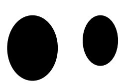
UNIT OPERATIONS OF PARTICULATE SOLIDS

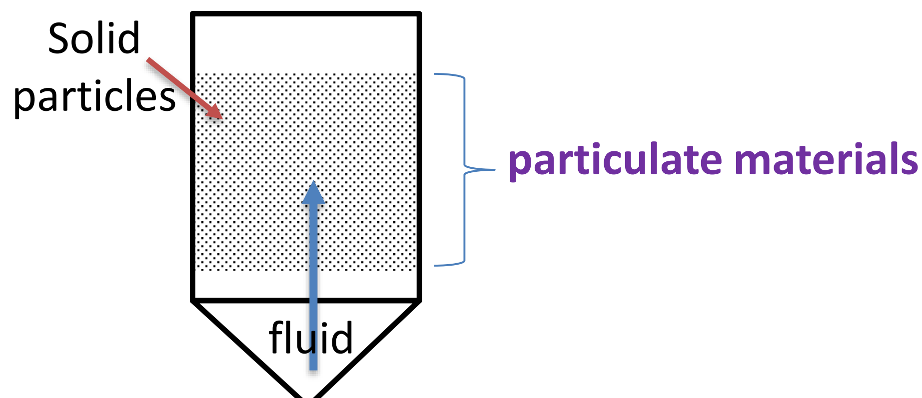
Some Definitions & applications in chemical industries

Introduction

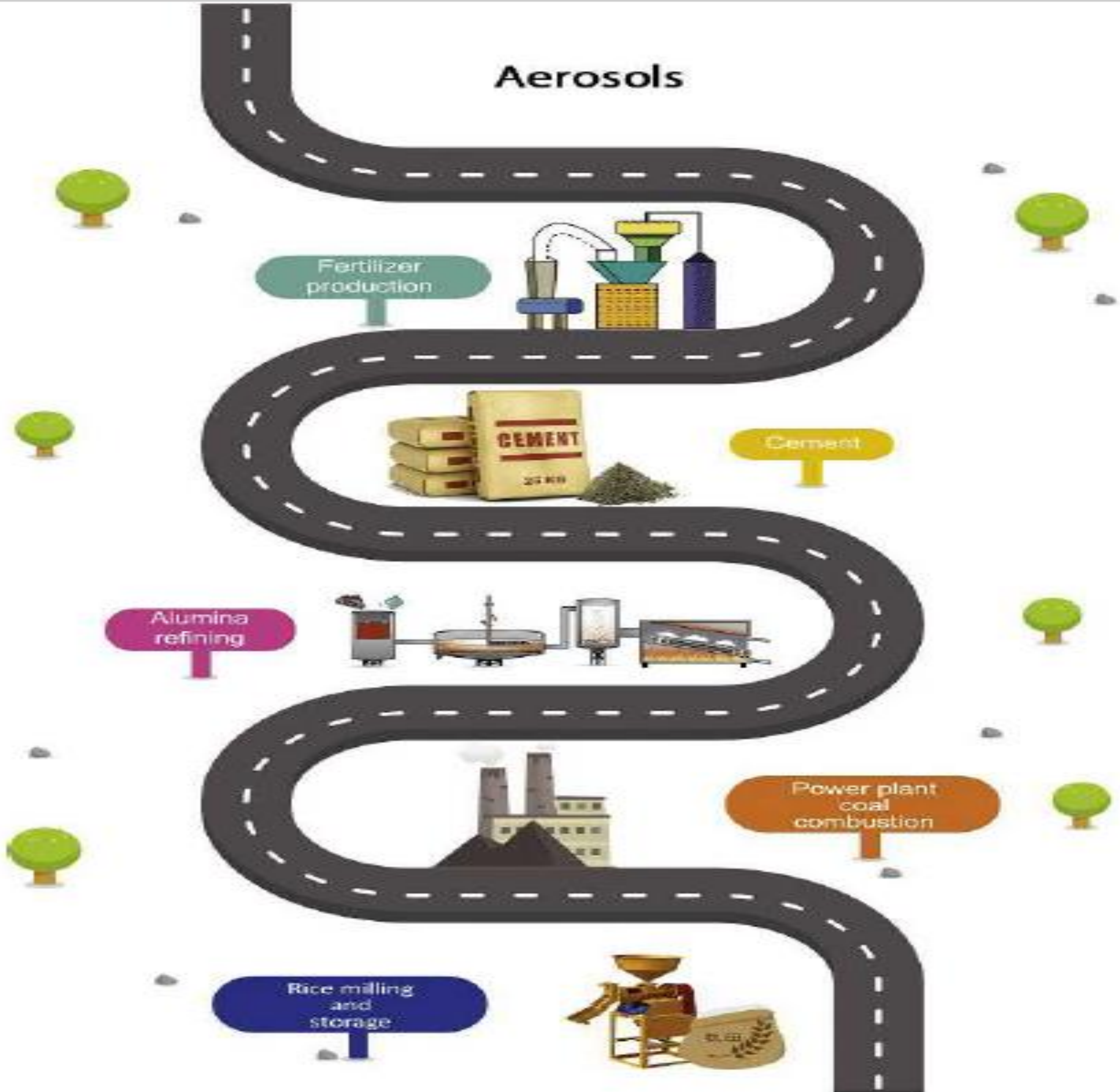
- Solid materials and their dispersion in fluids are the subject matter of this course.
- Individual unit of dispersed solid material is referred to as a **particle.**
- Solid-fluid material systems are referred to as particulate systems, **particulate materials** or **particulates**.



