Electrical

Summer
2025/2026

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Chapter 1

Chapter (1)

[1] Units :

Time → sec.

Electrical current → Ampere → A

voltage → volt

mass → Kg

length → m

power → watt

charge → coloum

[2] Prefixes :

Pico → 10^{-12}

nano → 10^{-9}

Micro → 10^{-6}

milli → 10^{-3}

Kilo → 10^{3}

Mega → 10^{6}

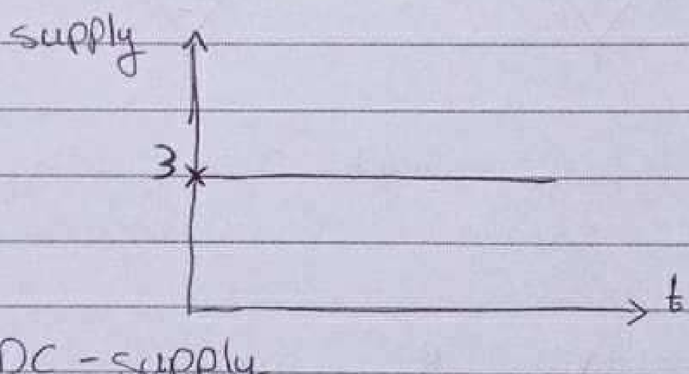
giga → 10^{9}

[3] Power supply :

a) types : 1) voltage source

2) current source

b) output signal : 1) DC output signal : value of the supply is fixed with time.



Ex : supply V = 3 volt

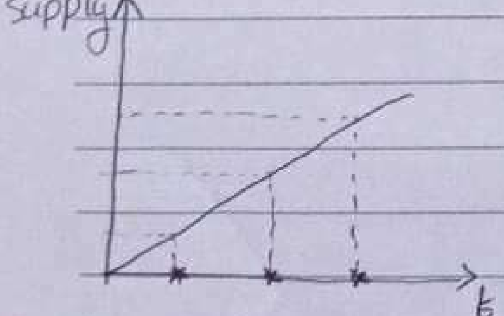
I = 2 A = constant

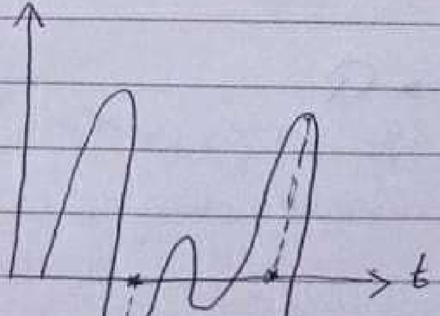
DC - supply

Ex : $V(t) = 3\sin(4t)$

$I(t) = 2e^{-t}$

2) AC output signal : it's value vary with time.





بسم الله الرحمن الرحيم

lect. (1) , مفاهيم أساسية   13 /Jule/2025

* charge , current → 1.3

* I → التيار هو سيل من الإلكترونات

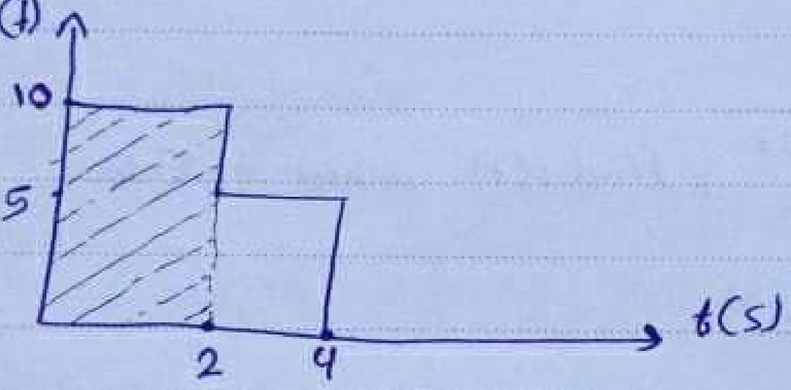
$e = 1.6 \times 10^{-19}$ C. , $Q = ne$

شحنة الالكترون   ← عدد الـ(e)

$i = \frac{dq}{dt}$

$$Q = \int_{t_0}^{t_1} i \, dt$$

Ex1:

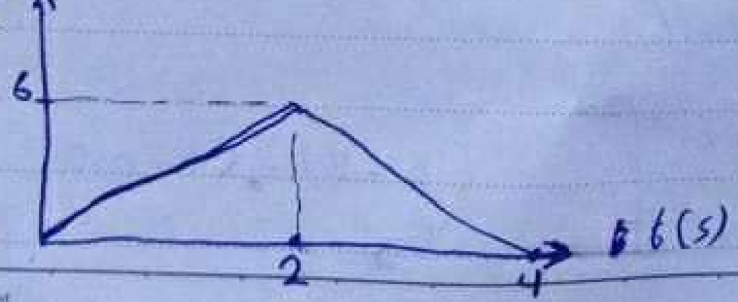


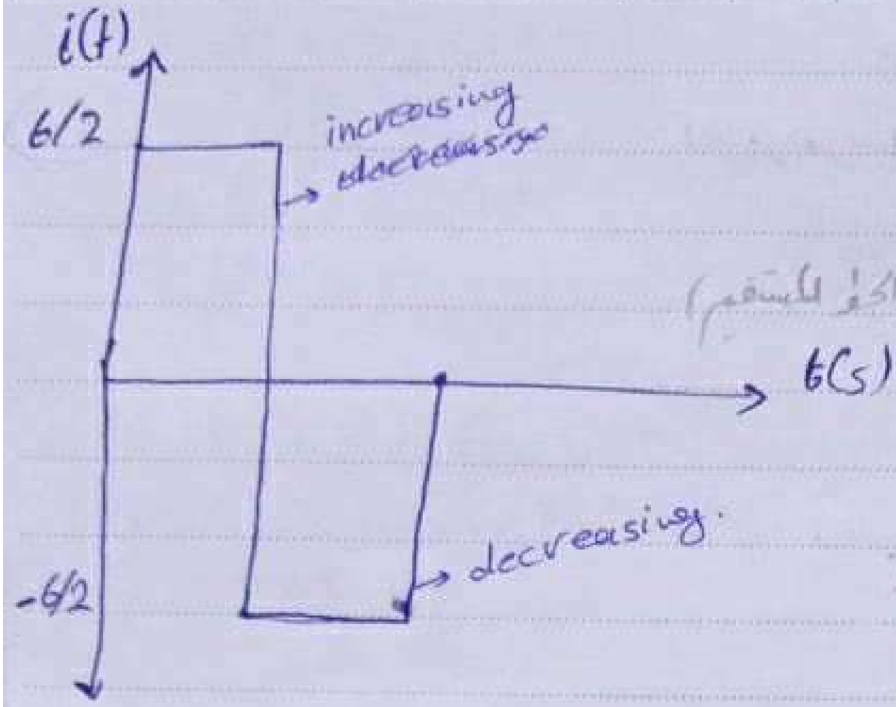
* Find Q. t 0→2

$$Q = \int_0^2 i(t)\, dt \Rightarrow \text{Area}$$

$Q = 2 \times 10 = 20$ coloum.

Ex2: Find i(t)





* current wave form

هو التيار بدأ من الصفر ثم ازداد
وصل قيمة معينة ثم ثبت (عبارة عن الخط المستقيم)
وبعدها بدأ بالانخفاض ليصل
إلى قيمة معينة ثم يثبت (عبارة عن
خط مستقيم) ويعود إلى الصفر.

Ex:3 $Q(t) = 10 - 10e^{-2t}$, Find $i(t)$ when $t=5$ sec.

$$i = \frac{dq}{dt} = 0 + 20e^{-2t}$$

$$i(t) = 20e^{-2t}$$

$$i(5) = 20e^{-2\times5} = 9.1\times10^{-4}\,A.$$

Ex (4):

$i(t) = (3t^2 - t)\,A$, Find Q when $t=1$, $t=2$

$$Q = \int i(t)\,dt$$

$$= \int_1^2 (3t^2 - t)\,dt$$

$$= \frac{3t^3}{3} - \frac{t^2}{2}\Big]_1^2 = (8-2) - (1-0.5)$$

$$= 5.5\,C$$

/ /

* voltage :- work

* $V_a \rightarrow$ voltage of point ⓐ

* $V_b \rightarrow$ voltage of point ⓑ

*توضيح الـ voltage :

If I have in (∞) charge equal 1 coloum.

* the work need to bring a charge of 1C to point ⓐ from ∞ إذا بدي أجيب شحنة مقدارها 1 كولوم موجودة بالـ (∞) بدي أبذل عليها شغل لأحركها للنقطة (a).

* $V = \frac{dw}{dq}$, w: work

q: charge.

$\bullet V_a = 5V$

$\bullet V_c = -4V$ $\quad\quad \bullet V_b = 3V$

$V_{ab} = V_a - V_b = 5 - 3 = 2V$

$V_{ac} = V_a - V_c = 5 - -4 = 9V$

* Power and Energy :

$\vec{P}$ $\quad\quad \vec{W}$

P(w) $\quad\quad$ E(J)

$P = \frac{dw}{dt}$

$W = \int P.dt$

Ex:(5) a• $\quad$ •b , Find v if $q = 2C$.

$-30J$

$v = \frac{w}{q} = \frac{-30}{2} = -15V$

/ /

Ex(6):- find the power if $t = 3ms$,

$i = 5 \cos(60\pi t)$

$v = 3i$

$P = I . V$

$p = i . v = 5 \cos(60\pi t)\,(3)(5 \cos 60\pi t)$

$= 75 \cos^2(60\pi t)$

at 3ms

$P = 75 \cos^2(60\pi(3 \times 10^{-3}))$ → الآلة الحاسبة بال Radian.

$= 53.5\ w$

Ex(7):- we have lamp has $P = 100w$, work in 2 hours

How much energy?

$W = Pt$

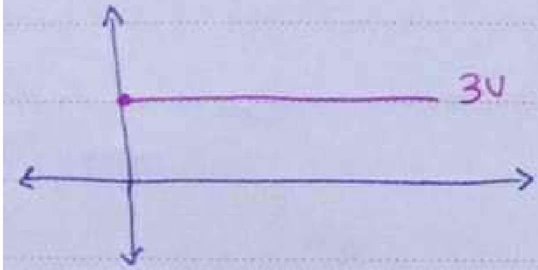
$= 100 \times 2 \times 3600$

$= 72 \times 10^4\ J$

* Circuit Elements:
  - → Power source (Produce Power)
  - → load (consums Power).
  - → concentration

* Power source

① Independent voltage source. ±

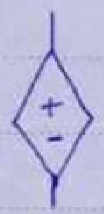


, Direct current (continous)

DC

Independent current source. ↑

② Dependent power source.



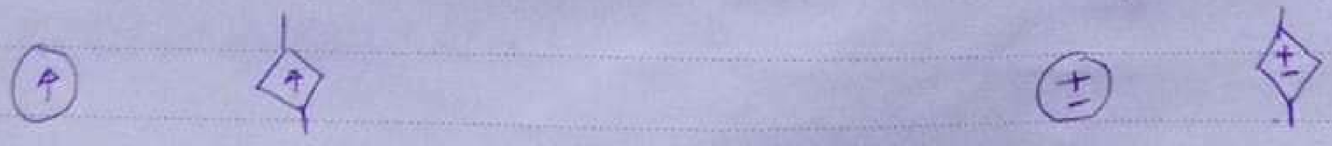
dep. ⇒ voltage source. (يعتمد على قيمة تيار معينة مثل 3jx)

dep. ⇒ current source. (يعتمد على قيمة جهد معين في الدائرة مثل ...)

Power supply.

- current
  - Indp
  - dep
- voltage
  - indep
  - dep

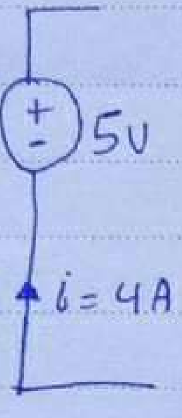
* Inde ⇒ قيمة محددة
* dep ⇒ تعتمد على التيار أو فرق الجهد



* For any electrical circuit

$$\sum P = 0$$

$$\sum P_{in} = \sum P_{out}$$

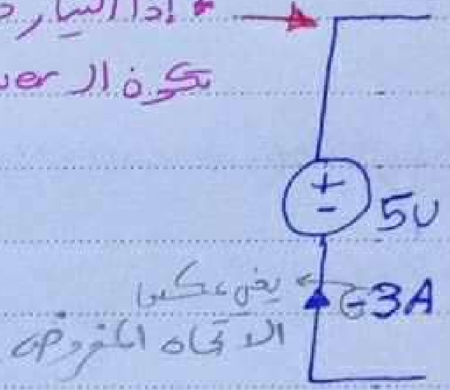


$$P = -5 \times 4 = -20\ W$$

* إذا دخل التيار في القطب السالب لمصدر فرق الجهد تكون ال Power سالبة، وتسمى
produce, generate, deliver power.

* إذا التيار دخل في القطب الموجب لمصدر فرق الجهد تكون ال power موجبة، وتسمى
consumed power.

$$P = -(5)(-3)$$
$$= 15\ W$$



particle problem.

/ /

$1.1 \Rightarrow Q = ?$, $n = +6\times10^{6}$ proton

$Q = n\boxed{e}$ → نعوض القيمة دون إشارة

$= 6\times10^{6} \times +1.6\times10^{-19}$

$= +9.6\times10^{-13}\ C$

$1.3 \Rightarrow$

$$i = \begin{cases} 4A, & 0<t<1 \\ 4t^2 A, & t>1 \end{cases} \qquad Q,\ t=0 \rightarrow t=2$$

$$Q = \int_{t_1}^{t_2} i\, dt$$

$$= \int_0^1 4\, dt + \int_1^2 4t^2\, dt$$

$$= 4t\Big]_0^1 + \frac{4t^3}{3}\Big]_1^2$$

$$= 4 + \frac{4}{3}(8-1)$$

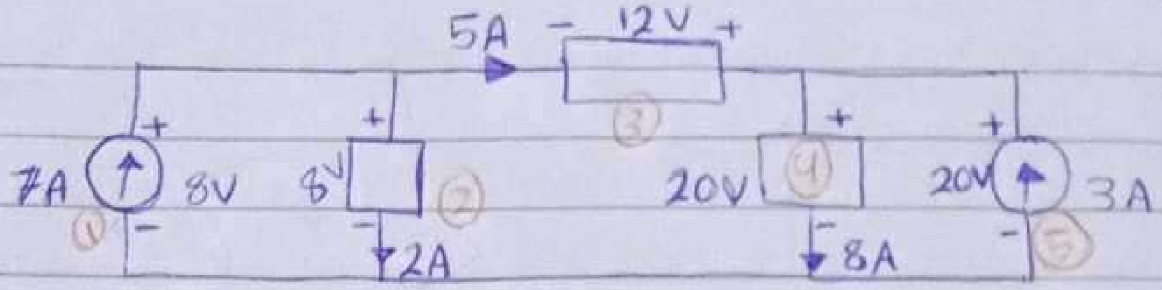
$$= 13.33\ C$$

$1.5 \Rightarrow P = ?$, $i = 5\cos 60\pi t$, $t = 5\,ms$, $v = 10 + 5Q$

$$P = IV = 5\cos(60\pi t) \times \left(10 + 5\int_0^5 5\cos 60\pi t\right) = 29.7\,w$$

$$\frac{25\sin(60\times\pi\times5)}{60\times\pi}$$

Ex:



*Find power for each element

$P_1 = (-)(7)(8) = -56\ W \rightarrow$ generated

$P_2 = (+)(2)(8) = 16\ W \rightarrow$ consumed

* الإشارة تدل على الاتجاه

$P_3 = (-)(5)(12) = -60\ W \rightarrow$ generated

$P_4 = (+)(8)(20) = +160 \rightarrow$ consumed

$P_5 = (-)(3)(20) = -60 \rightarrow$ generated

Law of conservion energy $\Rightarrow \sum P_{consumed} = \sum P_{generated}$

$16 + 160 \stackrel{?}{=} +56 + +60 + +60$

$176 \stackrel{?}{=} 176$ ✓

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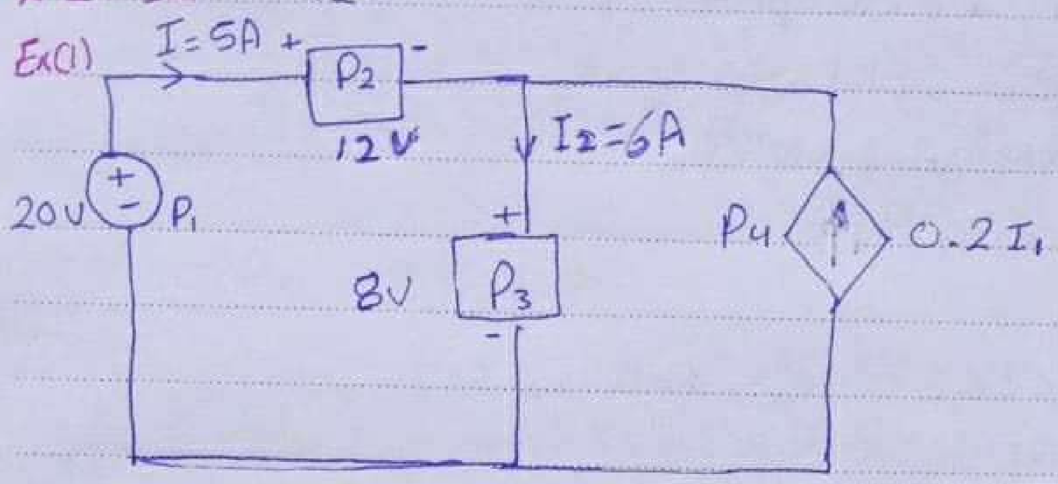
* $\Sigma P = 0$

Ex(1)



$P_1 = (-20)(5) = -100$ w (proundce, diliner)

$P_2 = +(5)(12) = +60$ w (cons.)

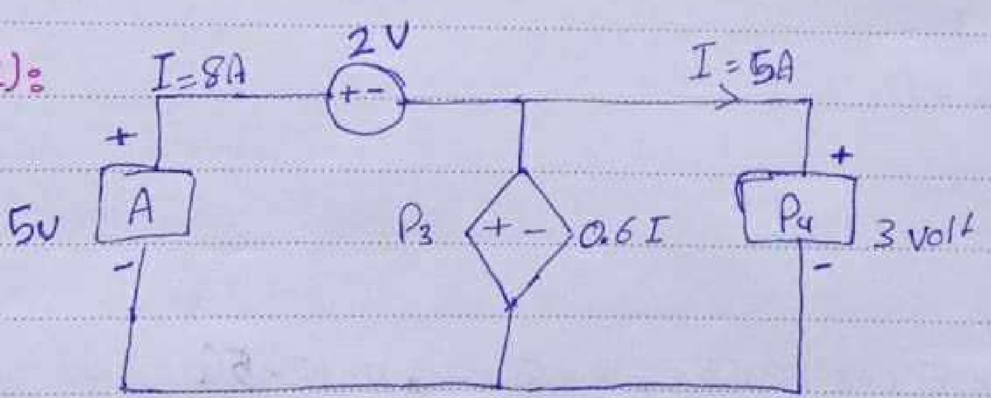
$P_3 = +(6)(8) = +48$ (cons.)

$\Sigma P = 0$, so $P_4 = -8$ W

* إذا دخل التيار القطب السالب إذاً (−) ⇐ P

* إذا دخل التيار القطب الموجب إذاً (+) ⇐ P

Ex (2):



$P_1 = (-5)(8) = -40$ W

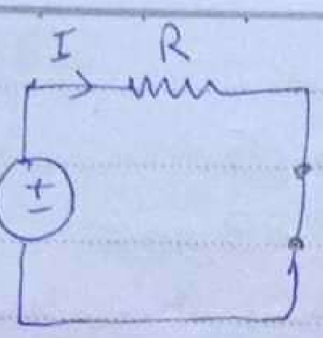
$P_2 = (+2)(8) = +16$ W $\Rightarrow -40 + 16 + P_3 + 15 = 0$

$P_3 = ?$ $P_3 = +9$ W

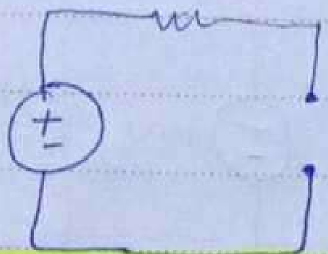
$P_4 = +(5)(3) = +15$

End of chapter one

Chapter 2

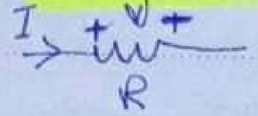


* have a current. $V=0$, $R=0$



* open circuit no current.

Ohm's law (2.2)



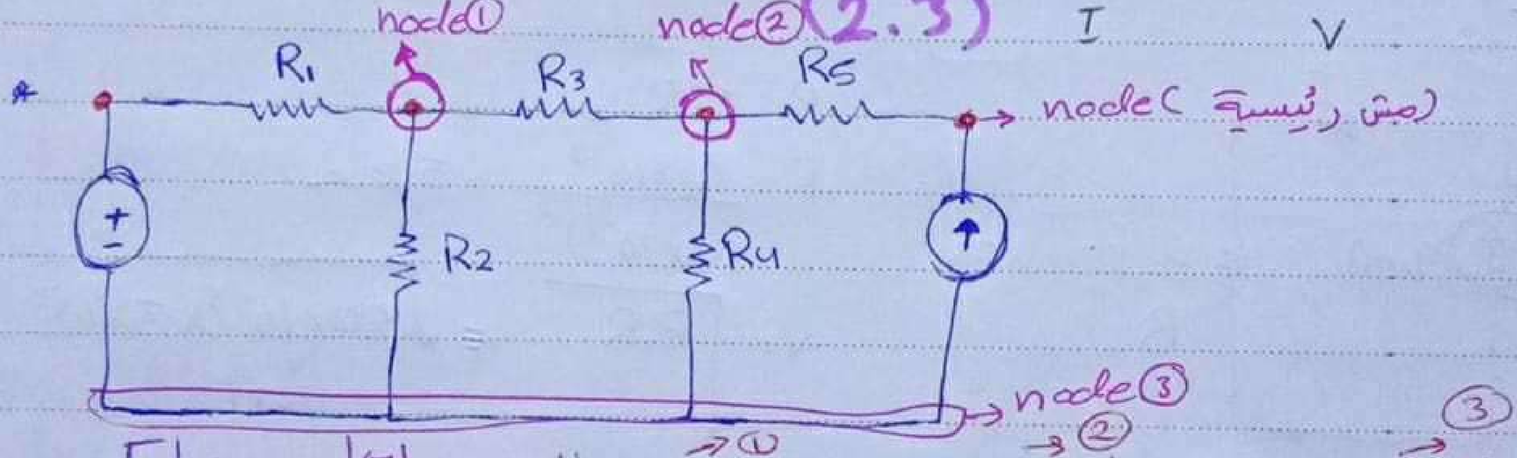
poliarty هو تحديد الـ ← للبطارية

حيث مكان خروج التيار : $V = IR$

هو القطب الموجب $P = IV = I^2R = V^2/R$

* Conductance: $G = 1/R$, $V = IR \Rightarrow R = \frac{V}{I}$, $G = \frac{I}{V}$

(2.3)



→ Element (7) → voltage source, current source, Resistance

→ Branch (7,5) → any part has an element.

→ Node → conection point of two branches or more.

→ loop (6) → any closed path (علّم نقطة وأرجع إلها)

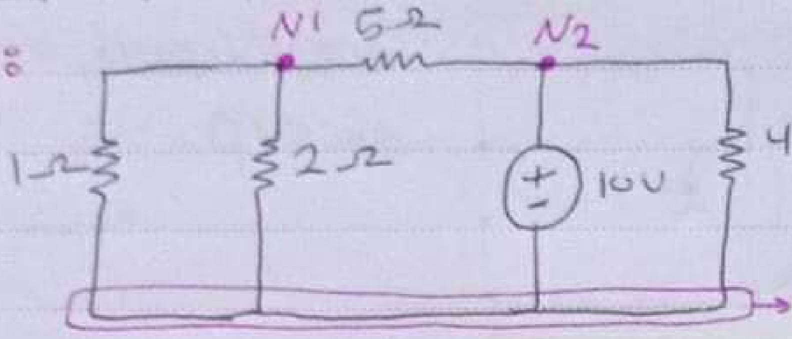
→ mesh → any closed path that doesn't have another closed loop or path.

* كل mesh هو loop

* مش كل loop هو mesh

/ /

Ex:

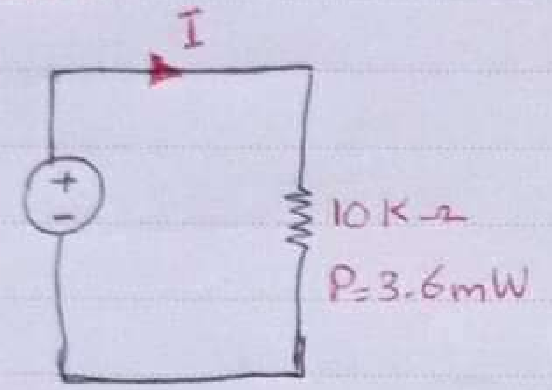


* Element = 5
* branch = 5
* Node = 3 node

→ N3 * سلك مافيه اي element بقدر اعتبرو نقطة واحدة.

* loop → 6 * mesh → 3

Ex:

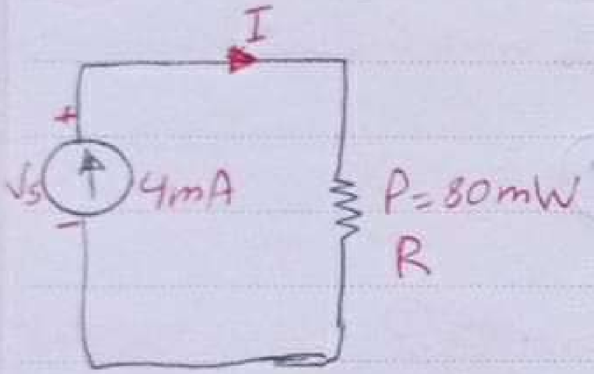


$$P = IV = \frac{V^2}{R} = I^2R$$

$$V = \sqrt{3.6\times10^{-3}\times10\times10^{3}} = 6 \text{ volt}$$

$$I = \sqrt{\frac{3.6\times10^{-3}}{10\times10^{3}}} = 6\times10^{-4}\,A$$

Ex:



$$P = I^2R =$$

$$R = \frac{80\times10^{-3}}{(4\times10^{-3})^2} = 5\,K\Omega$$

$$V = \sqrt{PR} = \sqrt{80\times10^{-3}\times5\times10^{3}}$$

$$= 20 \text{ volt}$$

Ex:

A 100 Watt high bulb is illuminated on for one hour only. How many joules of electrical energy have been used by the lamp.

Energy = Power x time

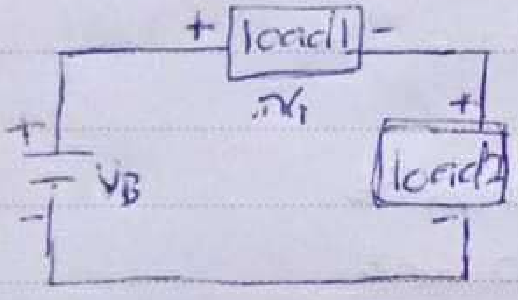
= 100 x60 x60

= 360 KJ.

Ex: How much energy does a 1200W dishwasher use when it runs for 30 minutes (1800s)?

$$\text{Energy} = \text{Power} \times \text{Time}$$
$$= 1200 \times 1800$$
$$= 2160\,KJ,\ 2.16\,MJ$$

Ex: Power dissipated or generated by each element.



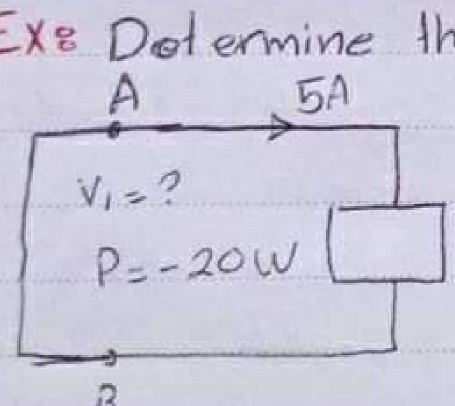
$V_1 = 8V$ $P_B = -V_B \times i = -12 \times 0.1 = -1.2W$

$V_2 = 4V$ $P_1 = V_1 \times i = 8 \times 0.1 = 0.8W$

$V_B = 12V$ $P_2 = V_2 \times i = 4 \times 0.1 = 0.4W$

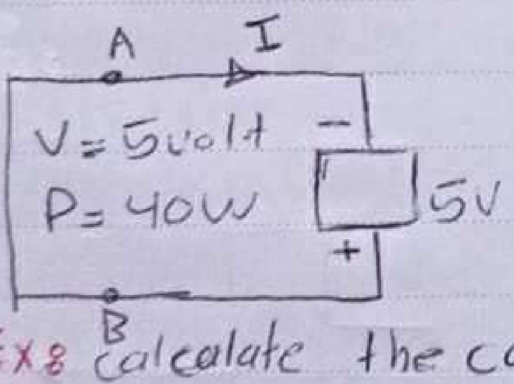
$i = 0.1A$

Ex: Determine the unknown voltage or unknown current.



$P = IV$

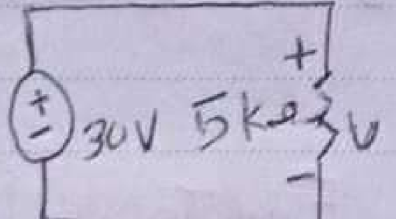
$V = \frac{P}{I} = \frac{-20}{5} = -4\ \text{volt}$



$P = IV$

$I = \frac{P}{V} = \frac{40}{5} = 8A$

Ex: Calculate the current (i), the conductance (G)



$V = IR \Rightarrow I = V/R = 30/5\times10^3 = 6\times10^{-3}\ \text{volt}$

$G = 1/R \Rightarrow 1/5\times10^3 = 2\times10^{-4}\,S$

$P = IV = 30 \times 6 \times 10^{-3} = 0.18\,W$

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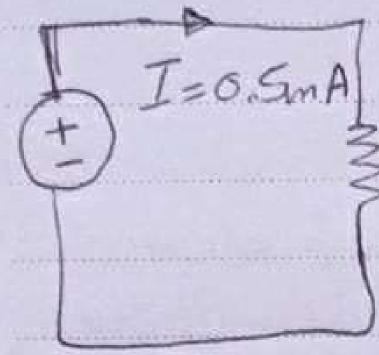
Ex: Given the following circuit find, the value of the voltage source and the power abosorbed by the R.



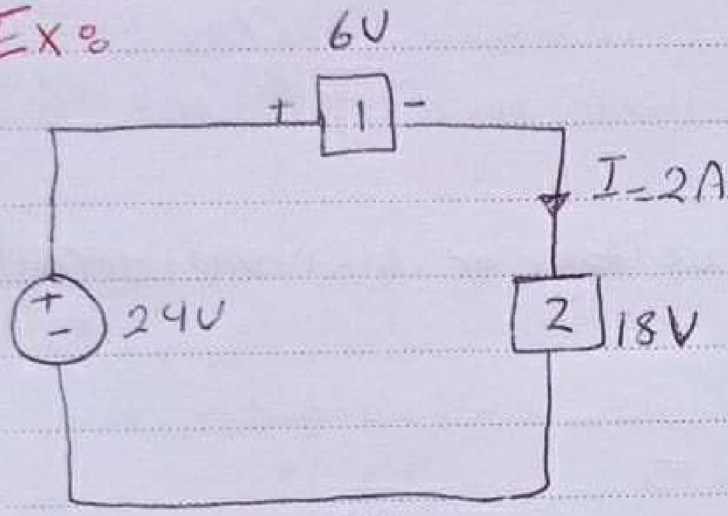
$$V = \frac{I}{G} = \frac{0.5\times10^{-3}}{50\times10^{-6}}$$

$= 10$ volt

$P = IV = 10 \times 0.5 \times 10^{-3}$

$= 5\times10^{-3}$ W

Ex:



$P_1 = IV = 2\times24 = 48$ W

$P_1 = IV = 2\times6 = 12$ W

$P_2 = IV = 18\times2 = 36$ W

Ex:

$P = 20\cos^2 t$

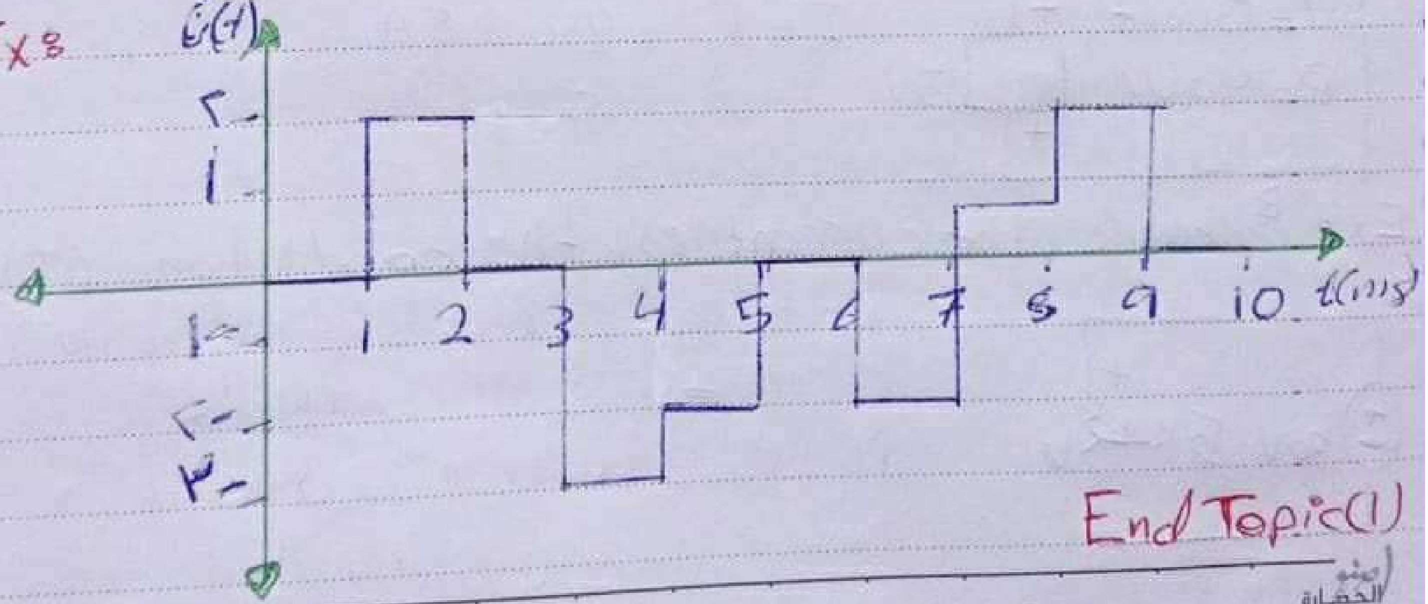
$v = 10\cos t$

$P = IV$

$$I = \frac{P}{V} = \frac{20\cos^2 t}{10\cancel{\cos t}} = 2\cos t$$

$$R = \frac{V}{I} = \frac{10\cancel{\cos t}}{2\cancel{\cos t}} = 5\,\Omega$$

Ex:



End Topic(1)

Topic(2)

2.4 : kirchof law.   /   /

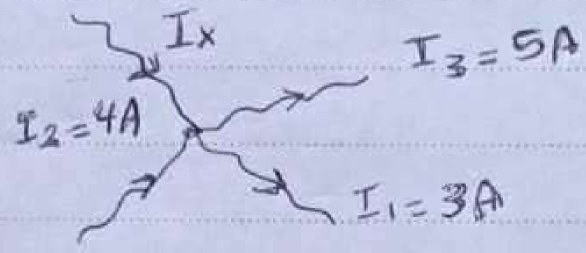
① kirchof current law (KCL)

② kirchof voltage law (KVL)

→ **KCL : for any node $\sum I = 0$**

$$\sum I_{in} = \sum I_{out}$$

Ex : Find I_x.

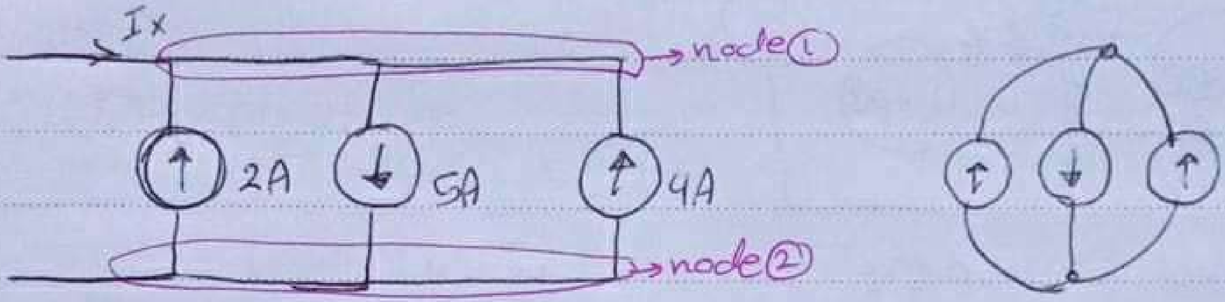


$$I_x + I_2 = I_1 + I_3$$

$$I_x + 4 = 3 + 5$$

$$I_x = 4A$$

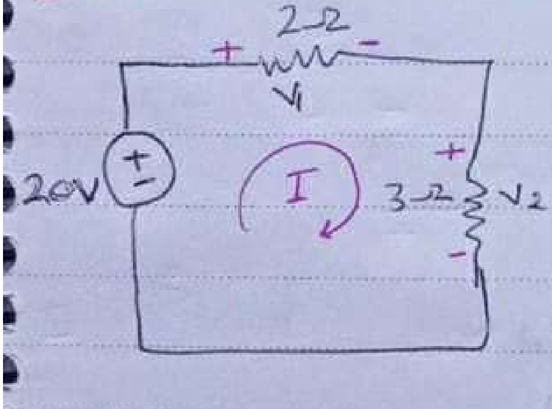
Ex :



$$I_x + 2 + 4 = 5 \Rightarrow I_x = -1A$$

→ **KVL : for any loop (Any closed loop) , $\sum V = 0$**

Ex :



في البطارية، ويخرج من القطب السالب في المقاومة.

① Assing the current (التيار يخرج من القطب الموجب

② Assing the polarity of each voltage

③ Apply KVL.

$$-20 + V_1 + V_2 = 0 \qquad V_1 = I \times 2$$

$$-20 + 2I + 3I = 0 \qquad V_2 = I \times 3$$

$$I = 4A$$

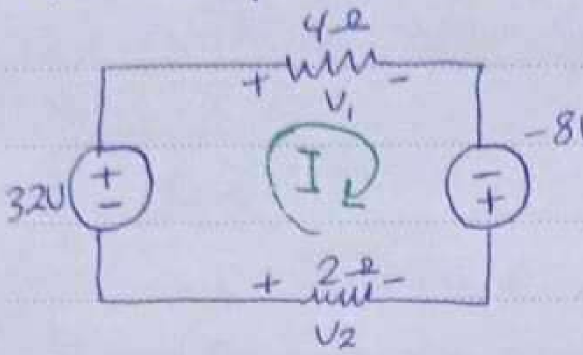
$$V_1 = 2 \times 4 = 8 \text{ volt}$$

$$V_2 = 3 \times 4 = 12 \text{ volt}$$

(2.4)

/ /

*Particle problem (2.5)



$-32 + V_1 - 8 - V_2 = 0$

$V_1 - V_2 = 24$ volt

$2 + 4 = 6\ \Omega$

$24 = I \times 6$

$I = 4A$

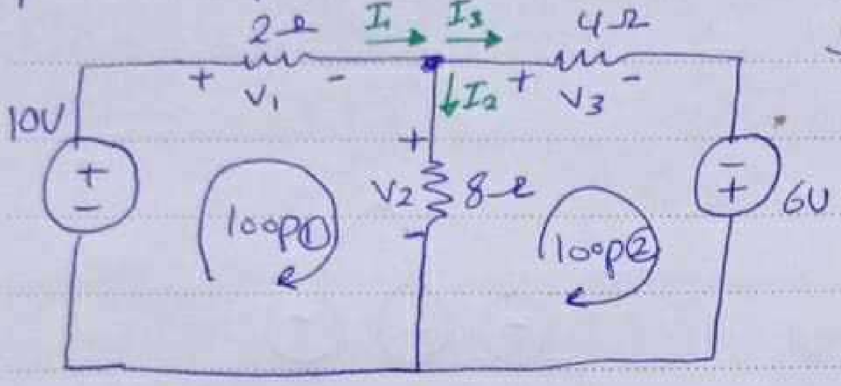
$V_1 = 4 \times 4$

$= 16$ volt

$V_2 = 4 \times -2$

$= -8$ volt.

particle problem (2.8)



$I_1 = ?$, $I_2 = ?$, $I_3 = ?$

$V_1 = ?$

*apply KVL at loop ①:

$-10 + 2I_1 + 8I_2 = 0$

$2I_1 + 8I_2 = 10$ ---①

$-10 + V_1$

*apply KVL at loop ②:

$-8I_2 + 4I_3 - 6 = 0$

$-8I_2 + 4I_3 = 6$ ---②

$I_1 = I_2 + I_3$ ---③ → $I_1 - I_2 - I_3 = 0$

$I_1 = 3A$

$2(I_2 + I_3) + 8I_2 = 0$

$I_2 = 0.5A$

$10I_2 + 2I_3 = 0$ --- ④

$I_3 = 2.5A$

solving equ. ② & ④:

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* Series elements have same current. • التوالي يمر فيها نفس التيار

① $R_{equ} = R_1 + R_2 + R_3 ---$

② voltage devision $\Rightarrow V_1 = \frac{R_1}{R_{equ}} \times V$ و $V_2 = \frac{R_2}{R_{equ}} V$

لم يطبق فقط على التوالي

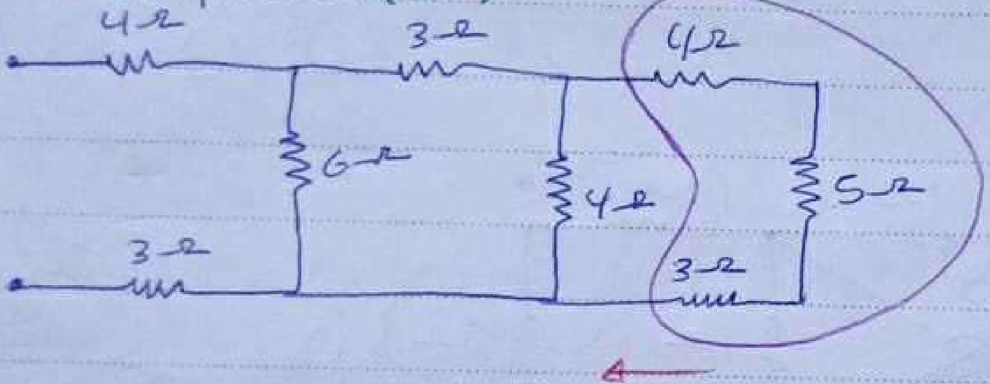
• parallel elements have same voltage • التوازي يمر فيها نفس الجهد

① $\frac{1}{R_{equ}} = \frac{1}{R_1} + \frac{1}{R_2} + --- + \frac{1}{R_N}$ و $R_{equ} = \frac{R_1 \times R_2}{R_1 + R_2} \Rightarrow$ two resistance

$G_{equ} = G_1 + G_2 + ---. + G_N$

② current devision $\Rightarrow I_1 = \frac{R_2}{R_1 + R_2} I$ و $I_2 = \frac{R_1}{R_1 + R_2} I$

Particle problem (2.9)

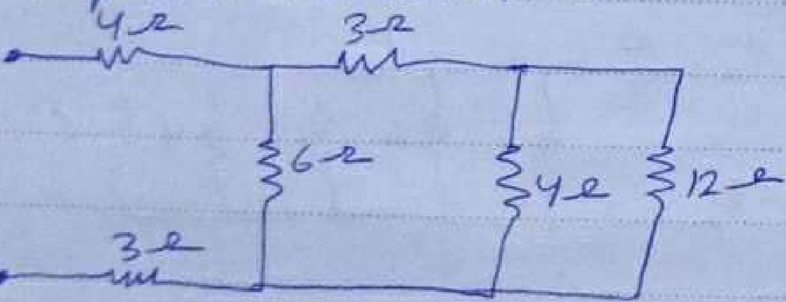


Device:

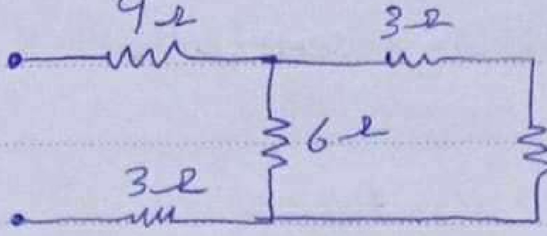
① ابدأ من اليمين

② الرسم أول بأول

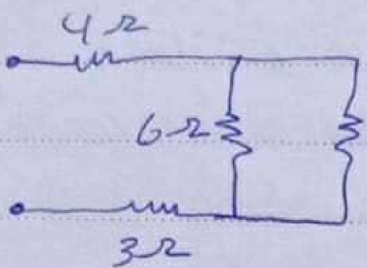
Requ.① $= 4 + 5 + 3 = 12\,\Omega$



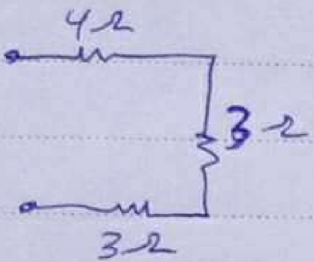
Requ.② $= 12 // 4 = \frac{12 \times 4}{12 + 4} = 3\,\Omega$



$R_{equ.}$ ③ $= 3 + 3 = 6\,\Omega$

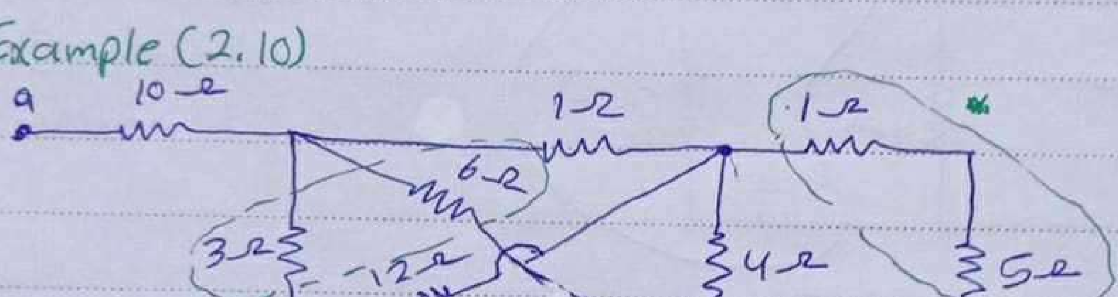


$R_{equ.}$ ④ $= \dfrac{6 \times 6}{6+6} = 3\,\Omega$



$R_{equ.}$ ⑤ $= 4 + 3 + 3 = 10\,\Omega$

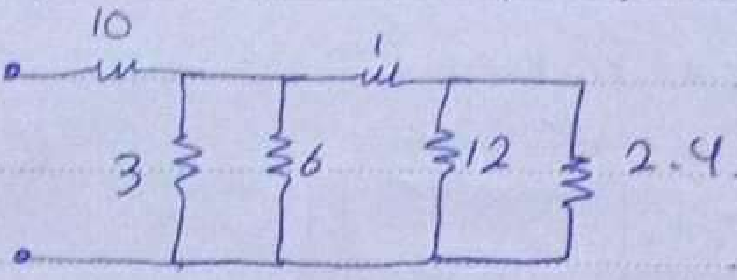
Example (2.10)



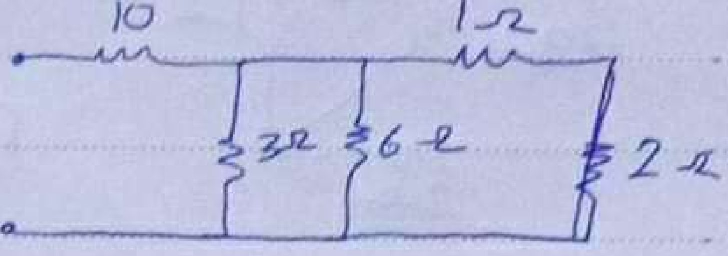
$R_{equ.}$ ① $= 5 + 1 = 6\,\Omega$

10Ω

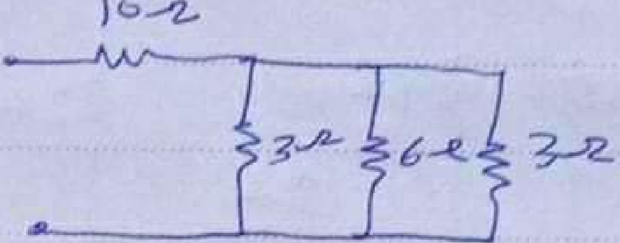
Requ. ② $= \dfrac{6 \times 4}{6+4} = 2.4\,\Omega$



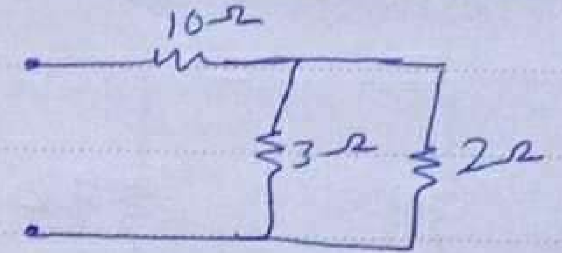
Requ. ③ $= \dfrac{2.4 \times 12}{12+2.4} = 2\,\Omega$



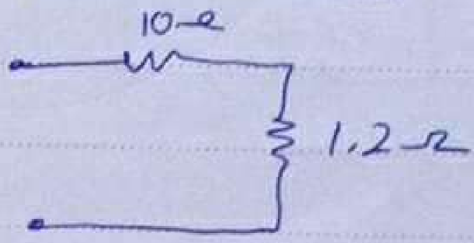
Requ. ④ $= 2 + 1 = 3\,\Omega$



Requ. ⑤ $= \dfrac{3 \times 6}{3+6} = 2\,\Omega$

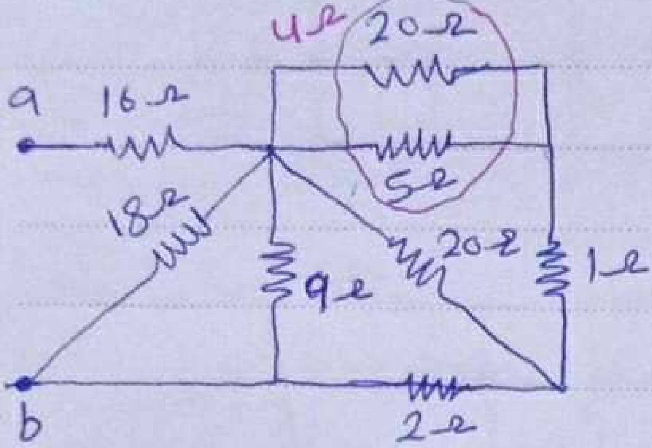


Requ. ⑥ $= \dfrac{2 \times 3}{2+3} = 1.2\,\Omega$



Requ. ⑦ $= 10 + 1.2 = 11.2\,\Omega$

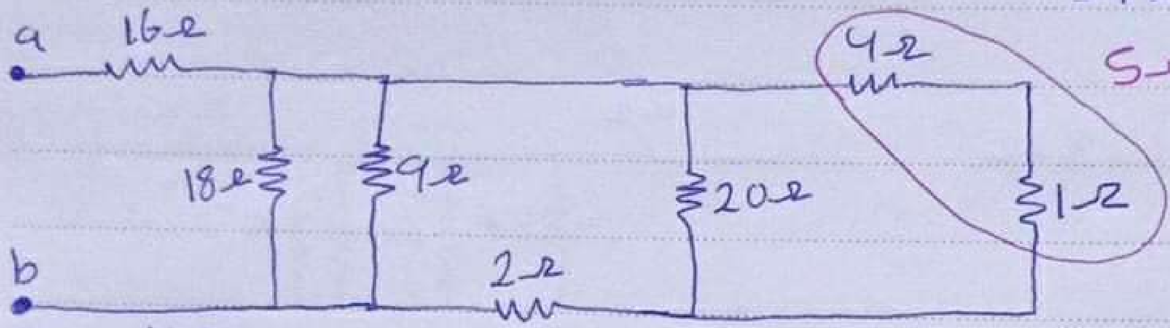
Practice Problem (2.10)

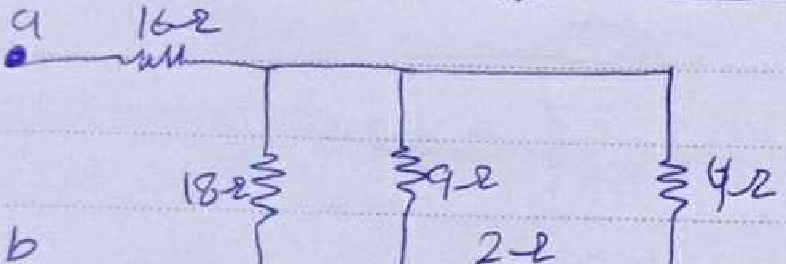


$$R_{equ.①} = \frac{20 \times 5}{20+5} = 4\,\Omega$$

$$R_{equ.②} = 4 + 1 = 5\,\Omega$$

$$R_{equ.③} = \frac{5 \times 20}{5+20} = 4\,\Omega$$



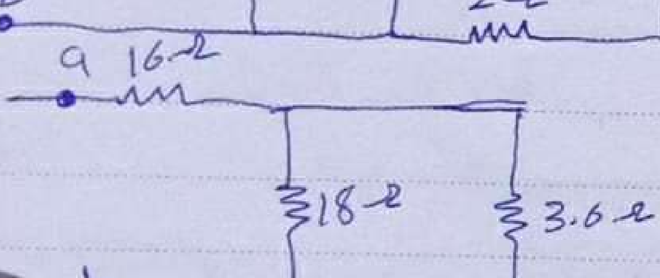


$$R_{equ.④} = 4 + 2 = 6\,\Omega$$

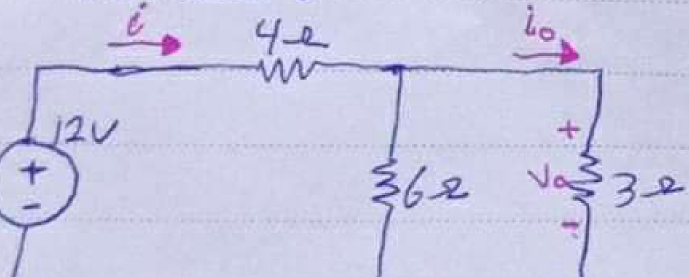
$$R_{equ.⑤} = \frac{6 \times 9}{6+9} = 3.6\,\Omega$$

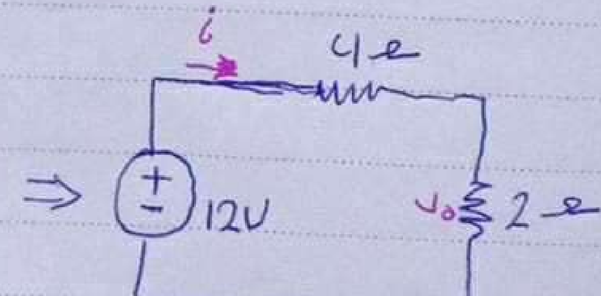
$$R_{equ.⑥} = \frac{3.6 \times 18}{3.6+18} = 3\,\Omega$$

$$R_{equ.⑦} = 3 + 16 = 19\,\Omega$$



Example (2.12): Find i_o and v_o. Calculate the power in $3\,\Omega$.





/ /

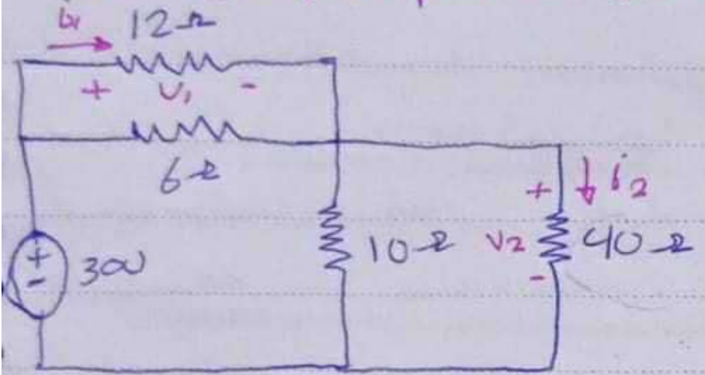
voltage division $\Rightarrow$ $V_o = \frac{2 \times 12}{2+4} = 4$ volt.

