (a - C_v)D - C_f$$

→ Total costs (C_T) = fixed cost (C_f) + variable cost (C_v)

To maximize profit $\dfrac{\partial(\text{Profit})}{\partial D} = a - C_v - 2bD = 0$

Optimal D $D^* = \dfrac{a - C_v}{2b}$

→ Two Breakeven points $D' = \dfrac{-(a - C_v) \pm \sqrt{(a-C_v)^2 - 4(-b)(-C_f)}}{2(-b)}$

Demand is independent of Price

Profit = Total revenue − total costs

$$\text{profit} = pD - (C_f + C_v D)$$

→ only one breakeven point

→ no optimal Demand

Example 15

A company produces an electronic timing switch that is used in consumer and commercial products. The fixed cost (C_F) is $73,000 per month, and the variable cost (c_v) is $83 per unit. The selling price per unit is

$$p = \underbrace{\$180}_{a} - \underbrace{0.02}_{b}(D)$$

1) Determine the optimal volume for this product and confirm that a profit occurs (instead of a loss) at this demand.

2) Find the volumes at which breakeven occurs; that is, what is the range of profitable demand?

optimal Demand $D^* = \frac{a - c_v}{2b} = \frac{180 - 83}{2(0.02)} = 2425$

$\text{Profit} = -bD^2 + (a - c_v)D - C_F$

$-0.02D^2 + (180 - 83)D - 73{,}000 = 0$

Derive $\rightarrow$ $-0.04D + 97 = 0$

negative to indicate profit

$D = 2425$

b) Break even points $D_1 = 932$

$D_2 = 3914$

Example 16

An engineering consulting firm measures its output in a standard service hour unit. The variable cost (c_v) is $62 per standard service hour and the charge-out rate [i.e., selling price (p)] is $85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost (C_F) is $2,024,000 per year.

1. What is the breakeven point in standard service hours and in the percentage of total capacity?
2. what is the percentage reduction in the breakeven point (sensitivity) if fixed costs are reduced 10%; if variable cost per hour is reduced 10%; and if the selling price per unit is increased by 10%?

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$\text{Profit} = pD - (C_F + C_vD)$

$0 = 85.56(D) - (2{,}024{,}000 + 62(D))$ $D = 85908$ h to break even

% of total capacity $\frac{85908}{160{,}000} = 53.69\%$

2) C_F reduced by 10%

$$0 = 85.56D - (0.9(2{,}024{,}000) + 62(D))$$

$$D = 77317$$

% reduction in Service $\frac{85908 - 77317}{85908} = 10\%$ reduction

C_V reduced by 10%

$$0 = 85.56D - (2024000 + 0.9(62)(D))$$

$$D = 68011$$

% reduction $\frac{85908 - 68011}{85908} = 20\%$

Price increased by 10%

$$0 = 1.1(85.56)D - (2024000 + 62D)$$

$$D = 63014$$

% reduction $\frac{85908 - 63014}{85908} = 26.6\%$

Example 17

The demand for a certain part is 100,000 units. The part is produced on a highspeed turret lathe, using screw-machine steel costing $0.30 per pound. A study was conducted to determine whether it might be cheaper to use brass screw stock, costing $1.40 per pound. Because the weight of steel required per piece was 0.0353 pounds and that of brass was 0.0384 pounds, the material cost per piece was $0.0106 for steel and $0.0538 for brass. However, when the manufacturing engineering department was consulted, it was found that, although 57.1 defect-free parts per hour were being produced by using steel, the output would be 102.9 defect-free parts per hour if brass were used. Assuming the machine attendant is paid **$15.00 per hour**, and the variable (i.e., traceable) overhead costs for the turret lathe are estimated to be **$10.00 per hour**. Which material should be used for this part?

Unknown or constant revenue (demand is constant) → compare the cost per defect-free unit

Steel

Material $0.3 \ \$/\text{Pound} \times 0.0353 \text{ Pounds} = 0.01059$

Labour $15 \ \$/\text{hour} \times \frac{1 \text{ hour}}{57.1 \text{ defect free}} = 0.26278$

overhead $10 \ \$/h \times \frac{h}{57.1} = 0.1751$

total per peice = 0.4484

Brass

Material $1.4 \ \$/\text{Pound} \times 0.0384 \text{ Pounds} = 0.05376$

Labour $15 \ \$/h \times \frac{h}{102.9} = 0.1458$

overhead $10 \ \$/h \times \frac{h}{102.9} = 0.0972$

total per peice = 0.2968

select Brass

Ch 3: Cost estimation techniques

→ Cost estimation is useful for:

- Setting up a selling price for a quote or abid
- Determining if a product will be profitable
- Justifying capital for process changes or improvements
- Setting benchmarks for productivity improvements

Top down approach

Good for early estimates when developing alternatives

Uses historical data from similar projects with adjustments to account for inflation, deflation & other factors [one entity project (no details)]

Bottom up approach

More detailed approach

Project is broken down into small units

The estimated overall cost is the sum of the units costs + other costs [detailed, more accurate value]

Top down

Suppose that the published cost of attending your university is $15,750 for the current year. This figure is anticipated to increase at the rate of 6% (inflation Rate) per year and includes fulltime tuition and fees and a weekly meal plan.

Not included are the costs of books, supplies, and other personal expenses. For the initial estimate, these "other" expenses are assumed to remain constant at $5,000 per year.

		other	tot
year 1	$15,750 \times (1+0.06) = 16695$	5,000	21,695
year 2	$16,695 \times (1.06) = 17,697$	5,000	22,697
year 3	$17,697 \times (1.06) = 18759$	5,000	23,759
year 4	$18,759 \times (1.06) = 19885$	5,000	24,885
			Σ 93,036

Bottom up

Break down anticipated expenses into the typical catagories for each of the four years.

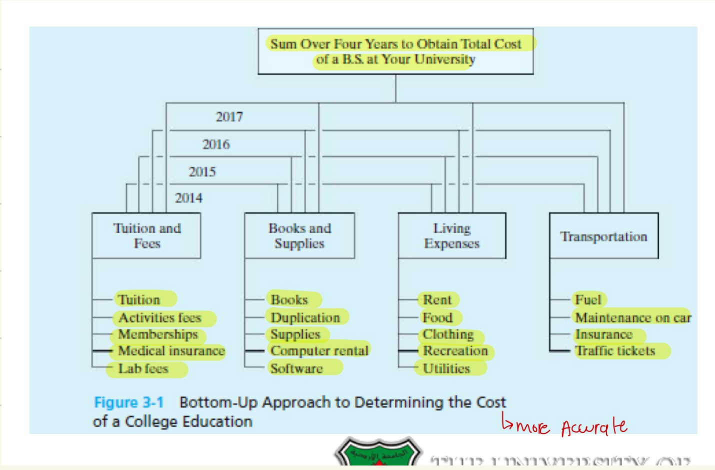


Figure 3-1 Bottom-Up Approach to Determining the Cost of a College Education

Integrated cost estimation approach

1. Work Breakdown Structure (WBS)

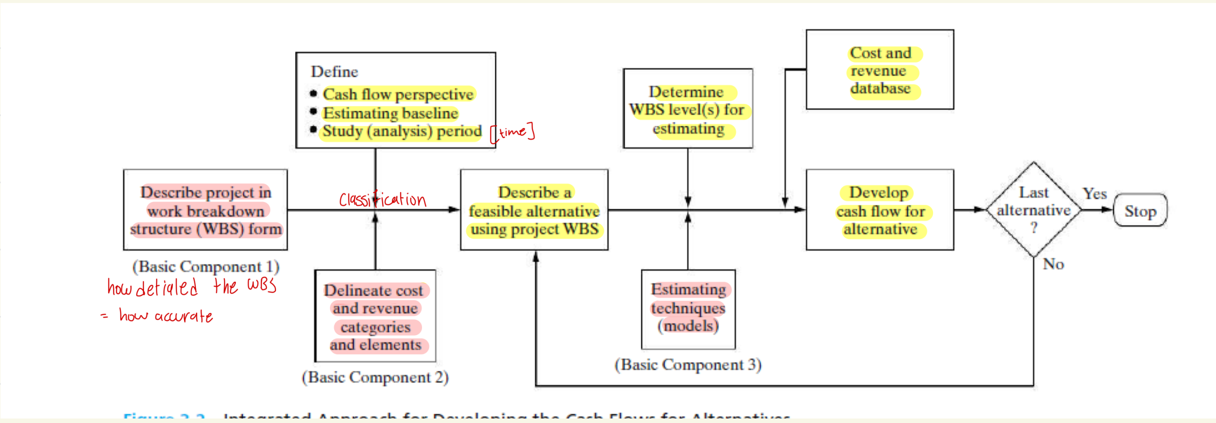
   Successive levels of the work elements & their interrelationships (details to achive max level of accuracy)

2. Cost & Revenue Structure (classification)

   Projection of cost & revenue catagories & elements for different WBS levels

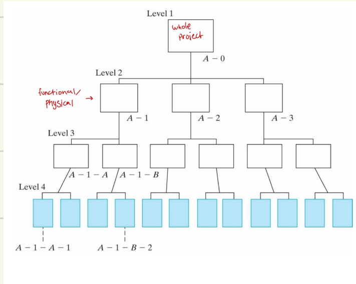
3. Estimating Techniques (Models)

   Selected mathematical models to estimate future costs & revenues.



1- Work Breakdown Structure

- WBS defines all project elements & their interrelationships, collecting & organizing information, & developing relevant cost & revenue data & managemet activities.



→ Recurring (maintenance)

→ Nonrecurring (initial construction)

WBS includes functional & physical work elements

→ Functional (logistic support, project management, & marketing)

→ Physical (labor, materials, & resources)

* Level 1 → Total project
* Level 2 → Physical & functional work elements
* Level 3 → Sub elements as required

↪ Important terms in WBS:

Level of Effort (LOE): how much work is required to complete a task

WBS code: unique identifier assigned to each element in a WBS for the purpose of designating the elements hierarchical location

Work Package: Deliverable or work component at the lowest level of it's WBS branch.

WBS component: A component located at any level

WBS element: Single WBS component & its associated attributes located anywhere.

2. Cost & Revenue Structure

• Cost & Revenue → Identified
↳ Catagorized

The most serious source of errors in developing cashflows is overlooking important catagories of cost & revenue.

• Cost & Revenue Structure is prepared by : checklist, life cycle concept & WBS.

