## Relation between thermodynamics & heat transfer

→ consider heat transfer between two blocks $T_A > T_B$

→ Applying the 1'st Law or Energy balance gives:-

$$\text{heat gain}_B = \text{heat loss}_A$$

→ Energy Transfer is exchange of internal energy

→ To know the direction of heat transfer, apply the 2'nd law of thermodynamics

$$S_{final} > S_{initial}$$ irreversible process

### → Modes of heat Transfer

Conduction

Convection

Radiation

## Conduction

→ Transfer of Energy from the more energetic to less Energetic particles of a substance by collisions between atoms &/or molecules

heat flux, heat transfer rate

$$q''_x = -K \frac{\Delta T}{L}$$ ] → fourier's Law

thermal conductivity

→ heat rate $q''_x \cdot A = q_x$

## Convection

$$q''_x = \underset{\text{convective heat transfer coefficient}}{hA} (T_s - T_\infty)$$ ] Newton's Law of cooling

→ The heat transfer coefficient depends on surface geometry, nature of the fluids motion, as well as fluid properties

→ Forced convection heat transfer, where fluid is forced over a surface by any mechanical means (pumps, compressors, fans)

→ Natural or free convection, driven by density differences resulting from boyancy forces, temp gradient

→ convection could be either turbulent or laminar depending on conditions

## Radiation

→ Thermal radiation is energy emitted by matter, it travels through vacuum

$$q''_{emitted} = \sigma T_s^4$$ ] Stefan - Boltzmann Law

$$q''_{emitted} = \varepsilon \sigma T_s^4$$

↳ Emessivity

$$q''_{incident} = \alpha \sigma T_s^4$$

↳ absorptivity

L03

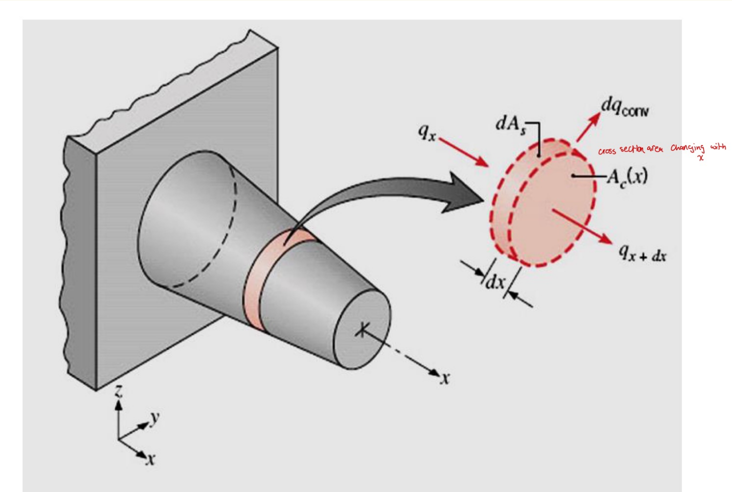
# Extended Surfaces

→ for heat transfer enhancement :-

1) increase fluid velocity to increase $h$
2) reduce fluid temp $(T_\infty)$
3) increase surface area in which heat transfer occurs, by employing fins

$$q = hA(T_s - T_\infty)$$

→ $h$ & $T$ are limited in increasing & reducing ; therfore changing surface area is more efficient



## General Energy Equation for fins

Start by Applying the conservation of Energy over the differential element:

$$E_{in} = -E_{out} - gen$$

surface area of differential element

$$q_x = -q_{x+dx} - dq_{conv} \rightarrow h\,dA_s(T_s - T_\infty)$$

Fourier's Law

$$q_x = -KA\frac{dT}{dx}$$

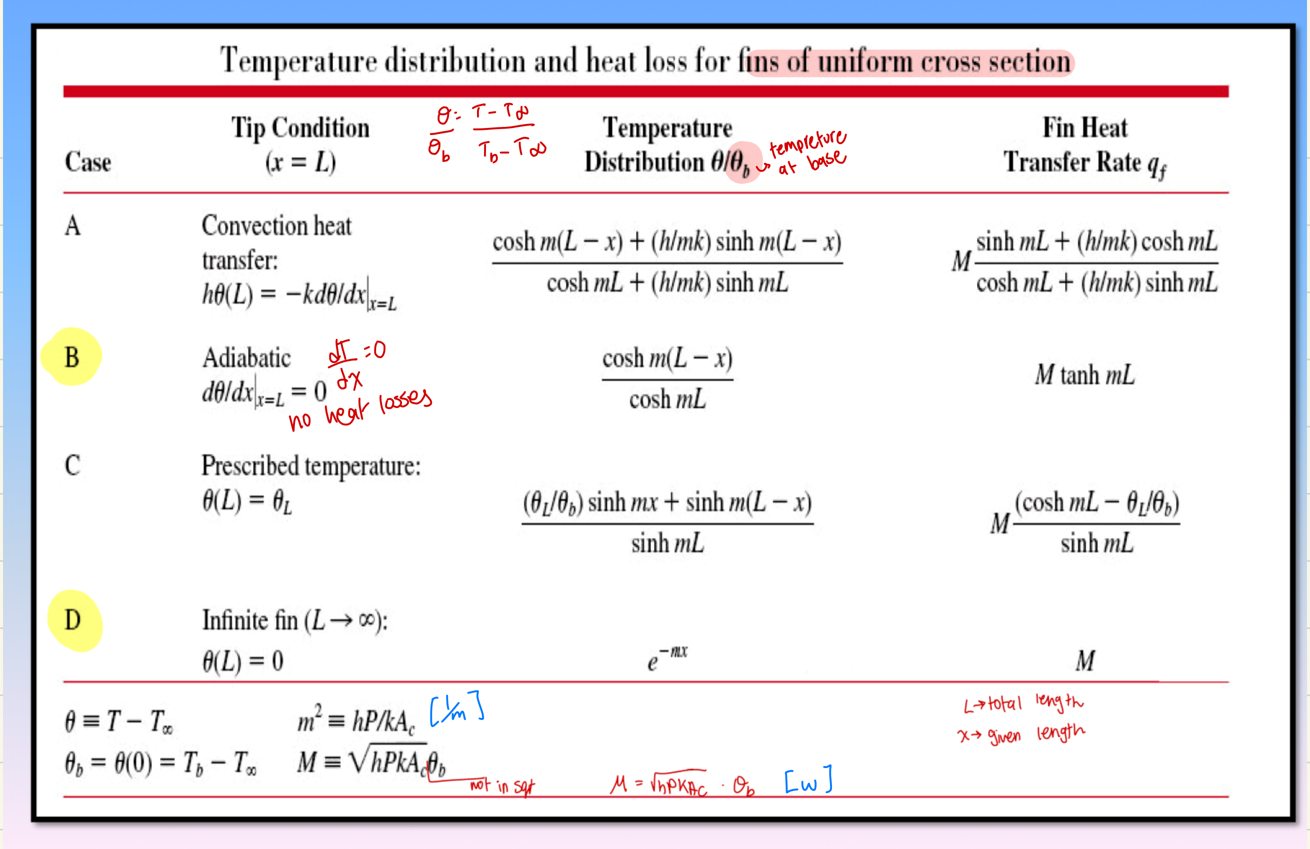
$$q_x + \frac{dq_x}{dx}dx$$

cross sectional area varies with $x$

$$-KA\frac{dT}{dx} - K\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)dx$$

$$-KA\cancel{\frac{dT}{dx}} = K\cancel{A\frac{dT}{dx}} + K\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)dx - h\,dA_s(T_s - T_\infty)$$

$$\left[\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T_s - T_\infty) = 0\right]$$



### Temperature distribution and heat loss for fins of uniform cross section

$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty}$

| Case | Tip Condition ($x = L$) | Temperature Distribution $\theta/\theta_b$ ↪ tempreture at base | Fin Heat Transfer Rate $q_f$ |
|---|---|---|---|
| A | Convection heat transfer:<br>$h\theta(L) = -k\,d\theta/dx\rvert_{x=L}$ | $\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ | $M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ |
| B | Adiabatic $\frac{dT}{dx} = 0$<br>$d\theta/dx\rvert_{x=L} = 0$<br>no heat losses | $\frac{\cosh m(L-x)}{\cosh mL}$ | $M \tanh mL$ |
| C | Prescribed temperature:<br>$\theta(L) = \theta_L$ | $\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ | $M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ |
| D | Infinite fin ($L \rightarrow \infty$):<br>$\theta(L) = 0$ | $e^{-mx}$ | $M$ |

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$ [1/m]

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$ not in sqt

$M = \sqrt{hPKA_c} \cdot \theta_b$ [W]

L → total length

x → given length

→ Fin Performance [fin effectivity $\varepsilon$]

$$\varepsilon = \frac{q_f}{h A_c \theta_b}$$

$\uparrow\varepsilon \quad \uparrow k$ , $\uparrow\varepsilon \quad \downarrow h$ , $\uparrow\varepsilon \quad \uparrow \frac{P}{A_c}$

Assume Adiabatic $\quad 0.98\,q_{f,max} = q_{f,adiabatic}$

$$0.98\,(hPKA_c)^{1/2}\,\theta_b = (hPKA_c)^{1/2}\,\theta_b \cdot \tanh(mL)$$

$$0.98 = \tanh(mL)$$

$$mL = \tanh^{-1}(0.98) = 2.3$$

$$L = 2.3/m \rightsquigarrow$$

to obtain most reasonable Length for max heat transfer

## → Effectiveness & Thermal Resistance

Assume infinite $q_f = M = \sqrt{hPKA_c}\ \theta_b \rightsquigarrow T_b - T_\infty$

$$q_f = \frac{\theta_b}{\underbrace{\left[\frac{1}{\sqrt{hPKA_c}}\right]}_{}} \rightsquigarrow \text{Resistance "infinite" convective} \qquad = \frac{\theta_b}{R_f}$$

$$R_{infinite} = \frac{\theta_b}{q_f} = \frac{1}{\sqrt{hPKA_c}}$$

without fin $q = hA_c\theta_b$

$$q = \frac{\theta_b}{\left[\frac{1}{hA_c}\right]} \rightsquigarrow \text{Resistance conductive} \qquad = \frac{\theta_b}{R}$$

$$\boxed{\varepsilon = \frac{\text{conduction}}{\text{convection}}} = \frac{q_{f,\infty}}{q} = \frac{\theta_b}{R_f} \cdot \frac{R}{\theta_b}$$

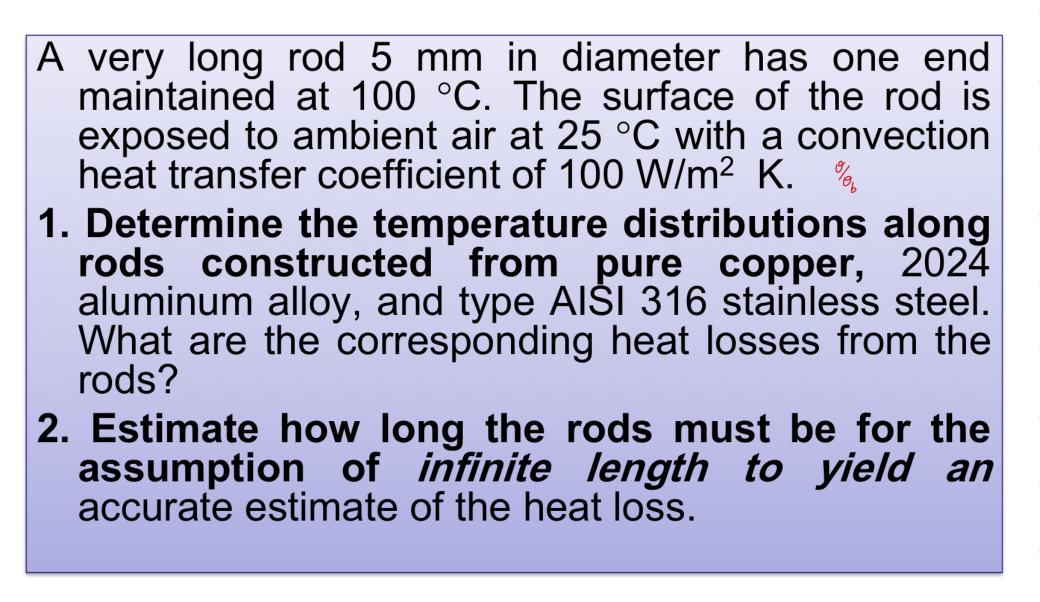
$$\varepsilon = \frac{R}{R_f} \quad \begin{matrix} \rightarrow \text{conductive "without fin"} \\ \rightarrow \text{convective "infinite fin"} \end{matrix}$$

⇒ if $\varepsilon > 1$ adding fins enhances heat transfer

$\varepsilon < 1$ adding fins decreases heat transfer (insulator)

$\varepsilon = 0$ adding fins has no effect

Ex:



A very long rod 5 mm in diameter has one end maintained at 100 °C. The surface of the rod is exposed to ambient air at 25 °C with a convection heat transfer coefficient of 100 W/m² K. $\theta/\theta_b$

1. **Determine the temperature distributions along rods constructed from pure copper,** 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. **Estimate how long the rods must be for the assumption of *infinite length to yield an*** accurate estimate of the heat loss.

→ infinite $D = 5mm$ $T_\infty = 25°c$

$h = 100$ $T_b = 100°c$

1- Temp distribution & heat losses

1- Pure copper

2- 2024 AL

3- AISI 316 stainless steel.