$i = \frac{V}{R_{equ}} = \frac{12}{6} = 2A$

$V_o = I_o R \Rightarrow i_o = \frac{V_o}{R} = \frac{4}{3} A$

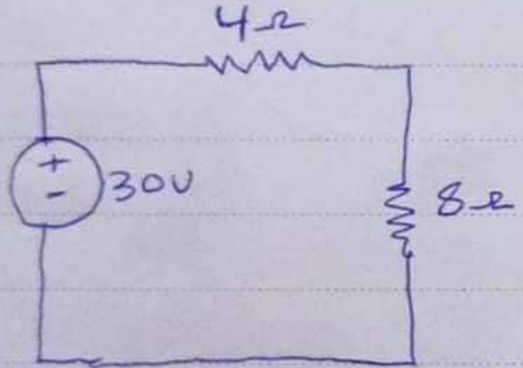
$P_o = V_o i_o = 4 \times \frac{4}{3} = 5.333W$

Particle ~~past~~ problem (2.12):



$R_{equ.①} = \frac{40 \times 10}{40+10} = 8\Omega$

$R_{equ.②} = \frac{12 \times 6}{12+6} = 4\Omega$



voltage devision on seriess:

$V_1 = \frac{30 \times 4}{12} = 10$ volt $\Rightarrow$ same in 12Ω

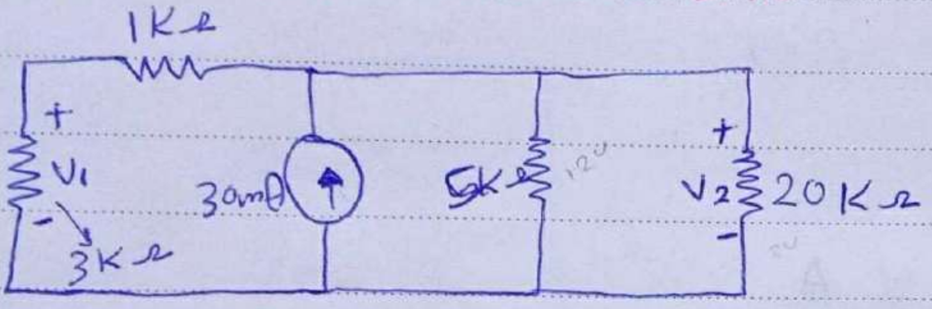
$V_2 = \frac{30 \times 8}{12} = 20$ volt $\rightarrow$ same in 40Ω

$V = IR \Rightarrow I = V/R$

$i_1 = 10/12 = 0.833A \rightarrow P_1 = i_1 V_1 = 10 \times 0.833 = 8.3W$

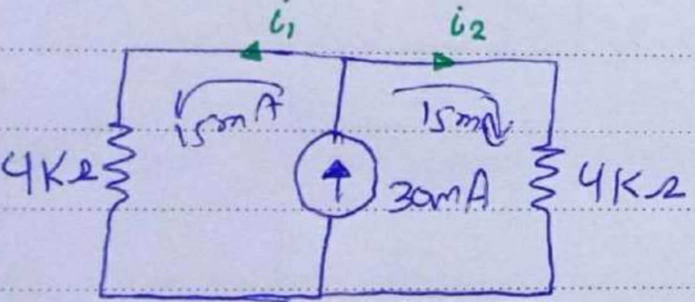
$i_2 = 20/40 = 0.5A \rightarrow P_2 = i_2 V_2 = 0,5 \times 20 = 10W$

Practice Problem (2.13): → 76



$$R_{equ.}① = \frac{20 \times 5}{20+5} = 4K\Omega$$

$$R_{equ.}② = 3+1 = 4K\Omega$$



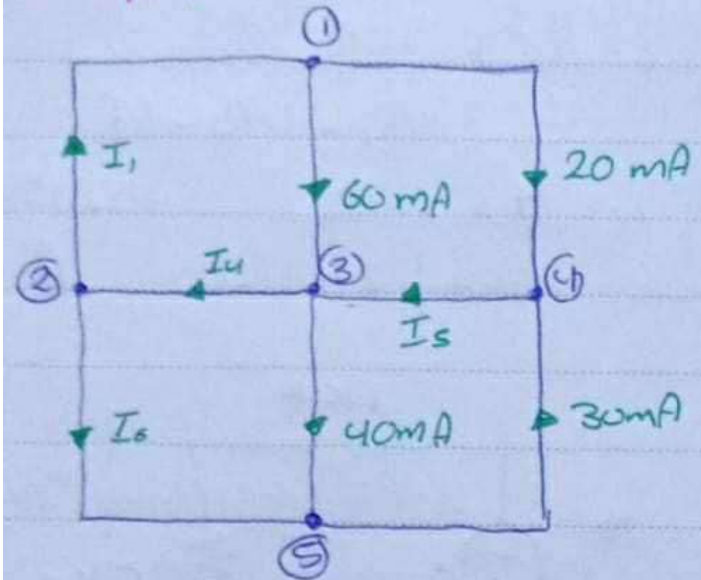
$$i_1 = \frac{30 \times 4}{8} = 15\,A$$

$$i_2 = \frac{30 \times 4}{8} = 15\,A.$$

$$V_1 = i_1 R = 15 \times 3 = 45V$$

$$V_2 = i_2 R = 15 \times 4 = 60V$$

Example:



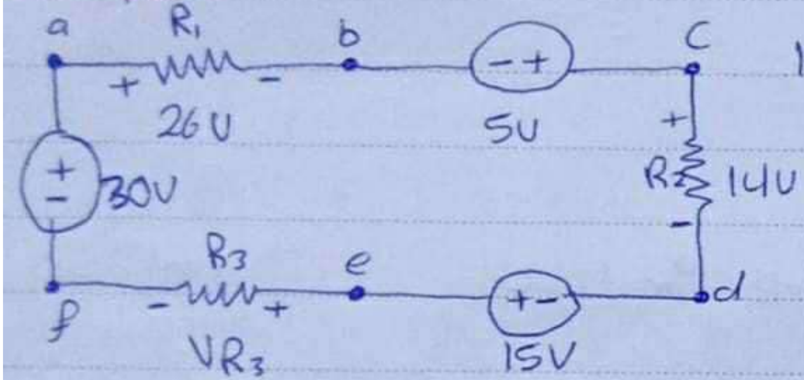
$I_5 = 50 mA = 20 + 30$

$I_4 = 60 + 50 - 40 = 70 mA$

$I_1 = 20 + 60 = 80 mA$

$I_6 = 70 - 80 = -10 mA$

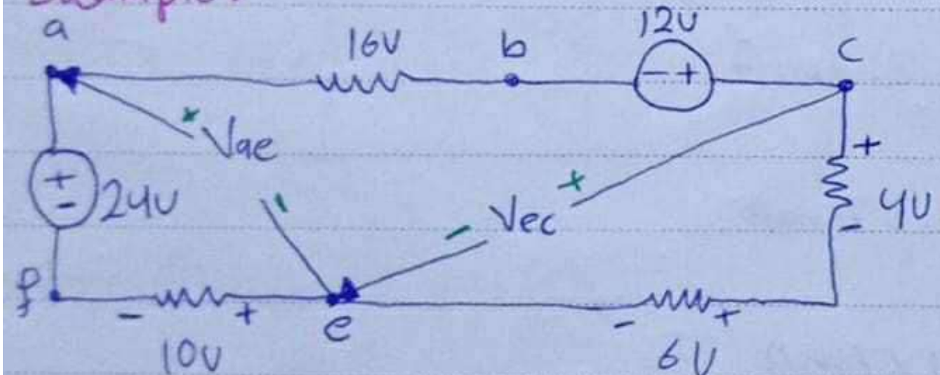
Example:



KVL $\Rightarrow 26 - 5 + 14 - 15 + V_{R_3} - 30 = 0$

$V_{R_3} = 10$ volt.

Example:



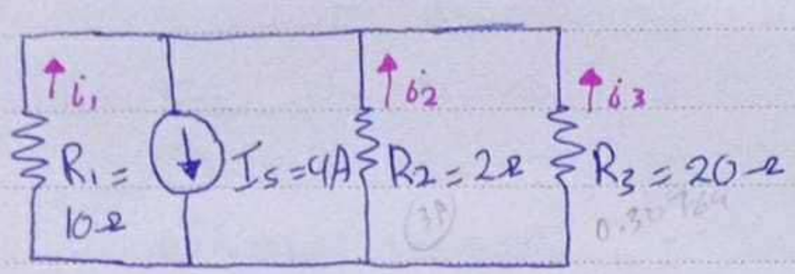
KVL $\Rightarrow -24 + V_{ae} + 10 = 0$

$V_{ae} = 14$ volt

KVL $\Rightarrow 4 + 6 - V_{ec} = 0$

$V_{ec} = 10$ volt

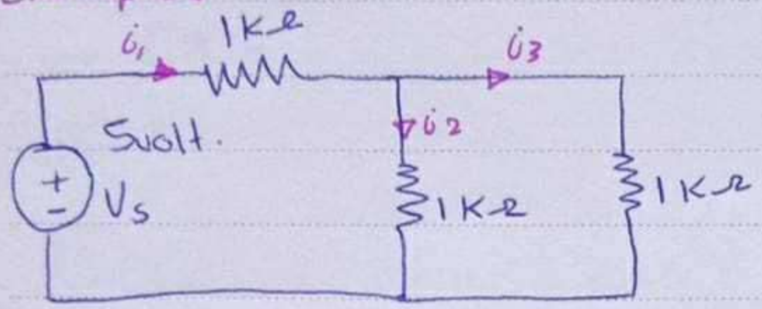
Example:



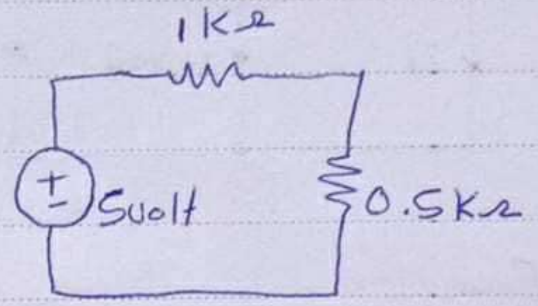
$$i = I_s \times \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3}$$

$$i_1 = \frac{4 \times 1/10}{1/10 + 1/2 + 1/20} = 0.6154 \text{ A.}$$

Example:



⇒



$$v_1 = \frac{5 \times 1}{1.5} = 3.3 \text{ volt}$$

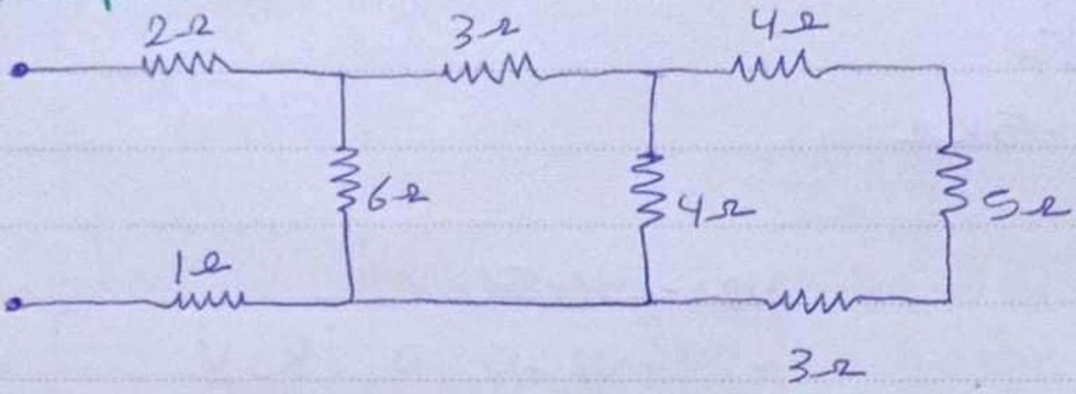
$$v_2 = \frac{5 \times 0.5}{1.5} = 1.67 \text{ volt} \Rightarrow \text{volt for } 1K\Omega$$

$$i_1 = \frac{v_1}{R_1} = \frac{3.3}{1 \times 10^3} = 3.3mA$$

$$i_2 = \frac{v_2}{R_2} = \frac{1.67}{1 \times 10^3} = 1.67 \text{ mA.}$$

$$i_3 = \frac{v_3}{R_3} = \frac{1.67}{1 \times 10^3} = 1.67mA.$$

Example:



Requ.① $= 4 + 5 + 3 = 12\,\Omega$

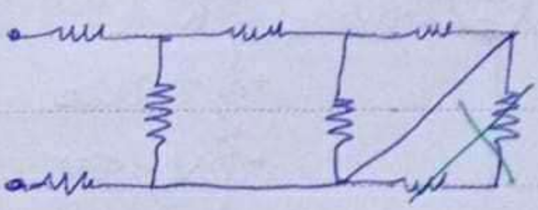
Requ.② $= \dfrac{12 \times 4}{12 + 4} = 3\,\Omega$

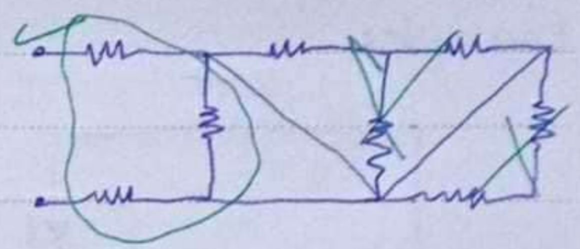
Requ.③ $= 3 + 3 = 6\,\Omega$

Requ.④ $= \dfrac{6 \times 6}{6 + 6} = 3\,\Omega$

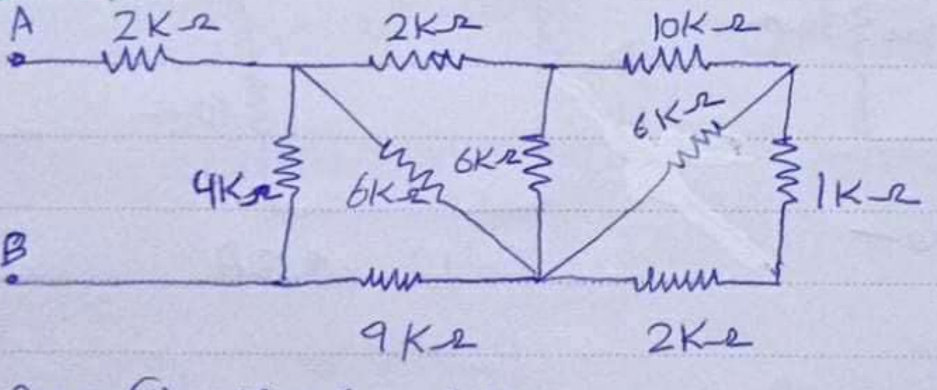
Requ.⑤ $= 3 + 1 + 2 = 6\,\Omega$

Treak $\Rightarrow$





Example:

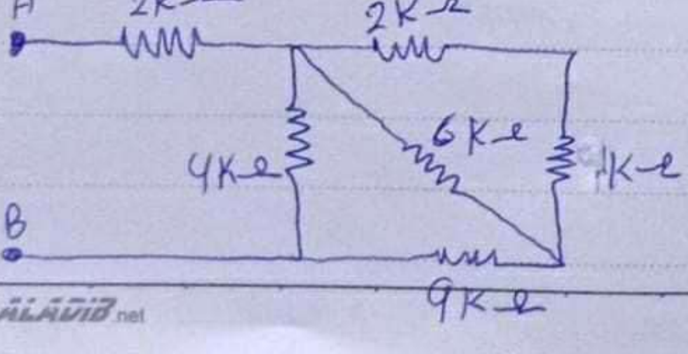


Requ.① $= 2 + 1 = 3\,K\Omega$

Requ.② $= \dfrac{3 \times 6}{6 + 3} = 2\,K\Omega$

Requ.③ $= 10 + 2 = 12\,K\Omega$

Requ.④ $= \dfrac{12 \times 6}{6 + 12} = 4\,K\Omega$



Requ.⑤ $= 4 + 2 = 6\,K\Omega$

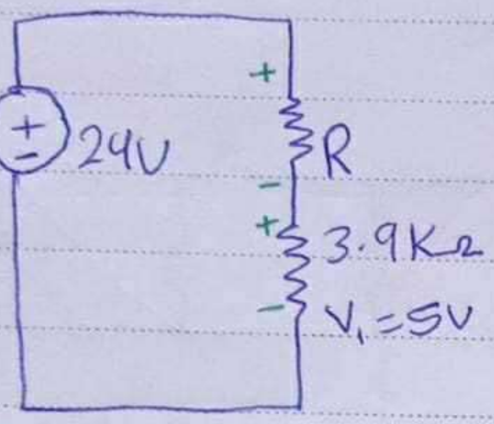
Requ.⑥ $= \dfrac{6 \times 6}{6 + 6} = 3\,K\Omega$

Requ.⑦ $= 3 + 9 = 12\,K\Omega$

$R_{equ.}(8) = \frac{\cancel{13.2}\,12 \times 4}{4 + \cancel{13.2}\,12} = 3\,K\Omega$

$R_{equ.}(9) = 3 + 2 = 5\,K\Omega$

Examples



$\cancel{V_R = 24 + 5}$

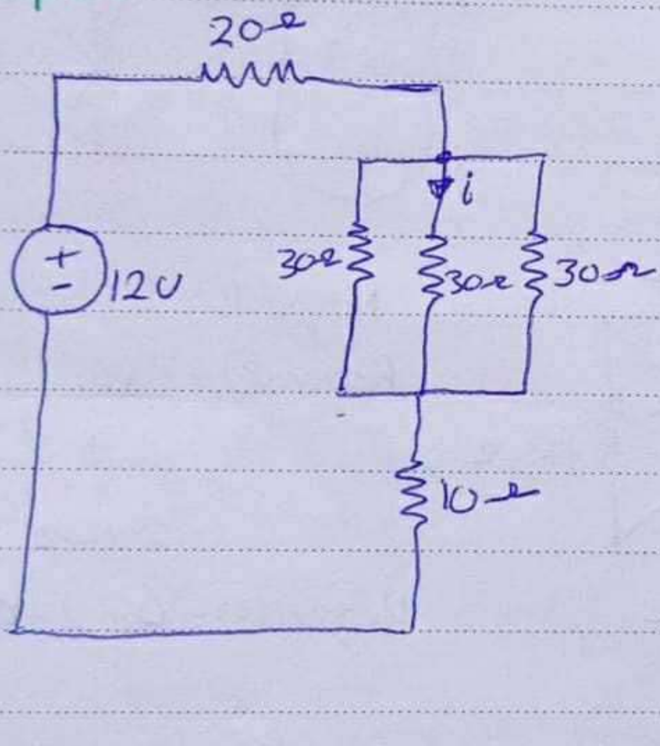
$-24 + V_R + 5 = 0$

$V_R = 19$ volt

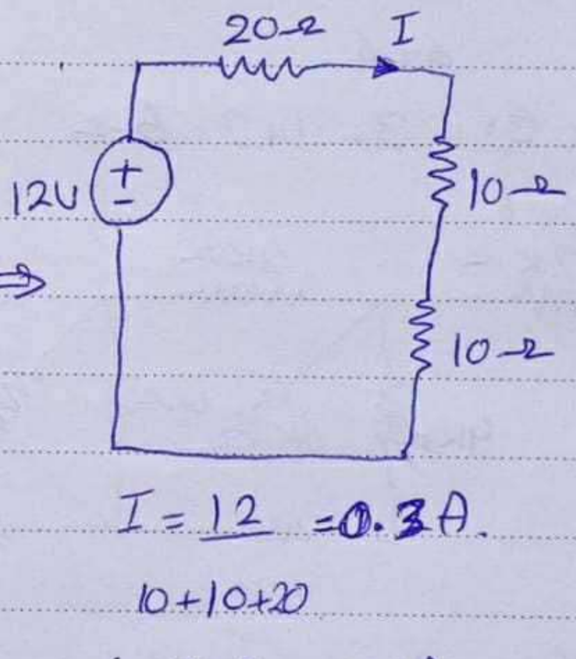
$I = \frac{5}{3.9} = 1.3\,A$

$R = \frac{V}{I} = \frac{19}{1.3} = 14.6\,K\Omega$

Example:



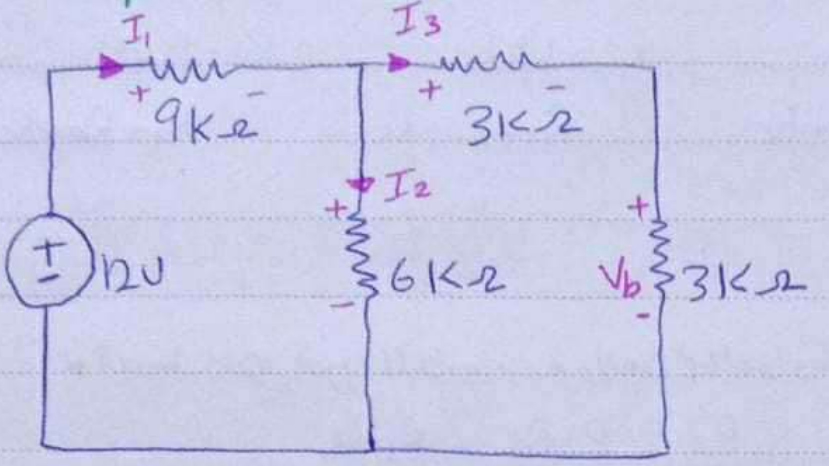
⇒



$I = \frac{12}{10+10+20} = 0.3\,A$

$i = \frac{0.3}{3} = 0.1\,A$

Example:



KVL loop ①:

$-12 + 9I_1 + 6I_2 = 0 \rightarrow ①$

KVL loop ②:

$3I_3 + 3I_3 - 6I_2 = 0$

$-6I_2 + 6I_3 = 0$ ... ②

nodes

$I_1 = I_2 + I_3$

$I_1 - I_2 + I_3 = 0$ ... ③

$I_1 = 1A$ $\quad I_2 = 0.5A$ $\quad I_3 = 0.5A$

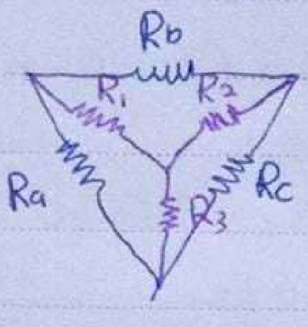
$V_b = IR = 0.5 \times 3 = 1.5$ volt.

Topic(2) → ∇ to Y conection

17 /Jul / 2025

* from ∇ to Y ⇒

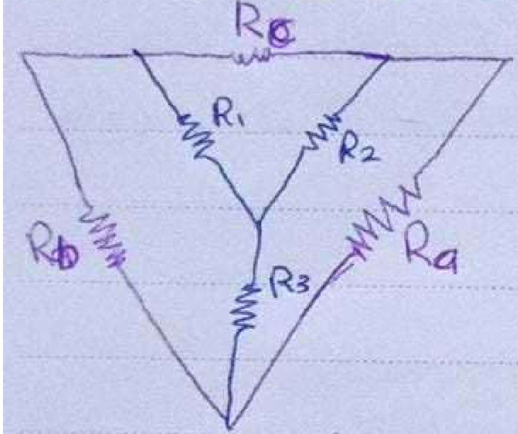
* البسط ناتج ضرب المقاومتين المجاورات مقاومة Y، والمقام جمعهم



$$R_1 = \frac{R_aR_b}{R_a+R_b+R_c}, \quad R_2 = \frac{R_bR_c}{R_a+R_b+R_c}, \quad R_3 = \frac{R_aR_c}{R_a+R_b+R_c}$$

* From Y to Δ ⇒

* البسط ناتج ضرب المقاومات، والمقام المقاومة البعيدة.



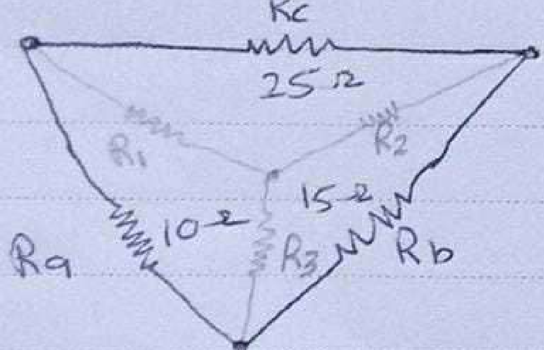
$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

* In case $R_1 = R_2 = R_3$ → $R_Y = \frac{R_\Delta}{3}$, $R_\Delta = 3R_Y$ *

Example:



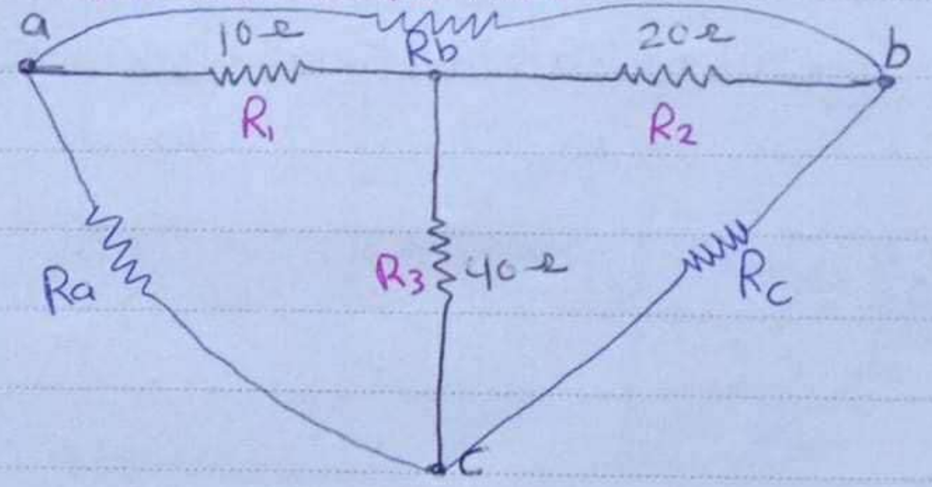
$$R_1 = \frac{R_aR_c}{R_a+R_b+R_c} = \frac{25 \times 10}{25+10+15} = 5\,\Omega$$

$$R_2 = \frac{R_bR_c}{R_a+R_b+R_c} = \frac{15 \times 25}{25+10+15} = 7.5\,\Omega$$

$$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} = \frac{15 \times 10}{25+10+15} = 3\,\Omega$$

/ /

Particle Problem (2.14):

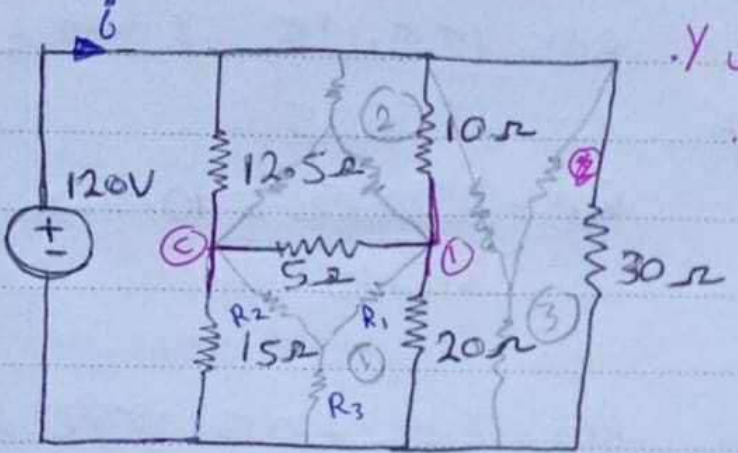


$$R_a = \frac{(10\times 40)+(10\times 20)+(20\times 40)}{20} = 70\,\Omega$$

$$R_b = \frac{1400}{40} = 35\,\Omega$$

$$R_c = \frac{1400}{10} = 140\,\Omega$$

Example (12.15):



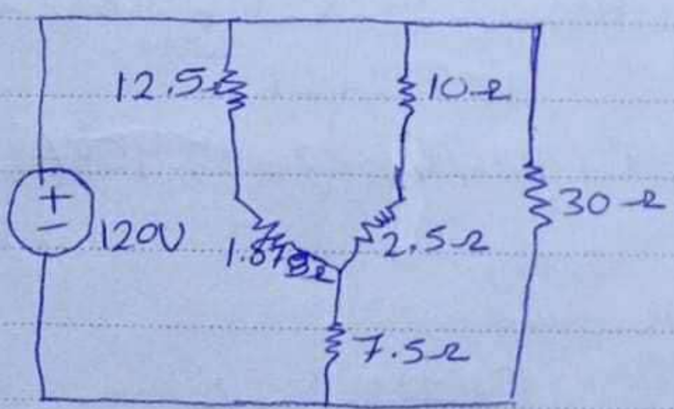
• طرق للحل ← ١ طريقة للتحويل من Δ إلى Y.

٢ طريقة تحويل من Y إلى Δ.

$$R_1 = \frac{20\times 5}{20+15+5} = 2.5\,\Omega$$

$$R_2 = \frac{5\times 15}{20+15+5} = 1.875\,\Omega$$

$$R_3 = \frac{20\times 15}{20+15+5} = 7.5\,\Omega$$



$$R_{equ.1} = 2.5 + 10 = 12.5\,\Omega$$

$$R_{equ.2} = 12.5 + 1.875 = 14.375\,\Omega$$

$$R_{equ.3} = \frac{12.5\times 14.375}{12.5+14.375} = 6.686\,\Omega$$

$$R_{equ.4} = 6.686 + 7.5 = 14.186\,\Omega$$

$$R_{equ.5} = \frac{14.186\times 30}{14.186+30} = 9.63\,\Omega$$

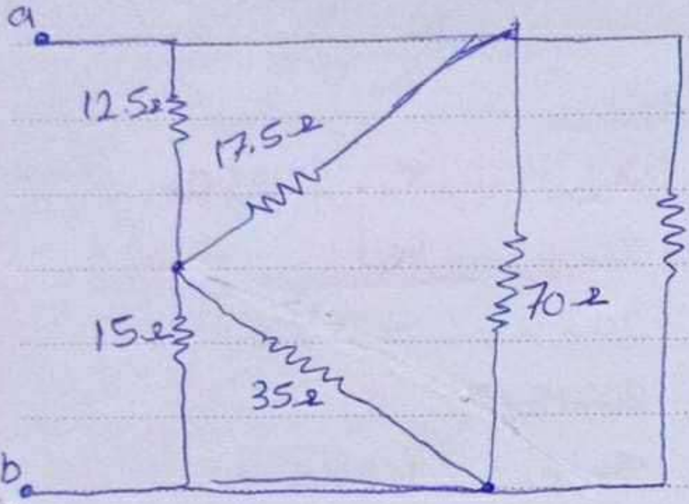
$$I = \frac{V}{R} = \frac{120}{9.63} = 12.46\,A.$$

* Conuert Y to Δ

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} = \frac{(10\times20)+(20\times5)+(5\times10)}{10} = \frac{350}{10} = 35\Omega$$

$$R_b = \frac{350}{20} = 17.5\Omega$$

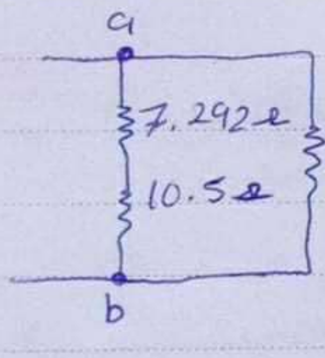
$$R_c = \frac{350}{5} = 70\Omega$$



$$R_{(1)} = \frac{30\times70}{30+70} = 21\Omega$$

$$R_{(2)} = \frac{17.5\times12.5}{17.5+12.5} = 7.292\Omega$$

$$R_{(3)} = \frac{35\times15}{35+15} = 10.5\Omega$$



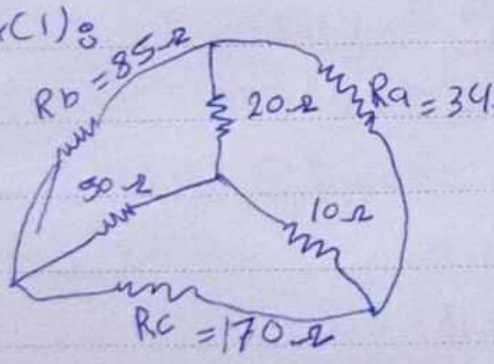
$$R_{(4)} = 7.292+10.5 = 17.792\Omega$$

$$R_{(5)} = \frac{17.792\times21}{17.792+21} = 9.632\Omega$$

$$I = 120/9.632 = 12.458A$$

مثالين بالسلايدات

Ex(1):



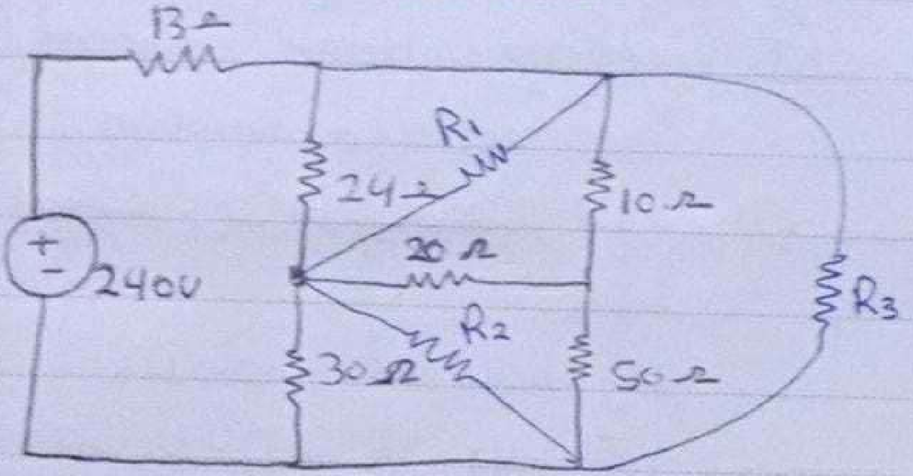
$$R_3 = \frac{144}{6} = 24K\Omega$$

Ex(2): $Y \rightarrow \Delta$

$$R_1 = \frac{(6\times1)+(4\times12)+(12\times7)}{4} = 36K\Omega$$

$$R_2 = \frac{144}{12} = 12K\Omega$$

Practice Problem (2.15):

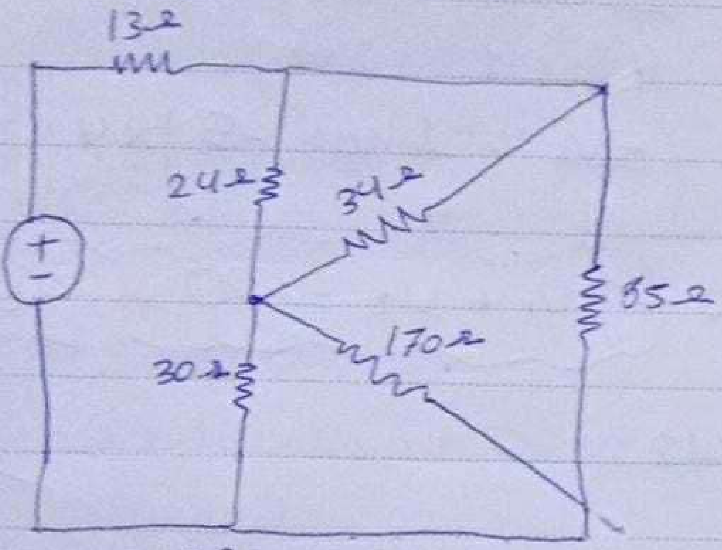


$R_1 = (10\times 50)+(50\times 20)+(10\times 20)/50 =$

$= \dfrac{1700}{50} = 34\,\Omega$

$R_2 = \dfrac{1700}{10} = 170\,\Omega$

$R_3 = \dfrac{1700}{20} = 85\,\Omega$



$R① = \dfrac{34\times 24}{34+24} = 14\,\Omega$

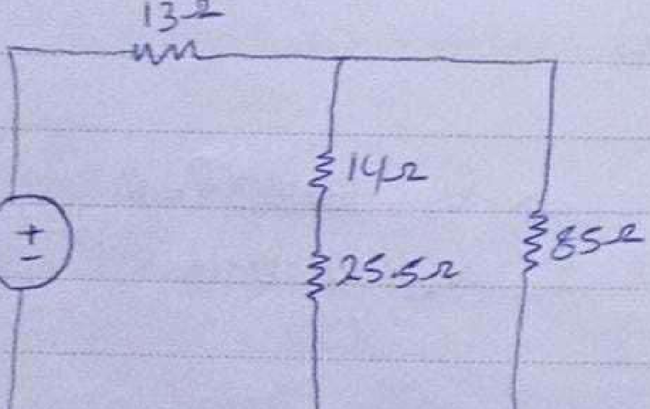
$R② = \dfrac{30\times 170}{30+170} = 25.5\,\Omega$

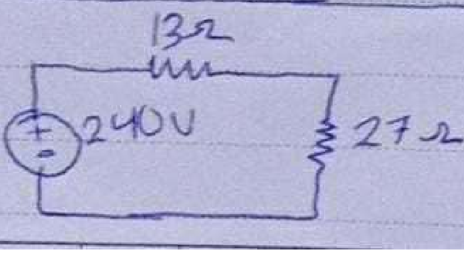
$R③ = 25.5 + 14 = 39.5\,\Omega$

$R④ = \dfrac{85\times 39.5}{85+39.5} = 27\,\Omega$

$R⑤ = 27 + 13 = 40\,\Omega$

$I = \dfrac{240}{40} = 6\,A.$





Chapter 3

CH3

/ /

Nodal analysis

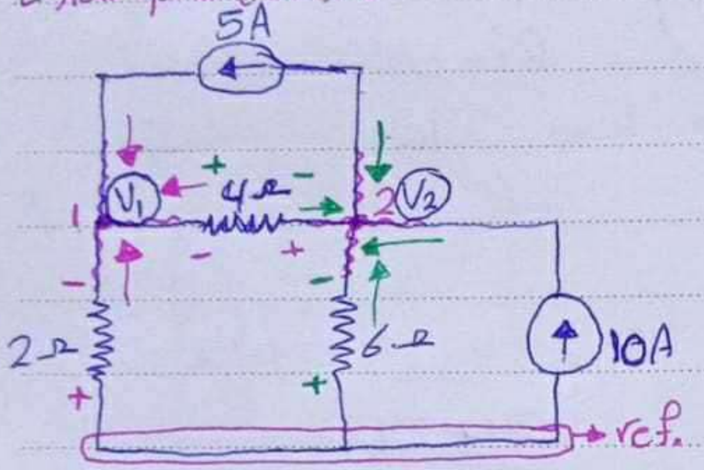
* we are interested in finding the node voltage.

→ Nodal analysis for CKTs with no voltage sources.

→ " " " " " voltage sources.

* عند فرض الاتجاه للتيار القطب الموجب عنده جهد أكبر من القطب السالب

Example (3.1) :



apply KCL at node ① :

$$5 + \frac{V_2 - V_1}{4} + \frac{0 - V_1}{2} = 0$$

$$\frac{V_2}{4} - \frac{V_1}{4} - \frac{V_1 \times 2}{2 \times 2} = -5$$

$$\left(\frac{-3V_1}{4} + \frac{1}{4} V_2 = -5\right) \times 4$$

$$-3V_1 + V_2 = -20 \quad \dots ①$$

* KCL at node ② :

$$-5 + 10 + \frac{V_1 - V_2}{4} + \frac{0 - V_2}{6} = 0$$

$$\left(\frac{1}{4} V_1 - \frac{10}{24} V_2 = -5\right) \times -12$$

$$-3V_1 + 5V_2 = 60 \quad \dots ②$$

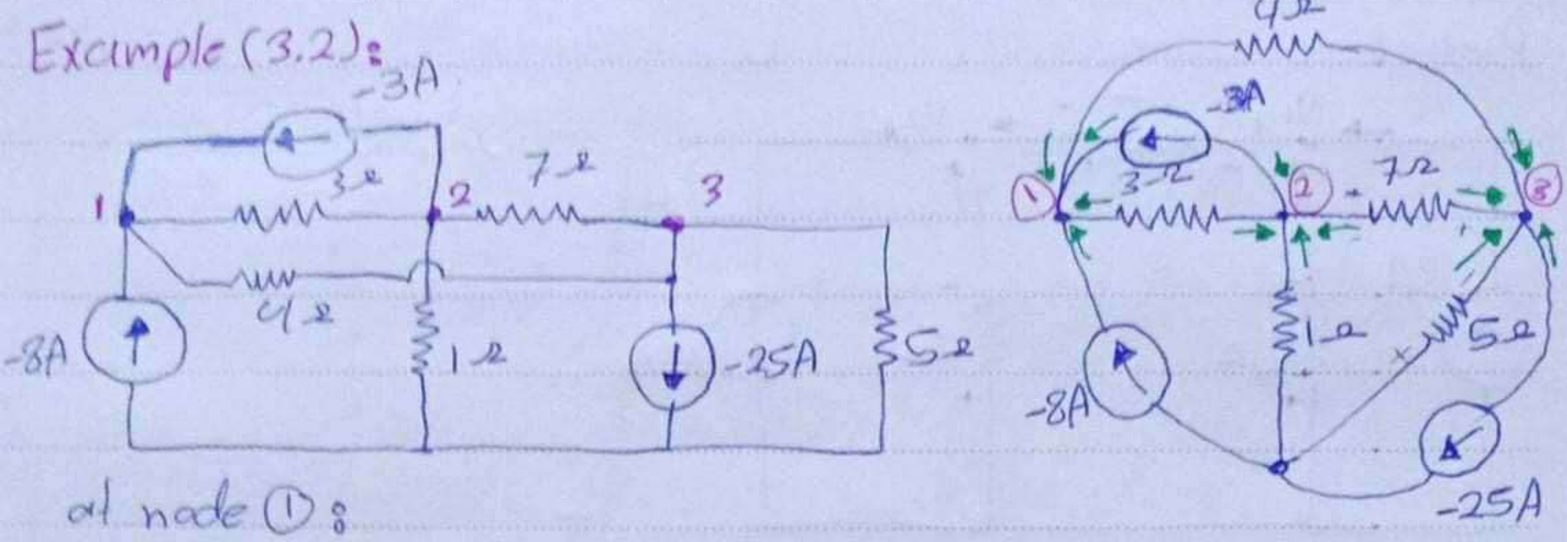
$$\begin{bmatrix} -3 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -20 \\ 60 \end{bmatrix}$$

$\Rightarrow V_1 = 13.33$ volt

$V_2 = 20$ volt.

/ /

Example (3.2):



at node ①:

$$\left(-3+-8+\frac{V_2-V_1}{3}+\frac{V_3-V_1}{4}=0\right)\times 12$$

$$-\frac{V_1}{3}-\frac{V_1}{4}+\frac{V_2}{3}+\frac{V_3}{4}=11 \Rightarrow -0.583V_1 - 0.333V_2$$

$$-0.25V_3 = 11 \sim ①$$

$$-7V_1+4V_2+3V_3=132 \ldots ①$$

at node ②:

$$3+\frac{V_1-V_2}{3}+\frac{0-V_2}{1}+\frac{V_3-V_2}{7}=0$$

$$0.333V_1 + 1.476V_2 + 0.1429V_3 = 3 \ldots ②$$

at node ③:

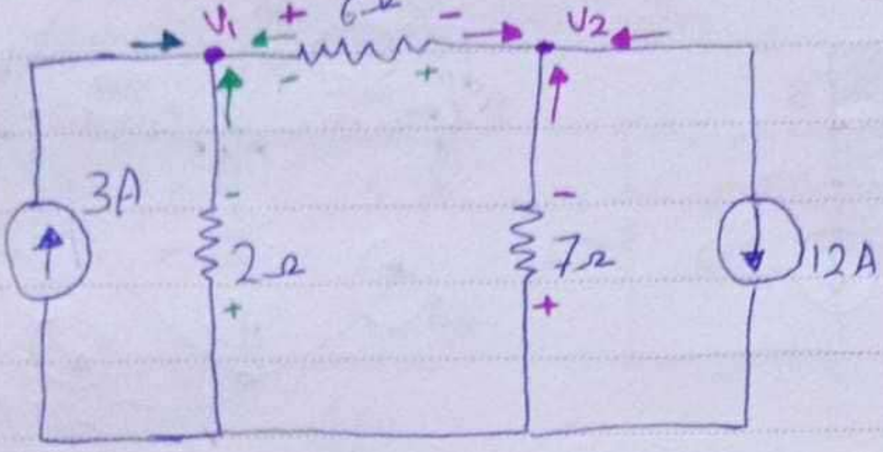
$$25+\frac{0-V_3}{5}+\frac{V_2-V_3}{7}+\frac{V_1-V_3}{4}=0$$

$$0.25V_1 + 0.1429V_2 + 0.5929V_3 = -25 \ldots ③$$

$V_1 = 5.4$ volt $\quad V_2 = 7.7$ volt $\quad V_3 = 46.3$ volt.

/ /

Particle problem (3.1): $V_1 = -6\,volt$, $V_2 = -42\,volt$.



* at node ① KCL:

$$3 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{6} = 0$$

$$\left(-\frac{V_1}{2} - \frac{V_1}{6} + \frac{V_2}{6} = -3 \right) \times -6$$

$$3V_1 + V_1 - V_2 = 18 \quad ⓧ$$

$$\boxed{4V_1 - V_2 = 18} \quad ①$$

at node ② KCL:

$$-12 + \frac{0 - V_2}{7} + \frac{V_1 - V_2}{6} = 0$$

$$\left(\frac{V_1}{6} - \frac{V_2}{6} - \frac{V_2}{7} = 12 \right) \times 42$$

$$7V_1 - 7V_2 - 6V_2 = \cancel{288}\ 504$$

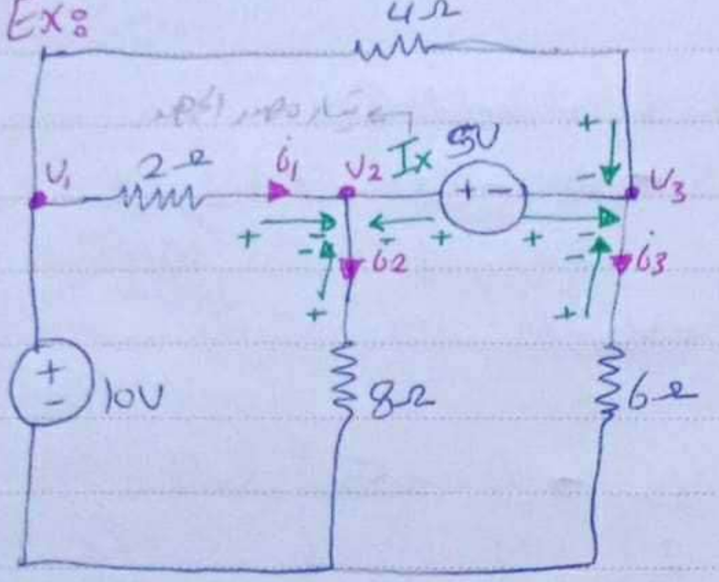
$$7V_1 - 13V_2 = \cancel{288}\ 504 \quad ②$$

$V_1 = -6\ volt$ $\quad V_2 = -42\ volt$.

Supernode

/ /

Ex:



Case 1: If a voltage source is connected between the reference node and a nonreference node, voltage equal the voltage source

Case 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, called supernode

case(1) $\Rightarrow V_1 = 10$ volt

case(2) $\Rightarrow V_2 - V_3 = 5$ volt. $\Rightarrow V_2 = 5 + V_3 \Rightarrow V_2 = 5 - 5.84$
$= -0.84$

apply KCL node ①:

$$\frac{10 - V_2}{2} + \frac{0 - V_2}{8} + \cancel{I_x} = 0$$

apply KCL node ②:

$$\frac{10 - V_3}{4} + \frac{-V_3}{6} - \cancel{I_x} = 0$$

عشان أتخلص من ال Ix أجمع المعادلتين

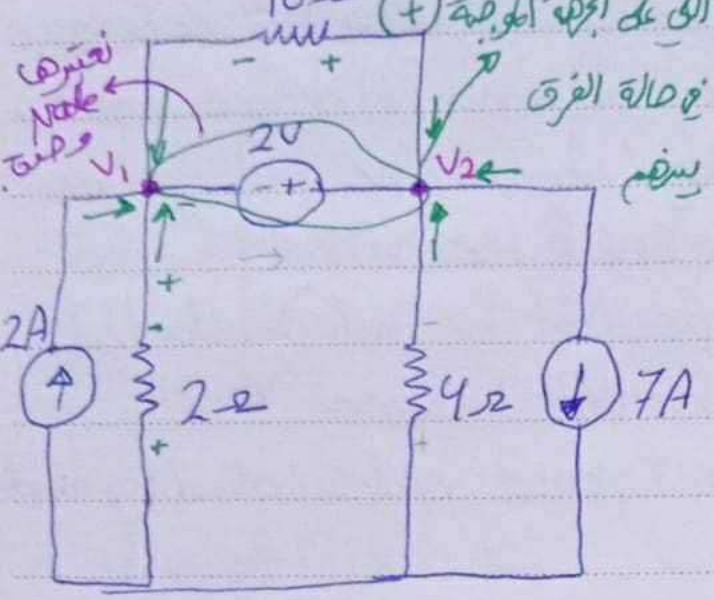
$$\frac{10 - V_2}{2} - \frac{V_2}{8} + \frac{10 - V_3}{4} - \frac{V_3}{6} = 0$$

$$\frac{10 - 5 - V_3}{2} \; \frac{-5 + V_3}{8} + \frac{10 - V_3}{4} \; \frac{-V_3}{6} = 0$$

$$\frac{5}{2} - \frac{V_3}{2} - \frac{5}{8} + \frac{V_3}{8} + \frac{10}{4} - \frac{V_3}{4} - \frac{V_3}{6} = 0$$

$$-\frac{19}{24} V_3 = \frac{37}{8} \Rightarrow V_3 = -5.84 \text{ volt}$$

Example 3.3: الي على الجهة السالبة (-)
الي على الجهة الموجبة (+)



KCL at node ①:

$$2 - \frac{v_1}{2} - \frac{v_2}{4} - 7 + \cancel{\frac{v_1 - v_2}{10}} + \cancel{\frac{v_2 - v_1}{10}} = 0$$

$$\left(-\frac{v_1}{2} - \frac{v_2}{4} = 5\right) \times -4$$

$$2v_1 + v_2 = -20 \quad ①$$

supernodes

$$v_2 - v_1 = 2 \rightarrow v_2 = v_1 + 2 \quad ②$$

$v_1 = -7.333$ volt

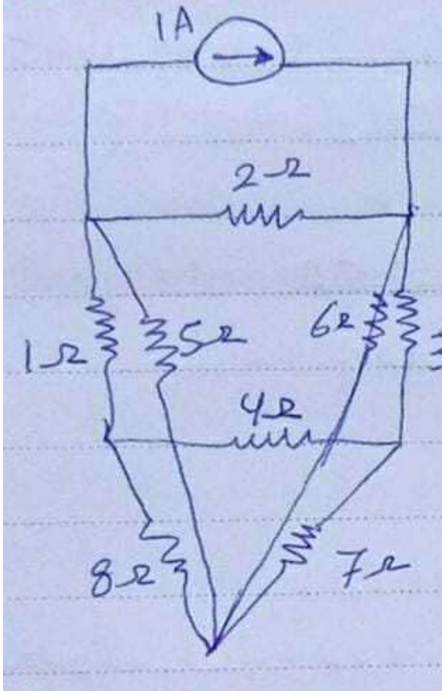
$v_2 = -5.333$ volt

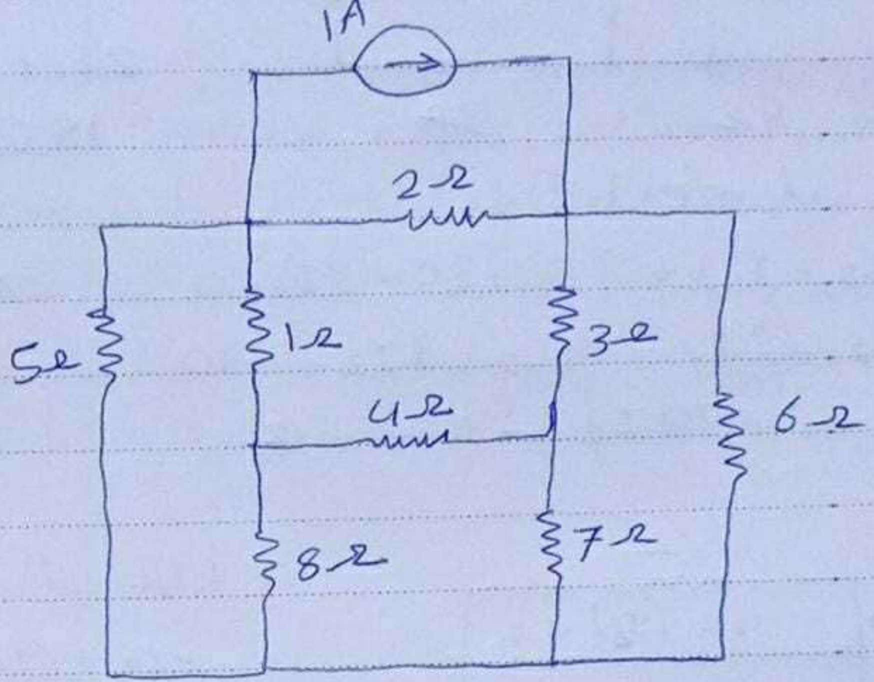
Mesh analysis.

/ /

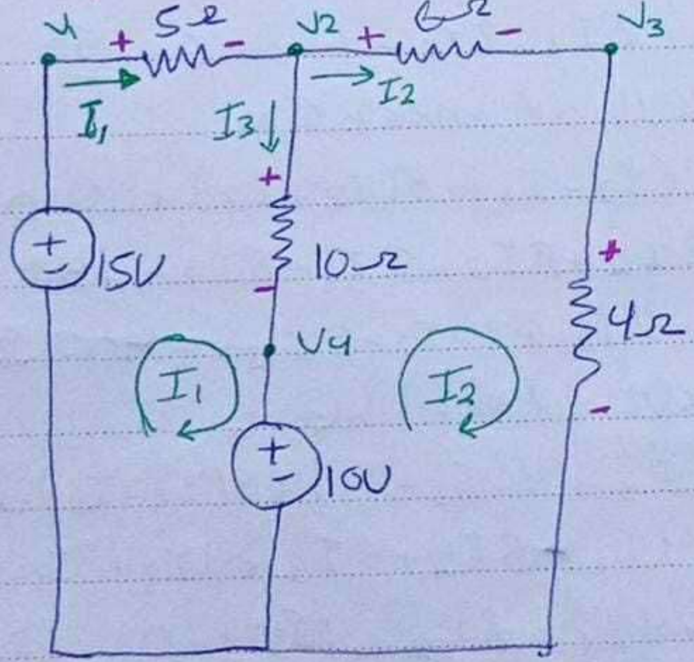
① Mesh analysis for CKTs with no current sources.
② " " " " " " "

* mesh is a loop which does not contain any other loops with in it.





Example (3.5):



KVL at mesh ①:

$-15 + 5I_1 + 10(I_1 - I_2) + 10 = 0$

$15 I_1 - 10 I_2 = 5$ ....①

KVL at mesh ②:

$6 I_2 + 4 I_2 - 10 - 10(I_2 - I_1) = 0$

$10 I_1 = 10$

$I_1 = 1A$

$15 \times 1 - 10 \times I_2 = 5$

$-10 I_2 = -10$

$I_2 = 1 A$

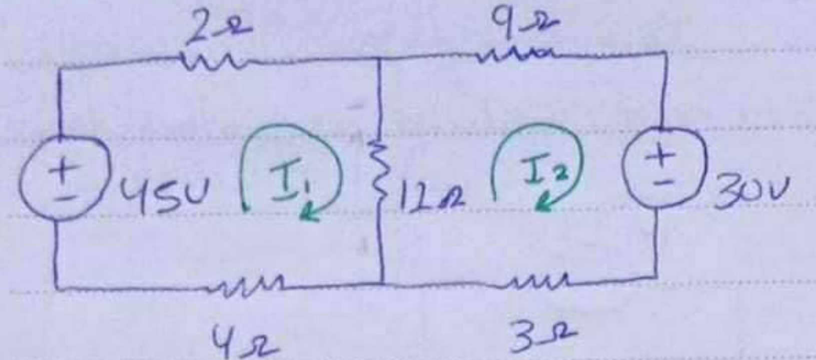
$I_3 = I_1 - I_2 = 1 - 1 = 0 A$

① تحديد اتجاه التيار المفروض في ال mesh

② تحديد ال polarty على المقاومات .

/ /

P.P(3.5)



KVL at mesh ①:

$-45 + 2I_1 + 12(I_1 - I_2)$
$+ 4I_1 = 0$

$2I_1 + 12I_1 - 12I_2 + 4I_1 = 45$

$18I_1 - 12I_2 = 45$ ⋯ ①

KVL at mesh ②:

$12(I_2 - I_1) + 9I_2 + 30 + 3I_2 = 0$

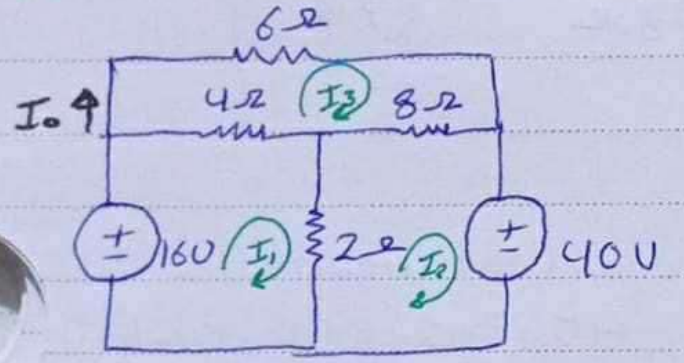
$12I_2 - 12I_1 + 9I_2 + 3I_2 = -30$

$-12I_1 + 24I_2 = -30$ ⋯ ②

$I_1 = 2.5A$

$I_2 = 0$

Ex:



$I_1 = 3A$

$I_2 = -2.1A$

$I_3 = 1.61A.$

$I_0 = 3 - -1.61 = 4.6A$

KVL at mesh ①:

$-16 + 4(I_1 - I_3) + 2(I_1 - I_2) = 0$

$4I_1 + 2I_1 - 2I_2 - 4I_3 = 16$

$6I_1 - 2I_2 - 4I_3 = 16$ ⋯ ①

KVL at mesh ②:

$2(I_2 - I_1) + 8(I_2 - I_3) + 40 = 0$

$2I_2 + 8I_2 - 2I_1 - 8I_3 = -40$

$-2I_1 + 10I_2 - 8I_3 = -40$ ⋯ ②

KVL at mesh ③:

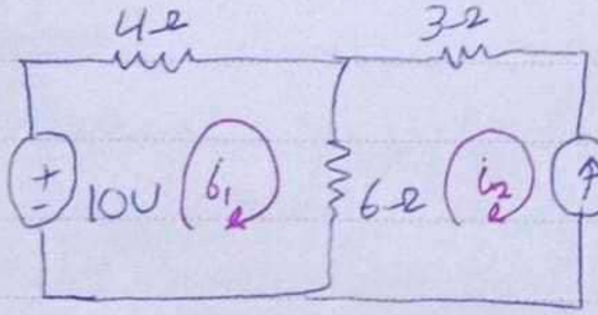
$6I_3 + 4(I_3 - I_1) + 8(I_3 - I_2) = 0$

$-4I_1 - 8I_2 + 6I_3 + 4I_3 + 8I_3 = 0$

$-4I_1 + 8I_2 + 18I_3 = 0$ ⋯ ③

Mesh analysis with current source. / /

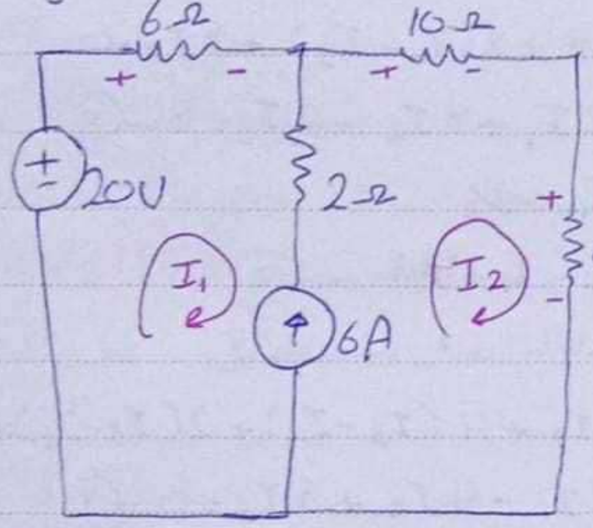
case1: when a current source exists only in one mesh



*لما بقدر أطبق KVL على mesh فيها current source

$i_2 = -5A$

case2: when a current source exists between two meshes:
We create a supermesh by excluding the current source any elements connected in series with it.