3. Models

The goal is to develop cash flow projections, not exact future data

→ Order of magnitude estimates

• Planning an initial evaluation of a project to select feasible alternatives (± 30 – 50% accuracy)
level 1 or 2 on WBS

→ Semi detailed (or budget) estimates

• Preliminary or conceptual design stage of a project (± 15% accuracy)
• level 2 or 3 on WBS

→ Definative (detailed) estimates

• Detailed design estimates from drawing, specs, quotes (± 5% accuracy)
• level 3 & beyond

* The level of detail & Accuracy depends on:

1) Time & effort available
2) Difficulty of estimating the items in question
3) Methods or techniques employed
4) Qualification of the estimators
5) Sensitivity of study results to particular factor estimates

* Sources of Estimating Data

- Accounting Records:
  → not suitable for direct, unadjusted use
- Other Sources inside the firm:
  → Engineering, sales, production, quality & purchasing departments.
- Sources outside the firm:
  → Published information / personal contacts
- Research & Development (R & D)

* Estimating Techniques:

1) Indexs (Ratio technique)
2) Unit technique
3) Factor technique
4) Parametric cost estimating
   ↳ Power sizing technique
   ↳ learning curve.

Indexes [Ratio Technique]

An Index is a dimensionless number used to estimate present & future costs from historical data

$$C_n = C_k \times \frac{I_n}{I_k}$$

Index value in year n

Cost in year n

Cost in year k

Index value in year k

Ex) A company wants to install a new boiler. The price of the boiler in the year 2016 was $525,000 when the index was 468. What is the price of the boiler in 2021 given that the index value is 542 in the year 2021?

$C_{2016} = 525{,}000$

$I_{2016} = 468$

$C_{2021} = ?$

$I_{2021} = 542$

$$C_{2021} = 525{,}000 \times \frac{542}{468}$$

$$C_{2021} = 608{,}013$$

Unit Technige

→ Widely used & understood, Good for preliminary estimates

Ex) Suppose the Air Force's B-2 aircraft costs $68,000 per hour to own, operate, and maintain. A certain mission requires two B-2 aircraft to fly a total round-trip time of 45 hours.

$$68{,}000 \ \$/\text{hour} \times 45 \text{ hours} \times 2 = \$6{,}120{,}000$$ (2 aircrafts per mission)

Such average value can be misleading.

Factor Technique.

→ Extension of the unit technique, good for perliminary estimates.

$$C = \sum C_d + \sum f_m U_m$$

cost being estimated

cost of d

cost per unit m

number of units m

Ex) Suppose that we need a slightly refined estimate of the cost of a house consisting of 2,000 square feet, two porches, and a garage. Using a unit factor of \$85 per square foot, \$10,000 per porch, and \$8,000 per garage

Calculate the total estimate:

$$2{,}000 \text{ ft}^2 \times 85 \, \$/\text{ft}^2 + 10{,}000 \, \$/\text{poarch} \times 2 \text{ poarches} + 8000 \, \$/\text{garage} \times 1 \text{ garage} = 198{,}000 \, \$$$

Parametric cost estimation

- utilizing historical cost data & statistical techniques to predict future costs
- Used in early design stages to get an estimate of a product or project cost based on a few physical characteristics
  - → Power sizing technique
  - → learning curve

i) Power Sizing Technique

exponential model, used for industrial plants & equipment

$$C_A = C_B \left(\frac{S_A}{S_B} \right)^X$$

X: cost capacity factor [depends on the type of plant]

S_A, S_B: size of plant A & B respectively

The purchase price of a commercial boiler (capacity S) was \$181,000 eight years ago. Another boiler of the same basic design, except with a capacity of 1.42 S, is currently being considered for purchase. If the cost index was 162 for this type of equipment when the capacity S boiler was purchased and is 221 now, and the applicable cost capacity factor is 0.8, what is your estimate of the purchase price for the new boiler?

$$C_A = 181{,}000 \times \frac{221}{162} \qquad C_A = 246{,}920 \, \$$$

$$C = 246{,}920 \times \left(\frac{1.42S}{S} \right)^{0.8} = \$ \, 326{,}879$$

Suppose that an aircraft manufacturer desires to make a preliminary estimate of the cost of building a 600-MW fossil-fuel plant for the assembly of its new long-distance aircraft. It is known that a 200-MW plant cost $100 million, 20 years ago when the approximate cost index was 400, and that cost index is now 1,200. The cost-capacity factor for a fossil-fuel power plant is 0.79.

$$C_{today} = C_{20\,years\,ago} \times \frac{I_{today}}{I_{20\,years\,ago}}$$

$$C_T = 100{,}000{,}000 \times \frac{1200}{400} = 300 \text{ mill}$$

$$C_{new} = C_{old} \times \left(\frac{S_{new}}{S_{old}} \right)^x$$

$$C_{new} = 300 \text{ mill} \times \left(\frac{600}{200} \right)^{0.79} = 714 \text{ mill}$$

2) Learning Curve

Expirience curve or manufacturing progress function

Reflects increased efficiency & performance with repetitive production

$$Z_u = K(u^n)$$

K → number of input resources needed to produce the first output

u → output unit number

Z_u → number of input resources to produce output u

n → learning curve exponent $= \dfrac{\log S}{\log 2}$

S → learning curve slope parameter (decimal)

The time required to assemble the first car is 100 hours and the learning rate is 80%. What is the time required to assemble the 10th car?

$K = 100h$ $\quad S = 0.8$ $\quad u = 10$

$$Z_u = K(u^n)$$

$$Z_u = 100\,(10)^{\frac{\log 0.8}{\log 2}} = 47.65 \text{ hours}$$

You have been asked to estimate the cost of 100 prefabricated structures, each structure provides 1,000 sq.ft of floor space, with 8-ft ceilings. In 2018, you produced 70 similar structures consisting of the same materials and having the same ceiling height, but each provided only 800 sq.ft of floor space. The material cost for each structure was \$25,000 in 2018, and the **cost capacity factor is 0.65**. The cost index values for 2018 and 2023 are 200 and 289, respectively. The estimated manufacturing cost for the first 1,000 sq.ft structure is \$12,000. Assume a learning curve of 88% and use the cost of the 50th structure as your standard time for estimating manufacturing cost.

Estimate the total material cost and the total manufacturing cost for the 100 prefabricated structures?

cost →

$$C_{2023} = C_{2018} \frac{I_{2023}}{I_{2018}}$$

$$C_{2023} = 25000 \frac{289}{200} = 36,125$$

Sizing →

$$C_{new} = C_{old} \left(\frac{S_{new}}{S_{old}} \right)^{X}$$

$$36,125 \left(\frac{1000}{800} \right)^{0.65} = 41,764$$

cost

$K = \$1200$

$S = 0.88$

$$Z_{50} = k\,(u^{n})$$

$$1200\,(50)^{\frac{\log 0.88}{\log 2}} = 5,832 \text{ /unit}$$

Total cost: $(41,764 + 5,832) \times 100 = 4,759,600$

Chapter 4. Time Value of Money

→ Capital: refers to the wealth in the form of money or property that can be used to produce more money

Interest

→ Simple Interest: Not commonly used

Total interest is linearly proportional to the initial loan amount (principle)

→ Compound Interest: more common in personal & professional financing

Interest is based on the remaining principle + any accumulated Interest.

Simple Interest

when total interest earned or charged is linearly proportional to the initial amount of the loan (principle), interest rate, & the number of interest periods

$$I = P \times N \times i$$

P: Principle amount
N: Periods of interest
i: Interest Rate

* The total amount paid at the end of N periods = I + P

Example:
A $1,000 loan for 3 years at a simple interest rate of 10% per year.

$$I = P \times N \times i$$

$$1{,}000 \times 3 \times 0.1 = \$300$$

→ Total amount Repaid = P + I

1000 + 300 = $1300

Example: You borrowed $5,000 at a simple interest rate = 0.5% per month to be repaid after 4 years.

How much will you pay back? Or

What is the future equivalent of the borrowed $5,000?

$\rightarrow$ 4 years $\times \frac{12 \text{ months}}{\text{year}} = 48$ months

$I = P \times N \times i$

$5{,}000 \times 48 \times 0.005 = \$1200$

total amount Repaid : $1200 + 5000 = \$6200$

Compound Interest

→ Interest is based on the amount of remaining principle + accumulated interest

* The more you stretch the loan, the higher the interest.

Example: $1,000 loan for 3 years at a compound interest rate of 10% per year.

At year 1 $1000 \times 0.1 = 100$ Amount owed $= 1100$

At year 2 $1100 \times 0.1 = 110$ Amount owed $= 1210$

At year 3 $1210 \times 0.1 = 121$ Amount owed $= 1331$

→ Simple VS compound Interest

interest on amount of money after each period

Compound Interest > Simple Interest

↳ Interest on total amount of money

Repayment of $17,000 in Four Months with Interest at 1% per Month:

Simple

$17000 \times 4 \times 0.01 = 680$

$17000 + 680 = \$17680$

Compound

$17000 \times 0.01 = 170 \rightarrow 17170$

$17170 \times 0.01 = 171.7 \rightarrow 17341.7$

$17341.7 \times 0.01 = 173.417 \rightarrow 17515.117$

$17515.117 \times 0.01 = 175.15117 \rightarrow \$17690.27$

The concept of Economic Equivalence.

→ Used for comparing alternatives when time value of money is a factor (compound interest is involved)

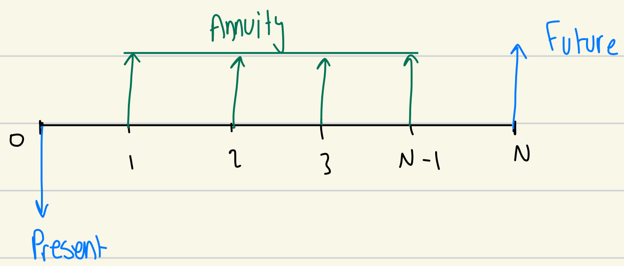
→ Each alternative can be reduced to an equivilent basis dependent on:

- Interest Rate
- Amount of money involved
- Timing of monetary reciepts or expenses.

→ Using these elements we can move cash flows so that we can compare them at particular points in time.