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty}$$

$$e^{-mx} = \frac{T - T_\infty}{T_b - T_\infty} \qquad e^{-mx}(T_b - T_\infty) = T - T_\infty$$

$$T = e^{-mx}(T_b - T_\infty) + T_\infty$$

$$q_f \text{ (Pure copper)} = \sqrt{hPKA_c}\ \theta_b$$

$$A_c = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.005)^2$$

$$P = \pi D = \pi(0.005)$$

$$K_{copper} = 398$$

↳ from tables

Appendix A

$$= \sqrt{100 \cdot \pi(0.005)\ 398 \cdot \frac{\pi}{4}(0.005)^2}\ (100 - 25)$$

$$= 8.3\ w$$

$$\left[ T_{fin} = \frac{T_b + T_\infty}{2} \right] \Longrightarrow$$ to find properties from tables

2- Estimate length

Assume 99% $\tanh(mL) = 0.99$

$mL = 2.65$

$L = \frac{2.65}{m}$

$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{100 \cdot \pi(0.005)}{398\ \pi/4\ (0.005)^2}}$

$L_\infty = \frac{2.65}{14.177} = 0.186\,m$

$m = 14.177$

→ Efficiency of Fins ($\eta$)

$$\eta = \frac{q_f}{q_{max}} = \frac{q_f}{h\underbrace{A_f}_{\text{surface Area}}\theta_b}$$

surface Area $= PL = \pi D L$

→ Max heat transfer when surface temp of pin = base temperature

⇝ for convection ; could be solved using adiabatic

$$q_{f,adiabatic} = M \tanh(mL) = \sqrt{hPKA_c} \cdot \theta_b \cdot \tanh(m\underbrace{L_c}_{})$$

rectangular → (flat plate) $L_c = L + t/2$

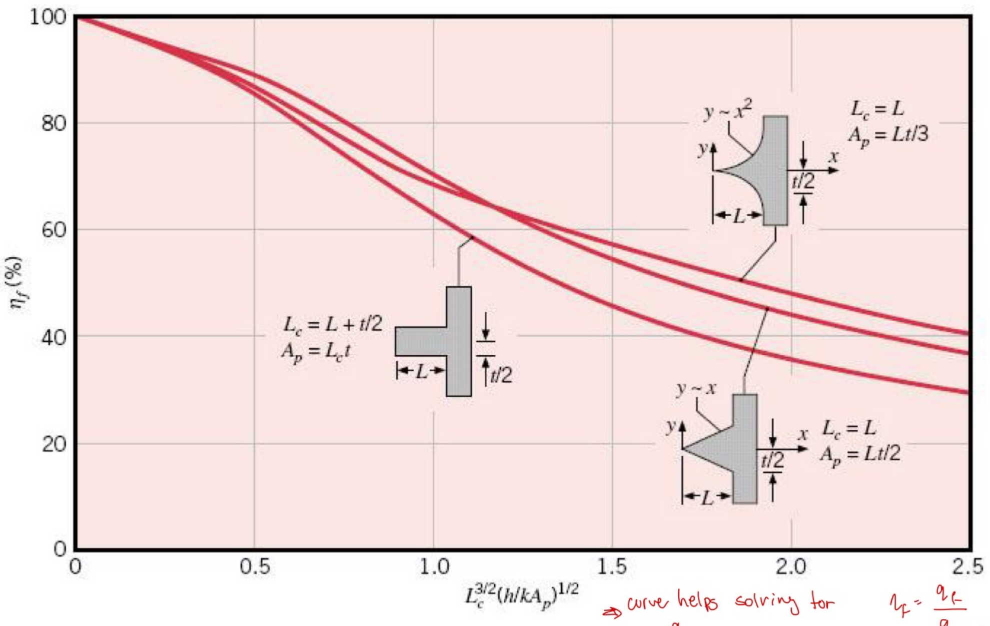
→ pin fin $L_c = L + D/4$

$$\eta_f = \frac{\tanh(mL)}{mL}$$

→ Error : rectangular : $\frac{ht}{K}$

pin : $\frac{hD}{2K}$

$\leqslant 0.0625$



⇒ curve helps solving for $q_f$ $\eta_f = \frac{q_f}{q_{max}}$

**FIGURE 3.18** Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

→ solve for $L_c^{3/2}(h/kA_p)^{1/2}$ depending on given shape to find efficiency

$$\eta_f = \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$

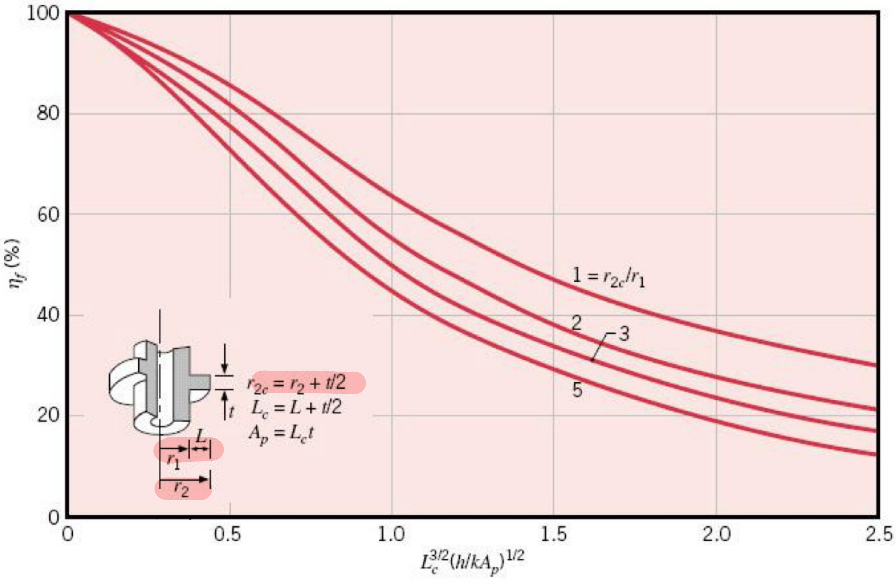


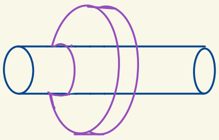
**FIGURE 3.19** Efficiency of annular fins of rectangular profile.

→ Annular fin

$$r_{2c} = \underbrace{r_2}_{r_1+L} + t/2$$

→ solve $L_c^{3/2}(h/kA_p)^{1/2}$ & connect it to corresponding value of r to find efficiency

$$q_f = \eta_f q_{max} = \eta_f(hA_f\theta_b)$$



Ex: An annular Aluminum fin of rectangular profile is attached to a circular tube having an outside diameter of 25mm & surface tempreture of 250°c, The fin is 1mm thick & 10 mm long, & the tempreture & the convection coefficient associated with the adjoining fins are 25°c & 25 w/m².k, what is the heat loss per fin?

→ $L_c^{3/2} \left( \frac{h}{K A_p} \right)^{1/2}$

$L_c = L + t/2$

$L_c = 0.010 + \frac{0.001}{2} = 0.0105$

$(0.0105)^{3/2} \left( \frac{25}{240 \cdot 1.05\times10^{-5}} \right)^{1/2}$

$= 0.11$

$K \rightarrow \frac{T_b + T_\infty}{2} = \frac{250 + 25}{2} = 137.5\mathring{C}$

410.5 K

↙

$K = 240 \ w/m \cdot K$

$A_p = L_c \cdot t = 0.0105(0.001) = 105\times10^{-5}$

$r_1 = D/2 = 0.0125$

→ $r_2 = r_1 + L$

$0.0125 + 0.010 = 0.0225 m$

→ $r_{2c} = r_2 + t/2$

$0.0225 + \frac{0.001}{2} = 0.023 m$

→ $\frac{r_{2c}}{r_1} = \frac{0.023}{0.012} = 1.84$

→ from curve $\eta \approx 96\%$

$A_f = 2\pi \left( r_{2c}^2 - r_1^2 \right)$

$2\pi \left( 0.023^2 - 0.0125^2 \right)$

$A_f = 0.00234$

$q_f = \eta \, q_{max}$

$0.96 \left( h A_f \theta_b \right) = 0.96 \left( 25 . 0.00234 . (250 - 25) \right)$

$q_f = 12.64 W$

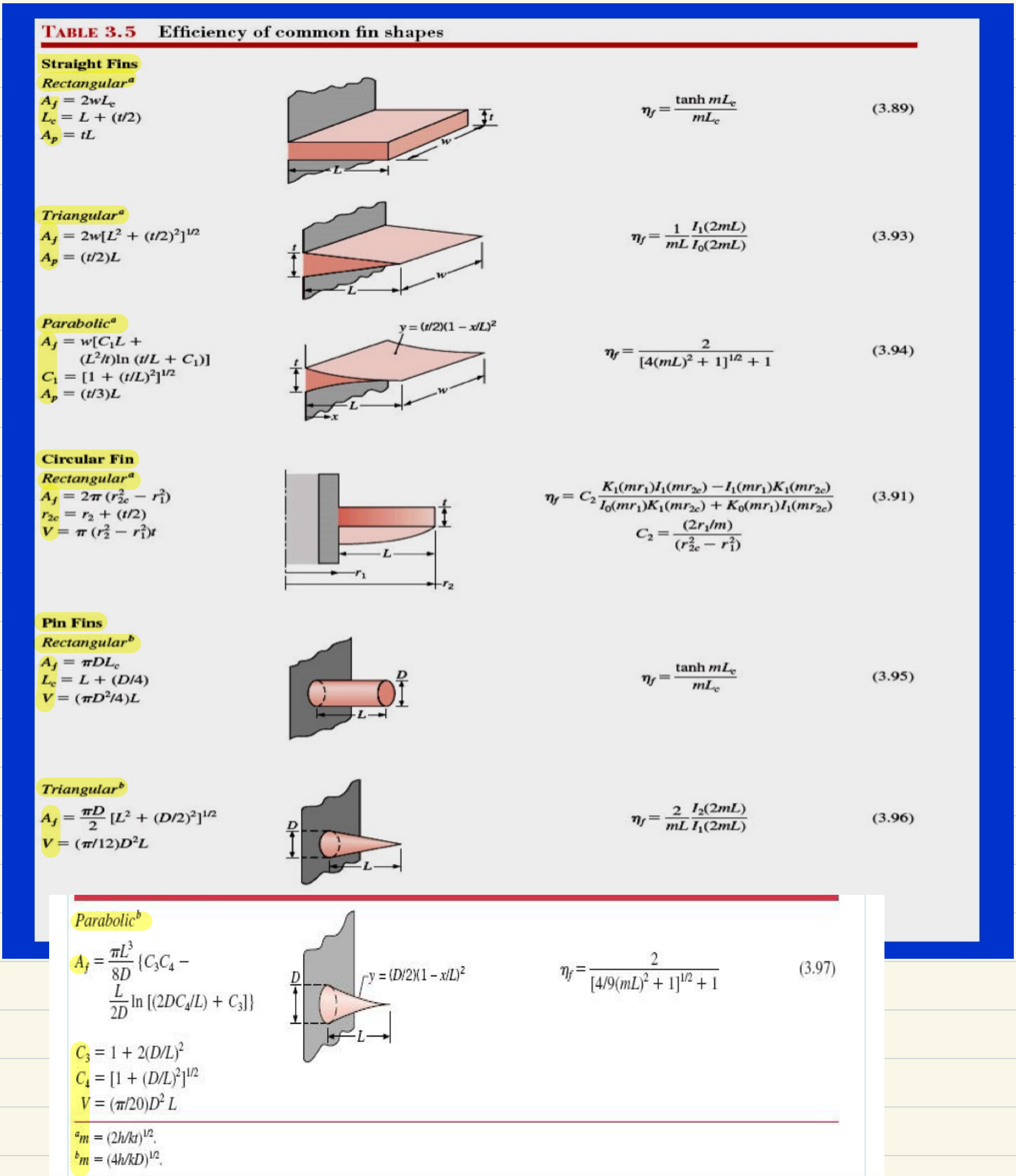


**TABLE 3.5 Efficiency of common fin shapes**

| | | |
|---|---|---|
| **Straight Fins** | | |
| *Rectangular*[a]<br>$A_f = 2wL_c$<br>$L_c = L + (t/2)$<br>$A_p = tL$ | $\eta_f = \frac{\tanh mL_c}{mL_c}$ | (3.89) |
| *Triangular*[a]<br>$A_f = 2w[L^2 + (t/2)^2]^{1/2}$<br>$A_p = (t/2)L$ | $\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$ | (3.93) |
| *Parabolic*[a]<br>$A_f = w[C_1L +$<br>$(L^2/t)\ln(t/L + C_1)]$<br>$C_1 = [1 + (t/L)^2]^{1/2}$<br>$A_p = (t/3)L$ | $\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$ | (3.94) |
| **Circular Fin** | | |
| *Rectangular*[a]<br>$A_f = 2\pi(r_{2c}^2 - r_1^2)$<br>$r_{2c} = r_2 + (t/2)$<br>$V = \pi(r_2^2 - r_1^2)t$ | $\eta_f = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$<br>$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$ | (3.91) |
| **Pin Fins** | | |
| *Rectangular*[b]<br>$A_f = \pi DL_c$<br>$L_c = L + (D/4)$<br>$V = (\pi D^2/4)L$ | $\eta_f = \frac{\tanh mL_c}{mL_c}$ | (3.95) |
| *Triangular*[b]<br>$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$<br>$V = (\pi/12)D^2L$ | $\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$ | (3.96) |
| *Parabolic*[b]<br>$A_f = \frac{\pi L^3}{8D}\{C_3C_4 -$<br>$\frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$<br>$C_3 = 1 + 2(D/L)^2$<br>$C_4 = [1 + (D/L)^2]^{1/2}$<br>$V = (\pi/20)D^2L$ | $\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$ | (3.97) |

---

[a] $m = (2h/kt)^{1/2}$.

[b] $m = (4h/kD)^{1/2}$.