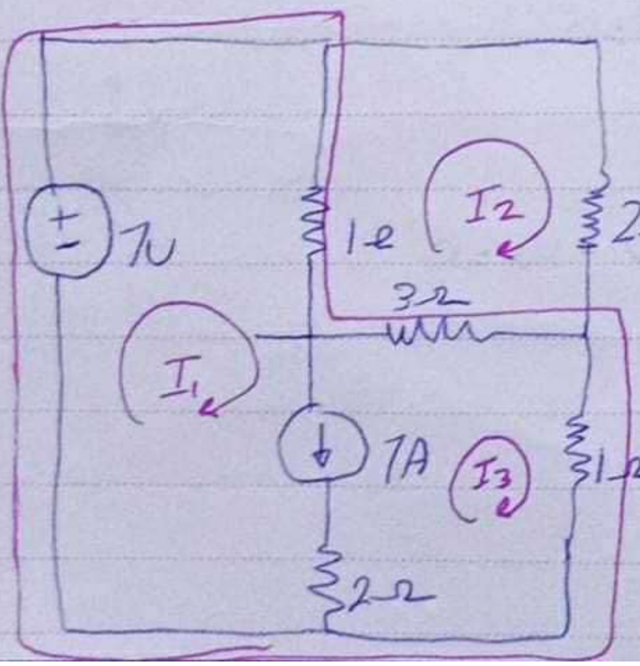


* في حالة وجود current source بين two meshes
اعتبر الـ two meshes عبارة عن one mesh وبطبق KVL.

$$-20 + 6I_1 + 10I_2 + 4I_2 = 0$$

$$I_1 - I_2 = 6$$

Ex:



طريقتين للحل:

الطريقة الأولى: أفرض Vx للـ current source و
اعمل KVL للـ mesh ① & ③ وأطلع معادلتين
وأتخلص من Vx.

الطريقة الثانية: super mesh at ① & ③

$$-7 + 1(I_1 - I_2) + 3(I_3 - I_2) + 1I_3 = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 \quad \ldots ①$$

mesh 2:

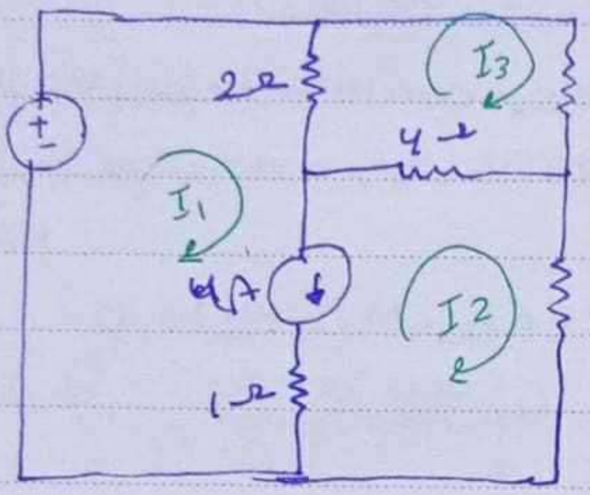
$1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$

$-I_1 + 6I_2 - 3I_3 = 0$ ~ (2)

super mesh: $I_1 - I_3 = 7$ ~ (3)

$I_1 = 9A$ $I_2 = 2.5A$ $I_3 = 2A$.

P.P(3.7): find I_1 & I_2 & I_3.



* supermesh:

$-8 + 2(I_1 - I_3) + 8I_2 = 0$

$2I_1 + 8I_2 - 2I_3 = 8$ ~ (1)

$\cancel{I_1 - I_2}$

$I_2 - I_1 = 4A$ ~ (2)

KUL at mesh (3)

$2I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$

$-2I_1 - 4I_2 + 8I_3 = 0$ ~ (3)

KUL at mesh (1):

$-8 + 2(I_1 - I_3) + V_x + 1(I_1 - I_2) = 0$

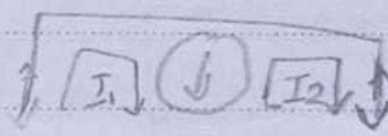
$3I_1 - I_2 - 2I_3 + V_x = 8$

KUL at mesh (2):

$4(I_2 - I_3) + 8I_3 + 1(I_2 - I_1) - V_x = 0$

$-I_1 + 5I_2 + 4I_3 - V_x = 0$

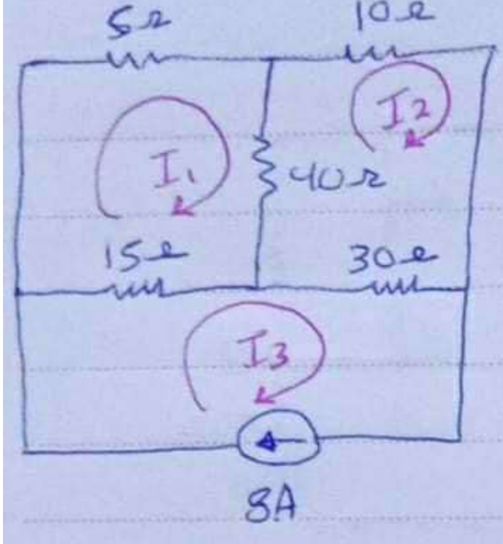
$2I_1 + 4I_2 + 2I_3 = 0$



$I_1 - I_2 = 4A$

CH3

Example



① By inspection $\Rightarrow I_3 = 8A$

② KVL at mesh #①:

$5I_1 + 40(I_1 - I_2) + 15(I_1 - 8) = 0$

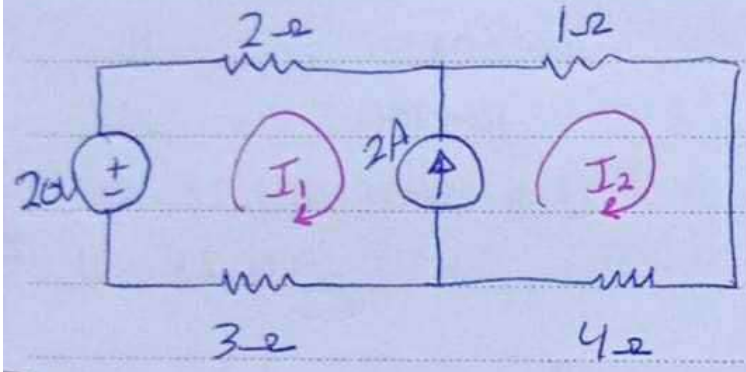
$60I_1 - 40I_2 = 120$ ~ ①

③ KVL at mesh #②:

$10I_2 + 30(I_2 - 8) + 40(I_2 - I_1) = 0$

$-40I_1 + 80I_2 = 240$ ~ ②

$I_1 = 6\,A$, $I_2 = 6\,A$.



* super mesh:

$-I_1 + I_2 = 2\,A$ ~ ①

KVL at supermesh:

$-20 + 2I_1 + 1I_2 + 4I_2 + 3I_1 = 0$

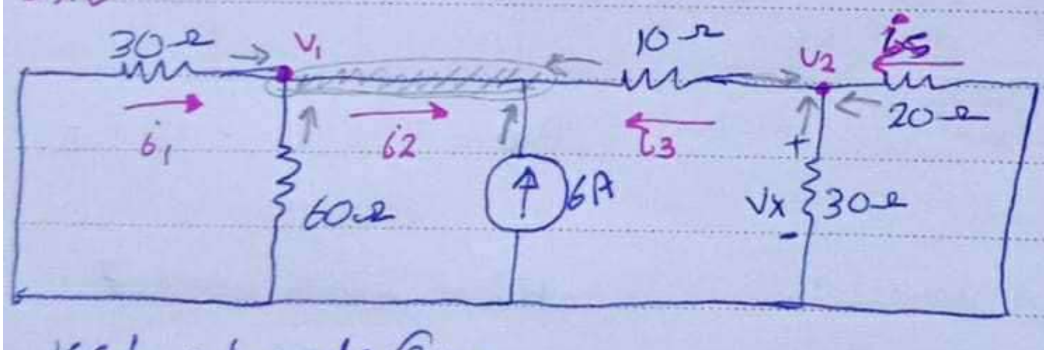
$5I_1 + 5I_2 = 20$ ~ ②

$I_2 = 3A$

$I_1 = I_2 - 2 = 3 - 2 = 1A$

$V = 15\,\text{Volt}$.

Ex:



KCL at node ②:

$\frac{V_1 - V_2}{10} + \frac{0 - V_2}{3} + \frac{0 - V_2}{20} = 0 \Rightarrow 6V_1 - 11V_2 = 0$ ~ ②

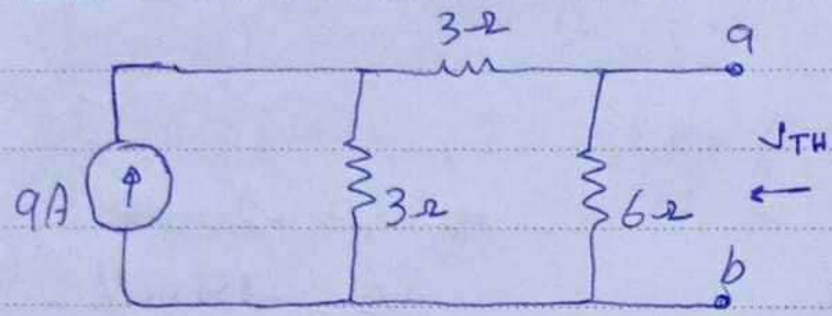
KCL at node ①:

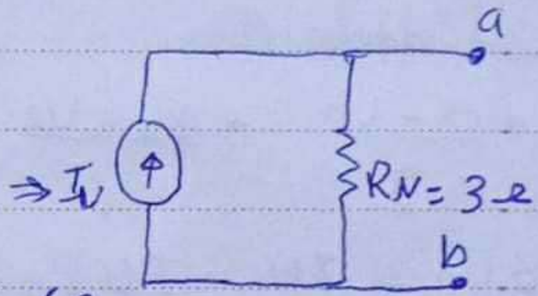
$$-\frac{V_1}{30} + \frac{V_1}{60} + \frac{V_2 - V_1}{10} + 6 = 20 \rightarrow \left(\frac{-3}{20} V_1 + \frac{V_2}{10} = \cancel{20}\ -6\right) \times 20$$

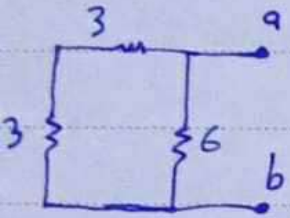
$$-3V_1 + 2V_2 = \cancel{400}\ -120 \quad ①$$

$V_1 = 62.9$ volt $\qquad V_2 = 34.2$

Ex:

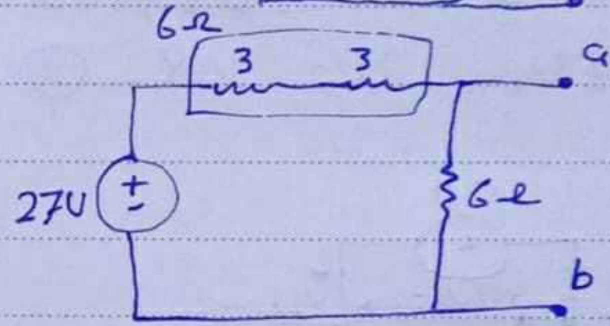






$R① = 3 + 3 = 6$

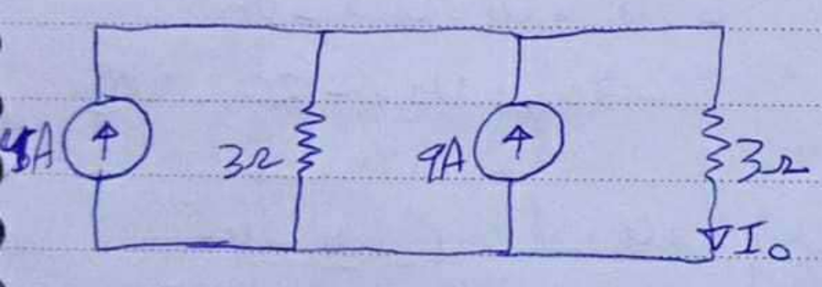
$R② = \frac{6 \times 6}{6+6} = 3Ω$

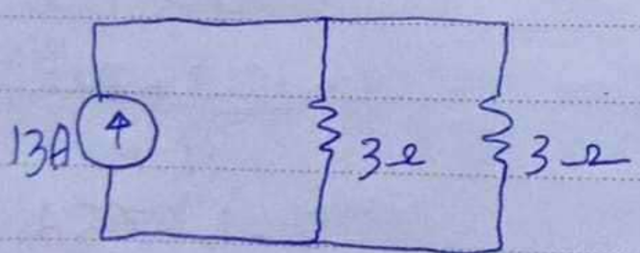


$I_N = \frac{13.5}{3} = 4.5A.$

$V_{ab} = \frac{27 \times 6}{12} = 13.5$ volt.

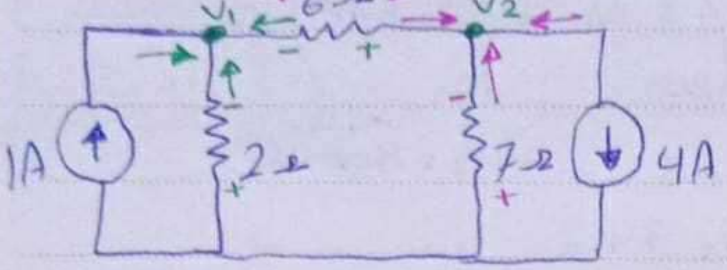
Ex:





$I_N = \frac{13.5}{1.5} = 8.7A$

Ex:



KCL at node ①:

$$\left(1 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{6} = 0\right) \times 6$$

$$-3V_1 + V_2 - V_1 = -6$$

$$-4V_1 + V_2 = -6 \quad \text{①}$$

KCL at node ②:

$$\left(-4 + \frac{0 - V_2}{7} + \frac{V_1 - V_2}{6} = 0\right) \times 42$$

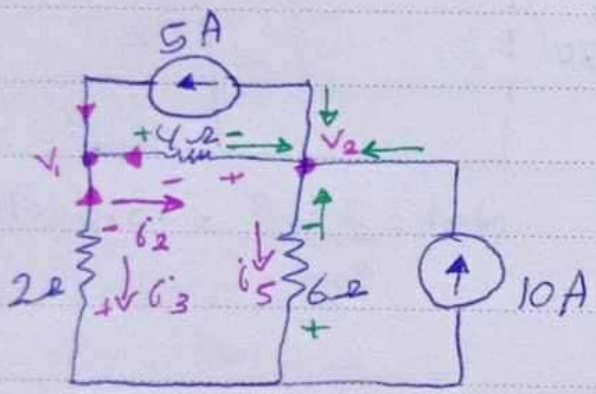
$$-6V_2 + 7V_1 - 7V_2 = 168$$

$$7V_1 - 13V_2 = 168 \quad \text{②}$$

$\Rightarrow V_1 = -2$ volt

$V_2 = -14$ volt.

Ex:



KCL at node ①:

$$\left(5 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{4} = 0\right) \times 4$$

$$-2V_1 + V_2 - V_1 = -20$$

$$-3V_1 + V_2 = -20 \quad \text{①}$$

KCL at node ②:

$$\left(10 - 5 + \frac{0 - V_2}{6} + \frac{V_1 - V_2}{4} = 0\right) \times 24$$

$$-4V_2 + 6V_1 - 6V_2 = -120$$

$$6V_1 - 10V_2 = -120 \quad \text{②}$$

$V_1 = 13.3$ volt

$V_2 = 20$ volt.

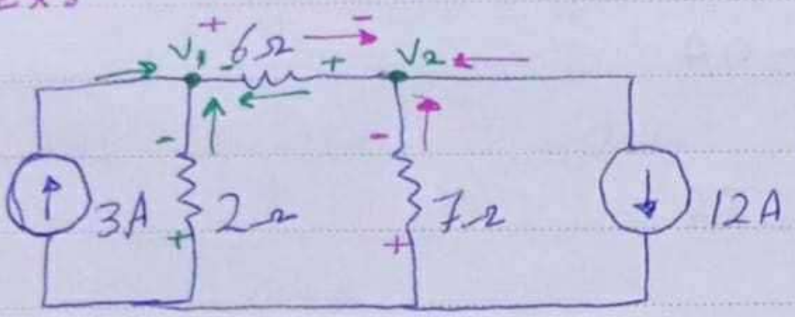
$$i_2 = \frac{V_1 - V_2}{4} = \frac{13.3 - 20}{4} = -1.675\,\text{A}$$

/ /

$i_3 = \frac{V_1 - 0}{2} = \frac{13.3}{2} = 6.65A$

$i_5 = \frac{V_2 - 0}{6} = \frac{20 - 0}{6} = 3.33A.$

Ex:



KCL at node ①:

$6\times\left(3 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{6} = 0\right)$

$-3V_1 + V_2 - V_1 = -18$

$-4V_1 + V_2 = -18$ ①

$\left(-12 + \frac{0 - V_2}{7} + \frac{V_1 - V_2}{6} = 0\right)\times 42$

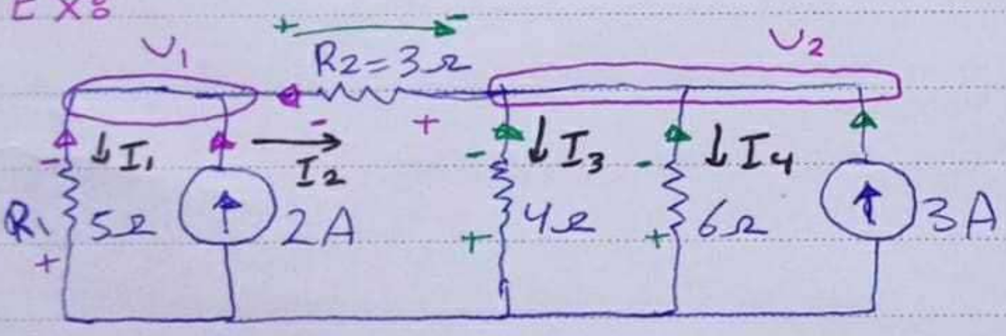
$-6V_2 + 7V_1 - 7V_2 = +504$

$7V_1 - 13V_2 = 504$ ②

$V_1 = -6$ volt

$V_2 = -42$ volt.

Ex:



KCL at node ①:

$\left(2 + \frac{0 - V_1}{5} + \frac{V_2 - V_1}{3} = 0\right)\times 15 \Rightarrow -3V_1 + 5V_2 - 5V_1 = -30$

$-8V_1 + 5V_2 = -30$ ①

KCL at node ②:

$\left(\frac{V_1 - V_2}{3} + \frac{0 - V_2}{4} + \frac{0 - V_2}{6} + 3 = 0\right)\times 12 \Rightarrow 4V_1 - 4V_2 - 3V_2 - 2V_2 + 36 = 0$

$4V_1 - 10V_2 = -36$ --- 2

$V_1 = 8$ volt $\qquad V_2 = 6.8$ volt.

$$I_1 = \frac{V_1 - 0}{5} = \frac{8-0}{5} = 1.6A$$

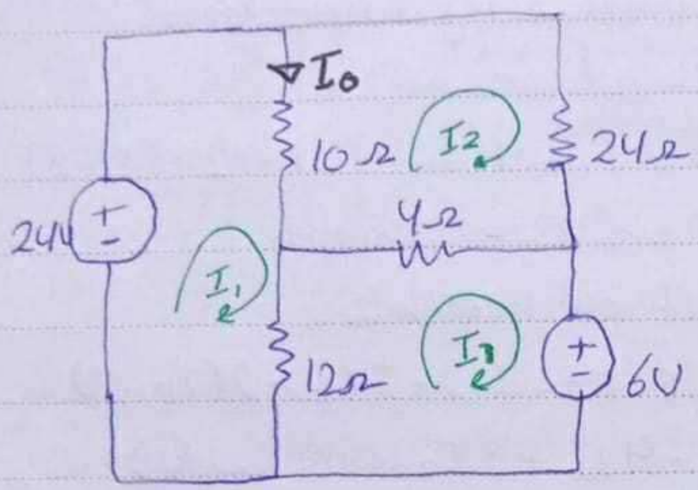
$$I_2 = \frac{V_1 - V_2}{3} = \frac{8-6.8}{3} = 0.4A$$

$$I_3 = \frac{V_2 - 0}{4} = \frac{6.8-0}{4} = 1.7A$$

$$I_4 = \frac{V_2 - 0}{6} = \frac{6.8-0}{6} = 1.13A$$

/ /

Ex:



KVL at mesh ①:

$-24 + 10(I_1 - I_2) + 12(I_1 - I_3) = 0$

$10I_1 + 12I_1 - 10I_2 - 12I_3 = 24$

$22I_1 - 10I_2 - 12I_3 = 24$ — ①

KVL at mesh ②:

$10(I_2 - I_1) + 24I_2 + 4(I_2 - I_3) = 0$

$-10I_1 + 10I_2 + 24I_2 + 4I_2 - 4I_3 = 0$

$-10I_1 + 38I_2 - 4I_3 = 0$ ... ②

KVL at mesh ③:

$12(I_3 - I_1) + 4(I_3 - I_2) + 6 = 0$

$-12I_1 - 4I_2 + 12I_3 + 4I_3 = -6$

$-12I_1 - 4I_2 + 16I_3 = -6$ — ③

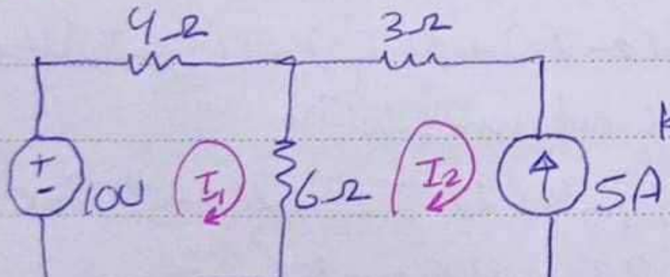
$I_1 = 2.25A$

$I_2 = 0.75A$

$I_3 = 1.5A$

$I_o = I_1 - I_2 = 2.25 - 0.75 = 1.5A$

Ex:



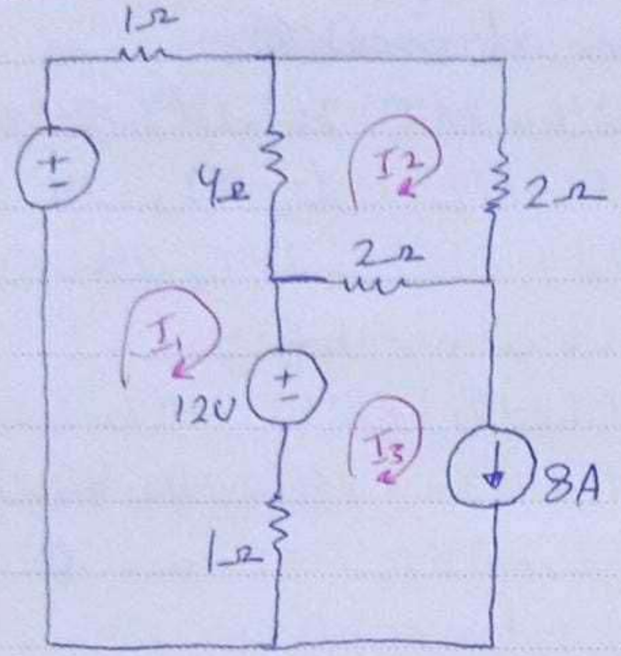
$I_2 = -5A$

KVL ⇒ $-10 + 4I_1 + 6(I_1 + 5) = 0$

$I_1 = -2A.$

/ /

Ex:



$I_3 = 8A \Rightarrow$ By Inspection

KVL at mesh ①:

$28 = I_1 + 4(I_1 - I_2) + 12 + (I_1 - 8)$

$= 6I_1 - 4I_2 + 4$ ~ ①

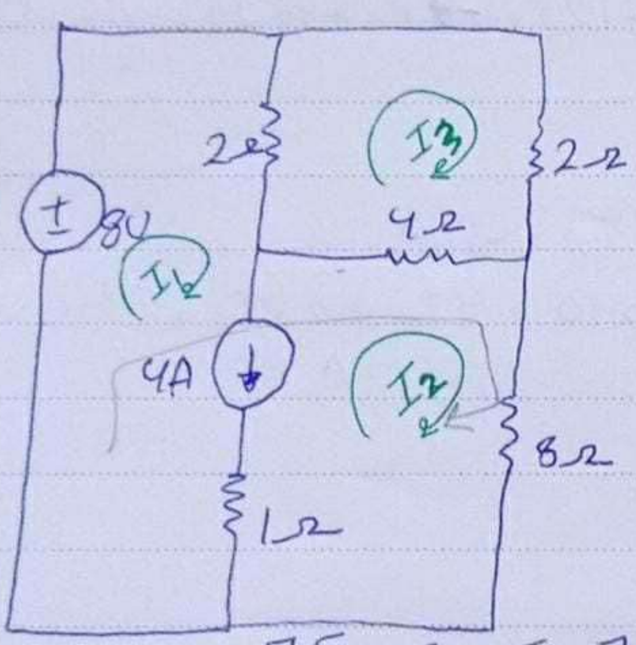
KVL at mesh ②:

$0 = 4(I_2 - I_1) + 2I_2 + 2(I_2 - 8)$

$= -4I_1 + 8I_2 - 16$ ~ ②

$I_1 = 8A \quad I_2 = 6A.$

Ex:



KVL at mesh ①:

$-8 + 2(I_1 - I_3) + V + 1(I_1 - I_2) = 0$

KVL at mesh ②:

$4(I_2 - I_3) + 8(I_2) + 1(I_2 - I_1) - V = 0$

KVL at mesh ③:

$2I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$

$-2I_1 - 4I_2 + 8I_3 = 0$

Supermesh:

$I_1 - I_2 = 4, \; -8 + 2(I_1 - I_3) + 4(I_2 - I_3) + 6I_2 = 0$

$$\begin{bmatrix} 2 & 12 & -6 \\ -2 & -4 & 8 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$$

$\Rightarrow I_1 = 4.63A$

$I_2 = 0.63A$

$I_3 = 1.47A.$

Chapter 4

Linearity Property.

/ /

* linearity is the property of an element describing a linear relationship between cause (input) and effect (response or output).
* The linearity property is a combination of both:

① the homogeneity (scaling) property $\Rightarrow \xrightarrow{x} \boxed{\quad} \rightarrow y \rightarrow$ in = out

$\xrightarrow{kx} \boxed{\quad} \rightarrow ky$

② the additivity property.

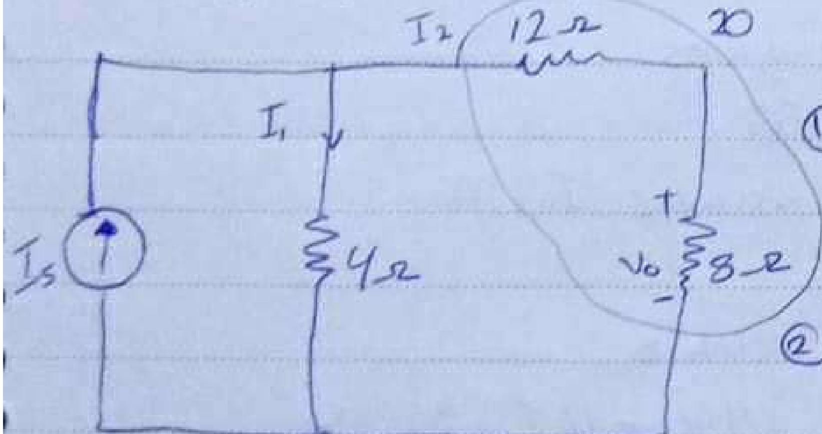
وأي تغيير
على ال in رح تنعكس على ال out بنفس النسبة

$\xrightarrow{x_1} \boxed{\quad} \rightarrow y_1$

$\xrightarrow{x_2} \boxed{\quad} \rightarrow y_2$ $\xrightarrow{x_3 = x_1 + x_2} \boxed{\quad} \rightarrow y_3 = y_1 + y_2$

* The independent source are linear elements.
* The dependent source is linear ← مرفوعة لقوة واحد if its output current or voltage is proportional only to first power of a specified current or voltage variable in the current

Ex: $I_s = 30\,A$, $I_s = 45\,A$



$R_{eq4} = 12 + 8 = 20\,\Omega$

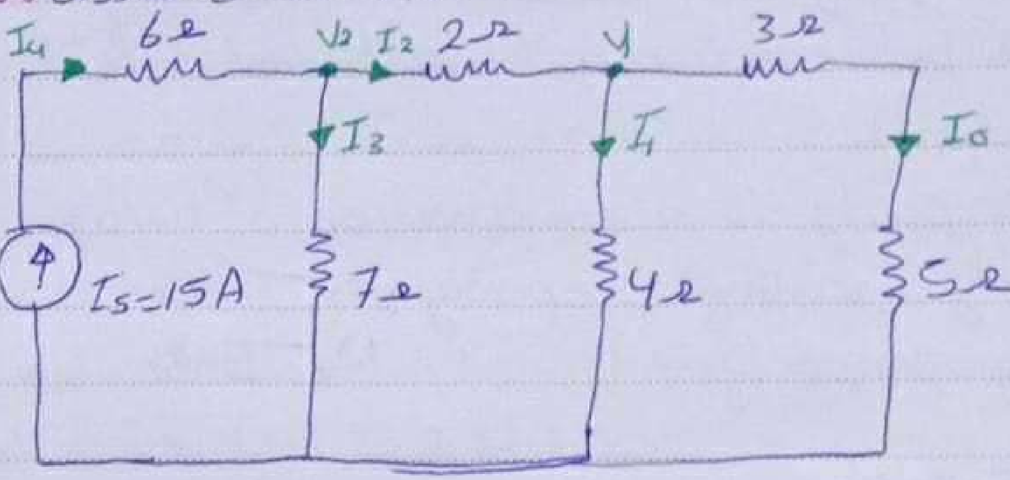
① $I_2 = \dfrac{30 \times 4}{4 + 20} = 5A$

$V_s = 5 \times 8 = 40\,volt.$

② $I_2 = \dfrac{45 \times 4}{4 + 20} = 7.5\,A.$

$V_s = 7.5 \times 8 = 60\,volt.$

Ex: assume $I_o = 1A$



apply KCL at node ①

$V_1 = (3+5) I_o$

$= 8 \text{ volt}$.

$I_1 = \frac{V_1}{4} = 2A$.

$I_2 = I_1 + I_o = 3A$

$\frac{V_2 - V_1}{2} = 3$

$\frac{V_2 - 8}{2} = 3 \Rightarrow V_2 = 14 \text{ volt}$.

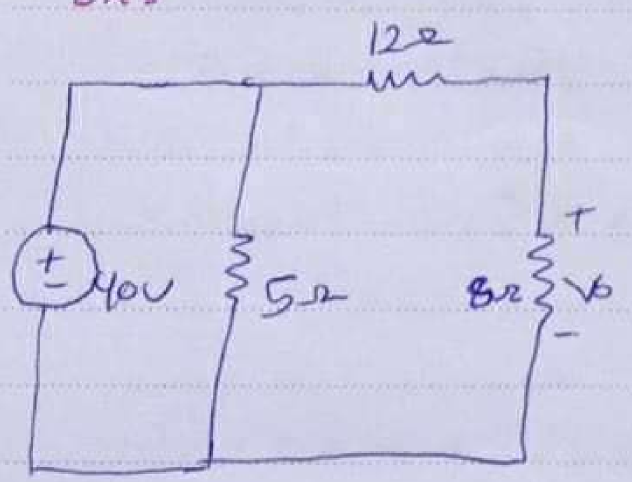
$I_3 = \frac{V_2}{7} = 2A$.

* apply KCL at node ② gives

$I_4 = I_3 + I_2 = 5A$

$I_s = 5A$ when assume $I_o = 1A$.

Ex:



KVL:

$-40 + 12I + 8I = 0$

$40 = 20I$

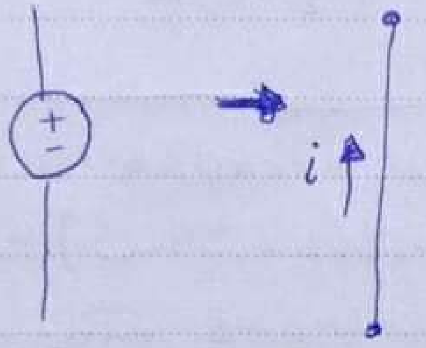
$I = 2A$

$V_o = IR = 2 \times 8 = 16 \text{ volt}$.

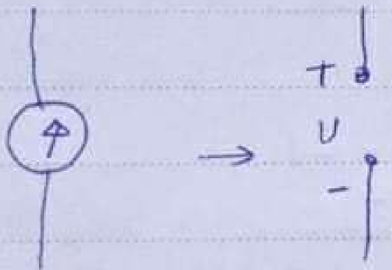
Superposition 24 / 7 / 2025

* To apply superposition must have two or more independent sources.

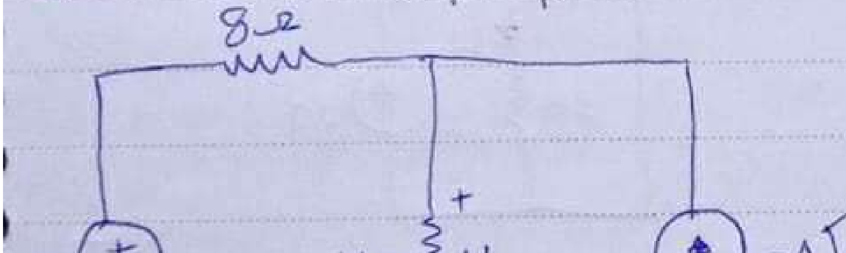
→ To Kill (Turn off) an independent voltage source, replace it by short circuit.

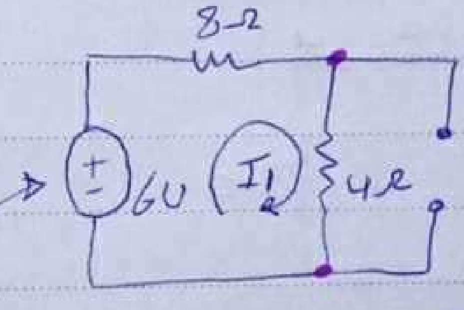


→ To Kill (Turn off) current source, replace it by Open.C

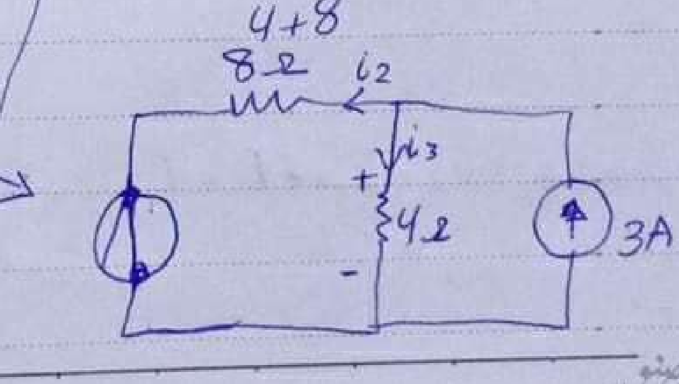


Ex: Use the superposition theorem to find v in the C.





$$v_1 = \frac{4}{4+8} \times 6 = 2 \text{ volt}$$

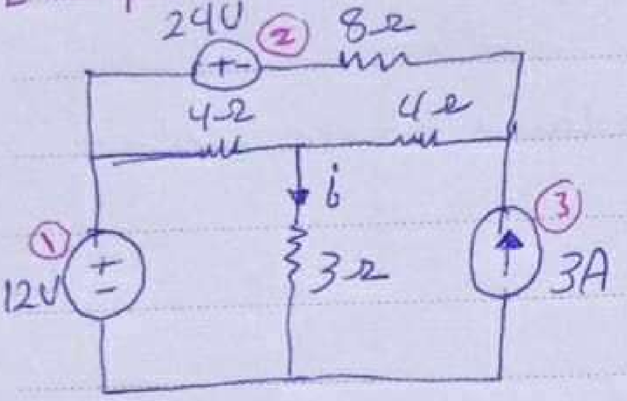


$$i_3 = \frac{8 \times 3}{8+4} = 2A$$

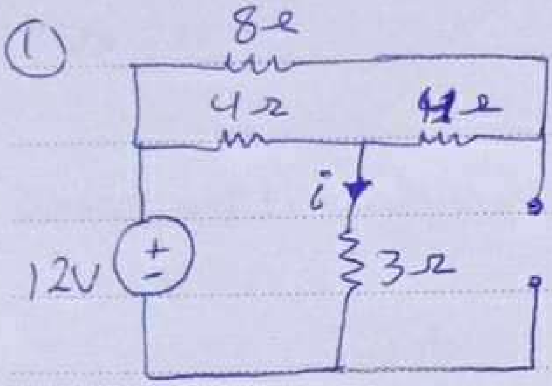
$$v_2 = 4 \times 2 = 8 \text{ volt}$$

$$v_{total} = v_1 + v_2 = 8 + 2 = 10 \text{ volt}$$

Example : Use the superposition to find i.

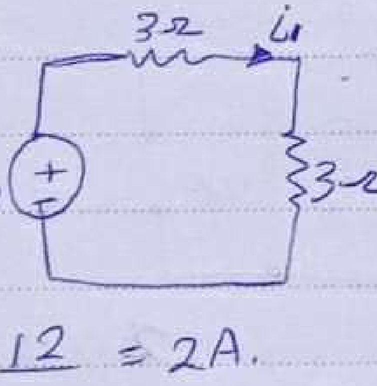


①



$R_{equ.}① = 4 + 8 = 12\Omega$

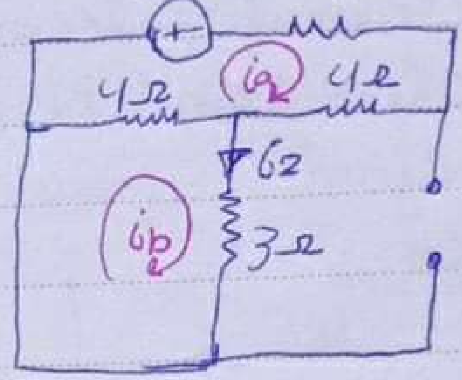
$R_{equ.}② = \frac{12 \times 4}{4 + 12} = 3\Omega$



$i_1 = \frac{12}{6} = 2A.$

$i = i_1 + i_2 + i_3 = 2 - 1 + 1$
$= 2A$

②



KVL at mesh ① :

$+4(i_a - i_b)$

$24 + 8i_a + 4i_a$

$(16 i_a - 4 i_b + 24 = 0) \div 4$

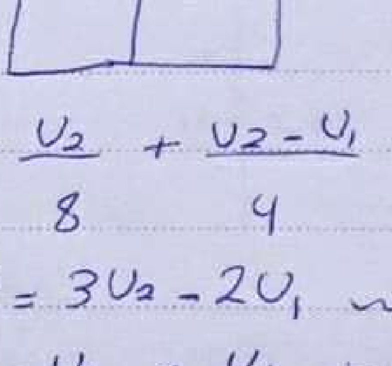
$4 i_a - i_b = -6$ ~ ①

$7 i_b - 4 i_a = 0$

$i_a = \frac{7}{4} i_b$

$i_2 = i_b = -1$

③



$3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4}$

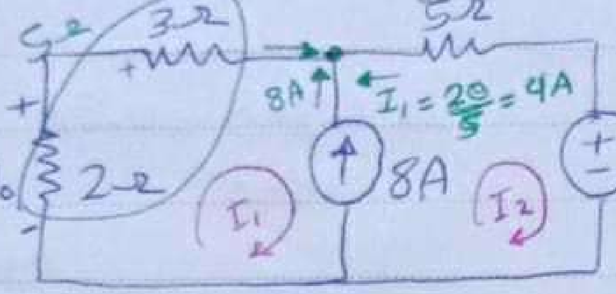
$24 = 3V_2 - 2V_1$ ~ ①

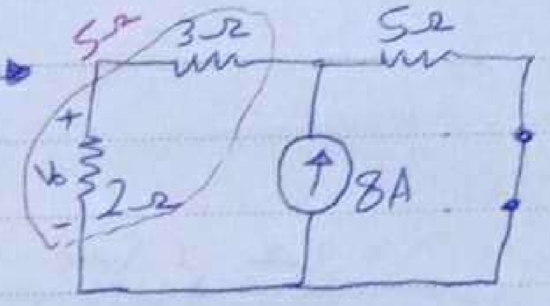
$\frac{V_2 - V_1}{4} = \frac{V_1}{4} + \frac{V_1}{3}$

$V_2 = \frac{10}{3} V_1$ ~ ②

superposition

Ex: Consider the effects of 8A and 20V one by one, then add the two effects together for final V_o





KVL at mesh ①:

$+2I_1 + 3I_1 + 5I_2 + 20 = 0$

$5I_1 + 5I_2 = -20$ ...①

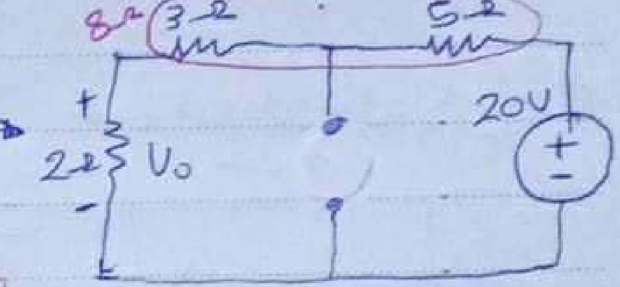
$I_2 - I_1 = 8$

$I_1 = -6A \quad I_2 = 2A$

$V_o = 6 \times 2 = 12$ volt.

$I = \frac{8 \times 5}{5+5} = 4A$

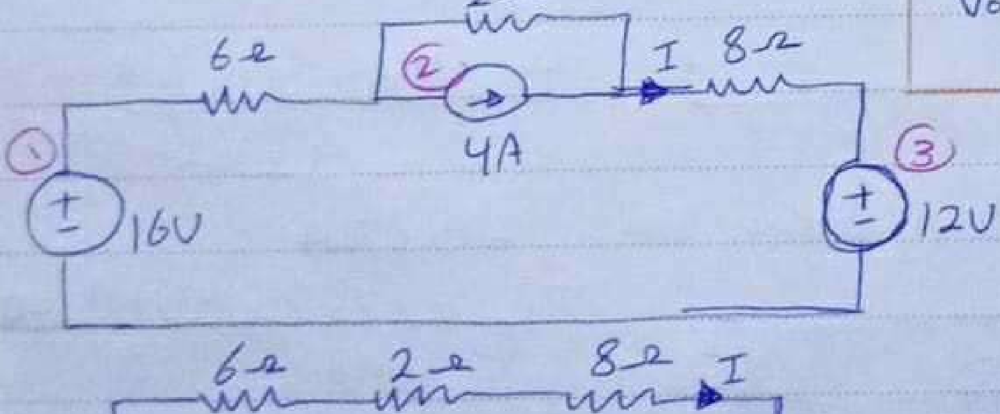
$V_{o①} = IR = 2 \times 4 = 8$ volt.



Ex: Using superposition principle find the current I in the following circuits

$V_{o②} = \frac{2 \times 20}{2+8} = 4$ volt

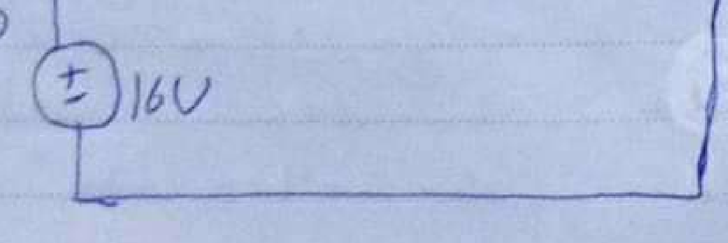
$V_o = 4 + 8 = 12$ volt.



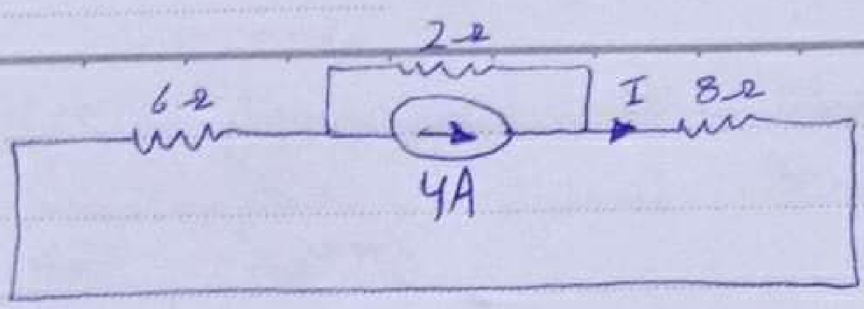
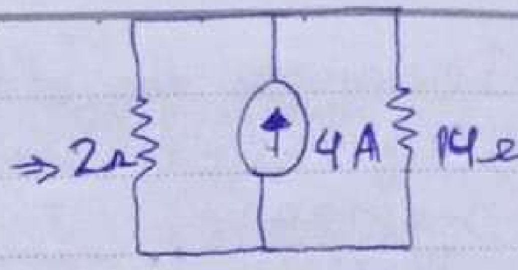
$\rightarrow I = I_1 + I_2 + I_3$

$= 1 + 0.5 - 0.75$

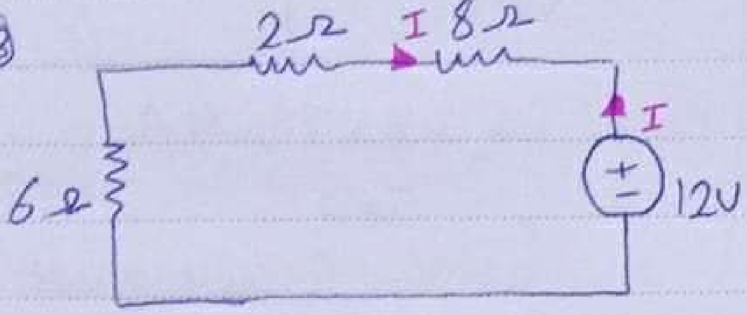
$= 0.75A$



$I = \frac{16}{16} = 1A$

②  ⇒ 

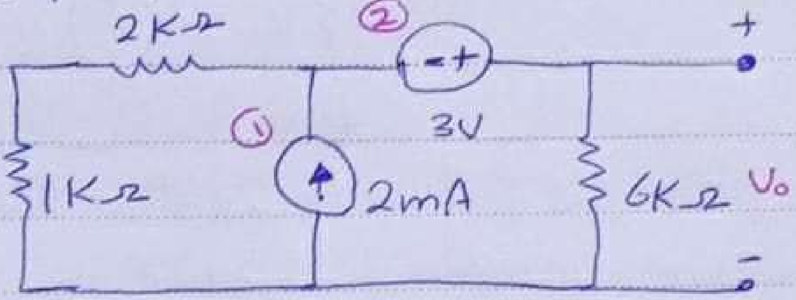
$$I_2 = \frac{4 \times 2}{14+2} = 0.5A$$

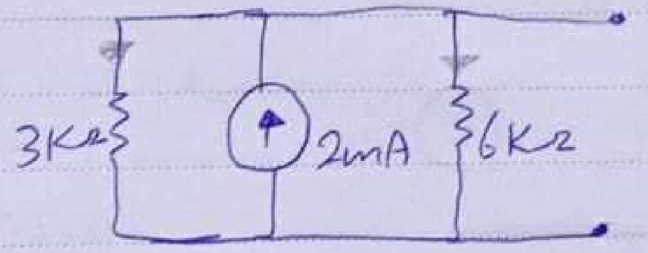
③ 

$$V_8 = \frac{12 \times 8}{16} = 6 \text{ volt} \; ([illegible])$$

$$I = -12/16 = -0.75$$

Ex: Superposition:

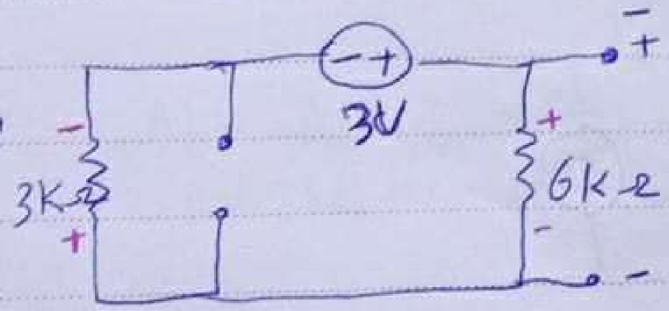


① 

$$I_o = \frac{2 \times 10^{-3} \times 3 \times 10^{3}}{9 \times 10^{3}}$$

$$= 0.67mA$$

$$V_{o_①} = 0.67 \times 6 \times 10^{3} \times 10^{-3} = 4V$$

② 

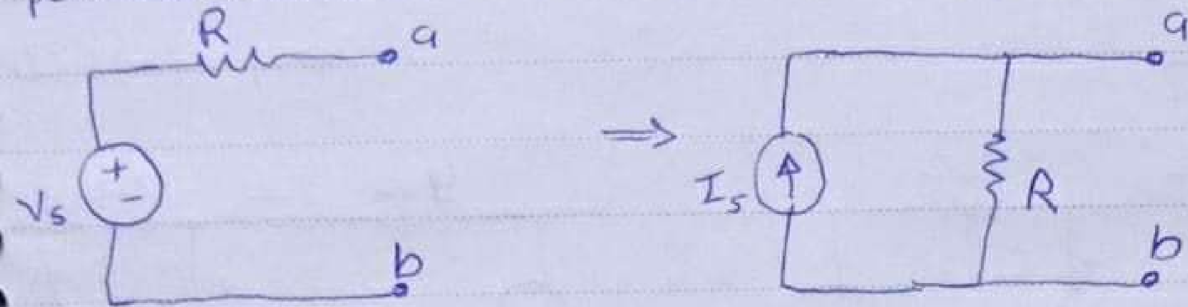
$$V_{o_②} = \frac{3 \times 6}{3+6} = 2 \text{ Volt}$$

$$V_o = 2 + 4 = 6 \text{ volt}$$

Source Transformation.

/ /

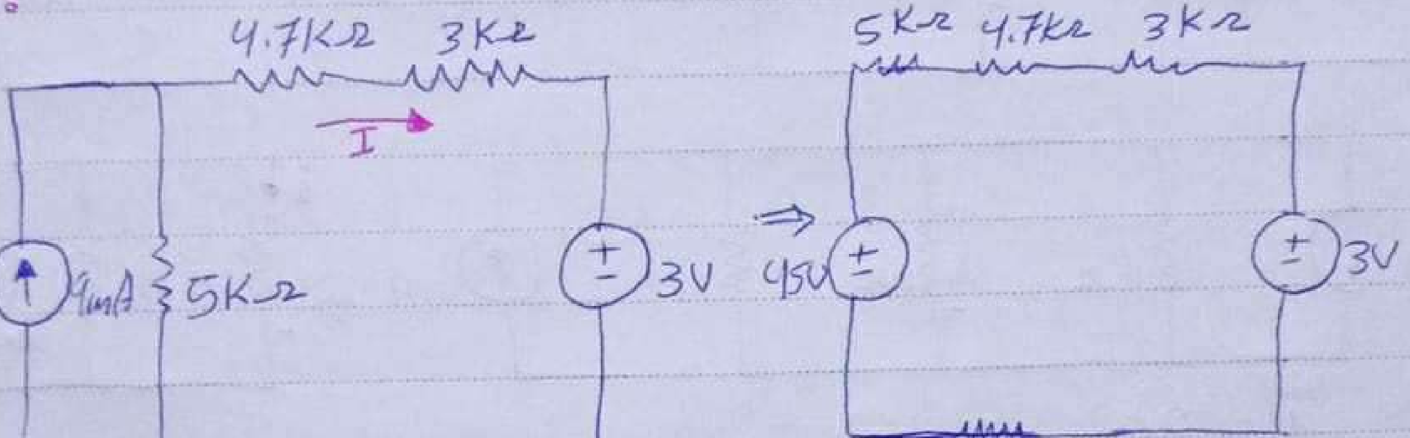
is the process of replacing a voltage source V_s in series with a resistor R by a current source I_s in parallel with a resistor R or vice versa.



$V_s = I_s R$

$I_s = \dfrac{V_s}{R}$

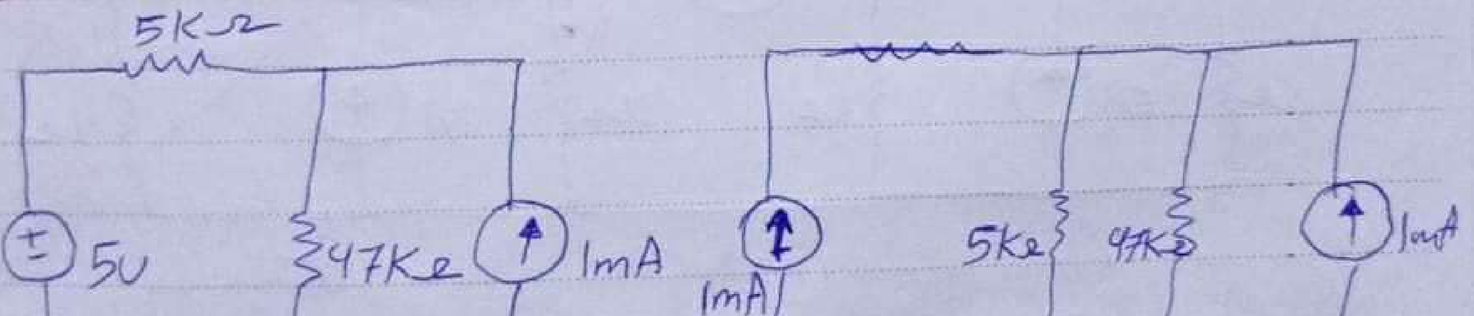
Ex:

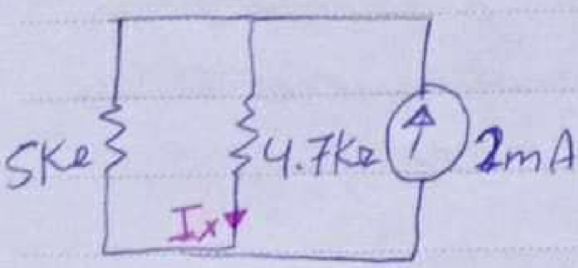


$3 + -45 + 5000I + 4700 + 3000 = 0$

$I = 3.307\ mA.$

Ex:

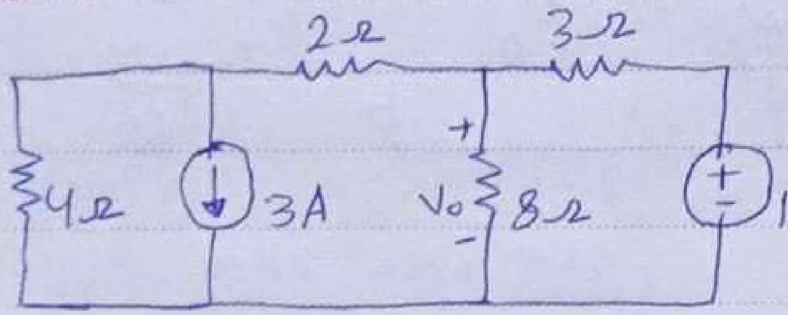


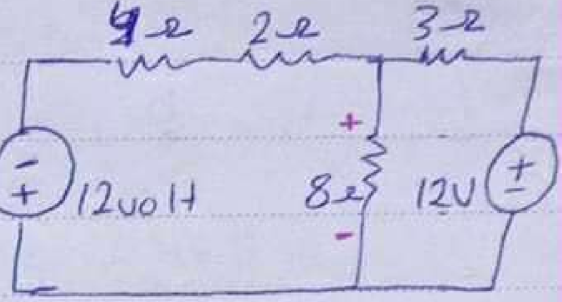


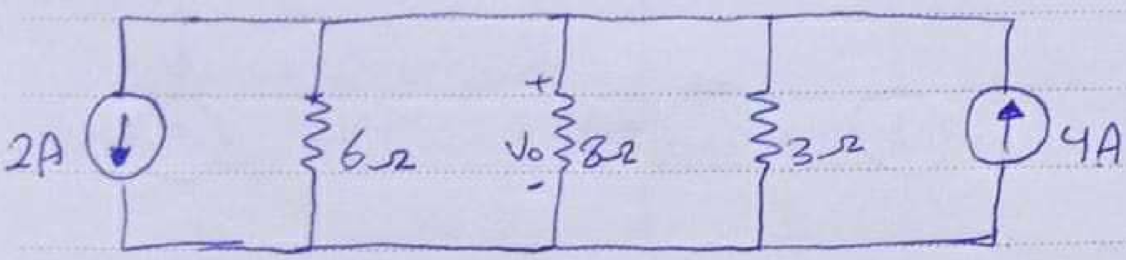
$$I_x = \frac{2 \times 5}{4.7+5} = 1A$$

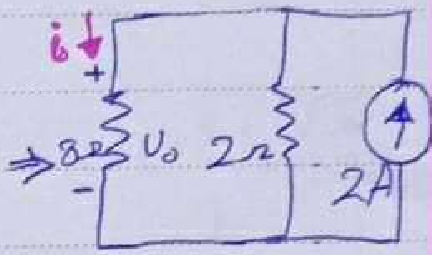
* لما يكون عندي
two current source
parallel متصلين
ونفس الاتجاه نجمعهم .