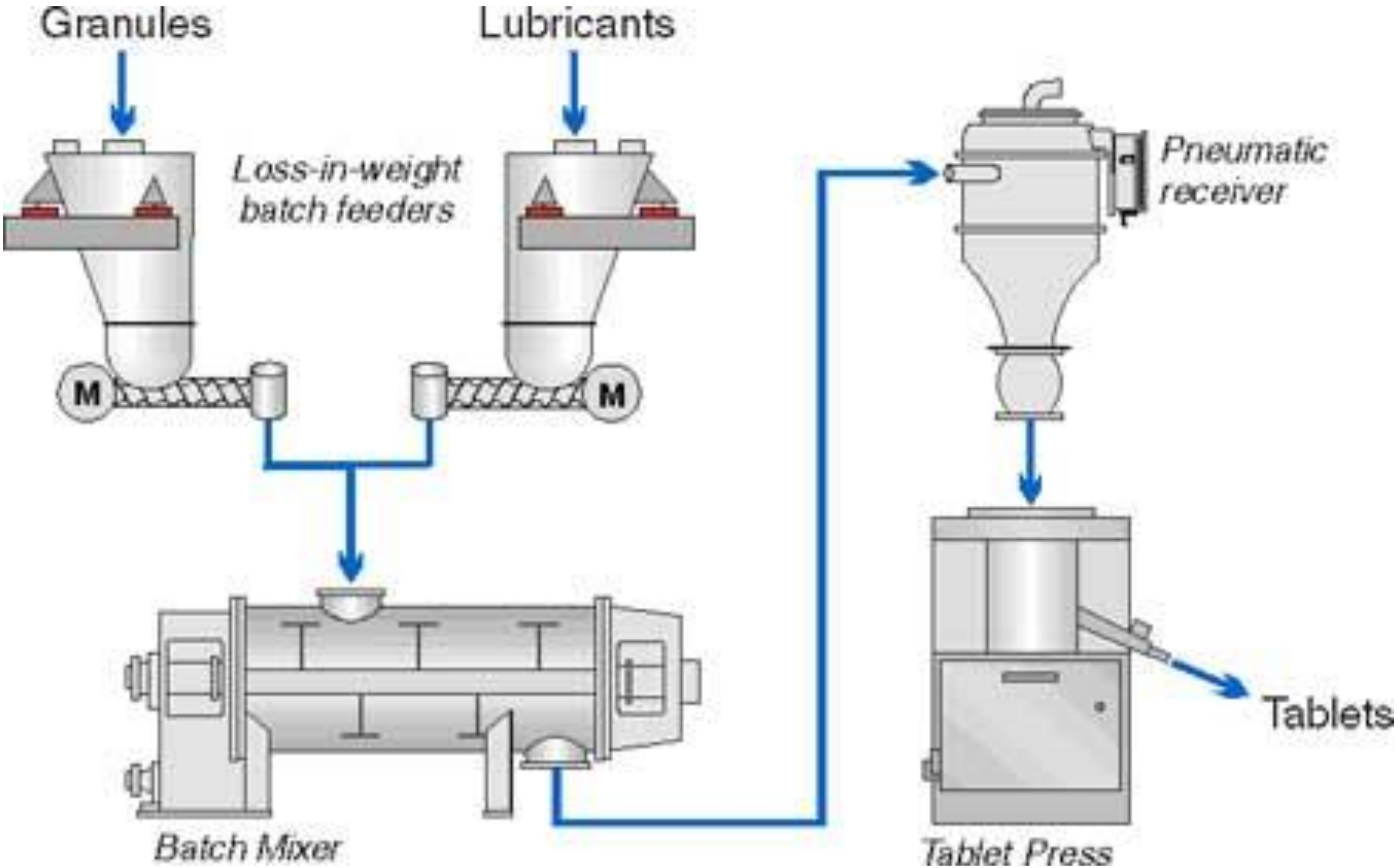
Solid particle



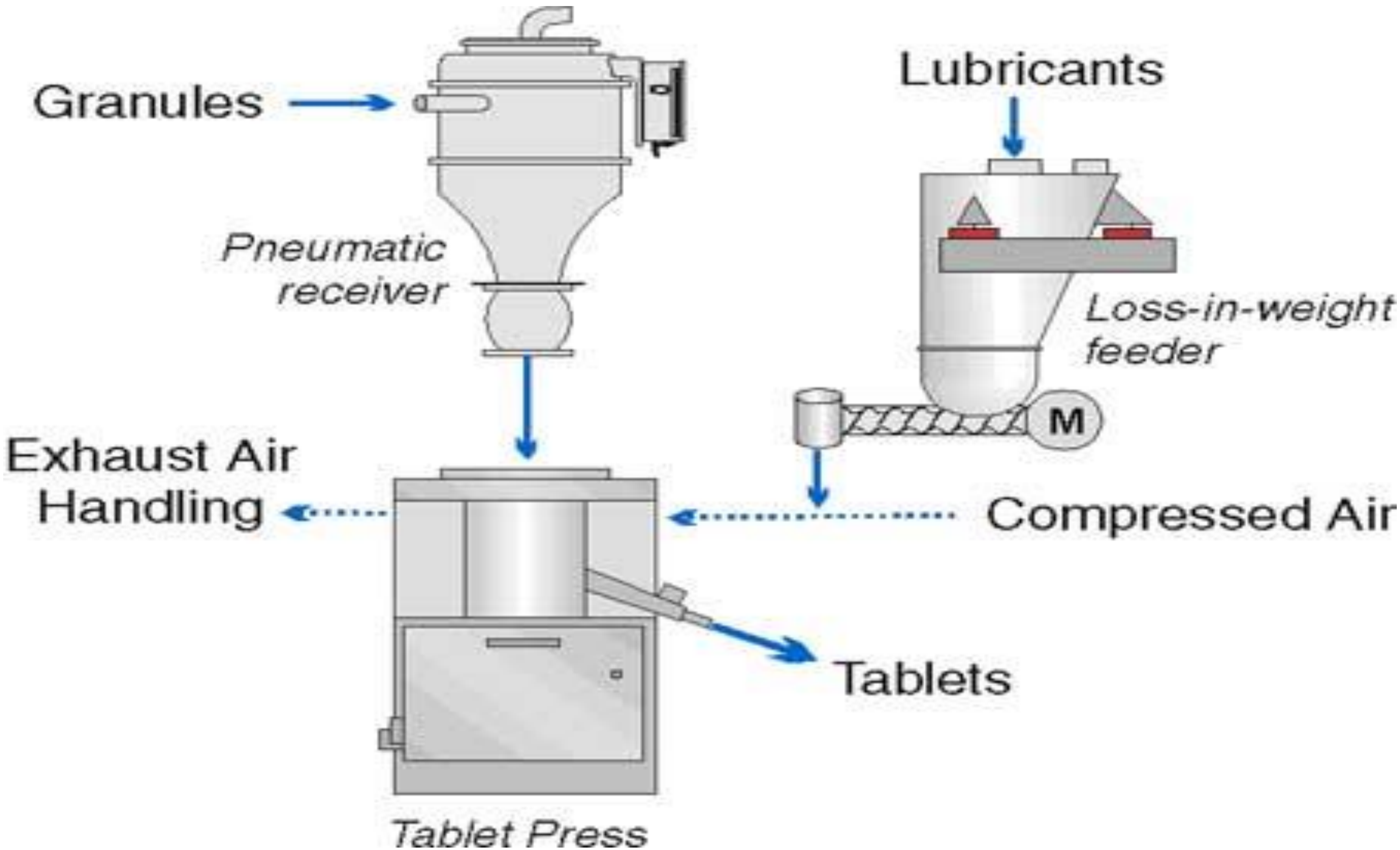
Examples of particulate systems in industrial applications.