Ex. A metal rod $D = 2\,cm$ $L = 10\,cm$ $K = 50\,w/m\cdot K$ $T_\infty = 20^\circ c$ $h = 30\,w/m^2\cdot K$ $T_b = 70^\circ C$
other end has negligable heat losses [adiabatic], calculate the heat losses from the rod

$$q_f : M\tanh(mL)$$

$$P = \pi D = \pi(2\times10^{-2}) = 0.0628$$

$$A_c = \frac{\pi D^2}{4} = 0.0157$$

$$q_f = \sqrt{hPKA_c}\;\theta_b \cdot \tanh\left(\sqrt{\frac{hP}{KA_c}}\cdot L\right)$$

$$q_f = \sqrt{30\cdot(\pi(2\times10^{-2}))\cdot 50\left(\pi\frac{(2\times10^{-2})^2}{4}\right)}\;(70-20)\cdot\tanh\left(\sqrt{\frac{30\cdot\pi\, 2\times10^{-2}}{50\left(\pi\frac{(2\times10^{-2})^2}{4}\right)}}\cdot 10\times10^{-2}\right)$$

$$q_f = 6.87\ W$$

⇒ Overall fin efficiency

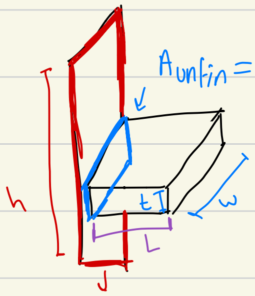
$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{tot,fin}}{\dot{Q}_{tot,nofin}} = \frac{h\,(A_{unfin} + \eta_{fin} A_{fin})\,(T_b - T_\infty)}{h\,A_{nofin}\,(T_b - T_\infty)}$$

$$Q_{tot,fin} = \dot{Q}_{unfin} + \dot{Q}_{fin}$$

$$hA_{unfin}(T_b - T_\infty) + \eta_{fin}\,h\,A_{fin}\,(T_b - T_\infty)$$

unfin → Area on surface

fin 〃 〃



$A_{unfin} = wh - (wt)$

$A_{nofin} = w\cdot h$

$A_{fin} = 2\cdot L\cdot w$

L04 Forced convection External & Internal Design Problems

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \begin{array}{l} \nearrow \text{turbulent} \\ \rightarrow \text{Laminar} \end{array}$$

⇝ Depending on flow regime & geometry of shape, find Nusselt number from tables

$$Nu = \frac{hL}{k} \qquad q = hA(T_s - T_\infty)$$

→ Forced convection ↗ outside plates, pipes, & spheres "outside diameter"
↘ inside pipes & ducts "inside diameter"

→ Forced convection in external flow

Flat Plate, Cylinder, Sphere

$Re < 5\times10^5$ Laminar

$Re > 5\times10^5$ Turbulent

$$T_f = \frac{T_\infty + T_s}{2}$$

fin tempreture

to find the properties, except the ones depending on the surface

* $Pe_x = Re_x \, Pr$

↳ Peclet number

$$** \quad Pr = \frac{C_p \mu}{k}$$

Ex: Air at pressure of $6KN/m^2$ & a tempreture of 300°c flow with a velocity of 10m/s over a flat plate 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface tempreture of 27°c

$P = 6kN/m^2$

$T_\infty = 300°c$

$v = 10m/s$

$L = 0.5m$

$T_s = 27°c$

$$T_F = \frac{T_s + T_\infty}{2} = \frac{300+27}{2} = 163.5°c \rightarrow 436.65\,K$$

$$Re = \frac{\rho v L}{\mu} = \frac{vL}{\nu}$$

→ from appendix table A.4 $\nu = 30.84 \times 10^{-6}\ m^2/s$

$$30.84 \times 10^{-6} \times \frac{101.325\ kPa}{6\ kPa} = 5.21 \times 10^{-4}\ m^2/s$$

$$Re = \frac{10 \cdot (0.5)}{5\,21 \times 10^{-4}} = 9596 \Rightarrow \text{Laminar}$$

$$\overline{Nu}_x = 0.664\, Re^{1/2} \cdot Pr^{1/3}$$

$$0.664\,(9596)^{1/2}\,(0.687)^{1/3} = 57.4$$

$$Nu = \frac{hL}{k}$$

$$57.4 = \frac{h \cdot (0.5)}{0.0364} \qquad \bar{h} = 4.18\ W/m^2 \cdot K$$

$$q = hA(T_s - T_\infty)$$

$$\frac{q}{w} = 3.253\,(0.5)\,(27-300)$$

$$\frac{q}{w} = 570\ W$$

Ex: Air at 20°c is blown over a 6cm OD of pipe that has surface tempreture of 140°c , The free stream air velocity is 10m/s . what is the rate of heat transfer per meter of pipe ?

$T = 20°c$  $T_s : 140$

$$T_f = \frac{T_\infty + T_s}{2} = \frac{20+140}{2} = 80°c + 273$$
$$= 353$$

$OD = 6cm$

$v = 10m/s$

⇝ properties from tables $\frac{400+350}{353+350} = \frac{26.41 + 20.92}{\gamma + 20.92}$

$\nu = 23.44 \times 10^{-6}$

$$Re = \frac{\rho v d}{\mu} = \frac{v d}{\gamma} = \frac{10 \times 6 \times 10^{-2}}{23.44 \times 10^{-6}} = 25597.26$$

Laminar Flow

$Pr = 0.6993$

$$\overline{Nu}_D = 0.3 + \left( 0.62\, Re^{1/2}\, Pr^{1/3} \underbrace{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{-1/4}}_{0.8772381843} \underbrace{\left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}}_{1.174900395} \right)$$

$\overline{Nu}_D = 91.0437$

$$\overline{Nu} = \frac{hD}{k} \qquad 91.0437 = \frac{h \times 6 \times 10^{-2}}{30.228 \times 10^{-3}} \qquad h = 45.867 \quad w/m^2k$$

$$q = hA\,(T_s - T_\infty)$$
$$45.867 \cdot \pi(6 \times 10^{-2})(140 - 20)$$

$$\frac{400-350}{353-350} = \frac{33.8-30}{k-30} \qquad k : 30.228 \times 10^{-3}$$

$$q = 1037.50 \text{ W}$$

Ex: A duct carries hot waste gas from a process unit to a pallution control device. The duct cross section is 4ft x 4ft & it has surface tempreture of 140°C. Ambient air at 20°C blows across the duct with a wind speed of 10 m/s. Estimate the rate of heat loss per meter of duct length.

$A = 4ft \times 4ft \rightsquigarrow \underline{1.2192 m}$ $\qquad T_f = \frac{140+20}{2} = 80 + 273 = 353 K$

$T_s = 140 \qquad T_\infty = 20$

$v = 10 m/s$ $\qquad \overline{Nu}_D = C Re_D^m Pr^{1/3}$

Properties:- $\nu = 23.44 \times 10^{-6}$ $\quad K = 30.228 \times 10^{-3}$ $\quad Pr = 0.6993$

$$Re_D = \frac{vL}{\nu} = \frac{10 \times 1.2192}{23.44 \times 10^{-6}} = 5.20136518 \times 10^5$$

Turbulent

$C = 0.102$

$m = 0.675$

$$\overline{Nu} = 0.102 \left(5.20136518 \times 10^5\right)^{0.675} (0.6993)^{1/3}$$

$$Nu = 653.4322$$

$Nu = \frac{hL}{K}$ $\qquad 653.4322 = \frac{h \times 1.2192}{30.228 \times 10^{-3}}$ $\qquad h = 16.2 \ W/m^2.K$

$$q = hA(T_s - T_\infty)$$

$$q = 16.2 \ (1.2192 \times 1.2192)(140 - 20)$$

$q = 2889.65$ → per meter of duct length

$q = 2370.11 \ W$

## Internal flow

$Re = \frac{\rho v L}{\mu}$ → Laminar
→ Transition
→ Turbulent

→ $Re < 2100$ ⇒ Laminar

$$Nu = 1.86 \left[ Re\, Pr \left( D/L \right) \right]^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14}$$

→ $0.5 < Pr < 17000$

→ if $\left[ Re.\, Pr \left( \frac{D}{L} \right) \right]^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14}$ ↗ $< 2$ use Nu eq

↘ $> 2$ $Nu = 3.66$

→ $2100 < Re < 10^4$ ⇒ Transition

$$Nu = 0.116 \left[ Re^{2/3} - 125 \right] Pr^{1/3} \left( \mu/\mu_w \right)^{0.14} \left[ 1 + \left( D/L \right)^{2/3} \right]$$

→ an alternative equation:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7 (f/8)^{1/2} (Pr^{2/3} - 1)} \left[ 1 + (D/L)^{2/3} \right]$$

$0.6 < Pr < 2000$

$Re > 2300$

$$f = (0.782 \ln Re - 1.51)^{-2}$$

↙ Darcy friction factor

→ $Re \geqslant 10^4$ ⇒ Turbulent

$$Nu = 0.027\, Re^{0.8}\, Pr^{1/3} \left( \mu / \mu_w \right)^{0.14}$$

for developed enterance :- $10 < L/D < 60$ → $q = h \underset{\pi D L}{A} (T_s - T_\infty)$

↳ check L

$$\therefore \bar{Nu} = 0.027\, Re^{0.8}\, Pr^{1/3} \left( \mu / \mu_w \right)^{0.14} \left[ 1 + \left( D/L \right)^{2/3} \right]$$

→ All properties except surface properties from $T_{avg} = \frac{T_i + T_o}{2}$

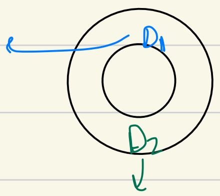
→ $T_{wall} = \frac{T_{s1} + T_{s2}}{2}$

→ Flow in ducts & conduits with non-circular cross section

equivilent Diameter $D_e = 4 \times \overset{\text{area}}{A} / \underset{\text{wetted parimeter}}{P}$

$$D_e = \cancel{4} \times \frac{\cancel{\pi}/\cancel{4} \left( D_2^2 - D_1^2 \right)}{\cancel{\pi} \left( D_1 + D_2 \right)}$$

$$= \frac{(D_2 - D_1)\cancel{(D_2 + D_1)}}{\cancel{(D_1 + D_2)}}$$



outside diameter of inner pipe

inside diameter of outer pipe

$A = \pi/4 \left( D_2^2 - D_1^2 \right)$

$P = \pi \left( D_1 + D_2 \right)$

$$D_e = D_2 - D_1$$

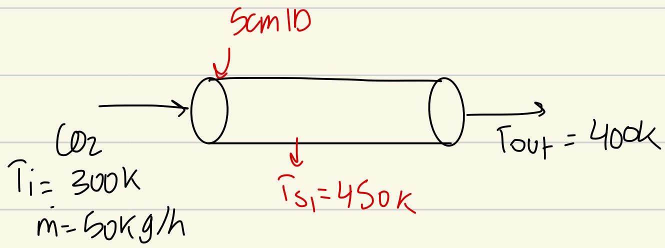
$$Nu = 3.66 + 1.2 \left( D_2 / D_1 \right)^{0.8} + \frac{0.19 \left[ 1 + 0.14 \left( D_2 / D_1 \right)^{1/2} \right] \left[ Re\, Pr.\, De / L \right]^{0.8}}{1 + 0.117 \left[ Re.\, Pr.\, De / L \right]^{0.467}}$$

External → Nu → $q = hA(T_s - T_\infty)$

Internal → Nu → $q = hA\,\Delta T_{lm}$ & $q = C_p m\,\Delta T$ ↳ $T_{out} - T_{in}$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]} \quad \longrightarrow \Delta T_1 = T_{s1} - T_{in} \quad \longrightarrow \Delta T_2 = T_{s2} - T_{out}$$

Ex: Carbon dioxide at 300K & 1atm is to be pumped through a 5cm ID pipe at a rate of 50 kg/h. The pipe wall will be maintained at a tempreture of 450k in order to raise the carbon dioxide temp to 400k. what length of pipe will be required



$q = hA\,\Delta T_{LM}$

① $c_p m\,\Delta T = h(\pi D L)\,\Delta T_{Lm}$

↳ for properties $T_{avg} = \frac{T_{in} + T_{out}}{2}$

$\frac{300 + 400}{2} = 350$

$0.8995 \frac{kJ}{\cancel{kg}\cdot\cancel{K}} \times \frac{50\cancel{kg}}{h} \times (400-300)\cancel{K} = 4497.5\ kJ/h$

$4997.5 \frac{kJ}{h} \cdot \frac{1h}{3600s} = 1.2493\ kW$

$c_p @ 350$ "Interpolation" = 0.8995 kJ/kg

② $$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(450-300) - (450-400)}{\ln\left[\frac{(450-300)}{(450-400)}\right]} = 91.02\ K$$

③ $Re = \frac{\rho v D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4\rho Q}{\pi D \mu}$ $\mu \rightarrow$ interpolation

$$Re = 4 \left| \frac{50\ \cancel{kg}}{\cancel{h}} \right| \frac{}{\pi(5\times10^{-2})\cancel{m}} \left| \frac{\cancel{m^2}}{169\times10^{-7}\ \cancel{N}\cdot\cancel{s}} \right| \frac{1\cancel{N}\cdot\cancel{s^2}}{\cancel{kg}\cdot\cancel{m}} \left| \frac{1\cancel{h}}{3600\cancel{s}} \right.$$

$= 20927.67$

⇒ Turbulent

$Pr = 0.7435$ $\mu_w = 210 \times 10^{-7}$ @T=450

$$Nu = 0.027\ Re^{0.8} \cdot Pr^{1/3} \left( \mu / \mu_w \right)^{0.14}$$

$$0.027 \,(2093828)^{0.8} \,(0.7435)^{1/3} \left( \frac{169 \times 10^{-7}}{210 \times 10^{-7}} \right)^{0.14} = 67.89$$

$$\overline{Nu} = \frac{hD}{k}$$ $k = 20.45 \times 10^{-3}$ W/m K

$$67.92 = \frac{h\,(5 \times 10^{-2})}{20.45 \times 10^{-3}}$$ $h = 27.76$ W/m$^2$.K

④ $q = h(\pi D L)\,\Delta T_{LM}$

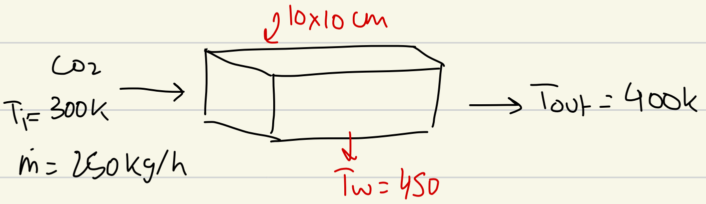
$1249.3 = 27.78\,(\pi\ 5 \times 10^{-2}\ L)(91.02)$ $L = 3.145$ m

⑤ Imp $10 < \frac{L}{D} < 60$

$$\frac{3.145}{5 \times 10^{-2}} = 62.9 > 60$$ no effect from enterance

⇒ developed

Ex:- Carbon dioxide at 300K & 1atm is to be pumped through a duct with a 10cm x 10cm Square cross section at a rate of 250 kg/h. The walls of the duct will be at a temp of 450k. what distance will the $CO_2$ travel through the duct before it's temp reaches 400k