Ex:





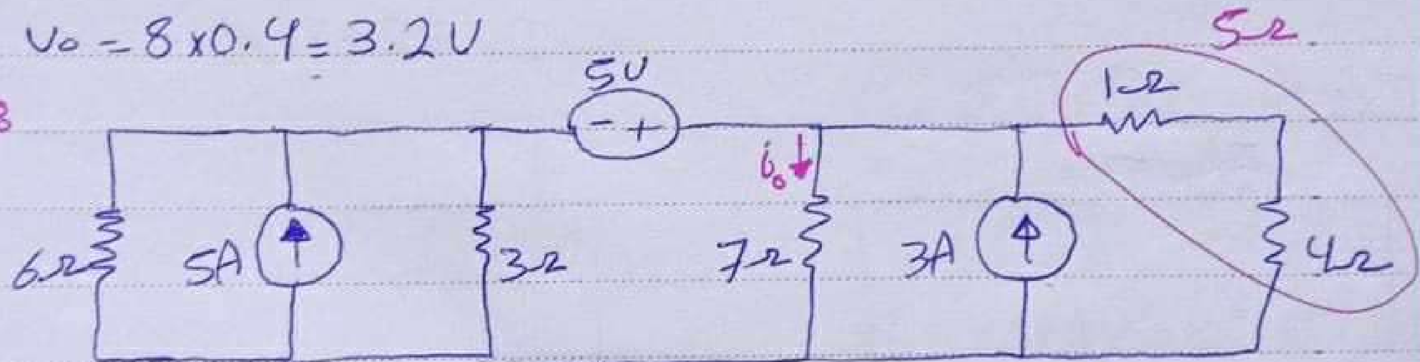


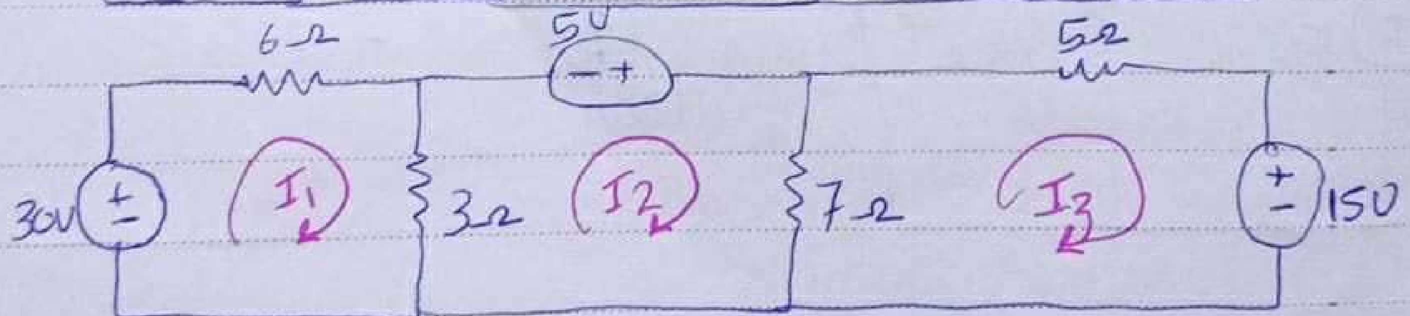


$$i = \frac{2 \times 2}{2+8} = 0.4\,A$$

$$V_o = 8 \times 0.4 = 3.2\,V$$

Ex:





KVL at mesh ①:

$-30+6I_1+3(I_1-I_2)=0$

$-30+6I_1+3I_1-3I_2=0$

$9I_1-3I_2=30$ --- ①

KVL at mesh ②:

$3(I_2-I_1)-5+7(I_2-I_3)=0$

$3I_2-3I_1-5+7I_2-7I_3=0$

$-3I_1+10I_2-7I_3=5$ --- ②

KVL at mesh ③:

$7(I_3-I_2)+5I_3+15=0$

$7I_3-7I_2+5I_3+15=0$

$-7I_2+12I_3=-15$ --- ③

$I_1=3.76\ A$

$I_2=1.27\ A$

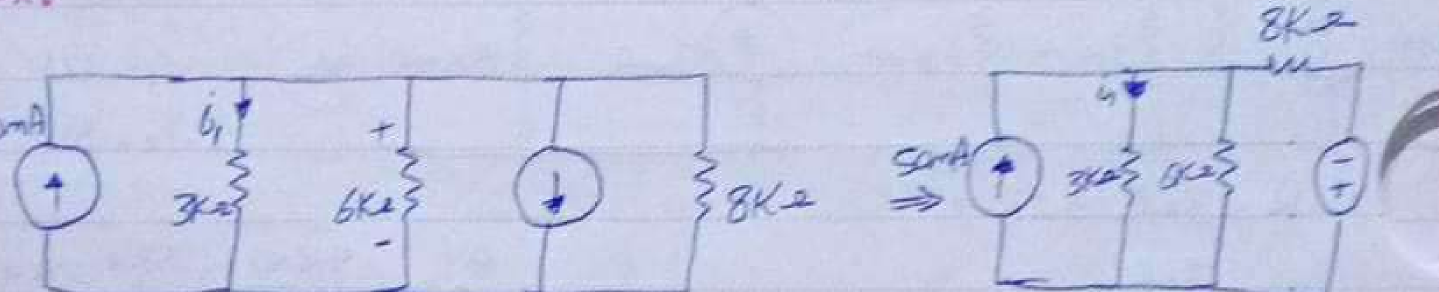
$I_3=-0.5A$

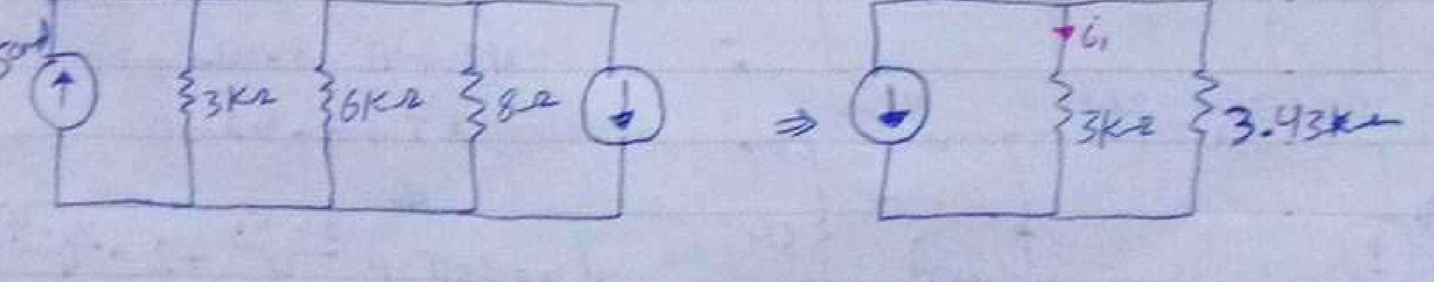
$i_o=I_3-I_2$

$=-0.5-1.27$

$=-1.77A$

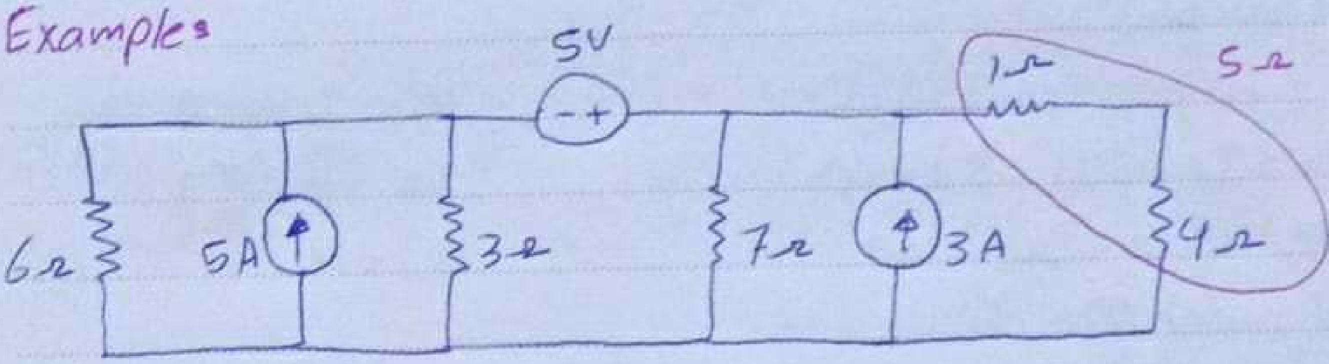
Ex:



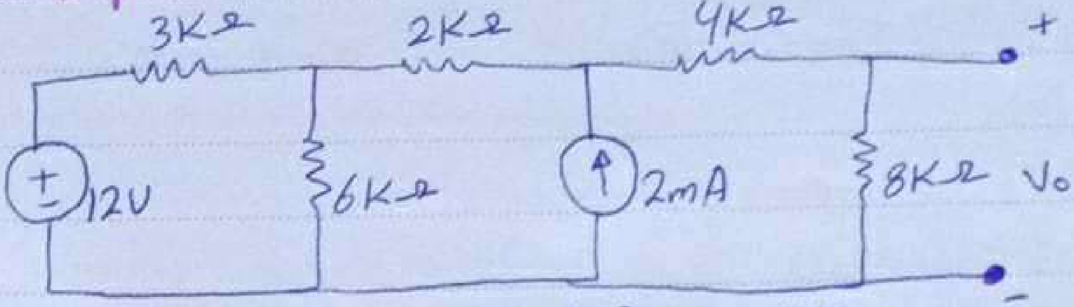


Example of slide.

Examples



Examples

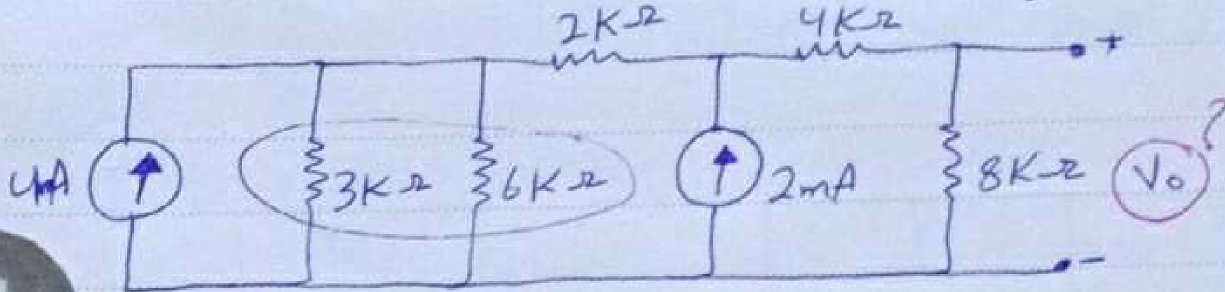


at node ② :

$8 \times \left(\frac{V_1 - V_2}{4} + \frac{0 - V_2}{8} = 0 \right)$

$2V_1 - 2V_2 - V_2 = 0$

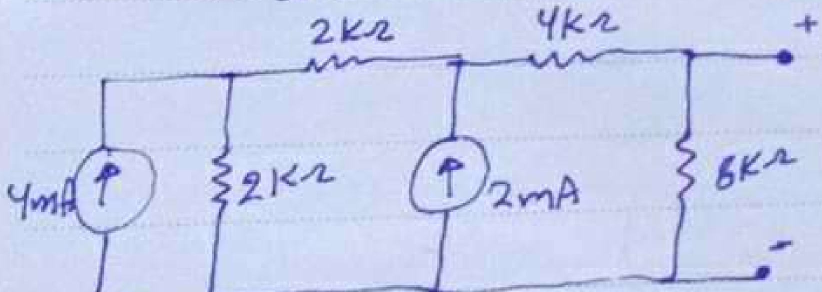
$2V_1 - 3V_2 = 0$ ②

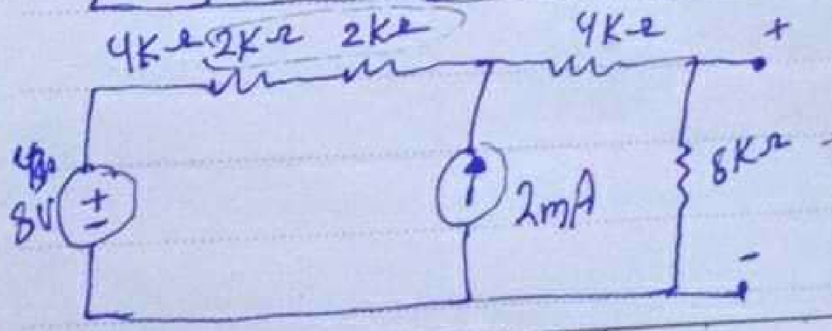


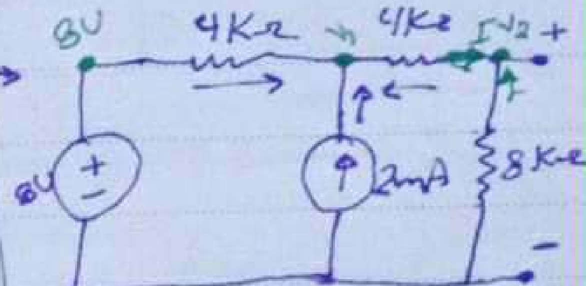
$V_1 = 12V$

$V_2 = 8V$

$R = \frac{3 \times 6}{3+6} = 2K\Omega$







$4I + \cancel{4I + 8 - V_o} = 0$

$8I + 8 - 8I = 0$

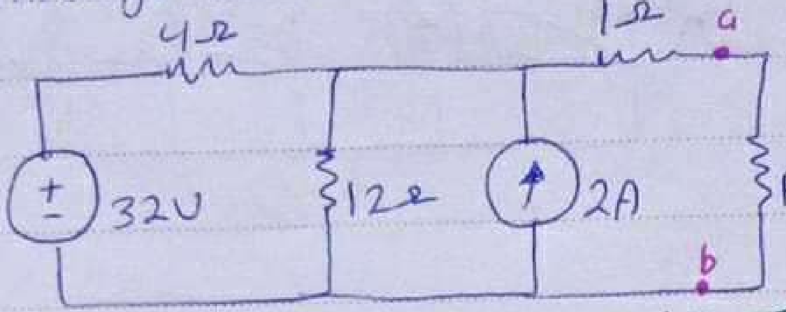
at node ① : $\frac{8 - V_1}{4} + 2 + \frac{V_2 - V_1}{4}$

$-2V_1 + V_2 = -16$ ①

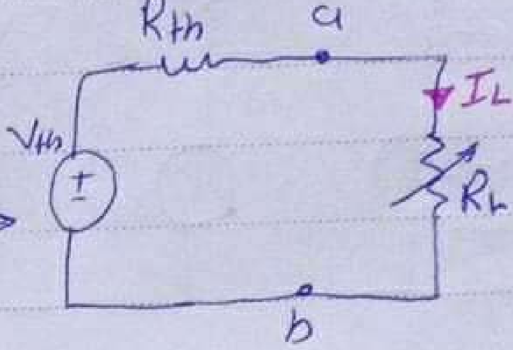
Thevenin's Equivalent Theorem. 28 / 7 / 2025

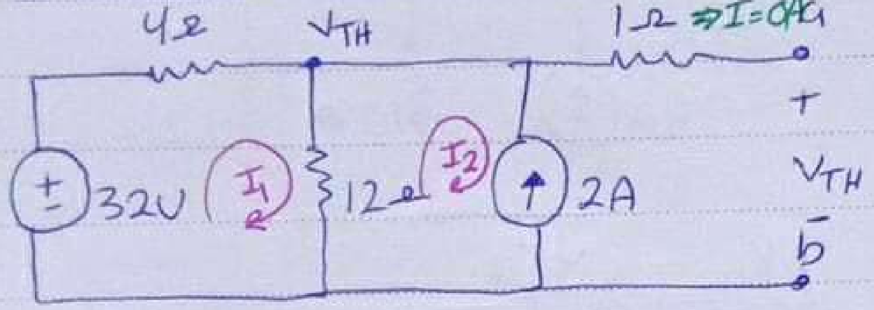
Example:

Find the Thevenin equivalent circuit, Then find the current through $R_L = 6$.



⇒



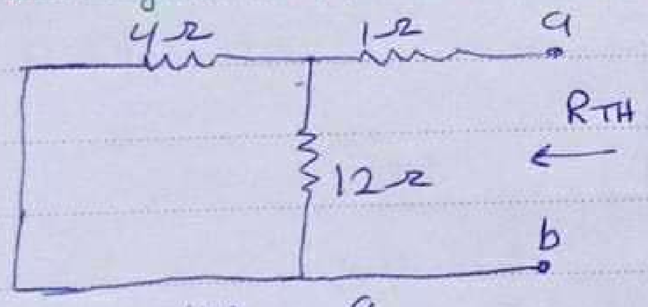


$$-32 + 4I_1 + 12(I_1 - I_2) = 0$$

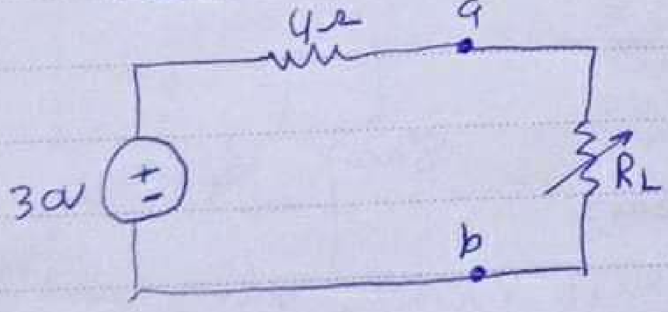
$$I_2 = -2A$$

$$V_{TH} = 12(I_1 - I_2) = 12(0.5 + 2) = 30V$$

* To get R_{TH} we must Kill all sources.



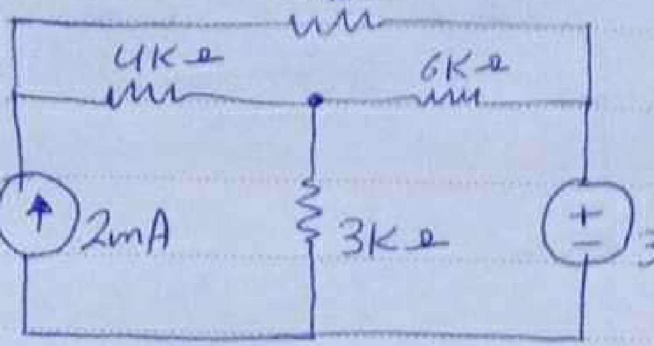
$$R_{TH} = \frac{12 \times 4}{4 + 12} + 1 = 4\Omega$$



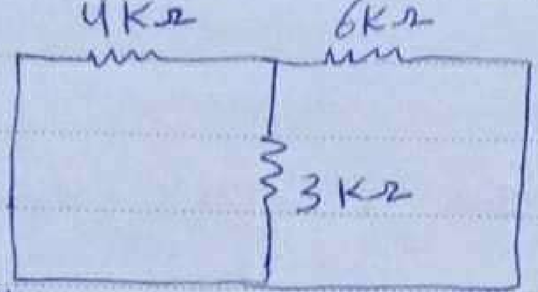
$$I_L = \frac{V_{TH}}{R_L + R_{TH}} = \frac{30}{6+4} = 3A$$

/ /

Ex: Find R_{TH} & I.



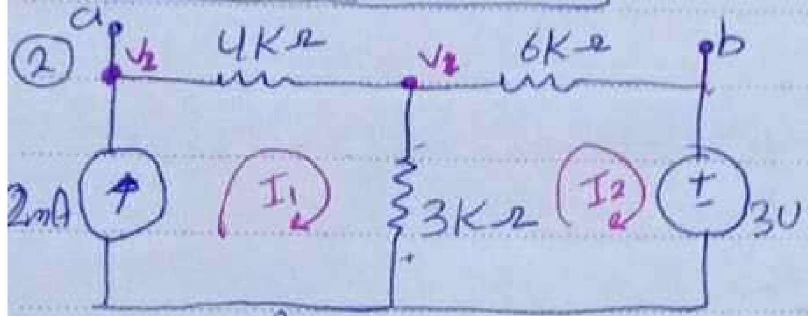
①



① $R_{equ} = \frac{6 \times 3}{3+6} = 2K\Omega$

② $R_{TH} = 2 + 4 = 6K\Omega$

②



$I_1 = 2mA$.

$3(I_2 - I_1) + 6 I_2 + 3 = 0$.

$3(I_2 - 2) + 6 I_2 + 3 = 0$.

$I_2 = 0.3mA$.

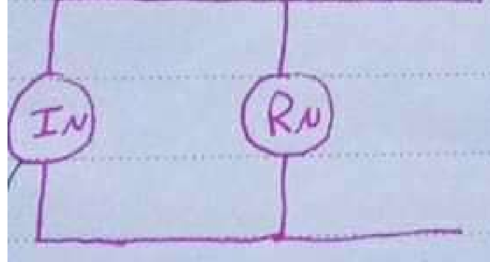
$V_1 = 6 \times 10^3 \times \frac{1}{3} \times 10^{-3} = 2 volt$

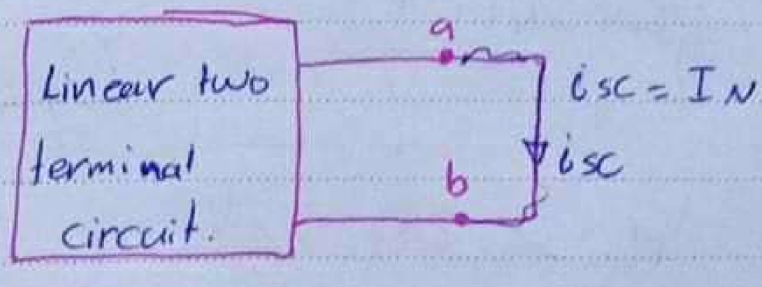
$V_{oc} = V_1 + V_2 = 2 + 8 = 10 volt$.

$V_2 = 4 \times 10^3 \times 2 \times 10^{-3} = 8 volt$

Norton's Theorem

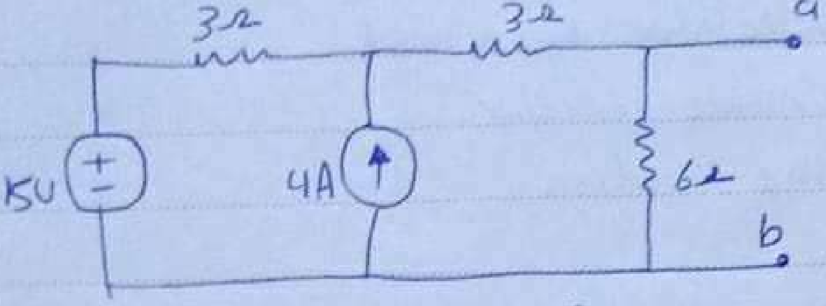
* $R_N = R_{TH}$ → it's same.

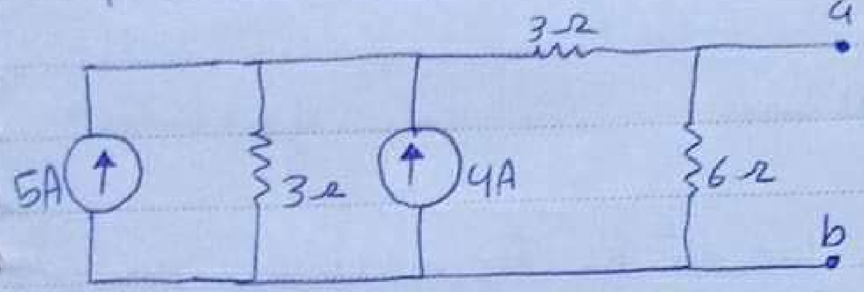


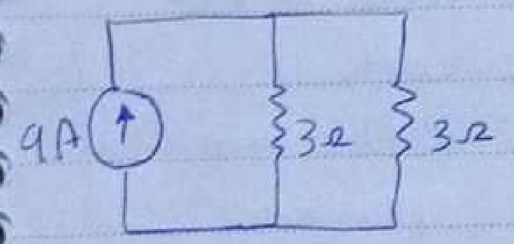


→ $\frac{V_{TH}}{R_{TH}}$ → إذا عرفت قيمة كلاهما

Ex:

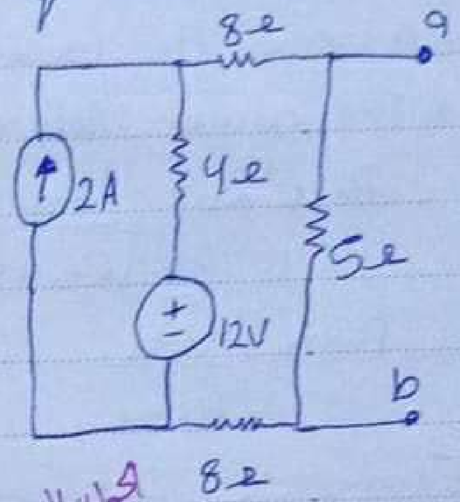




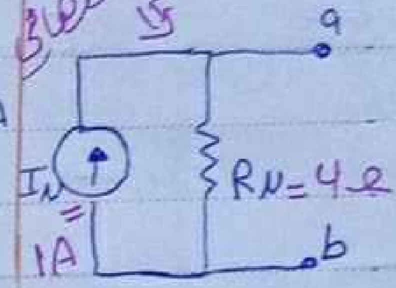


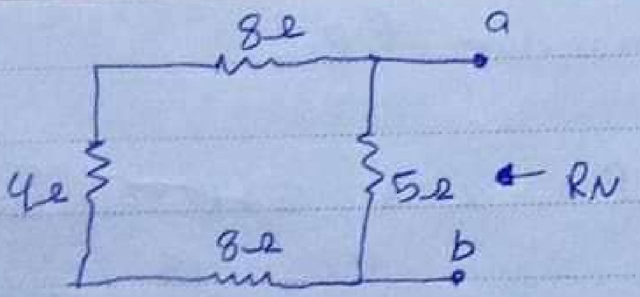
$$I = \frac{3 \times 9}{3+3} = 4.5A$$

Ex: Find the Norton equivalent circuit.



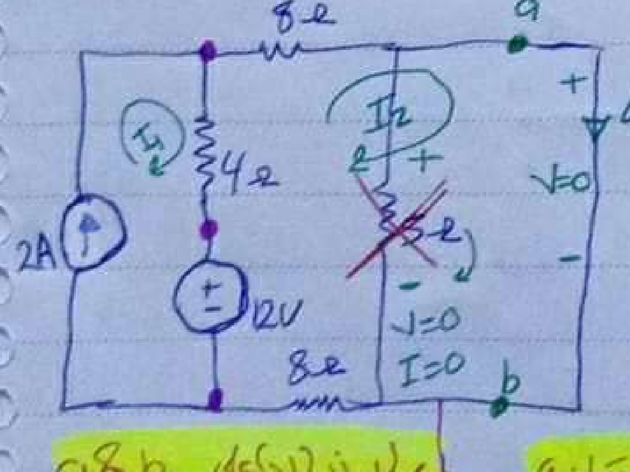
الجواب النهائي





$$R_{equ.①} = 8 + 8 + 4 = 20\Omega$$

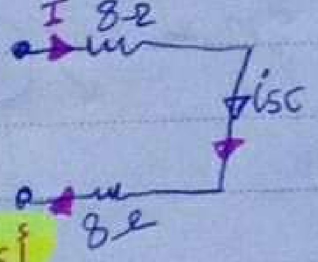
$$R_{equ.②} = \frac{20 \times 5}{20+5} = 4\Omega$$



$I_1 = 2A$

$20I_2 - 4I_1 - 12 = 0$

$I_2 = 1A = i_{sc} = I_N$



نفس التيار

a & b لإن (V) على متساوية

أي مقاومة متوازية مع S.C التيار فيها صفر

Maximum power Transfer.

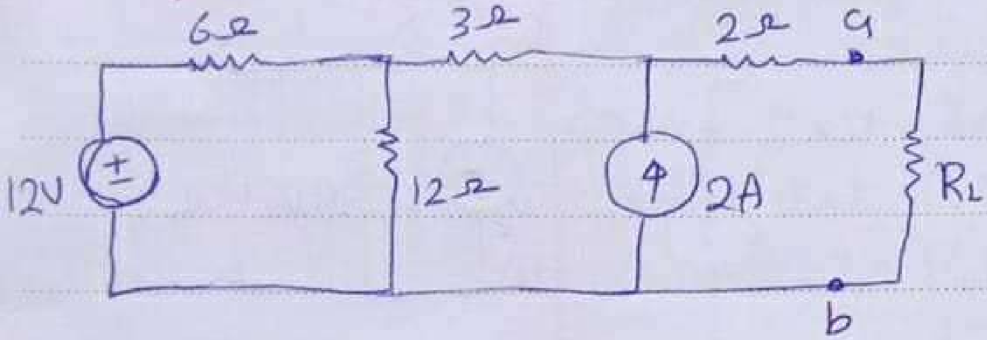
/ /

* The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.
* Assume the load has variable resistor.
* The power delivered to the load is:

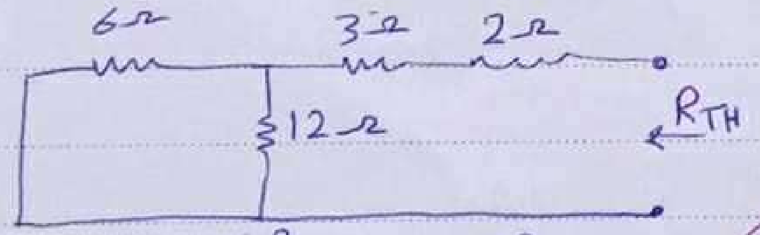
$$P = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} \Rightarrow R_L = R_{max}$$

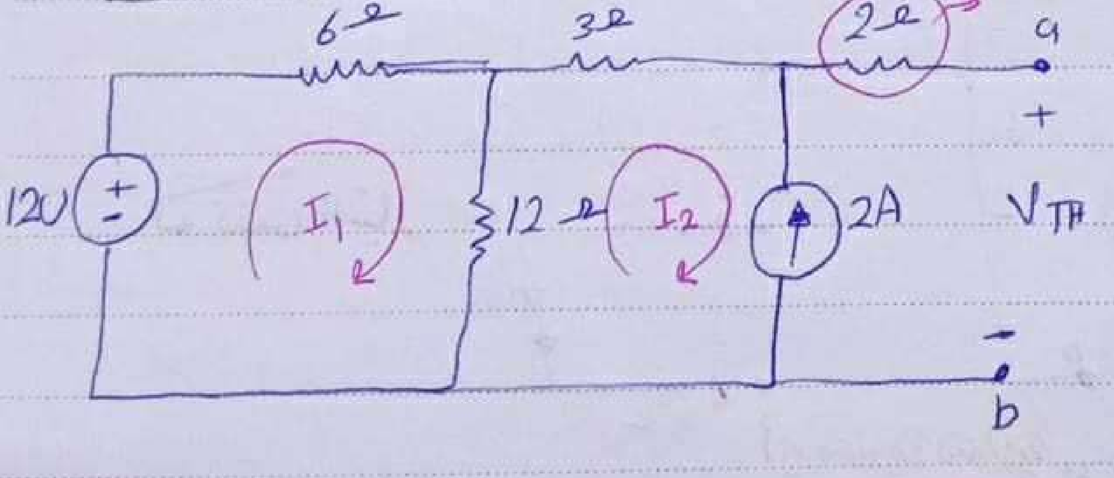
Example(4.13): Find the value of R_L for maximum power Transfer in the circuit.
Find the maximum power.



$\boxed{R_L = R_{TH}}$ → maximum value of R_L.



$$R_{TH} = 2 + 3 = 5 + \frac{12 \times 6}{18} = 9\Omega$$



$-12 + 6I_1 + 12(I_1 - I_2) = 0$

$I_2 = -2A$

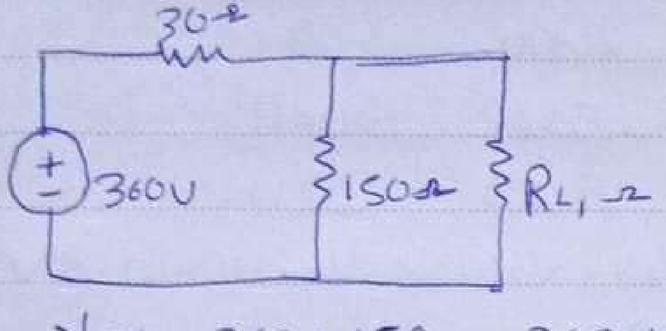
$18I_1 - 12I_2 = 12$ --- ⊛

$I_1 = -2/3 A$

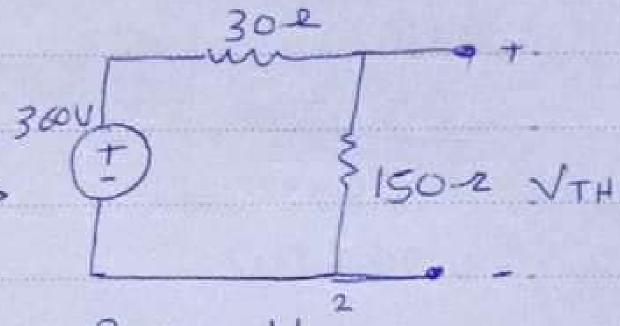
$-12 + 6I_1 + \frac{3}{2}I_2 + 2(0) + V_{TH} = 0 \Rightarrow V_{TH} = 22V$

$P_{max} = \frac{V_{TH}^2}{4R_L} = \frac{(22)^2}{4 \times 9} = 13.44W$

Ex3



⇒

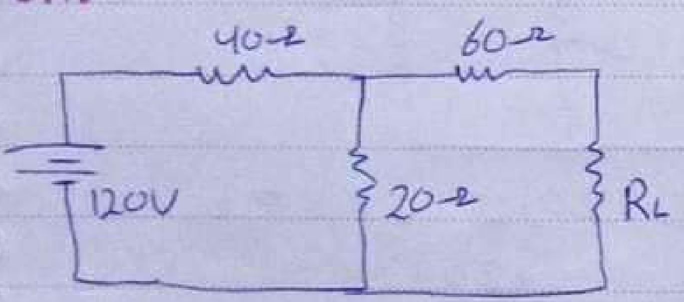


$V_{TH} = \frac{360 \times 150}{150 + 30} = 300$ volt.

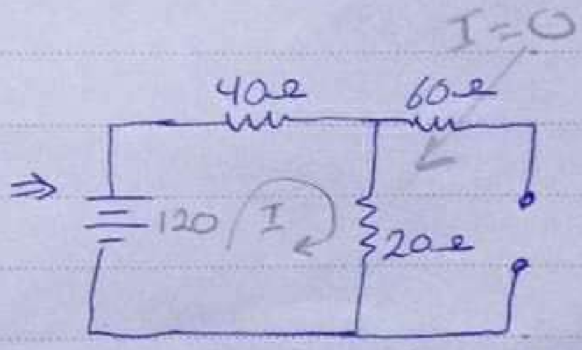
$R_{TH} = \frac{150 \times 30}{150 + 30} = 25\Omega$

$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(300)^2}{4 \times 25} = 900W$

Ex3



⇒



$R_{TH} = \frac{40 \times 20}{40 + 20} + 60 = 73.3\Omega$

$V_{TH} = \frac{20}{40+60}(120)$

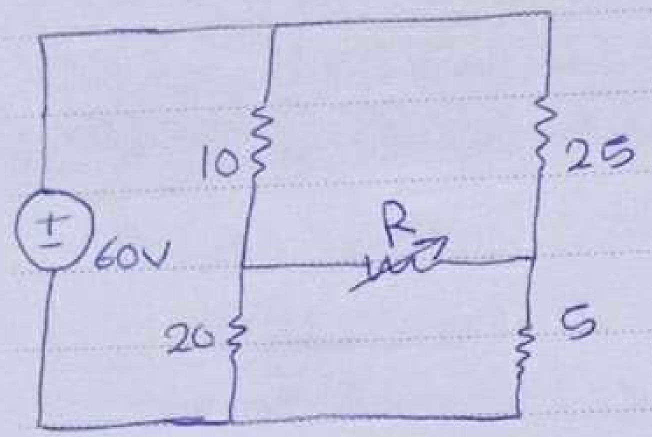
$-120 + 40I + 20I = 0 \quad = 40V$

$P_{max} = \frac{(40)^2}{4 \times 73.3} = 5.46W$

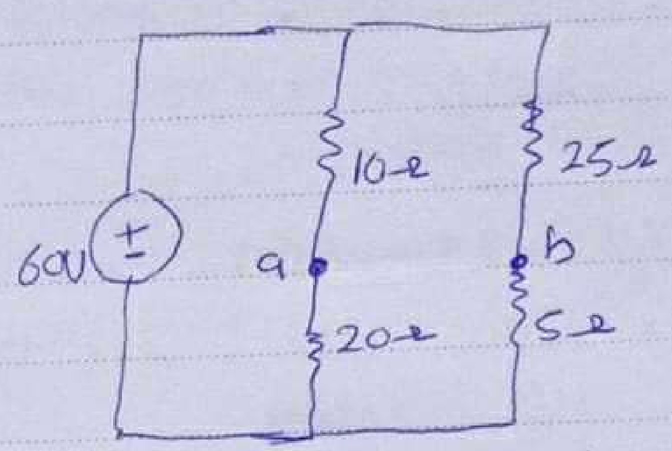
$60I = 120 \rightarrow I = 2A \quad V = IR = 2 \times 20$

/ /

Ex 8



① evaluate V_{TH}

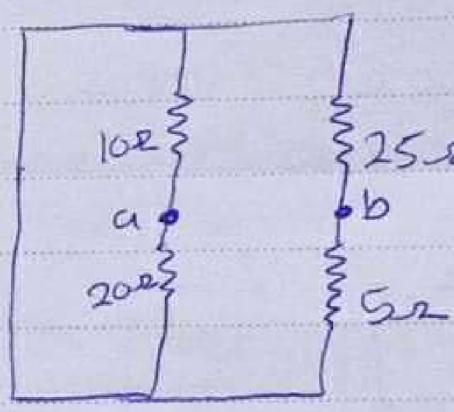


$$V_a = \frac{20 \times 60}{20+10} = 40 \text{ volt}.$$

$$V_b = \frac{5 \times 60}{5+25} = 10 \text{ volt}$$

$$V_{th} = V_{ab} = V_a - V_b = 40 - 10 = 30V$$

② R_{TH}



$$R_① = \frac{20 \times 10}{10+20} = 6.67\,\Omega$$

$$R_② = \frac{25 * 5}{25+5} = 4.16\,\Omega$$

$$R_{TH} = 6.67 + 4.16 = 10.83\,\Omega$$

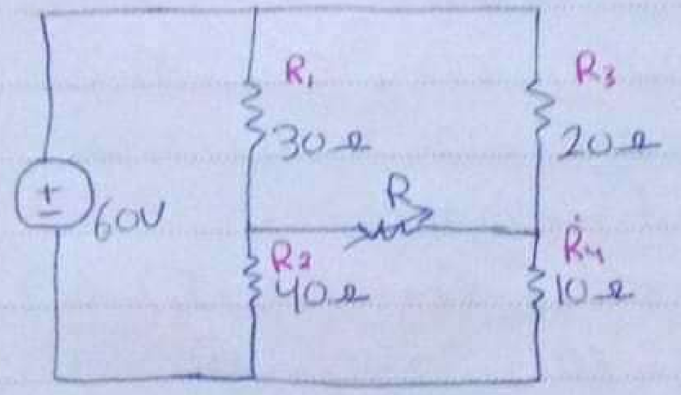
③

$$P_{max} = \frac{(V_{TH})^2}{4R_L}$$

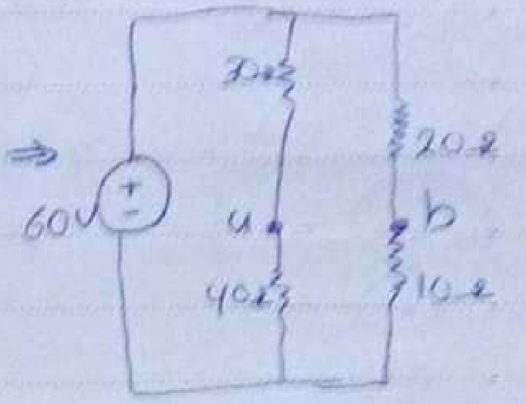
$$= \frac{(30)^2}{4 \times 10.83}$$

$$= 20.77W$$

Ex:



$\Rightarrow$

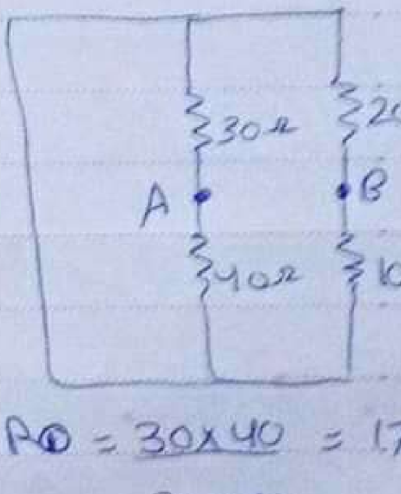


$$V_A = \frac{60 \times 40}{40+70} = 34.28 \text{ volt}$$

$$V_B = \frac{60 \times 10}{10+20} = 20 \text{ volt}$$

$$V_{AB} = V_{TH} = 34.28 - 20$$
$$= 14.28 \text{ volt}$$

$$P_{max} = \frac{(V_{TH})^2}{4R_L}$$
$$= \frac{(14.28)^2}{4 \times 23.8}$$
$$= 2.14W$$



$$R_1 = \frac{30 \times 40}{30+40} = 17.1\Omega$$

$$R_2 = \frac{20 \times 10}{10+20} = 6.7$$

$$R_{TH} = 17.1 + 6.7 = 23.8\Omega$$

Midterm-Exam

	بيانات الطالب	
[redacted] الرقم الجامعي	هداية عبدالحميد النوابية	اسم الطالب باللغة العربية

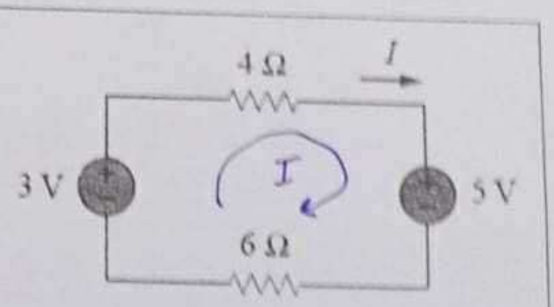
Grade:	26 ....../30

ANSWER SHEET

QUESTION #	ANSWER			
1.	a	b	c	d
2.	a	b	c	d
3.	a	b	c	d
4.	a	b	c	d
5.	a	b	c	d
6.	a	b	c	d
7.	a	b	c	d
8.	a	b	c	d
9.	a	b	c	d
10.	a	b	c	d
11.	a	b	c	d
12.	a	b	c	d
13.	a	b	c	d
14.	a	b	c	d
15.	a	b	c	d
16.	a	b	c	d

Select the correct answer for each of the following statements and fill the correct answer in the Answer Sheet provided. Show the calculations for all Questions

1.	The current I in the circuit shown below is:….		
a.	$I = -0.8$ A	b.	$I = 0.8$ A
c.	$I = -0.2$ A	d.	$I = 0.2$ A



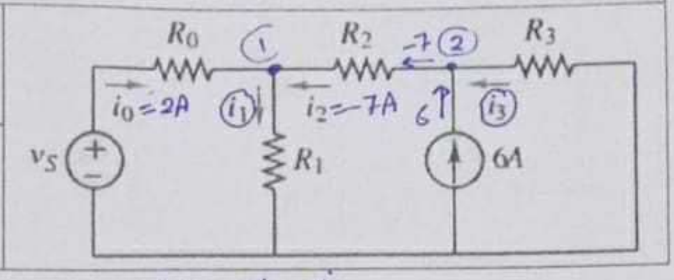
$-3 + 4I + 5 + 6I = 0$

$\frac{10I}{10} = \frac{-2}{10}$

$I = -0.2$ A

2.	For the circuit shown below, assuming $i_0 = 2$ A and $i_2 = -7$A, use KCL to determine the unknown currents i_1 and i_3 in the circuit shown below.

a.	$i_1 = 9$ A, $i_3 = -7$A	b.	$i_1 = 5$ A, $i_3 = 1$A	
c.	$i_1 = -5$ A, $i_3 = 13$A	d.	$i_1 = -5$ A, $i_3 = -13$A	



1) $2 - 7 = i_1$

$i_1 = -5A$

2) $i_3 + 6 = -7 \rightarrow i_3 = -13A.$

$-6 \quad -6$

$-V_0 - 7 + 25 = 0$

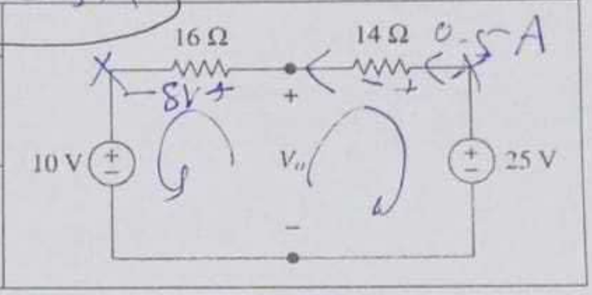
$V_0 = 18$

$\frac{25 - 10}{30} = 0.5 A$

طريقة حل أخرى صحيحة لفرع (3)

3.	For the circuit below, the voltage V_o is….

a.	$V_o = 15$ V	b.	$V_o = 32$ V
c.	$V_o = 2$ V	d.	$V_o = 18$ V



$-V_0 + 8 + 10 = 0$

$V_0 = 18$

الحل الصحيح

$25 - 10 = 15$

$-10 + 16I + 14I + 25 = 0$

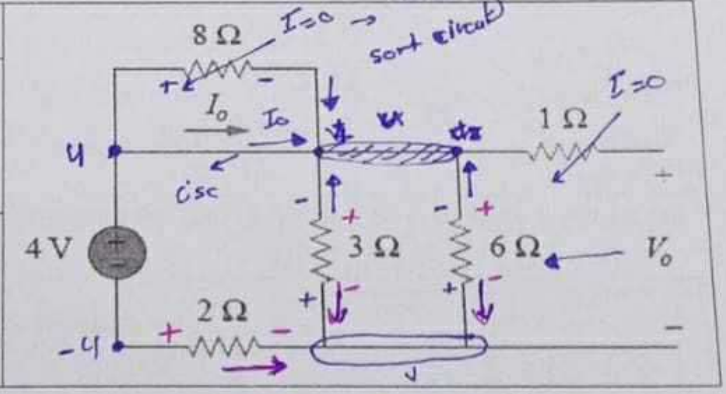
$15 + 30I = 0$

$30I = -15 \rightarrow I = -0.5$

موازية

4.	The voltage V_o in the circuit below is:….

a.	$V_o = 1$ V	b.	$V_o = 2$ V
c.	$V_o = 4$ V	d.	$V_o = 0.5$ V



$I_o + \frac{0 - v}{3} + \frac{0 - v}{6} = 0$

$\left(\frac{-4 - v}{2} + \frac{0 - v}{3} + \frac{0 - v}{6} = 0 \right) \times 6$

$-2 - \frac{v}{2} - \frac{v}{3} - \frac{v}{6} = 0$

$\frac{-v}{2} - \frac{v}{3} - \frac{v}{6} = 2$

$-v = 2$

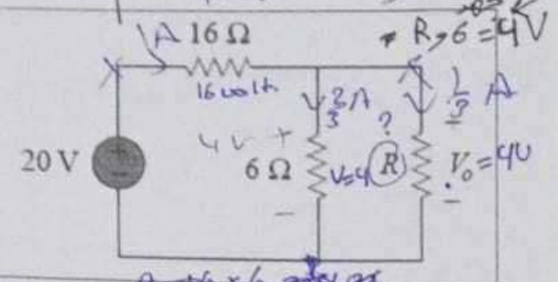
$v = -2$ volt.

① $V_{16\Omega} = \frac{16\times 20}{16+4} = 16V$

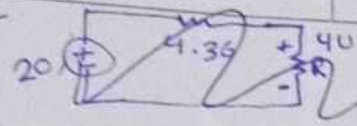
② $I_{total} = I_{16\Omega} = 16/16 = 1A$; $\frac{2}{3}A \to 6\Omega$

$1/3 A \to R_o$; 16V ; $\frac{4}{1/3} = 12\Omega$; $R, 6 = 4V$

5.	In the circuit shown below, if $V_o = 4V$, then $R = \ldots$		
a.	$R = 12\ \Omega$	b.	$R = 1\ \Omega$
c.	$R = 2.4\ \Omega$	d.	$R = 4\ \Omega$



$R = \frac{4V}{1/3 A} = 12\Omega$



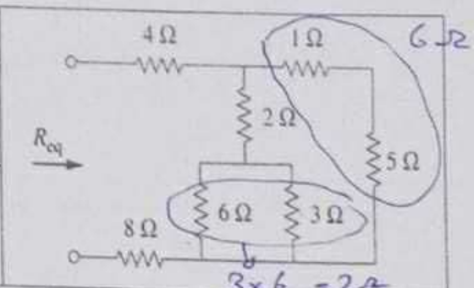
$-20 + 4.36 I + 4 = 0$

$4.36 I = 16$

$I = 3.67$

$V = IR \to R = \frac{V}{I} = \frac{4}{3.67} =$

6.	For the circuit below, the equivalent resistance R_{eq} is		
a.	$R_{eq} = 14.4\ \Omega$	b.	$R_{eq} = 22\ \Omega$
c.	$R_{eq} = 2.4\ \Omega$	d.	$R_{eq} = 10\ \Omega$

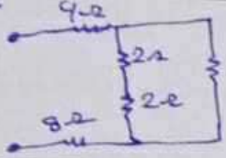


6Ω ; $\frac{3\times 6}{6+3} = 2\Omega$

$R_1 = 5+1 = 6\Omega$

$R_2 = 2\Omega$

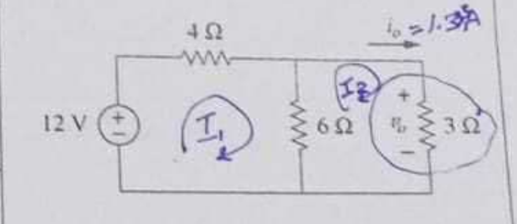
$R_3 = 4\Omega$



$R_4 = \frac{4\times 6}{6+4} = 2.4$

$R_5 = 2.4 + 8 + 4 = 14.4$

7.	For the circuit shown below, the power dissipated in the 3-Ω resistor is		
a.	$P_{3\Omega} = 8$ W	b.	$P_{3\Omega} = 5.33$ W
c.	$P_{3\Omega} = 48$ W	d.	$P_{3\Omega} = 1.33$ W



KVL mesh ①:

$-12 + 4I_1 + 6(I_1 - I_2) = 0$

$10I_1 - 6I_2 = 12$ — ①

KVL mesh ②:

$6(I_2 - I_1) + 3I_2 = 0$

$-6I_1 + 9I_2 = 0$ — ②

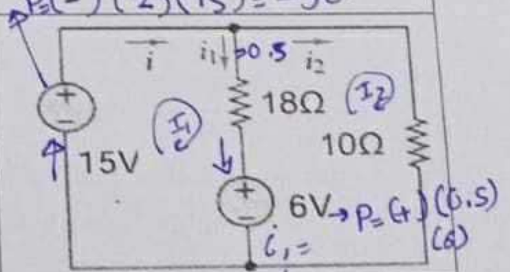
$I_1 = 2A$

$I_2 = 1.3A$

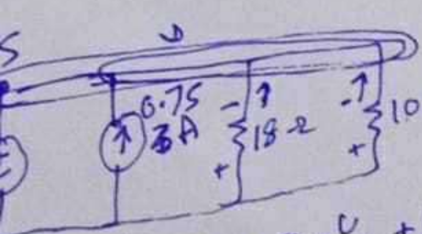
$V_o = 1.3\times 3 = 3.9$

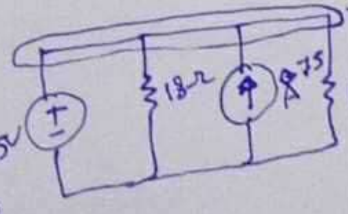
8.	For the circuit below, the power supplied/consumed by the 15 V- and 6 V-voltage sources are		
a.	$P_{15V} = 30$ W supplied $P_{6V} = 3$ W consumed	b.	$P_{15V} = 15$ W consumed $P_{6V} = 6$ W supplied
c.	$P_{15V} = 15$ W supplied $P_{6V} = 6$ W consumed	d.	$P_{15V} = 30$ W consumed $P_{6V} = 3$ W supplied

$P = (-)(2)(15) = -30$



$P = (+)(0.5)(6)$





$V = IR$

$I = \frac{V}{R}$

$15 + 0.75 + \frac{0-V}{18} + \frac{0-V}{10} = 0$

$V = 101.25$

$2 = 1.5 + 6$

$i = 15.7$

$i_1 = 5.6$

$i_2 = 10.1$

$-15 + 18(I_1 - I_2) + 6 = 0$

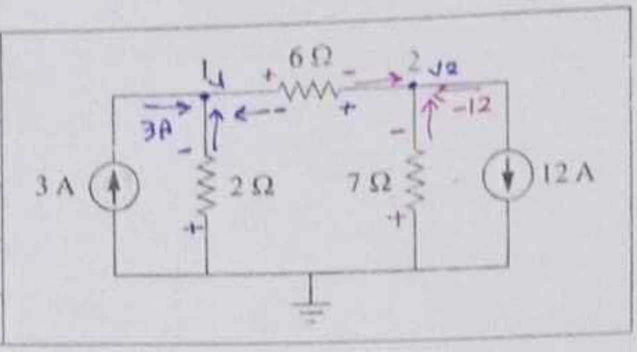
$18I_1 - 18I_2 = 9$ — ①

$-6 + 18(I_2 - I_1) + 10I_2 = 0$

$-18I_1 + 28I_2 = 6$ — ②

$I_1 = 2$ $I_2 = 1.5$

9.	Using the nodal analysis, the node voltage V_1 and V_2 are .....
a.	$V_1 = -6$ V and $V_2 = 84$ V
b.	$V_1 = 6$ V and $V_2 = 42$ V
c.	$V_1 = -6$ V and $V_2 = -42$ V
d.	$V_1 = 6$ V and $V_2 = -84$ V



node ① :

$$3 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{6} = 0 \Big) \times 6$$

$$-3V_1 + V_2 - V_1 = -18$$

$$-4V_1 + V_2 = -18 \sim ①$$

$$\boxed{-\frac{2}{3}V_1 + \frac{V_2}{6} = -3}$$

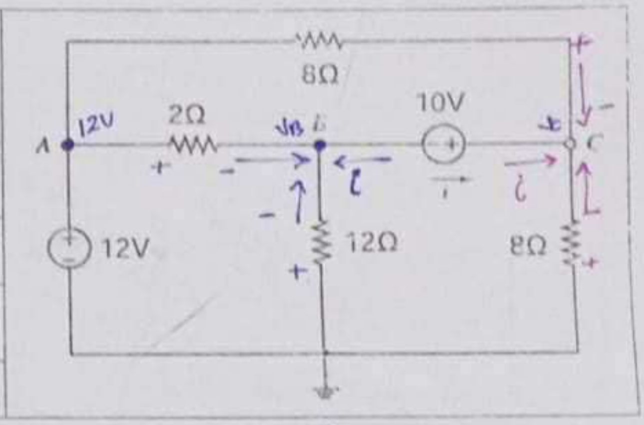
node ② :

$$\Big(-12 + \frac{0 - V_2}{7} + \frac{V_1 - V_2}{6} = 0\Big) \times 42 \to \frac{V_1}{6} - \frac{13}{42}V_2$$

$$-6V_2 + 7V_1 - 7V_2 = 504$$

$$7V_1 - 13V_2 = 504 \sim ①$$

10.	Using the node voltage analysis method in the circuit shown below, the current i through the 10 V-voltage source is ....
a.	$i = 2.5$ A
b.	$i = 1.5$ A
c.	$i = -2.5$ A
d.	$i = 3$ A



node B → $\frac{12 - V_B}{2} + \frac{0 - V_B}{12} + i = 0 \to -\frac{7}{12}V_B - i = -6$ ①

$V_C - V_B = 10$ ②

$V_B = 6$ volt.
$V_C = 16$ volt

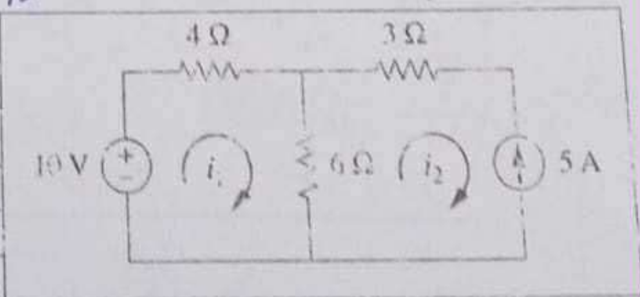
node C → $\frac{12 - V_C}{8} + \frac{0 - V_C}{8} + i = 0$

$$-\frac{1}{4}V_C + i = \frac{-12}{8} \to 1 + 2 = \frac{-7}{12}V_B - \frac{1}{4}V_C = \frac{-15}{2} \to ①$$

$$\frac{12-6}{2} - \frac{6}{12} - i = 0$$

$$i = 2.5$$

11.	For the circuit shown below, using mesh analysis, the mesh currents i_1 and i_2 are .....
a.	$i_1 = 2$ A and $i_2 = 5$ A
b.	$i_1 = -2$ A and $i_2 = -5$ A
c.	$i_1 = 2$ A and $i_2 = -5$ A
d.	$i_1 = -2$ A and $i_2 = 5$ A

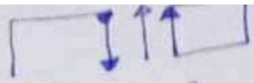


KVL at mesh ① :

$$-10 + 4I_1 + 6(I_1 - I_2) = 0 \to I_1 = -2A.$$

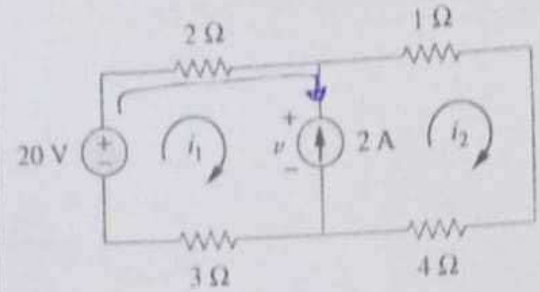
KVL at mesh ② :

$I_2 = -5 \to$ by intersection.