[cash flow diagram]

* comparision of cash flow can't happen at different time spaces [future, present]

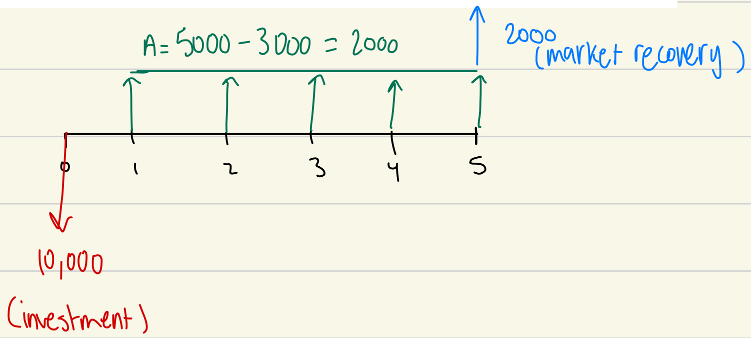


* past is not accounted for

↑ (upward arrows) : positive cash flow, cash inflow

↓ (downward arrows) : expenses, negative cash flow, cash outflow

Example: An investment of \$10,000 will produce a uniform annual revenue of \$5,000 for 5 years and have a market (recovery) value of \$2,000 at the end of year (EOY) five. Annual operating and maintenance expenses are estimated at \$3,000 at the end of each year. Draw a cash-flow diagram from the corporation's viewpoint.



In a company's renovation of a small office building, two feasible alternatives for upgrading the heating, ventilation, and air conditioning (HVAC) system have been identified. Either Alternative *A or* Alternative *B* must be implemented. The costs are as follows:

➢ Alternative *A: Rebuild (overhaul) the existing HVAC system*

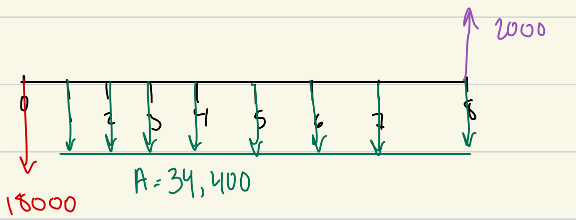
- Equipment, labor, and materials to rebuild : \$18,000
- Annual cost of electricity : \$32,000
- Annual maintenance expenses : \$2,400

➢ Alternative *B: Install a new HVAC system that utilizes existing ductwork*

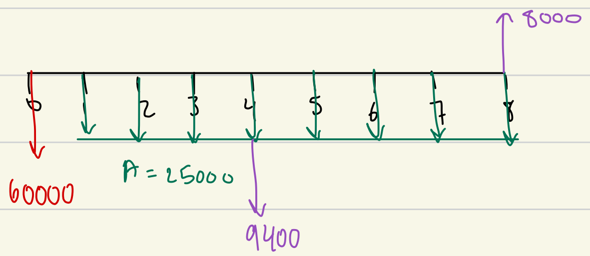
- Equipment, labor, and materials to install : \$60,000
- Annual cost of electricity : \$9,000
- Annual maintenance expenses : \$16,000
- Replacement of a major component after four years : \$9,400

At the end of eight years, the estimated market value for Alternative *A* is \$2,000 and for Alternative *B* it is \$8,000. Assume that both alternatives will provide comparable service (comfort) over an eight-year period, and assume that the major component replaced in Alternative *B* will have no market value at EOY eight.

Alternative A:



Alternative B



Relating Present & future equivalent values.

for a single cash flow & using the compound interest rate formula using the standard notation, we can find that a present amount (P), can grow into a future amount (F), in N time periods at interest rate i

$F = P(1+i)^N$ or $F = P(F/P, i\%, N)$

$P = F(1+i)^{-N}$ or $P = F(P/F, i\%, N)$

→ from tables Appendix C

Example:

Suppose that you borrow $8,000 now, promising to repay the loan principal plus accumulated interest in four years at i = 10% per year. How much would you repay at the end of four years?

$F = P(F/P, 10\%, 4)$ or $F = P(1+i)^N$

$F = 8000(1.4641) = 11,713$ $\quad$ $F = 8000(1+0.1)^4 = 11,713$

Finding the interest Rate (i) Given P, F, & N

$$i = \sqrt[N]{F/P} - 1$$

Example: What is the interest rate that will double an investment of $50,000 in 10 years?

$$i = \sqrt[10]{\frac{100,000}{50,000}} - 1 = 0.0718$$

$i = 7.18\%$

Finding N when given P, F, & i

$$N = \frac{\log(F/P)}{\log(1+i)}$$

Example: How many years does it take to double my money at an interest rate of 5% per year?

$$N = \frac{\log(2)}{\log(1+0.05)} = 14.2 \text{ years}$$

The concept of Economic Equivalence

Annuity: A series of uniform (equal) payments occurring at the end of each period for N periods

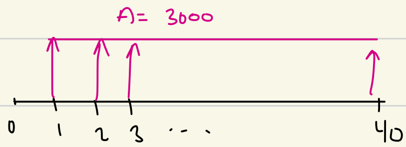
$$F = A\left[\frac{(1+i)^N - 1}{i}\right] \quad \text{or} \quad F = A(F/A, i\%, N)$$

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \quad \text{or} \quad P = A(P/A, i\%, N)$$

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] \quad \text{or} \quad A = F(A/F, i\%, N)$$

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] \quad \text{or} \quad A = P(A/P, i\%, N)$$

Example: How much will you have in 40 years if you invest $3,000 of your income each year in a project that earns 8% per year?



$F = A(F/A, 8\%, 40)$

$F = 3000(259.0565) = \$777,169.5$

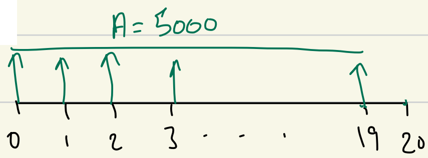
Example: You took a loan that is to be repaid in uniform payments over 4 years. Assuming the interest rate is 1% per month, and your monthly payment is $300. What is the principal amount (the amount of money borrowed)?

4 years x 12 months = 48 months

$P = A(P/A, 0.01, 48)$

$P = 300(37.9740) = 11,392.2$

Example: Calculate the compounded future value at EOY 20 of 20 annual payments of $5,000 each into a savings account that earns 6% per year. All 20 payments are made at the beginning of each year.



→ Future Annuity at year 19 $(0 \rightarrow 19 = 20)$ ↓ to convert Present to year 20

$F = A(F/A, 0.06, 20)(F/P, 0.06, 1)$

$5000(36.7856)(1.060) = \$194,963.68$

Example: A loan of $10,000 is to be repaid in 4 equal payments (over 4 years) and the interest rate is 10% per year. Determine the interest paid and principal repayment every year.

$P = 10{,}000 \quad N = 4 \quad i = 10\%$

$A = P(A/P, 10\%, 4)$

$10{,}000\,(0.3155) = 3155$ per year

		Interest	Annuity	Principle repayment
year 1	$10{,}000 \times 0.1$	= 1000	3155	2155
year 2	7845×0.1	= 785	3155	2370
year 3	5475×0.1	= 547	3155	2608
year 4	2867×0.1	= 287	3155	2868

Example: You borrowed $100,000 at an interest rate of 7% per year. If the annual payment is $8,000, how many years does it take to repay the loan?

$P = A(P/A, 7\%, N)$

$100\,000 = 8000\,(P/A, 7\%, N)$

$(P/A, 7\%, N) = 12.5$

→ By Interpolation

$$\frac{35-30}{N-30} = \frac{12.9477 - 12.4090}{12.5 - 12.4090}$$

$N = 30.73$

Example: You invested $20,000 in a project and you are expected to gain $4,000 annually. At a 10% interest rate, when will you recover your investment?

$$P = A\,\frac{(1+i)^N - 1}{i(1+i)^N}$$

$$20{,}000 = 4000\,\frac{(1+0.1)^N - 1}{0.1(1+0.1)^N}$$

$N = 7.27$ years

Example: Your company has a $100,000 loan for a new security system it just bought. The annual payment is $8,880 and the interest rate is 8% per year for 30 years. Your company decides that it can afford to pay $10,000 per year. After how many payments (years) will the loan be paid off?

$P = 100{,}000 \quad A = 8880 \quad i = 8\% \quad N = 30$

$A = 10{,}000$

$$P = A\,(P/A,\ 8\%,\ N)$$

$$100{,}000 = 10{,}000\,(P/A,\ 8\%)\ N$$

$$10 = (P/A,\ 8\%,\ N) \rightarrow \text{By interpolation} \quad \frac{21-20}{N-20} = \frac{10.0168 - 9.8181}{10 - 9.8181}$$

$$N = 20.9$$

Solving for i

Example: You wanted to start saving so that you will have $60,000 in your bank account eight years from now. Each year, you deposit $6,000 in your bank account. What should be the interest rate so you can achieve your goal?

$$F = A\,(F/A,\ i,\ N)$$

$$60{,}000 = 6{,}000\,(F/A,\ i,\ 8)$$

$$10 = (F/A,\ i,\ 8) \rightarrow \text{By interpolation} \quad \frac{0.07 - 0.06}{i - 0.06} = \frac{10.2598 - 9.8975}{10 - 9.8975}$$

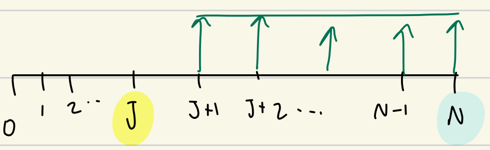
$$i = 6.29\%$$

Deferred Annuities

- Ordinary Annuity (uniform series) appears at the end of the first period
- Deffered Annuity (also uniform series) begins at a later time

Finding the value at time 0 of a deffered Annuity is a two step process

$$P = A\,(P/A,\ i\%,\ N-J)\,(P/F,\ i\%,\ J)$$

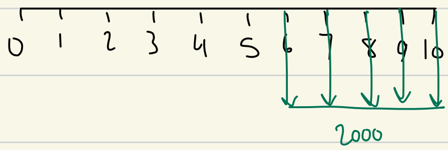


Example: You just purchased a new sports car and want to also set aside cash for future maintenance expenses. The car has a bumper-to-bumper warranty for the first five years. It was estimated that the car will need approximately $2,000 per year in maintenance expenses for years 6-10, at which you will sell the vehicle. How much money should you deposit into an account today, at 8% per year, so that you will have sufficient funds in that account to cover the projected maintenance expenses?

Present at year 5

$P = A(P/A, 8\%, 5)$

$P = 2000\ (3.9927) = 7985.4$

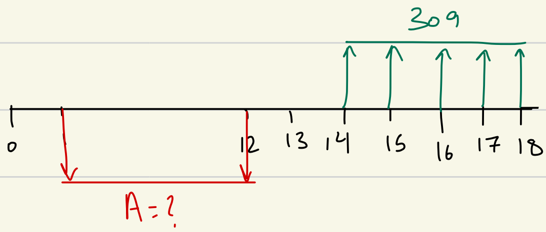


Present at year zero

$P = F(P/F, 8\%, 5)$

$P = 7985.4\ (0.6806) = 5434.86$

Example: How much money should be deposited each year for 12 years if you wish to withdraw $309 each year for five years, beginning at the end of the 14th year? Assume the interest rate is 8% per year.