$q = h(P.L)\ \Delta T_{LM}$

① $q = c_p \dot{m}\ \Delta T$

$0.8995 \frac{kJ}{kg} \cdot 250 \frac{kg}{h} \cdot (100)k \cdot \frac{1h}{3600}$

$q = 6.2465\ kW$

$= 6246.52\ W$

All properties same as last ex

$T_{avg} = 350$ $\quad$ $Pr = 0.7435$

$k = 0.02045$ $\quad$ $\mu = 169 \times 10^{-7}$

$c_p = 0.8995\ kJ/kg$ $\quad$ $\mu_w = 210 \times 10^{-7}$

$\Delta T_{LM} = 91.02\ K$

② $Re = \frac{\rho v D_e}{\mu}$ $\qquad$ $v = \frac{Q}{A}$ $\quad$ $\rho = \frac{m}{Q}$

$Re = \frac{D_e \dot{m}}{\mu A}$ $\qquad$ $D_e = 4 \times \frac{A}{P} = 4 \times \frac{(10 \times 10^{-2})^2}{4 \times 10 \times 10^{-2}} = 0.1m$

$Re = \frac{0.1\ (250)}{169 \times 10^{-7}\ (10 \times 10^{-2})^2} \cdot \frac{1h}{3600s} = 41091.38$ Turbulent

③ $\overline{Nu} = 0.027 \cdot Re^{0.8} \cdot Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$

$= 0.027\ (41091.38)^{0.8}\ (0.7435)^{1/3} \left(\frac{169 \times 10^{-7}}{210 \times 10^{-7}}\right)^{0.14}$

$\overline{Nu} = 116.48 = \frac{h\ D_e}{k}$

$116.48 = \frac{h\ (0.1)}{0.02045}$ $\qquad$ $h = 23.82\ W/m^2 K$

④ $6246.52 = 23.82\ (4 \times 10 \times 10^{-2} \cdot L)\ (91.02)$

$L = 7.202\ m$

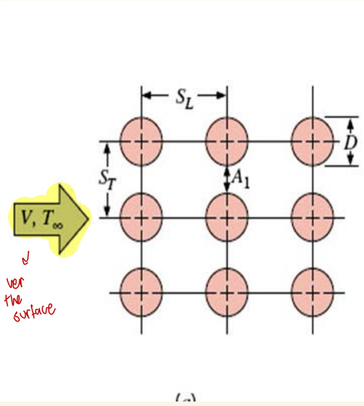
③ ⇒ check L $\quad 10 < \frac{L}{De} < 60$

$$\frac{7.202}{0.1} = 72 \rightarrow \text{no enterance effect}$$

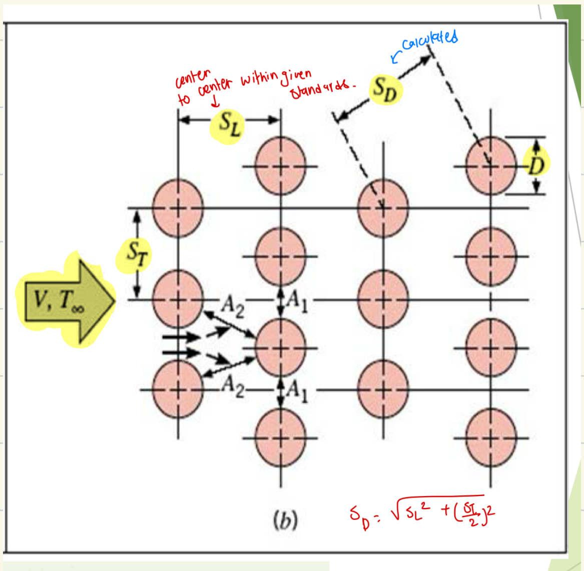
L5

## Tube Arrangements in a bank

Aligned



Staggered



$S_D = \sqrt{S_L^2 + (\frac{S_T}{2})^2}$

$S_T$: Transverse pitch

$S_L$: Longitudinal pitch

→ for large $S_L$, the influence of upstream rows decreases, & heat transfer in the downstream rows is not enhanced. $S_T/S_L < 0.7$ undesirable

→ for the staggared tube array, the path of the main flow is more tortous, & mixing of the cross flowing fluid is increased relative to the aligned tube arrangement.

→ Correlations of heat transfer

$$\overline{Nu}_D = C_1 \, Re_{D,max}^{m} \, Pr^{0.36} \left( Pr/Pr_s \right)^{1/4}$$

number of rows ↳

$N_L \geq 20$

$0.7 \leq Pr \leq 500$

$10 \leq Re_D \leq 2\times10^{6}$

↳ all properties except $Pr_s$ are evaluated at arithmatic mean of temp $\frac{T_i+T_o}{2}$

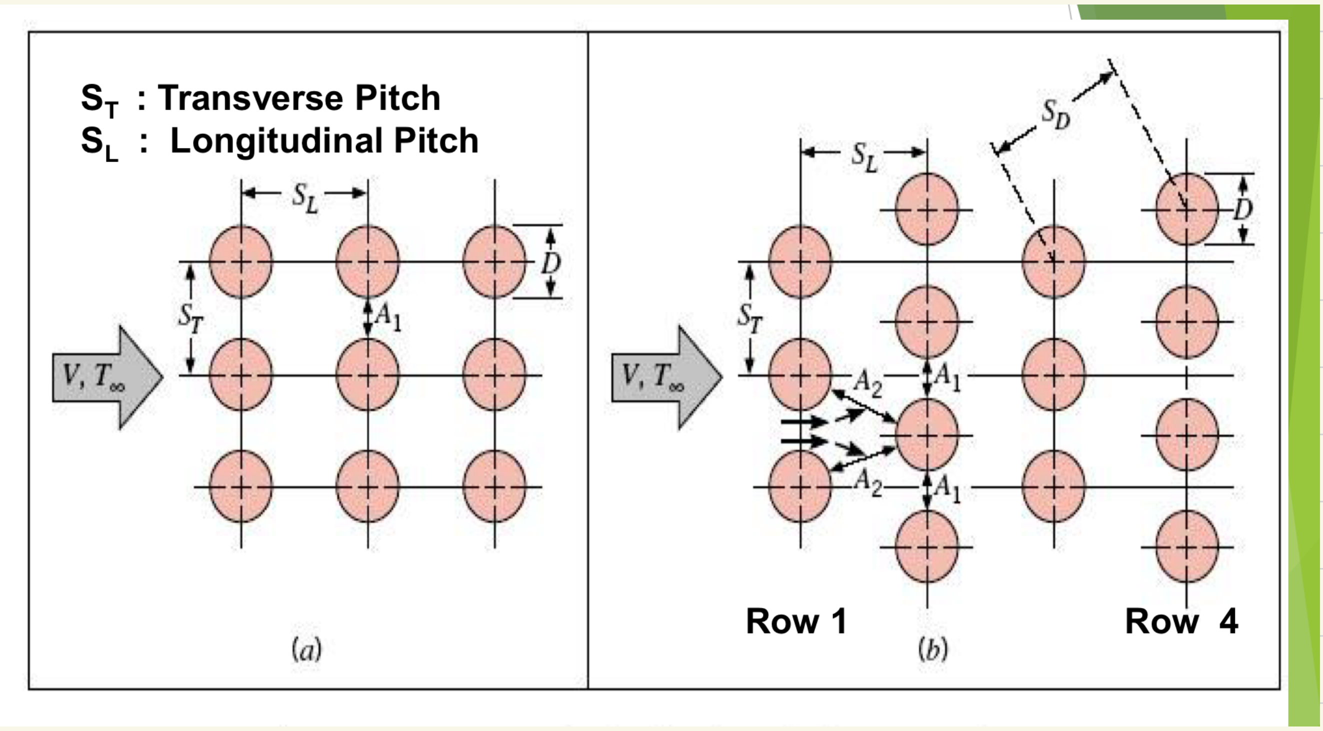
↳ $C_1$ & $m$ are tabulated data.

↳ if 20 or fewer rows ⇒ correction factor is applied

$$\overline{Nu}_{D(N_L<20)} = C_2 \, \overline{Nu}_D$$

↳ correction factor ⇒ tabulated data

→ $V_{maximum}$



→ for aligned arrangement, $V_{max}$ occurs at the transverse plane $A_1$

$$V_{max} = \frac{S_T}{(S_T - D)} \cdot \text{velocity}$$

→ for staggared arrangement, maximum velocity may occur at either the transverse plane $A_1$ or the diagonal plane $A_2$

⇒ if $2A_2 < A_1$ $\quad S_D < \frac{S_T + D}{2}$

$$V_{max} = \frac{S_T}{2(S_D - D)} \cdot \text{velocity}$$

⤳ Energy Balance

$$q = hA\,\Delta T_{LM} = h(\underset{\text{total number of tubes}}{N}\pi D L)\,\Delta T_{LM} = m\,C_p\,\Delta T$$

$$= (\rho V \underset{S_T \cdot N_T \cdot L}{A})\,C_p\,\Delta T$$

↳ $S_T \cdot N_T \cdot L$

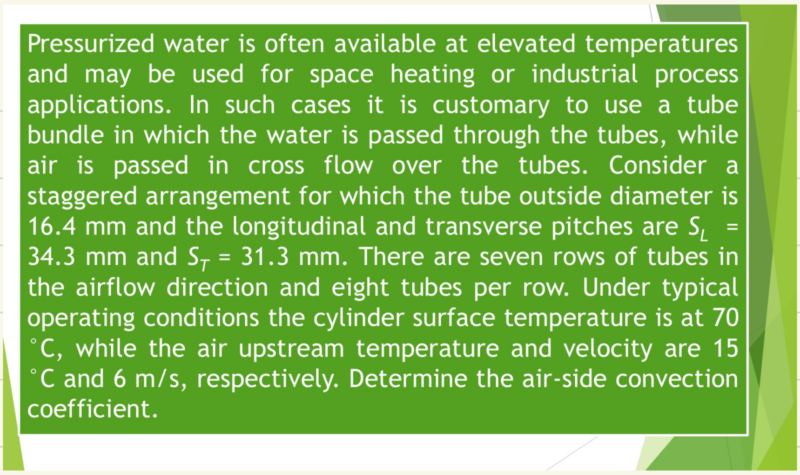
↳ number of tubes per row

$N = N_T \cdot N_L$

$$\Rightarrow \frac{T_s - T_o}{T_s - T_i} = \exp\left[\frac{-\pi D N h}{\rho V N_T S_T C_p}\right]$$

$$\rightarrow S_D = \sqrt{S_L^2 + \left(\frac{S_T}{2}\right)^2}$$

Ex:



Pressurized water is often available at elevated temperatures and may be used for space heating or industrial process applications. In such cases it is customary to use a tube bundle in which the water is passed through the tubes, while air is passed in cross flow over the tubes. Consider a staggered arrangement for which the tube outside diameter is 16.4 mm and the longitudinal and transverse pitches are $S_L$ = 34.3 mm and $S_T$ = 31.3 mm. There are seven rows of tubes in the airflow direction and eight tubes per row. Under typical operating conditions the cylinder surface temperature is at 70 °C, while the air upstream temperature and velocity are 15 °C and 6 m/s, respectively. Determine the air-side convection coefficient.

Staggared Arrangement

OD = 16.4 mm

$S_L = 34.3$ mm $\quad S_T = 31.3$ mm

$N_L = 7 \quad N_T = 8$ (per row)

$T_S = 70°C$

air $T = 15°C \quad v = 6$ m/s

$$Nu = C_1 C_2 Re^m Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{\frac{1}{4}}$$

→ properties $15+273 = 288$

$\rho = 1.217 \quad K = 25.34 \times 10^{-3} \quad \nu = 14.822 \times 10^{-6} \quad Pr = 0.71012 \quad Pr_s = 0.70098$

To find where Vmax occurs:

$$S_D = \sqrt{S_L^2 + \left(\frac{S_T}{2}\right)^2} \quad ☐ \quad \frac{S_T + D}{2}$$

$$\sqrt{(0.0343)^2 + \left(\frac{0.0313}{2}\right)^2} \quad ☐ \quad \frac{0.0313 + 0.0164}{2}$$

$$0.0377 > 0.02385$$

$V_{max}$ at $A_1$

$$V_{max} = \frac{S_T}{S_T - D} \cdot V$$

$$\frac{0.0313}{0.0313 - 0.0164} (6) = 12.6 \text{ m/s}$$

$$Re = \frac{V_{max} D}{\nu} = \frac{12.6(0.0164)}{14.822 \times 10^{-6}} = 13941.44$$

$$Nu = C_1 C_2 Re^m Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{\frac{1}{4}}$$

→ to Find constants

$$\frac{S_T}{S_L} = \frac{31.3}{34.3} = 0.912 < 2$$

$C_1 = 0.35\left(\frac{S_T}{S_L}\right)^{\frac{1}{5}}$ $C_2 = 0.95$ $m = 0\ 6$

$$0.35\left(\frac{31.3}{34.3}\right)^{\frac{1}{5}} = 0.34$$

$$Nu = 0.34 \times 0.95 \times (13941.44)^{0.6} \times (0.71012)^{0.36}\left(\frac{0.71012}{0.70098}\right)^{\frac{1}{4}}$$

$$Nu = 87.83$$

$87\ 83 = \frac{h(0.0164)}{25.34\times10^{-3}}$ $h = 135.71\ w/m^2.n$

→ find $\Delta P$

$$\Delta P = N_L x\left(\frac{\rho V_{max}^2}{2}\right) f$$

$P_T = \frac{S_T}{D} = \frac{31.3}{16.4} = 1.9$ ⇝ from charts $f \simeq 0.35$

$P_L = \frac{S_L}{D} = \frac{34\ 3}{16.4} = 2.09$

$\rightarrow \frac{P_T}{P_L} = \frac{1.9}{2.09} = 0.9$ ⇝ from charts $x \simeq 1.04$

$$\Delta P = 7(1\ 04)\left(\frac{1.217\ (12.6)^2}{2}\right)(0.35)$$

$$\Delta P = 246.15\ N/m^2$$

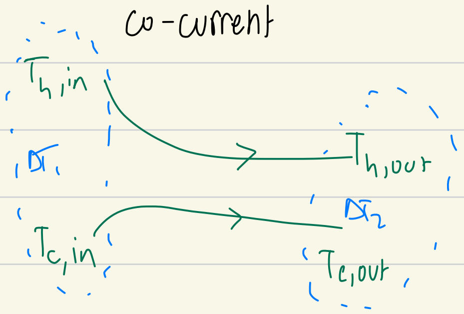
L6

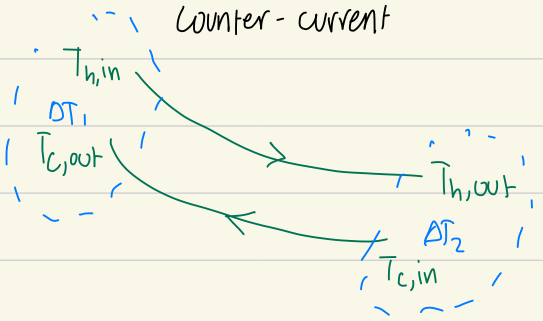
# Basic Theory of Heat exchangers

→ Concentric heat Exchanger

Double pipe heat exchanger ↗ co-current
↘ counter current

Annulus → inside pipes / Annular → outside pipe inside shell


Co-current


Counter-current

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

→ if $\Delta T_1 = \Delta T_2$ $\quad \Delta T_{LM} = \Delta T_{1 \text{ or } 2}$

$$Q = \underbrace{U}_{\text{overall coefficient}} A \, \Delta T_{LM}$$

→ Thermal Resistance

$$R = \frac{1}{UA}$$

$$\frac{1}{UA} = \underbrace{\frac{1}{h_i A_i}}_{\text{by convection}} + \underbrace{\frac{\ln(D_o/D_i)}{2k\pi L}}_{\text{by conduction}} + \underbrace{\frac{1}{h_o A_o}}_{\text{by convection}}$$