$I_2 - I_1 = 2$

12.	Using mesh analysis in the circuit shown below, the power supplied/absorbed by the 2-A current source is		
a.	$P_{2A} = 30$ W absorbed	b.	$P_{2A} = 50$ W supplied
c.	$P_{2A} = 30$ W supplied	d.	$P_{2A} = 6$ W absorbed



$I_2 - I_1 = 2 \sim (1) \rightarrow -I_1 + I_2 = 2 \sim (1)$

Supermesh:

$-20 + 2I_1 + 1I_2 + 4I_2 + 3I_1 = 0$

$5I_1 + 5I_2 = 20 \sim (2)$

$+ \rightarrow ab$

$I_1 = 1A$

$I_2 = 3A$

KVL:

$-20 + 2 + V + 3 = 0$

$V = 20 - 3 - 2$

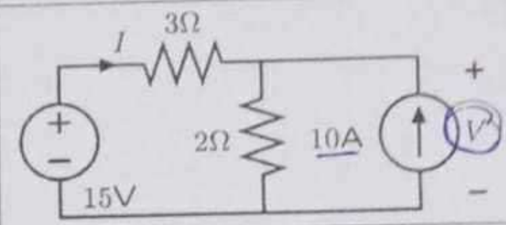
$= 15V$

$P = I\ V = 15 \times 2$

$= 30$ volt.

$P = (+)(-2) I\ (15) V$

13.	Using source transformation, the current I and the voltage V are		
a.	$I = -1$ A, $V = 18$ V.	b.	$I = 3$ A, $V = 12$ V.
c.	$I = -1$ A, $V = 20$ V.	d.	$I = 3$ A, $V = 20$ V.



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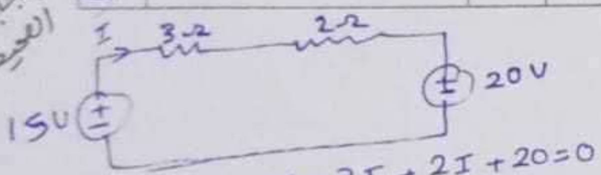


KVL $\rightarrow -15 + 3I + 2I + 20 = 0$

$5I + 5 = 0$

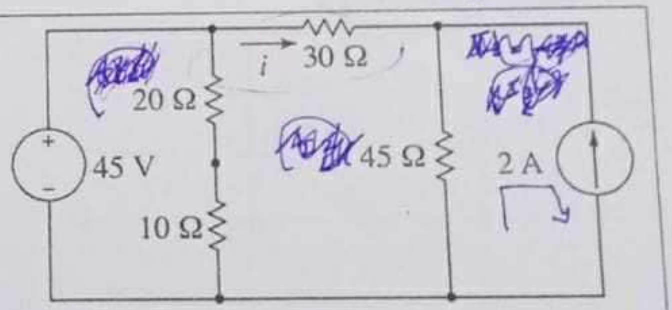
$I = -1A$

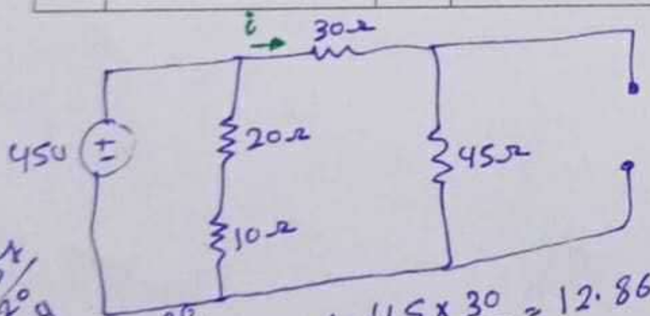
$-45 + 20(I_1 - I_2) + 10(I_1 - I_2) = 0$

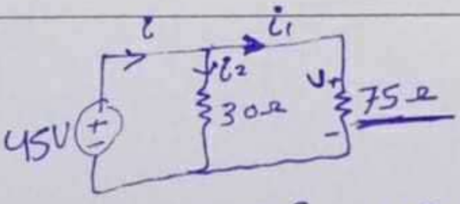
$30I_1 - 30I_2 = 45 \sim (1)$

$10(I_2 - I_1) + 20(I_2 - I_1) + 3I_2 + 45(I_2 + 2) = 0$

14.	Using superposition principle, the contribution of the 45 V-voltage source to the current i through the 30 Ω resistor is		
a.	$i_{45V} = -0.6$ A.	b.	$i_{45V} = 0.6$ A.
c.	$i_{45V} = 1.5$ A.	d.	$i_{45V} = -1.2$ A.







$V = \frac{45 \times 75}{75 + 30} = 32.14$

$I = \frac{V}{R} = \frac{32.14}{75} = 0.4A$

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$V_{30} = \frac{32.14 \times 30}{75} = 12.856$

$I = V/R$

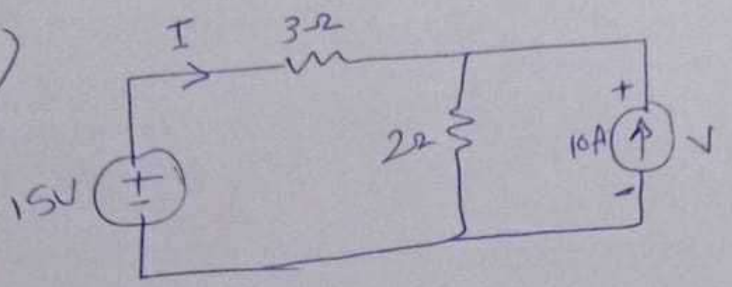
$V = \frac{45 \times 30}{75 + 30} = 12.86$

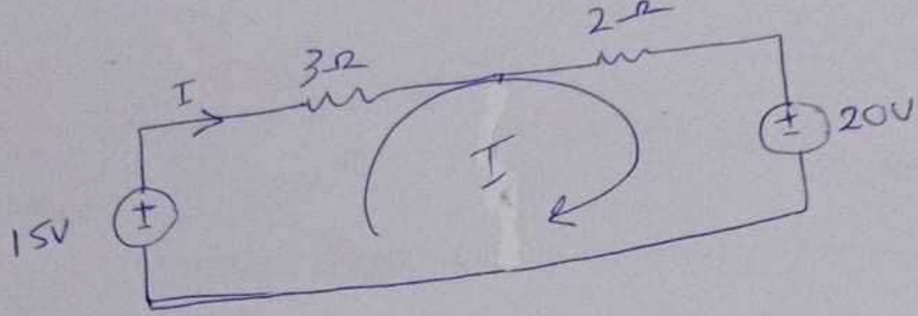
$I = \frac{12.86}{30} = 0.4A$

$V = \frac{12.86 \times 20}{30} = 8.57$

$i_{20} = \frac{8.57}{20} = 0.4$

13



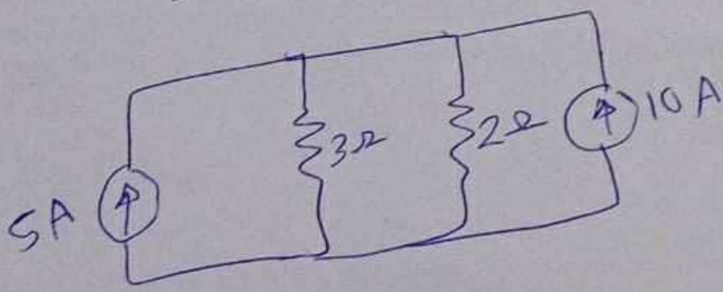


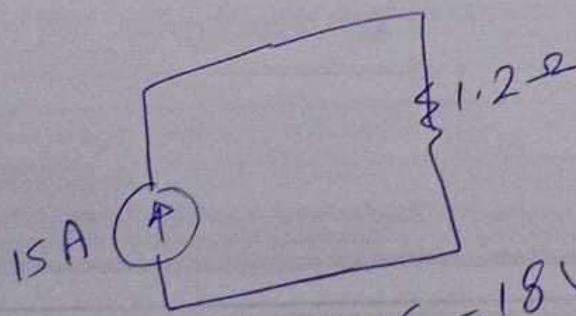
KVL :

$-15 + 3I + 2I + 20 = 0$

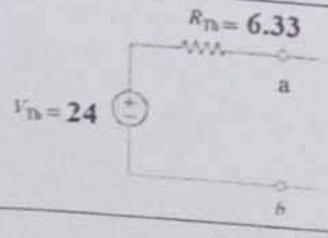
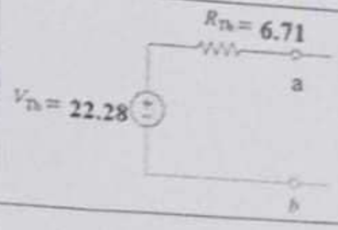
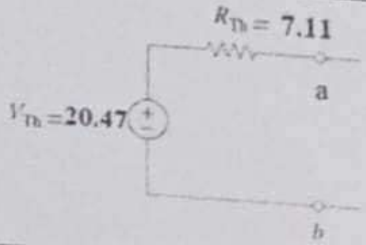
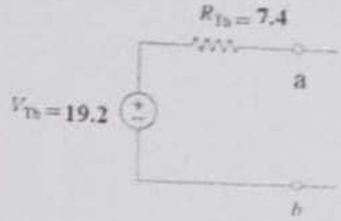
$5I = 5$

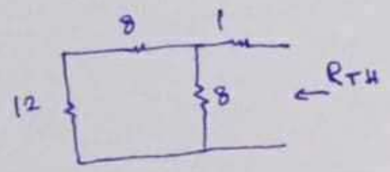
$I = 1A \rightarrow I = -1A.$



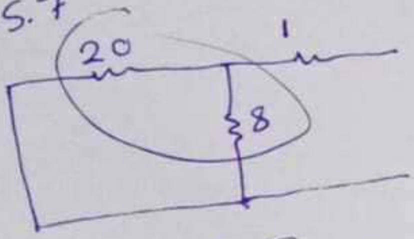


$V = 1.2 \times 15 = 18V$ #

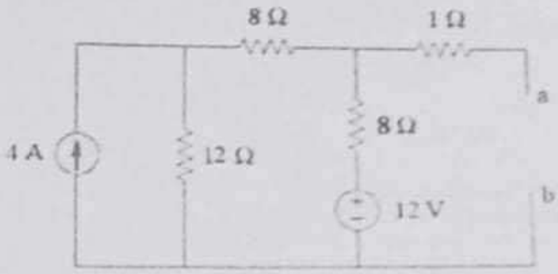
15.	For the circuit below, the Thevenin's equivalent seen between the terminals ab is given as ..		
a.		b.	
c.		d.	

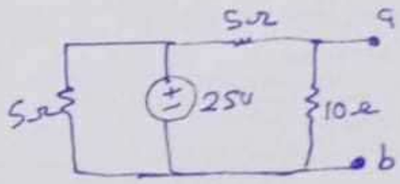


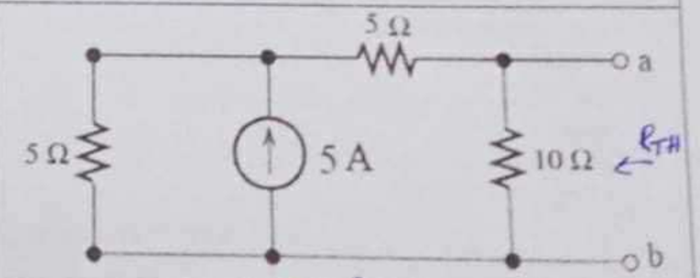
5.7



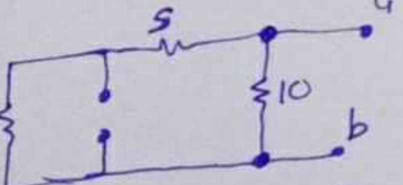
$R_T = 5.7 + 1 = 6.7$





16.	For the circuit shown below, what is the value of the resistor load to be connected between terminal ab to transfer a maximum power? what is the maximum power?			
a.	R_L=5 Ω, P_{max}=125 W	b.	R_L=20 Ω, P_{max}=31.3 W	
c.	R_L=5 Ω, P_{max}=7.8 W	d.	R_L=20 Ω, P_{max}=62.5 W	

$R_{TH} = R_L$



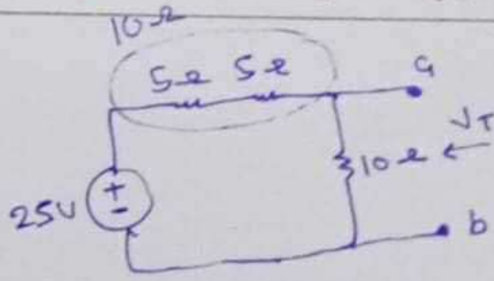
$V_{TH} = 10 \times 2.5$
$= 25$ volt

$R_1 = 10\,\Omega$

$R_2 = \frac{10 \times 10}{10+10} = 5\,\Omega$

$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$

$= \frac{(12.5)^2}{4 \times 20} \rightarrow (12.5)^2$



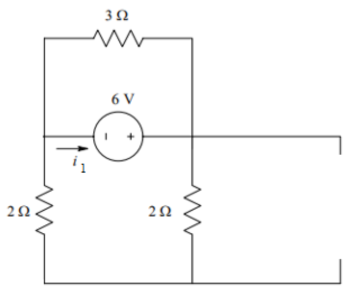
$V_{TH} = V_{10} = \frac{25 \times 10}{10 + 10}$

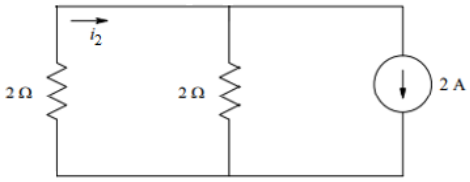
$= 12.5V$

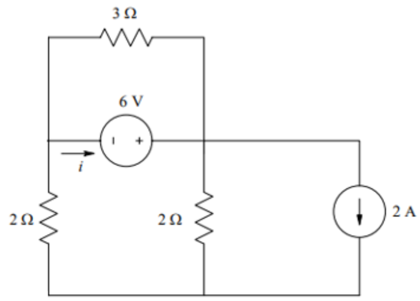
Question # 1 (6 points)

For the circuit shown below, use the **principle of superposition** to find the current i. Draw the corresponding circuit diagrams

a.	Find the current i_1 due to the 6-V voltage source.	i_1 =	3.5	A
b.	Find the current i_2 due to the 2-A current source.	i_2 =	1	A
c.	Find the total current i due to both sources.	i =	4.5	A







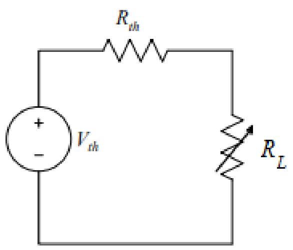
Solution:

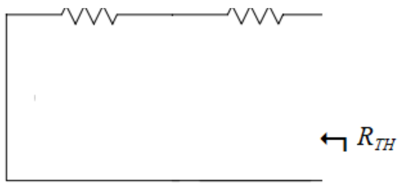
i_1= 6/3+6/4=3.5 A $\quad$ i_2= 2×2(2+2) =1 A $\quad$ $i = i_1 + i_2$ = 3.5 + 1 =4.5 A

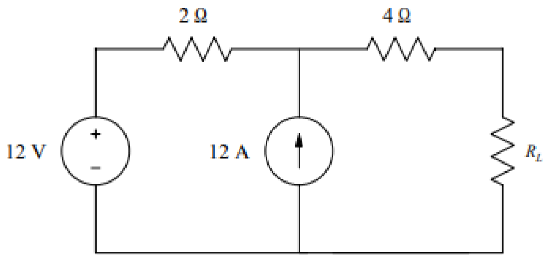
Question # 2 (6 points)

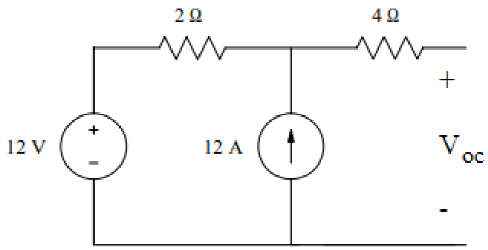
For the circuit shown below, find

a.	the load resistance R_L for maximum power transfer	R_L =	6	Ω
b.	the Thevenin equivalent voltage using source transformation technique.	V_{th} =	36	V
c.	the maximum power absorbed by the load R_L.	P_{max} =	54	W





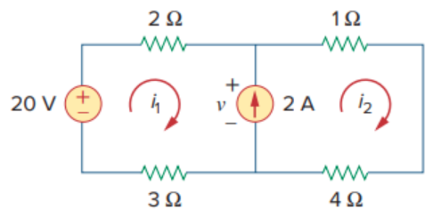




Solution:

R_{th} =2+4= 6 Ω $\quad$ V_{th}= V_{oc} = 12×2+12=36 V

$P_{max} = V_{th}^2/(4×R_{th})$ = (36×36)/(4×6) = 54 W



a) Mesh Equations

From Mesh 1:

$$5i_1 + 5(i_1 + 2) = 20$$

Simplifying:

$$5i_1 + 5i_1 + 10 = 20 \Rightarrow 10i_1 = 10 \Rightarrow i_1 = \boxed{1 \text{ A}}$$

From the current source constraint:

$$i_2 = i_1 + 2 = 1 + 2 = \boxed{3 \text{ A}}$$

b) Voltage across the current source

Method 1:

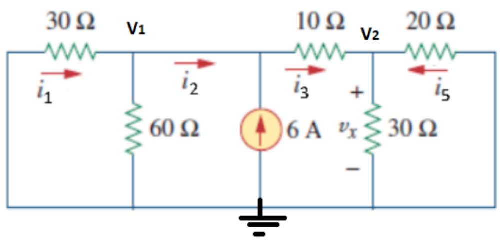
$$v = 5i_2 = 5 \cdot 3 = \boxed{15 \text{ V}}$$

Method 2:

$$v = 20 - 5i_1 = 20 - 5 \cdot 1 = \boxed{15 \text{ V}}$$

Final Answers:

$$i_1 = 1 \text{ A}, \quad i_2 = 3 \text{ A}, \quad v = 15 \text{ V}$$



Nodal Equations:

$$\left(\frac{1}{60}+\frac{1}{30}+\frac{1}{10}\right)V_1-\frac{1}{10}V_2=6$$
$$-\frac{1}{10}V_1+\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{20}\right)V_2=0$$

Simplified Form:

$$\frac{3}{20}V_1-\frac{1}{10}V_2=6$$
$$-\frac{1}{10}V_1+\frac{11}{60}V_2=0$$

Cleared of Fractions: $3V_1-2V_2=120, \quad -6V_1+11V_2=0$

Solutions:

$$V_1=\frac{440}{7}=62.86\text{ V}, \quad V_2=\frac{240}{7}=34.29\text{ V}$$

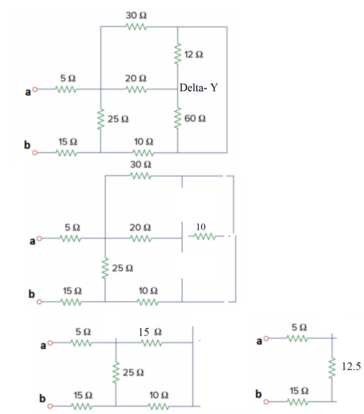
Current through the $10\ \Omega$ resistor:

$$I_3=\frac{V_1-V_2}{10}=\frac{62.86-34.29}{10}=\frac{20}{7}=2.857\text{ A}$$

Power supplied by the 6A current source:

$$P_{6A}=-V_1\cdot 6=-62.86\cdot 6=\frac{-2640}{7}=-377.14\text{ W}$$

Equivalent Resistance and Circuit Analysis Questions



To find the equivalent resistance between terminals a and b, we simplify the network step by step using series-parallel combinations and an effective transformation based on recognizing a short circuit.

The triangle formed by $12\,\Omega$, $60\,\Omega$, and a short circuit ($0\,\Omega$) can be replaced by:

$$R = \frac{12 \cdot 60}{12 + 60 + 0} = \frac{720}{72} = 10\,\Omega$$

This $10\,\Omega$ resistor is in series with a $20\,\Omega$ resistor:

$$R_1 = 10 + 20 = 30\,\Omega$$

The result is in parallel with a $30\,\Omega$ resistor:

$$R_2 = \frac{30 \cdot 30}{30 + 30} = \frac{900}{60} = 15\,\Omega$$

Adding $10\,\Omega$ in series:

$$R_3 = 15 + 10 = 25\,\Omega$$

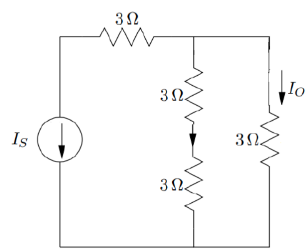
Now this is in parallel with a $25\,\Omega$ resistor:

$$R_4 = \frac{25 \cdot 25}{25 + 25} = \frac{625}{50} = 12.5\,\Omega$$

Adding the final series resistors $5\,\Omega$ and $15\,\Omega$:

$$R_{\text{eq}} = 12.5 + 5 + 15 = \boxed{32.5\,\Omega}$$

Current Divider (Q2_B)



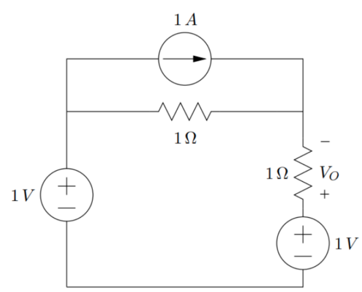
In this circuit, the total current I_s splits between two branches:

- Left branch: two $3\,\Omega$ resistors in series $\rightarrow R_1 = 6\,\Omega$ - Right branch: one $3\,\Omega$ resistor $\rightarrow R_2 = 3\,\Omega$

Using the current divider rule:

$$\frac{I_o}{I_s} = \frac{R_1}{R_1 + R_2} = -\frac{6}{6+3} = -\frac{6}{9} = -\boxed{\frac{2}{3}}$$

textbfVoltage Calculation (Figure 4)



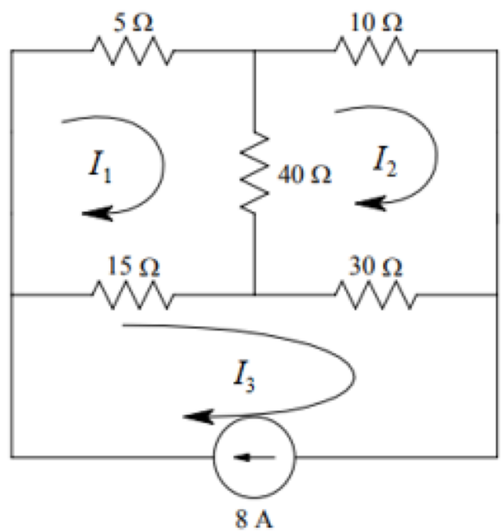
Walking through the loop clockwise:

- Bottom voltage source: $+1\,\text{V}$ - Middle resistor: $1\,\text{A} \cdot 1\,\Omega = +1\,\text{V}$ - Top voltage source: $-1\,\text{V}$

Using KVL or voltage division:

$$V_o = (1\,\text{V} - 1\,\text{V} - 1\,\text{A} \cdot 1\,\Omega) \cdot \frac{1}{1+1} = -1 \cdot \frac{1}{2} = -\boxed{0.5\,\text{V}}$$

Question 3: Mesh Analysis



a) Mesh Equations:

$$60i_1 - 40i_2 - 15 \cdot 8 = 0$$
$$-40i_1 + 80i_2 - 30 \cdot 8 = 0$$

Answer:

$$i_1 = 6 \text{ A}, \quad i_2 = 6 \text{ A}, \quad i_3 = 8 \text{ A}$$

b) Current through the 30 Ω resistor:

$$i_{30\Omega} = i_2 - i_3 = -2 \text{ A}$$

c) Power dissipated by the 15 Ω resistor:

$$P_{15\Omega} = (i_1 - i_3)^2 \cdot 15 = (-2)^2 \cdot 15 = 60 \text{ W}$$

d) Power balance check:

Voltage across the 8 A current source:

$$V_{\text{source}} = \frac{P_{\text{absorbed}}}{I} = \frac{2340}{8} = 292.5 \text{ V}$$

Power supplied by the current source:

$$P_{\text{source}} = V \cdot I = 292.5 \cdot 8 = 2340 \text{ W}$$

Total power absorbed by resistors:

$$P_{15\Omega} + P_{60\Omega} + P_{30\Omega} = 60 + 2160 + 120 = 2340 \text{ W}$$

Conclusion: Power supplied equals power absorbed. $\sum P = 0$ is satisfied.

Chapter 6

DC-CIRCUIT

30 / 7 / 2025

[1] Capacitors and inductors are called energy storage elements.

[2] A capacitor is a passive element designed to store energy in it's electric field.

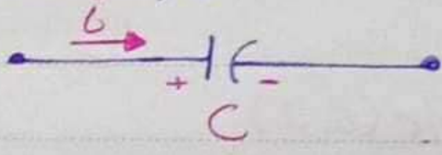
[3] A capacitor consists of two conducting plates separated by an insulator (or dielectric).

[4] Capacitance C is the ratio of the charge q on one plate of a capacitor to the voltage difference v between the two plates, measured in farads (F).

$$q = CV \qquad C = \frac{\boxed{\varepsilon} A}{d}$$ permittivity.

[5] If i is flowing into the +ve terminal of C

* Charging $\Rightarrow i$ is (+ve)
* Discharging $\Rightarrow i$ is (-ve)



[6] The current-voltage relationship of capacitor according to above convention is

$$i = C\frac{dV}{dt} \qquad V = \frac{1}{C}\int_{t_0}^{t} i\,dt + V_0(t_0)$$

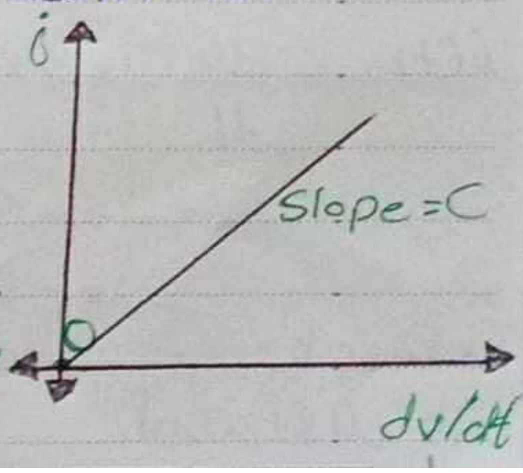
[7] The energy, w, stored in the capacitor is:-

$$W = \frac{1}{2}CV^2$$

[8] A capacitor is

* an open circuit to dc ($dV/dt = 0$).
* its voltage cannot change abruptly

Units: F, PF (10^{-12}), nF (10^{-9}), μF (10^{-6})



Example (6.1)

a) Calculate the charge stored on 3 PF capacitor with 20V across it.

b) Find the energy stored in the capacitor.

sol:

a) $q = CV = 3 \times 10^{-12} \times 20 = 60\ PC$.

b) $w = \frac{1}{2} CV^2 = \frac{1}{2} \times 3 \times 10^{-12} \times (20)^2 = 600\ PJ$.

* What is the Power? $P = \frac{dw}{dt} = 0$

مشتقة الثابت صفر

$P = vi = (20)\left(C \frac{dv}{dt}\right) = 0$

Example (6.2)

The voltage across a 5-μF capacitor is

$v(t) = 10 \cos 6000t\ V$ $\quad \omega = 2\pi f$

Calculate the current through it. $\quad f = \frac{1}{T}$

sol:-

$i(t) = C \frac{dv}{dt} = 5 \times 10^{-6} \times \frac{d(10 \cos 6000t)}{dt}$

$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t$

$= -0.3 \sin 6000t\ A$.

* Discharge capacitor because the sign of v & I is different.

/ /

Example (6.3):

Determine the voltage across 2-μF capacitor if the current through is

$$i(t) = 6e^{-3000t} \text{ mA}$$

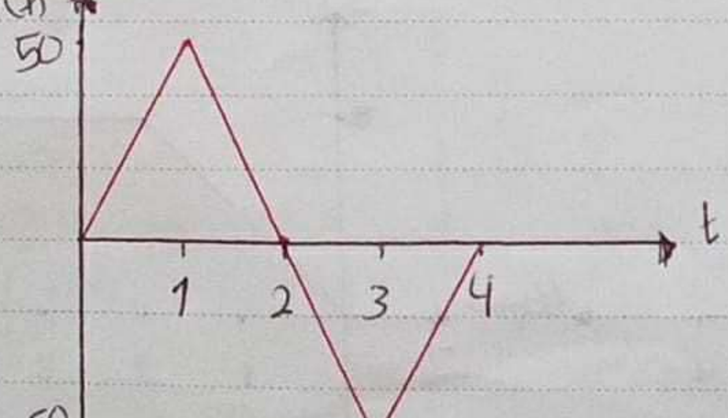
Assume that the initial capacitor voltage is zero.

$$v = \frac{1}{C}\int_0^t i \, dt + v(0)$$

$$v = \frac{1}{2}\int_0^t 6e^{-3000t} \, dt \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000t}\Big|_0^t = (1 - e^{-3000})V$$

Example (6.4):

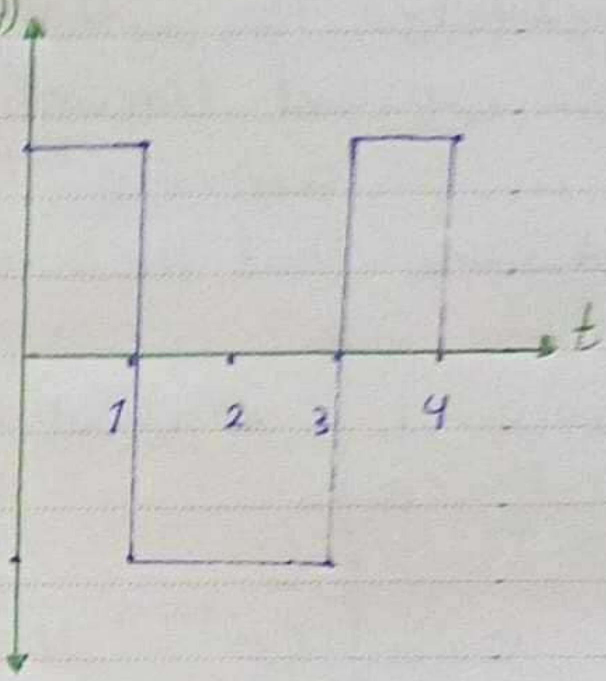


Determine the current through 200-μF capacitor.

$$v(t) = \begin{cases} 50t, & 0<t<1 \\ 100-50t, & 1<t<3 \\ -200+50t, & 3<t<4 \\ 0, & \text{otherwise} \end{cases}$$

$i = C\, dv/dt$, $C = 200\,\mu F$

$$i(t) = 200\times10^{-6} \times \begin{cases} 50 & 0<t<1 \\ -50 & 1<t<3 \\ 50 & 3<t<4 \\ 0 & \text{otherwise}\end{cases}$$

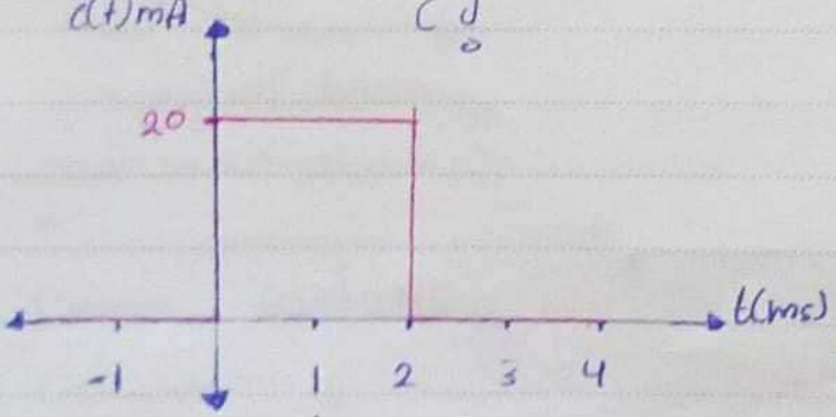


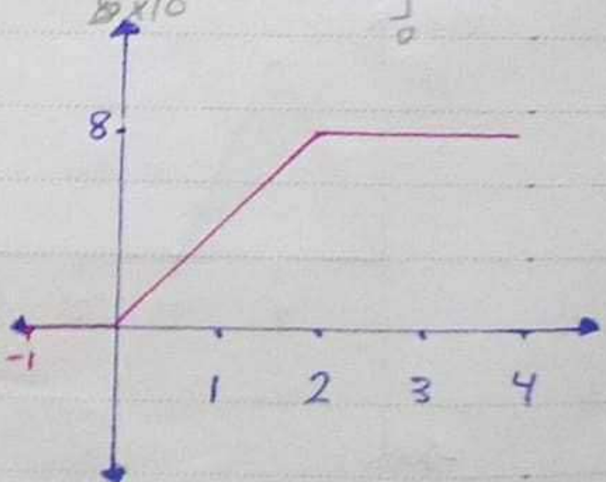
$$= \begin{cases} 10\,mA & 0<t<1 \\ -10\,mA & 1<t<3 \\ 10\,mA & 3<t<4 \\ 0 & \text{otherwise}\end{cases}$$

Exit Example:

Find the capacitor voltage that is associated with the current shown graphically. The value of the cap. is $5\,\mu F$.

$v = \frac{1}{C}\int_0^t i(t)\,dt + v(0)$ $= \frac{1}{5\times10^{-6}} \times 20 \Big]_0^2 +$





$i = C\,dv/dt$

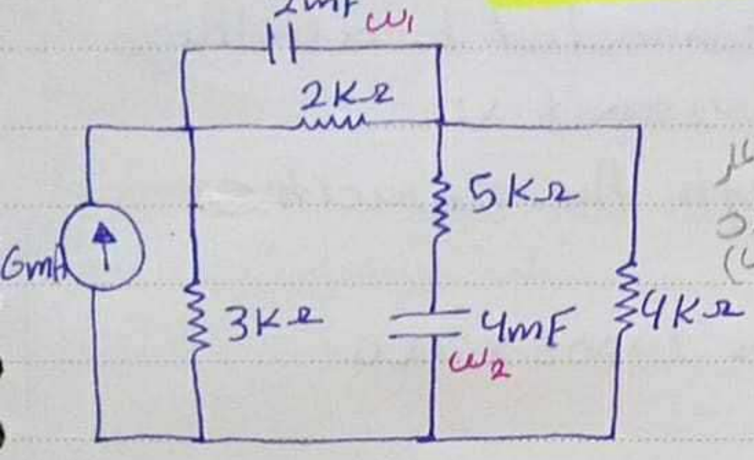
$dv = \frac{i}{C}\,dt$

$v = i/C\; t$

$$\begin{cases} v(t) = 0 & t<0 \\ \quad = 4000t & 0\le t\le 2\ ms \\ \quad = 8 & t\ge 2\ ms \end{cases}$$

$\hookrightarrow \frac{1}{\ldots}(0)\Big] + 8000\times10^{-3}$

Example (6.5): In DC circuit each capacitor replace O.C.



$$w = \frac{1}{2} C v^2$$

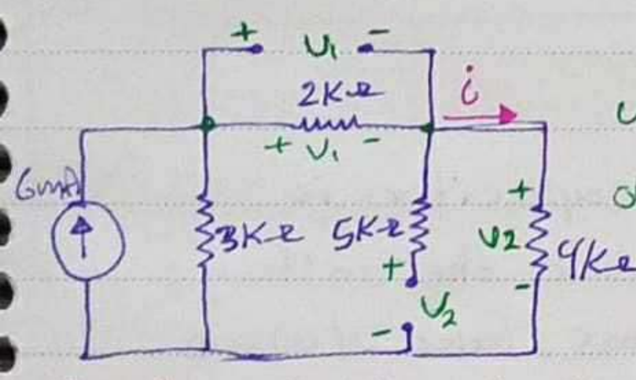
التيار المار بالمقاومة (4+2)

$$i = \frac{3}{3+2+4}(6mA) = 2mA$$

$$v_1 = 2000i = 4V$$

$$v_2 = 4000i = 8V$$

$$w_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2}(2\times10^{-3})(4)^2 = 16mJ$$



v_1 / v_2 مع المقاومات المتوازية لها

$$w_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(4\times10^{-3})(8)^2 = 128 mJ$$

Practice Problem (6.1)

What is the voltage across a 4.5-μF capacitor if the charge on one plate is 0.12mC? How much energy is stored?

$$i = C\frac{dv}{dt}$$

$$v = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(0)$$

$$q = Cv$$

$$v = \frac{q}{C} = \frac{0.12\times10^{-3}}{4.5\times10^{-6}} = 26.67 \text{ volt}$$

$$w = \frac{1}{2} C v^2 = \frac{1}{2} \times 4.5 \times 10^{-3} \times (26.67)^2$$

$$= 1.6 mJ$$

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Parctice Problem 6.2:

If a 10-μF capacitor is connected to a voltage source with $v(t) = 75 \sin 2000t$ V determine the current through the capacitor.

$$i = c\frac{dv}{dt} = 10\times10^{-6}\times 75\times\cos 2000t \times 2000$$

$$= 1.5\cos(2000t)\ A.$$

Paractice Problem 6.3:

The current through a 100-μF capacitor is $i(t) = 50\sin(120\pi t)$ mA. Calculate the voltage across it at $t = 1$ms and $t = 5$ms. Take $v(0) = 0$

$$V = \frac{1}{c}\int_0^t 50\sin(120\pi t)\,dt + 0$$

$$= \frac{1}{100\times10^{-6}} \times 50 \times \frac{-\cos(120\pi t)}{120\pi}$$

$$v(1) = \frac{1}{100\times10^{-6}} \times 50 \times \frac{\cos(120\pi\times1\times10^{-3})}{120\pi}$$

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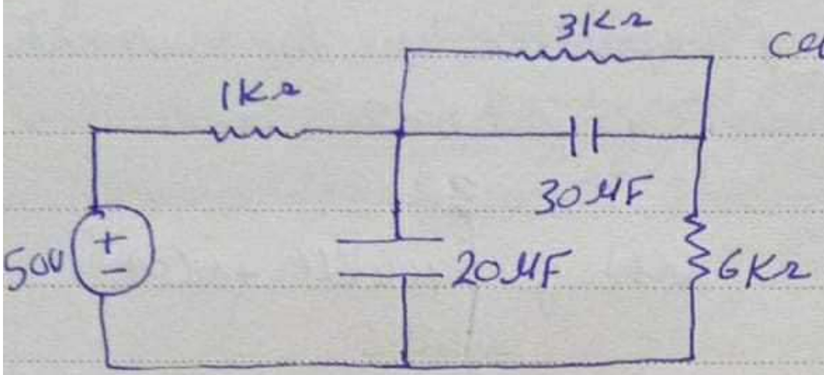
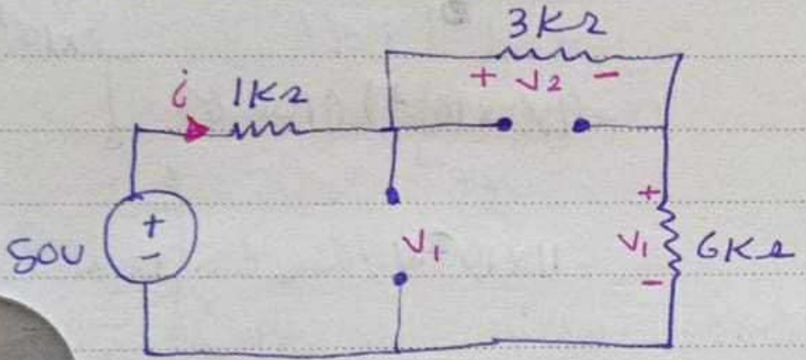
Practice Problem 6.5: Find the energy stored in the capacitor.



$i = \frac{50}{1+3+6} = 5mA$

$v_1 = 6\times10^3 \times 5\times10^{-3} = 30\,volt$

$v_2 = 3\times10^3\times5\times10^{-3} = 15\,volt.$



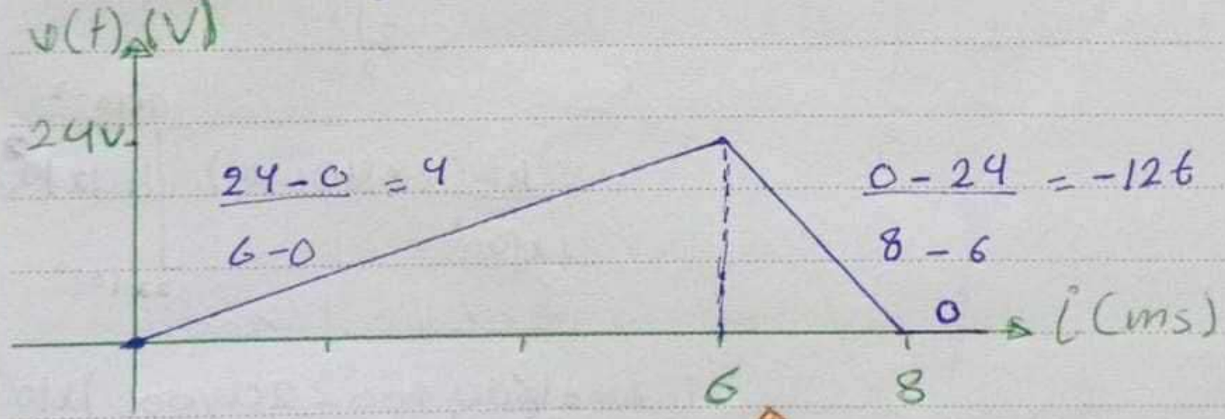
$w = \frac{1}{2}CV^2 = \frac{1}{2}\times20\times10^{-6}\times(50)^2$

$= 0.025\,J$

$w = \frac{1}{2}\times3\times10^3\times(15)^2 = 3.375mJ.$

Exercise:

The voltage across a 5-μF capacitors is given.

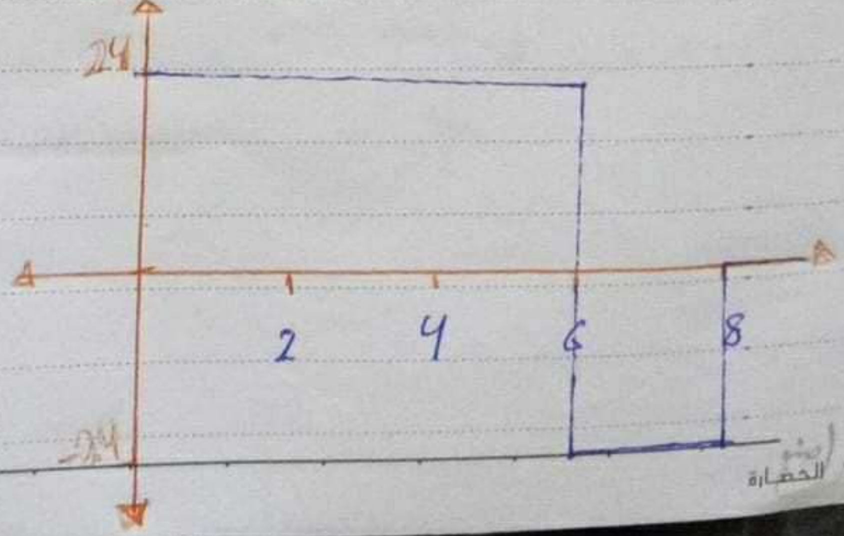


$\frac{24-0}{6-0} = 4$

$\frac{0-24}{8-6} = -12t$

$i = C\frac{dv}{dt}$

$$i(t) = \begin{cases} 4t, & 0 \le t \le 6 \\ -12t, & 6 \le t \le 8 \\ 0, & t > 8 \end{cases}$$



Series and Parallel capacitors.

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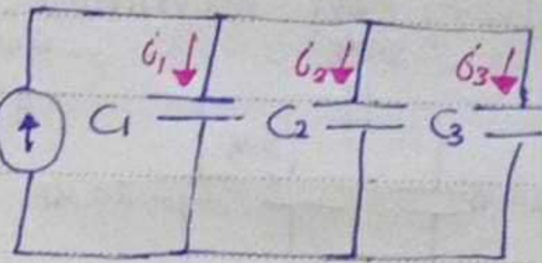
* Capacitors in Parallel:

$C_{equ} = C_1 + C_2 + C_3 + \dots + C_N$

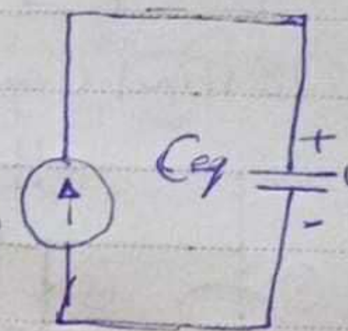


Proof:

$i = i_1 + i_2 + i_3 + \dots + i_N$

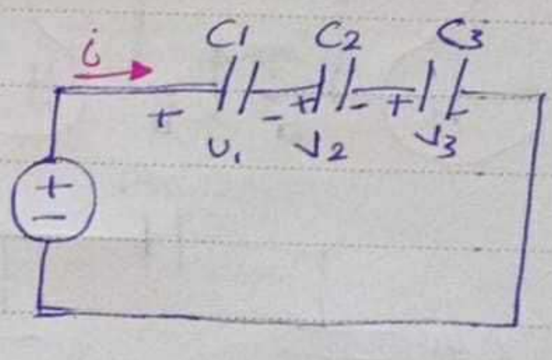
$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

$$= \sum_{k=1}^{N} C_K \frac{dv}{dt} = C_{equ} \frac{dv}{dt}$$

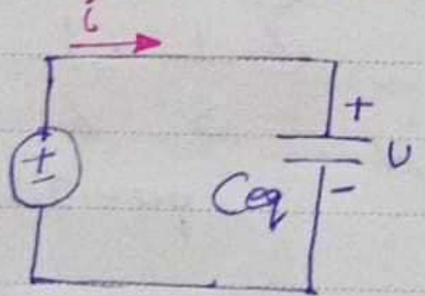


* Capacitors in series:

$$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



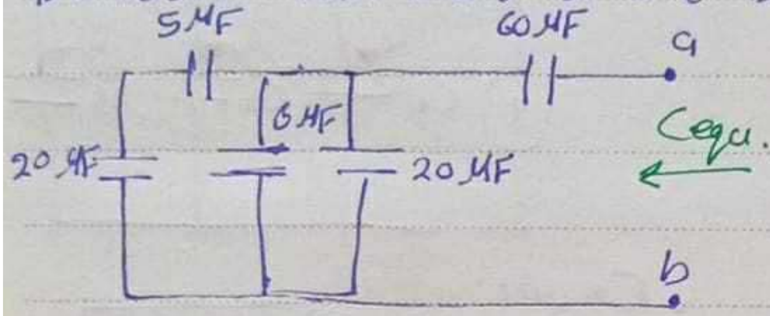
Proof:

$v = v_1 + v_2 + v_3 + \dots + C_N$

$$v = \frac{1}{C_1} \int_{t_0}^{t} i(\tau)\, d\tau + v_1(t_0) + \frac{1}{C_2} \int_{0}^{t} i(\tau)\, d\tau + v_2(t_0) + \dots$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N} \right) \int_{t_0}^{t} i(\tau)\, d\tau + v_1(t_0) + v_2(t_0) = \frac{1}{C_{equ}} \int_{t_0}^{t} i(\tau)\, d\tau + v_0$$

Example (6.6): Find the equivalent capacitance seen between terminals a and b.

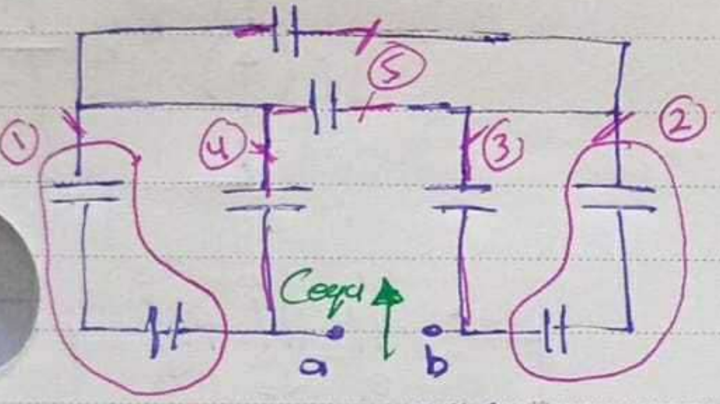


$\frac{20\times5}{20+5} = 4\,\mu F$

$4+6+20=30$

$C_{equ} = (30\times60)/(30+60) = 20\,\mu F$

6.18: Find C_{eq} in the circuit if all capacitors are $4\,\mu F$.



① $\frac{4\times4}{4+4} = 2\,\mu F$

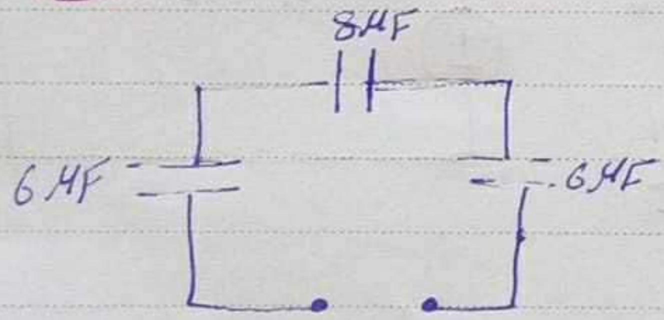
② $2\,\mu F$

③ $2+4=6\,\mu F$

④ $2+4=6\,\mu F$

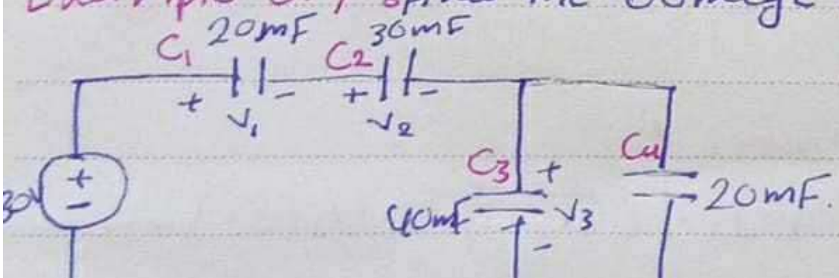
⑤ $4+4=8\,\mu F$

⑥ $\frac{1}{8}+\frac{1}{6}+\frac{1}{6} = 0.4583\,\mu F$



$C_{equ} = 2.18\,\mu F$.

Example 6.7: Find the voltage across each capacitor.



$v_3 \rightarrow$ ~~are~~ is the same at C_3 & C_4 because it's parallel

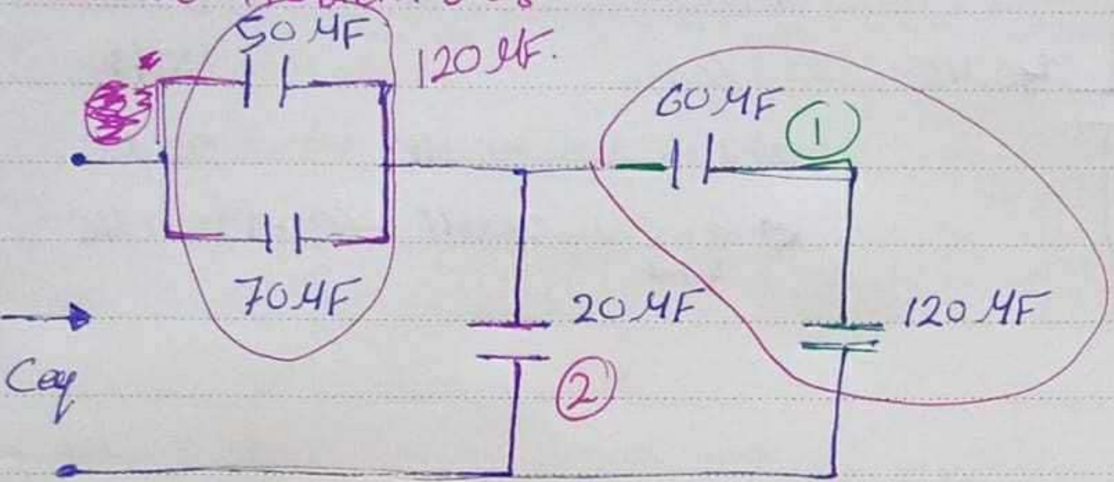
$C_{equ} = \frac{1}{20}+\frac{1}{30}+\frac{1}{60} = 10\,mF$

$q_{total} = C_{eq}(30) = 0.3C$, charge acts like current, $q_1 = q_2 = q_{total}$

$v_1 = \frac{q_1}{C_1} = 15V$, $v_2 = \frac{q_2}{C_2} = 10V$, $v_3 = 30 - v_2 - v_2$

$= 5V$

Practice Problem 6.6:

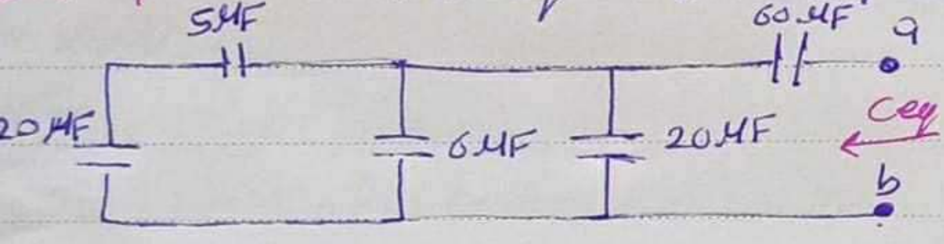


$$C_1 = \frac{60 \times 120}{60 + 120} = 40\,\mu F$$

$$C_2 = 40 + 20 = 60\,\mu F$$

$$C_3 = \frac{[illegible] \times 120}{120 + 60} = 40\,\mu F$$

Example 7: Find the equivalent capacitance between a and b.



[illegible]

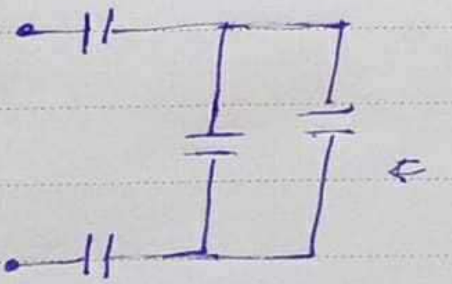
Exercise: Comput the equivalent capacitance.

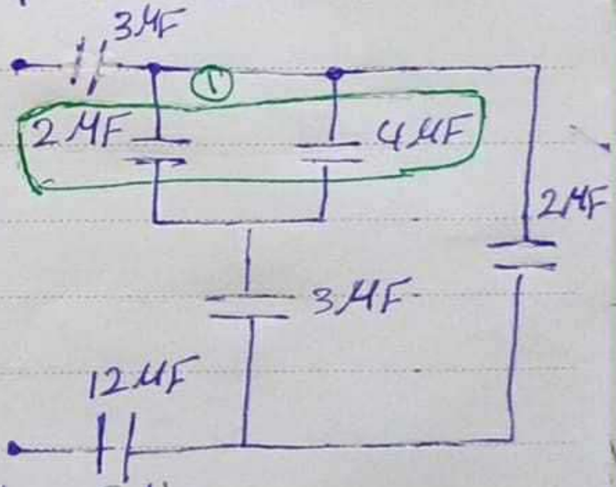
1) $2 + 4 = 6\,\mu F$

2) $\frac{6 \times 3}{6 + 3} = 2\,\mu F$

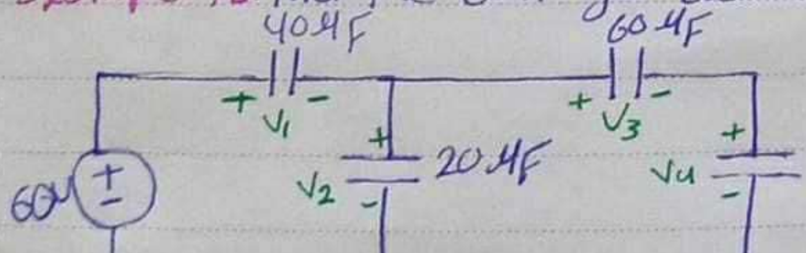
3) $2 + 2 = 4\,\mu F$

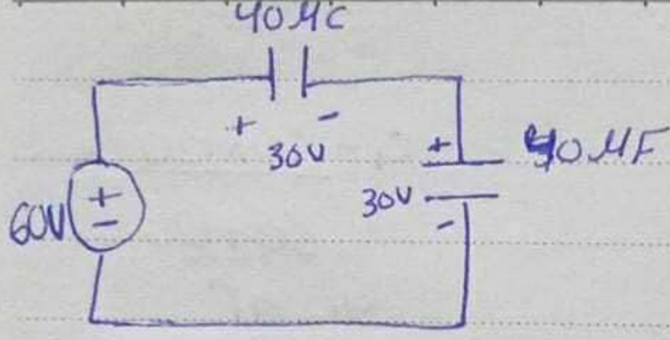
4) $C_{equ} = \frac{1}{4} + \frac{1}{12} + \frac{1}{3} = 1.5\,\mu F$





Example 9: Find the voltage across each of the





1) $\dfrac{60\times 30}{60+30} = 20\,\mu F$

2) $20 + 20 = 40\,\mu F$

$q_{total} = CV = 20\,\mu F \times 60 = 1200\,\mu C$

3) $\dfrac{40\times 40}{40+40} = 20\,\mu C$

Inductors

/ /

① passive element designed to store energy in it's magnetic field.

② An inductor consists of a coil of counducting wire.

③ Inductance is the property where by an inductor exhibits opposition to the change of current flowing through it.

* Measured in henrys (H).

$V = L \frac{di}{dt}$ and $L = \frac{N^2 \mu A}{L}$

$N \rightarrow$ number of turns
$L \rightarrow$ length
$A \rightarrow$ cross-section area.
$\mu \rightarrow$ permeability of the core.