At year 13

$P = A(P/A, 8\%, 5)$

$P = 309(3.9927) = 1233.7443$

At year zero

$P = F(P/F, 8\%, 13)$

$P = 1233.7443\ (0.3677) = 453.6477791$

Annuity

$P = A(P/A, 8\%, 12)$

$453.6477791 = A(7.5361)$

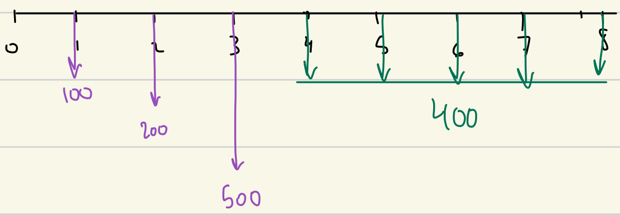
$A = 60.2$

Compounding Interest factor

* EXAMPLE 4-16: The cash flow below have a problem with a series of year-end cash flow extending over eight years. The amounts are $100 for the first year, $200 for the second year, $500 for the third year, and $400 for each year from the fourth through the eighth. These could represent something like the expected maintenance expenditures for a certain piece of equipment or payments into a fund.

Find the present equivalent expenditure if the annual interest rate is 20%.

* single & uniform payments



→ for single payments

$$P = F_1(P/F, 20\%, 1) + F_2(P/F, 20\%, 2) + F_3(P/F, 20\%, 3)$$

$$P = 100(0.833) + 200(0.6944) + 500(0.5787) = 511.53$$

→ Annuity

$$P = A(P/A, 20\%, 5)(P/F, 20\%, 3)$$

$$P = 400(2.9906)(0.5787) = 692.264088$$

$$P_{tot} = 692.264088 + 511.53 = 1203.8$$

Uniform (Arithmetic) gradient of cash flows

Cash flow that changes by a constant amount (G) each period

Present equivalent

$$P = G\left\{\frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]\right\}$$

$$P = G(P/G, i\%, N)$$

Annuity Equivalent

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = G\,(A/G, i\%, N)$$

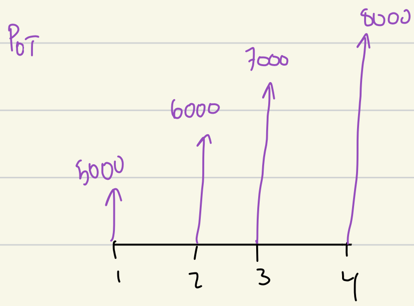
Future Equivalent

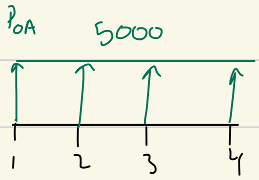
$$F = \frac{G}{i}\,(F/A, i\%, N) - \frac{N \times G}{i}$$

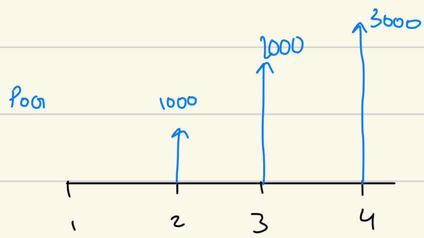
Example: suppose that we have cash flows as follows:
Calculate their present equivalent at *i* = 15% per year

End of Year	Cash Flows ($)
1	5,000
2	6,000
3	7,000
4	8,000

$G = 1000$







$$P_{oT} = P_{oA} + P_{oG}$$

$$P_{oA} = A\,(P/A, 15\%, 4) = 5000\,(2.855) = 14275$$

$$P_{oG} = G\,(P/G, 15\%, 4) = 1000\,(3.786) = 3786$$

$$P_{oT} = 18061$$

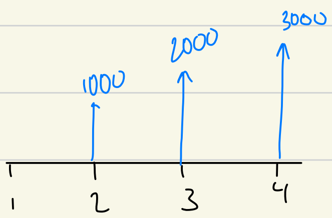
Example: suppose that we have cash flows as follows:

Calculate their present equivalent at i = 15% per year

- POT = POA – POG

End of Year	Cash Flows ($)
1	8,000
2	7,000
3	6,000
4	5,000

$G = -1000$



$$P_{OT} = A(P/A, 15\%, 4) + G(P/G, 15\%, N)$$

$$8000(2.855) + -1000(3.786)$$

$$P_{OT} = 19054$$

Geometric Sequance of Cash flows

Cash flow that changes by a constant rate $\bar{f}$ each period

→ First payment at EOY 1

$$P = \begin{cases} \dfrac{A_1\left[1-(1+i)^{-N}(1+\bar{f})^{N}\right]}{i-\bar{f}} & \bar{f} \neq 1 \\ A_1 N (1+i)^{-1} & \bar{f} = 1 \end{cases}$$

$$P = \begin{cases} \dfrac{A_1\left[1-(P/F, i\%, N)(F/P, 1\%, N)\right]}{i-f} & f \neq i \\ A_1 N (P/F, i\%, 1) & f = i \end{cases}$$

Example: Assume that a payment of $1,000 is made at EOY 1 and decreases by 20% per year after the first year for 4 years. At a 25% interest rate, Determine the present equivalent, A, and F.

$\bar{f} = -20\%$

$i = 25\%$

$\bar{f} \neq i$

$$P = \frac{A_1 \left[1 - (P/F, 25\%, 4)(F/P, -20\%, 4)\right]}{i - \bar{f}}$$

$$P = \frac{1000 \left[1 - (0.4096)(1 + -0.2)^4\right]}{0.25 - (-0.20)} = 1849.38$$

$$A = P(A/P, 25\%, 4)$$

$$1849.58(0.4234) = 783.03$$

$$F = P(F/P, 25\%, 4)$$

$$1849.38(2.4414) = 4515.08$$

Example: On your 23rd birthday you decide to invest $4,500 (10% of your annual salary) in a mutual fund earning 7% per year. You will continue to make annual deposits equal to 10% of your annual salary until you retire at age 62 (40 years after you started your job). You expect your salary to increase by an average of 4% each year during this time. How much money will you have accumulated in your mutual fund when you retire?

$A_1 = 4500$ $\quad i = 7\%$ $\quad N = 40$

$\bar{f} = 4\%$

$$P = \frac{4500 \left[1 - (P/F, 7\%, 40)(F/P, 4\%, 40)\right]}{0.07 - 0.04}$$

$$P = \frac{4500 \left[1 - (0.0668)(4.8010)\right]}{0.07 - 0.04} = 101,894$$

$$F = P(F/P, 7\%, 40)$$

$$101,894(14.9745) = 1,525,812$$

Interest Rates that vary with time

Interest rates often change with time

→ Resort to moving cash flows one period at a time, reflecting the interest rate for that single period

The present equivalent of a cash flow occurring at the end of period N can be computed with the equation:

$$P = \frac{F_N}{\prod (1+i_k)}$$

F_N ← future equivalent

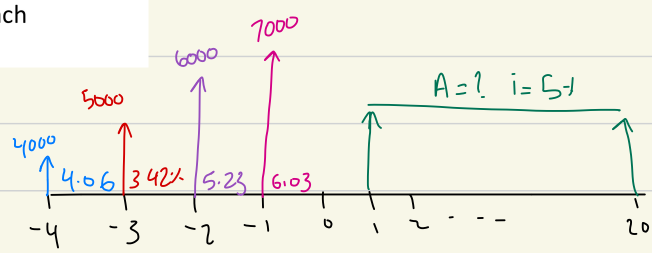
i_k ↳ interest rate for k^{th} period

If F_3 = \$2,500 and i_1=8%, i_2=10%, and i_3=11%, then

$$P = \frac{2500}{(1+0.08)(1+0.1)(1+0.11)} = 1896$$

EXAMPLE 4-27: Ashea Smith is a 22-year-old senior who used the Stafford loan program to borrow \$4,000 four years ago when the interest rate was 4.06% per year. \$5,000 was borrowed three years ago at 3.42%. Two years ago she borrowed \$6,000 at 5.23%, and last year \$7,000 was borrowed at 6.03% per year. Now she would like to consolidate her debt into a single 20-year loan with a 5% fixed annual interest rate. If Ashea makes annual payments (starting in one year) to repay her total debt, what is the amount of each payment?

$F_0 = P_0$



From (-4 → -3) $\quad F = P(F/P, 4.06\%, 1)$

$F = 4000(1+0.0406)^1 = 4162.4$

at year -3 $\quad 4162.4 + 5000 = \$9162.4$

from (-3 → -2) $\quad F = P(F/P, 3.42, 1)$

$F = 9162.4(1+0.0342) = 9475.75$

at year -2 $\quad 9475.75 + 6000 = \$15,475.75$

from $(-2 \rightarrow -1)$ $F = P(F/P, 5.23\%, 1)$

$F = 15{,}475.75\,(1 + 0.0523) = 16{,}285.13$

at year -1 $16{,}285.1317 + 7000 = \$23285.13$

from $(-1 \rightarrow 0)$ $F = P(F/P, 6.03, 1)$

$23{,}285.13\,(1 + 0.0603) = 24{,}689.22$

$\rightarrow$ Annuity $A = P(A/P, 5\%, 20)$

$A = 24{,}689.22\,(0.0802) = \$1980.08$ per year

Nominal & effective Interest rates

If the Compounding period is less than a year

Annual rate is called nominal interest rate or Annual percentage Rate (APR)

Actual or exact rate is called effective interest rate

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

i: effective interest rate per year

r: nominal Interest rate

M: compounding periods per year

Suppose that a \$100 lump-sum amount is invested for 10 years at a nominal interest rate of 6% compounded quarterly. How much is it worth at the end of the 10th year?

$P: 100$ $r = 6\%$

$M = 4$ $N = 10$ years

$$i = \left(1 + \frac{0.06}{4}\right)^4 - 1 = 6.14\%$$

$F = P(F/P, 6.14\%, 10)$

$100\,(1 + 0.0614)^{10} = 181.46$

Example: A credit card company charges 1.375% per month on the unpaid balance. They claim that the annual interest rate is (12 × 1.375% = 16.5%). Is that true?

$$i = \left(1 + \frac{0.165}{12}\right)^{12} - 1 \quad = 17.8\%$$

Does this card provide a better deal than another card which charges 16.8% annual rate compounded bimonthly?

$$i = \left(1 + \frac{0.168}{6}\right)^{6} - 1 \quad = 18.02\%$$

not a better deal.