⇒ Multiplying by $A_o$ & inverting yeilds:-

$$U = \left[ \frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o} \right]^{-1}$$

⇒ correct for new & clean heat exchanger

→ Fouling process & Fouling factor

$$U_D = \left[ \frac{D_O}{D_i h_i} + \frac{D_O (\ln D_O / D_i)}{2k} + \frac{1}{h_o} + \frac{R_{Di} D_O}{D_i} + R_{DO} \right]^{-1}$$

↙
overall after
fouling factor

$R_{DO}$ & $R_{Di}$ ⇒ Tabulated data

## L7 Shell & Tube Heat Exchanger

TEMA →Tubular Exchange Manufacturers Association ⇒ Employes a three letter code to specify the front-end, shell, & rear-end types.

TEMA codes:

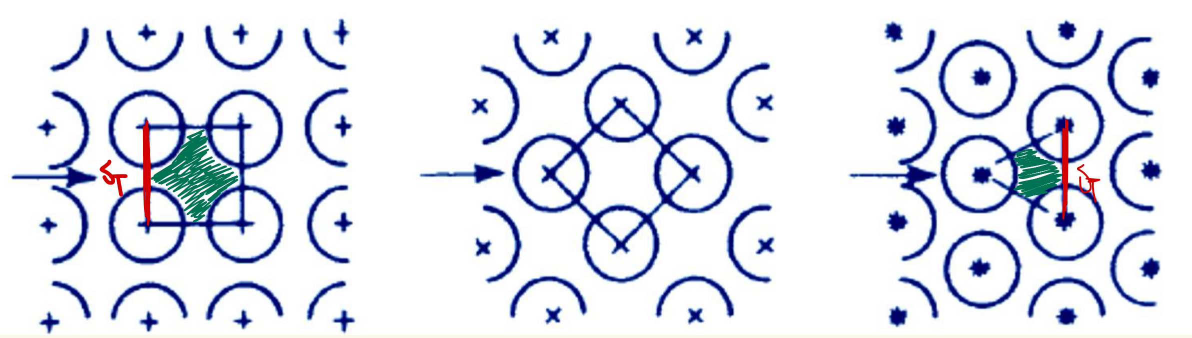
| TEMA Class: | Application |
|---|---|
| R | Severe requirments of petroleum & related process applications |
| C | Moderate requirments of commercial & general process applications |
| B | Chemical process service |

→ Shell : Available with inside diameters with discrete sizes upto 120 in. Shells upto 24in diameters are generally made from steel pipes, larger sizes from rolled steel plates.

→ Tubes : Sizes from ¼ to 2in → properties from appendix B

→ Tube layout

↓ Square (90°)     ↓ Rotated Square (45°)     ↓ Equilateral Traingle



⇒ Equivilent Diameter $De = 4 \times \frac{A}{P}$ ↳ wetted Perimeter

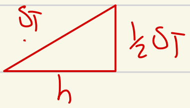
↳ for square $De = 4 \times \frac{[S_T^2 - \pi/4\, D^2]}{\pi D}$

Traingle → $\frac{180°}{360°} = \frac{1}{2}$

circle

Area of Traingle

↳ for equilateral Traingle $De = 4 \times \frac{([\frac{1}{2} \cdot S_T \cdot 0.86 S_T] - [\pi/8\, D^2])}{\pi/2 \quad D}$



Baffles:

- Baffles are a number of discs installed in the shell side of the heat exchanger.

  → Used to support the tube bundle & to withstand bending

Tube & shell passes:

→ Tube side

multiple passes are achived by U-tubes or Partioning the headers

1, 2, 4 ... 16

→ shell side

multiple passes by partioning the shell with logitudinal baffle

1 — 6

$$(\underset{\substack{\downarrow \\ \text{shell} \\ \text{pass}}}{1} - \underset{\substack{\downarrow \\ \text{tube} \\ \text{pass}}}{2})$$

→ (3 - 6) is not practicle therfore 3(1 − 2)

- flow pattern in shell & tube is a sinous motion both transverse & parallel to the tubes

# L8 Heat Exchanger Calculations

The LMTD correction factor

$$\Delta T_m = F(\Delta T_{Lm})$$

$$R = \frac{T_a - T_b}{t_b - t_a} \qquad P = \frac{t_b - t_a}{T_a - t_a}$$

$T_a$ = shell inlet temp

$T_b$ = shell outlet temp

$t_a$ : tube inlet temp

$t_b$ = tube outlet temp

$F > 0.8$ if else change the design

→ Tw for h correction

$$T_w = \frac{t_{avg}\, h_i + T_{avg}\, h_o \left( D_o / D_i \right)}{h_i + h_o \left( D_o / D_i \right)}$$

$T_{avg}$ → shell

$t_{avg}$ → tube

$$\phi_1 = \left( \mu / \mu_w \right)^{0.14}$$

$$\phi_o = \left( \mu / \mu_w \right)^{0.14}$$

→

$$h_{i,corrected} = \phi_i\, h_i$$

$$h_{o,corrected} = \phi_i\, h_o$$

$\left( \mu / \mu_w \right)^{0.14}$ → Turbulent

$\left( \mu / \mu_w \right)^{0.25}$ → Laminar

# L9 Heat Exchangers Design of Double pipe heat exchanger

→ Two purposes must be satisfied during the design process

1) Low capital cost ( small area, high U)

2) low operating cost ( low $\Delta P$, allowable range)

* for low viscosity liquids → 7-20 psi

* for gases → 1-5 psi

↳ series - parallel configurations of hair pins are considered to justify the allowable $\Delta P$.

## Heat transfer coefficient for heat exchangers without fins:-

For Turbulent flow Re $> 10^4$

$$Nu = 0.023 \, Re^{0.8} \, Pr^{1/3} \left( \mu / \mu_w \right)^{0.14}$$

For Transient flow $2100 < Re < 10^4$

$$Nu = 0.116 \left[ Re^{2/3} - 125 \right] Pr^{1/3} \left( \mu / \mu_w \right)^{0.14} \left[ 1 + \left( D_i / L \right)^{2/3} \right]$$

* Both eq work in tubes & annuli , with De replacing Di

For Laminar flow $Re < 2100$

$$Nu = 1.86 \left[ Re \, Pr \, (D/L) \right]^{1/3} \left( \mu / \mu_w \right)^{0.14}$$

→ in annulus:-

$$Nu = 3.66 + 1.2 \left( D_2 / D_1 \right)^{0.8} + \frac{0.19 \left[ 1 + 0.14 \left( D_2 / D_1 \right)^{0.5} \right] \left[ Re \, Pr \, De/L \right]^{0.8}}{1 + 0.117 \left[ Re \, Pr \, De/L \right]^{0.467}}$$

$$\Delta P_f = \frac{\overset{\text{friction factor}}{f}\ \overset{\text{pipe length}}{L}\ \overset{\text{fluid density}}{\rho}\ V^2 \leftarrow \text{avg fluid velocity}}{\underset{\text{gravitational acceleration}}{2g_c}\ D_i \rightarrow \text{inner pipe ID}}$$

→ Laminar flow in inner pipe

$$f = \frac{64}{Re}$$

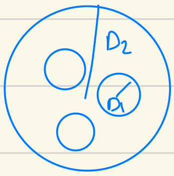
→ Laminar flow in annulus

$$f = \frac{64}{Re}\left[\frac{(1-k)^2}{1+k^2+(1-k^2)/\ln k}\right] \qquad k = \frac{D_1}{D_2}$$

→ for turbulent & Transient $Re > 3000$

$$f = 0.3673\ Re^{-0.2314}$$

Multi tube heat exchanger



Flow area $= A_f = \frac{\pi}{4}\left(D_2^2 - \underset{\hookrightarrow \text{number of tubes}}{n_t}\, D_1^2\right)$

Wetted Perimeter $P = \pi\left(D_2 + n_t D_1\right)$

$$\therefore\ De = 4\ \frac{\frac{\pi}{4}\left(D_2^2 - n_t D_1^2\right)}{\pi\left(D_2 + n_t D_1\right)}$$

$$De = \left(D_2^2 - n_t D_1^2\right)\ \left(D_2 + n_t D_1\right)$$

Over Surface [over design]

$U_{Design} < U_{Calculated}$ → due to turbulance

* higher fouling → smaller surface area [inner]

## 4.10 Preliminary design of Shell & Tube Exchangers

→ One simply estimates a value for the overall coefficient based on the tabulated values & then computes the required heat transfer area from heat transfer eq

$$q = UA\,\Delta T_{LM}$$

→ In computing the tube-side coefficient ($h_i$), it is assumed that all tubes in the exchanger are exposed to the same thermal & hydraulic conditions. $h_i$ → same for all tubes, & the calculations can be made for single tube

$$\dot{m}_{\text{per tube}} = \frac{\overset{\text{total } \dot{m} \text{ for tube side}}{\dot{m}_t}\ \overset{\text{\# of tube side passes}}{n_p}}{\underset{\text{\# of tubes}}{n_t}} = \frac{\dot{m}}{n_t/n_p}$$

→ ho calculations:
1) Heat transfer reaserch Inc. (HTRI) method
2) Delware method
3) Kern method

**Delware method:**

→ graph of modified Colburn factor $J_H$ vs Shell side Reynold's number

→ valid for segmental baffles with 20% cut, also based on TEMA standards

$$h_o = J_H \left(\frac{K}{D_e}\right) Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

→ flow area across shell

$$a_s = \frac{\overset{\text{shell inside D}}{d_s}\ \overset{\text{clearance}}{C}\ B \rightarrow \text{Baffle spacing}}{144\ \underset{\text{Pitch}}{P_T}}$$

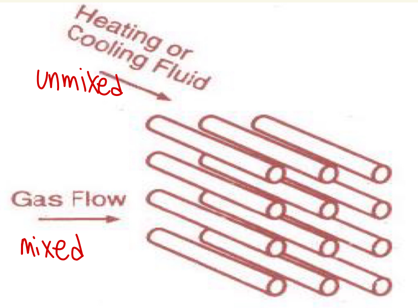
→ shell mass flux $G = \frac{\dot{m}}{a_s}$

→ Reynolds number for shell side fluid $Re = \frac{G D_e}{\mu}$

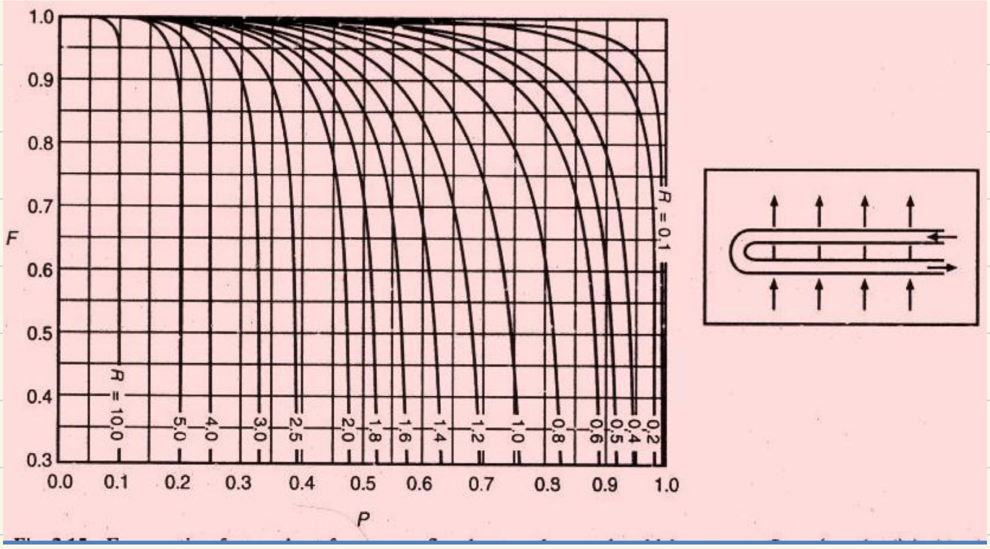
## L.11 Cross flow exchanger & exchanger effectiveness

- Counter current > cross flow > co-current

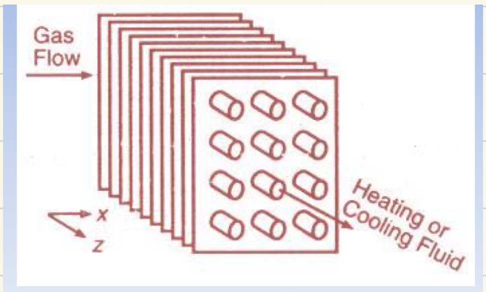
  - cross flow mixed - unmixed flow



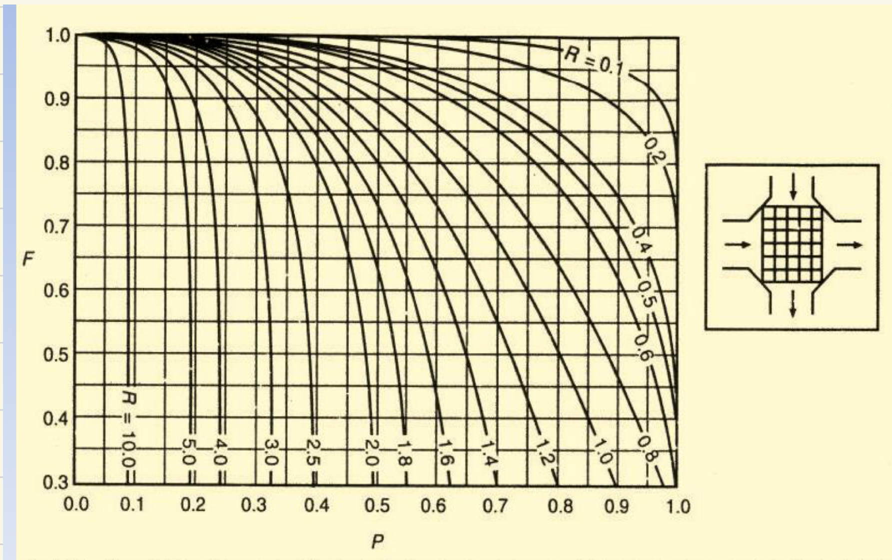
Unmixed fluid → inside channel



- cross flow exchanger both fluids unmixed



Both fluids unmixed → both in channels



# Heat Exchanger Analysis: The NTU method

- preferable when only inlet temperatures are known
- Max possible heat transfer Rate $q_{max}$, can be acheived in case of counter-current flow of infinite length, But infinite length is (unachievable, costining, ...)