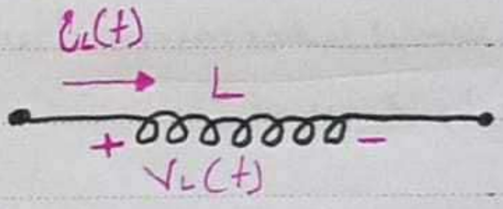
$\mu = \mu_r \mu_0$

$\mu_0 = 4\pi \times 10^{-7}$ (H/m)

* The unit of inductors (mH) and (μH).

* The current-voltage relationship of an inductor

$$i = \frac{1}{L} \int_0^t v(t)\,dt + i(t_0)$$



* The energy stored by an inductor is: $W = \frac{1}{2} L i^2$

* An inductors acts like a short circuit to dc ($di/dt = 0$) and its current cannot change abruptly.

* $V = L \frac{di}{dt}$

/ /

Example 6.8: The current through a 0.1 H inductor is $i(t) = 10e^{-5t}$. Find the voltage across the inductor and the energy stored

$$V = L\frac{di}{dt}$$

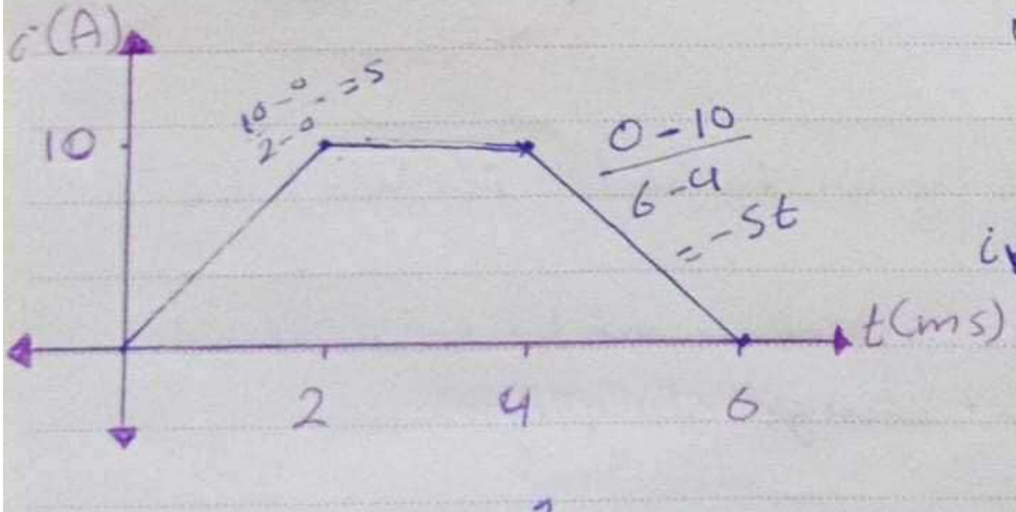
$L = 0.1\,H$

$$v = 0.1\frac{d(10te^{-5t})}{dt} = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1-5t)\,V$$

* The energy stored is:-

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)(100)t^2e^{-10t} = 5t^2e^{-10t}\,J.$$

6.40: The current through a 5-mH inductors is shown in Figure. Determine the voltage across the inductor at $t = 1, 3$ and 5.

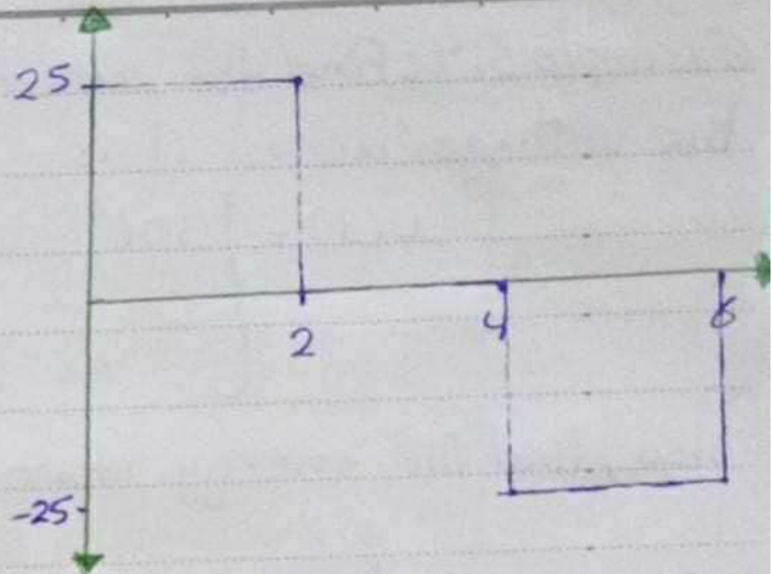


$$v(t) = L\frac{di}{dt}$$

$$i(t) = \begin{cases} 5t, & 0 < t < 2ms \\ 10, & 2 < t < 6ms \\ 30-5t, & 4 < t < 6ms \end{cases}$$

$$V = L\frac{di}{dt} = \frac{5\times10^{-3}}{10^{-3}}\begin{cases} 5, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -5, & 4 < t < 6ms \end{cases}$$

$$v = \begin{cases} 25, & 0<t<2\,ms \\ 0, & 2<t<4\,ms \\ -25, & 4<t<6\,ms. \end{cases}$$



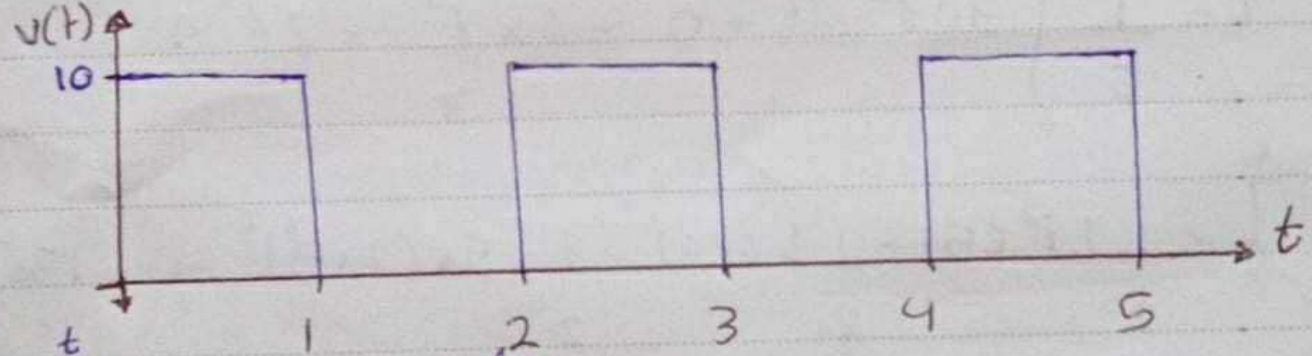
At $t = 1\,ms$, $v = 25V$

At $t = 3\,ms$, $v = 0V$

At $t = 5\,ms$, $v = -25V$

6.42: If the voltage is applied across the terminals of 5-H inductor calculate the current through the inductor.

• Assume $i(0) = -1A$.



$$i = \frac{1}{L}\int_0^t v\,dt + i(0) = \frac{1}{5}\int_0^t v\,d(t) - 1$$

For $0<t<1$, $i = \frac{10}{5}\int_0^1 t\,dt - 1 = 2t - 1\,A$.

For $1<t<2$, $i = 0 + i(1) = 1A$

For $2<t<3$, $i = \frac{1}{5}\int_2^3 10\,dt + i(2) = 2t\Big]_2^3 + 1 = 3A$

For $3<t<4$, $i = 0 + i(3) = 3A$

For $4<t<5$, $i = \frac{1}{5}\int_4^5 10\,dt + i(4) = 2t\Big]_4^t + 3 = 2t - 5\,A$

Example 6.9: Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2 & , t > 0 \\ 0 & , t < 0 \end{cases}$$

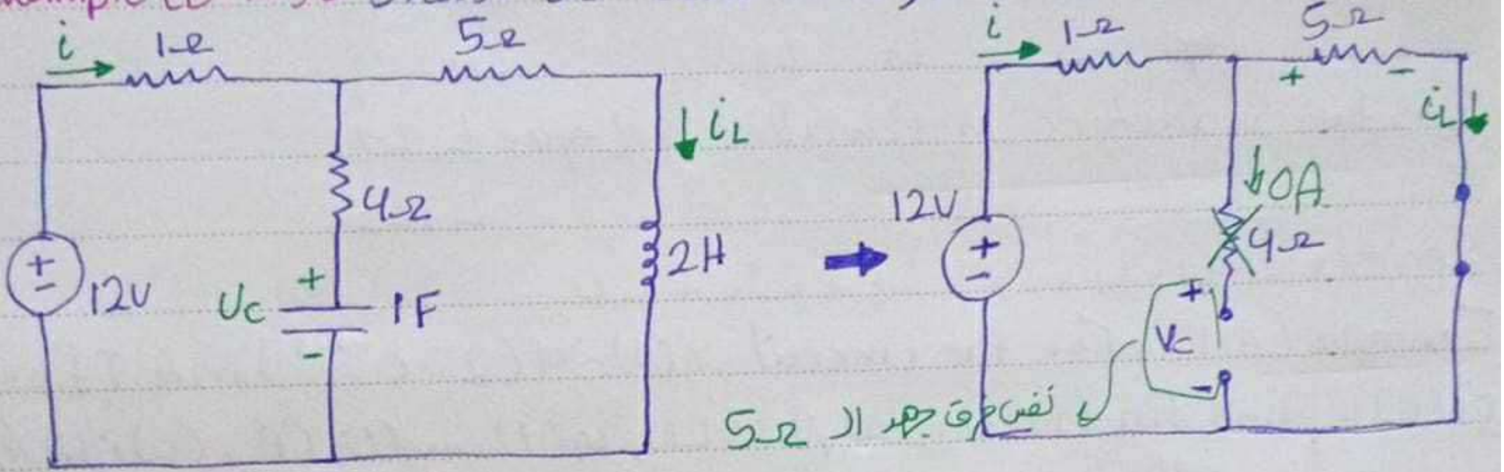
Also, find the energy stored at $t = 5s$. Assume $i(v) > 0$.

Since $i = \frac{1}{L}\int_0^t v(t)\,dt + i(t_0)$ and $L = 5H$.

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3\ A.$$

$$w\Big]_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2} \times 5 \times (2 \times 5^3)^2 - 0 = 156.25\,kJ$$

Example (6.10): Under dc conditions, find a) i, v_C and i_L.



$$i = i_L = \frac{12}{5+1} = 2A$$

$$v_C = 5i = 10A$$

$$w_C = \frac{1}{2} C v^2 = \frac{1}{2}(1)(10^2) = 50J$$

$$w_L = \frac{1}{2} L i_L^2 = \frac{1}{2}(2)(2^2) = 4J$$

Series and Parallel Inductors.

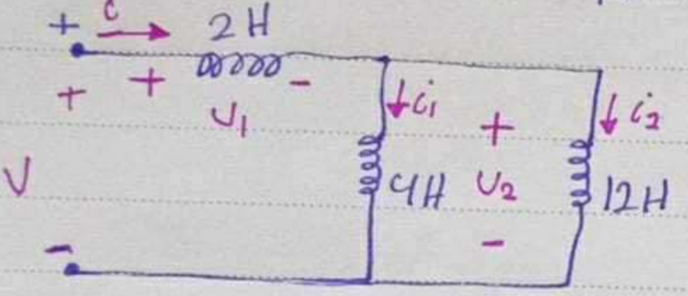
/ /

Parallel: $\frac{1}{L_{equ}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$

For two inductors in Parallel: $L_{equ} = \frac{L_1 L_2}{L_1 + L_2}$

Series: $L_{equ} = L_1 + L_2 + L_3 + \dots + L_N$

Example (6.12): For the circuit, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA. Find: (a) $i_1(0)$, (b) $v(t)$, $v_1(t)$ and $i_2(t)$, (c) $i_1(t)/i_2$

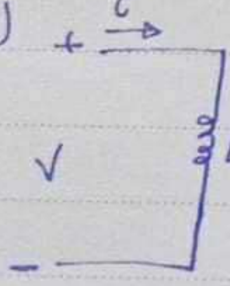


a) $i = i_1 + i_2$

$i(0) = i_1(0) + i_2(0)$

$4 = i_1(0) + -1 \Rightarrow i_1(0) = 5$ mA.

b)



$L_{equ} = 5H$ $\quad v = L_{equ}\frac{di}{dt} = (5)(4)(-10)e^{-10t} = 200e^{-10t}$ mV

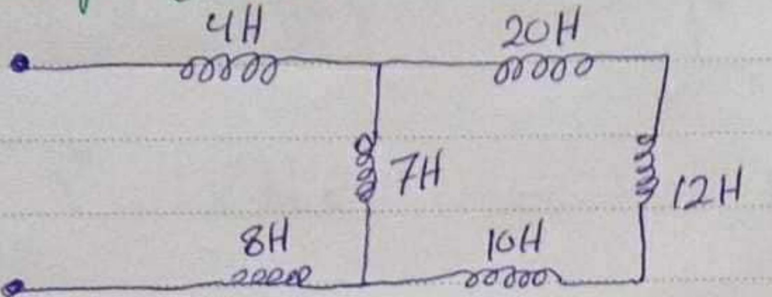
$v_1 = 2\frac{di}{dt} = 2(4)(-10)e^{-10t}$

$v_2 = v - v_1 =$

c) $i_1(t) = \frac{1}{L}\int_0^t v_2(t')\,dt = \frac{1}{4}\int_0^t$

$i_2(t) = i(t) - i_1(t)$

Example (6.11):

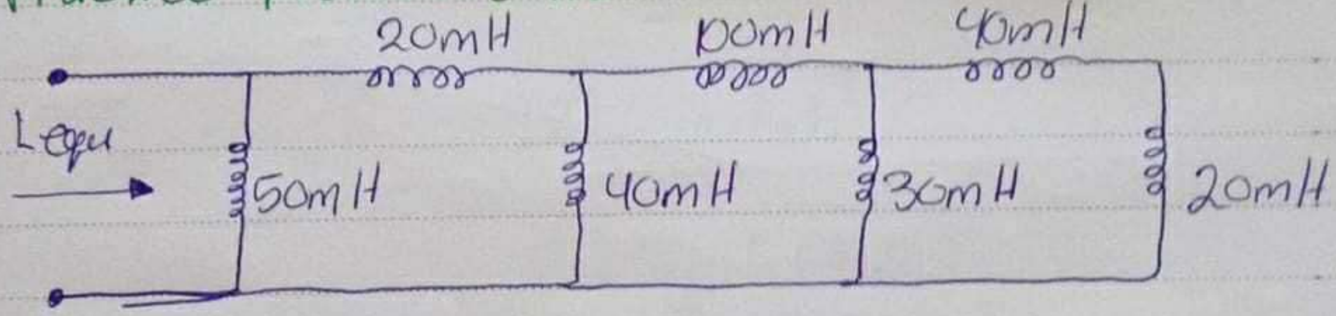


$R_1 = 20 + 12 + 10 = 42H$

$L_2 = \frac{42 \times 7}{42 + 7} = 6H$

$L_3 = 4 + 6 + 8 = 18H$

Practice Problem 6.11:



$L_1 = 20 + 40 = 60mH$

$L_2 = \frac{60 \times 30}{60 + 30} = 20mH$

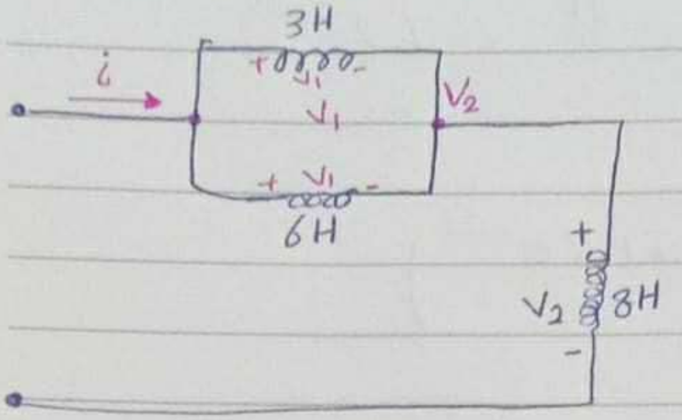
$L_3 = 20 + 100 = 120mH$

$L_4 = \frac{40 \times 120}{40 + 120} = 30mH$

$L_5 = 30 + 20 = 50mH$

$L_6 = \frac{50 \times 50}{50 + 50} = 25mH$

Particle Problem 6.12:



$i_1(t) = 0.6e^{-2t}$

$i(0) = 1.4A$

a) Find $i_2(0)$.

b) Find $i_2(t)$ and $i(t)$.

c) Find $v_1(t)$, $V_2(t)$ and $v(t)$

a) $i_2(0) = i(0) - i_1(0) \rightarrow i_1(0) = 0.6\cancel{e^{-2\times 0}} = 0.6$

$= 1.4 - \boxed{0.6}$

$= 0.8A$

b) $i_2 = \frac{1}{L_2}\int v_1\,dt$

v_1 For 6H ⇒ * لأن (3H,6H) موصولات على التوازي بالتالي V1 متساوي بينهما.

$v_1(t) = L\frac{di}{dt}$

$= 6\times -2\times 0.6e^{-2t}$

$= -7.2e^{-2t}$

v_1 For 3H ⇒

$-7.2e^{-2t} = 3\frac{di_2(t)}{dt} \rightarrow$

$\frac{di_2(t)}{dt} = \frac{-7.2e^{-2t}}{3}$

$= -2.4e^{-2t}$

$i_2(t) = \int -2.4e^{-2t}\,dt$

$= \frac{-2.4e^{-2t}}{-2} + C$

$= +1.2e^{-2t} + C$

$i_2(0) = 0.8$

$i_2(0) = +1.2\cancel{e^{-2\times 0}} + C = 0.8$

$C = 0.8 - 1.2 = -0.4$

$i_2(t) = 1.2e^{-2t} - 0.4$

$i(t) = i_1(t) + i_2(t)$

$= 0.6e^{-2t} + (-0.4 + 1.2e^{-2t})$

$= -0.4 + (0.6 + 0.4)e^{-2t}$

$= -0.4 + 1.8e^{-2t}$

$i(t) = 1.8e^{-2t} - 0.4$

c)

$$v_1(t) = L\frac{di}{dt} = 6 \times 0.6 \times -2 \times e^{-2t} = -7.2e^{-2t}$$

$$v_2(t) = L\frac{di}{dt} = 8\frac{d}{dt}\left(-0.4 + 1.8e^{-2t}\right)$$

$$= 8(0 + 1.8 \times -2 \times e^{-2t})$$

$$= 8 \times (-3.6e^{-2t})$$

$$= -28.8e^{-2t}\ V$$

$$V(t) = V_1(t) + V_2(t)$$

$$= (-7.2e^{-2t}) + (-28.8e^{-2t})$$

$$= (-7.2 - 28.8)e^{-2t}$$

$$= -36e^{-2t}\ V$$

Chapters-
9 + 10

CH9 :-

AC circuits Analysis.

/ /

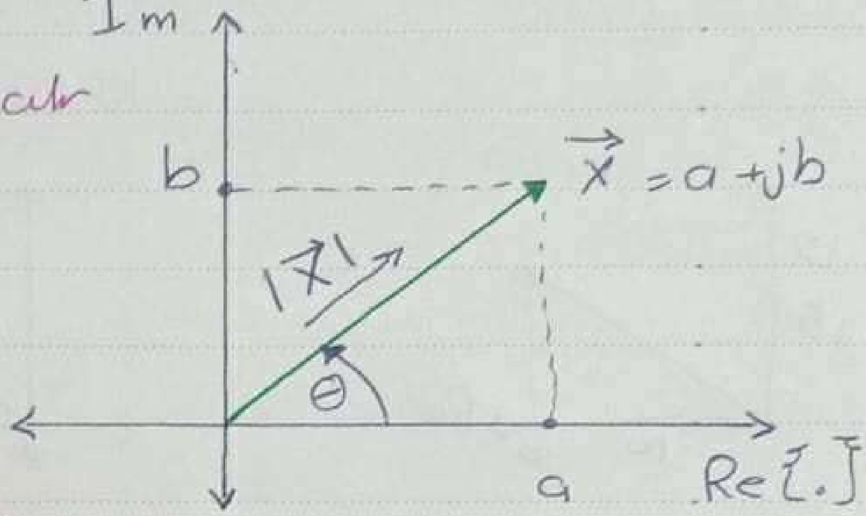
* Complex number:

$\sqrt{-1} = j$

$\vec{X} = a + jb$ (rectangular form)

$a = Re\{\vec{X}\}$

$b = Im\{\vec{X}\}$



$\vec{X} = |\vec{X}| \angle \theta$ (Polar form).

* convert from rectangular form to polar form:

$\vec{X} = a + jb$, $|\vec{X}| = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}(b/a)$

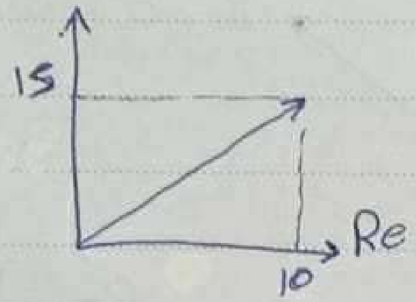
* Convert from Polar form to rectangular form:

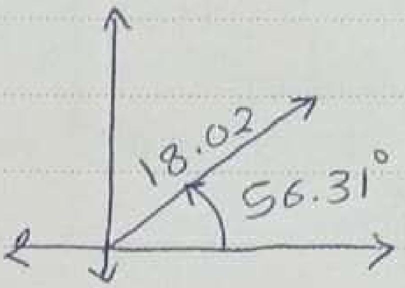
$\vec{X} = |\vec{X}| \angle \theta \Rightarrow a = |\vec{X}| \cos\theta$

$b = |\vec{X}| \sin\theta$

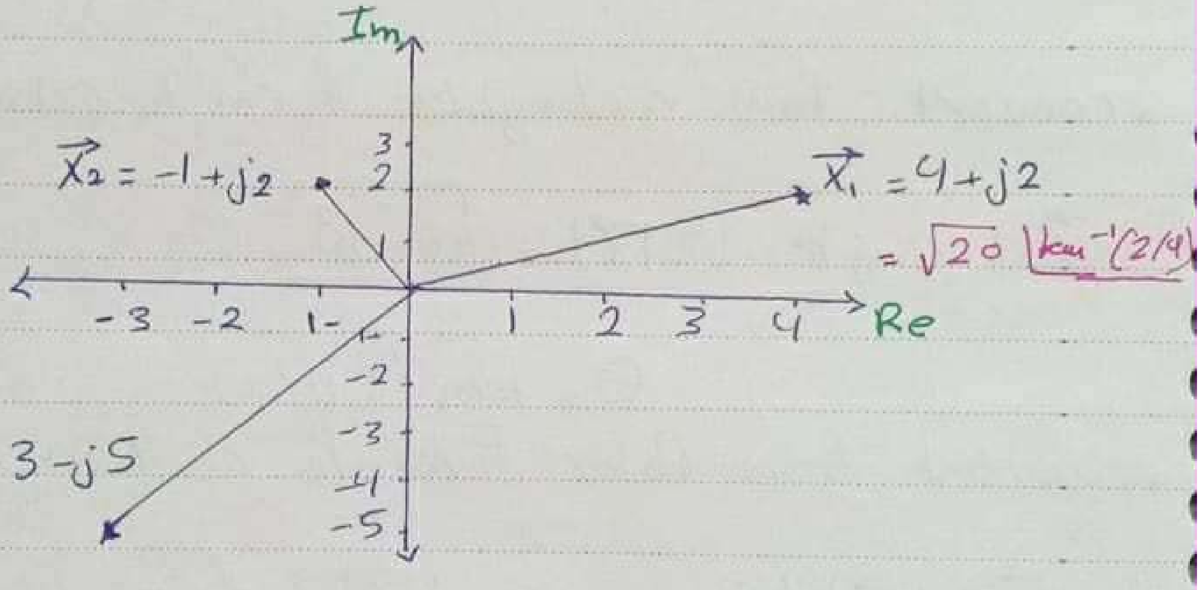
Ex:- $\vec{X} = 10 + j15$, $\triangleq \sqrt{10^2 + 15^2} \angle \tan^{-1}(15/10)$

$$= 18.02 \angle 56.31^\circ$$

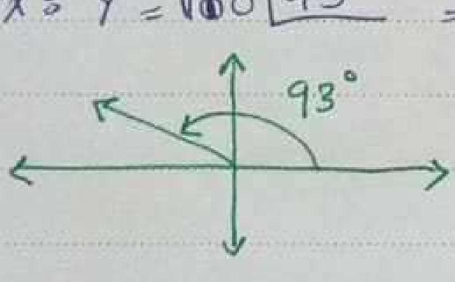




Ex: Find the polar form for each of the complex number.



Ex: $\vec{Y} = 110\angle 93^\circ \Rightarrow \vec{Y} = 110\cos(93^\circ) + j110\sin(93^\circ)$

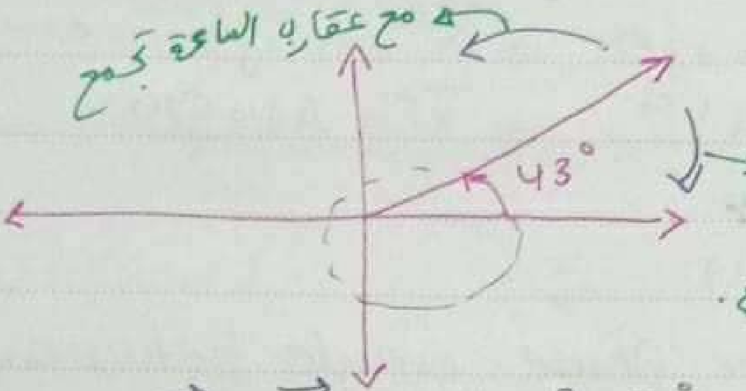


$$= -5.76 + j109.849$$

Ex:- $\vec{X} = 12\angle 43^\circ \triangleq 12\angle 43 \mp 360 = 12\angle 403^\circ = 12\angle -317$

٣ متساويين في القيمة

مع عقارب الساعة تجمع



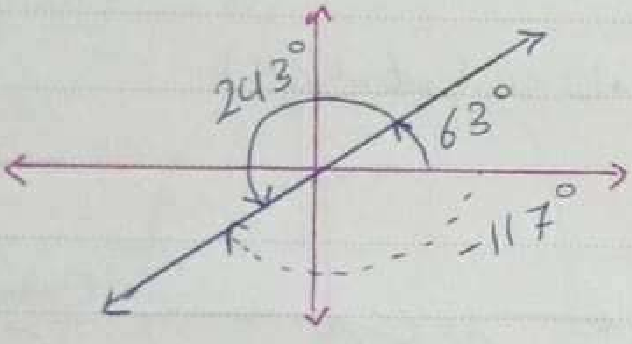
الزاوية لما تزيد 360 ترجع نفسها

عكس عقارب الساعة تطرح.

Ex: $\vec{X}_1 = 5\angle 63^\circ \Rightarrow \vec{X}_2 - \vec{X}_1 = 5\angle 63^\circ \mp 180^\circ$

$= 5\angle 243^\circ$

$= 5\angle -117^\circ$



Sum & subtraction of two complex number (Preferred in the rectangular form).

لأن الأسهل rectangular form

Ex:- $\vec{X}_1 = 3 + j4$, $\vec{X}_2 = -9 + j2$

$\vec{X}_1 + \vec{X}_2 = (3 + (-9)) + j(4 + 2) = -6 + j6$

$\vec{X}_1 - \vec{X}_2 = (3 - -9) + j(4 - 2) = 12 + j2$

Multiplication & division (Preferred in Polar form).

Ex: $\vec{X}_1 = 120\angle -89^\circ$ $\vec{X}_2 = 60\angle 112^\circ$

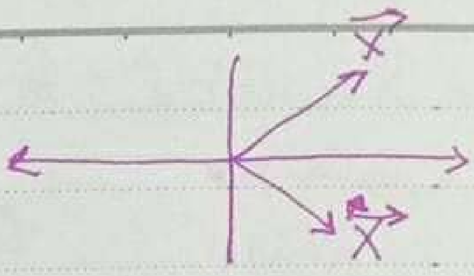
$\vec{X}_1 \vec{X}_2 = (120)(60)\angle -89^\circ + 112^\circ = 7200\angle 23^\circ$

$\dfrac{\vec{X}_1}{\vec{X}_2} = \dfrac{120\angle -89}{60\angle 112^\circ} = \dfrac{120}{60}\angle (-89)^\circ - (112^\circ) = 2\angle -201$

* Conject for complex number:

لـ نضرب $\vec{X} = a + jb \Rightarrow \vec{X}^* = a - jb$

الزاوية بسالب $\vec{X} = A\angle\theta \Rightarrow \vec{X}^* = A\angle-\theta$



Sinusoidals:

Example: (9.2)

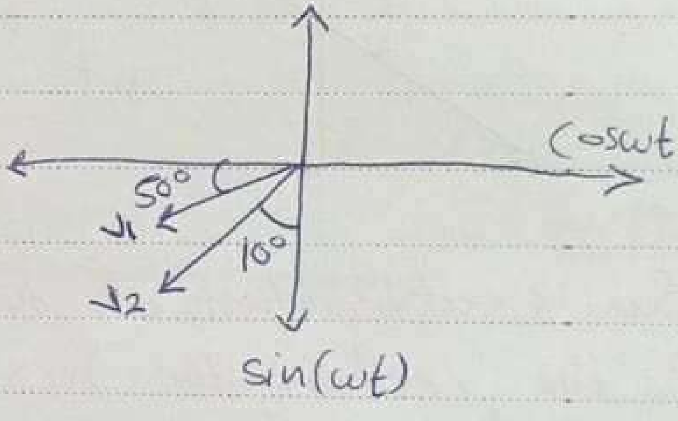
Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$.

$v_1 = +10\sin(\omega t + 50 - 90) = +10\sin(\omega t - 40°)$

$v_2 = 12\sin(\omega t - 10°)$

v_2 leads v_1 by 30°

90 − 10 − 50°.



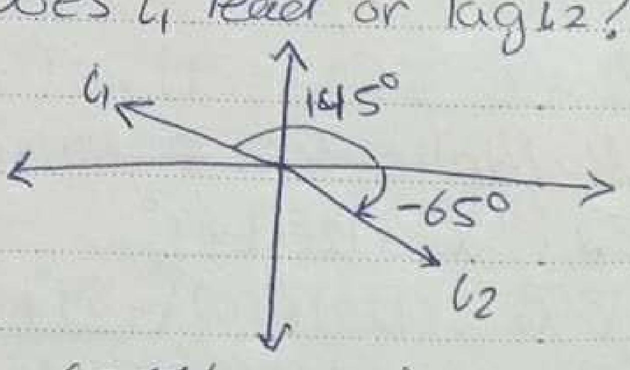
Particle Problem (9.2)

Find the phase angle between

$i_1 = -4\sin(377t + 55°)$

$i_2 = 5\cos(377t - 65°)$

Does i_1 lead or lag i_2?



$i_1 = -4\cos(377t + 55 - 90)$

$= -4\cos(377t - 35) = -4\cos(377t + 145°)$.

Sinsudal characteristics $\rightarrow$ sin( ) / cos( )

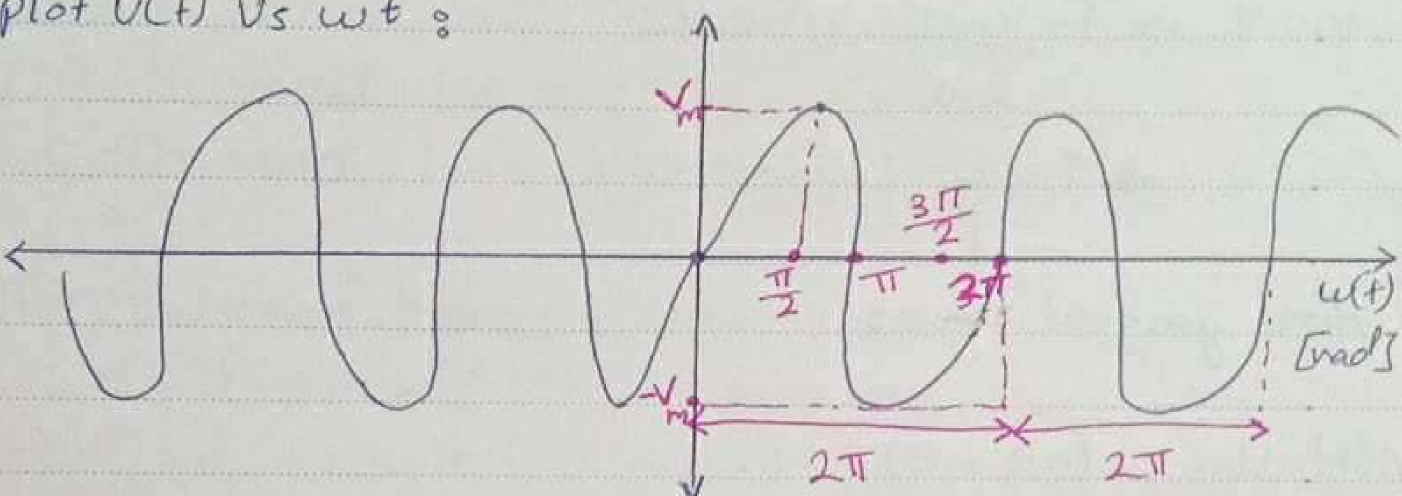
$v(t) = V_m \sin(\omega t)$

$V_m \Rightarrow$ Peak value (amplitude)

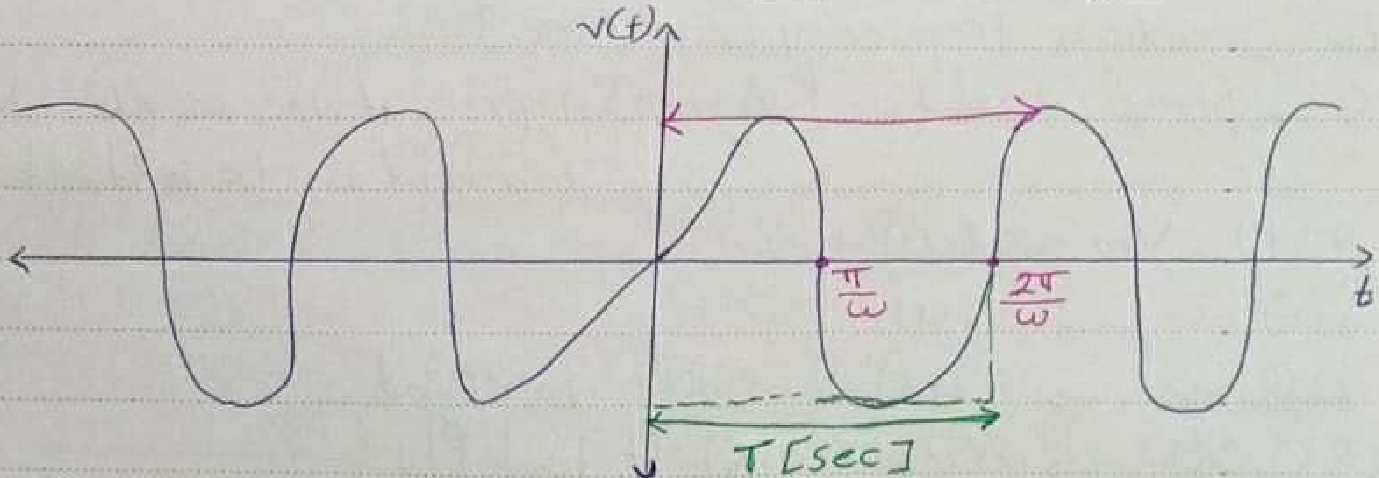
$\omega t \rightarrow$ argument [rad]

$\omega \Rightarrow$ radian frequency [rad/sec]

$\rightarrow$ plot $v(t)$ Vs ωt :



$\rightarrow$ Plot $v(t)$ Vs t : divide the (ωt) - axis by ω :



$f = \frac{1}{T}$ (HZ) $\rightarrow$ frequency $\Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = 2\pi f$

Ex: $v(t) = 20 \sin(1000t)$, find $\underline{f}$ & $\underline{T}$.

frequency

sol: $\omega = 1000 \Rightarrow f = \frac{\omega}{2\pi} = \frac{1000}{2\pi} = 159.15\ Hz$

Period $\Rightarrow T = \frac{1}{f} = 6.283\ msec$

Ex: $i(t) = 10\cos(100\pi t)$ [A]

$\omega = 100\pi \Rightarrow f = \frac{100\pi}{2\pi} = 50\ Hz$

$\Rightarrow T = \frac{1}{f} = 20 msec$.

More general case:

$v(t) = V_m \sin(\omega t + \theta)$

$V_m \rightarrow$ Peak value or amplitude

$\omega t + \theta \rightarrow$ argument [rad] / [degree]

$\omega \rightarrow$ radian frequency [rad/sec]

$\theta \rightarrow$ phase [rad] or [degree] $\rightarrow$ represent, the single shift to right $\theta(-)$ or to left $\theta(+)$

$v(t) = V_m \sin(\omega t + \theta)$

① $\theta = 0 \rightarrow v_1(t)$

② $\theta < 0 \rightarrow v_2(t)$ shift to right

③ $\theta > 0 \rightarrow v_3(t)$ Shift to left.

(متأخر) ← → (متقدم)

lagging & leading.

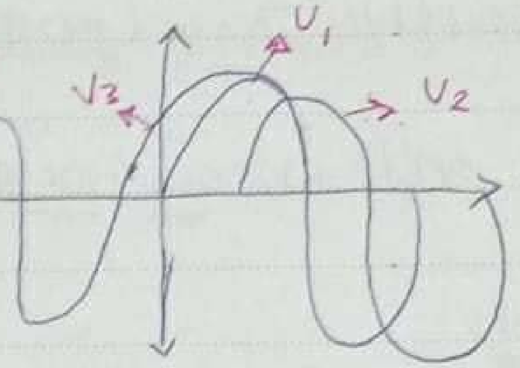
$v_1(t)$ leads $v_2(t)$ by θ.

OR $v_2(t)$ lags $v_1(t)$ by θ

OR $v_1(t)$ lags $v_2(t)$ by $-\theta^\circ = 360^\circ - \theta$

OR $v_2(t)$ leads $v_1(t)$ by $-\theta^\circ = 360^\circ - \theta$

لما تعطى ال lag to be leads تعرف الزاوية بسالب



* Two compare two sinusoidal:

① Both must have same ω.

② Both must be "cosine" or "sine".

③ Both must have +ve amplitude.

The conversion between them is:

$-\sin(\omega t) = \sin(\omega t \pm 180^\circ)$ — إشارة ال sin/cos *

$-\cos(\omega t) = \cos(\omega t \mp 180^\circ)$ — مع إشارة الزاوية

$\mp \cos(\omega t) = \sin(\omega t \mp 90^\circ)$ — نفس الترتيب

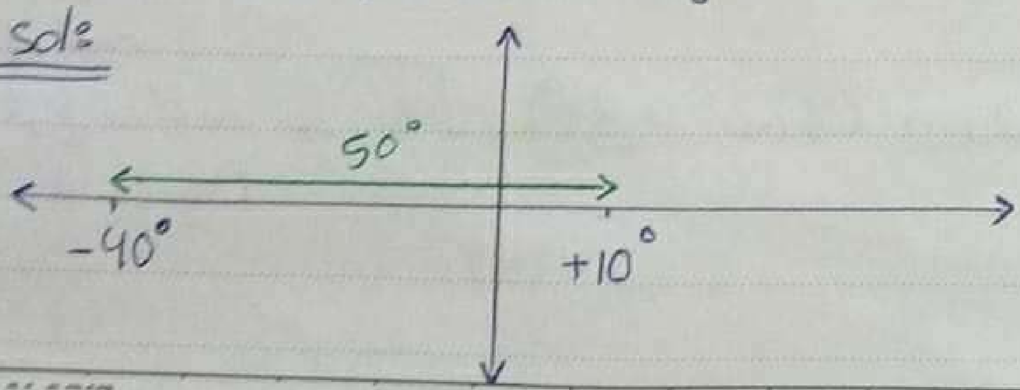
$\mp \sin(\omega t) = \cos(\omega t \pm 90^\circ)$

Ex: $v_1(t) = +\underset{\text{amplited}}{10} \cos(1000\pi t^\circ - 10^\circ)$ — to the right

$v_2(t) = +2\cos(1000\pi t^\circ + 40^\circ)$ — to the left.

which leads? which lags.

sol:



V_1 lags V_2 by 50°.

V_2 leads V_1 by 50°

V_1 leads V_2 by $-50^\circ = 310^\circ$

/ /

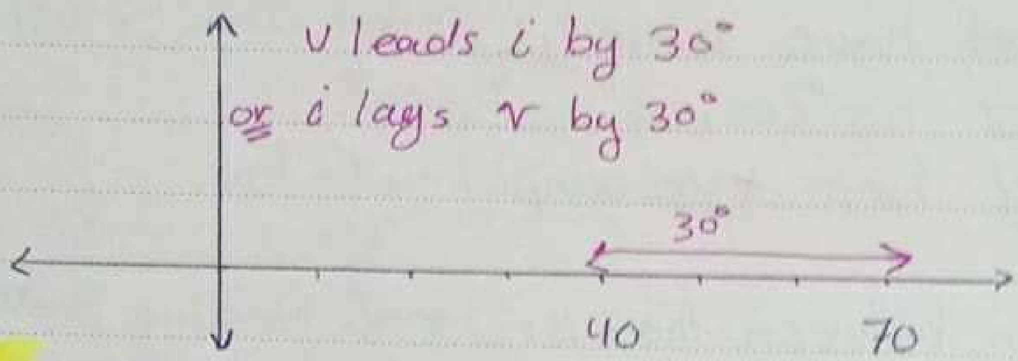
Ex: $i(t) = -2\cos(100\pi t + 20^\circ)$ [A]

$v(t) = +2\sin(100\pi t - 40^\circ)$ [V]

sol:

$i(t) = 2\sin(100\pi t + 20^\circ - 90^\circ)$

$= 2\sin(100\pi t - 70^\circ)$



Ex: $v(t) = 2\cos(200\pi t + 30^\circ)$

Find $v(0.01)$?

sol:

$v(0.01) = 2\cos(200 \times \underbrace{\pi \times 0.01}_{\text{rad.}} + \underbrace{30^\circ}_{\text{degree}})$

your calculater on rad $\rightarrow \triangleq 2\cos\left(2\pi + 30 \times \frac{\pi}{180}\right) = \sqrt{3}$ → حوّلت على Degree

your calculater on degree. $\triangleq 2\cos(360^\circ + 30^\circ) = \sqrt{3}$ → لو rad حوّل من rad لـ Degree

AC - circuits.

/ /

* Electric circuits that are driven by AC (sinusoidal)

* **Phasors:**

<u>Def:</u> a phasor is a complex number that represent the amplitude and phase of a sinusoid.
(the sinusoid has to be written as cosine with the amplitude).

let: $v(t) = V_m \cos(\omega t + \theta_v)$

$= Re\left[V_m \underline{\angle \omega t + \theta_v} \right]$

$= Re\left[V_m \underline{\angle \theta_v} \cdot 1 \underline{\angle \omega t} \right]$

للتحويل من Time do. إلى phaser
① لازم cos
② amplitude (+)
③ تحول إلى phasor.

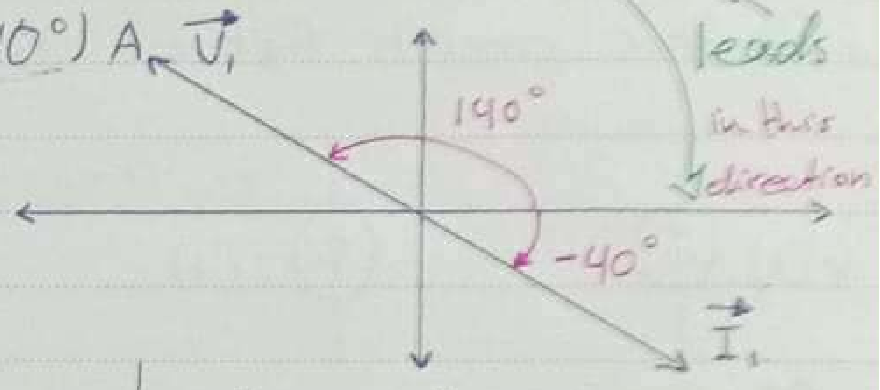
this is the phasor $\vec{V} = V_m \underline{\angle \theta_v}$

$v(t) = V_m \cos(\omega t + \theta_v) \Leftrightarrow \vec{V} = \underline{V_m} \underline{\angle \theta_v}$

the same of currents

$i(t) = I_m \cos(\omega t + \theta_i) \Leftrightarrow \vec{I} = \underline{I_m} \underline{\angle \theta_i}$

Ex: $i_1(t) = 6\cos(50t - 40^\circ)$ A



$\vec{I} = 6 \angle -40^\circ$ A

$= 6 \angle 320^\circ$ A

↳ phasor diagram.

$v_1(t) = -4\sin(30t + 50^\circ)$

$= 4\cos(30t + 50^\circ + 90^\circ)$

$= 4\cos(30t + 140^\circ)$

$\vec{V}_1 = 4 \angle 140^\circ \Rightarrow v_1(t)$ leads $i_1(t)$ by 180°.

Ex: $\vec{I} = 30 \angle -20$ mA

sol:- $i(t) = 30\cos(\omega t - 20^\circ)$ mA.

Ex: $\vec{V} = -3 + j4$ V, Find $v(t)$.

sol:- $\vec{V} = \sqrt{9+16} \angle \tan^{-1}(4/-3) = 5 \angle \overset{-53.1^\circ}{\cancel{126.87^\circ}}$

$v(t) = 5\cos(\omega t + \overset{-53.1^\circ}{\cancel{126.87^\circ}})$

Ex:- Find $i(t) = 4\cos(\omega t + 30^\circ) + 5\sin(\omega t - 20)$.

Sol:- $i(t) = i_1(t) + i_2(t)$

$i_1(t) = 4\cos(\omega t + 30^\circ) \Rightarrow I_1 = 4\angle 30^\circ$

$i_2(t) = 5\sin(\omega t - 20^\circ)$

$= 5\cos(\omega t - 110^\circ) \Rightarrow I_2 = 5\angle 110^\circ$

$i(t) = \vec{I} = \vec{I_1} + \vec{I_2}$

$= 4\angle 30^\circ + 5\angle -110^\circ$

$= 3.218\angle -56.97^\circ$

$i(t) = 3.218\cos(\omega t - 56.97^\circ)$.

Impedance

/ /

* What is Impedance? can be delt as complex number.

R ⇒ Z_R , $\vec{Z}$ = number such as 2

C ⇒ Z_C , $\vec{Z}$ = complex and number(+) → j2

L ⇒ Z_L , $\vec{Z}$ = complex(−) → −j2

$$\vec{Z} = \underline{R} + j\boxed{X} = |\vec{Z}| \angle \theta \quad [\Omega]$$

resistance (dissiaple energy), originally

Reactance (stores energy) originally or

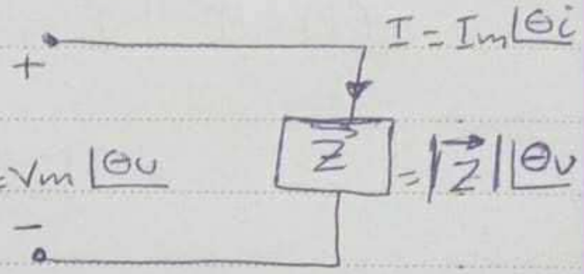
* **Ohm's law directly applied:-**

$\vec{V} = \vec{Z}\vec{I}$

$V_m \angle \theta_v = (|\vec{Z}| \angle \theta_z)(I_m \angle \theta_i)$

$V_m = |\vec{Z}| I_m$

$\theta_v = \theta_i + \theta_z$



time domain	complex domain
R	$R = Z_R = R \angle 0^\circ$
L	$j\omega L = \vec{Z}_L = \omega L \angle 90^\circ$
C	$\dfrac{-j}{\omega C} = Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{\omega C} \angle 90^\circ$

/ /

Time	$\vec{Z}$	load typical	Θ_Z	$\vec{I}$ & $\vec{V}$
R	$R\angle 0^\circ$	pure resistance	0	In phase
L	$j\omega L = jX_L$	pure inductive	90°	$\vec{I}$ lags $\vec{V}$ by 90°
C	$-j1/\omega C = -jX_C$	pure capacitive	-90°	$\vec{I}$ leads $\vec{V}$ by 90°
R L	$R + jX_L$	inductive (not purity)	+ve	$\vec{I}$ lags $\vec{V}$ by Θ_Z
R C	$R - jX_C$	capacitive (not purity)	-ve	$\vec{I}$ leads V by Θ_Z

Ex: If $\omega = 10$ rad/sec, specify the equivalent load for each of the following impedanes:

① $\vec{Z} = 5\,\Omega$ (Pure resistance) $\Rightarrow R = 5\,\Omega$, $R = 5\,\Omega$

② $\vec{Z} = j20\,\Omega$ (Pure inductive) $\Rightarrow L = 2\,H$, $L = 2\,H$

$j20 = j\omega L \Rightarrow L = 20/\omega = 2\,H$

③ $\vec{Z} = \dfrac{1}{j20}\,\Omega \Rightarrow Z = -j\dfrac{1}{20} \Rightarrow$ pure capacitive, $C = 2F$

$-j\dfrac{1}{20} = -j\dfrac{1}{\omega C} \Rightarrow \omega C = 20 \rightarrow C = 2F$

④ $\vec{Z} = 4 + j3 \Rightarrow$ inductive load $R = 4$ $L = 3/10\,H$

$\omega L = 3 \Rightarrow L = 3/10\,H$

⑤ $\vec{Z} = 2 - j2\,\Omega \Rightarrow$ capacitive load $R = 2\,\Omega$ $C = 1/20\,F$

$-j2 = -j\dfrac{1}{\omega C}$

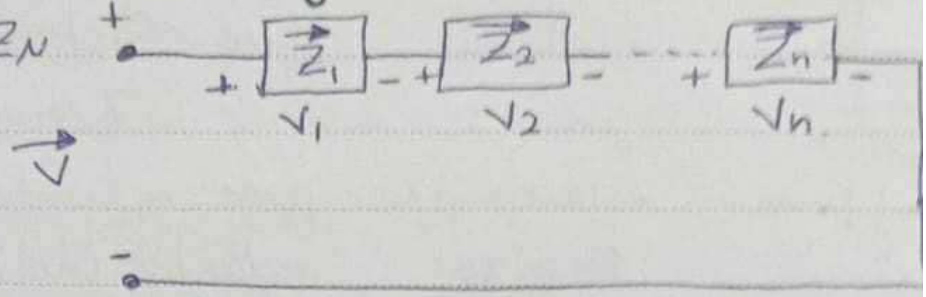
$2 = \dfrac{1}{\omega C} \Rightarrow \dfrac{1}{20} = C$

/ /

Impedanes in series & voltage division:-

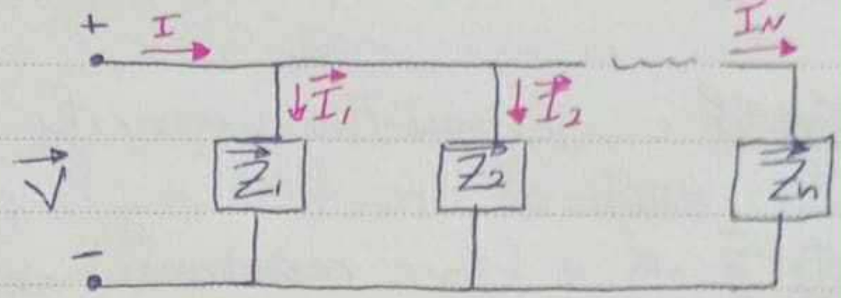
$$\vec{Z}_{equ} = Z_1 + Z_2 + Z_3 \dots + Z_N$$

$$\vec{V}_n = \frac{\vec{Z}_n}{\vec{Z}_1 + \vec{Z}_2 + \dots + \vec{Z}_n} \vec{V}$$



* Impedanes in Parallel & current division:

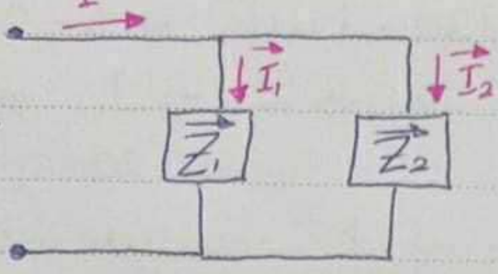
$$\vec{Z}_{equ} = \frac{1}{\frac{1}{\vec{Z}_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$



$$I_N = \frac{1/Z_n}{1/Z_{equ}} I$$ (current division).

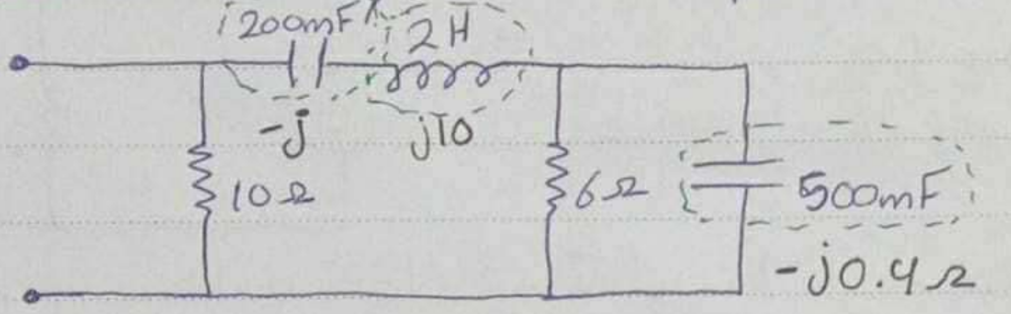
$$\vec{Z}_{equ} = \frac{\vec{Z}_1 \vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}$$

$$I_1 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{I} \quad \Big\{ \quad I_2 = \frac{\vec{Z}_1}{\vec{Z}_1 + \vec{Z}_2} \vec{I}$$



/ /

Ex: Find the equivalent impedance:



$R \to R$

$H \to j\omega L$

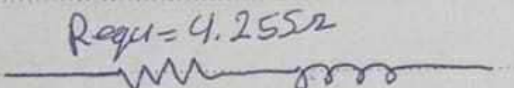
$C \to \frac{-j}{\omega C}$

$\omega = 5 rad/sec$

① $6 // -j0.4 = \frac{(6)(-j0.4)}{6-j0.4} = 0.3991\angle -86.18^\circ$

② $0.3991\angle -86.18^\circ + j10 - j = 8.602\angle 89.823^\circ$

③ $8.602\angle 89.823^\circ // 10^\circ = 6.511\angle 49.19^\circ$
$= 4.255 + j4.93\Omega$



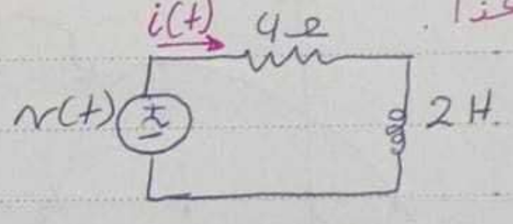
→ lagging impedence

لأن إشارة الـ (j) موجبة دائمًا نسميها هكذا .

Ex: Find $i(t)$

ⓐ using time-domain

ⓑ using complex-domain.



$v(t) = 10\cos(50t + 20)$

ⓐ KVL: $-v(t) + 4i(t) + 2\frac{di}{dt} = 0$

$2\frac{di}{dt} + 4i(t) = 10\cos(50t + 20^\circ)$

*we need to solve 1st order diff-equation!

يكمل الحل لكن ليس ضروري

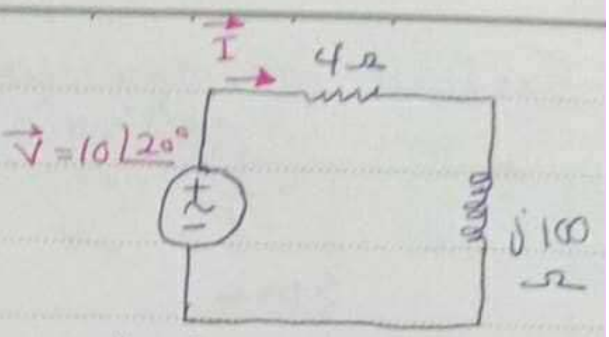
لا نستخدمه

الطريقة المستخدمة والمعتمدة

(b) using complex domain:

$R = 4\Omega \rightarrow Z_R = 4\Omega$

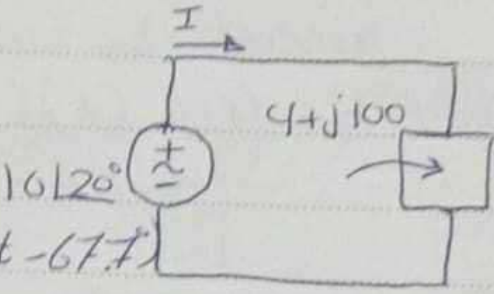
$L = 2H \rightarrow Z_L = j\omega L = j100\Omega$

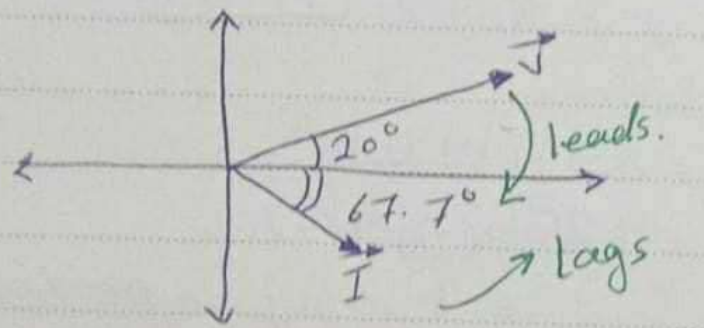


$v(t) = 10\cos(50t + 20^\circ) \rightarrow \vec{V} = 10\angle 20^\circ$ volt.

