L3

Example 2.3

A rectangular aluminum alloy (Duralumin) fin is 2 in. long, 0.1 in. thick, and 40 in. wide. It is attached to a prime surface at 150°F and is surrounded by a fluid at 100°F with a heat-transfer coefficient of 75 Btu/h · ft^2 · °F. Calculate the fin efficiency and the rate of heat transfer from the fin.

$L = 2in$

$t = 0.1\ in$ → Assume adiabatic $\eta = \frac{\tanh(mL_c)}{mL_c}$

$W = 40in$

$T = 150°F$

$T_\infty = 100°F$

$h = 75\ Btu/h.ft^2.°F$

$\rightarrow L_c = L + t/2 = 2 + \frac{0.1}{2} = \frac{2.05\ in}{12ft} = 0.1708\ ft$

$\Rightarrow w >> t \rightarrow m = \left(\frac{2h}{kt}\right)^{1/2} = \frac{2 \times 75}{94.76 \times (\frac{0.1}{12})} = 13.782$

$K = 94.76$

$mL_c = 13.782 \times 0.1708 = 2.354$

$\eta = \frac{\tanh(2.354)}{2.354} = 0.417$

$q = \eta q_{max}$

$0.417\ (h A \theta_b)$ $A = P L_c$

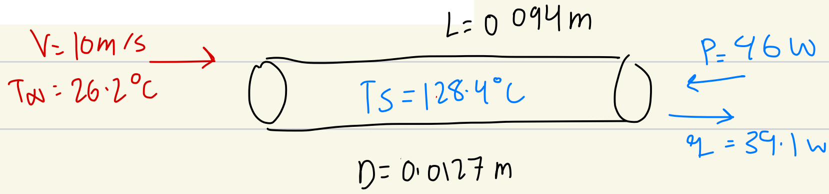
$0.417\ (75 \times 1.1415\ (150 - 100))$ $2(w + t) L_c$

$2(\frac{40 + 0.1}{12}) \times 0.1708 = 1.1415$

$q = 1785.04\ Btu/h$

1.4

Experiments have been conducted on a metallic cylinder 12.7 mm in diameter and 94 mm long. The cylinder is heated internally by an electrical heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the upstream air velocity and temperature were maintained at $V = 10$ m/s and 26.2°C, respectively, the heater power dissipation was measured to be $P = 46$ W, while the average cylinder surface temperature was determined to be $T_s = 128.4$°C. It is estimated that 15% of the power dissipation is lost through the cumulative effect of surface radiation and conduction through the endpieces.



$0.015(46) = 6.9$

$46 - 6.9 = 39.1$

Properties: $\dfrac{128.4 + 26.2}{2} = 77.3°C \rightarrow 350.3\ K$

$\nu = 20.92 \times 10^{-6}$ $\quad k = 30 \times 10^{-3}$ $\quad Pr = 0.7$

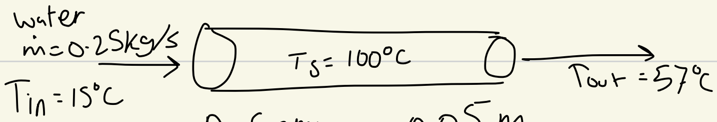
$q = hA(T - T_\infty)$

$39.1 = h(\pi(0.0127)(0.094))(128.4 - 26.2)$

$h = 102.01\ W/m^2 \cdot K$

Q4

Steam condensing on the outer surface of a thin-walled circular tube of diameter $D = 50$ mm and length $L = 6$ m maintains a uniform outer surface temperature of 100°C. Water flows through the tube at a rate of $\dot{m} = 0.25$ kg/s, and its inlet and outlet temperatures are $T_{m,i} = 15$°C and $T_{m,o} = 57$°C. What is the average convection coefficient associated with the water flow?



$D = 50mm = 0.05 m$

$L = 6m$

$$\dot{m} c_p \Delta T = h A \Delta T_{LM}$$

Properties $\frac{57+15}{2} = 36°C \rightarrow 309 K$

$c_p = 4.178 kJ/kg \rightarrow 4178 J/kg$ ⇒ Table A.6

$$\Delta T_{LM} = \frac{(100-15) - (100-57)}{\ln\left[\frac{100-15}{100-57}\right]} = 61.63$$

$$0.25(4184)(57-15) = h(\pi(0.05)(6))(61.63)$$

$$h = 756.34 \ W/m^2 \cdot K$$

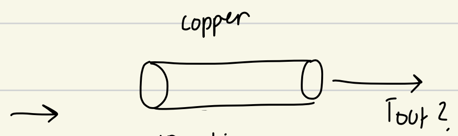
Example 2.6

1000 lbm/h of oil at 100°F enters a 1-in. ID heated copper tube. The tube is 12 ft long and its inner surface is maintained at 215°F. Determine the outlet temperature of the oil. The following physical property data are available for the oil:

$$C_P = 0.5 \ \text{Btu/lbm}\cdot{}^\circ\text{F}$$
$$\rho = 55 \text{lbm/ft}^3$$
$$\mu = 1.5 \text{lbm/ft}\cdot\text{h}$$
$$k = 0.10 \ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

$\dot{m} = 1000 \text{lbm/h}$

$T = 100°F$



ID = 1in

L = 12ft

$T_s = 215°F$

$$\dot{m} c_p \Delta T = h A \Delta T_{LM}$$

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \ 1000}{\pi \left(\frac{1}{12}\right)(1.5)} = 10185.9$$

Turbulent

$$Nu = 0.027 \ Re^{0.8} \ Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad Pr = \frac{c_p \mu}{k}$$

$$0.027 (10185.9)^{0.8} \left(\frac{0.5 \times 1.5}{0.1}\right)^{1/3} (1)$$

$$Nu = 85$$

$$85 = \frac{h \left(\frac{1}{12}\right)}{0.1} \qquad h = 102 \ W/m^2 \cdot K$$

Assume $\Delta T_{LM} = \Delta T_{avg}$

$$0.25 (0.5)(T_{out} - 100) = 102 \left(\pi \left(\frac{1}{12}\right)(12)\right) \left(\frac{(215-100) - (215 - T_{out})}{2}\right)$$

$$T_{out} = 155.8°F$$

$$\Rightarrow \Delta T_{LM} = \frac{(215-100) - (215-155.8)}{\ln\left[\frac{215-100}{215-155.8}\right]} = 84°F$$

$q = hA\,\Delta T_{LM}$

$102(\pi(\frac{1}{12})\,12)\,84$ $\qquad q = 26917$ Btu/h·°F

$26917 = \dot{m}\,c_p\,\Delta T$

$26917 = 1000\,(0.5)(T_{out} - 100)$

$T_{out} = 154\ °F$

LS

Example

Pressurized water is often available at elevated temperatures and may be used for space heating or industrial process applications. In such cases it is customary to use a tube bundle in which the water is passed through the tubes, while air is passed in cross flow over the tubes. Consider a staggered arrangement for which the tube outside diameter is 16.4 mm and the longitudinal and transverse pitches are S_L = 34.3 mm and S_T = 31.3 mm. There are seven rows of tubes in the airflow direction and eight tubes per row. Under typical operating conditions the cylinder surface temperature is at 70 °C, while the air upstream temperature and velocity are 15 °C and 6 m/s, respectively. Determine the air-side convection coefficient.

h for Air

Air →

Tube bundle — water →

Staggered Arrangement.

OD = 16.4 mm $\quad S_L = 34.3$ mm $\quad$ rows $N_L = 7$

$S_T = 31.3$ mm $\quad$ tubes per row = 8

T_s Cylinder 70 °C

Air V: 6 m/s & 15 °C

Air :

$$\overline{Nu}_D = C_1 C_2 Re_{D,max}^{m} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2}\right)^2\right]^{1/2} \quad \text{or} \quad \frac{S_T + D}{2}$$

$$\left[34.3 + \left(\frac{31.3}{2}\right)^2\right]^{1/2} > \frac{31.3 + 16.4}{2}$$

$$37.70 > 23.85 \Rightarrow V_{max} \text{ at } A_1$$

→ V_{max} at A_1

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{31.3 \times 10^{-3}}{(31.3 - 16.4)\times 10^{-3}}(6) = 12.60 \text{ m/s}$$

Properties:- at $T_{avg} = 15+273 = 288$

$\rightarrow$ Interpolation $\nu = 14.822 \times 10^{-6} m^2/s$

$\Rightarrow Re_{D,max} = \dfrac{V_{max} D}{\nu}$

$$Re_{max} = \frac{12.6(16.4\times10^{-3})}{14.822\times10^{-6}} = 13941.438$$

$\rightarrow$ Constants $C_2 \Rightarrow Re \geq 10^3$ $N_L = 7$

$C_2 = 0.95$

$C_1 \rightarrow$ $\dfrac{S_T}{S_L} = \dfrac{31.3}{34.3} = \boxed{0.912 < 2}$

$$C_1 = 0.35\left(\frac{S_T}{S_L}\right)^{1/5}$$

$$C_1 = 0.35\left(\frac{34.4}{31.3}\right)^{1/5} = 0.34$$

$$\overline{Nu}_D = C_1 C_2 Re_{D,max}^{m} Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Properties:

$m \rightarrow 0.6$

Pr at $T_{avg} = 0.71012$

Pr_s at $T_s = 343K = 0.70098$

$$\overline{Nu}_D = 0.34(0.95)(13941.4)^{0.6}(0.71012)^{0.36}\left(\frac{0.71012}{0.70098}\right)^{1/4} = 87.83$$

$\rightarrow$ to find h

properties: $T_{avg} = 288K$

$k = 25.34 \times 10^{-3}$ W/mK

$$\overline{Nu}_D = \frac{hD}{k}$$

$$87.83 = \frac{h(16.4\times10^{-3})}{25.34\times10^{-3}}$$ $h = 135.7$ W/m²·K

L7

Example 3.1

A double-pipe heat exchanger will be used to cool a hot stream from 350°F to 250°F by heating a cold stream from 80°F to 120°F. The hot stream will flow in the inner pipe, which is 2-in. schedule 40 carbon steel with a thermal conductivity of 26 Btu/h · ft^2 · °F. Fouling factors of 0.001 h · ft^2 · °F/Btu should be provided for each stream. The heat-transfer coefficients are estimated to be h_i = 200 and h_o = 350 Btu/h · ft^2 · °F, and the heat load is 3.5 × 10^6 Btu/h.