$\Rightarrow$ if $C_c < C_h$ $\quad C_c \Delta T_c = C_h \Delta T_h$ $\quad \therefore \Delta T_c > \Delta T_h$

→ The cold fluid would be heated to inlet temperature of hot fluid

$$q_{max} = C_c \left(T_{h,in} - T_{c,in}\right)$$

$\Rightarrow$ if $C_h < C_c$ $\quad C_h \Delta T_h = C_c \Delta T_c$ $\quad \Delta T_h > \Delta T_c$

→ The hot fluid would be cooled to the inlet of cold fluid

$$q_{max} = C_h \left(T_{h,in} - T_{c,in}\right)$$

$\Rightarrow$ In General $\quad q_{max} = C_{min} \left(T_{h,in} - T_{c,in}\right)$

↳ $C_{min} = C_h$ or $C_c$ whichever is the smallest

$\Rightarrow$ Effectiveness $\varepsilon$ = The ratio of actual heat Transfer for the exchanger to the maximum possible heat transfer rate

$$\varepsilon = \frac{q_{act}}{q_{max}} = \frac{C_h \left(T_{h,in} - T_{h,out}\right)}{C_{min} \left(T_{h,in} - T_{c,in}\right)}$$

or

$$\frac{C_c \left(T_{c,out} - T_{c,in}\right)}{C_{min} \left(T_{h,in} - T_{c,in}\right)}$$

* effectiveness must be in the range $0 \leq \varepsilon \leq 1$

→ Assume a parallel flow for which $C_{min} = C_h$

$$\varepsilon = \frac{(T_{h,in} - T_{h,out})}{(T_{h,in} - T_{c,in})}$$ ⇒ if $\varepsilon$, $T_{h,in}$ & $T_{c,in}$ are known

$$q_{act} = \varepsilon \, q_{max}$$

$$q_{act} = \varepsilon \, C_{min} \, (T_{h_{in}} - T_{c,in})$$

→ for any exchanger it can be shown that

$$\varepsilon = f\left(\underbrace{NTU}_{\text{number of Transfer Unit}}, \underbrace{\frac{C_{min}}{C_{max}}}_{= \frac{C_c}{C_h} \text{ or } \frac{C_h}{C_c}}\right)$$

depending on the relative values of heat capacity rates

- Number of Transfer unit

$$NTU = \frac{UA}{C_{min}}$$

* $\varepsilon$ & NTU → tables & charts.

* for $C_r = 0$ , as in boiler, condensor, or single stream heat exchanger, is given by eq

$$[\ \varepsilon = 1 - \exp(-NTU)\ ]$$ for all flow arrangements

Hence for this special case, it follows that heat exchanger behaviour is independent of flow arrangement

* To determine NTU for shell & Tube heat exchanger with multiple shell passes, $\varepsilon$ would first be calculated for the entire heat exchanger F & $\varepsilon_1$ ⇒ 11.31 c & 11.31 b

$\varepsilon$ ⇒ 11.30 c ⇝ 11.30 b to find NTU

(result)n = NTU ⇒ 11.31 d

## L.12 Condensation [ Phase Change heat Transfer Problems ]

- Condensation & boiling ⇒ phase change of a fluid
- The primary driving force for heat transfer is not a temperature difference across a boundary layer, but rather the latent heat of the fluid

Latent Heat = Heat of vaporization = Enthalpy of vaporization.

→ Condensors:

Feed : Saturated vapor or super heated vapor

Condensate: saturated liquid or subcooled liquid

### Condensation Types & Mechanisms:

- Condensation occurs when the temperature of the vapor is reduced below its saturation temperature
- A solid surface with temperature below vapor's sat temp is needed for condensation

Types of Condensation:

1- Film -wise condensation

2- Drop - wise condensation

### Filmwise Condensation:

→ liquid condensate forms a continous film over the surface , this film flows down the surface under the action of gravity , shear force due to vapor flow , or other forces (Bouancy)

→ more common type of condensation ←

→ The liquid layer of condensate acts as a barrier to heat flow due to its very low thermal conductivity & hence low heat transfer rate. [Thermal resistant for heat transfer]

### Dropwise Condensation:

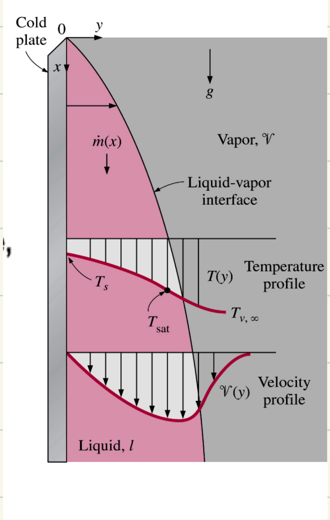
→ when liquid condensate doesn't wet the solid surface , droplets don't spread but form seperate drops.

→ The drops in turn coalesce to form large drops & sweeping clean a portion of the surface, where new droplets generate.

→ The average heat transfer coefficient for dropwise condensation is much higher than filmwise condensation.

[ part of the solid surface will have droplets & Blank space ⇒ doesn't create a heat transfer barrier (no heat transfer resistance) ⇒ heat transfer rate ↑ ]

## Film wise Condensation Nusselt Analysis



- The flow of condensate in film is laminar
- Constant fluid properties
- Subcooling of condensate may be negligable
- Momentum changes through the film may be negligable
- The vapor is stationary & exerts no drag on the condensate
- Heat transfer is by conduction only
- Isothermal surface

### → Force Balance on an Element

Gravity force = Bouyant force + Friction forces ( due to viscosity )

* $T_g$ → sat Temp     $T_g < \text{ambient } T_\infty$  or  $= T_\infty$

⇒ Velocity profile $V_z = \dfrac{\delta^2 g}{\nu_f}\left(1 - \dfrac{\rho_v}{\rho_f}\right)\left(\dfrac{y}{\delta} - \dfrac{y^2}{2\delta^2}\right)$ ①

⇒ mass flow rate of condensate $\dot{m}_f = \dfrac{W \delta^3 g \rho_f}{3 \nu_f}\left(1 - \dfrac{\rho_v}{\rho_f}\right)$ ②

⇒ heat flow at the wall within the area $W\,dz$

$$q_y = -k_f\,(W\,dz)\,\frac{dT}{dy}\Big|_{y=0}$$

⇒ Assume linear Temperature profile from $T_{wall}$ to $T_g$

$$q_y = -k_f\,W\,dz\,\frac{T_w - T_g}{\delta}$$

⇒ The amount of condensate between $z$ & $dz$ is given by added condensate

$$= \dot m_f\Big|_{z+dz} - \dot m_f\Big|_{z}$$

constant in $dz$

$$\frac{d\dot m_f\,dz}{dz} = \frac{d\dot m_f}{d\delta}\cdot\frac{d\delta}{dz}\cdot dz$$

$$= \frac{d\dot m_f}{d\delta}\cdot d\delta$$

→ from ②

$$\frac{d\dot m_f}{d\delta}\cdot d\delta = \frac{W\rho_f\,\delta^2 g}{\nu_f}\left(1-\frac{\rho_v}{\rho_f}\right)d\delta \qquad ③$$

→ eq ③ x $h_{fg}$

$$\frac{W\rho_f\,\delta^2 g}{\nu_f}\left(1-\frac{\rho_v}{\rho_f}\right)h_{fg}\,d\delta = k_f\,W\,dz\,\frac{T_g - T_w}{\delta}$$

or

$$\delta^3\,d\delta = \frac{k_f\,\nu_f\,(T_g - T_w)}{\rho_f\,g\,h_{fg}\left(1-\frac{\rho_v}{\rho_f}\right)}\,dz$$

⇒ Integrating

$$\delta^4 = \frac{4k_f\,\nu_f\,z\,(T_g - T_w)}{\rho_f\,g\,h_{fg}\left(1-\frac{\rho_v}{\rho_f}\right)} + C_1$$

$z=0 \quad \delta=0 \quad C_1=0$

$$\boxed{\delta = \left[\frac{4\,k_f\cdot\nu_f\cdot z\cdot(T_g - T_w)}{\rho_f\cdot g\cdot h_{fg}\cdot\left(1-\frac{\rho_v}{\rho_f}\right)}\right]^{\frac{1}{4}}} \qquad ④$$

⇒ Express heat transfer in terms of local convective coefficient "$h_z$"

$$h_z \, W \, dz \, (T_w - T_g) = -K_f \, W \, dz \, \frac{T_g - T_w}{\delta}$$

or $$h_z = \frac{K_f}{\delta}$$

→ Substitute in eq ④

$$h_z = K_f \left[ \frac{\rho_f \, g \, h_{fg} \left(1 - \frac{\rho_v}{\rho_f}\right)}{4 k_f \, \nu_f \, z \, . (T_g - T_w)} \right]^{\frac{1}{4}}$$ ⑤

⇒ local Nusselt number is

$$Nu = \frac{h_z \, z}{K_f} = \left[ \frac{\rho_f . g . z^3 . h_{fg} \left(1 - \frac{\rho_v}{\rho_f}\right)}{4 K_f \, \nu_f \, (T_g - T_w)} \right]^{\frac{1}{4}}$$ ⑥

Average coefficient over the entire surface

$$\bar{h} = \frac{4}{3} K_f \left[ \frac{\rho_f . g . h_{fg} . \left(1 - \frac{\rho_v}{\rho_f}\right)}{4 . K_f \, \nu_f . L \, (T_g - T_w)} \right]^{\frac{1}{4}} = \frac{4}{3} h_L$$ ⑦

↳ h local at L = $h_L$

By derivation $$\bar{h} = 0.943 \left[ \frac{\rho_f . K_f^3 . h_{fg} \left(1 - \frac{\rho_v}{\rho_f}\right)}{\nu_f . L . (T_g - T_w)} \right]^{\frac{1}{4}}$$ ⑦

⇒ Reynolds

$$Re = \frac{V \, D_h}{\nu_f}$$

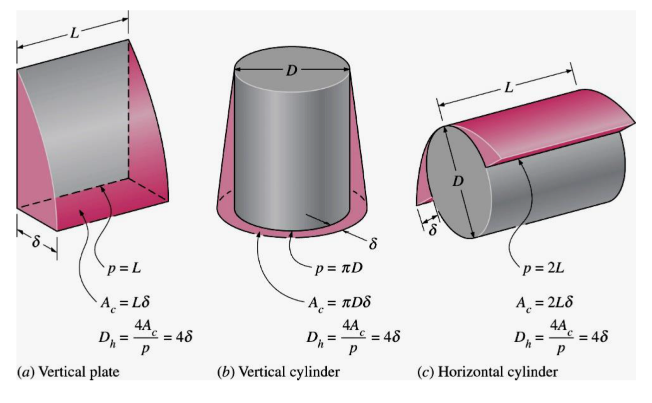
$D_h$ = hydraulic diameter $= \frac{4A}{P} = \frac{4 \delta W}{W} = 4\delta$

V = avg film velocity

$$Re = \frac{V(4\delta)}{\nu_f}$$

$$= \frac{\dot m_f}{\rho_f A} \frac{4\delta}{\nu_f} = \frac{4\dot m_f}{\rho_f(\delta w)} \frac{\delta}{\nu_f} = \frac{4\dot m_f}{w \rho_f \nu_f} \quad ⑧$$

for Laminar flow $Re < 1800$



→ $D_h$ for some geometries

→ Note !

Re can be written in terms of convective coefficient

$$q = h_L A_s (T_g - T_w)$$

↳ Surface Area of plate in contact with film.

but $q = m_f h_{fg}$

$$\dot m_f = \frac{h_L A_s (T_g - T_w)}{h_{fg}}$$

$$\therefore Re = \frac{4 h_L A_s (T_g - T_w)}{W h_{fg} \rho_f \nu_f}$$

→ not very common

→ Note 2

Expiremental values of the convective coefficient can be as much as 20% higher than those predicted in eq 7 for Laminar flow

$$\bar{h} = 1.13 \left[ \frac{\rho_f . k_f^3 . h_{fg} \ g . (1 - \frac{\rho_v}{\rho_f})}{\nu_f . L . (T_g - T_w)} \right]^{1/4}$$

## ⇒ Turbulent film Condensation on a vertical flat surface ⇐

$$Nu = \frac{\bar{h}_L (\nu_l^2/g)^{1/3}}{k_L} = \frac{Re}{8750 + 58 Pr^{-0.5} (Re^{0.75} - 253)}$$

for $Re \geq 1800$ Turbulent

$Pr \geq 1$

$$\bar{h} = 0.0077 k_f \left[ \frac{g \cdot (1 - \frac{\rho_v}{\rho_f})}{\nu_f^2} \right]^{1/3} Re_\delta^{0.4}$$

$$Re_\delta = \frac{4 \dot{m}_f}{W \rho_f \nu_f} > 1800$$

$$q = \bar{h} A_s (T_g - T_w)$$

$$\dot{m}_f = q / h_{fg}$$

## ⇝ Laminar film condensation on an inclined film surfacc

→ Replace $g$ with $g \sin\theta$ ⇝ Same as vertical plate correlations.