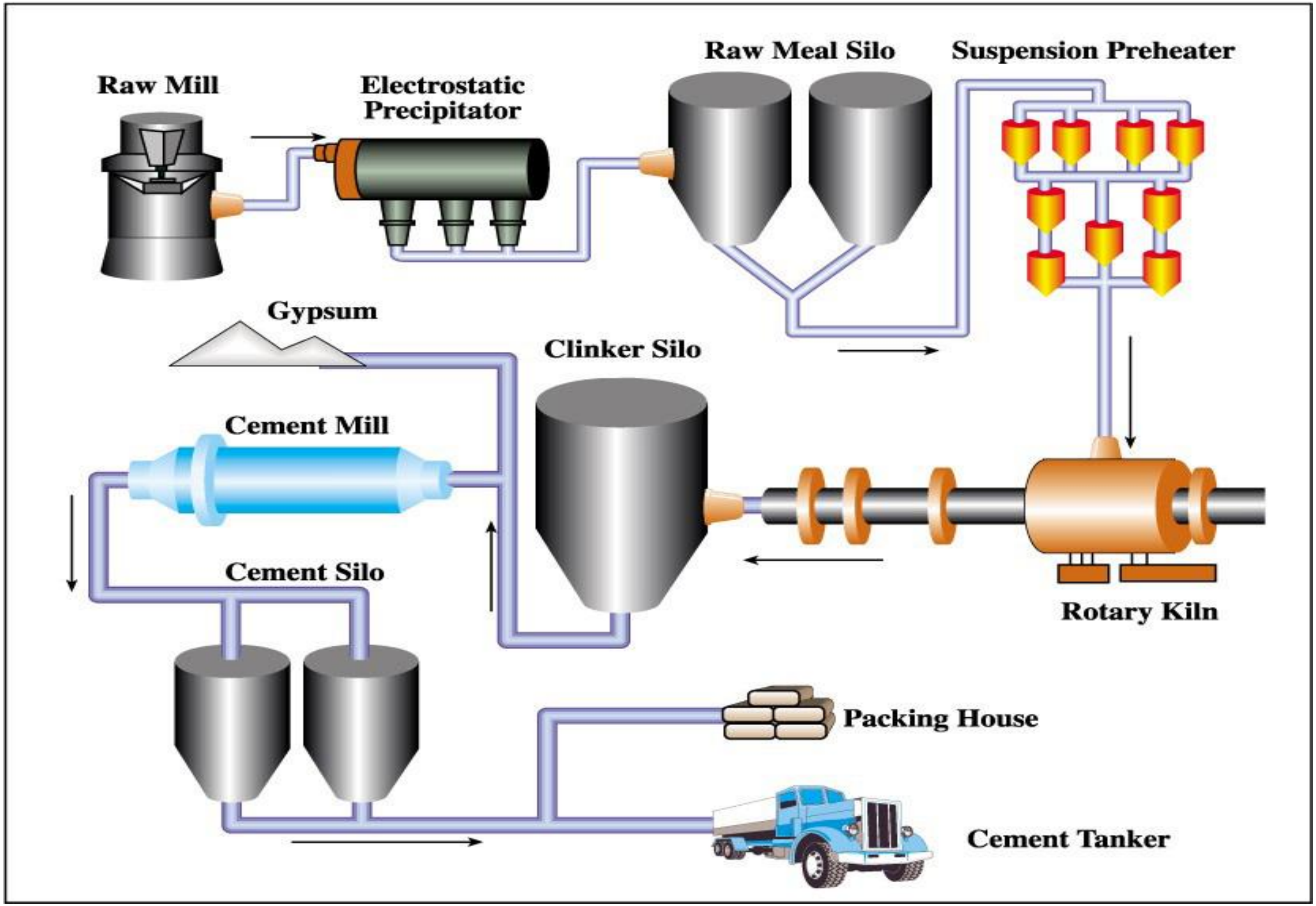
Application Example: Twin Screw Feeders in Pharmaceutical Tablet Press Lubrication Applications