(a) For counter-current operation, what surface area is required?
(b) For co-current operation, what surface area is required?

inner pipe → hot stream   2in schedule 40 → OD = 2.375 in, ID = 2.067 in

$h_o = 350$ Btu/ft^2·h $h_i = 200$ Btu/ft^2·h $k = 26$ Btu/h·ft^2·°F

$R_D = 0.001$ h·ft^2·°F/Btu → fouling factor for Both streams

heat load $= 3.5 \times 10^6$ Btu/h

→ counter current

$$Q = UA\,\Delta T_{LM}$$

$$\Delta T_{LM} = \frac{(250-80) - (350-120)}{\ln\left[\frac{250-80}{350-120}\right]} = 198.5\,°F$$

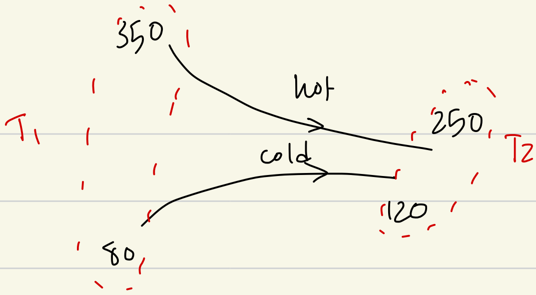
$$U = \left[\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o} + \frac{R_{Di} D_o}{D_i} + R_{Do}\right]^{-1}$$

$$U = \left[\frac{2.375}{200(2.067)} + \frac{(2.375/12)\ln(2.375/2.067)}{2 \times 26} + \frac{1}{350} + \frac{0.001(2.375)}{2.067} + 0.001\right]^{-1}$$

$U = 88.65$ Btu/h·ft^2·°F

$3.5 \times 10^6 = 88.65\ A\,(198.5)$

$A = 198.9$ ft^2



→ Co-current

$$Q = UA\,\Delta T_{LM}$$

$$\Delta T_{LM} = \frac{(350-80) - (250-120)}{\ln\left[\frac{350-80}{250-120}\right]} = 191.5$$

U → same as counter-current $= 88.65$ Btu/ h.ft^2 °F

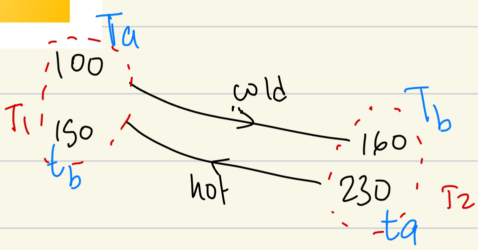
$$3.5\times10^6 = 88.65\,(A)\,191.5$$

$$A = 206.16\ ft^2$$

A fluid is to be heated from 100°F to 160°F by heat exchange with a hot fluid that will be cooled from 230°F to 150°F The heat-transfer rate will be 540,000 Btu/h and the hot fluid will flow in the tubes. Will a 1-2 exchanger (i.e., an exchanger with one shell pass and a multiple of two tube passes) be suitable for this service? Find the mean temperature difference in the exchanger.

$q = 540{,}000$ Btu/h

hot fluid → inside tubes



$$\Delta T_{LM} = \frac{(150-100)-(230-160)}{\ln\left[\frac{150-100}{230-160}\right]} = 59.44$$

$$\Delta T_m = F\,\Delta T_{LM}$$

$$= 0.94(59.44) = \underline{55.87}$$

$$R = \frac{T_a - T_b}{t_b - t_a} = \frac{100-160}{150-230} = 0.75$$

$$P = \frac{t_b - t_a}{T_a - t_a} = \frac{150-230}{100-230} \approx 0.615$$

⇓ from charts

$F(1\text{-}2) \simeq 0.72$

$0.72 < 0.8$ (1-2) shouldn't be used

$F(2\text{-}4) \simeq 0.94$

$0.94 > 0.8$ (2-4) should be used.

10,000 lb/h of benzene is to be heated from 60°F to 120°F by heat exchange with an aniline stream that will be cooled from 150°F to 100°F A number of 16 ft hairpins consisting of 2 in. by 1.25 in. schedule 40 stainless steel pipe (type 316, k = 9.4 Btu/h. ft. °F are available and will be used for this service. how many hairpins will be required?

$L = 16\,ft$

2in (outer) → 2.067 in

1.25in (inner) → 1.38 in , 1.66 out

$k = 9.4$ Btu/h ft.°F

Benzene

$\dot{m} = 10{,}000$ lb/h

60°F → 120°F

Aniline

150°F → 100°F

Fluid property	Benzene ($t_{ave} = 90°F$)	Aniline ($T_{ave} = 125°F$)
μ (cp)	0.55	2.0
C_P (Btu/lbm · °F)	0.42	0.52
k (Btu/h · ft · °F)	0.092	0.100

① Energy Balance

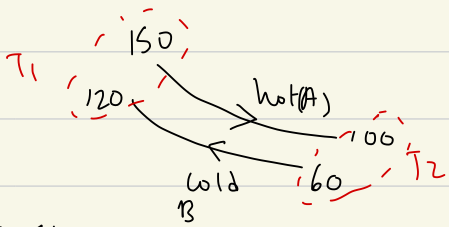
$$q_B = q_A$$

$$\dot{m}\, c_p\, \Delta T = \dot{m}\, c_p\, \Delta T$$

$$10{,}000\,(0.42)(120-60) = \dot{m}\,(0.52)(150-100)$$

$\dot{m} = 9692.3$ lb/h

② LMTD



→ Assume counter-current

$$\Delta T_{LM} = \frac{(150-120)-(100-60)}{\ln\left[\frac{150-120}{100-60}\right]} = 34.76$$

③ hi Calculation

→ Assume Benzene in inner

$$Re = \frac{4\dot{m}}{\pi D_i \mu}$$

$$D_i = \frac{1.38}{12} = 0.115\ ft$$

$$Re = \frac{4(10{,}000)}{\pi(0.115)(0.55)(2.419)} = 83217.3$$

⇒ Turbulent flow

$$Pr = \frac{c_p \mu \cdot 2.419}{k}$$

$$Nu_D = 0.027\, Re^{0.8}\, Pr^{1/3}$$

$$0.027\,(83217.3)^{0.8}\left(\frac{0.42 \times 0.55 \times 2.419}{0.092}\right)^{1/3} = 425.29$$

$$Nu = \frac{h D}{k}$$

$$425.29 = \frac{h_i\,(0.115)}{0.092}$$

$h_i = 340\ Btu/ft^2{}^\circ F \cdot h$

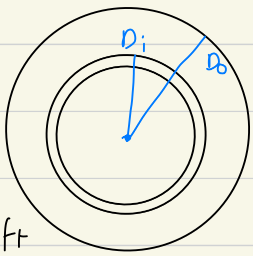
$\Rightarrow$ Check enterance effects

$$\frac{L}{D} > 60$$

$$\frac{32}{0.115} = 278.26 > 60 \quad \Rightarrow \text{enterance effects are negligable}$$

entire length

④ h_o calculation



$$Re = \frac{D_e\,(\dot{m}/A_f)}{\mu}$$

$$D_e = D_o - D_i$$

$$\frac{2.067 - 1.66}{12} = 0.0339\,ft$$

$$A_f = \frac{\pi}{4}(D^2 - D_i^2) = \frac{\pi}{4}\left[\left(\frac{2.067}{12}\right)^2 - \left(\frac{1.66}{12}\right)^2\right]$$

$$A_f = 0.00827\ ft^2$$

$$Re = \frac{0.0339\,(9692.3/0.00827)}{2 \times 2.419} = 8212.11$$

$\Rightarrow$ Transient Flow

$$Nu = 0.116\left[Re^{2/3} - 125\right] Pr^{1/3}$$

$$= 0.116\left[(8212.11)^{2/3} - 125\right]\left(\frac{2 \times 0.52 \times 2.419}{0.1}\right)^{1/3}$$

$$Nu = 95.86$$

$$Nu = \frac{h\,De}{k}$$

$$95.86 = \frac{h_o\,(0.0339)}{0.1}$$

$h_o = 282.7\ Btu/h \cdot ft^2 \cdot {}^\circ F$

$\Rightarrow$ check enterance effects

$$L/De > 60$$

length of one leg of hair pin

$$\frac{16}{0.0339} = 471.97 > 60$$

⑤ calculate Pipe wall temperature for corrections:

$$T_w = \frac{t_{avg}\,h_i + T_{avg}\cdot h_o\,(D_o/D_i)}{h_i + h_o\,(D_o/D_i)}$$

$$t_{avg} = \frac{60+120}{2} = 90$$

$$T_{avg} = \frac{100+150}{2} = 125$$

$$T_w = \frac{90\,(340) + 125\,(282.7)\left(1.66/1.38\right)}{340 + 282.7\left(\frac{1.66}{1.38}\right)} = 107.5\ {}^\circ F$$

⑥ calculate corrections & correct h_o & h_i

$$\phi_{in\,(B)} = \left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{0.55}{0.47}\right)^{0.14} = 1.022$$

$$\phi_{out\,(A)} = \left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{2}{2.4}\right)^{0.14} = 0.97$$

$$h_i = \phi_i\,h_i = 1.022\,(340) = 348\ Btu/ft^2 \cdot h \cdot {}^\circ F$$

$$h_o = \phi_o\,h_o = 0.97\,(282.7) = 276\ Btu/ft^2 \cdot h \cdot {}^\circ F$$

⑦ fouling factor

$$R_{Do} = R_{Di} = 0.001 \ h \cdot ft^2 \ °F / Btu$$

⑧ solve for U

$D_o = 1.66$

$D_i = 1.38$

$$U = \left[\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2K} + \frac{1}{h_o} + \frac{R_{Di} D_o}{D_i} + R_{Do} \right]^{-1}$$

$$U = \left[\frac{1.66}{348 \times 1.38} + \frac{\left(\frac{1.66}{12}\right) \ln\left(1.66/1.38\right)}{2 \times 94} + \frac{1}{276} + \frac{0.001 \times 1.66}{1.38} + 0.001 \right]^{-1}$$

$U = 94 \ Btu / h \cdot ft^2 \cdot °F$

⑨ calculate Required surface area.

$$q = UA \ \Delta T_{lm}$$

$252{,}000 = 94 \ A \ (34.76)$ $A = 77.12 \ ft^2$

$$L = \frac{77.12 ft^2}{0.435 ft} = 177.3 \ ft$$

number of hair pins $= \frac{177.3}{32} = 5.5 \rightarrow 6$

In a petroleum refinery, it is required to cool 30,000 lb/h of kerosene from 400 °F to 250 °F by heat exchange with 75,000 lb/h of gas oil, which is at 110 °F A shell-and-tube exchanger will be used, and the following data are available:

Fluid property	Kerosene	Gas oil
C_P (Btu/lbm · °F)	0.6	0.5
μ (cp)	0.45	3.5
k (Btu/h · ft · °F)	0.077	0.08

For the purpose of making a preliminary cost estimate, determine the required heat-transfer area of the exchanger.