Example: A loan of $2,000 at 10% annual interest rate for 8 years is to be repaid in two equal payments, @ EOY 4 and EOY 8. What is the value of the payments?

$r = 10\%$ $\quad N = 8$ years

$10\% \times 4 = 40\%$

$$i = \left(1 + \frac{0.4}{4}\right)^{4} - 1 \quad = 46.41\%$$

$$A = P(A/P, 46.4, 2)$$

$$2000\left(\frac{0.4641(1+0.4641)^2}{(1+0.4641)^2 - 1}\right) = 1739.9$$

If the monthly interest rate is 1%, what is the effective semi-annual rate?

semiannual = 6 per year

6 per year × 1% per month = 6%

$$i = \left(1 + \frac{0.06}{6}\right)^{6} - 1 \quad = 6.15\%$$

EXAMPLE 4-32: A loan of $15,000 requires monthly payments of $477 over a 36-month period of time. These payments include both principal and interest.

$P = 15{,}000$ $N = 36$ month

$A = 477$

(a) What is the nominal interest rate (annual percentage rate (APR)) for this loan?

(b) What is the effective interest rate per year?

(c) Determine the amount of unpaid loan principal after 20 months.

a) $P = A(P/A, i\%, 36)$

$$15{,}000 = 477 \left[\frac{(1+i)^{36} - 1}{i(1+i)^{36}} \right]$$

$i = 0.75\%$ (per month)

$r = 0.75\% \times 12 = 9\%$ per year

b) $i = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 9.38\%$ per year

36-20 = 16

c) $P = A(P/A, 0.75\%, 16)$

$P = 477\,(15.0243) = 7{,}166.59$

Continous Compounding

Allowing interest to compound continously throughout the period $\Rightarrow$ M approaches ∞

$$i = e^r - 1$$

$$(F/P, r\%, N) = e^{rN}$$

$$(P/F, r\%, N) = e^{-rN} = \frac{1}{e^{rN}}$$

$$(F/A, r\%, N) = \frac{e^{rN} - 1}{e^r - 1}$$

$$(P/A, r\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^{rN} - 1)}$$

Example: A bank offers loans at an annual interest rate of 12% compounded continuously,

- What is the effective annual interest rate?

$$i = e^r - 1$$

$$i = e^{0.12} - 1 = 12.75\%$$

- What is the effective monthly interest rate?

$$r = \frac{0.12}{12} = 0.01 \text{ (nominal monthly)}$$

$$i = e^{0.01} - 1 = 1.0051$$

- If you borrowed $10,000 on these terms, what is the future equivalent of this loan after 5 years?

$$F = P(F/P, 0.12, 5)$$

$$F = 10{,}000\,(e^{0.12 \times 5}) = 18{,}221$$

Example: A nominal interest rate of 8% is compounded continuously.

- What is the uniform EOY amount for 10 years that is equivalent to $8,000 at EOY 10?

$$F = A\,(F/A,\ 8\%,\ 10)$$

$$8000 = A\,(14.7147) \qquad A = 543.68$$

- What is the present equivalent value of $1,000 per year for 12 years?

$$P = A\,(P/A,\ 8\%,\ 12)$$

$$P = 1000\,(7.4094) = 7409.4$$

- What is the future equivalent at the end of the 6th year of $243 payments **made every 6 months** during the 6 years (first payment occurs 6 months from the present and the last occurs at EOY 6)?

6months every year = 2times per year

2 times per year x 6 years = 12 times.

8% per year → 2 times a year

$$= \frac{8}{2} = 4\%$$

$$F = A\,(F/A,\ 4\%,\ 12)$$

$$F = 243\left[\frac{e^{0.04\times 12} - 1}{e^{0.04} - 1}\right] = 3668.3$$

Chapter 5 : Evaluating a single project

Methods for Evaluating a single project

- Minimum attractive Rate of Return (MARR) : The lowest internal rate of return that the organization would consider it to be a good investment

→ Present worth (PW)

→ Future worth (FW)

→ Annual worth (AW)

→ Internal Rate of Return (IRR)

→ External Rate of Return (ERR)

→ Payback period

* A project must provide a return that is equal to or greater than the MARR

Present worth

All cash inflows & outflows are discounted to the present time at an interest rate [MARR]

$PW(i = MARR) \geq 0$ → Acceptable project

<u>Example</u>: A project has a capital investment of $50,000 and returns $18,000 per year for 4 years. At a 12% MARR, is this a good investment?

$PW = -50{,}000 + 18{,}000\,(P/A, 12\%, 4)$

$-50{,}000 + 18{,}000\,(3.0373) = 4{,}671.4$ → Good investment

* The higher the interest rate, the lower the present worth

A new heating system is to be purchased and installed for $110,000. This system will save approximately 300,000 kWh of electric power each year for 6 years with no additional O&M costs. Assume the cost of electricity is $0.10 per kWh, the company's MARR is 15% per year, and the system's market value will be $8,000 at EOY 6. Using the PW method, is this a good idea?

A: $300{,}000\,kWh \times 0.10\,\frac{\$}{kWh} = 30{,}000$

$PW = -110{,}000 + 30{,}000(P/A, 15\%, 6) + 8{,}000(P/F, 15\%, 6)$

$-110{,}000 + 30{,}000(3.7845) + 8{,}000(0.4323) = 6{,}993.4$ → Good investment

→ Present worth Assumptions

1. Assume we know the future with certainty
2. Assume we can borrow & lend money at the same interest rate

Future worth

→ Maximize the future wealth of the owners

→ Equivalent of all cash inflows & outflows at the end of the study period at MARR

→ $FW \geq 0$, project is economically justified

Example: A $45,000 investment in a new conveyer system is projected to improve throughout and increase revenue by $14,000 per year for five years. The estimated market value of the conveyer at the end of five years is $4,000. Using the FW method at a MARR of 12%, is this a good investment?

$FW = 4{,}000 + -45{,}000(F/P, 12\%, 5) + 14{,}000(F/A, 12\%, 5)$

$4000 + -45000(1.7623) + 14000(6.3528) = 13635.7$ → Good investment

Example: A $110,000 retrofitted space-heating system was projected to save $30,000 per year in electrical power and be worth $8,000 at the end of the six-year study period. Use the FW method to determine whether the project is still economically justified if the system has zero market value after six years. The MARR is 15% per year.

$FW = -110{,}000(F/P, 15\%, 6) + 30{,}000(F/A, 15\%, 6)$

$-110{,}000(2.3131) + 30{,}000(8.7537) = 8{,}170$ → economically justified

Annual worth

→ Equivalent to cash inflows & outflows at (MARR)

→ AW ≥ 0 → economically justified

Annual equivalent Revenue or savings minus Annual equivalent expenses, less its annual capacity Recovery amount (CR)

$$AW(i\%) = R - E - CR(i\%)$$

$$CR(i\%) = I\,(A/P, i\%, N) - S(A/F, i\%, N)$$

Initial cost ↲ ↳ Salvage value

Example: A project requires an initial investment of $45,000, has a salvage value of $12,000 after six years, incurs annual expenses of $6,000, and provides annual revenue of $18,000. Using a MARR of 10%, determine the AW of this project.

$$AW = 18{,}000 - 6000 - CR$$

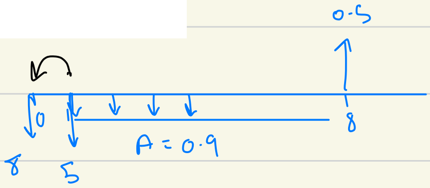
$$CR = 45{,}000\,(A/P, 10\%, 6) - 12000\,(A/F, 10\%, 6)$$

$$45000\,(0.2296) - 12\,000\,(0.1296) = 8777$$

$$AW = 18000 - 6\,000 - 8777 = 3\,223$$

Example: Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in opening up new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of $13 million, with $8 million committed now and the remaining $5 million expended at the end of year 1 of the project. Annual operating costs for the system are expected to start the first year and continue at $0.9 million per year. The useful life of the tracker is 8 years with a salvage value of $0.5 million. Calculate the CR and AW values for the system, if the corporate MARR is 12% per year.

$$AW = R - E - CR$$



$$CR = [8M + 5M\,(P/F, 12\%, 1)]\,(A/P, 12\%, 8) - 0.5M\,(A/F, 12\%, 8)$$

$$CR = [8M + 5M\,(0.8929)]\,(0.2013) - 0.5M\,(0.0813) = 2.47\ M$$

$$AW = -0.9 - 2.47 = -3.37\ M$$

PW Application. Bonds

redemtion or disposal price

$$V_N = C(P/F, i\%, N) + rZ(P/A, i\%, N)$$

Value of Bond — V_N; bond yield — $i\%$; bond rate (nominal) — r

Example: A bond with a face value of $5,000 pays interest of 8% per year. This bond will be redeemed at par value at the end of its 20-year life, and the first interest payment is due one year from now.

(a) How much should be paid now for this bond in order to receive a yield of 10% per year on the investment?

(b) If this bond is purchased now for $4,600, what annual bond yield would the buyer receive?

A) $V_N = C(P/F, 10\%, 20) + rZ(P/A, 10\%, 20)$ $\quad i = \text{yield} = 10\%$

$$V_N = 5000(0.1486) + 0.08(5000)(8.5136) = 4148.44$$

B) $$4600 = 5000\left[\frac{1}{(1+i)^{20}}\right] + 0.08(5000)\left[\frac{(1+i)^{20}-1}{i(1+i)^{20}}\right]$$

$$i = 8.87\%$$

Example: A bond has a face value of $10,000 and matures in 8 years. The bond stipulates a fixed nominal interest of 8% per year, but interest payments are made to the bondholder every 3 months. The bondholder wishes to earn 10% nominal annual interest (compounded quarterly). Assuming the redemption value is equal to the face value, how much should be paid for the bond now?

$N = 8\text{ years} \times 4 \frac{\text{quarters}}{\text{year}} = 32$

$r = \frac{8\%}{4} = 2\%$

$i = \frac{10\%}{4} = 2.5\%$

$$V_N = 10{,}000(P/F, 2.5\%, 32) + 0.02(10000)(P/A, 2.5\%, 32)$$

$$V_N = 10000\left[\frac{1}{(1+2.5\%)^{32}}\right] + 0.02(10000)\left[\frac{(1+2.5\%)^{32}-1}{(1+2.5\%)^{32}\,2.5\%}\right]$$

$$V_N = 8907.55$$

Example: What is the value of a 6%, 10-year bond with a par (and redemption) value of $20,000 that pays dividends semi-annually, if the purchaser wishes to earn an 8% return?