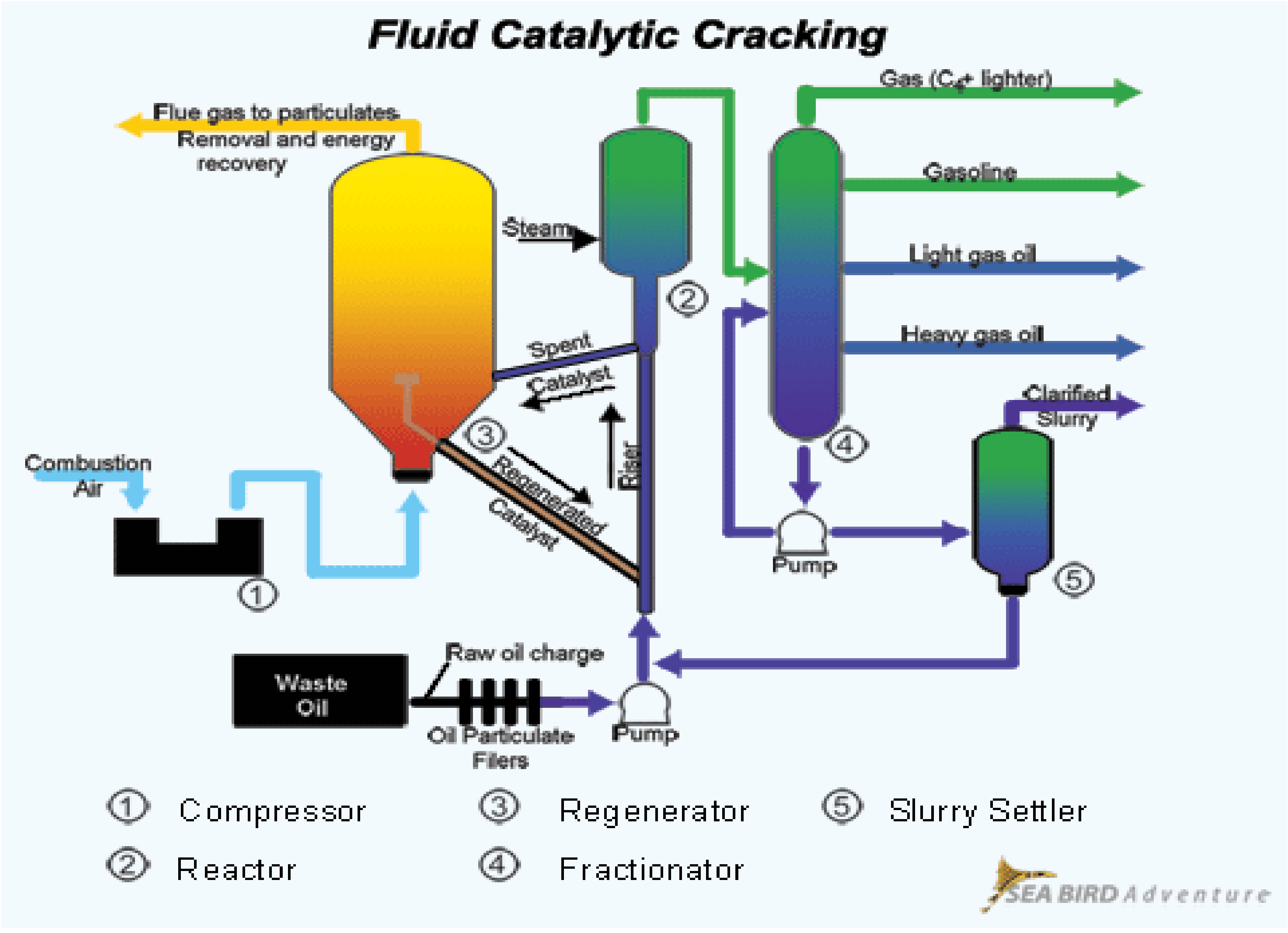
Pharmaceutical application tablet Press



Cement Production Process



FCC process



Fluid Catalytic Cracking
Flue gas to particulates Removal and energy recovery
Gas (C4+ lighter)
Gasoline
Light gas oil
Heavy gas oil
Clarified Slurry
Steam
Spent Catalyst
Regenerated Catalyst
Riser
Combustion Air
Pump
Raw oil charge
Waste Oil
Oil Particulate Filers
Pump
① Compressor
② Reactor
③ Regenerator
④ Fractionator
⑤ Slurry Settler
SEA BIRD Adventure

Examples of particulate systems in consumer applications.

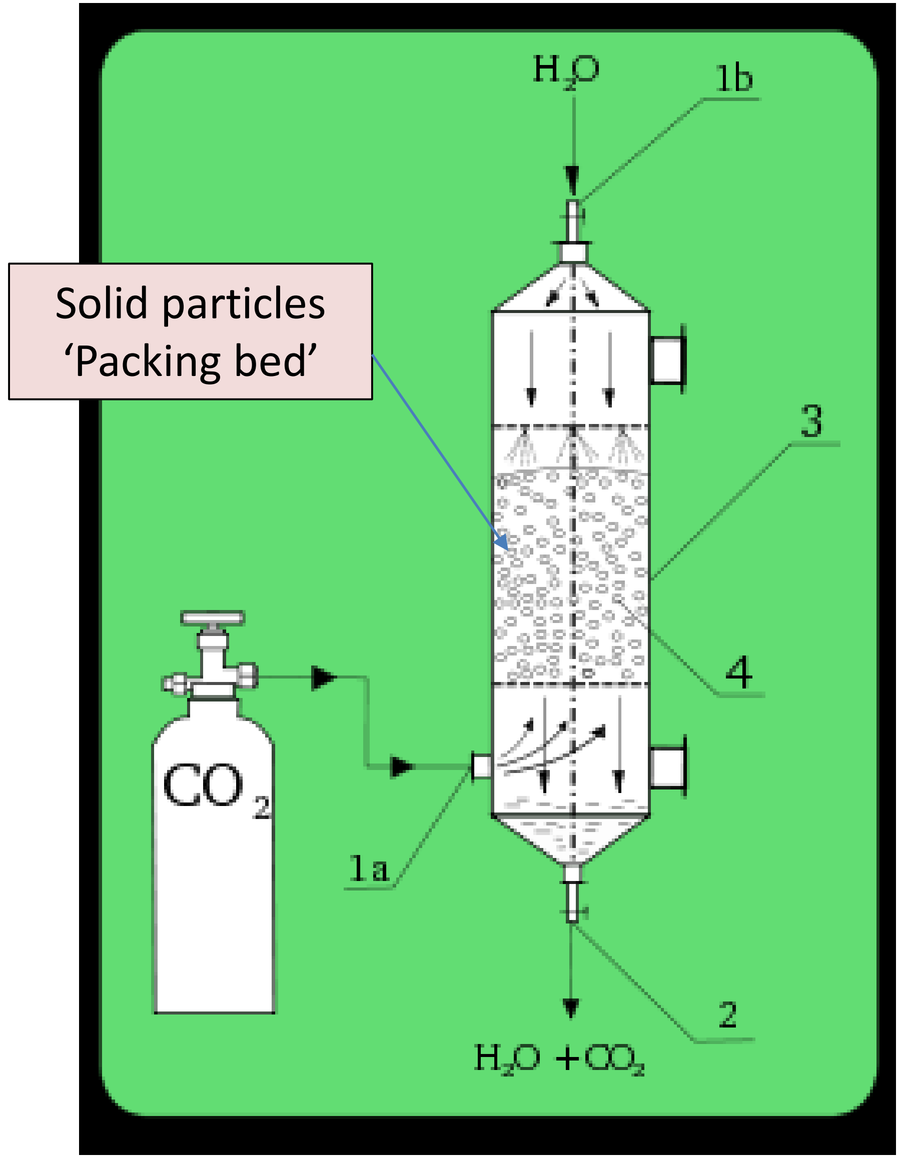
Absorption column "lab experiment"

Solid particles
'Packing bed'