Kerosene

$\dot{m} = 30{,}000$ lb/h

$400°F \rightarrow 250°F$

Gas oil

$\dot{m} = 75{,}000$ lb/h

$T \rightarrow 110°F$

① Energy Balance

$$q_k = q_g$$

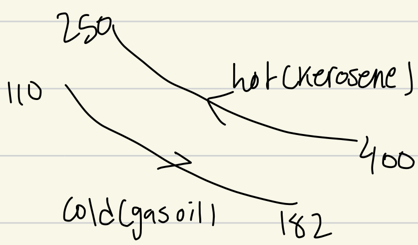
$$\dot{m} c_p \Delta T = \dot{m} C_p \Delta T$$

$$30{,}000\,(0.6)(400-250) = 75{,}000\,(0.5)(T-110)$$

$$T = 182°F$$

② ΔT_{LM}

$$\Delta T_{LM} = \frac{(400-182)-(250-110)}{\ln\left[\frac{400-182}{250-110}\right]}$$



$$\Delta T_{LM} = 176$$

③ ΔT_{LM} correction factor [shell & tube]

→ Assume Kerosen in shell

$$R = \frac{T_a - T_b}{t_b - t_a} = \frac{400-250}{182-110} = 2.08$$

$$P = \frac{t_b - t_a}{T_a - t_a} = \frac{182 - 110}{400 - 110} = 0.25$$

$\Rightarrow$ from charts $F \simeq 0.93$

4) Calculate U_D

$$q = U_D A F (\Delta T_{LM})$$

$U_D \rightarrow$ from tables $= 25$ Btu/h.ft^2.°F

$$30{,}000\ (0.6)\ (400-250) = 25\ (A)\ (0.93)\ (176)$$

$$A = 659.8 \simeq 660\ ft^2$$

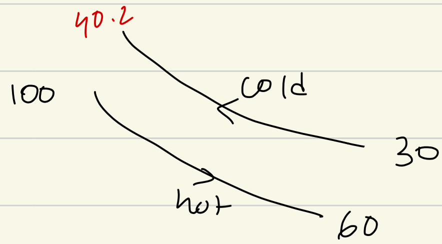
Example 11.1

A counterflow, concentric tube heat exchanger is used to cool the lubricating oil for a large industrial gas turbine engine. The flow rate of cooling water through the inner tube (D_i = 25 mm) is 0.2 kg/s, while the flow rate of oil through the outer annulus (D_o = 45 mm) is 0.1 kg/s. The oil and water enter at temperatures of 100 and 30°C, respectively. How long must the tube be made if the outlet temperature of the oil is to be 60°C?

turbine gas:- $100^\circ C \longrightarrow 60^\circ C$

$\dot{m} = 0.1$ kg/s

OD = 45 mm



water :- $30 \longrightarrow T_2$

$\dot{m} = 0.2$ kg/s

ID = 25 mm

① Energy Balance

$$\dot{m} c_p \Delta T = \dot{m} c_p \Delta T$$

$$0.1(2131)(100-60) = 0.2(4178)(T_2 - 30)$$

$$T_2 = 40.2$$

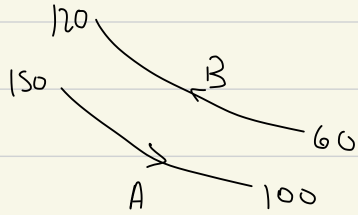
② ΔT_{LM}

$$\Delta T_{LM} = \frac{(100-40.2)-(60-30)}{\ln\left[\frac{100-40.2}{60-30}\right]} = 43.2^\circ C$$

10,000 lb/h of benzene will be heated from 60°F to 120°F by heat exchange with an aniline stream that will be cooled from 150°F to 100°F. A number of 16-ft hairpins consisting of 2-in. by 1.25-in. schedule 40 stainless steel pipe (type 316, $k = 9.4$ Btu/h·ft·°F) are available and will be used for this service. A maximum pressure drop of 20 psi is specified for each stream. The specific gravity of benzene is 0.879 and that of aniline is 1.022. Determine the number and configuration of hairpins that are required.

length hair pins outer inner

tables for dimensions



10,000 lb/h Benzene

$60°F \rightarrow 120°F$

$SG = 0.879$

Analine

$150°F \rightarrow 100°F$

$SG = 1.022$

16ft hairpins 2in 1.25in sch 40

OD = 2.375 ID = 2.067 ID = 1.38 OD = 1.66

$k = 9.4$ Btu/h·ft·°F

max $\Delta P = 20$ psi

Fluid property	Benzene ($t_{ave} = 90°F$)	Aniline ($T_{ave} = 125°F$)
μ (cp) $\times 2.419$	0.55	2.0
C_P (Btu/lbm·°F)	0.42	0.52
k (Btu/h·ft·°F)	0.092	0.100

1) Energy Balance

$$m c_p \Delta T = m c_p \Delta T$$

$$10{,}000(0.42)(120-60) = \dot{m}(0.52)(150-100)$$

$$\dot{m} = 9692.3 \text{ lbm/h}$$

$$q = UA\,\Delta T_{LM}$$

$$q = 10{,}000(0.42)(120-60) = 252000 \text{ Btu/h}$$

$$\Delta T_{LM} = \frac{(150-120)-(100-60)}{\ln\left[\frac{150-120}{100-60}\right]} = 34.76$$

$$U = \left[\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o} + \frac{R_{Di} D_o}{D_i} + R_{Do}\right]^{-1}$$

R_{Di} & $R_{Do} = 0.001$

$D_o = 1.66$ $D_i = 1.38$

Assume Benzene in inner tube

$\rightarrow h_i$: $Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4(10{,}000)}{\pi(\frac{1.38}{12})(0.55) \times 2.419} = 83217.3$

Turbulent

$Nu = 0.023\, Re^{0.8}\, Pr^{1/3}$ $\qquad Pr = \frac{c_p \mu}{k}$

$0.023\,(83217.3)^{0.8}\left(\frac{0.55 \times 2.419 \times 0.42}{0.092}\right)^{1/3}$

$Nu = 362.28 = \frac{h(\frac{1.38}{12})}{0.92}$

$h_i = 290$ Btu/ h.ft^2.°F

$\rightarrow h_o$ $\quad D_e = D_2 - D_1$

$2.067 - 1.66 = \frac{0.407}{12}$ ft

$A_f = \pi/4\,(D_2^2 - D_1^2)$

$= 0.00827$ ft2

$Re = \frac{D_e \dot{m}}{\mu A} = \frac{(\frac{0.407}{12})(9692.3)}{2 \times 2.419 \times 0.00827} = 8216.15$

Transient

$Nu = 0.116\,[Re^{2/3} - 125]\,Pr^{1/3}$

$Nu = 0.116\,[(8216.5)^{2/3} - 125]\left(\frac{0.52 \times 2 \times 2.419}{0.1}\right)^{1/3} = 95.91$

$95.91 = \frac{h(\frac{0.407}{12})}{0.1}$ $\qquad h_o = 283$ Btu/h.ft^2.°F

$\rightarrow T_w$ for correction

$T_w = \frac{t_{avg}\, h_i + T_{avg}\, h_o\,(D_o/D_i)}{h_i + h_o\,(D_o/D_i)}$

$t_{avg} = \frac{60+120}{2} = 90$

$T_{avg} = \frac{150+100}{2} = 125$

$T_w = \frac{90(290) + 125(283)(\frac{1.66}{1.38})}{290 + 283(\frac{1.66}{1.38})} = 108.89$°F

$$\phi_i = \left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{0.55}{0.47}\right)^{0.14} = 1.022 \times h_i = \boxed{1296.5}$$

$$\phi_o = \left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{2}{2.4}\right)^{0.14} : 0.974 \times h_o = \boxed{1276}$$

$U = 89 \; Btu/ft^2 . h . °F$

$$q = UA \, \Delta T_{LM}$$

$$252000 = 89 (A)(34.76) \qquad A = 81.5 \; ft^2$$

$$L = \frac{81.5 ft^2}{0.435 \, ft^2/ft} = 187.4 \; ft$$

# of hair pins: $\frac{187.4}{32} = 5.9 \Rightarrow 6$

inner Pipe

$$\Delta P_f = \frac{f L G^2}{7.5 \times 10^{12} . S . D_i \, \phi_i}$$

$$f = 0.3673 Re^{-0.2314} = 0.0267$$

$$A_f = \frac{\pi}{4} D^2 = 0.0104$$

$$G = \frac{\dot{m}}{A_f} = \frac{10,000}{0.0104} = 961538.46$$

$$\Delta P_f = \frac{0.0267 (6 \times 32)(961538.46)^2}{7.5 \times 10^{12} (0.879)\left(\frac{1.38}{12}\right)(1.022)}$$

$$\Delta P_f = 6.11 \; psi$$

Bends: $\Delta P_r = 1.6 \times 10^{-13} . (2 \times N_{HP} - 1) \; G^2/s$

$$1.6 \times 10^{-3} . (2 \times 6 - 1) \frac{(961538.46)^2}{0.879} = 1.85 \; psi$$

$\Delta P = \Delta P_f + \Delta P_r$

$6.11 + 1.85 = 7.96$ psi < 20 $\Rightarrow$ works for inner

Shell:

$$\Delta P_f = \frac{f \, L \, G^2}{7.5 \times 10^{12} \cdot S \cdot \phi \cdot D_e}$$

$f = 0.3673 \, Re^{-0.2314} = 0.0456$

$A_f = \frac{\pi}{4}(D_2^2 - D_1^2) = 0.00827$

$G = \frac{\dot{m}}{A_f} = \frac{9692}{0.00827} = 1171472.813$

$$\Delta P_f = \frac{0.0456 \times (6 \times 32)(1171472.8)^2}{7.5 \times 10^{12} \times 1.022 \times 0.974 \times \left(\frac{0.407}{12}\right)}$$

$\Delta P_f = 47.5$ > 20 ✗

30,000 lb/h of kerosene are to be cooled from 400°F to 250°F by heat exchange with 75,000 lb/h of gas oil which is at ll0 °F Available for this duty is a shell-and-tube exchanger having 156 tubes in a 21 1/4-in. ID shell. The tubes are 1-in. OD, 14 BWG, 16 ft long on a $1\frac{1}{4}$-in. square pitch. There is one pass on the shell side and six passes on the tube side. The baffles are 20% cut segmental type and are spaced at 5-in. intervals. Both the shell and tubes are carbon steel having k-26 Btu/h · ft² ·°F. Fluid properties are given in the previous example. Will the exchanger be thermally suitable for this service?