$N = 10\text{years} \times 2 = 20$

$i = \frac{8\%}{2} = 4\%$

$r = \frac{6\%}{2} = 3\%$

$V_N = 20{,}000(P/F, 4\%, 20) + 0.03(20{,}000)(P/A, 4\%, 20)$

$20{,}000(0.4564) + 0.03(20{,}000)(13.5903) = 17{,}282.18$

PW Applications : Capitalized worth

→ Revenues or expenses occur over an infinite length of time

→ if only expenses are considered ⇒ Capitalized cost

→ A CW of a series of end of period uniform payments A, with interest rate i% per period, is

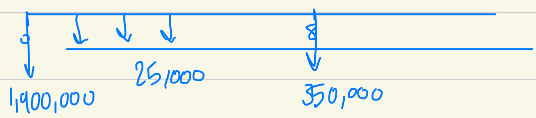
$$A(P/A, i\%, N)$$

As N becomes very large the (P/A) term approaches $\frac{1}{i}$

$$CW = A\left[\frac{1}{i}\right]$$

Example: A bridge was constructed at a cost of $1,900,000 and the annual upkeep cost is $25,000. It is also estimated that maintenance will be required at a cost of $350,000 every 8 years. What is the capitalized worth of the bridge over its life assuming MARR = 8%?

A/F ,8%,8 $25,000



$$CW = -1{,}900{,}000 - \frac{350{,}000(A/F, 8\%, 8)}{0.08} - \frac{25{,}000}{0.08}$$

$$-1{,}900{,}000 - \frac{350{,}000(0.0940)}{0.08} - \frac{25000}{0.08} = -2{,}623{,}750$$

- If the bridge has an expected life of 50 years, what is the capitalized worth (CW) of the bridge over a 100-year study period?

$$CW = -1{,}900{,}000 - 1{,}900{,}000(P/F, 8\%, 50) - \left[\frac{350{,}000(A/F, 8\%, 8)}{0.08}\right] - \frac{25000}{0.08}$$

$$-1{,}900{,}000 - 1{,}900{,}000(0.0213) - \frac{350{,}000(0.0940)}{0.08} - \frac{25000}{0.08} \approx -2{,}664{,}220$$

Betty has decided to donate some funds to her local community college. Betty would like to fund an endowment that will provide a scholarship of $25,000 each year in perpetuity, and a special award, "Student of the Decade," each ten years (again, in perpetuity) in the amount of $50,000. How much money does Betty need to donate today, in one lump sum, to fund the endowment? Assume the fund will earn a return of 8% per year.

$50{,}000\ (A/F, 8\%, 10)$

$50{,}000\ (0.069) = 3450$

$$CW = \frac{25000 + 3450}{0.08} = 355{,}625$$

Internal Rate of Return

called investors method, discounted cash flow method, probability index

The IRR is the interest rate that equates the equivalent worth of an alternative's cash inflows (Revenue) to the equivalent worth of cash outflows (Expenses)

IRR → breakeven interest rate

if IRR ≥ MARR → economically justified

Equivalent worth of cash inflows = Equivalent worth of cash outflows

$$\sum R_k (P/F, i'\%, k) = \sum E_k (P/F, i'\%, k)$$

Example: A company is considering the purchase of a digital camera for the maintenance of design specifications by feeding digital pictures directly into an engineering workstation. The capital investment requirement is $345,000 and the estimated market value of the system after a six-year study period is $115,000. Annual revenues attributable to the new camera system will be $120,000, whereas additional annual expenses will be $22,000. You have been asked by management to determine the IRR of this project and to make a recommendation. The corporation's MARR is 20% per year.

$$PW = -345000 + 115000(P/F, i'\%, 6) + (120000 - 22000)(P/A, i'\%, 6)$$

$$0 = -345000 + 115000\left[\frac{1}{(1+i'\%)^6}\right] + (120000 - 22000)\left[\frac{(1+i'\%)^6 - 1}{i'\%(1+i'\%)^6}\right]$$

$$i = 22.1\%$$

Example: A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is $25,000, and the equipment will have a market (salvage) value of $5,000 at the end of its expected life of five years. Increased productivity attributable to the equipment will amount to $8,000 per year after extra operating costs have been subtracted from the value of the additional production. Use a spreadsheet to evaluate the IRR of the proposed equipment. Is the investment a good one? Recall that the MARR is 20% per year.

$$0 = -25000 + 5000(P/F, i'\%, 5) + 8000(P/A, i'\%, 5)$$

$$-25000 + 5000\left[\frac{1}{(1+i'\%)^5}\right] + 8000\left[\frac{(1+i'\%)^5 - 1}{i'\%(1+i'\%)^5}\right]$$

$$i' = 21.58\%$$

$21.58\% > 20\% \Rightarrow$ Good investment

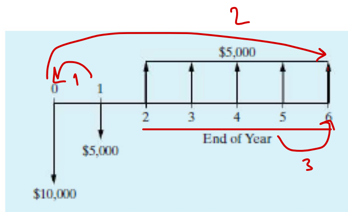
External Rate of Return

1. All net cash outflows are discounted to time zero at $\varepsilon\%$

All net cash inflows are compounded to period N at $\varepsilon\%$

$$ERR: \quad \sum R_k(F/P, \varepsilon\%, N-k) = \sum E_k(P/F, \varepsilon\%, k)(F/P, i'\%, N)$$

Example: When ∈=15% and MARR = 20% per year, determine whether the project (whose net cash-flow diagram appears next) is acceptable.



$$[10{,}000 + 5{,}000\,(P/F, 15\%, 1)]\,(F/P, i'\%, 6) = 5000\,(F/A, 15\%, 5)$$

$$[10{,}000 + 5000\,(0.8696)]\,(1+i'\%)^6 = 5000\,(6.7424)$$

$$14{,}348\,(1+i\%)^6 = 33712 \qquad i'\% = 15.3\%$$

15.3 < MARR → Unacceptable

Example: For the cash flows given below, find the ERR when the external reinvestment rate (ε) = MARR = 12%.

Year	0	1	2	3	4
Cashflow	($15,000)	($7,000)	$10,000	$10,000	$10,000

$$[15000 + 7000\,(P/F, 12\%, 1)]\,(F/P, i'\%, 4) = 10000\,(F/A, 12\%, 3)$$

$$[15000 + 7000\,(0.8929)]\,(1+i'\%)^4 = 10\,000\left[\frac{(1+12\%)^3 - 1}{12\%}\right]$$

$$i'\% = 12.26\%$$

Payback (payout Period)

The payback method, which is often called the simple payout method, mainly indicates a project's liquidity rather than its profitability

→ liquidity deals with how fast an investment can be recovered

Simple payback: ignores the time value of money

$$\sum (R_k - E_k) - I \geq 0$$

Discounted payback : time value of money is considered

$$\sum (R_k - E_k)(P/F, i\%, k) - I \geq 0$$

I → capital investment

Example: An investment of $5,000,000 yields net annual revenue of $1,500,000. What is *the simple* payback period?

$5,000,000

$$\text{Payback period} = \frac{I}{A} = \frac{5\,000\,000}{1\,500\,000} = 3.33 = 4 \text{ years}$$

* Payback period can produce misleading results, & it's recommended as supplemental information only in conjuction with one or more of the five methods [PW, FW, AW, IRR, ERR]

Example: For the following cash flows, what is the simple and discounted payback periods at i = 6%?

EOY	0	1	2	3	4	5
Net cash flow	-$42,000	$12,000	$11,000	$10,000	$10,000	$9,000

EOY	Net cash flow	Cumulative PW (simple)	PW of cash flow (6%)	Cumulative PW (6%)
0	- 42 000	-42000	- 42000	-42 000
1	12 000	- 30 000	11 320.8	-30 679
2	11 000	-19 000	9790	- 20 889
3	10 000	- 9000	8396	-12 493
4	10 000	1000	7921	-4572
5	9 000		6725.7	2153.7

θ = 4 years

θ' = 5 years

Column 1 End of Year k	Column 2 Net Cash Flow	Cumulative (simple)	PW (cash flow 20%)	Cumulative (20%)
0	−$25,000	-25000	-25000	-25000
1	8,000	-17000	6667	-18333
2	8,000	-9000	5556	-12777
3	8,000	-1000	4630	-8147
4	8,000	7000	3858	-4289
5	13,000		5223	934

$\theta = 4$ years

$\theta' = 5$ years

Chapter 6 : Comparison & selecting Among Alternatives

Mutually exclusive : selection of one alternative excludes the others

Independent : Selection of one alternative does not exclude the other alternatives

* Acceptable alternative with the least capital investment → base alternative

↳ Investment Alternatives (positive cash flow)

↳ Cost Alternatives (negative cash flows)

Use a MARR of 10% and useful life of 5 years to select between the investment alternatives below:

Alternative	Capital investment	Annual revenues less expenses
A	-\$100,000 → Base	\$34,000
B	-\$125,000	\$41,000

$PW_A = -100{,}000 + 34000\,(P/A, 10\%, 5) = 28{,}887$

$PW_B = -125000 + 41000\,(P/A, 10\%, 5) = 30{,}423$ → More revenue, better alternative

Use a MARR of 12% and useful life of 4 years to select between the cost alternatives below:

Alternative	Capital investment	Annual expenses
C	-\$80,000	-\$25,000
D	-\$60,000 → Base	-\$30,000

$PW_C = -80{,}000 - 25{,}000\,(P/A, 12\%, 4) = -155{,}933$

$PW_D = -60{,}000 - 30{,}000\,(P/A, 12\%, 4) = -151{,}119$ → less cost, better alternative

Study Period

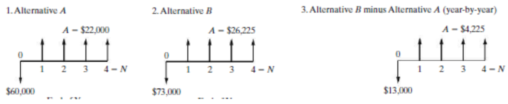
→ Selected time period over which mutually exclusive alternatives are compared

→ Useful lives of all mutually Exclusive Alternatives = Study period : No cashflow adjustment

→ Useful lives are unequal ∴ Repeatability Assumption

Co-terminated Assumption

- If PW (B-A) is positive, the additional capital invested in *B* is justified. @ MARR = 10%



	Alternative		
	A (Base)	*B*	$\Delta(B-A)$
Capital investment	−$60,000	−$73,000	−$13,000
Annual revenues less expenses	22,000	26,225	4,225

$PW_A = -60{,}000 + 22000\,(P/A, 10\%, 4) = 9{,}737.8$

$PW_B = -73{,}000 + 26\,225\,(P/A, 10\%, 4) = 10\,131$ → highest revenue

$PW_{B-A} = -13000 + 4225\,(P/A, 10\%, 4) = 393$

- If PW (D-C) is positive, the additional capital invested in *D* is justified. @ MARR = 10%.
- Alternative C is the *base alternative* (lowest capital).