$\theta$ = angle of inclination of the plate with horizontal

## ⇝ film condensation on vertical tube

→ same correlations of vertical Plate if film thickness < outside diameter

$$Re_\delta = \frac{4 \dot{m}_f}{\pi D \mu_f}$$

→ tube loading $\Gamma = \frac{\dot{m}_f}{\pi D}$

→ Plate loading $\Gamma = \frac{m_f}{W}$

$$Re_\delta (\text{tube}) = \frac{4\Gamma}{\mu_f}$$

imp

→ for tube bundle $\Gamma = \frac{\dot{m}_f}{N_f \pi D}$

↳ film condensation on a horizontal tube & horizontal tube Bank:

→ for one tube $$\bar{h} = 0.728 \left[ \frac{g \cdot \rho_f \cdot \left(1 - \frac{\rho_v}{\rho_f}\right) k_f^3 \ h_{fg}}{\nu_f \ (T_g - T_w) \ \underset{\text{↳ outside Diameter}}{D}} \right]^{\frac{1}{4}}$$

→ for j tubes vertically above each other

$$\bar{h} = 0.728 \left[ \frac{g \cdot \rho_f \cdot \left(1 - \frac{\rho_v}{\rho_f}\right) k_f^3 \ h_{fg}}{\nu_f \ (T_g - T_w) \ j \ D} \right]^{\frac{1}{4}}$$

or $\bar{h}_j = \bar{h} \cdot j^{-\frac{1}{4}}$

# L.13 Boiling Heat Transfer

Boiling .

- large heat transfer rates with small temperature differences (nearly isothermal)
- High heat transfer coefficient
- Excellent for high heat fluxes [compact]

→ Boiling is evaporation at solid - liq interface , & occurs when $T_s > T_{sat}$ where $T_{sat}$ is the temperature for liquid to gas phase change , & is a function of pressure

The rate eq [Newton's law of cooling]

$$\underset{\text{flux of heat}}{q_s''} = h(T_s - T_{sat}) = h\underbrace{\Delta T_e}_{\text{excess temperature.}}$$

⇒ Modes of Boiling

① Pool Boiling : quiescent liquid , motion near the surface is due to free convection & mixing due to bubble growth & detachment.

② Forced Convection (flow boiling) . external means drive fluid motion

→ Subcooled (local) Boiling . $T_{liq} < T_{sat}$ ⇒ bubbles formed at solid surface condense in the liquid

→ Saturated Boiling . $T_{liq} > T_{sat}$ ⇒ bubbles can rise & escape.

## → Dimensionless Parameters:-

Nusselt number = $hL/k$

Prandalt number = $\mu c_p / K$

Jakob number = $Ja = (C_p \Delta T)/h_{fg}$  $\Delta T = T_s - T_{sat}$

Bond number = $Bo = [g(\rho_L - \rho_v) L^2]/\sigma$

Grashot-like number = $[\rho g (\rho_L - \rho_v) L^3]/\mu^2$

## → Different Boiling Regimes in pool Boiling

a) Natural Convection

b) Nucleate Boiling  * $\Delta T_{excess} = T_s - T_{sat}$

c) Transition Boiling [vapor pockets]  if $\Delta T_{excess} < 5 \Rightarrow$ no Boiling

d) Film Boiling [Radiation]

## → Pool Boiling Correlations:

1- Nucleate Boiling

$$q = \mu_L h_{fg} \left[ \frac{g(\rho_L - \rho_v)}{\underbrace{\sigma}_{\text{surface tension}}} \right]^{\frac{1}{2}} \left[ \frac{C_{p_f} (T_s - T_{sat})}{\underbrace{C_{s,f}}_{\text{surface fluid combination factor [tabulated]}} h_{fg} \, P_{r_f}^{\,n}} \right]^3$$

2. Critical heat flux

$$q_{max} = \frac{\pi}{24} h_{fg} \rho_v \left[ \frac{\sigma g (\rho_L - \rho_v)}{\rho_v^2} \right]^{\frac{1}{4}} \left( \frac{\rho_L + \rho_v}{\rho_L} \right)^{\frac{1}{2}}$$

3- Minimum heat flux

$$q_{min} = 0.09 \rho_v h_{fg} \left[ \frac{g \sigma (\rho_L - \rho_v)}{(\rho_L + \rho_v)^2} \right]^{\frac{1}{4}}$$

4- Film Boiling

$$Nu = \frac{h_{conv} D}{K_v} = C \left[ \frac{g(\rho_f - \rho_v) h_{fg}' D^3}{\nu_v K_v (T_s - T_{sat})} \right]^{\frac{1}{4}}$$

$C = 0.62$ horz cylinder

$C = 0.67$ sphere

$$h_{fg}' = h_{fg} + 0.8 C_{p_v} (T_s - T_{sat})$$

* At high temp $T_s \geqslant 300°C$ ; Radiation mode affects the process , Total H.T.C

$$\bar{h}^{4/3} = h_{conv}^{4/3} + \bar{h}_{rad}\, h^{1/3}$$

if $h_{rad} < h_{conv}$

$$h = h_{conv} + \frac{3}{4} h_{rad}$$

$$\therefore h_{rad} = \frac{\varepsilon \sigma (T_s^4 - T_{sat}^4)}{(T_s - T_{sat})}$$

σ = stefan-Boltzman constant

ε: emissivity of solid.

## → Forced Convection Boiling

**Region A** ( only liq , no phase change )

↝ Turbulent $Nu = \frac{hD}{K} = 0.023\, Re^{0.8}\, Pr^{0.4}$

$L/D > 60$ , $Re > 10{,}000$

$$q'' = h (T_w - T_l)$$

**Region B** (subcooling Boiling)

$$\frac{C_{p_l} \Delta T}{h_{fg}\, Pr^n} = C_{sf} \left[ \frac{q''}{\mu_l h_{fg}} \sqrt{\frac{g_c\, \sigma}{g(\rho_l - \rho_v)}} \right]^{0.33}$$

Regions C, D, E, & F

$$\underset{\text{Two phase}}{h_{TP}} = \underset{\text{Nucleate Boiling}}{h_{NB}} + \underset{\text{forced convection}}{h_c}$$

$$h_c = 0.023 \left( \frac{k_L}{D} \right) Re_l^{0.8} \, Pr_l^{0.4} \, \underbrace{F}_{\text{charts.}}$$

$$Re_L = \frac{G(1-x)D}{\mu_L}$$

$$h_{NB} = 0.00122 \left[ \frac{k_L^{0.79} \, C_{p_L}^{0.45} \, \rho_L^{0.49}}{\sigma^{0.5} \, \mu_L^{0.29} \, h_{fg}^{0.24} \, \rho_v^{0.24}} \right] \Delta T_{sat}^{0.24} \, \Delta P_{sat}^{0.75} \, \underbrace{S}_{\text{suppression factor}}$$

Charts

Heat Flux

$$q'' = h_{TP} \, \Delta T_{sat} = h_{TP} \left( T_s - T_{sat} \right)$$

* for Boiling all properties at $T_{sat}$

# L.14 Radiation

* All objects with a temperature above absolute zero Radiate. Radiation $\alpha T$

* Radiation propagates in vacuum

## Physical Concepts:-

→ mechanism of Radiation

1) Electromagnetic waves [solar Radiation, X-rays, radio waves]
2) Photons [discrete packets of energy]

→ The wave nature of thermal Radiation:

wave length ($\lambda$) is associated with the frequancy of radiation ($v$)

wave length [m] $\leftarrow \lambda = \frac{c}{v}$ $c$ → speed of propagation → in vacuum = speed of light = $3 \times 10^8$ m/s

$v$ → frequancy [Hz]

→ Thermal Radiation ⇒ Radiation due to body's temperature

## Characteristics of a Blackbody :-

1. for a given surface temp & $\lambda$, no surface can emit more radiation than a black body

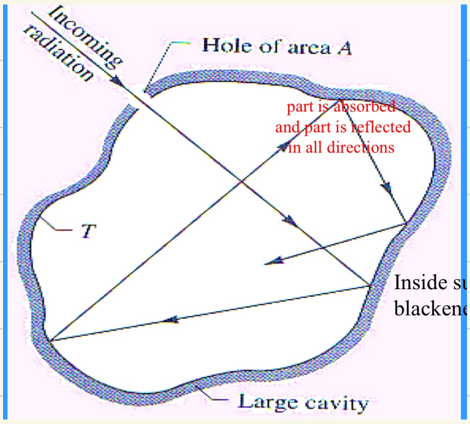
⇒ Blackbody is a perfect emitter

2. A blackbody absorbs all incident radiation regardless of its direction & wavelength

⇒ Blackbody is a perfect absorber

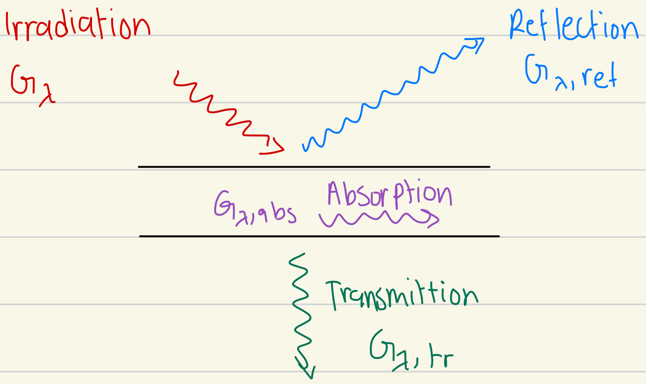
3. A blackbody emits radiation in all directions

⇒ Diffuse emitter



## Cavity model of Blackbody

⇒ Reflection process continues untill all of the energy entering is absorbed & the area of the hole acts as a perfect black body



→ Irradiation: Radiation from other Surfaces [w/m²]

$$G_\lambda = G_{tr} + G_{abs} + G_{ref}$$

→ Reflection.

$$G_{ref} = \rho G_\lambda$$

↳ reflectivity $0 \leq \rho \leq 1$

→ Absorption :-

$$G_{abs} = \alpha G_\lambda$$

↳ absorptivity $0 \leq \alpha \leq 1$

→ Transmittion :-

$$G_{tr} = \tau G_\lambda$$

↳ Transmissivity

$0 \leq \tau \leq 1$

* $G_{tr} = 0$ for opaque Surfaces

$G_{tr} \neq 0$ for semitransparent medium.

$\rho + \alpha + \tau = 1$

→if $\tau = 0$ $\rho + \alpha = 1$

## ⇒ Black body Emissive Power

$$\boxed{E_{b\lambda}(T) = \frac{C_1}{\lambda^5\left[e^{C_2/\lambda T} - 1\right]}}$$

plank's Law

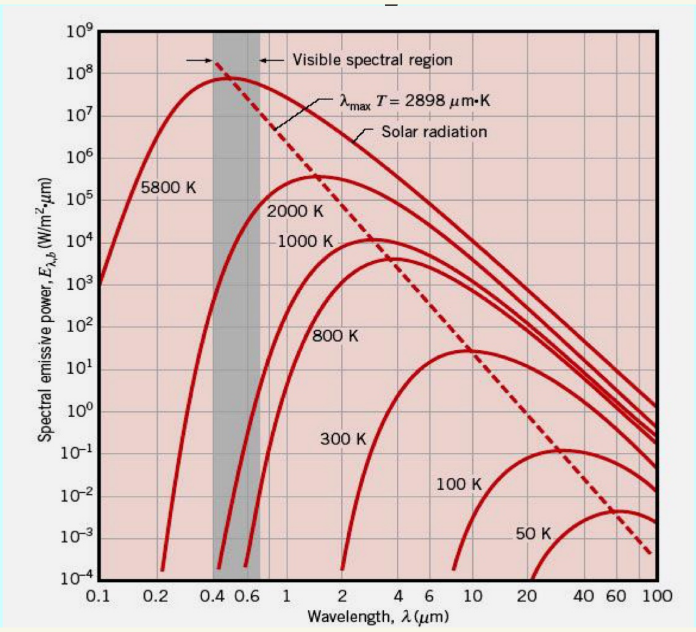
$C_1 = 2\pi h c^2 = 3.74 \times 10^8$ $[W\mu m^4/m^2]$

$C_2 = 1.4387 \times 10^4$ $[\mu m \cdot K]$

$T$ = absolute temp $[K]$

$h$ = plank's constant

$\lambda$ = wave length $[\mu m]$



Spectral blackbody emissive power

Wien's Displacement Law:

$(\lambda T)_{max} = 2897.6$ $[\mu m \cdot K]$

$$E_b(T) = \int_{\lambda=0}^{\lambda=\infty} E_{b\lambda}(T)\, d\lambda$$

⇒ Area under the curve.

$$E_b(T) = \sigma T^4$$

⇒ stefan-boltzmann Law

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

## ⇒ Heat Transfer Rates: Radiation

- Heat transfer involves Radiation emittion from the surface & absorption + Convection if $(T_s \neq T_\infty)$

## Energy outflow due to Emission:-

emissive power of Blackbody

emissive power $\leftarrow E = \varepsilon E_b = \varepsilon \sigma T^4$

$\hookrightarrow = 5.67 \times 10^{-8}$

surface emissivity

## Energy absorption due to irradiation:-

$G_{abs} = \alpha G \leftarrow$ Irradiation

absorbed incident Radiation

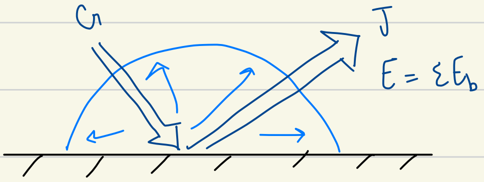
absorptivity

* Irradiation Special case :- Surface exposed to large surroundings of uniform Temp ($T_{sur}$)

$$G = G_{sur} = \sigma T_{sur}^4$$

$\rightarrow$ if $\alpha = \varepsilon$

$\rightarrow$ Consider an opaque surface



$J \rightarrow$ radiosity [W/m²]

energy leaving the surface

$$J = \varepsilon E_b + \rho G$$

$q'' \rightarrow$ net radiation heat flux leaving the surface

$= J - G \qquad = \varepsilon E_b + \rho G - G$

out in

$$q''_{rad} = \varepsilon E_b - (1-\rho) G \quad = \varepsilon E_b(T_s) - \alpha G \quad = \varepsilon \sigma (T_s^4 - T_{sur}^4)$$