Fluid property	Kerosene	Gas oil
C_P (Btu/lbm · °F)	0.6	0.5
μ (cp)	0.45	3.5
k (Btu/h · ft · °F)	0.077	0.08

Shell & Tube

156 tubes $21\frac{1}{4}$ in   ID shell

tubes → 1 in OD

14 BWG $1\frac{1}{4}$ in ☐ pitch

16 ft long

(1-6)

Kerosene

30000 lb/h

(inside tube) → gas oil

75000 lb/h

$T_{in} = 110 \rightarrow 182$

20% Baffles   5 in

k = 26

$q = UA\,\Delta T_{LM}$

$q = 30000\,(0.6)(400-250)$

$q = 2.7 \times 10^6$ Btu/h

$\Delta T_{LM} = 176$ $F = 0.93$

$(\Delta T_{ln})_{cf} = 176\,(0.93) = 163.68$

$A = 156 \text{ tubes} \times 16 \text{ ft} \times 0.2618 \frac{ft^2}{ft\,tube} = 653\ ft^2$

⇓

BWG tables

$q = UA\,(F\,\Delta T)$

$2.7 \times 10^6 = U\,(653)(176 \times 0.93)$ $U = 25.3$ Btu/ft²·h °F

→ Calculate U_c

$$U_c = \left[\frac{D_o}{D_i h_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o} \right]$$

$\rightarrow h_i$

$$\dot{m}_{per\,tube} = \frac{\dot{m}_t}{n_t/n_p} = \frac{75000}{156/6} = 2884.61$$

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4\,(2885)}{\pi\left(\frac{0.834}{12}\right) \times 3.5 \times 2.419} = 6243 \rightarrow \text{Transition}$$

$$Nu = 0.116\left[Re^{2/3} - 125\right] Pr^{1/3} \left(\mu/\mu_w\right)^{0.14} \left[1 + \left(D/L\right)^{2/3}\right]$$

$$0.116\left[(6243)^{2/3} - 125\right] \left(\frac{0.5 \times 3.5 \times 2.419}{0.08}\right)^{1/3} (1) \left[1 + \left(\frac{0.834}{12 \times 16}\right)^{2/3}\right] = \frac{h_i\left(\frac{0.834}{12}\right)}{0.08}$$

$$h_i = 110 \ \text{Btu/h.ft}^2.°F$$

$\rightarrow h_o$

$$\left.\begin{array}{l} C = 0.25 \\ d_e = 0.99 \end{array}\right\} \text{Figure}$$

$$D_e = d_e/12 = \frac{0.99}{12} = 0.0825 \ \text{ft}$$

$$a_s = \frac{d_s C B}{144 P_T} = \frac{21.25\,(0.25)(5)}{144\,(1.25)} = 0.1476 \ \text{ft}^2$$

$$G = \frac{\dot{m}}{a_s} = \frac{30000}{0.1476} = 203294 \ \text{lbm/ft}^2.\text{h}$$

$$Re = \frac{D_e G}{\mu} = \frac{0.0825\,(203294)}{0.45 \times 2.419} = 15407$$

$h_0 \rightarrow \quad B/d_S = \frac{5}{21.25} = 0.235$

L.11

EXAMPLE 11.3

Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at 300°C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125°C. The exhaust gas specific heat is approximately 1000 J/kg · K, and the overall heat transfer coefficient based on the gas-side surface area is U_h = 100 W/m² · K. Determine the required gas-side surface area A_h using the NTU method.

Finned Tube - cross flow - Both unmixed

$T_{h,in} = 300°C$ $\quad$ $c_{p,h} = 1000\,J/kg\cdot K$ $\quad$ Gas side surface

$T_{h,out} = 100°C$ $\quad$ $U_h = 100\,W/m^2\cdot K$ $\quad$ Area A_h

$\dot{m}_{water} = 1\,kg/s$ $\quad$ $T_{c,in} = 35°C$

$T_{c,out} = 125°C$ $\quad$ $T_{avg} = \frac{35+125}{2} = 80°C = 353\,K$

Properties $c_{p,c} = 4197\,J/kg\cdot K$

$C_c = c_{p,c}\,\dot{m}$

$4197\,(1) = 4197\,W/K$

$$C_h = \dot{m}_h c_{p,h} = C_c \frac{(T_{c,out} - T_{c,in})}{(T_{h,in} - T_{h,out})} = 4197 \frac{(125-35)}{(300-100)} = 1888.65\,W/K$$

$C_{min} = C_h = 1888.65\,W/K$

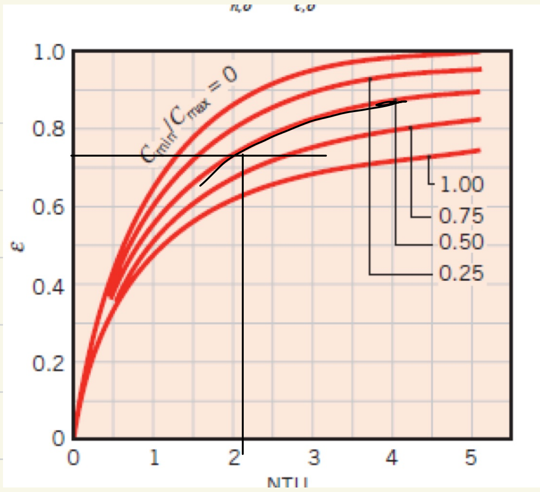
$\therefore q_{max} = C_{min}\,(T_{h,in} - T_{c,in})$

$1888.65\,(300-35) = 500492.25 = 5\times10^5\,W$

$\therefore q_{act} = \dot{m}_c\,c_{p,c}\,\Delta T_c = 1\,(4187)(125-35) = 3.76\times10^5\,W$

$$\varepsilon = \frac{q_{act}}{q_{max}} = \frac{3.76\times10^5}{5\times10^5} = 0.752$$

$$\frac{C_{min}}{C_{max}} = \frac{1888.65}{4197} = 0.45$$



$NTU \simeq 2.1$

$$NTU = \frac{U\,A}{C_{min}}$$

$$2.1 = \frac{100\,(A)}{1888.65} \qquad A = 39.66\,m^2$$

Consider the heat exchanger design of Example 11.3, that is, a finned-tube, cross-flow heat exchanger with a gas-side overall heat transfer coefficient and area of 100 $W/m^2 \cdot K$ and 40 m^2, respectively. The water flow rate and inlet temperature remain at 1 kg/s and 35°C. However, a change in operating conditions for the hot gas generator causes the gases to now enter the heat exchanger with a flow rate of 1.5 kg/s and a temperature of 250°C. What is the rate of heat transfer by the exchanger, and what are the gas and water outlet temperatures?

Solution

Cross flow - finned tube — Both unmixed

gas side $A = 40\,m^2$ $\quad m = 1.5\,kg/s$ $\quad C_p = 1000\ J/kg\cdot K$

$U = 100\ W/m^2\cdot K$ $\quad T_{h\,in} = 250°c$

water $\dot{m} = 1\,kg/s$

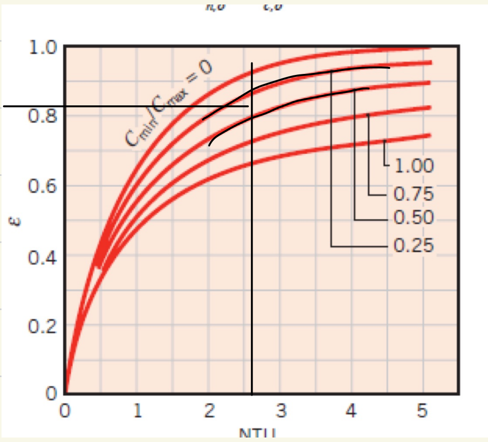
$T_{c\,in} = 35$ $\quad C = 4197\ J/kg\cdot K$

$C_h = \dot{m}\,C_p = 1.5\,(1000) = 1500\ W/K = C_{min}$

$C_c = \dot{m}\,C_p = 1\,(4197) = 4197\ W/K$

$$\frac{C_{min}}{C_{max}} = \frac{1500}{4197} = 0.357$$

$$NTU = \frac{UA}{C_{min}} = \frac{100\,(40)}{1500} = 2.667$$



$\varepsilon \approx 0.82$

$\therefore q_{max} = C_{min}\,(T_{h\,in} - T_{c\,in})$

$1500\,(250 - 35) = 3.23 \times 10^5\ W$

$$\varepsilon = \frac{q_{act}}{q_{max}}$$

$$0.82 = \frac{q_{act}}{3.23 \times 10^5}$$ $\quad q_{act} = 2.65 \times 10^5\ W$

$$q = \dot{m}_h C_p \Delta T_h$$

$$2.65 \times 10^5 = 1.5 (1000)(250 - T_{hout})$$

$T_{hout} = 73.33°C$

$$q = \dot{m}_c C_p \Delta T_c$$

$$2.65 \times 10^5 = 1(4197)(T_{cout} - 35)$$

$T_{cout} = 98.14°C$

EXAMPLE 11.5

The condenser of a large steam power plant is a heat exchanger in which steam is condensed to liquid water. Assume the condenser to be a *shell-and-tube* heat exchanger consisting of a single shell and 30,000 tubes, each executing two passes. The tubes are of thin wall construction with $D = 25$ mm, and steam condenses on their outer surface with an associated convection coefficient of $h_o = 11{,}000\ \mathrm{W/m^2 \cdot K}$. The heat transfer rate that must be effected by the exchanger is $q = 2 \times 10^9$ W, and this is accomplished by passing cooling water through the tubes at a rate of 3×10^4 kg/s (the flow rate per tube is therefore 1 kg/s). The water enters at 20°C, while the steam condenses at 50°C. What is the temperature of the cooling water emerging from the condenser? What is the required tube length L per pass?

steam → liq water / shell & Tube

single shell – 30,000 tube two passes

(1-2)

$D_{tubes} = 25\,mm$

$h_o = 11{,}000\ W/m^2 \cdot K$

$q = 2\times10^9\ W$ (steam)

$\dot{m}_{water\ in\ tubes} = 3\times10^4\ kg/s$

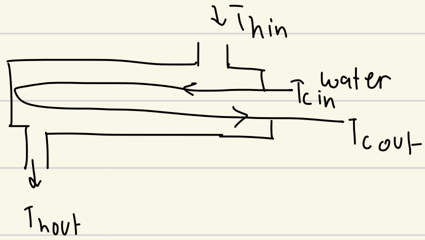
$\dot{m}_{per\ tube} = 1\ kg/s$

$\boxed{T_{h\,in} = 50}$

$T_{c\,in} = 20°C$

$T_{c\,out} = ?$

$N = 30{,}000$
$D = 25mm$
$h_o = 11{,}000$
$\dot{m} = 1kg/s$



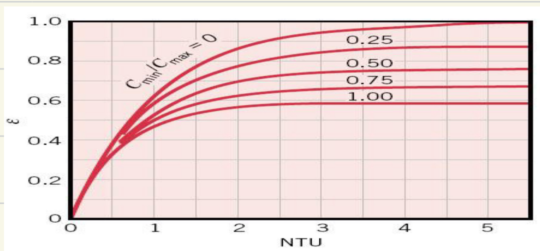
$$q = \dot{m}\, c_{p_c}\, \Delta T_c$$

$$2\times10^9 = 1(3\times10^4)(4197)(T_{c\,out} - 25)$$

$T_{c\,out} = 36°C$

→ NTU method:

$$NTU = \frac{AU}{C_{min}}$$



$$U = \frac{1}{\left(\frac{1}{h_i}\right)+\left(\frac{1}{h_o}\right)}$$

properties $\rightarrow T_{avg}$

$$\frac{36+25}{2}$$

$\Rightarrow h_i$

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4(1)}{\pi(25\times10^{-3})(855\times10^{-6})} = 59566.76$$