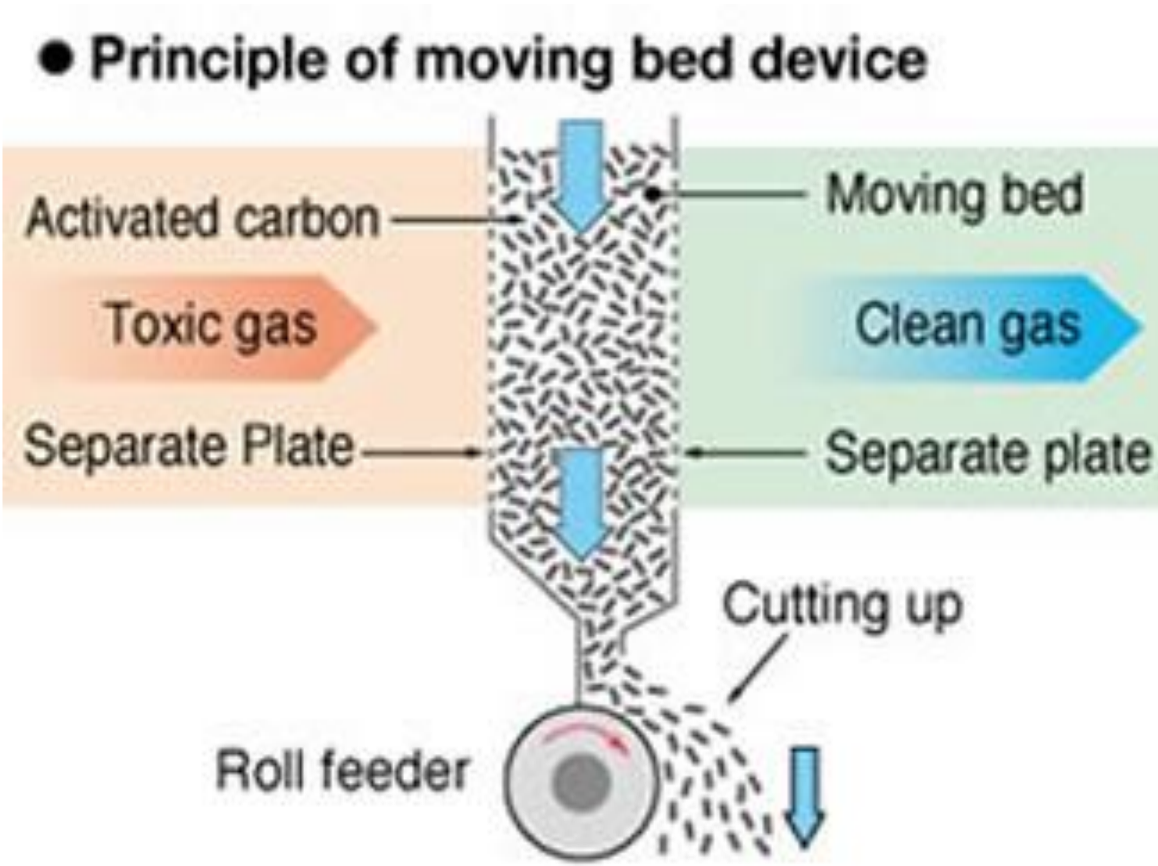
Absorption column

Laboratory absorber

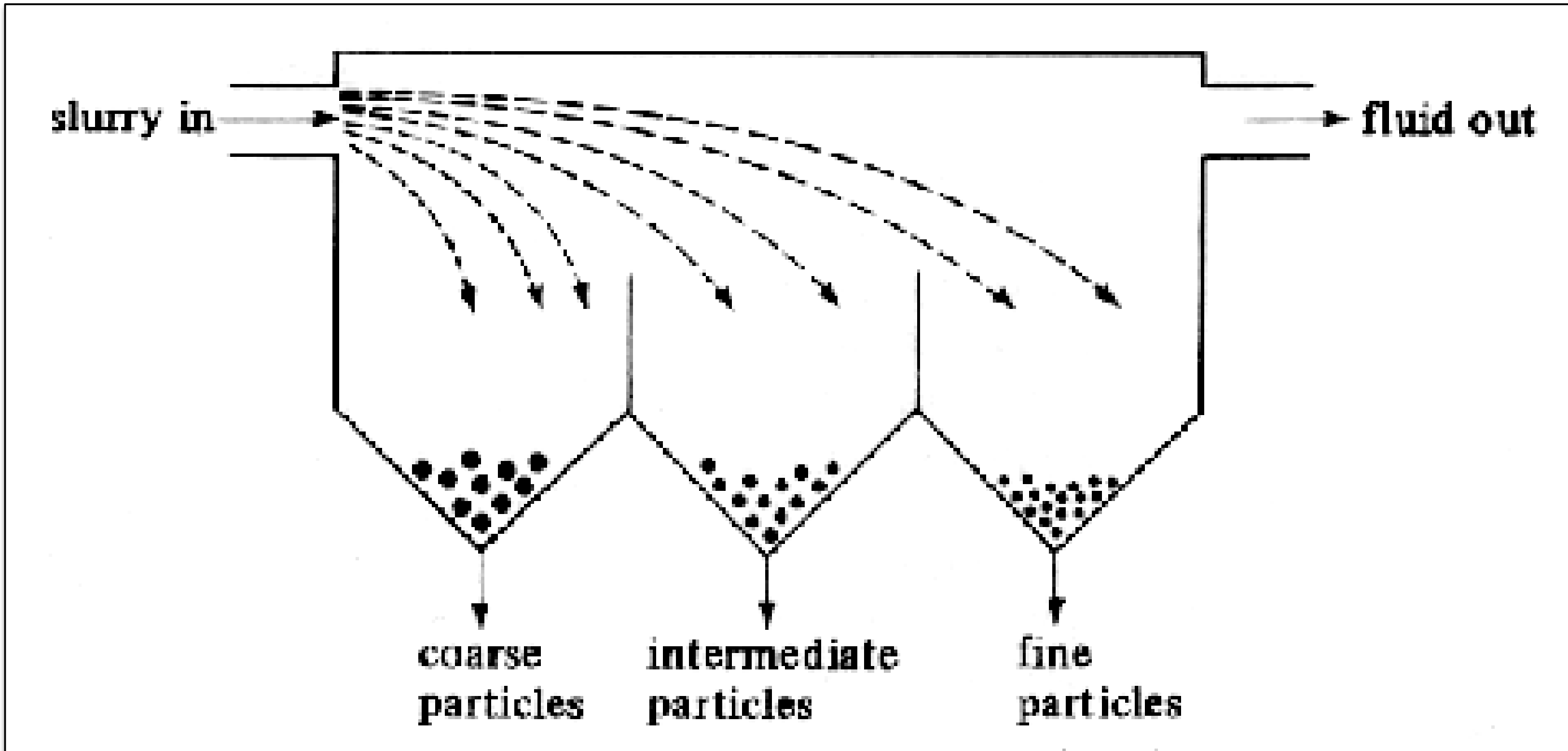
(1a): CO_2 inlet;
(1b): H_2O inlet;
(2):outlet;
(3): absorption column;
(4): **packing**.