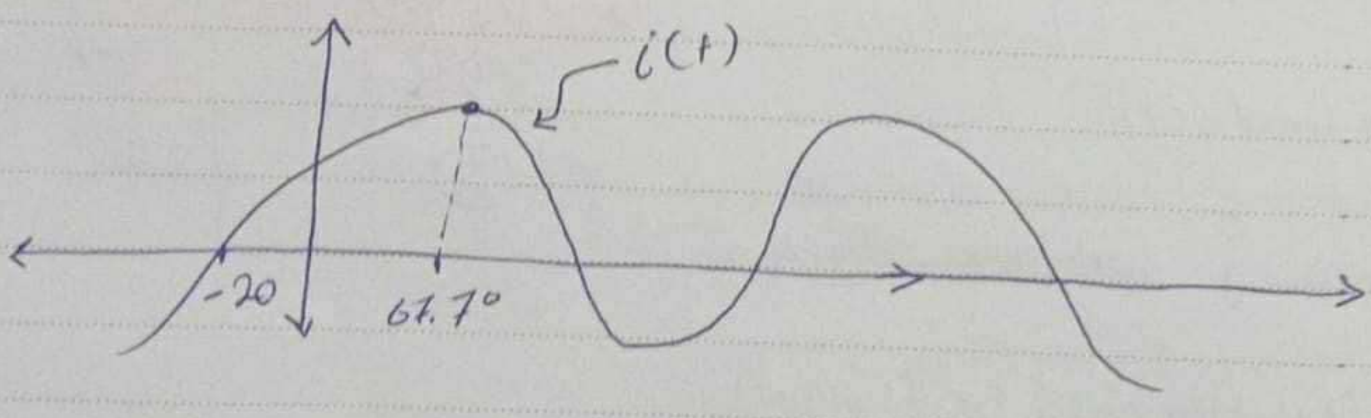
$$\vec{I} = \frac{10\angle 20^\circ}{4 + j100}$$

$$= 0.1\angle -67.7^\circ \Rightarrow i(t) = 0.1\cos(50t - 67.7)$$



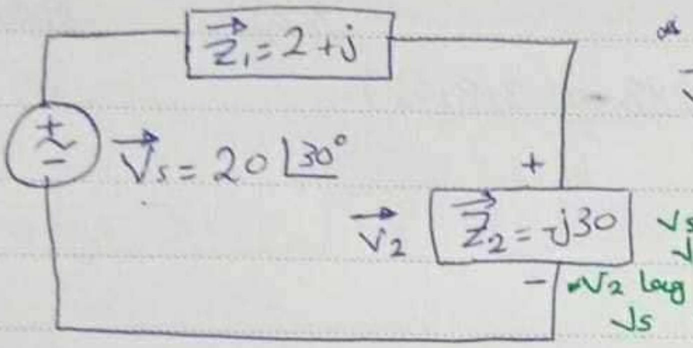


i lag v by 87.7°



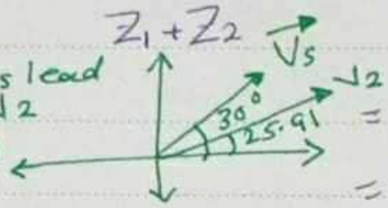
/ /

Ex: Find $\vec{V}_2$?



* By voltage division:

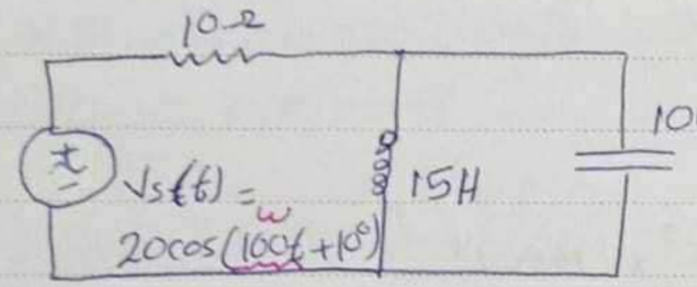
$$\vec{V}_2 = \frac{\vec{Z}_2}{Z_1 + Z_2}\vec{V}_s = \frac{-j30}{(2+j)+(-j30)}\,20\angle 30^\circ$$



$$= 19.244 + j9.34$$

$$= 21.374\angle 25.91^\circ$$

Ex: Find $i_s(t)$?



$$Z_L = j\omega L$$
$$= j\times 100\times 15 = j1500$$
$$Z_C = \frac{-j}{\omega C} = \frac{-j}{100\times 10\times 10^{-3}}$$
$$= -j$$

$$Z_{equ} = (-j \,//\, j1500) + 10$$
$$= \frac{(-j)(j1500)}{-j + j1500} + 10$$
$$= 10 - j1$$
$$= 10.05\angle -5.71^\circ$$

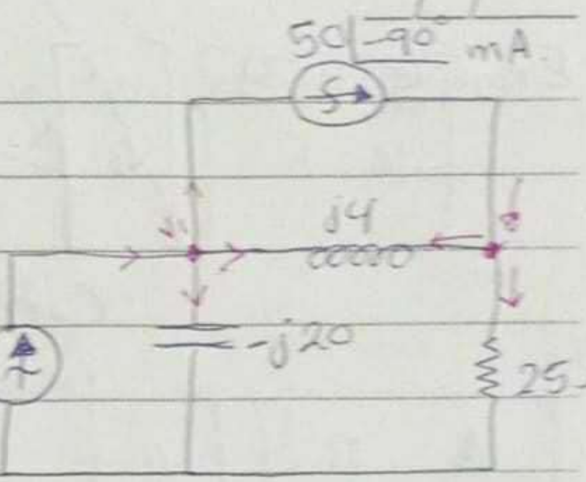
~~$i_s(t) = 10.05\cos(100 - 5.71^\circ)$~~

$$I_s = \frac{20\angle 10^\circ}{10.05\angle -5.71}$$
$$\approx 2\angle 15.71^\circ$$

$$i_s(t) = 2\cos(100t + 15.71^\circ)$$

10.7

Ex: Nodal analysis, Find $\vec{V}_1$ & $\vec{V}_2$



① at node ①:

$$20\text{mA} = \frac{\vec{V}_1 - 0}{-j20} + \frac{\vec{V}_1 - \vec{V}_2}{j4} + 50\angle -90° \text{ mA}$$

$$\left(\frac{1}{-j20} + \frac{1}{j4}\right)\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 20\text{ mA} - 50\angle -90° \text{ mA}$$

$$\left(\frac{1}{j4} + -j\frac{1}{5}\right)\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 20\text{ mA} - 50\angle -90° \text{ mA}$$

$$-j\frac{1}{5}\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 53.85\angle 68.19° \quad ①$$

② at node ②:

$$50\angle -90° \text{ mA} = \frac{\vec{V}_2 - \vec{V}_1}{j4} + \frac{\vec{V}_2 - 0}{25}$$

$$j0.25\vec{V}_1 + 0.253\angle -80.91° \vec{V}_2 = +50\angle -90° \text{ mA} \quad ②$$

$$\begin{bmatrix} \overset{A}{-j/5} & \overset{B}{-1/j4} \\ j0.25\,C & \underset{D}{0.253\angle -80.91°} \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} \overset{E}{53.85\angle 68.19°} \\ -50\angle -90\;F \end{bmatrix}$$

$$\Delta = (-j/5) \times (0.253\angle \cancel{-80.91}) - (j0.25 \times -1/j4) = \cancel{0.51\angle -96.98°}$$
$$0.95\angle -32.53°$$

$$\begin{bmatrix} \overset{E}{53.85\angle 68.19°} & -1/j4\;B \\ -50\angle -90°\;F & 0.253\angle -80.91\;D \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} -j/5 \\ j0.25 \end{bmatrix}$$

$$V_1 = (53.85\angle -68.19°) \times 0.253\angle -80.91 = \cancel{13.62\angle -149.1°}/\cancel{0.51\angle -96.98°} = \cancel{26.7}$$
$$= 206.8\angle -42.7 \qquad \angle \cancel{-52.12°}$$

$$V_2 = \frac{\begin{bmatrix} A & E \\ B & F \end{bmatrix}}{\Delta} = \frac{5.59\angle -63.45^\circ}{0.015\angle -32.53^\circ} = 372.66\angle -30.92$$

Cramer's rule

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}$$

$$V_1 = \frac{\Delta 1}{\Delta} = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{(E \cdot D) - (B \cdot F)}{(A \cdot D) - (B \cdot C)}$$

$$V_2 = \frac{\Delta 2}{\Delta} = \frac{\begin{vmatrix} A & E \\ C & F \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{(A \cdot F) - (E \cdot C)}{(A \cdot D) - (B \cdot C)}$$

① تفعيل complex mode على الآلة الحاسبة

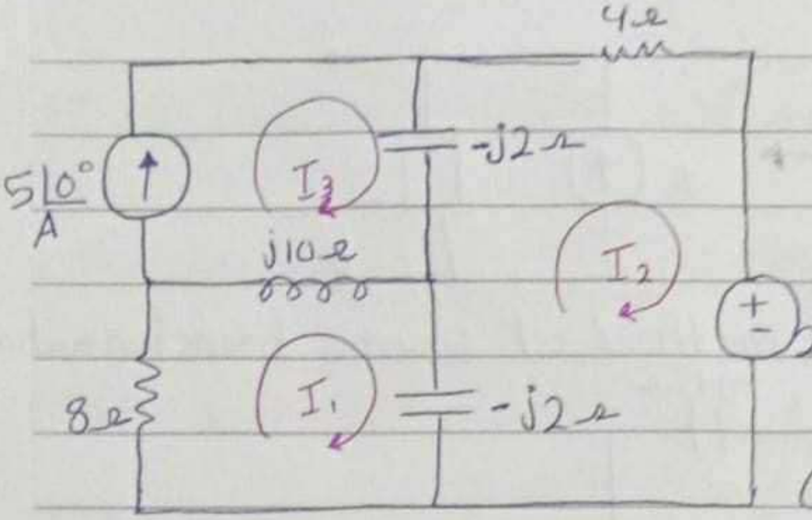
② إدخال الـ complex number وتخزينها على الآلة الحاسبة بـ (Shift + RCL)

STO

③ تطبيق على قانون V_1 & V_2 عن طريق الضغط على RCL ومن ثم الحرف المطلوب.

→ 10.3

Ex: Determine current I_o using mesh analysis.



Apply KVL at mesh ①:

$8I_1 + j10(I_1 - I) - j2(I_1 - I_2) = 0$

$(8 + j10 - j2)I_1 - -j2I_2 - j10I_3 = 0$

$(8 + j8)I_1 - (-j2)I_2 - j10I_3 = 0$ ... ①

KVL at mesh ②:

$-j2(I_2 - I_1) - (-j2)(I_2 - I_3) + 4I_2 + 20\angle 90^\circ = 0$

$-(-j2)I_1 - (4 - j2 - j2)I_2 - (-j2)I_3 + 20\angle 90^\circ = 0$ ...... ②

KVL at mesh ③:

by intersection → $I_3 = 5$ → نعوضها في المعادلة ① & ②

$(8 + j8)I_1 + j2I_2 = j50$ ... ①

$j2I_1 + (4 - j4)I_2 = -j10 - j20$ ... ②

$$\begin{bmatrix} \overset{A}{8+j8} & \overset{B}{2j} \\ \underset{C}{j2} & \underset{D}{(4-j4)} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \overset{E}{j50} \\ \underset{F}{-j30} \end{bmatrix}$$

$\Delta = (AD) - (BC)$

$= (8 + j8)(4 - j4) - (2j \times 2j)$

$= 68$

$I_1 = \dfrac{\begin{bmatrix} E & B \\ F & C \end{bmatrix}}{\Delta} = \dfrac{-160}{68} = -2.35$

$I_2 = \dfrac{\begin{bmatrix} A & E \\ C & F \end{bmatrix}}{\Delta} = \dfrac{416.17\angle -35^\circ}{68}$

$= 6.12\angle +144.78$

$= 6.12\angle -35.22^\circ$ → ⊖

DOMS

10.4

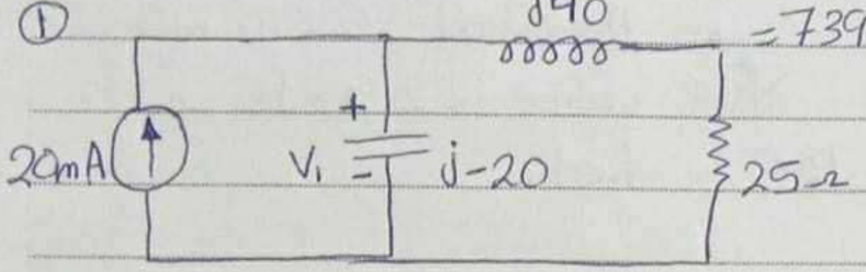
Ex: Find $\vec{V_1}$ using superposition: → بستخدمها عند اختلاف قيم (w)

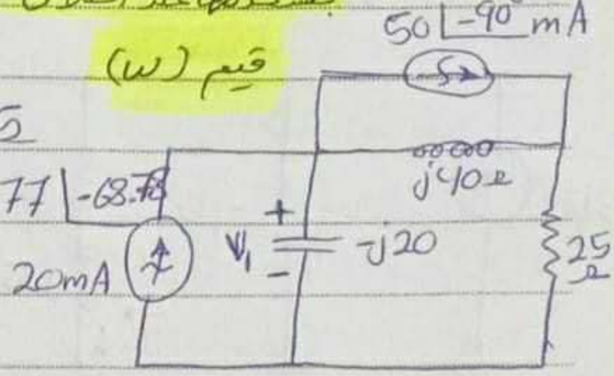
$\vec{V_1} = V_1' + V_2''$

$= 589.336\angle{-70.66} + 151.99\angle{-61.45}$

$= 739.77\angle{-68.78}$

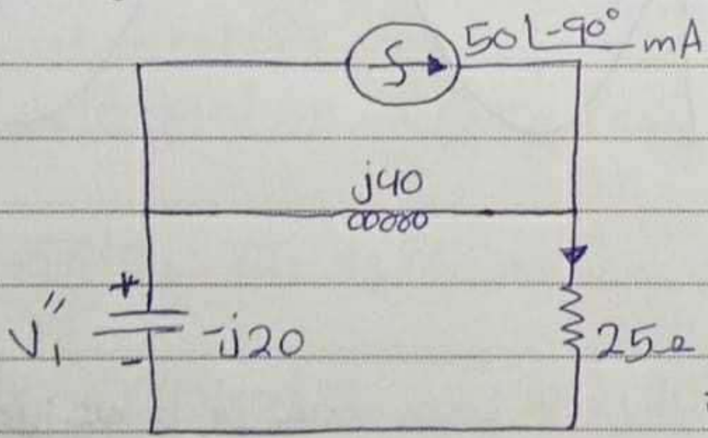
①





$$\vec{I_c} = \frac{25 + j40}{(25+j40) - j20} \times 20mA = 29.467\angle{19.334^\circ}\ mA$$

$\vec{V_1}' = (-j20)\vec{I_c} = 589.336\angle{-70.66^\circ}\ mA$



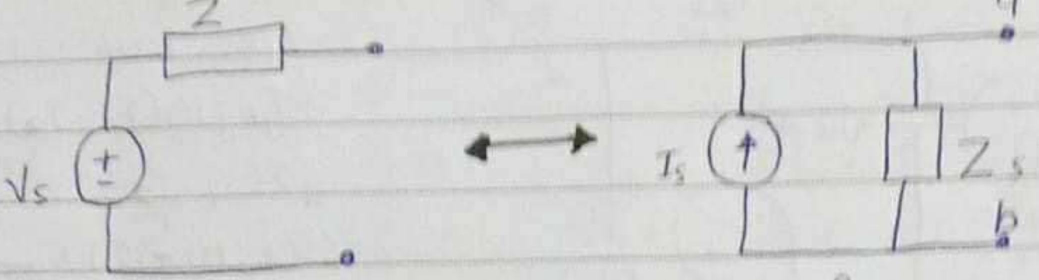
$$\vec{I} = \frac{(j40)(50\angle{-90^\circ})}{j40 + (25 - j20)}$$

$= 6.738\angle{-147.38}$

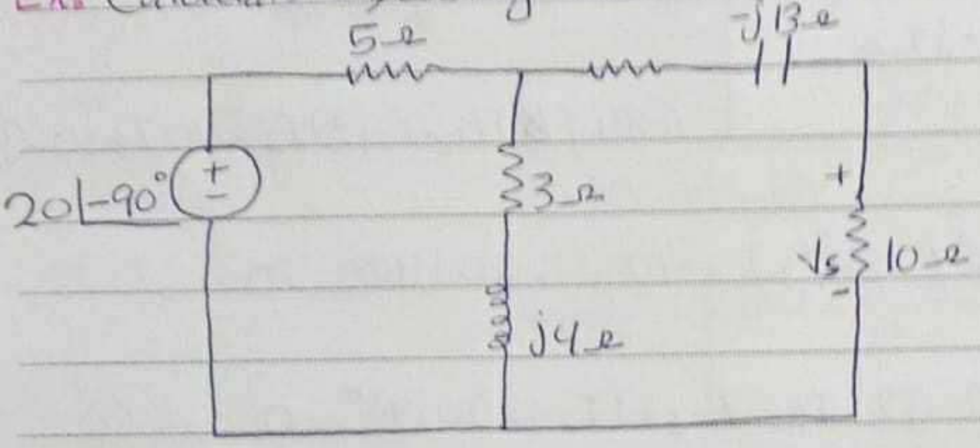
$\vec{V_1}'' = (-j20)\vec{I_x}$

$\vec{V_1}'' = (-j20) \times (1 - 6.738\angle{-147.38})$

10.5:- Source Transformation.

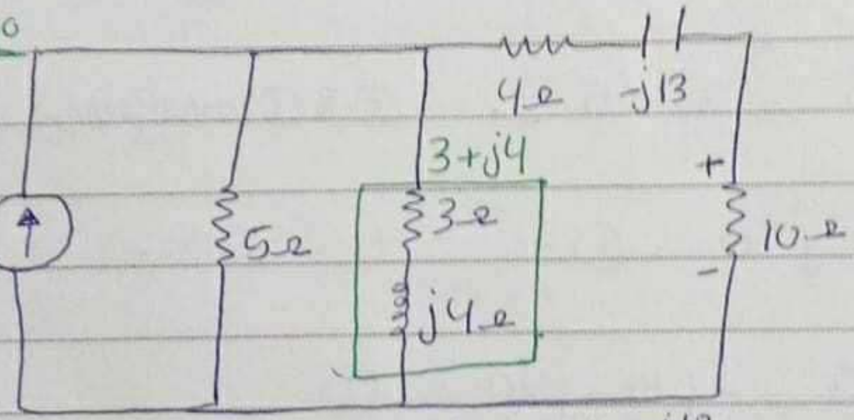


Ex: Calculate V, using the method of source transformation



$I_s = \frac{20\angle -90}{5}$

$I_s = -j4\,A$



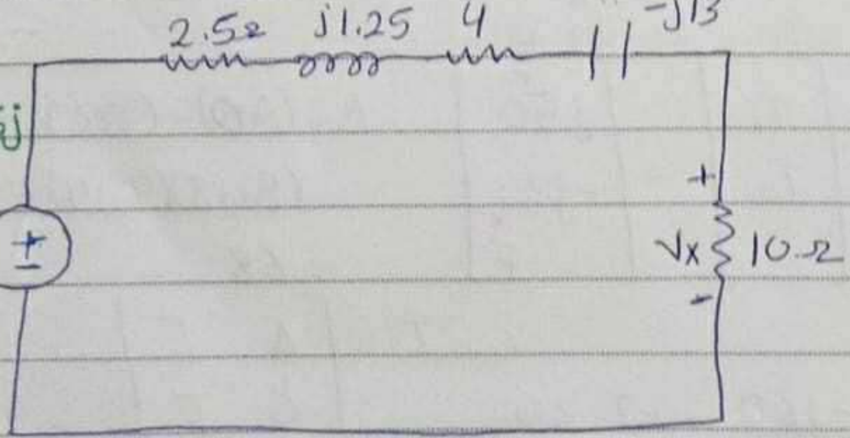
$Z_{equ} = \frac{5 \times (3+j4)}{5+3+j4}$

$= 2.795\angle 26.565$

$= 2.5 + 1.25j\ \Omega$

$V_s = -j4 \times (2.5+1.25j)$

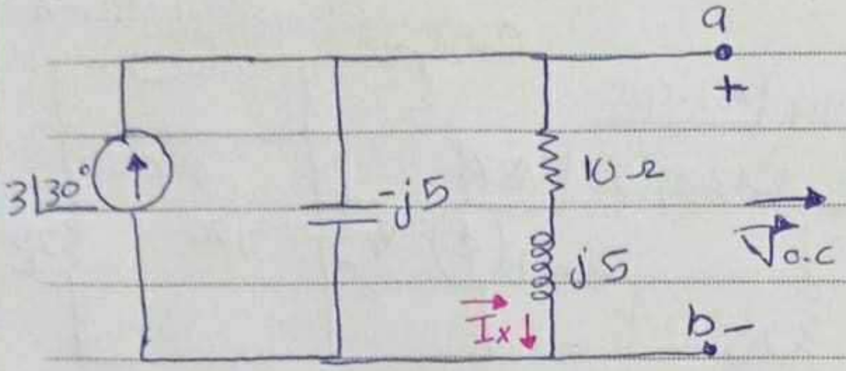
$V_s = 5 - j10\ V$



$$V_x = \frac{(5-j10)(10)}{10+4+2.5+j1.25-j13} = 5.519\angle -28^\circ\ V$$

10.6

Ex: For the circuit shown find the Thevenin equivalent

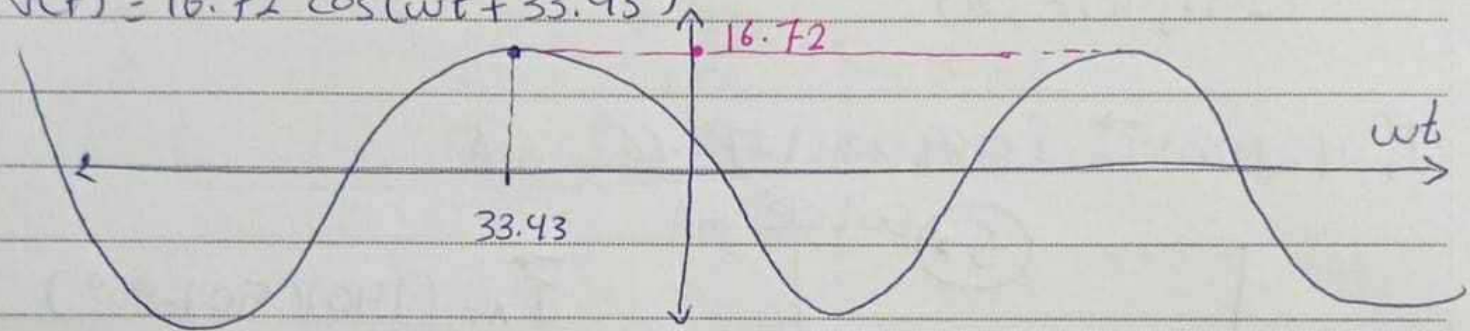


$$\underline{Z_{th}} = (-j5) // (10 + j5) = \frac{(-j5)(10+j5)}{-j5 + 10 + j5} = 2.5 - j5 \ \Omega$$

Kill the current source first.

$$\vec{V}_{th} = \vec{V}_{oc} = (10+j5)\left[\frac{-j5}{-j5+10+j5} \; 3\angle 30^\circ\right] = 16.72\angle 33.43^\circ \ V$$

$v(t) = 16.72\cos(\omega t + 33.43^\circ)$



$\vec{Y} = \dfrac{1}{\vec{Z}}$ (admedance)

1) حل أمثلة قدر الإمكان
2) تحول من Time domain → AC ونطبق طرق الحل القديمة
3) نعرف lead & lag
4) تحويل إلى phasor
5) حساب الـ I, V, Z.

Example 10.4

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

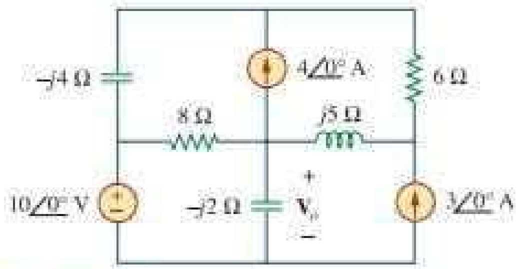


Figure 10.9
For Example 10.4.

Solution:
As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \qquad \textbf{(10.4.1)}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \qquad \textbf{(10.4.2)}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \qquad \textbf{(10.4.3)}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \qquad \textbf{(10.4.4)}$$

■ **METHOD 1** Instead of solving the above four equations, we reduce them to two by elimination.

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \qquad \textbf{(10.4.5)}$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \qquad \textbf{(10.4.6)}$$

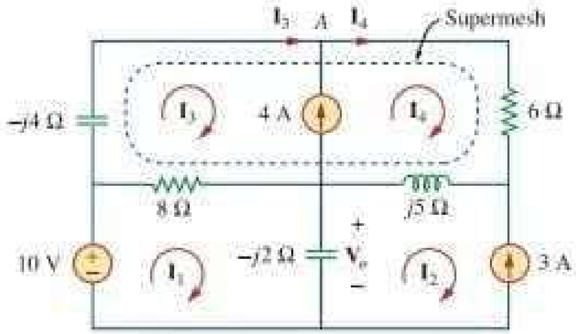


Figure 10.10
Analysis of the circuit in Fig. 10.9.

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_0$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$

$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

METHOD 2 We can use *MATLAB* to solve Eqs. (10.4.1) to (10.4.4). We first cast the equations as

$$\begin{bmatrix} 8 - j2 & j2 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ -8 & -j5 & 8 - j4 & 6 + j5 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 0 \\ 4 \end{bmatrix} \tag{10.4.7a}$$

or

$$\mathbf{AI} = \mathbf{B}$$

By inverting $\mathbf{A}$, we can obtain $\mathbf{I}$ as

$$\mathbf{I} = \mathbf{A}^{-1}\mathbf{B} \tag{10.4.7b}$$

We now apply *MATLAB* as follows:

```
>> A = [(8-j*2) j*2    -8       0;
        0       1      0        0;
        -8      -j*5   (8-j*4)  (6+j*5);
        0       0      -1       1];
>> B = [10 -3 0 4]';
>> I = inv(A)*B

I =
  0.2828 - 3.6069i
  -3.0000
  -1.8690 - 4.4276i
   2.1310 - 4.4276i
>> Vo = -2*j*(I(1) - I(2))

Vo =
  -7.2138 - 6.5655i
```

as obtained previously.

Example 10.6

Find v_o of the circuit of Fig. 10.13 using the superposition theorem.

Figure 10.13
For Example 10.6.

Solution:

Since the circuit operates at three different frequencies ($\omega = 0$ for the dc voltage source), one way to obtain a solution is to use superposition, which breaks the problem into single-frequency problems. So we let

$$v_o = v_1 + v_2 + v_3 \tag{10.6.1}$$

where v_1 is due to the 5-V dc voltage source, v_2 is due to the 10 cos 2t V voltage source, and v_3 is due to the 2 sin 5t A current source.

To find v_1, we set to zero all sources except the 5-V dc source. We recall that at steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. There is an alternative way of looking at this. Since $\omega = 0$, $j\omega L = 0$, $1/j\omega C = \infty$. Either way, the equivalent circuit is as shown in Fig. 10.14(a). By voltage division,

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V} \tag{10.6.2}$$

To find v_2, we set to zero both the 5-V source and the 2 sin 5t current source and transform the circuit to the frequency domain.

$$\begin{aligned}
10\cos 2t \quad &\Rightarrow \quad 10/\underline{0^\circ}, \qquad \omega = 2 \text{ rad/s} \\
2 \text{ H} \quad &\Rightarrow \quad j\omega L = j4\ \Omega \\
0.1 \text{ F} \quad &\Rightarrow \quad \frac{1}{j\omega C} = -j5\ \Omega
\end{aligned}$$

The equivalent circuit is now as shown in Fig. 10.14(b). Let

$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

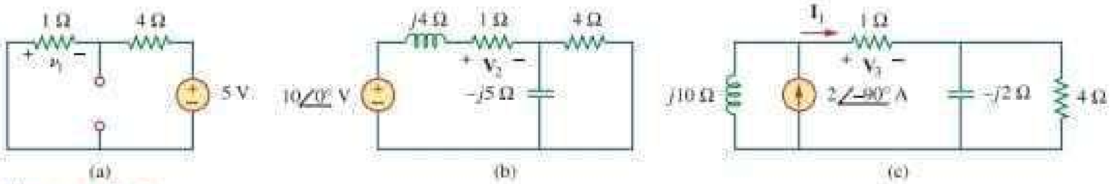


Figure 10.14
Solution of Example 10.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

By voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10/\underline{0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498/\underline{-30.79^\circ}$$

In the time domain,

$$v_2 = 2.498\cos(2t - 30.79^\circ) \tag{10.6.3}$$

To obtain v_3, we set the voltage sources to zero and transform what is left to the frequency domain.

$$\begin{aligned}
2\sin 5t \quad &\Rightarrow \quad 2/\underline{-90^\circ}, \qquad \omega = 5 \text{ rad/s} \\
2 \text{ H} \quad &\Rightarrow \quad j\omega L = j10\ \Omega
\end{aligned}$$

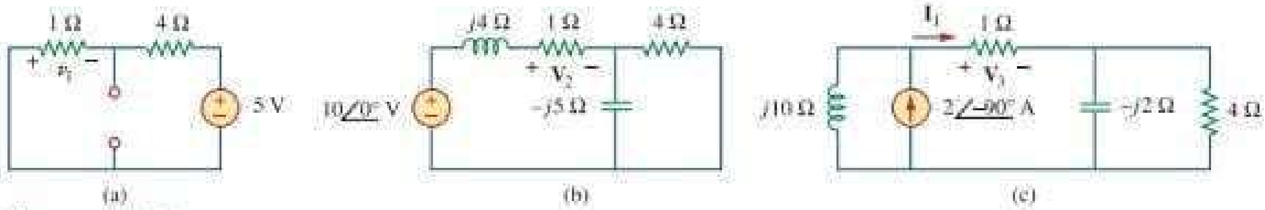


Figure 10.14
Solution of Example 10.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

By voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

In the time domain,

$$v_2 = 2.498\cos(2t - 30.79^\circ) \qquad \textbf{(10.6.3)}$$

To obtain v_3, we set the voltage sources to zero and transform what is left to the frequency domain.

$$2\sin 5t \quad \Rightarrow \quad 2\underline{/-90^\circ}, \qquad \omega = 5\text{ rad/s}$$

$$2\text{ H} \quad \Rightarrow \quad j\omega L = j10\ \Omega$$

$$0.1\text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2\ \Omega$$

The equivalent circuit is in Fig. 10.14(c). Let

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

By current division,

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ})\text{ A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ}\text{ V}$$

In the time domain,

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ)\text{ V} \qquad \textbf{(10.6.4)}$$

Substituting Eqs. (10.6.2) to (10.6.4) into Eq. (10.6.1), we have

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ)\text{ V}$$

Example 10.8

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.22.

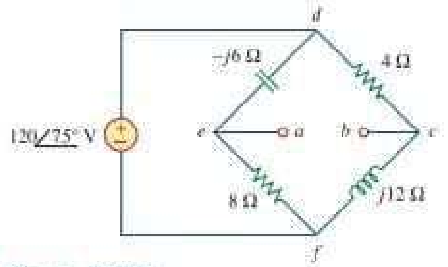


Figure 10.22
For Example 10.8.

Solution:

We find $\mathbf{Z}_{Th}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

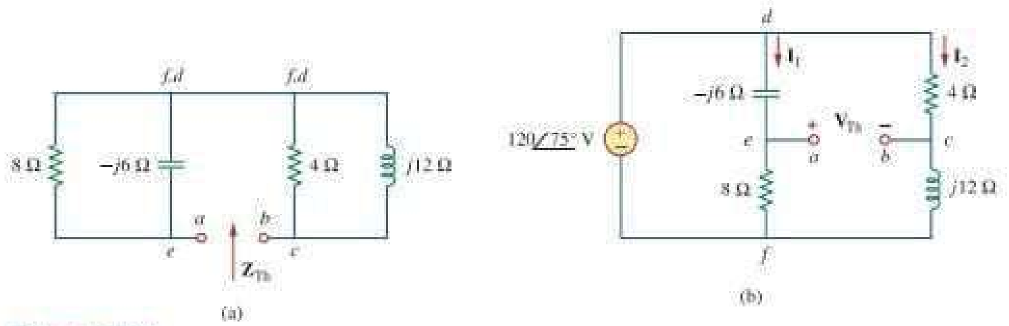


Figure 10.23
Solution of the circuit in Fig. 10.22: (a) finding $\mathbf{Z}_{Th}$, (b) finding $\mathbf{V}_{Th}$.

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\underline{/75^\circ}}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\underline{/75^\circ}}{4 + j12}\ \text{A}$$

Applying KVL around loop *bcdeab* in Fig. 10.23(b) gives

$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

$$\begin{aligned}\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\underline{/75^\circ}}{4 + j12} + \frac{720\underline{/75^\circ + 90^\circ}}{8 - j6} \\ &= 37.95\underline{/3.43^\circ} + 72\underline{/201.87^\circ} \\ &= -28.936 - j24.55 = 37.95\underline{/220.31^\circ}\ \text{V}\end{aligned}$$

Obtain current $\mathbf{I}_o$ in Fig. 10.28 using Norton's theorem.

Example 10.10

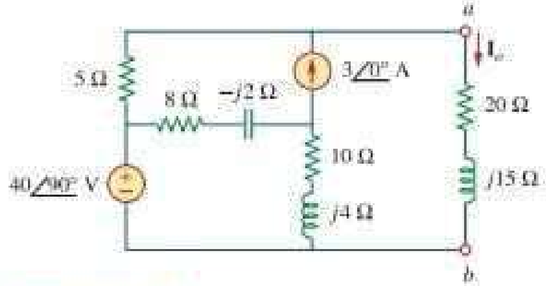


Figure 10.28
For Example 10.10.

Solution:

Our first objective is to find the Norton equivalent at terminals a-b. $\mathbf{Z}_N$ is found in the same way as $\mathbf{Z}_{Th}$. We set the sources to zero as shown in Fig. 10.29(a). As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that

$$\mathbf{Z}_N = 5\ \Omega$$

To get $\mathbf{I}_N$, we short-circuit terminals a-b as in Fig. 10.29(b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad \textbf{(10.10.1)}$$

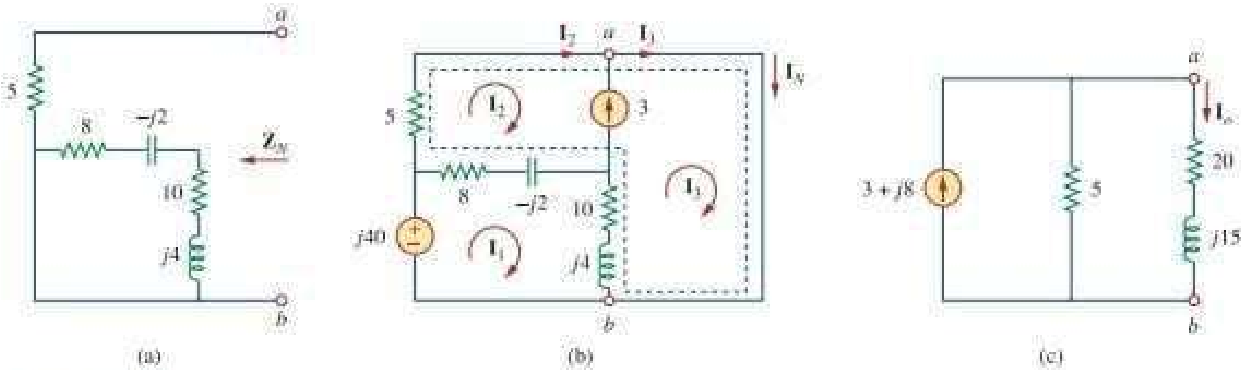


Figure 10.29
Solution of the circuit in Fig. 10.28: (a) finding $\mathbf{Z}_N$, (b) finding $\mathbf{V}_N$, (c) calculating $\mathbf{I}_o$.

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad \textbf{(10.10.2)}$$

At node a, due to the current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \qquad \textbf{(10.10.3)}$$

Adding Eqs. (10.10.1) and (10.10.2) gives

$$-j40 + 5\mathbf{I}_2 = 0 \quad \Rightarrow \quad \mathbf{I}_2 = j8$$

From Eq. (10.10.3),

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is

$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\ \text{A}$$

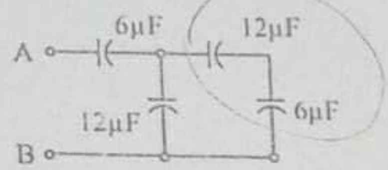
Figure 10.29(c) shows the Norton equivalent circuit along with the impedance at terminals a-b. By current division,

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\underline{/38.48^\circ}\ \text{A}$$

C توالي ← جمع مقلوب
توازي ← جمع عادي

Question # 3 (7 points)

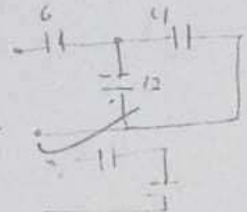
a. For the circuit shown below, find the equivalent capacitance $\underline{C_{eq}}$ seen between terminal A-B.



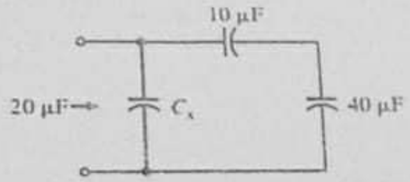
$C_{equ}① = \dfrac{12 \times 6}{12+6} = 4\,\mu F$

$C_{equ}② = 12 + 4 = 16\,\mu F$

$C_{eq}③ = \dfrac{16 \times 6}{16+6} = 4.4\,\mu F$



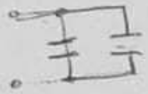
b. For the circuit shown below, determine $\underline{C_x}$ if the equivalent capacitance is $\boxed{20\ \mu F.}$



$C_{equ}① = \dfrac{10 \times 40}{10+40} = 8\,\mu F$

$20 = 8 + C_x$

$C_x = 12\,\mu F$



c. For the circuit shown below, find the equivalent inductance $\underline{L_T}$.

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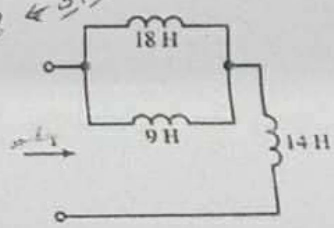
توالي ← جمع عادي



$L_{equ}① = \dfrac{18 \times 9}{18+9} = 6\ H$

$L_{equ}② = 14 + 6 = 20\,H.$

d. If the voltage $\underline{v_c(t)}$ across a $\underline{2F}$ capacitor is shown below, draw the waveform for the capacitor current $i_c(t)$.

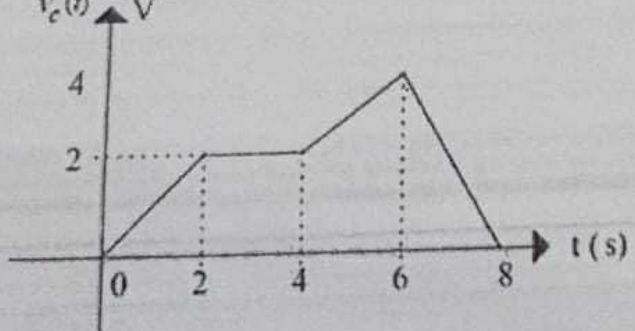
$I = C \cdot V$

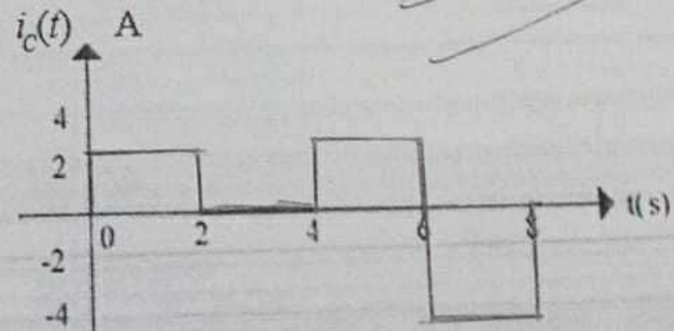
$0 < t < 2 \rightarrow V_c = 1$ volt $(0,0)$ $(2,2)$
$\rightarrow i_c(t) = 2A$

$2 < t < 4 \rightarrow V_c = 0$ volt $(2,2)$ $(4,2)$
$\rightarrow i_c = 0$

$4 < t < 6 \rightarrow V_c = 1$ volt $(4,2)$ $(6,4)$
$\rightarrow i_c = 2$ volt

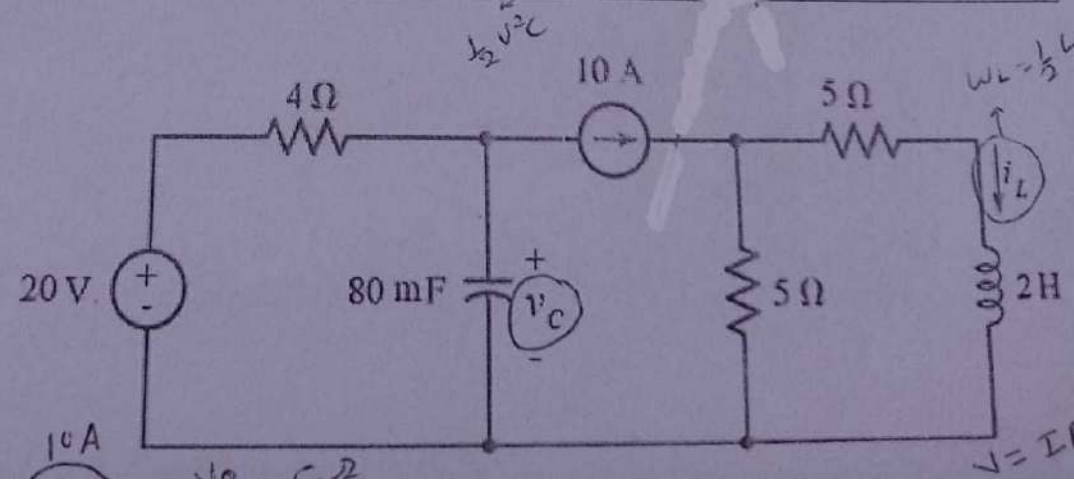
$6 < t < 8 \rightarrow V_c = -2$ volt. $(6,4)$ $(8,0)$
$\rightarrow i_c = 2 \times 2 = 4A$





For the circuit shown below, the sources have been connected for very long time. Find under this condition:

a.	the current through the inductor.	$I_L =$	A
b.	the voltage across the capacitor.	$v_C =$	V
c.	the energy stored in the inductor.	$W_L =$	J
d.	the energy stored in the capacitor.	$W_C =$	J



Question # 5 (6 points)

a. In a linear circuit, if the voltage source is $v_s(t) = 12\sin(314t + 30°)$ V, find the frequency f and the period T of the voltage waveform.

$T = \frac{2\pi}{\omega}$

$\omega = 2\pi f$

$314 = 2\pi f \rightarrow f = \frac{314}{2\pi} = 49.97 \simeq 50$

$T = \frac{1}{f} = 0.02\,s$

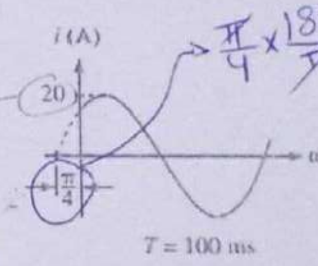
1

b. Write the equation of the sinusoidal current waveform $i(t)$ shown below with the phase angle θ expressed in degrees.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{100 \times 10^{-3}} = 62.8$ X

$i(t) = 20 \sin(62.8t + \cancel{3.4/2})$

$\frac{\pi}{4} \times \frac{180}{\pi} = 45°$



الإجابة الصحيحة $\Rightarrow 20 \sin(\omega t + \frac{\pi}{4}) = 20 \sin(\omega t + 45°)$

c. Obtain the sinusoidal waveforms corresponding to each of the following phasors:

i. $\overline{V}_1 = 60\angle 15°\ V$, $\omega = 10$ rad/s

$v(t) = 60 \sin(10t - 75)$ X

الإجابة الصحيحة $\rightarrow 60 \cos(10t + 15°)$

ii. $\overline{V}_2 = 6 + j8\ V$, $\omega = 40$ rad/s

$V = \sqrt{6^2 + 8^2} = 10$

$\theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.1$

$v(t) = 10\cos(40t + 53.1)$

d. Given $v_1(t) = 20\sin(\omega t + 60°)$ V and $v_2(t) = 60\cos(\omega t - 10°)$, determine the phase angle θ

Chapter 11

CH(11): AC POWER Analysis

11.2: Instantanous and average Power

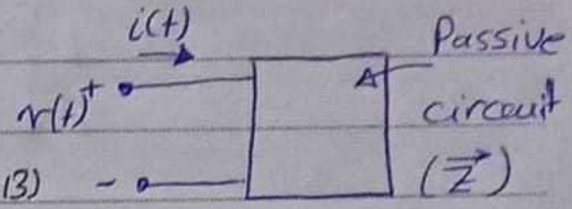
* The instantanous Power: $P(t)$ → تتغير قيمتها بتغير الزمن

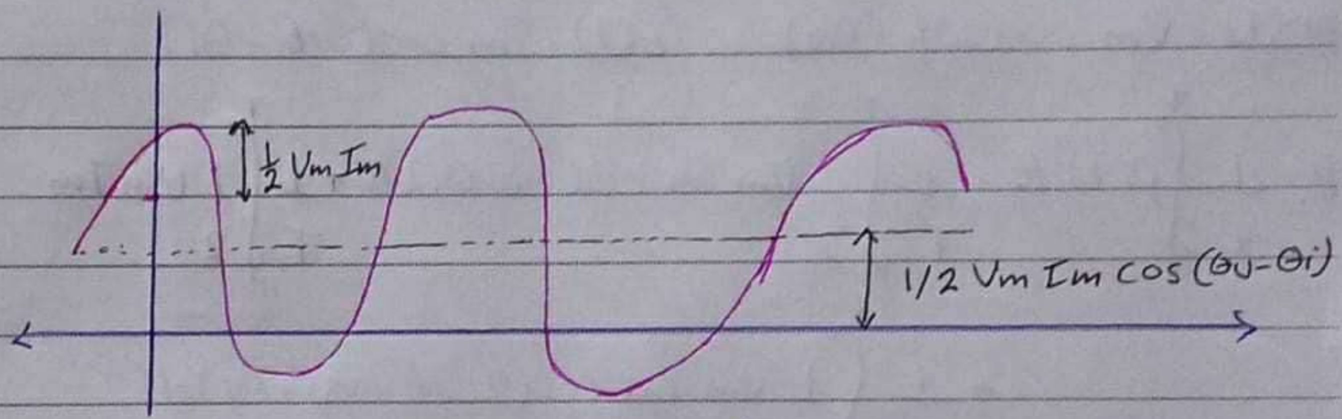
$$P(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$
$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$i(t) = I_m \cos(\omega t + \theta_i)$

$v(t) = V_m \cos(\omega t + \theta_v)$

$\cos(A)\cos(B) = 1/2 \cos(A-B) + 1/2 \cos(A+B)$

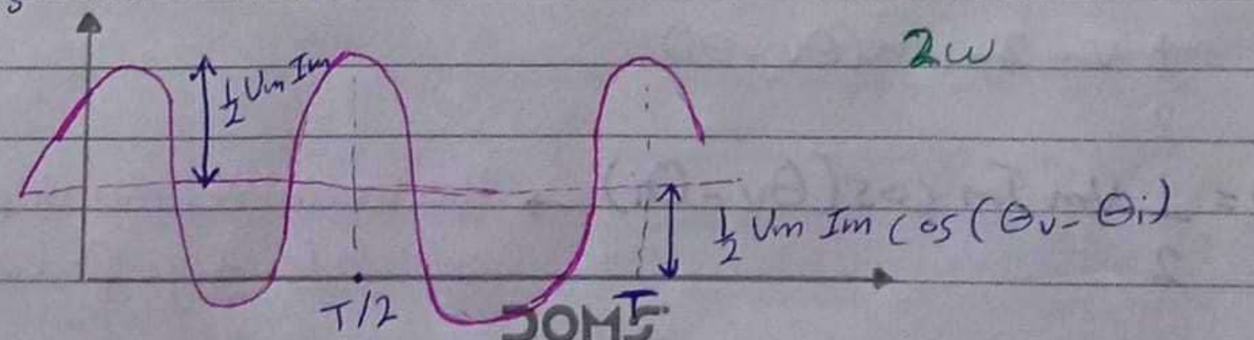




التمثيل البياني لل Instenous Power عبارة عن رسمة ال cos المعتادة ولكن مع (shift) للأعلى وهي ال Peak value

* Average Powers

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$



NOTE:

1. P is not time dependent.
2. When $\theta_v = \theta_i$, it is a purely resistive load case.
3. When $\theta_v - \theta_i = \mp 90^\circ$, it is a purely reactive load case.
4. $P = 0$ means that the circuit absorbs no average Power.

The average Power P is the average of the instantanous power over one Period.

$$p(t) = v(t)\,i(t) \rightarrow \text{Intantanous Power}$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt \rightarrow \text{Average Power}$$

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt$$

$$+ \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \cancel{\frac{1}{2} V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}^{\,0}$$

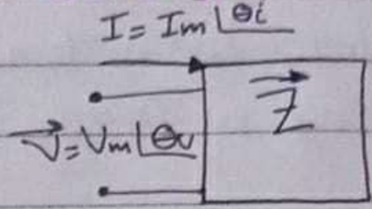
Integral of sinusoidal $= 0$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \rightarrow \text{The main relationship to calculate Avg. Power}$$

* If I have impedance with value of I & V will be 3 cases:

① If Impendance pure resistor:



$\vec{Z} = R \quad , \theta_v - \theta_i = 0$

$P_{avg} = \frac{1}{2} V_m I_m$

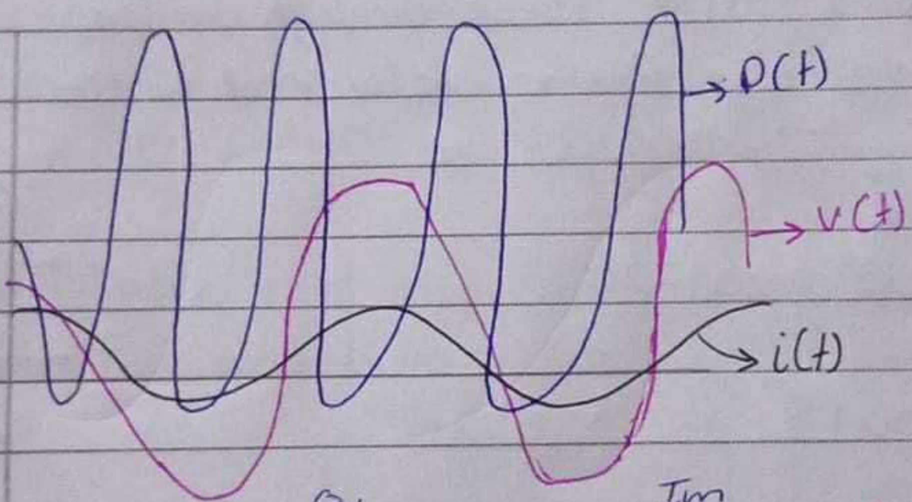
② $\vec{Z} = j\omega L = \omega L \angle 90° \quad , \theta_v - \theta_i = 90°$

$P_{avg} = 0$ → Don't have inductor Power.

③ $\vec{Z} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90°$

$P = 0$

Ex:



$v(t) = \underset{V_m}{\underline{120\sqrt{2}}} \cos(\underset{\omega}{\underline{377}}t + \underset{\theta_v}{\underline{60°}}) \qquad i(t) = \underset{I_m}{\underline{24\sqrt{2}}} \cos(\underset{\omega}{\underline{377}}t + \underset{\theta_i}{\underline{30°}})$

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \times 120\sqrt{2} \times 24\sqrt{2} \times \cos(60° - 30°)$

$= 2494.2 + \frac{1}{2} \times 120\sqrt{2} \times 24 \times \sqrt{2} \times \cos(\underline{754}t + 90°)$

لأنها دورة كاملة فتصبح 2ω

$= 2494.2 + 2880 \cos(754t + 90°)$

Example:

Calculate the instantaneous Power and average Power absorbed by Passive linear network if:

$v(t) = 80\cos(10t + 20^\circ)$ يجب تحولها

$i(t) = 15\sin(10t + 60^\circ) \xrightarrow{\text{لـ cos}} 15\cos(10t + 60^\circ - 90^\circ)$

$\sin = \cos - 90$ $\qquad$ 2ω

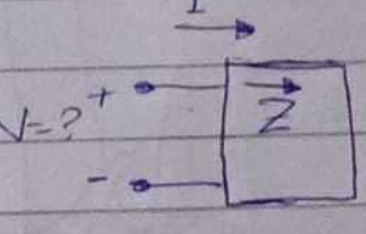
$$P(t) = v(t)\,i(t) = \frac{1}{2} \times 80 \times 15\cos(50) + \frac{1}{2} \times 80 \times 15 \times \cos(20t - 10^\circ)$$

$$= \underbrace{385.7}_{P_{avg}} + 600\cos(20t - 10^\circ)\ W$$

Example:

A current $I = 10\angle 30^\circ$ flows through an impedance. Find the average Power delivered to the impedance

$$Z = 20\angle -22^\circ\ \Omega$$



① $\vec{V} = \vec{I}\,\vec{Z}$

$= 10\angle 30^\circ \times 20\angle -22^\circ$

$= 200\angle 8^\circ$

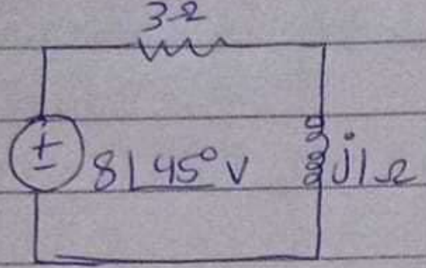
② $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= \frac{1}{2} \times 200 \times 10 \times \cos(8 - 30)$

$= 927.2W$

③ $P(t) = 927.2 + \frac{1}{2} \times 200 \times 10 \times \cos(2\omega t + 38^\circ)$

Example: Find the average absorbed by resistor and inductor. Find the average power supplied by the source



① For the resistor:

$I_R = I = 2.53\angle 26.57^\circ$

$V_R = 3I = 7.59\angle 26.57^\circ$

$P_R = \frac{1}{2} V_m I_m = \frac{1}{2}(2.53)(7.59) = 9.6\,W$

$\left\{ I = \frac{8\angle 45^\circ}{3+j} = 2.53\angle 26.57^\circ \right.$

② For the inductor, $I_L = 2.53\angle 26.57^\circ$

$V_L = j I_L = 2.53\angle 26.57 + 90^\circ$

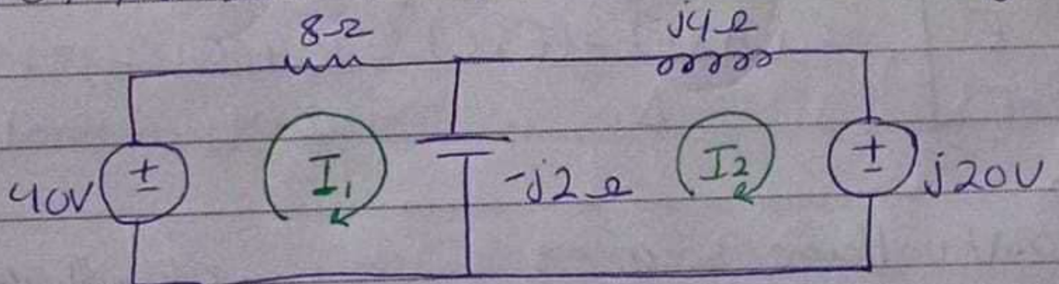
$= 2.53\angle 116.57^\circ$

$P_L = \frac{1}{2}(2.53)^2 \cos(90^\circ) = 0\,W$

③ The average Power supplied is:

$P = \frac{1}{2}(8)(2.53)\cos(45^\circ - 26.57^\circ) = 9.6\,W$

Example: Calculate the average Power absorbed by each of the five elements in the circuit given.



For mesh ①:

$-40 + 8I_1 + -j2(I_1 - I_2) = 0$

$-40 + (8-j2)I_1 + (-j2)I_2 = 0$

$((8-j2)I_1 + (-j2)I_2 = 40) \div 2$

$(4-j)I_1 + (-j)I_2 = 20$ ---- ①

For mesh ②:

$-j20 + (j4 - j2) I_2 + (-j2) I_1 = 0$

$(j4 - j2) I_2 + (-j2) I_1 = j20 \;/\; 2$

$-j I_1 + j I_2 = j10$

In matrix Form:

$$\begin{array}{cc} A & B \\ C & D \end{array} \qquad \begin{array}{c} E \\ F \end{array}$$

$$\begin{bmatrix} 4-j & -j \\ -j & 0-j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

$\Delta = (A \times D) - (C \times B) = (1 + 4 \times j) - (-1) = 2 + 4j$

-20j 10 -10-20j -5-6j

$\Delta_1 = \dfrac{\begin{bmatrix} E & B \\ F & C \end{bmatrix}}{\Delta} = \dfrac{(E \times C) - (B \times F)}{\Delta} = \dfrac{-10+20j}{2+4j} = 3+4j$

$I_1 = 5 \angle 53.14^\circ =$

$7.8 \angle -129.8$

* الجواب اللي بالأسود هيك طلع معي على الآلة الحاسبة

$\Delta 2 = \dfrac{\begin{bmatrix} A & E \\ C & F \end{bmatrix}}{\Delta} = \dfrac{(A \times F) - (E \times C)}{\Delta} = \dfrac{10+60j}{2+4j} = 13 + 4j = 13.6 \angle 17.11^\circ$

For the 40-V voltage source:

$V_s = 40 \angle 0^\circ \qquad I_1 = 5 \angle 53.14^\circ$

$P_s = \dfrac{-1}{2} (-4)(5) \cos(-53.14^\circ) = -60\,W$

* كملت الحسابات بناءً على جواب الدكتور للأمانة في النقل

For the j20-V voltage source:

$V_s = 20 \angle 90^\circ \qquad I_2 = 13.6 \angle 17.11^\circ$

$P_s = -\frac{1}{2}(20)(13.6)\cos(90^\circ - 17.11^\circ) = -40W$

For the resistor:

$I = |I_1| = 5$ $\quad V = 8|I_1| = 40$ volt

$P = \frac{1}{2}(40)(5) = 100\,W$

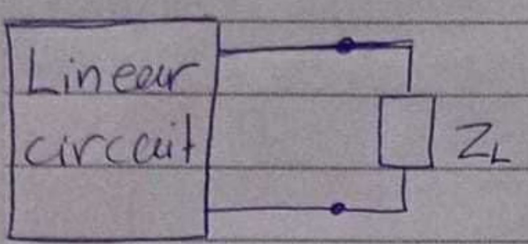
* The average Power absorbed by the inductor and capacitor is zero watts.