$\hookrightarrow$ Turbulent

$$Nu = 0.023\ Re^{4/5}\ Pr^{0.4} = 0.023(59566.76)^{4/5}(5.83)^{0.4} = 308$$

$$308 = \frac{h_i(0.025)}{613\times10^{-3}} \qquad h_i = 7552.16\ W/m^2.K$$

$$U = \frac{1}{\frac{1}{7552.16} + \frac{1}{11,000}} = 4477.84\ W/m^2.K$$

$$C_{min} = C_{pc}\cdot\dot{m}_c = 4197(3\times10^4) = 1.25\times10^8\ W/K$$

$$C_h = C_{max} = \infty$$

$$\frac{C_{min}}{C_{max}} = C_r = 0$$

$$\therefore q_{max} = C_{min}(T_{h,in} - T_{c,in})$$

$$1.25\times10^8(50-20) = 3.75\times10^9\ W$$

$$\varepsilon = \frac{q}{q_{max}} \qquad \varepsilon = \frac{2\times10^9}{3.75\times10^9} = 0.533$$

Eq 11.35b

$$NTU = -\ln(1-\varepsilon)$$

$$NTU = -\ln(1-0.533) \qquad NTU = 0.7614$$

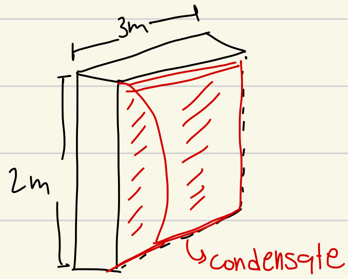
$$NTU = \frac{U(N.2.\pi.D.L)}{C_{min}}$$

$$0.7614 = \frac{4477.86\,(30{,}000 \times 2 \times \pi \times 0.025 \times L)}{1.25 \times 10^8}$$

$$L = 4.51 \text{ m}$$

L12 not from slides

Ex: sat steam at atmospheric pressure condenses on a 2m high & 3m wide vertical plate that is maintained at 80°C by circulating cooling water on the other side of the plate. Determine the rate of condensation heat transfer & the rate of water condensate.



$T_{sat} = 100°C$

$T_w = 80°C$

Given $Re = \left[4.81 + 3.7 \times L \times k_f \times \frac{(T_{sat} - T_{wall})}{\mu_L \cdot h_{fg}^*} \times \left(\frac{g}{\nu_L^2}\right)^{\frac{1}{3}}\right]^{0.82}$

$h = \frac{Re \times k_f}{(1.08 Re^{1.225} - 5.2)} \left(\frac{g}{\nu_L^2}\right)^{\frac{1}{3}}$ $\qquad h_{fg}^* = h_{fg} + 0.68\, C_{p_f}\, (T_{sat} - T_w)$

properties at $\frac{100+80}{2} = 90$

$\rho_L = 965.3\ kg/m^3$ $\quad \mu_L = 0.315 \times 10^{-3}$ $\quad k_f = 0.675$ $\quad C_{p_L} = 4206$

at $T_{sat} = 100°C$ $\quad h_{fg} = 2257 \times 10^3$

$\rightarrow h_{fg}^* = h_{fg} + 0.68\, C_{p_f}\, (T_{sat} - T_w)$

$2257 \times 10^3 + 0.68(4206)(100 - 80) = 2314201.6 = 2314 \times 10^3$

$\rightarrow Re = \left[4.81 + 3.7 \times 2 \times 0.675 \times \frac{(100-80)}{0.315 \times 10^{-3} \times 2314 \times 10^3} \times \left(\frac{9.81}{\left(\frac{0.315 \times 10^{-3}}{965.3}\right)^2}\right)^{\frac{1}{3}}\right]^{0.82}$

$* \ \nu_L = \frac{\mu_L}{\rho}$ $\qquad Re = 1287 < 1800 \rightarrow \underline{\text{Laminar}}$

$h = \frac{Re \cdot k_f}{1.08 Re^{1.225} - 5.2} \left(\frac{g}{\nu_L^2}\right)^{\frac{1}{3}}$

$h = \frac{1287 \times 0.675}{1.08 (1287)^{1.225} - 5.2} \left(\frac{9.81}{\left(\frac{0.315 \times 10^{-3}}{965.3}\right)^2}\right)^{\frac{1}{3}} = 5846\ W/m^2 \cdot C$

$A_s = W \cdot L = 3 \ 2 = 6 \, m^2$

$q = h A_s (T_{sat} - T_w)$

$5846 (6) (100 - 80) = 7 \times 10^5 \, W$

$\dot{m} = \frac{q}{h_{fg}^*} = \frac{7 \times 10^5}{2314 \times 10^3} = 0.303 \, kg/s$

Ex: sat steam at 1.4 bars condenses on the outer surface of a vertical 100 mm diameter pipe, 1m long. having a uniform surface temperature at 95°C, calculate the heat transfer rate of the condensation process. Assume Laminar film condensation

$$h = 0.943 \left[\frac{\rho_f \, k_f^3 \; h_{fg} \; \left(1 - \frac{\rho_v}{\rho_f}\right)}{\nu_f \cdot L \cdot (T_g - T_w)} \right]^{1/4}$$

properties at

T_{sat} at 1.4 bars = 382 K

$\frac{109 + 95}{2} = 102 = 375$

$\rho_f = 956.93$ $k_f = 681 \times 10^{-3}$ $h_{fg} = 2233.4 \times 10^3$ $\rho_v = 0.794$

$\nu_f = \left(\frac{274 \times 10^{-6}}{956.93}\right)$ $L = 1m$ $T_g = 109$ $T_w = 95$

$$h = 0.943 \left[\frac{956.93 \, (681 \times 10^{-3})^3 \, (2233.4 \times 10^3) \, \left(1 - \frac{0.794}{956.93}\right)}{\left(\frac{274 \times 10^{-6}}{956.93}\right) \; 1 \, . \, (109 - 95)} \right]^{1/4}$$

$$h = 3396.20 \; W/m^2 \cdot K$$

$$q = h A \Delta T$$

$$q = 3396.20 \, (\pi \times 100 \times 10^{-3} \times 1)(109 - 95)$$

$$q = 14937.29 = \underline{14.9 \, kW}$$

Example 11.1

A stream consisting of 5000 lb/h of saturated *n*-propyl alcohol (1-propanol) vapor at 207°F and approximately atmospheric pressure will be condensed using a tube bundle containing 3769 tubes arranged for a single pass. The tubes are 0.75 in. OD, 14 BWG, with a length of 12 ft. Physical properties of the condensate are as follows:

$k_L = 0.095$ Btu/h · ft · °F
$\rho_L = 49$ lbm/ft^3
$\mu_L = 0.5$ cp

Estimate the condensing-side heat-transfer coefficient for the following cases:

(a) The tube bundle is vertical and condensation occurs inside the tubes.
(b) The tube bundle is horizontal and condensation occurs outside the tubes.

$\dot{m}_f = 5000$ lb/h

$N = 3769$ tubes

OD = 0.75 14 BWG 12 ft $\rightarrow$ ID = 0.584 in

$$\Gamma = \frac{\dot{m}_f}{N\pi D} = \frac{5000}{3769\,\pi\,(0.584/12)} = 8.677 \text{ lbm/ft·h}$$

$$Re = \frac{4\Gamma}{\mu_L} = \frac{4(8.677)}{0.5 \times 2.419} = 28.7$$

$$h = 1.47\left[\frac{k_f^3\,\rho_L^2\,g}{\mu_L^2\,Re}\right]^{1/3}$$

$$h = 1.47\left[\frac{0.095^3\,(49)^2\,(4.17\times10^8)}{(0.5\times2.419)^2 \times 28.7}\right]^{1/3} \qquad h = 402 \text{ Btu/h·ft}^2\text{.F}$$

Example 10.4

A steam condenser consists of a square array of 400 tubes, each 6 mm in diameter. If the tubes are exposed to saturated steam at a pressure of 0.15 bar and the tube surface temperature is maintained at 25°C, what is the rate at which steam is condensed per unit length of the tubes?

$j = \sqrt{400} = 20$

$D = 6mm$

$P_{sat} = 0.15\ bar \rightarrow T = 327\ K = 54°C$

$T_s = 25°C = 298$

rate of condensation

$q = \dot{m}_f h_{fg}$

$$h = 0.728 \left[\frac{g \rho_f (1 - \rho_v/\rho_f) k_f^3 h_{fg}}{\nu_f (T_g - T_w) j D} \right]^{1/4}$$

Properties

$\rho_v = 0.0806$ $\quad h_{fg} = 2361.2 \times 10^3 \rightarrow T_{sat}$

$\rho_f = 997.4$ $\quad k_f^3 = 610.2 \times 10^{-3}$ $\quad \nu_f = \left(\frac{1000.6 \times 10^{-6}}{997.4} \right) \rightarrow$ at 298

$h = 4533.16\ W/m^2.K$

$q = hA(T_g - T_w)$

$\frac{q}{L} = 4533.16 (\pi \times 6 \times 10^{-3})(54 - 25)$

$q/L = 2477.99$

$2477.99 = m(2361.2 \times 10^3)$

$\dot{m} = 1.049 \times 10^{-3}$

EXAMPLE 10.1 polished copper

The bottom of a copper pan, 0.3 m in diameter, is maintained at 118°C by an electric heater. Estimate the power required to boil water in this pan. What is the evaporation rate? Estimate the critical heat flux.