$\rightarrow$ Alternatively:

$$q''_{rad} = h_r (T_s - T_{sur})$$

$\hookrightarrow$ Radiation heat transfer coefficient.

$$h_r = \varepsilon \sigma (T_s + T_{surr})(T_s^2 + T_{surr}^2)$$

→ for combined convection & Radiation:

$$\boxed{q'' = q''_{conv} + q''_{rad} = h(T_s - T_\infty) + h_r(T_s - T_{surr})}$$

⇒ Black body Radiation function

* Area under the curve from 0 → λ

$$E_{b0-\lambda}(T) = \int_0^\lambda E_{b\lambda}(T)\, d\lambda$$

$$f_{0\lambda}(T) = \frac{\int_0^\lambda E_{b\lambda}(T)\, d\lambda}{\int_0^\infty E_{b\lambda}(T) d\lambda} = \boxed{\frac{\int_0^\lambda E_{b\lambda}(T)\, d\lambda}{\sigma T^4}}$$

$f_{0\lambda}(T)$ ⇒ Blackbody Radiation function [tabulated] [table 12.2]

→ Over finite wavelength band $\lambda_1 - \lambda_2$

$$f_{\lambda_1 - \lambda_2}(T) = f_{0-\lambda_2}(T) - f_{0-\lambda_1}(T)$$

Ex: Find fractions of Solar energy in ultra violet, Visible, & infared regions

→ Sun behaves like a black body at 5800K

| | |
|---|---|
| Thermal UV | 0.1 → 0.4 |
| Visible | 0.4 → 0.7 |
| IR | 0.7 → 100 |

$$(\lambda_1 T) = (0.1)(5800) = 580$$

$$(\lambda_2 T) = (0.4)(5800) = 2320$$

$$(\lambda_3 T) = (0.7)(5800) = 4060$$

$$(\lambda_4 T) = (100)(5800) = 58000$$

→ Go to table [Interpolate]

| $\lambda T$ | $F_{(0-\lambda)}$ |
|---|---|
| 580 | 0 |
| 2320 | 0.12 |
| 4060 | 0.495 |
| 58000 | 1 → extrapolate |

$F_{UV} = f_{0-\lambda_2} - f_{0\lambda_1}$

$0.12 - 0 = 0.12$ (12%)

$F_{vis} = f_{0\lambda_2} - f_{0-\lambda_1}$

$0.495 - 0.12 = 0\ 375$ (37.5%)

$f_{IR} = f_{0-\lambda_2} - f_{0-\lambda_1}$

$1 - 0.495 = 0.505$ (50.5%)

## → Radiation Properties of surfaces:-

Absorption & emission of Radiation depend on:-

1. Type of material
2. Temp or wavelength

Types of materials → opaque to thermal Radiation [ $\varepsilon$ or $\alpha$ within very short distance ]

↳ semitransparent to Radiation [ penetrate into the depths ]

Emissivity $\varepsilon = \dfrac{\text{Energy Emitted by a real surface}}{\text{Energy Emitted by a blackbody at the same temp}}$ [ratio of two Areas]

$$\varepsilon = f(T, \lambda, \text{direction})$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{b_\lambda} \, d\lambda}{\sigma T^4}$$

absorptivity $\alpha = \dfrac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{G}$

reflectivity $\rho = \dfrac{\int_0^\infty \rho_\lambda G_\lambda \, d\lambda}{G}$

Transmissivity $\tau = \dfrac{\int_0^\infty \tau_\lambda G_\lambda \, d\lambda}{G}$

## Hemispherical Emissivity

$$\varepsilon(T) = \frac{\text{Radiation flux by real body over all wave length}}{\text{Energy emitted by black body at same } T}$$

into a hemispherical space

$$\varepsilon(T) = \frac{q(T)}{E_b(T)} = \frac{q(T)}{\sigma T^4}$$

$$q(T) = \varepsilon(T) \cdot E_b(T) = \varepsilon(T)\,\sigma T^4 \quad [W/m^2]$$

* Spectral hemispherical emissivity $\varepsilon_\lambda(T)$ → emissivity at a specific wave length $\lambda$ at certian temp.

obtained expirementally, can be used to obtain the avg value of $\varepsilon$

$$\varepsilon = \frac{\left[\int_0^\infty \varepsilon_\lambda E_{b\lambda} \; d\lambda\right]}{\sigma T^4} \rightarrow \text{Band of constant } \varepsilon$$

## Emission of Radiation

- Monochromatic Radiation = Radiation of a single wave length
- Beam of thermal Radiation → not monochromatic
- A temp > 500°C in visable spectrum → Significant 'Red heat' 'white heat'

* state of aggregation & molecular structure of the substance affect radiation

monoatomic & diatomic gases → Radiate weakly even at high T

polyatomic gases → emit & absorb radiation at several wavelengths

Solids & liquids → emit & absorb radiation over the entire spectrum in Thin layers.

* Monochromatic Energy emitted by an object depends on T & $\lambda$

$$E_T = \int_0^\infty E_\lambda \, d\lambda$$

## Types of bodies

| opaque Body | white body | Blackbody | Grey body |
|---|---|---|---|
| -no transmittion | - reflective | - absorptive | $0<\alpha<1$ |
| $\tau=0$ | $\rho=1$ | $\rho=0$ | |
| $\rho+\alpha=1$ | $\alpha=0$ | $\alpha=1$ | |

## Absorplivity:-

- Hemispherical absorptivity $\alpha$
- spectral hemispherical absorptivity $\alpha_\lambda$

$$\alpha = f(T_{source}, \lambda, material)$$

## Grey body Approximation

Grey body Approx → assume a uniform emissivity $\varepsilon$ over the entire wave length spectrum.

$$\text{Grey body} \neq f(\lambda)$$

## Kirchoff's Law

→ At thermal eq the power radiated by an object = Power absorbed

$$\varepsilon = \alpha$$

→ for body A in evacuated hollow sphere.

Intensity of Radiation from Source.

$$E_1 A_1 = I \alpha_1 A_1 \Rightarrow E_1 = I\alpha$$

→ for identical body $A_2$

$$E_2 = I\alpha_2$$

→ for black body $E_b = I$

$\boxed{\alpha=1}$ for black bodies

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = E_b$$ Kirchoff's Law

$$\varepsilon = \frac{E_1}{E_b} = \alpha_1$$

$\boxed{\varepsilon = \alpha}$ at eq.

$$\varepsilon_2 = \frac{E_2}{E_b} = \alpha_2$$

⇒ The rate of Energy from 1→2

$$\frac{q_{1-2}}{A} = \frac{\alpha_2 E_1}{1-\rho_1\rho_2}$$

* E → total emissive power

* $\alpha$ → absorptivity

* $\rho$ → reflectivity

* $\varepsilon$ → emissivity

⇒ The rate of Energy from 2→1

$$\frac{q_{2\to1}}{A} = \frac{\alpha_1 E_2}{1-\rho_1\rho_2}$$

$$\text{net } \frac{q_{1-2}}{A} = \frac{\alpha_2 E_1 - \alpha_1 E_2}{1-\rho_2\rho_1}$$

→ For Grey surfaces with zero transmissivity

$$\rho = 1-\alpha = 1-\varepsilon$$

$$\text{net } \frac{q_{1-2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

# Radiation Intensity

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{dq \;\to \text{Heat rate}}{\underbrace{dA_1 \cos\theta}_{\text{Projected Area.}} \; \underbrace{dw}_{\text{Solid angle}} \cdot d\lambda}$$

→ $\frac{dq}{d\lambda}$ $(dq_\lambda)$ is the rate at which radiation of wavelength λ leaves $dA_1$ & passes throug $dA_n$

$$dq_\lambda = I_{\lambda e}(\lambda,\theta,\phi)\, dA_1 \cos\theta \, dw$$

Solid angle $dw = \sin\theta \, d\theta \, d\phi$

$$\underbrace{dq''_\lambda}_{\text{Heat flux}} = I_{\lambda e}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

## Hemispherical Direction:-

→ The total hemispherical emissive power

$$E = \pi I_e$$

→ Suppose a diffusive surface (emitted radiation is independent of direction θ), intensity doesn't depend on θ & φ

$$I_{\lambda e}(\lambda,\theta,\phi) = I_{\lambda e}(\lambda)$$

## The net heat transfer between two Surfaces:-

$$\dot{Q}_{net} = A_1 \sigma (T_2^4 - T_1^4)$$

if $T_1 > T_2$ → blackbody cools down because of net loss

if $T_2 > T_1$ → blackbody heats up

[blackbody inside black body]

→ grey body inside grey body $\dot{Q}_{net} = A_1 \varepsilon_1 \sigma T_1^4 - A_1 \alpha_1 \sigma T_2^4$

# L15 View factor

→ Radiation between surfaces are strongly affected by

1. Orientation
2. size of surfaces

→ View factor is used to formulate the effects of orientation

View factor
- → Diffuse view factor : surfaces are diffuse reflectors & diffuse emitters
- → Specular view factor : Surfaces are diffuse emitters & specular reflectors.

* The veiw factor between two surfaces represents the fraction of the radiative energy leaving one surface that strikes the other surface directly.

⇒ View factor derivation:

$$dq_{1-2} = I\, dA_1 \cos\theta_1\, d\omega_{2-1}$$

$$d\omega_{2-1} = \frac{dA_2 \cos\theta_2}{r^2}$$

→ for Black surface $I = \frac{E}{\pi} = \frac{\sigma T^4}{\pi}$

$$\therefore dq_{1-2} = \frac{\sigma T_1^4}{\pi} \quad \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{r^2}$$

$$\therefore dq_{2-1} = \frac{\sigma T_2^4}{\pi} \quad \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{r^2}$$

⇒ net radiation between $dA_1$ & $dA_2$

$$dq_{1\to 2} = dq_{1-2} - dq_{21}$$

$$\sigma\, (T_1^4 - T_2^4) \quad \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}$$

→ on a per unit Area basis

View factor $F_A$

$$\frac{dq_{1-2}}{dA_1} = \sigma\left(T_1^4 - T_2^4\right)\left[\frac{dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}\right]$$

### → The elemental veiw factor

The elemental veiw factor ( $dF_{dA_1-dA_2}$ ) is the ratio of radiative energy leaving $dA_1$ that strikes $dA_2$ directly to the radiative energy leaving $dA_1$ in all directions into the hemispherical space.

$$dF_{dA_1-dA_2} = \frac{dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}$$

$$dF_{dA_2-dA_1} = \frac{dA_1 \cos\theta_1 \cos\theta_2}{\pi r^2}$$

Reciprocity Relation

$$dA_1 \, dF_{dA_1-dA_2} = dA_2 \, dF_{dA_2-dA_1}$$

### → View factor between two finite Surfaces:-

$$F_{A_1-A_2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} \, dA_2 \, dA_1$$

$$F_{A_2-A_1} = \frac{1}{A_2}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} \, dA_1 \, dA_2$$

Reciprocity Relation

$$A_1 F_{A_1-A_2} = A_2 F_{A_2-A_1}$$

### → Properties of View factor

Assume an enclosure of N zones, of surface area $A_1$. Each zone is isothermal a diffuse emitter & diffuse reflector

Surface of each zone may be [plane, convex, concave]

The view factor between $A_i$ & $A_j$

$$A_i F_{A_i-A_j} = A_j F_{A_j-A_i}$$

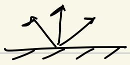
$$\sum_{K=1}^{N} F_{A_i-A_K} = 1$$

⇒ for surface $A_1$ with $N=6$

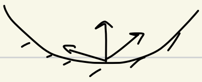
$$F_{A_1-A_1} + F_{A_1-A_2} + F_{A_1-A_3} + F_{A_1-A_4} + F_{A_1-A_5} + F_{A_1-A_6} = 1$$

→ for a plane or convex surface

$$F_{A_1-A_1} = 0$$



→ for concave surface



* radiation leaves & hits same surface

$$F_{A_1-A_1} \neq 0$$

## 2.16 Fired heaters:-

### Furnace heat duty & flame Temperature

→ Suppose that there were no heat sink & no losses. All the heat released by combustion would go into heating the gases produced

= Adiabatic Flame Temp $T_f$

$$\dot{Q}_f = \dot{M}_f \Delta h_f = M_g C_{pg} (T_f - T_0)$$

↳ inlet air temp.

* In case of real furnace the gases do not attain the adiabatic flame temp due to heat sink & wall loses

→ theoretical gas T

$$\dot{Q}_f - \dot{Q}_g = M_g C_{pg} (T_g - T_0)$$

→ neglecting losses but allowing sink loss $\dot{Q}_g$

$$\frac{Q_f - \dot{Q}_g}{Q_f} = \frac{T_g - T_0}{T_f - T_0}$$

### → Direct Radiation in Radiant section

$$q_r = \sigma \alpha A_{cp} F (T_g^4 - T_w)$$

↳ Radiant heat transfer

$\sigma$ → stefan Boltzmann

$\alpha$ → Relative effectiveness factor of the tube bank ⇒ Chart $\alpha$ vs. [center to center / Diameter]

$A_{cp}$ → cold plane area of the tube bank

$F$ → Exchange factor ⇒ chart F vs Gass emissivity

$T_g$ → Effective gas temp in fire box

$T_w$ → Avg tube wall temp.

* flue gas in fire box is a poor radiator

→ Acp for Single sided firing

$A_{cp} = N \times S \times L$

L = length

N = Number of tubes

S = tube spacing.

→ Acp for Double sided firing

$A_{cp} = N \times S \times L \times 2$

## Percent of absorbed heat by radiation section

$$R = \frac{1}{1 + \frac{G\sqrt{Q/\alpha A_{cp}}}{4200}}$$

R → fraction of heat libiration that is absorbed by cold surfaces in the combustion chamber.

G → air – fuel ratio

Acp → Area of furnace walls that has tubes mounted on it

α → factor by which Acp is multiplied to obtain the effective cool surface.