Moving Bed of solid particles



Sedimentation



Solid Particulate

Introduction, Definitions, particle size analysis, size distribution, mean particle size, and measuring techniques

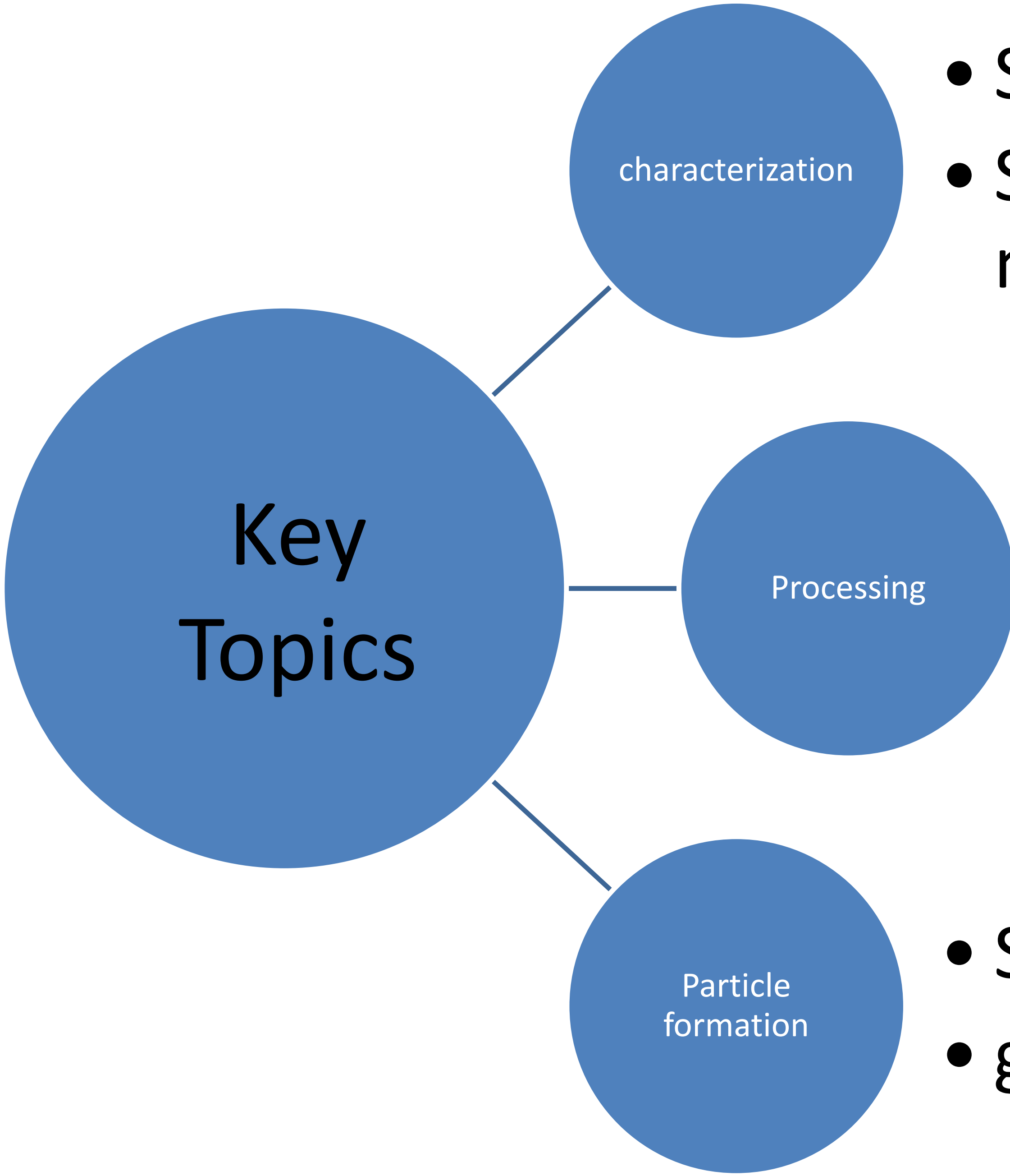
Topics of Particle Technology

❖ **Characterization of solids**

❖ **Motion of Particles through fluids**

❖ **Size Reduction**

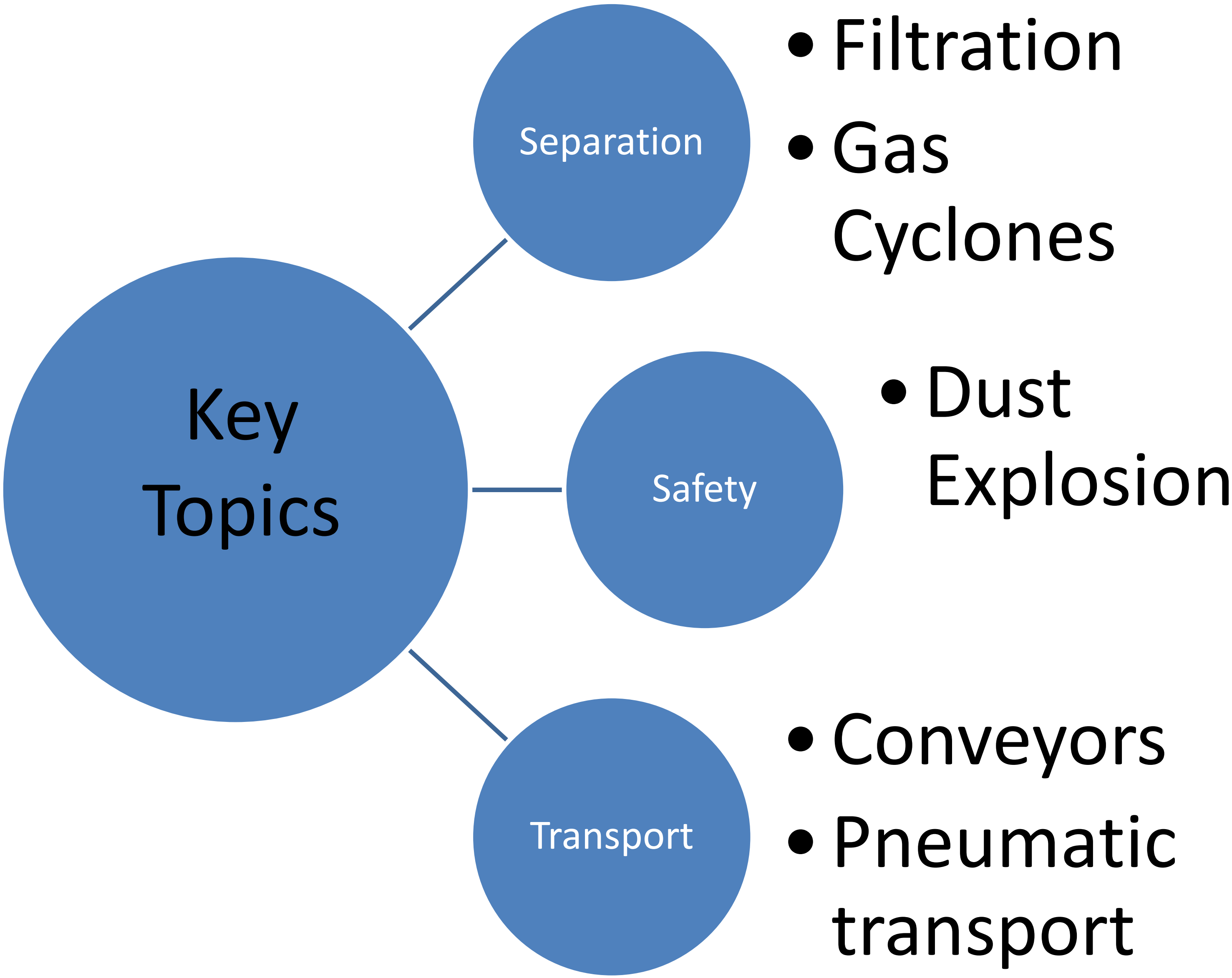