$D = 0.3\,m$

$T_s = 118°C$ $\qquad$ $\Delta T_{excess} = T_s - T_{sat}$

$T_{sat} = 100°C$ $\qquad$ $118 - 100 = 18°C \xrightarrow[\text{chart}]{\text{from}}$ [Nucleate Boiling]

$$q = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{p,l}\,(T_s - T_{sat})}{C_{sf}\, h_{fg}\, Pr_l^{\,n}} \right]^3$$

Properties: at $T_{sat} = 373.15\,K$

$\mu_l = 279 \times 10^{-6}$ $\quad$ $h_{fg} = 2257 \times 10^3$ $\quad$ $g = 9.81$ $\quad$ $\rho_l = 957.85$ $\quad$ $\rho_v = 0.5955$

$\sigma = 58.9 \times 10^{-3}$ $\quad$ $c_{p,l} = 4217$ $\quad$ $Pr_l = 1.76$ $\quad$ $C_{sf} = 0.0128$ $\quad$ $n = 1$

$$q'' = 279\times10^{-6}\,(2257\times10^3) \left[\frac{9.81\,(957.85 - 0.5955)}{58.9\times10^{-3}} \right]^{1/2} \left[\frac{4217\,(118-100)}{0.0128\,(2257\times10^3)(1.76)} \right]^3$$

$$q'' = 836550 = 836\,kW/m^2$$

→ heat transfer rate

$$q = q'' \times A$$

$$q = 836\times10^3 \times \frac{\pi}{4}(0.3)^2 \qquad q = 59\,kW$$

→ water evaporation Rate ($\dot{m}_b$)

$$q = \dot{m}_b h_{fg}$$

$$59\times10^3 = \dot{m}_b\,(2257\times10^3) \qquad \dot{m}_b = 0.0262\,kg/s$$

→ critical heat flux

$$q_{max} = \frac{\pi}{24} h_{fg} \rho_v \left[\frac{\sigma g (\rho_L - \rho_v)}{\rho_v^2} \right]^{1/4} \left(\frac{\rho_L + \rho_v}{\rho_v} \right)^{1/2}$$

$$q_{max} = \frac{\pi}{24} (2257 \times 10^3)(0.5955) \left[\frac{58.9 \times 10^{-3} \times 9.81 (957.85 - 0.5955)}{(0.5955)^2} \right]^{1/4} \left(\frac{957.85 + 0.5955}{0.5955} \right)^{1/2}$$

$$q_{max} = 1105984.301 \quad = \quad 1.1 \text{ MW}/\text{m}^2$$

EXAMPLE 10.2

A metal-clad heating element of 6-mm diameter and emissivity $\varepsilon = 1$ is horizontally immersed in a water bath. The surface temperature of the metal is 255°C under steady-state boiling conditions. Estimate the power dissipation per unit length of the heater.

$\varepsilon = 1$ $D = 6mm$

$T_s = 255°C$

$T_{sat} = 100°C$

$$\Delta T_{excess} = 255 - 100 = 155$$

→ Film Boiling

$$Nu = \frac{h_{conv} D}{k_v} = C \left[\frac{g (\rho_l - \rho_v) h_{fg}' D^3}{\nu_v k_v (T_s - T_{sat})} \right]^{1/4}$$

$$h_{fg}' = h_{fg} + 0.8 \, c_{p_v} (T_s - T_{sat})$$

$$h^{4/3} = h_{conv}^{4/3} + h_{rad} h^{1/3}$$

→ properties at T_{sat}:

$\mu_L = 279 \times 10^{-6}$ $h_{fg} = 2257 \times 10^3$ $g = 9.81$ $\rho_L = 957.85$ $\rho_v = 0.5955$

$\sigma = 58.9 \times 10^{-3}$ $c_{p_l} = 4217$ $Pr_L = 1.76$ $c_{p_v} = 2029$

$$Nu = 0.62 \left[\frac{9.81 \, (957.85 - 0.5955)(2502520)(6 \times 10^{-3})^3}{\left(\frac{12.02 \times 10^{-6}}{0.5955} \right) (24.8 \times 10^{-3})(255 - 100)} \right]^{1/4}$$

$h_{fg}' = 2257 \times 10^3 + 0.8 \, (2029)(255 - 100)$

$h_{fg}' = 2502520$

$$Nu = 55.759 = \frac{h_{conv} D}{k_v}$$

$h_{conv} = 230.47$

$$h_{rad} = \frac{\varepsilon \sigma \left(T_s^4 - T_{sat}^4\right)}{\left(T_s - T_{sat}\right)} = \frac{5.67 \times 10^{-8} \left(528^4 - 373^4\right)}{\left(528 - 373\right)}$$

k

$$h_{rad} = 21.34$$

$$h^{4/3} = (230.47)^{4/3} + 21.34\, h^{1/3}$$

$$h = 254.14 \text{ W/m}^2\cdot\text{K}$$

$$q' = h A \Delta T$$

$$q = 254.14 \times \pi \times 6\times10^{-3} \times L \times (255 - 100) \qquad \frac{q}{L} = 742 \text{ W/m}$$

EXAMPLE 12.3

Consider a large isothermal enclosure that is maintained at a uniform temperature of 2000 K. Calculate the emissive power of the radiation that emerges from a small aperture on the enclosure surface. What is the wavelength λ_1 below which 10% of the emission is concentrated? What is the wavelength λ_2 above which 10% of the emission is concentrated? Determine the maximum spectral emissive power and the wavelength at which this emission occurs. What is the irradiation incident on a small object placed inside the enclosure?

Black body emissive power $\Rightarrow$ $E = E_b(T)$ $\rightarrow$ independent of λ

$$E_b = \sigma T^4$$

$$5.67 \times 10^{-8} (2000)^4 = 907200 \text{ W/m}^2$$

$\rightarrow \lambda_1$ below 10%

$$F_{0\lambda_1} = 0.1$$

$$(\lambda (2000)) = 0.1$$

$\lambda T \rightarrow$ from tables :- interpolation = 2195

$$\lambda(2000) = 2195 \qquad \lambda_1 = 1.1$$

$\rightarrow \lambda_2$ above 10%

$$F_{\lambda_2 \rightarrow \infty} = 1 - F_{0\lambda_2} = 0.1$$

$$\hookrightarrow F_{0\lambda_2} = 0.9$$

$(\lambda T) \rightarrow$ from tables = 9382

$$\lambda(2000) = 9382 \qquad \lambda_2 = 4.69$$

$\rightarrow$ max spectral emissive power -

wein's law $\quad (\lambda T)_{max} = 2897.6 \ \mu m \cdot K$

$$\lambda(2000) = 2897.6 \qquad \lambda_{max} = 1.45 \ \mu m$$

$$E_{b_\lambda}(T) = \frac{3.743 \times 10^8}{(1.45)^5 \left[e^{\frac{1.4387 \times 10^4}{1.45 \times 2000}} - 1 \right]} = 411986.65$$

$$4.12 \times 10^5 \ \text{W/m}^2 \cdot \mu m$$

→irradiation on small object

$$G = E_b(T) = 907200 \ W/m^2$$

A small oxidized metal tube of diam = 2.5 cm and long = 60 cm, Ts = 500 K is in a very large furnace enclosure with fire-brick walls and surrounding air at 1100 K. Assume oxidized metal tube emissivity = 0.75. Calculate the heat transfer to the metal tube.

$D = 2.5 cm$ $\quad T_s = 500 K$

$L = 60 cm$ $\quad T_{surr} = 1100 K$

$\varepsilon = 0.75$

$$q''_{rad} = \varepsilon \sigma (T_s^4 - T_{surr}^4)$$

$$q''_{rad} = 0.75 (5.67 \times 10^{-8}) (500^4 - 1100^4)$$

$$q''_{rad} = -59603.04 \ W/m^2 \quad \times A = \pi D L$$

$$-59603 \times \pi \times 2.5 \times 10^{-2} \times 60 \times 10^{-2}$$

$$q_{rad} = -2802.72 \ W$$

→ with convection

$$q'' = q''_{conv} + q''_{rad}$$

$$q_{conv} = h (T_s - T_\infty)$$

→ h for horizontal cylinders:- $\quad h = 1.32 \left(\frac{\Delta T}{D} \right)^{1/4}$

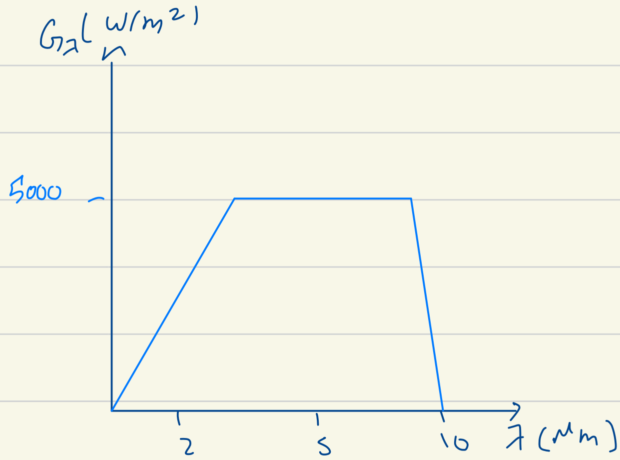
$$h = 1.32 \left(\frac{1100 - 500}{2.5 \times 10^{-2}} \right)^{1/4} = 16.43$$

$$q''_{conv} = -9858 \ W/m^2 \quad \times A$$

$$q_{conv} = -464.3118 \ W$$

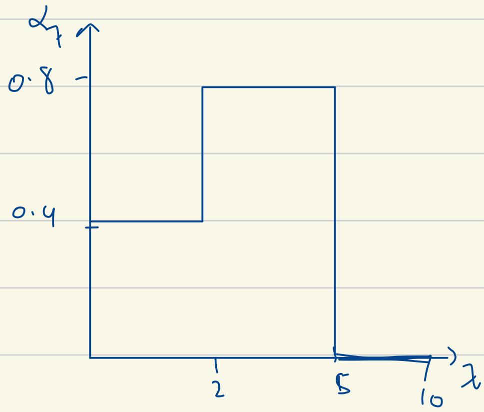
$$q = -464.3118 + -2802.72 = -3267.03 \ W$$

Ex: Find α & E



$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda}$$

$$\frac{0.4\int_0^2 G_\lambda d\lambda + 0.8\int_2^5 G_\lambda d\lambda + \int_5^{10} \cancel{\alpha_\lambda G_\lambda d\lambda} \rightarrow 0}{\int_0^\infty G_\lambda d\lambda}$$



$$\frac{0.4\left(\frac{1}{2}\right)(2)(500) + 0.8(3)(5000)}{\frac{1}{2}(2)(5000) + (10-2)(5000)} = 0.311$$

↳ G

Diffuse emitter $\alpha_\lambda = \varepsilon_\lambda$

to find $E = \varepsilon E_b \rightarrow \varepsilon$

$$\varepsilon = \frac{0.4\int_0^2 E_{b\lambda} d\lambda + 0.8\int_2^5 E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

$$0.4 \frac{\int_0^2 E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} + 0.8 \frac{\int_2^5 E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

$$0.4 \, F_{0-2} + 0.8 \, F_{2-5}$$

$(\lambda_1 T) = (2 \times 1250) = 2500 \qquad F_{0-2} = 0.162$ → from table 12.1