	Alternative		
End of Year	*C* (Base)	*D*	$\Delta(D-C)$
0	−$380,000	−$415,000	−$35,000
1	−38,100	−27,400	10,700
2	−39,100	−27,400	11,700
3	−40,100	−27,400	12,700
3[a]	0	26,000	26,000

[a] Market value.

$PW_C = -380{,}000 - 38{,}100\,(P/A, 10\%, 3) - 1000\,(P/G, 10\%, 3) = -477{,}077$

$PW_D = -415{,}000 - 27400\,(P/A, 10\%, 3) + 26\,000\,(P/F, 10\%, 3) = -463{,}607$ → least expenses

$PW_{D-C} = -35\,000 + 10700\,(P/A, 10\%, 3) + 1000\,(P/G, 10\%, 3) + 26000\,(P/F, 10\%, 3) = 13{,}470$

	Alternatives			
	A	B	C (Base)	D
Capital investment	-$150,000	-$85,000	-$75,000	-$120,000
Annual revenues	$28,000	$16,000	$15,000	$22,000
Annual expenses	-$1,000	-$550	-$500	-$700
Market Value (EOL)	$20,000	$10,000	$6,000	$11,000
Life (years)	10	10	10	10

MARR = 12%

$PW_A = -150{,}000 + 27\,000\,(P/A, 12\%, 10) + 20{,}000\,(P/F, 12\%, 10) = 8995$ → highest Revenue

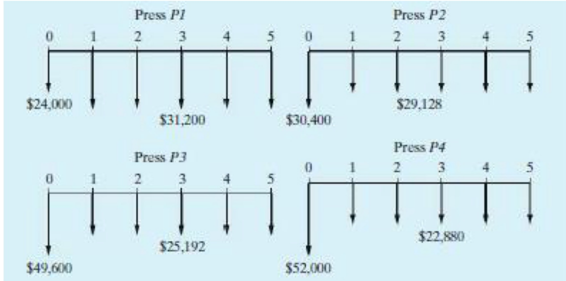
$PW_B = -85{,}000 + 15450\,(P/A, 12\%, 10) + 10{,}000\,(P/F, 12\%, 10) = 5516$

$PW_C = -75{,}000 + 14500\,(P/A, 12\%, 10) + 6000\,(P/F, 12\%, 10) = 8860$

$PW_D = -120\,000 + 21300\,\underbrace{(P/A, 12\%, 10)}_{=5.6502} + 11000\,\underbrace{(P/F, 12\%, 10)}_{0.322} = 3891$

Example 6.2: A company is planning to install a new automated plastic-molding press. Four different presses are available. The initial capital investments and annual expenses for these four mutually exclusive alternatives are as follows @ MARR = 10%:

	Press			
	Base P1	P2	P3	P4
Capital investment	$24,000	$30,400	$49,600	$52,000
Useful life (years)	5	5	5	5
Annual expenses				
Power	2,720	2,720	4,800	5,040
Labor	26,400	24,000	16,800	14,800
Maintenance	1,600	1,800	2,600	2,000
Property taxes and insurance	480	608	992	1,040
Total annual expenses	$31,200	$29,128	$25,192	$22,880



$PW_1 = -24{,}000 - 31200\,(P/A, 10\%, 5) = -142\,273$

$PW_2 : -30{,}400 - 29128\,(P/A, 10\%, 5) = -140\,818$

$PW_3 : -49600 - 25192\,(P/A, 10\%, 5) = -145098$

$PW_4 : -52000 - 22880\,\underbrace{(P/A, 10\%, 5)}_{3.7908} = -138734$ → less costs

$AW_1 = -24000\,(A/P, 10\%, 5) - 31200 = -37531$

$FW_1 = -24000\,(F/P, 10\%, 5) - 31200\,(F/A, 10\%, 5) = -229\,131$

* same answer using PW, AW, FW

Example: Three mutually exclusive design alternatives are being considered. The estimated cash flows for each alternative are given in the following table. At a MARR of 20% per year, which one will you select?

	Base A	B	C
Investment cost	$28,000	$55,000	$40,000
Annual expenses	$15,000	$13,000	$22,000
Annual revenues	$23,000	$28,000	$32,000
Market value	$6,000	$8,000	$10,000
Useful life	10 years	10 years	10 years
IRR	26.4%	24.7%	22.4%

$PW_A = -28000 + 8000\,(P/A, 20\%, 10) + 6000\,(P/F, 20\%, 10) = 6509$

$PW_B = -55000 + 15000\,(P/A, 20\%, 10) + 8000\,(P/F, 20\%, 10) = 9180$ → highest revenue

$PW_C = -40\,000 + 10\,000\,\underbrace{(P/A, 20\%, 10)}_{4.1925} + 10000\,\underbrace{(P/F, 20\%, 10)}_{0.1615} = 3540$

* Do not compare IRR only IRR ≥ MARR

→ Rate of Return Method

@ MARR = 10%. Calculate PW and IRR for both of the following alternatives:

N=4 years

	Base → A	B	Δ(B – A)
	Alternative		
Capital investment	–\$60,000	–\$73,000	–\$13,000
Annual revenues less expenses	22,000	26,225	4,225

A) $-60{,}000 + 22{,}000(P/A, i\%, 4) = 0$

$$-60{,}000 + 22{,}000\left(\frac{(1+i)^4 - 1}{i(1+i)^4}\right) = 0$$

IRR → $i\% = 17.23 > MARR$

B) $-73000 + 26225\ (P/A, i\%, 4) = 0$

IRR → $i\% = 16.31 > MARR$

B- A) $-13000 + 4225\ (P/A, i\%, 4) = 0$

IRR → $i\% = 11.4\%$

* All based on IRR are attractive, to select the best alternative use PW

$PW_A = -60{,}000 + 22{,}000\ (P/A, 10\%, 4) = 9738$

$PW_B = -73{,}000 + 26225\ (P/A, 10\%, 4) = 10{,}130$ → Best alternative, highest revenue

→ Incremental investment Analysis procedure

1. Arrange Alternatives based on increasing capital investment
2. Establish a base alternative
3. Evaluate differences [incremental cash flows]
4. Work up the order of ranked alternatives smallest to largest
5. lower Rank - higher rank
6. If attractive keep, if not eliminate

Six mutually exclusive alternatives with equal useful lives (10 years) are analyzed and compared using the IRR method. Assuming MARR = 10%, which alternative will you select?

	A	B	C	D	E	F
Capital investment	$900	$1,500	$2,500	$4,000	$5,000	$7,000
Net annual income	$150	$276	$400	$925	$1,125	$1,425
IRR	10.6%	13.0%	9.6%	19.1%	18.3%	15.6%

↳ IRR < MARR
Eliminate

A → Base

B is the new base & eliminate A

B − A	D − B	E − D	F − E
600	2500	1000	2000
126	649	200	300
16.4%	22.6%	15.1%	8.14%

↑ Choose E

↳ IRR < MARR
Eliminat F

B − A IRR: $-600 + 126\,(P/A, i'\%, 10) = 0 \qquad i'\% = 16.4\%$

Unequal Useful lives

→ if the useful life of an alternative is less than the study period:

- Cost alternatives: Contracting or leasing
  Repeatability Assumption
- Investment alternatives: reinvest at the MARR at the end of study period
  Replace with another asset, after the study period

→ if the useful life of an alternative is greater than the study period

- Truncate the alternative at the end of the study period [using estimated market value]
- Repeatability Assumption [when applicable]

1. Repeatability

Example: Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and using the repeatability assumption, which alternative would you pick?

	A	B
Capital investment	$3,500	$5,000
Annual net cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

least common multiple $4 \times 3 = 12$ A is repeated 3 times

$6 \times 2 = 12$ B is repeated 2 times

$PW_A = -3{,}500 + [-3500(P/F, 10\%, 4) + (P/F, 10\%, 8)] + 1255(P/A, 10\%, 12) = 1028$

$PW_B = -5000 + -5000(P/F, 10\%, 6) + 1480(P/A, 10\%, 12) = 2262$ → higher Revenue.

* if repeatability can be assumed ⇒ compare by AW of each alternative over its own useful life

$AW_A = -3500(A/P, 10\%, 4) + 1255 = 151$

$AW_B = -5000(A/P, 10\%, 6) + 1480 = 332$ → highest Revenue

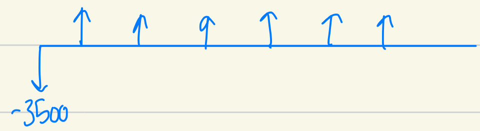
2. Co - terminated

Two mutually exclusive alternatives with different useful lives. If MARR = 10% per year, and the study period is 6 years, which alternative would you pick?

	A	B
Capital investment	$3,500	$5,000
Annual cash flow	$1,255	$1,480
Useful lives (years)	4	6
Market value at end of useful life	0	0

6 years [study period] is not a common multiple ⇒ repeatability is not applicable

Co-terminated Assumption. Money at EOY 4 is reinvested



$FW_A = [-3500(F/P, 10\%, 4) + 1255(F/A, 10\%, 4)](F/P, 10\%, 2) = 847$

$FW_B = -5000(F/P, 10\%, 6) + 1480(F/A, 10\%, 6) = 2561$ → Better Alternative

Example: Two mutually exclusive alternatives with different useful lives. At 5% per year MARR:

	A	B
Capital investment	$6,000	$14,000
Annual expenses	$2,500	$2,400
Useful lives (years)	12	18
Market value at end of useful life	0	$2,800

Determine which alternative to select assuming repeatability applies.