❖ **Mechanical Separations**

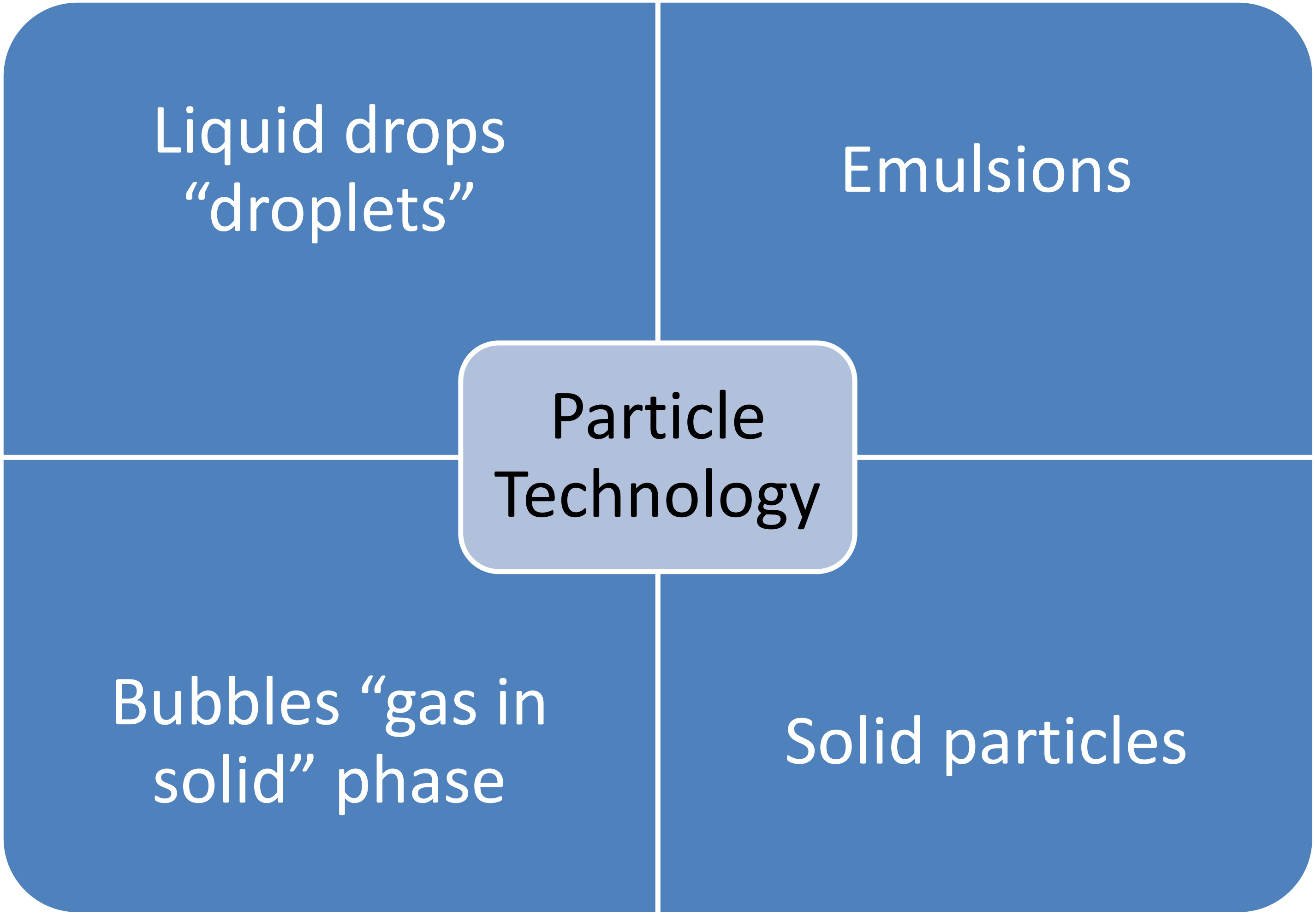


- Size analysis
- Size measurement

- F.B
- Granulation

- Size reduction
- granulation


Key Topics
Separation
Filtration
Gas Cyclones
Safety
Dust Explosion
Transport
Conveyors
Pneumatic transport


Liquid drops “droplets”
Emulsions
Particle Technology
Bubbles “gas in solid” phase
Solid particles