$(\lambda_2 T) = (5 \times 1250) = 6250 \qquad F_{0-5} = 0.757$

$\varepsilon = 0.4(0.162) + 0.8(0.757 - 0.162) = 0.540$

$E = \varepsilon E_b$

$E = 0.54 \times 5.67 \times 10^{-8} \times (1250)^4 = 74750 \ W/m^2$

$\rho + \alpha + \cancel{\tau}^{\,0} = 1$

$\rho = 0.689$

→ To find J

$J = E + \rho G$

$74750 + 0.689(45000) = 105750 \ W/m^2$

→ net heat flux q''

$q'' = J - G$

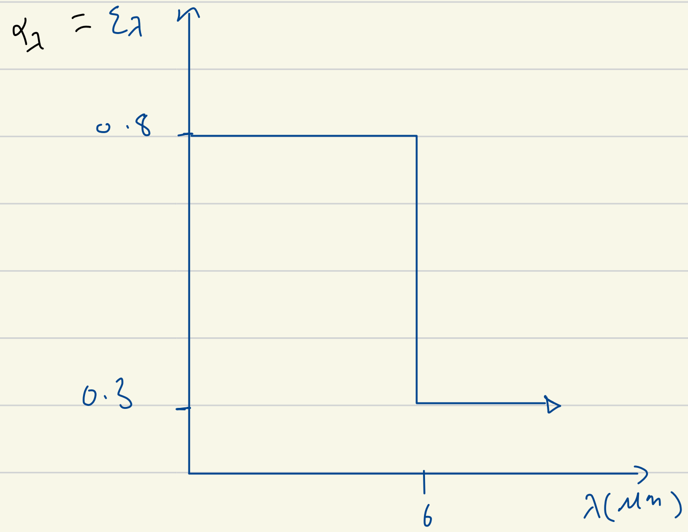
$60750 \ W/m^2$

→ Surface is irradiated by large surroundings at $T = 1500\,K$ & emissivity of 0.8

Ex: opaque Diffuse surface at 1000K

↳ $\tau = 0$ ↳ $\alpha_\lambda = \varepsilon_\lambda$

Find: 1. ε 2. α



$$\varepsilon = \frac{\int_0^6 \varepsilon_\lambda E_{b\lambda}\, d\lambda + \int_6^\infty \varepsilon_\lambda E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda}$$

$$0.8 \frac{\int_0^6 E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda} + 0.3 \frac{\int_6^\infty E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda} \rightarrow \lambda T = 6(1000) = 6000 \qquad F_{0-\lambda_1} = 0.737818$$

$(\lambda T) = 6(1000) = 6000 \qquad f_{0-\lambda_1} = 0.737818$

$$\varepsilon = 0.8(0.737818) + 0.3(1 - 0.737818) = 0.669$$

→ to find α

$$\alpha = 0.8 \frac{\int_0^6 G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda} + 0.3 \frac{\int_6^\infty G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda}$$

⇒ special case [large surroundings]

$G_\lambda = E_{b_\lambda}(T_{surr})$

$$= 0.8 \frac{\int_0^6 E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda} + 0.3 \frac{\int_6^\infty E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda_r}$$

$(\lambda T) = 6(1500) = 9000 \quad f_{0\lambda} = 0.890029$

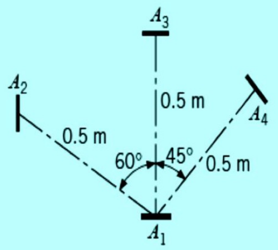
$(\lambda T) = 6(1500) = 9000 \quad f_{0\lambda} = 0.890029$

$\alpha = 0.8(0.890029) + 0.3(1 - 0.890029)$

$\alpha = 0.745$

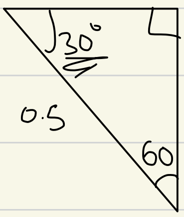
independent of θ

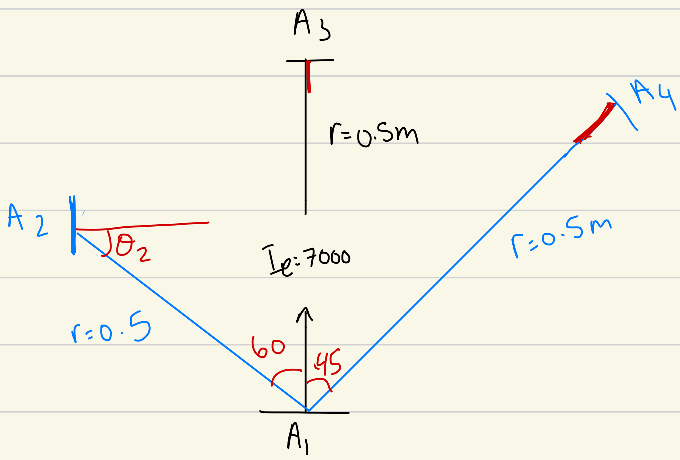
A small surface of area $A_1 = 10^{-3}$ m^2 is known to emit diffusely, and from measurements the total intensity associated with emission in the normal direction is $I_n = 7000$ W/m$^2 \cdot$ sr.



Radiation emitted from the surface is intercepted by three other surfaces of area $A_2 = A_3 = A_4 = 10^{-3}$ m^2, which are 0.5 m from A_1 and are oriented as shown. What is the intensity associated with emission in each of the three directions? What are the solid angles subtended by the three surfaces when viewed from A_1? What is the rate at which radiation emitted by A_1 is intercepted by the three surfaces?

* solid angle ⇒ between normal & Radiation line





Σangles of right triangle = 180°

$180 - (60 + 90) = 30$

1. Intensity in all Three directions

$I_e = 7000$ W/m$^2 \cdot$ sr ⇒ Diffuse Surface.

2. Solid angles

$$\omega_{2-1} \text{ (looking from } 1 \to 2) = \frac{A_2 \cos\theta_2}{r^2}$$

$$= \frac{10^{-3} \cos(30)}{0.5^2} = 3.464 \times 10^{-3} \text{ sr.}$$

$$\omega_{3-1} = \frac{A_3 \cos\theta}{r^2} = \frac{10^{-3} \cos 0}{(0.5)^2} = 4 \times 10^{-3} \text{ sr}$$

$$\omega_{4-1} = \frac{A_4 \cos\theta_4}{r^2} = \frac{10^{-3} \cos 0}{(0.5)^2} = 4 \times 10^{-3} \text{ sr}$$

3. Heat rates at three surfaces:-

$$q_{1-2} = I \cdot A_1 \cos\theta_1 \quad \omega_{2-1}$$
$$7000 \times 10^{-3} \cos 60 \times 3.464 \times 10^{-3} = 12.14 \times 10^{-3} \text{ W}$$

$$q_{1-3} = I \; A_1 \cos\theta \quad \omega_{3-1}$$
$$7000 \times 10^{-3} \cos 0 \; (4 \times 10^{-3}) = 28 \times 10^{-3} \text{ W}$$

$$q_{1-4} = I \; A_1 \cos\theta_2 \; \omega_{4-1}$$
$$7000 \times 10^{-3} \cos 45 \; (4 \times 10^{-3}) = 19.7 \times 10^{-3} \text{ W}$$

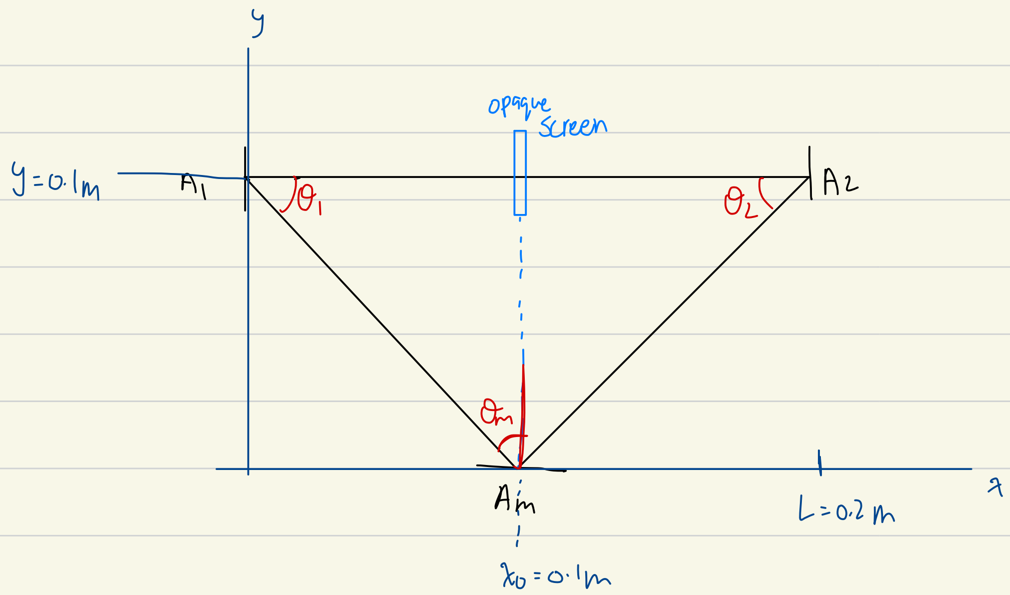
4. Irradiation on the three surfaces:-

$$G_2 = \frac{q_{1-2}}{A_2} = \frac{12.14 \times 10^{-3}}{10^{-3}} = 12.14 \text{ W/m}^2$$

$$G_3 = \frac{q_{1-3}}{A_3} = \frac{28 \times 10^{-3}}{10^{-3}} = 28 \text{ W/m}^2$$

$$G_4 = \frac{q_{1-4}}{A_4} = \frac{19.7 \times 10^{-3}}{10^{-3}} = 19.7 \text{ W/m}^2$$

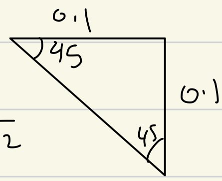
Ex2.