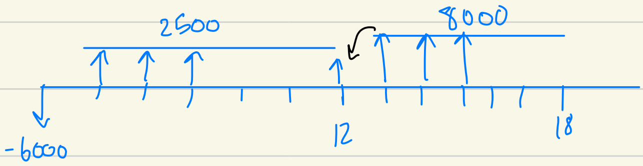
$AW_A = -6000(A/P, 5\%, 12) - 2500 = -3176.8$ → less costs

$AW_B = -14000(A/P, 5\%, 18) - 2400 + 2,800(A/F, 5\%, 18) = -3497.6$

Example: Two mutually exclusive alternatives with different useful lives. At 5% per year MARR:

	A	B
Capital investment	\$6,000	\$14,000
Annual expenses	\$2,500	\$2,400
Useful lives (years)	12	18
Market value at end of useful life	0	\$2,800

- Determine which alternative to select if the repeatability does not apply, study period is 18 years, and a new system can be leased for \$8,000 per year after the useful life of alternative A is over.



$$PW_A = -6000 - 2500(P/A, 5\%, 12) - [8000(P/A, 5\%, 6)](P/F, 5\%, 12) = -50,767.45$$

$$PW_B = -14000 - 2400(P/A, 5\%, 18) + 2800(P/F, 5\%, 18) = -40\,891.64$$ → lower cost

Example: Which alternative should be selected assuming MARR = 20%? Use the IRR method.

	Base → A	B
Capital investment	\$3,500	\$5,000
Annual cash flow	\$1,255	\$1,480
Useful lives (years)	4	6
Market value at end of useful lives	0	0

$$AW_A = AW_B$$

$$-3500(A/P, i'\%, 4) + 1255 = -5000(A/P, i'\%, 6) + 1480$$

$i'\% = 26\%$ → IRR > MARR

select B

Chapter 7: Depreciation & Income Taxes

Depreciation

Measures the decrease in value of physical properties with time & use

Begins once the property is placed in service for business

→ Depreciation Methods

Time:

Straight Line (SL) method

Sum of years digits (SOYD) method

Declining Balance (DB) method

Use:

Units of Production Method

Straight line Method (SL)

- constant amount is depreciated each year over the depreciable (useful) life

cost basis   salvage value

$$d_k = \frac{B - SV_k}{N}$$

Annual depreciation

$$d_k^* = k \cdot d_k$$

Cumulative depreciation

$$BV = B - d_k^*$$

Book value

Example: A tool has a cost basis of $200,000 and a five-year depreciable life. The estimated salvage value is $20,000 at the end of five years. Determine the annual depreciation using SL method and tabulate the annual depreciation amounts and book values at the end of each year.

$$d_k = \frac{200{,}000 - 20{,}000}{5} = 36000$$

EOY	d_k	BV
0	-	200,000
1	36000	164 000
2	36000	128 000
3	36000	92 000
4	36000	56000
5	36000	20,000

↳ salvage value

$d^*_1 = 1 \times 36000 = 36000$

$d^*_2 = 2 \times 36000 = 72000$

⋮

Sum of years Digits (SOYD) Method

$$\text{Depreciation Expense} = \frac{\text{Remaining useful life of asset}}{\text{Sum of the years digits}} \times \text{Depreciable cost}$$

$$SOYD = \frac{[N(N+1)]}{2}$$

$$d_k = \frac{[N-k+1]}{SOYD} \cdot (B - SV)$$

$$d_k^* = \sum d_k$$

$$BV = B - d_k^*$$

Example: a property has a cost basis of $33,000 and a salvage value of $3,000 with a 5-year useful life. Use the SOYD method to determine the annual depreciations and book values at end of each year.

EOY	d_k	BV
0	-	33 000
1	10 000	23 000
2	8000	15 000
3	6000	9000
4	4000	5000
5	2000	3000 ↳ salvage value

$$SOYD = \frac{[5(5+1)]}{2} = 15$$

$$d_{k_1} = \frac{[5-1+1]}{15}(33{,}000 - 3000) = 10\,000$$

Declining Balance (DB) Method

→ Also called constant-percentage method

$$d_k = B(1-R)^{k-1} R$$

$$d_k^* = B[1-(1-R)^k]$$

$$BV_k = B(1-R)^k$$

* R → percentage ratio

when 200% DB or double DB = $2/N$

when 150% DB = $1.5/N$

Example: A new cutting machine has a cost basis of $4,000 and a 10-year depreciable life. The machine has no market value at the end of its life. Use the DB method to calculate the annual depreciation when:

(a) $R = 2/N$ or 200% DB or DDB.

(b) $R = 1.5/N$ (150% DB).

A) $d_{k_1} = 4000\left(1 - \frac{2}{10}\right)^{1-1}(0.2) = 800$

→ Sample for year 7

$$d_{k_7} = 4000\left(1 - \frac{2}{10}\right)^{7-1}(0.2) = 209.7152$$

$$d_k^* = 4000[1-(1-0.2)^7] = 3161.14$$

$$BV = 4000(1-0.2)^7 = 838.86$$

EOY	d_k	BV_k
0	–	4000
1	800	3200
2	640	2560
3	512	2048
4	409.6	1638.4
5	327.68	1310.72
6	262.144	1048 576
⋮		
10		

* if Value at EOY (last year of the lifespan) doesnot equal Salvage value ⇒ switch to SL
* Switch to SL when d_k of SL > d_k of DB

Units of production Method

$$\text{Depreciation per unit of production} = \frac{B - SV}{\text{estimated life time production units}}$$

Example: An equipment has a basis of $50,000 and is expected to have a $10,000 SV when replaced after 30,000 hours of use. Find the depreciation rate per hour of use and find its book value after 10,000 hours of operation.

$$\text{Depreciation} = \frac{50{,}000 - 10{,}000}{30{,}000 \text{ hours of use}} = 1.333 \text{ \$ per hour}$$

→ After 10,000

$$BV = 50\,000 - (1.333 \times 10{,}000) = 36\,700$$

Taxes:

→ function of Gross revenue minus allowable deductions

Taxable income = Gross Income – All expenses (except capital investment) – Depreciation Deductions

Example: A company generates $1,500,000 of gross income during its tax year and incurs operating expenses of $800,000. Property taxes on business assets amount to $48,000. The total depreciation deductions for the tax year equal $114,000. What is the taxable income of this firm?

Taxable income = 1,500,000 – 800,000 – 114,000 – 48,000 = 538 000

* After Tax cash flow (ATCF)

$$T_k = \underset{\substack{\text{effective} \\ \text{income tax rate}}}{-t} \; \underbrace{(R_k - E_k - d_k)}_{\text{taxable income}}$$

$BTCF_k$

Example: A new equipment is estimated to cost $180,000 and is expected to reduce net annual expenses by $36,000 for 10 years and to have a $30,000 market value at the end of the 10th year. Using the SL depreciation method, and assuming a 40% effective income tax rate, develop the ATCF and BTCF.

EoY	R (no E) Capital	BTCF	d_k	Income tax	ATCF
0	−180 000	− 180000	−	−	−180 000
1	36 000	36 000	15000	−8400	27600
2	36 000	36 000	15000	−8400	27600
3	36 000	36 000	15000	−8400	27600
4	36000	36000	15000	−8400	27600
5	36 000	36 000	15000	−8400	27 600
6	36000	36000	15000	−8400	27600
7	36000	36000	15000	−8400	27600
8	36000	36000	15000	−8400	27600
9	36000	36000	15000	−8400	27600
10	36000	36000	15000	−8400	27 600
10ª	30,000	30,000	-	−	30,000

SV

$$\text{SL } d_k = \frac{180\,000 - 30\,000}{10} = 15000$$

$$\text{income tax} = -0.4\,(36000 - 15000) = -8400$$

$$\text{ATCF} = \text{BTCF} + \text{income tax}$$

Example: A company wants to purchase a machine with an initial cost of $100,000 with additional $10,000 installation and transportation costs and a salvage value after 10 years of $10,000. If the annual revenue is $20,000 and the annual expenses are $5,000, and using the SL depreciation method and a 30% income tax rate:

❖ What is the BTCF for the 3rd year?

BTCF = 20,000 − 5000 = 15000

ATCF for 2nd year

BTCF = 20,000 − 5,000 = 15000

$$SL\ d_k = \frac{(100\,000 + 10{,}000) - 10{,}000}{10} = 10\,000$$

income tax = − 0.3 (15000 − 10 000) = − 1500

ATCF = 15000 + − 1500 = 13500

* Interest rate > compounding → effective

Example 4: Finding the Effective Annual Interest Rate from Quarterly Compounding

A loan has a nominal annual interest rate of 7% compounded quarterly. What is the effective annual interest rate?

Steps:

1. **Use the formula** because you're asked to find the effective annual rate, which is over a longer period than the compounding frequency (quarterly).

$$i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{n}\right)^n - 1$$

Where:

- $i_{\text{nom}} = 7\%$ or 0.07
- $n = 4$ (since interest is compounded quarterly)

Substitute values:

$$i_{\text{eff}} = \left(1 + \frac{0.07}{4}\right)^4 - 1 = (1 + 0.0175)^4 - 1 = 1.071859 - 1$$

Convert to a percentage:

$i_{\text{eff}} = 7.19\%$

Annual interest > compound quarterly ⇒ $i_{eff} = (1 + \frac{r}{m})^m - 1$

* Interest rate < compounding → Divide

Example 7: Finding the Monthly Interest Rate from Annual Nominal Rate

A loan has a nominal annual interest rate of 15% compounded monthly. What is the monthly interest rate?

Steps:

1. **Simply divide** because you're asked for the rate over a single compounding period (monthly).

$$i_{\text{monthly}} = \frac{15\%}{12} = 1.25\%$$

Compounded monthly = monthly interest
⇒ Divide

* Compounded monthly

Example 8: Finding the Effective Quarterly Interest Rate from Monthly Compounding

A bank offers a nominal annual interest rate of 12% compounded monthly. What is the effective quarterly interest rate?

Steps:

1. **Use the formula** because you're asked for the effective rate over a period longer than the compounding frequency (monthly to quarterly).

First, calculate the monthly interest rate:

$$i_{\text{monthly}} = \frac{12\%}{12} = 1\% = 0.01$$

Now compound this for 3 months:

$$i_{\text{quarterly}} = (1 + 0.01)^3 - 1 = 1.030301 - 1 = 0.030301$$

Convert to a percentage:

$$i_{\text{quarterly}} = 3.03\%$$

compounded monthy → $\frac{\text{interest}}{12}$

$\frac{12 \text{ months}}{4 \text{ quarters}} = 3$

quartarly interest > monthly compounding

$i_{eff} = (1 + \frac{r}{m})^m - 1$

1.5 monthy, effective yearly

$1.5\% \times 12 = 18\%$

$i = (1 + \frac{18\%}{12})^{12} - 1 = 1.95\%$