11.3 → Maximum Average Power Transfer.

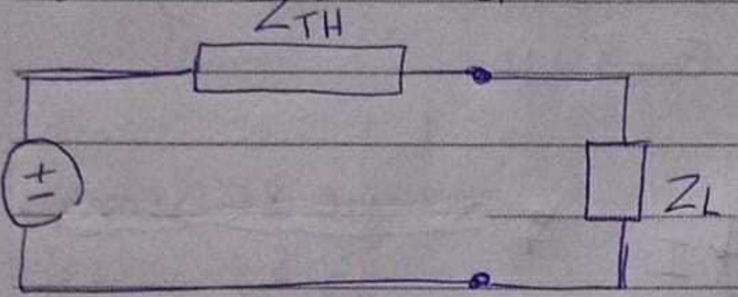
$Z_{TH} = R_{TH} + jX_{TH}$ $\quad$ * $R_L = R_{TH}$

$Z_L = R_L + jX_L$ $\quad$ * $X_L = -X_{TH}$

The maximum average power can be transferred to the load if



a) Circuit with a load



b) Thevenin Equivalent circuit.

$X_L = X_{TH}$ and $R_L = R_{TH}$

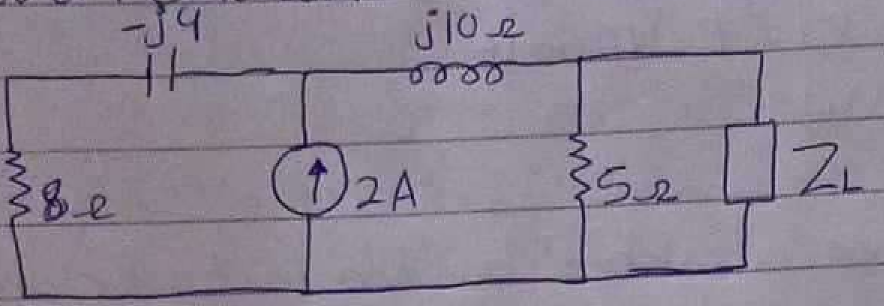
$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

* If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

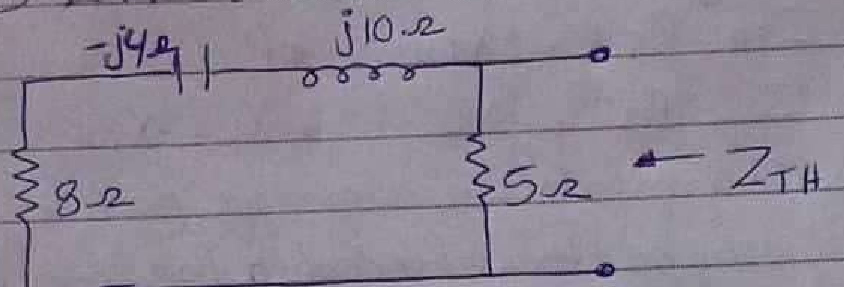
conject

$Z_L = R_{TH} - jX_{TH} = Z_{TH}^*$

Example: For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate the maximum average power.

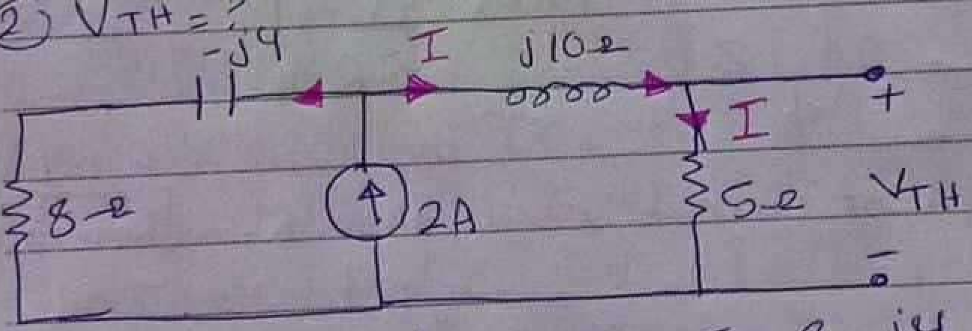


① $Z_{TH} = Z_L$ (We must kill all source and remove Z_L)



$$Z_{TH} = 5 \parallel (-j4 + 10j + 8) = \frac{5 \times (8 + j6)}{5 + 8 + j6} = 3.415 + j0.7317$$

② $V_{TH} = ?$



j10 & 5 → series
-j4 & 8 → series

By current division ⇒ $I = \dfrac{8 - j4}{8 - j4 + j10 + 5}$ (2)

$$V_{TH} = 5I = \frac{(2)(5)(8 - j4)}{13 + j6} = 6.25 \angle -51.34^\circ$$

$$Z_L = Z_{TH}^* = \underbrace{3.415}_{R_L} - j\underbrace{0.7317}_{X_L}\ \Omega$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_L} = \frac{(6.25)^2}{(8)(3.415)} = 1.429\text{ W}$$

Maximum Average Power for Resistive Load.

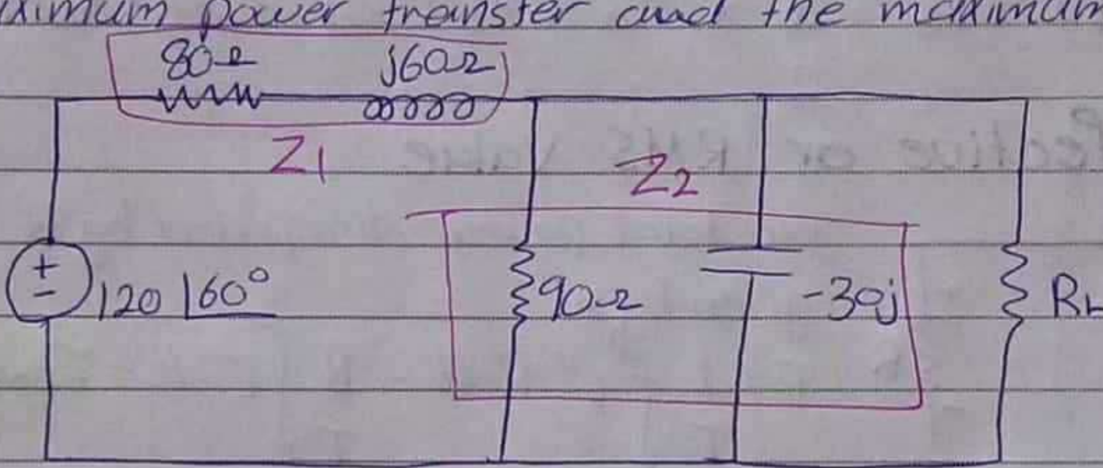
* When the load is Purely Resistive, the condition for maximum power transfer is:

$$R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$$

* Now the maximum power can not be obtained from P_{max} formula given befor.

$$P = \frac{1}{2} I_L^2 R_L = \frac{1}{2} I_L V_L = \frac{1}{2} \frac{V_L^2}{R_L}$$

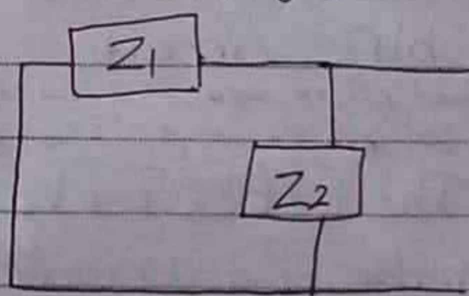
Example: Calculate the resistive load needed for maximum power transfer and the maximum avg. Power



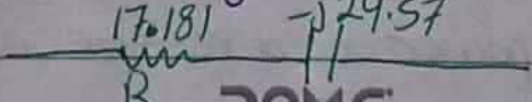
sol: We first find Z_{TH} and V_{TH} across R_L

Let $Z_1 = 80 + j60$ and $Z_2 = 90 \parallel -j30 = \frac{(90)(-j30)}{90 - j30}$
$= 9(1 - j3)$

$$Z_{TH} = Z_1 \parallel Z_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27}$$



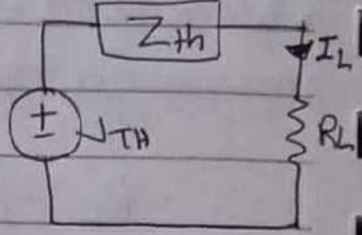
$$= 17.181 - j24.57\ \Omega$$



→ leading

$$V_{TH} = \frac{Z_2}{Z_1 + Z_2}(120\angle 60^\circ) = \frac{9(1-j3)}{89+j33}(120\angle 60^\circ)$$

$$= 35.98\angle -31.91^\circ$$



$R_L = |Z_{TH}| = 30\,\Omega$

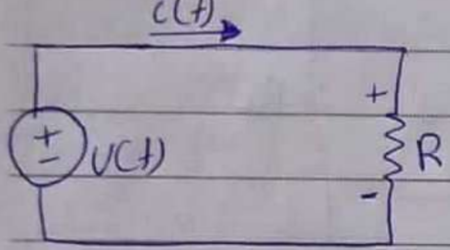
* The current through the load is

$$I = \frac{V_{TH}}{Z_{TH} + R_L} = \frac{35.98\angle -31.91}{47.181 - j24.57} = 0.6764\angle -4.4^\circ$$

The maximum average Power absorbed by R_L is

$$P_{max} = \frac{1}{2}|I^2|R_L = \frac{1}{2} \times (0.6764)^2(30) = 6.863\ W.$$

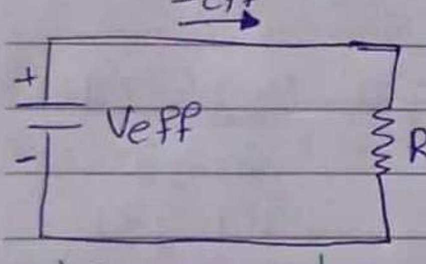
11.4: Effective or RMS Value



a) AC circuit

The total power dissipated by R given by:

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt = I_{rms}^2 R$$



Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = I_{rms}$$

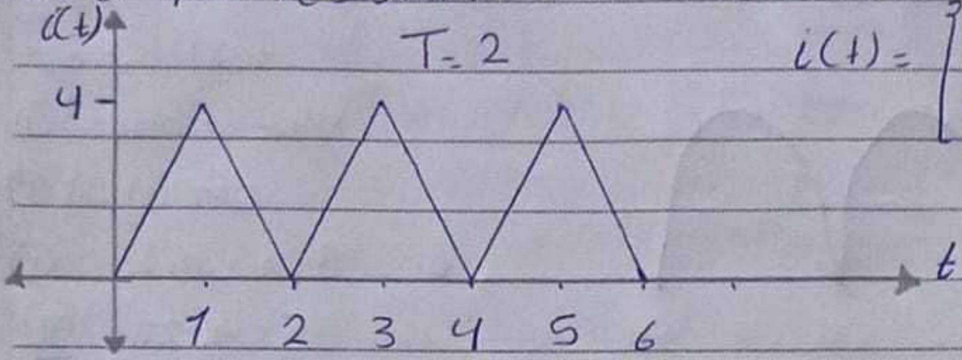
The rms value is a constant itself which depending on the shape of the function $i(t)$.

b) DC circuit

هو الاستفادة من مصدر فرق الجهد المتغير
بطريقة مكافئة لمصدر فرق الجهد الثابت

The EFFECTIVE Value or the Root Mean Squar (RMS) value of a periodic current is the DC value that delivers the same average power to a resistor as the periodic current.

Example: Find the RMS value of the current waveform. Calculate the average power if the current is applied to a 9 Ω resistor.



$$i(t) = \begin{cases} 4t & , 0<t<1 \\ 8-4t & , 1<t<2 \end{cases}$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\,dt = \frac{1}{2}\left[\int_0^1 (4t)^2\,dt + \int_1^2 (8-4t)^2\,dt\right]$$

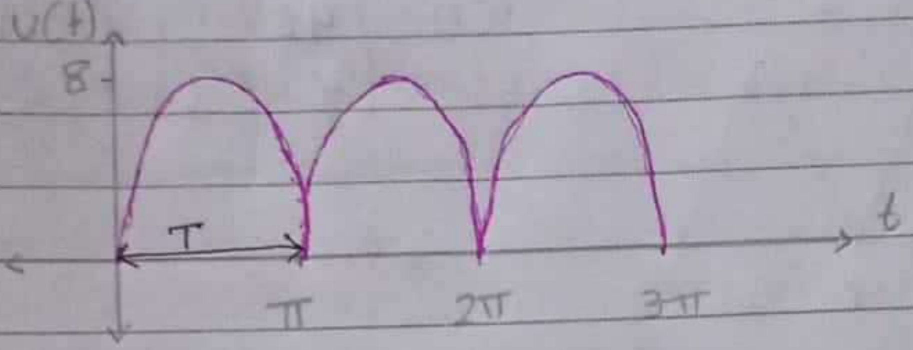
$$I_{rms}^2 = \frac{16}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (4-4t+t^2)\,dt\right]$$

$$I_{rms}^2 = 8\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{16}{3}$$

$$I_{rms} = \sqrt{\frac{16}{3}} = 2.309\,A$$

$$P = I_{rms}^2 R = \frac{16}{3}(9) = 48W$$

Example: Find the RMS value of the full-wave rectified sine wave. Calculate the average Power dissipated in a $6\,\Omega$ resistor.



كيف جبنا v(t) :

① Value of beak = 8

② اقتران sin

③ $8\sin(\omega t)$

$= 8\sin(2\pi t/2\pi)$

[الـ Period اللازمة لإكمال دورة كاملة

$= 8\sin(t)$

$T = \pi$, $v(t) = 8\sin(t)$, $0 < t < \pi$

$$V_{eff}^2 = \frac{1}{T}\int_0^T v^2\,dt = \frac{1}{\pi}\int_0^{\pi} (8\sin(t))^2\,dt$$

$$V_{eff}^2 = \frac{64}{\pi}\int_0^{\pi} \frac{1}{2}[1-\cos(2t)]\,dt = 32$$

$V_{eff} = 5.657\,V$

$$P = \frac{V_{eff}^2}{R} = \frac{32}{6} = 5.333\,W$$

$v(t) = V_m\cos(\omega t + \theta_v)$

$\vec{V} = V_m \angle \theta_v$ [V]

$V_{rms} = \frac{V_m}{\sqrt{2}}$ [V_{rms}]

11.5: Apparent Power and Power Factor.

→ Apparent Power, S, is the product of the r.m.s values of voltage and current.

→ It is measured in volt-amperes or VA

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = \underbrace{S}_{\text{Apparent Power}} \underbrace{\cos(\theta_v - \theta_i)}_{P.F}$$

مهم الجدول

Purely resistive load (R)	$\theta_v - \theta_i = 0$ $PF = 1$	$P/S = 1$, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \mp 90$ $PF = 0$ (+) → L (−) C	$P = 0$, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	Lagging → inductive load leading → capacitive load

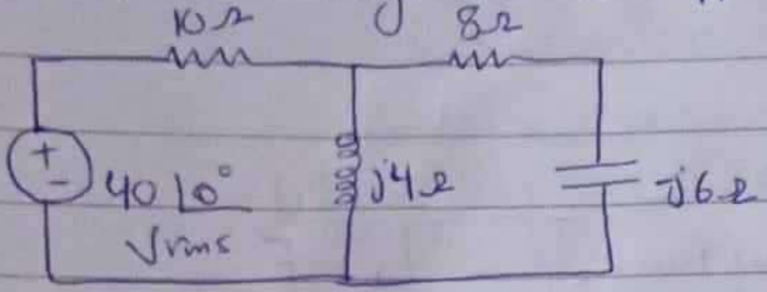
تلخيص العلاقات الثلاثة

① $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{Rms} I_{Rms} \cos(\theta_v - \theta_i)$ → [W]

② $PF = \frac{P}{S} = \cos(\theta_v - \theta_i)$

③ $S = \frac{1}{2} V_m I_m = V_{Rms} I_{Rms}$ [VA]

Example: Calculate the power factor seen by the source and the average Power supplied by the source.



* $V_m = V_{rms} \times \sqrt{2}$
$= 40\angle 0^\circ \times \sqrt{2}$
$= 56.56$ Volt

The total impedance as seen by the source is:

$$Z = 10 + [j4 \parallel (8 - j6)] = 10 + \frac{(j4)(8-j6)}{8 - j6 + j4} \Rightarrow Z = 12.69\angle 20.62^\circ$$

* The Power Factor

$PF = \cos(20.62^\circ) = 0.936$ (lagging) → lead / lag مهم شغلة ال بالامتحان

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40\angle 0^\circ}{12.69\angle 20.62^\circ} = 3.152\angle -20.62$$

* The average Power supplied by the source is equal to the power absorbed by the load.

$P = I_{rms}^2 R = (3.152)^2 (11.88) = 118\,W$

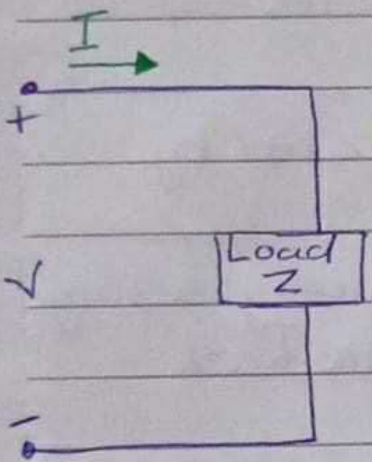
or

$P = V_{rms} I_{rms} R = (40)(3.152)(0.936) = 118\,W$

11.6: Complex Power → نزوقها عن ال app. بالسهم

*The COMPLEX Power ($\vec{S}$) contains all the information pertaining to the Power absorbed by a given load.

*Complex power (S) is the product of the voltage and the complex conjugate of the current:



$V = V_m \angle \theta_v$

$I = I_m \angle \theta_i$

→ conject is the oppiste sign of I.

$\frac{1}{2} V \boxed{I^*} = V_{rms} I_{rms} \angle \theta_v - \theta_i$ $\quad I^* = I_m \angle -\theta_i$

* الأشياء المستفادة من ال complex Power:

Ex. $\vec{S} = 2 \angle 30°$ → $S = 2$, $P.F = \cos(30)$, $P = S \times P.F$

$S = \frac{1}{2} V I^* = \underbrace{V_{rms} I_{rms}}_{S} \angle \underbrace{\theta_v - \theta_i}_{\theta_Z}$

$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$S = P + jQ = S \angle \theta_v - \theta_i$, $S = \sqrt{P^2 + Q^2}$

P: is the average Power in watts

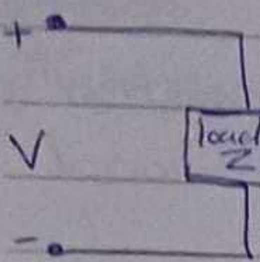
Q: is the reactive Power exchange between the source and the reactive part of the load.

Q = 0 (for resistive loads (unity PF)).

Q < 0 (for capacitive loads (leading PF)).

Q > 0 (for inductive loads (lagging PF)).

/ /



$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$S = P + jQ$

→ Apparent Power: $S = |S| = V_{rms} * I_{rms}$
$= \sqrt{P^2 + Q^2}$

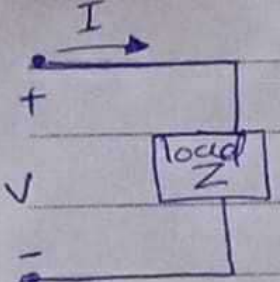
→ Real Power, $P = Re(S) = S\cos(\theta_v - \theta_i)$

→ Reactive Power, $Q = Im(S) = S\sin(\theta_v - \theta_i)$

→ Power Factor, $pf = P/S = \cos(\theta_v - \theta_i)$

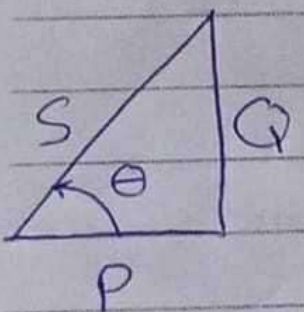
* Real Power is the actual power dissipated by the load.
* Reactive Power is a measure of the energy exchange between source and reactive part of the load.

Power Triangle

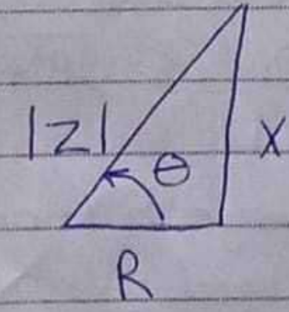


$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

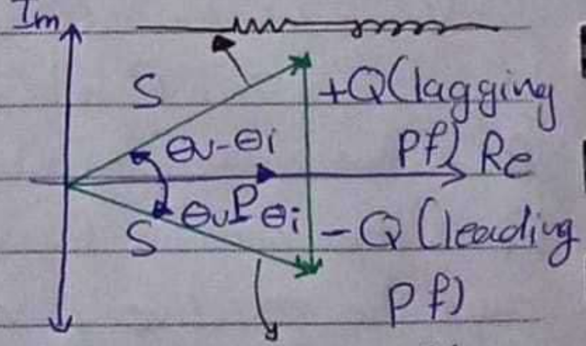
$S = P + jQ$



Power Triangle



Impedance Triangle



Power Factor

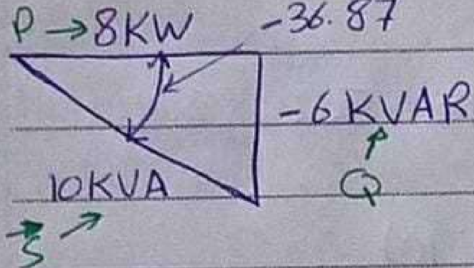
Real and Reactive Powers

* The unit of Q is volt-ampere reactive (VAR)

$$S = P + jQ = Re(S) + j\,Im(S)$$
$$= \text{Real Power} + \text{Reactive Power}$$
$$S = I_{RMS}^2 Z = I_{RMS}^2 (R + jX) = P + jQ$$
$$P = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) = Re\{S\} = I_{RMS}^2 R$$
$$Q = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i) = Im\{S\} = I_{RMS}^2 X$$

Unit: P[W] / $\vec{S}$, S[VA] / Q[VAR]

Use of Power Triangles



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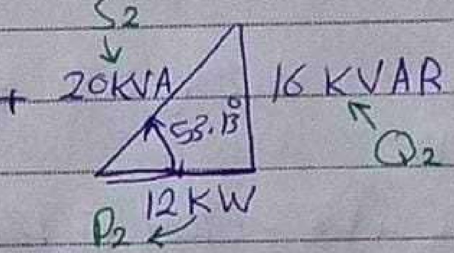


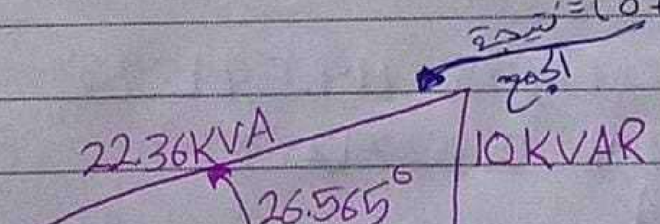
$\vec{S}_1 = 10\angle -36.87$
$= P + jQ$

leading

$\vec{S}_2 = 20\angle 53.13°$

* $S_{Total} = P + jQ = S_1 + S_2$
$= (P_1 + P_2) + j(Q_1 + Q_2)$
$= (8 + 12) + j(-6 + 16)$

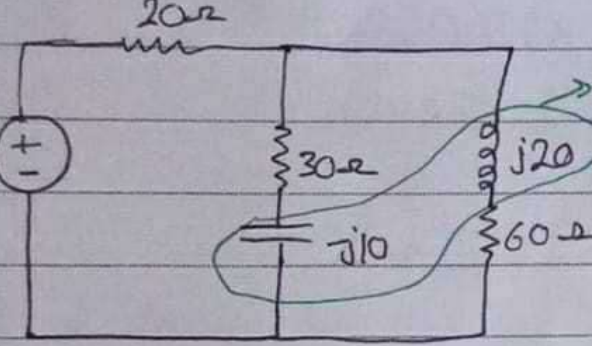
نتيجة الجمع



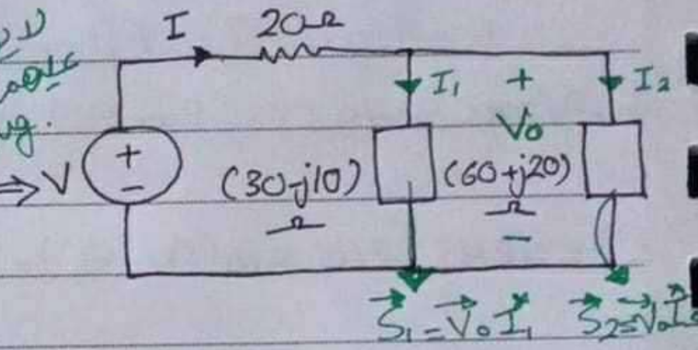
Power(W) ← قاعدة المثلث
Q(VAR) ← الضلع
$\vec{S}$ (VA) ← الوتر

Example:

The 60 Ω resistor absorbs 240 Watt of average Power. Calculate V and the complex power of each branch. What is the total complex power?



لإيجاد V عطانا Pavg:



$P = I_2^2 R \Rightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4 \Rightarrow I_2 = 2\angle 0^\circ$ (rms).

$V_0 = I_2(60 + j20) = 120 + j40 \rightarrow$ Vrms

$I = \frac{V_0}{30 - j10} = 3.2 + j2.4 \Rightarrow I = I_1 + I_2 = 5.2 + j2.4$

$V = 20I + V_0 = (104 + j48) + (120 + j40)$

$= 224 + j88$

$= 240.67\angle 21.45^\circ$ (rms)

For the 20 Ω resistor:

$V = 20I = 204 + j48 = 114.54\angle 24.8^\circ$

$I = 5.2 + j2.4 = 5.727\angle 24.8^\circ$

$S = 656$ VA

For the (30−j10) Ω impedance:

$V_0 = 120 + j40 = 126.5\angle 18.43^\circ$

$I_1 = 3.2 + j2.4 = 4\angle 36.87^\circ$

$S_1 = V_0 I^* = (126.5\angle 18.43^\circ)(4\angle -36.87)$

$\overset{P_{avg}}{}$ $Q = -160$

$S_1 = 506\angle{-18.44^\circ} = 480 - j160$ VA

For the $(60 + j20)\,\Omega$ impedance:

$I_2 = 2\angle{0^\circ}$

$S_2 = V_o I_2^* = (126.5\angle{18.43^\circ})(2\angle{-0^\circ})$

$= 253\angle{18.43^\circ}$

$= 240 + j80$ VA

The overall complex power supplied by the source is

$S_T = V I^* = (240.67\angle{21.45^\circ})(5.727\angle{-24.8^\circ})$

$= 1378.3\angle{-3.35^\circ}$

$= 1376 - j80$ VA

Example: Two loads are connected in Parallel. Load 1 has 2 KW, Pf = 0.75 leading and Load 2 has 4 KW, Pf = 0.95 lagging. Calculate the Pf of two loads and the complex power supplied by the source.

Load(1) $\Rightarrow P_1 = 2000$

$P_1 = S_1 \cos\theta_1$

$S_1 = \dfrac{P_1}{\cos\theta_1} = \dfrac{2000}{0.75}$

$= 2666.67$

$PF = 0.75 = \cos\theta_1$

$\theta = \cos^{-1}(0.75) = -41.41^\circ$

$Q_1 = S_1 \sin\theta_1 = -176.85$

$S_1 = P_1 + jQ_1$

$= 2000 - j176.85$ (leading)

load(2) $\Rightarrow P_2 = 4000$

$S_2 = \dfrac{P_2}{\cos\theta_2} = \dfrac{4000}{0.95}$

$= 4210.53$

$PF = 0.95 = \cos\theta_2$

$\theta_2 = \cos^{-1}(0.95) = 18.19^\circ$

$Q_2 = S_2 \sin\theta_2 = 1314.4$

$S_2 = P_2 + jQ_2$

$= 4000 + j1314.4$ (lagging).

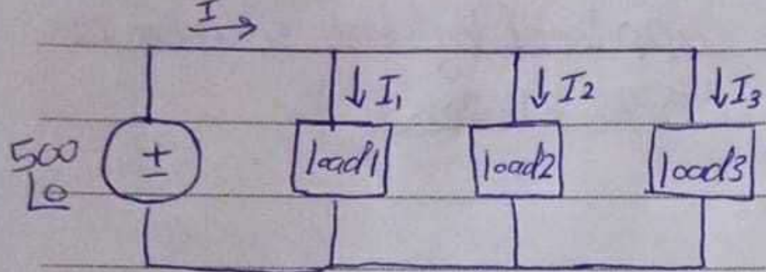
The total complex Power is

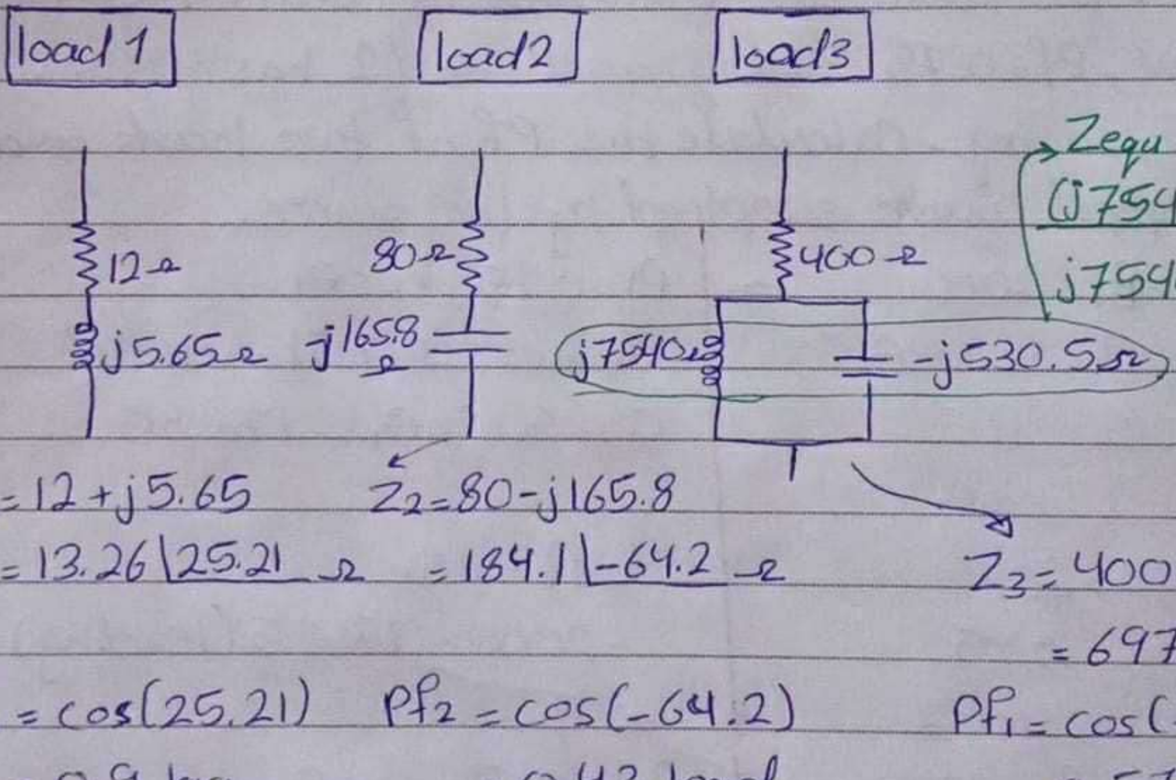
$S = S_1 + S_2$

$= 6 - j0.4495\,kVA$

$pf = \frac{P}{|S|} = \frac{6000}{6016.18} = 0.9972$ (leading).

Example:





$Z_{equ} = \frac{(j7540)(-j530.5)}{j7540 - j530.5} = -570.6 \approx 571\,\Omega$

$Z_1 = 12 + j5.65$
$= 13.26\angle 25.21\,\Omega$

$Z_2 = 80 - j165.8$
$= 184.1\angle -64.2\,\Omega$

$Z_3 = 400 + -j571$
$= 697\angle -55\,\Omega$

$pf_1 = \cos(25.21)$
$= 0.9$ lag

$pf_2 = \cos(-64.2)$
$= 0.43$ lead

$pf_1 = \cos(-55)$
$= 0.57$ lead

$I_1 = \frac{500\angle 0^\circ}{13.26\angle 25.21^\circ} = 37.7\angle -25.21^\circ = 34.11 - j16.06$

$I_2 = \dfrac{500\angle 0^\circ}{184.1\angle -64.2^\circ} = 2.72\angle 64.2^\circ = 1.18 + j2.45$

$I_3 = \dfrac{500\angle 0^\circ}{697\angle -55^\circ} = 0.72\angle 55^\circ = 0.41 + j0.59$

$I = I_1 + I_2 + I_3 = 35.7 - j13.02 = 38\angle -20^\circ$ A

Combined pf $= \cos(20) = 0.94$ lag.

$S_1 = \hat{V} I_1^* = 500 * 37.7\angle 25.21^\circ = 18850\angle 25.21^\circ = 17055 + j8029$ VA

$S_2 = \hat{V} I_2^* = 500 \times 2.72\angle -64.2^\circ = 1360\angle -64.2^\circ = 592 - j1224$ VA

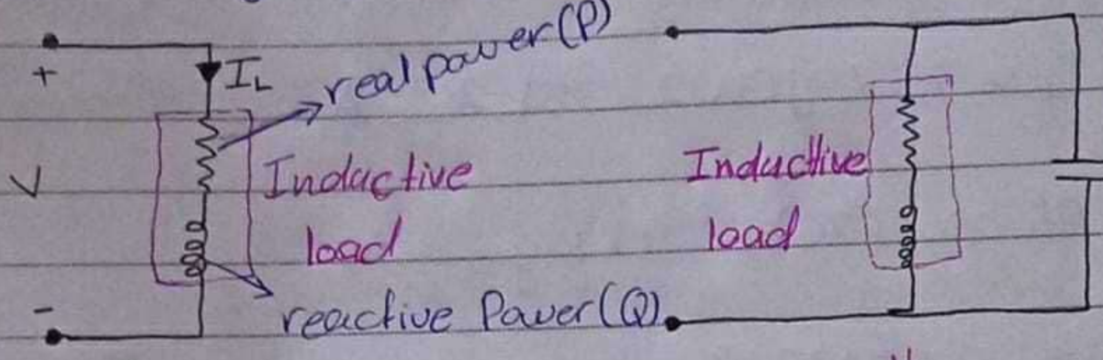
$S_3 = \hat{V} I_3^* = 500 * 0.72\angle -55^\circ = 360\angle -55^\circ = 207 - j295$ VA

$S = S_1 + S_2 + S_3 = 17854 - j6510 = 19000\angle -20^\circ$ VA

Check: $S = \hat{V} I^* = 500 \cdot 38\angle -20^\circ = 19000\angle -20^\circ$ VA.

Power Factor Correction

Power factor correction is the process of increasing the Power factor without altering the voltage or current to the original load.



$\vec{S}_1 = \vec{V}\vec{I}^*$

$= P + jQ$

$P_1 = S_1 \cos(\theta)$

$Q = S_1 \sin(\theta)$

$P.F = \cos(\theta_1)$.

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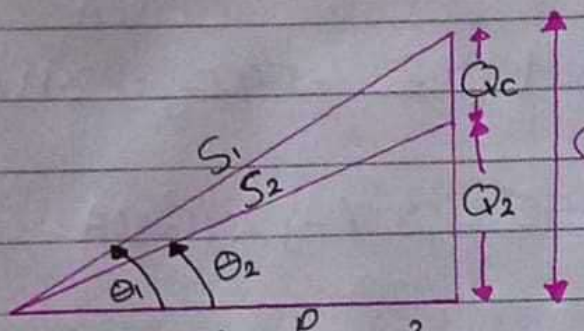
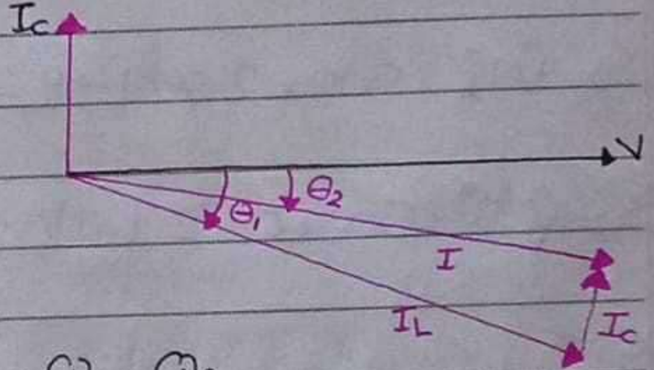




$$Q_c = \frac{V_{rms}^2}{X_c} = \frac{V_{rms}^2}{1/\omega C}$$

$$Q_2 = Q_1 + Q_c$$

$$Q_2 < Q_1$$

$$Q_c = Q_1 - Q_2$$
$$= P(\tan\theta_1 - \tan\theta_2)$$
$$= \omega C V_{rms}^2$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

$\uparrow P.F \downarrow \theta_1$

* الفكرة هي إيجاد قيمة C لترفع قيمة الـ Q وتنزل قيمة الزاوية.

Example: Find the value of the capacitance needed to correct a load of 140 KVAR at 0.85 lagging Pf to unity Pf. The load is supplied by 110 Volt (rms), 60 Hz line.

$Pf = 0.85 = \cos\theta \longrightarrow \theta = 31.79^\circ$

$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{140}{\sin(31.79^\circ)} = 265.8\,KVA$

$P = S\cos\theta = 225.93\,KW$

* For $Pf = 1 = \cos\theta_1 \longrightarrow \theta_1 = 0^\circ$

Since P remains the same:

$P = P_1 = S_1\cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 225.93$

$Q_1 = S_1\sin\theta_1 = 0$

The different between the new Q and the old Q is Qc.

$Q_c = 140\,KVAR = \omega C V_{rms}^2$

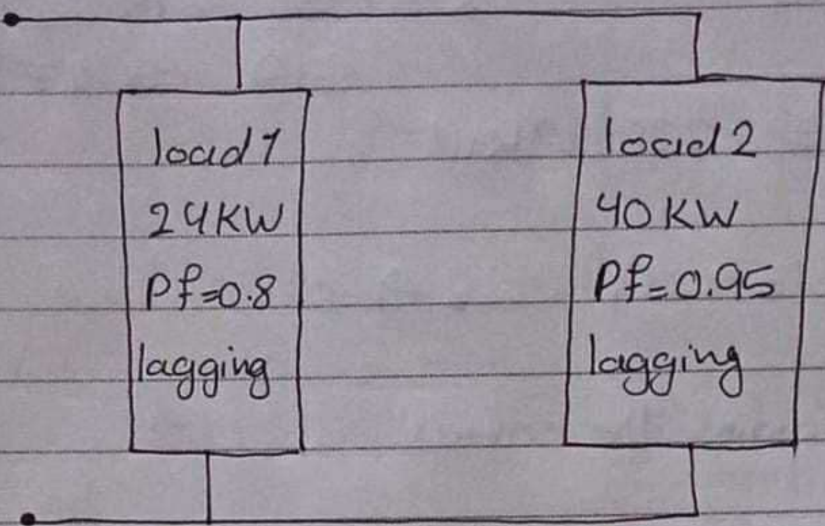
$C = \frac{140 \times 10^3}{(2\pi)(60)(110)^2} = 30.69\,mF.$

—/—/—

Examples: A 120-Vrms, 60HZ source supplies two loads connected in parallel.

a) Find the power Factor of the Parallel combination.

b) calculate the value of the capacitance connected in Parallel that will raise the Power factor to unity.



a) $\theta_1 = \cos^{-1}(0.8) = 36.87^\circ$, $S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30\,KVA$

$Q_1 = S\sin(\theta) = (30)(0.6) = 18\,KVAR$, $\vec{S}_1 = 24 + j18\,KVA$

$\theta_2 = \cos^{-1}(0.95) = 18.19^\circ$, $S_2 = \frac{P_1}{\cos\theta_1} = \frac{40}{0.95} = 42.105\,KVA$

$Q_2 = S_2\sin(\theta) = 13.144\,KVAR$, $\vec{S}_2 = 40 + j13.144\,KVA$

$S = S_1 + S_2 = 64 + j31.144\,KVA$

$\theta = \tan^{-1}(31.144/64) = 25.95^\circ$ $Pf = \cos\theta = 0.8992$ (lag)

b) $\theta_2 = 25.95^\circ$, $\theta_1 = 0^\circ$

$Q_C = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95^\circ) - 0] = 31.144\,KVAR$

$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{31.144}{(2\pi)(60)(120^2)} = 5.74\,mF$

Problem (1): The heating element in a soldering iron has a resistance of 30 Ω. Find the average power dissipated in the soldering iron if it is connected to a voltage source of 117 Vrms.

$\cancel{P_{avg} = I_{rms} V_{rms}}$ $\qquad$ $P = \frac{V_{rms}^2}{R} = \frac{(117)^2}{30}$

$\cancel{I_{rms} = \frac{V_{rms}}{R} = \frac{117}{30} = 3.9A}$ $\qquad$ $= 456.3W$

Problem (2):

A current of 4A flows when a neon light advertisment is supplied a 110 Vrms power system. The current lag the voltage by 60°. Find the Power dissipated by the circuit and the power factor.

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.82$ or $V_m = V_{rms} \times \sqrt{2} = 110 \times \sqrt{2}$

$= 155.56$

$P = I_{rms} V_{rms} \cos(\theta_v - \theta_i) = 2.82 \times 110 \times \cos(-60) = 155.1W$

$P.F = \cos(60) = 0.5$, $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = 155.56W$

Problem (3):

A current of 10 Arms flow when a single-phase circuit is placed acrross a 220-Vrms source. The current lags the voltage by 60°. Find the power dissipated by the circuit and the Power Factor.

$P = V_{rms} I_{rms} \cos(-60)$

$= 10 \times 220 \times \cos(-60)$

$= 1100 W$

$P.F = \cos(-60) = 0.5$

Problem: A current source $i(t)$ is connected to a $50\text{-}\Omega$ resistor. Find the average Power delivered to the resistor that $i(t)$ is:

a. $5\cos(50t)$ A. $I_{rms} = I_m/\sqrt{2}$

$i = 5\angle 0^\circ \rightarrow P_{avg} = I_{rms}^2 \times R = (5/\sqrt{2})^2 \times 50 = 625W$

b. $5\cos(5t - 45)$ A).

$i = 5\angle -45 \rightarrow P_{avg} = (5\angle -45/\sqrt{2})^2 \times 50 = 625\angle -90$ AW

c. $5\cos(50t) - 2\cos(50t - 0.873)$

$5\angle 0 - 2\angle -0.873 = 3\angle 0.582$

$I_{rms} = \dfrac{3\angle 0.582}{\sqrt{2}} = 2.12\angle 0.582$, $P_{avg} = (2.12\angle 0.582)^2 \times 50$

$= 225\angle 1.164$

D. $5\cos(50t) - 2$ A.

DC $\rightarrow -2A$

AC $\rightarrow 5\angle 0^\circ$

$P_{Total} = P_{AC} + P_{DC}$

$= (-2)^2 \times 50 + (5/\sqrt{2})^2 \times 50 = 200 + 625 = 825W$

Problem: Find the rms value of each of the following:

a. $i(t) = \cos(450t) + 2\cos(450t)$ A.

$= 3\cos(450t)$

$I_{rms} = 3/\sqrt{2} = 2.12A$

b. $i(t) = \cos(5t) + \sin(5t)$

$= \cos(5t) + \cos(5t - 90)$

$= 1\angle 0^\circ + 1\angle -90^\circ$

$= \sqrt{2}\angle -45^\circ$

$I_{rms} = \sqrt{2}\angle -45^\circ/\sqrt{2} = 1\angle -45$

c. $i(t) = \cos(450t) + 2$

$= 1\angle 0° + 2$

$I_{rms} = \sqrt{(2)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{4 + 0.5} = 2.12\,A$

d. $i(t) = \cos(5t) + \cos(5t + \pi/3)$.

$= 1\angle 0° + 1\angle 60°$

$= 1 + 0.5 + j0.866$

$= 1.5 + j0.866$

value: $\sqrt{(1.5)^2 + (0.866)^2} = \sqrt{3}$

$I_{rms} = \frac{\sqrt{3}}{\sqrt{2}} = 1.22A$

e. $i(t) = \cos(200t) + \cos(400t)$

$I_{rms} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{0.5 + 0.5} = 1\,A_{rms}$.

Problem: A residential electric power monitoring system rated for 120 V rms, 60 Hz, source registers power consumption of 1.2 KW, with P.F = 0.8. Find:-

① $I_{rms} = ?$ ② The phase angle ③ The system impedence ④ The system resistance.

② $\theta = \cos^{-1}(0.8) = 38.87°$

$S = \frac{P}{\cos\theta} = \frac{1.2}{0.8} = 1.5\,KVA$

$Q = S\sin\theta = 1.5 \times \sin(38.87°) = 0.9413\,KVAR$

$\vec{S} = 1.2 + j0.9413$

① $I_{rms} = \frac{P}{V \times \cos\theta} = \frac{1200}{120 \times 0.8} = 12.5\,A_{rms}$

③ $|Z| = V/I = 120/12.5 = 9.6\,\Omega$

④ $R = |Z| \times \cos\theta = 9.6 \times 0.8 = 7.68\,\Omega$

DOMS

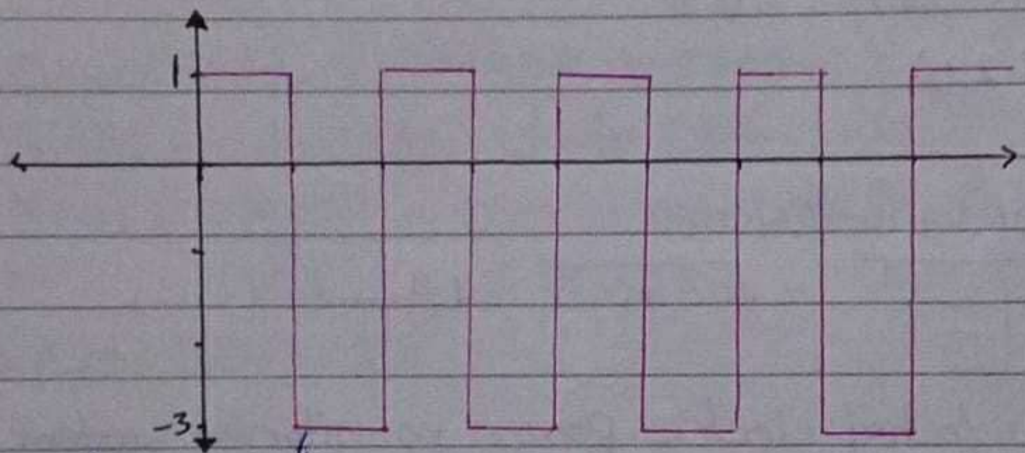
Problem #7.8:

Given the waveform of a voltage source shown below, find:

a. The steady DC voltage that would cause the same heating effect across a resistance.

b. The average current supplied to a 10 Ω resistor connected across the voltage source.

c. The average power supplied to a 1 Ω resistor across the voltage source.



a) $V_{rms}^2 = \frac{1}{T}\int_0^t v(t)^2 \, dt$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 (1)^2 \, dt + \int_1^2 (-3)^2 \, dt\right]$$

$$= \frac{1}{2}(1+9)$$

$$V_{rms}^2 = 5$$

$$V_{rms} = 2.24\,V$$

/ /

b) $V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$

$V_{avg} = \frac{1}{2}\left[\int_0^1 (1)\,dt + \int_1^2 (-3)\,dt\right]$

$= \frac{1}{2}(1-3)$

$= -1V$

$I_{avg} = \frac{V_{avg}}{R} = \frac{-1}{10} = -0.1A$

c) $P_{avg} = \frac{V_{rms}^2}{R}$

$= \frac{5}{1.5}$

$= 3.33W$

Problem: For the circuit shown below, determine the power factor for the load and state whether it is leading or lagging for the following conditions:

$\theta_z > 0 \rightarrow$ lag

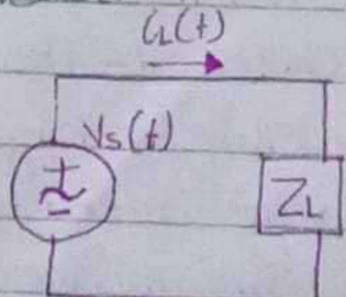
$\theta_z < 0 \rightarrow$ lead.

a. $V_s(t) = 540\cos(\omega t + 15^\circ)V$

$i_L(t) = 20\cos(\omega t + 47^\circ)A$

$V = 540\angle 15^\circ$, $Z = \frac{V}{I} = \frac{540\angle 15^\circ}{20\angle 47^\circ} = 27\angle -32$

$I = 20\angle 47^\circ$



$P.F = \cos(|-32|) = \cos(32) = 0.848$ leading

b) $v_s(t) = 155\cos(\omega t - 15^\circ)$

$i_L(t) = 20\cos(\omega t - 22^\circ)$

Phase difference: $\phi = -15 - -22 = 7$, $P.F = \cos(7^\circ) = 0.993$ lagging

c) $v_s(t) = 208\cos(\omega t)$

$i_L(t) = 1.7\sin(\omega t + 175^\circ) = 1.7\cos(\omega t + 175 - 90^\circ) = \cos(\omega t + 85)$

phase difference: $\phi = 0 - 85^\circ = -85^\circ$, $P.F = \cos(-85^\circ) = 0.087$ leading

d) $Z_L = (48 + j16)\,\Omega = 50.6\,\angle 18.4^\circ$, $\theta > 0$.

$P.F = \cos(18.4) = 0.949$, lagging

Problem: For the circuit shown below, determine whether the load is capacitive or inductive for the circuit shown

a) PF = 0.87 (leading), b) PF = 0.42 (leading). a & b same answer.

leading power factor means current leads voltage, which indicates a capacitive loads.

c) $v_s(t) = 42\cos(\omega t)\,V$

$i_L(t) = 4.2\sin(\omega t)\,A \rightarrow i_L(t) = 4.2\cos(\omega t - 90^\circ)$

current leads voltages by 90°, indicating a capacitive load.

d) $v_s(t) = 10.4\cos(\omega t - 12^\circ)\,V$

$i_L(t) = 0.4\cos(\omega t - 12^\circ)\,A$

→ No phase change means (neither capacitive nor ind.)

Problem:

A load impedance, $Z_L = 10 + j3\,\Omega$, is connected to a source with line resistance equal $1\,\Omega$, as shown below. Calculate the following values:

a) The average Power delivered to the total.

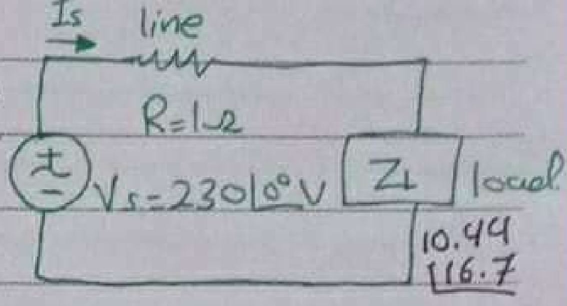
b) The average Power absorbed by the line.

c) The apparent power supplied by the generator.

d) The Power Factor to the load.

e) The power factor of line plus load.

$Z_{total} = 1 + 10 + j3 = 11 + j3 = 11.4\angle 15.26^\circ$



$I = \dfrac{V_s}{Z_{total}} = \dfrac{230\angle 0^\circ}{11.4\angle 15.26^\circ} = 20.18\angle -15.26^\circ$

a) $P_{load} = I^2 R = (20.18)^2 \times 10.44 = 4251.51$

b) $P_{line} = I^2 R = (20.18)^2 \times 1 = 407.23$

c) $S = VI = 230 \times 20.18 = 4641\,VA$

d) $P.F = \cos(16.7) = 0.958$, lagging

e) $P.F = \cos(15.26^\circ) = 0.965$, lagging.

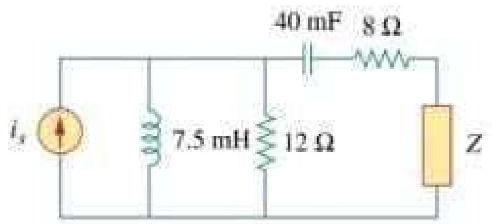
11. **Problem # 11**

Given the circuit shown below, find the average power supplied or absorbed by each element.

12. **Problem # 12**

It is desired to transfer maximum power to the load **Z** in the circuit shown below. If the source current, $i_s(t) = 5\cos 40t$ A, find:

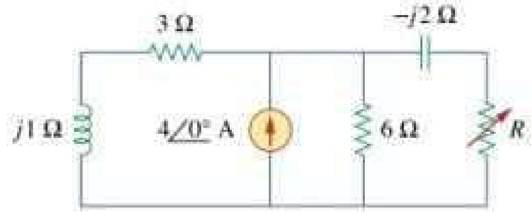
a. the load impedance **Z** for maximum power transfer.
b. the maximum average power.



13. **Problem # 13**

The variable resistor R in the circuit shown below is adjusted until it absorbs the maximum average power. Find:

a. the load resistance R for maximum power transfer.
b. the maximum average power.



Chapter 11, Problem 19.

The variable resistor *R* in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power. Find *R* and the maximum average power absorbed.

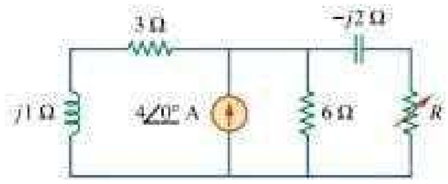


Figure 11.50
For Prob. 11.19.

Chapter 11, Solution 19.

At the load terminals,

$$\mathbf{Z}_{Th} = -j2 + 6 \,||\, (3+j) = -j2 + \frac{(6)(3+j)}{9+j}$$

$$\mathbf{Z}_{Th} = 2.049 - j1.561$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \underline{\mathbf{2.576\Omega}}$$

To get $\mathbf{V}_{Th}$, let $\mathbf{Z} = 6 \,||\, (3+j) = 2.049 + j0.439$.

By transforming the current sources, we obtain

$$\mathbf{V}_{Th} = (4\angle 0°)\,\mathbf{Z} = 8.196 + j1.756$$

$$P_{max} = \left|\frac{8.382}{2.049 - j1.561 + 2.576}\right|^2 \frac{2.576}{2} = \underline{\mathbf{3.798\ W}}$$

Chapter 11, Problem 14.

It is desired to transfer maximum power to the load **Z** in the circuit of Fig. 11.45. Find **Z** and the maximum power. Let $i_s = 5\cos 40t$ A.

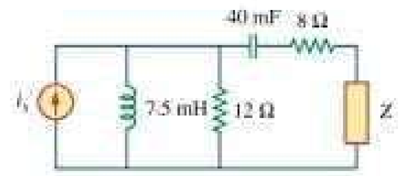


Figure 11.45
For Prob. 11.14.

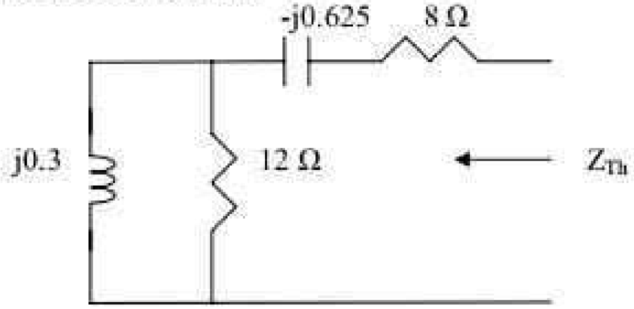
Chapter 11, Solution 14.

We find the Thevenin equivalent at the terminals of Z.

40 mF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j40x40x10^{-3}} = j0.625$

7.5 mH $\longrightarrow$ $j\omega L = j40x7.5x10^{-3} = j0.3$

To find Z_{Th}, consider the circuit below.



$$Z_{Th} = 8 - j0.625 + 12 // j0.3 = 8 - j0.625 + \frac{12x0.3}{12+0.3} = 8.0075 - j0.3252$$

$Z_L = (Z_{Thev})^* =$ **<u>8.008 + j0.3252Ω</u>**.

Chapter 11, Problem 19.

The variable resistor *R* in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power. Find *R* and the maximum average power absorbed.

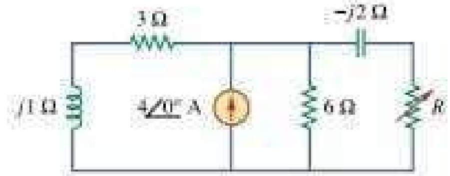


Figure 11.50
For Prob. 11.19.

Chapter 11, Solution 19.

At the load terminals,

$$\mathbf{Z}_{Th} = -j2 + 6 \,||\, (3+j) = -j2 + \frac{(6)(3+j)}{9+j}$$

$$\mathbf{Z}_{Th} = 2.049 - j1.561$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \underline{\mathbf{2.576\Omega}}$$

To get $\mathbf{V}_{Th}$, let $\mathbf{Z} = 6 \,||\, (3+j) = 2.049 + j0.439$.

By transforming the current sources, we obtain

$$\mathbf{V}_{Th} = (4\angle 0°)\,\mathbf{Z} = 8.196 + j1.756$$

$$P_{max} = \left|\frac{8.382}{2.049 - j1.561 + 2.576}\right|^2 \frac{2.576}{2} = \underline{\mathbf{3.798\ W}}$$