$I_1 = 1.2 \times 10^5 \ W/m^2.sr$

$A_1 = 1 \times 10^{-4} = A_2$

$A_m = 2A_1$



$A^2 = B^2 + C^2$

$\sqrt{0.1^2 + 0.1^2}$

1- $q_{1-m} = I_1 . A_1 \cos\theta \ w_{m-1}$

$$w_{m-1} = \frac{A_m \cos\theta_m}{r^2} = \frac{2\times10^{-4} \ \cos(45)}{(0.1^2 + 0.1^2)} = 7.07\times10^{-3} \ sr$$

$$q_{1-m} = 1.2\times10^5 \ (1\times10^{-4}) \cos(45) \times 7.07\times10^{-3} = 0.06W$$

$$G_m = \frac{q_{1-m}}{A_m} = \frac{0.06}{2\times10^{-4}} = 300 \ W/m^2$$

$\rightarrow G_m = \pi I_m$

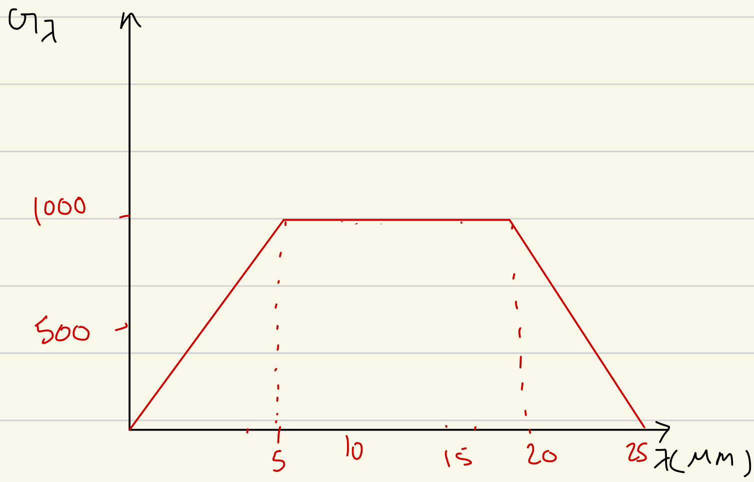
$300 = \pi \ I_m \qquad I_m = 95.5 \ W/m^2.sr$

2. $q_{m-2} = I_m A_m \cos\theta_m \cdot \omega_{2-m}$

$$\omega_{2-m} = \frac{A_2 \cos\theta_2}{r^2} = \frac{1\times10^{-4} \ \cos 45}{(0.1^2 + 0.1^2)} = 3.533\times10^{-3} \ sr.$$

$$95.5 \times 2\times10^{-4} \times \cos 45 \times 3.533\times10^{-3} = 4.77\times10^{-5} \ W$$

Ex: what is the total irradiation

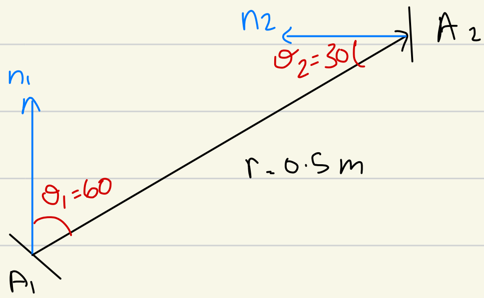


$$G = \int_0^\infty G_\lambda \, d\lambda$$

$$G = \int_0^{5\mu m} G_\lambda \, d\lambda + \int_{5\mu m}^{20\mu m} G_\lambda \, d\lambda + \int_{20\mu m}^{25\mu m} G_\lambda \, d\lambda$$

$$\frac{1}{2}(5) \times 1000 + 15(1000) + \frac{1}{2}(5)(1000) = 20000 \ W/m^2$$

Ex:



$$E = \pi I_e$$

$$5 \times 10^{-4} = \pi I_e$$

$$I_e = 1.5915 \times 10^{-4} \ W/m^2 . sr$$

$A_1 = 10^{-4} m^2$

$E = 5 \times 10^{4} \ W/m^2$

$A_2 = 5 \times 10^{-4} m^2$

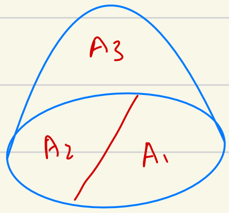
$$q_{1-2} = I_e . A_1 \cos\theta_1 . \omega_{2-1}$$

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{r^2} = \frac{5 \times 10^{-4} (\cos 30)}{0.5^2} = 1.73 \times 10^{-3} \ sr.$$

$$1.5915 \times 10^{-4} (10^{-4}) \cos 60 \times 1.73 \times 10^{-3} = 1.38 \times 10^{-11} \ W$$

$$G_2 = \frac{q_{1-2}}{A_2} = \frac{1.38 \times 10^{-11}}{5 \times 10^{-4}} = 2.75 \times 10^{-8} \ W/m^2$$

Ex: Find veiw factor



$N = 3$

$F = 3^2 = 9$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

$F_{11} \rightarrow$ plane surface $= 0$

$F_{12} = 0$

$F_{11} + F_{12} + F_{13} = 1$

$F_{13} = 1$

$F_{21} = 0$

$F_{22} = 0$

$F_{12} + F_{22} + F_{23} = 1$

$F_{23} = 1$

$A_3 F_{31} = A_1 F_{13}$

$F_{31} = \frac{A_1}{A_3} \, 1$

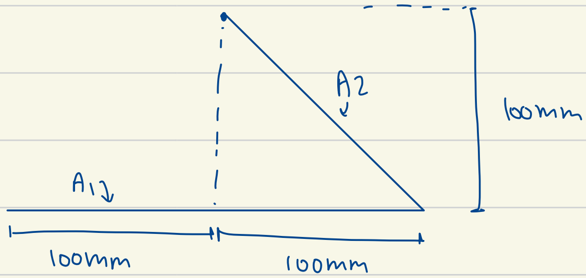
$$\frac{A_1 = \pi r^2}{A_3 = \pi \frac{D^2}{2}}$$

$F_{31} = F_{32}$

$F_{31} + F_{32} + F_{33} = 1$

$F_{33} = 1 - 2F_{31}$

Ex: Find F_{12} & F_{21}



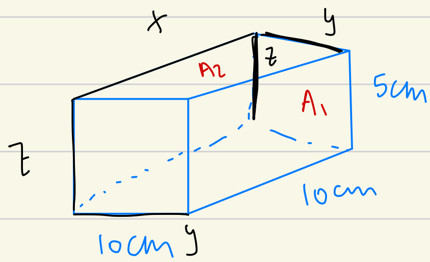
$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$F_{12} = 0.5$

$F_{12} = \frac{A_1}{A_2} F_{12}$

$F_{11} = 0$ $F_{22} = 0$

Ex:



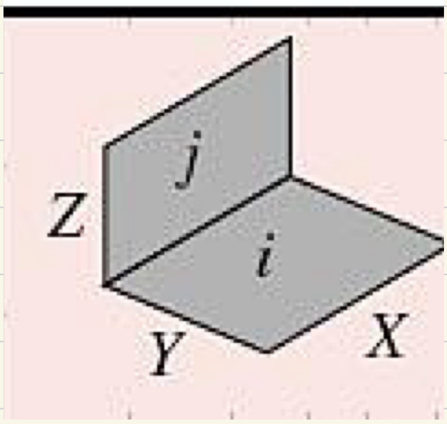
Find $F_{2\to1}$

graph C

$\frac{Z}{X} = \frac{0.05}{0.10} = 0.5$

$\frac{Y}{X} = \frac{0.10}{0.10} = 1$

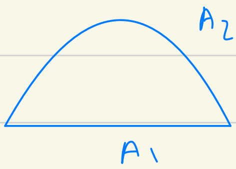
From graph $F_{21} = 0.15$



Ex: fraction of radiation leaving i that is intercepted by surface j

F_{11} F_{12}

F_{21} F_{22}



$F_{11} = 0$ by sumation Rule $\cancel{F_{11}}^{0} + F_{12} = 1$

$F_{12} = 1$

→ Reciprocity Rule $A_2 F_{21} = A_1 F_{12}$

$F_{21} = \frac{A_1}{A_2} F_{12}$

half circle $= \frac{1}{2} \pi d L$

$= \frac{1}{2} \pi r L$

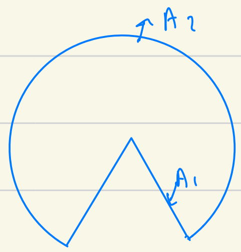
$F_{21} = \frac{2r\cancel{L}}{\pi \cancel{r} \cancel{L}}$

$F_{21} = \frac{2}{\pi} = 0.637$

→ $F_{22} + F_{21} = 1$

$F_{22} + 0.637 = 1$ $F_{22} = 0.363$

Ex



F_{11} F_{12}

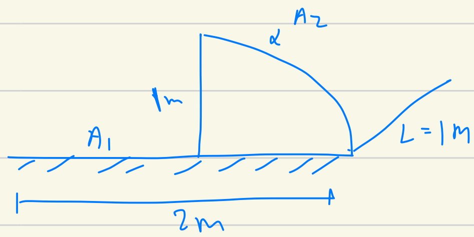
F_{21} F_{22}

$\rightarrow F_{11} = 0$ $F_{12} = 1$

$\rightarrow$ Reciprocity $A_2 F_{21} = A_1 F_{12}$

$$F_{21} = \frac{A_1}{A_2} = \frac{2 \times r \times L}{(\frac{3}{4})(2\pi r L)} = \frac{2}{\frac{3}{4}(2\pi)} = 0.424$$

$$F_{22} = 1 - 0.424 = 0.575$$



F_{11} F_{12} F_{13}

F_{21} F_{22}

$F_{11} = 0$ $F_{12} = 0.5$ $F_{13} = 0.5$ $\Rightarrow$ by inspection

$\rightarrow$ Reciprocity $A_2 F_{21} = A_1 F_{12}$

$$F_{21} = \frac{A_1}{A_2} F_{12}$$

$A_1 = L \times W$

$A_2 = \frac{\pi r^2}{4}$

$$F_{21} = \frac{2 \times 1}{\frac{\pi r^2}{4}} = \frac{2}{\frac{\pi (1)^2}{4}} =$$

L.16 Ex: Heat absorbed = 9500000 Btu/hr

T_w = 600°F

Excess Air = 15% → T_{Air} = 1500

Tube Diameter = 4.5 in

Tube spacing = 8 in

Length = 26 ft

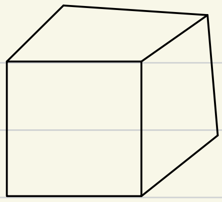
Number of tubes = 30

Area of flue gas exit = 42 ft²

Box Radiant Arrangement

A_r = 1076.644

gas emissivity = 0.427



$$q_r = \sigma \, \alpha A_{cp} \, F \, (T_g^4 - T_w^4)$$

$\sigma = 0.173 \times 10^{-8}$

→ α calculation

center to center / tube diameter

$8 / 4.5 = 1.77$

from graph $\alpha \cong 0.915$

→ A_{cp} calculation

$A_{cp} = N \cdot S \cdot L$

$$30 \times \frac{8}{12} \times 26 = 520 \text{ ft}^2$$

→ F calculations:-

$A_w = A_r - \alpha A_{cp}$ $A_r = 1076.644$

$A_w = 1076.644 - 0.915(520) = 600.844 \text{ ft}^2$

$$\frac{A_w}{\alpha A_{cp}} = \frac{600.844}{0.915(520)} = 1.26$$

$\rightarrow$ from chart

gas emissivity $= 0.427$

$F = 0.597$

$$\therefore q_r = 0.173 \times 10^{-8} \times 0.915 \times 520 \times 0.597 \times ((1500 + 460)^4 - (660 + 460)^4)$$

$\hookrightarrow$ conv from F to K

$$q_r = 6631794.66$$