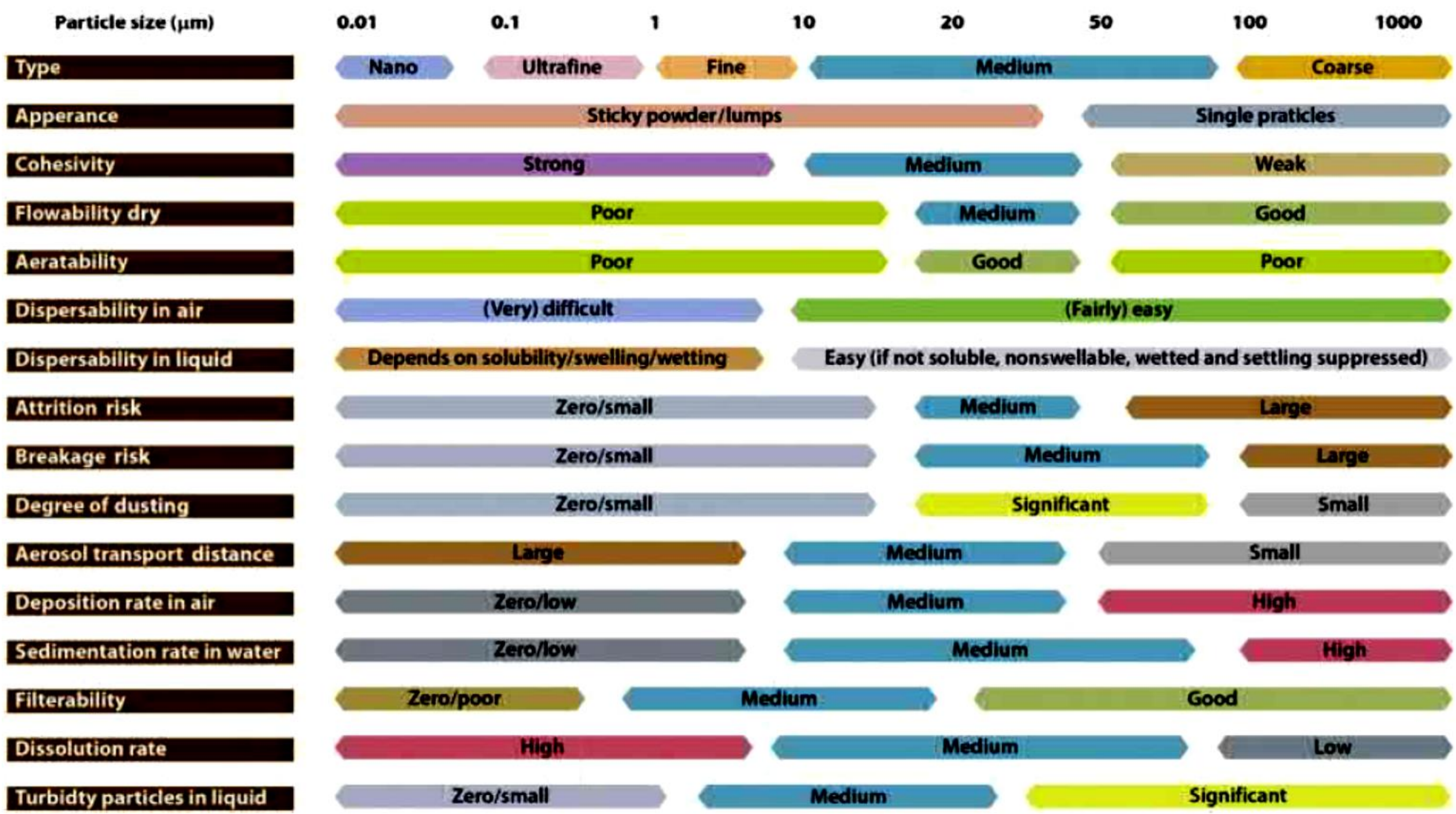
What is particle technology?

Techniques for processing and handling particulate solids. It plays a major role in the production of materials in industry.

Units used for particle size depend on the size of particles.

Particles	Size
Coarse particles	inches or millimeters
Fine particles:	screen size
Very fine particles	micrometers or nanometers
Ultra fine particles	surface area per unit mass, m^2/g

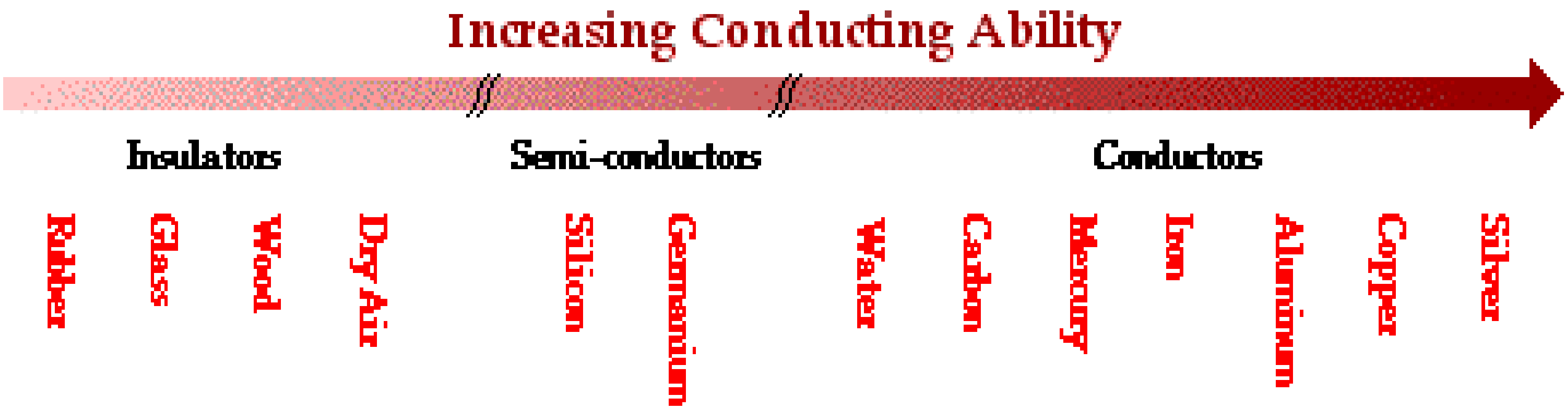
Influence of particle size on example characteristics of particulate solids



Characterization of solid particles

Individual solid particles are characterized by their size, shape, and density

Types of Material



CHARACTERIZATION OF SOLID MATERIAL

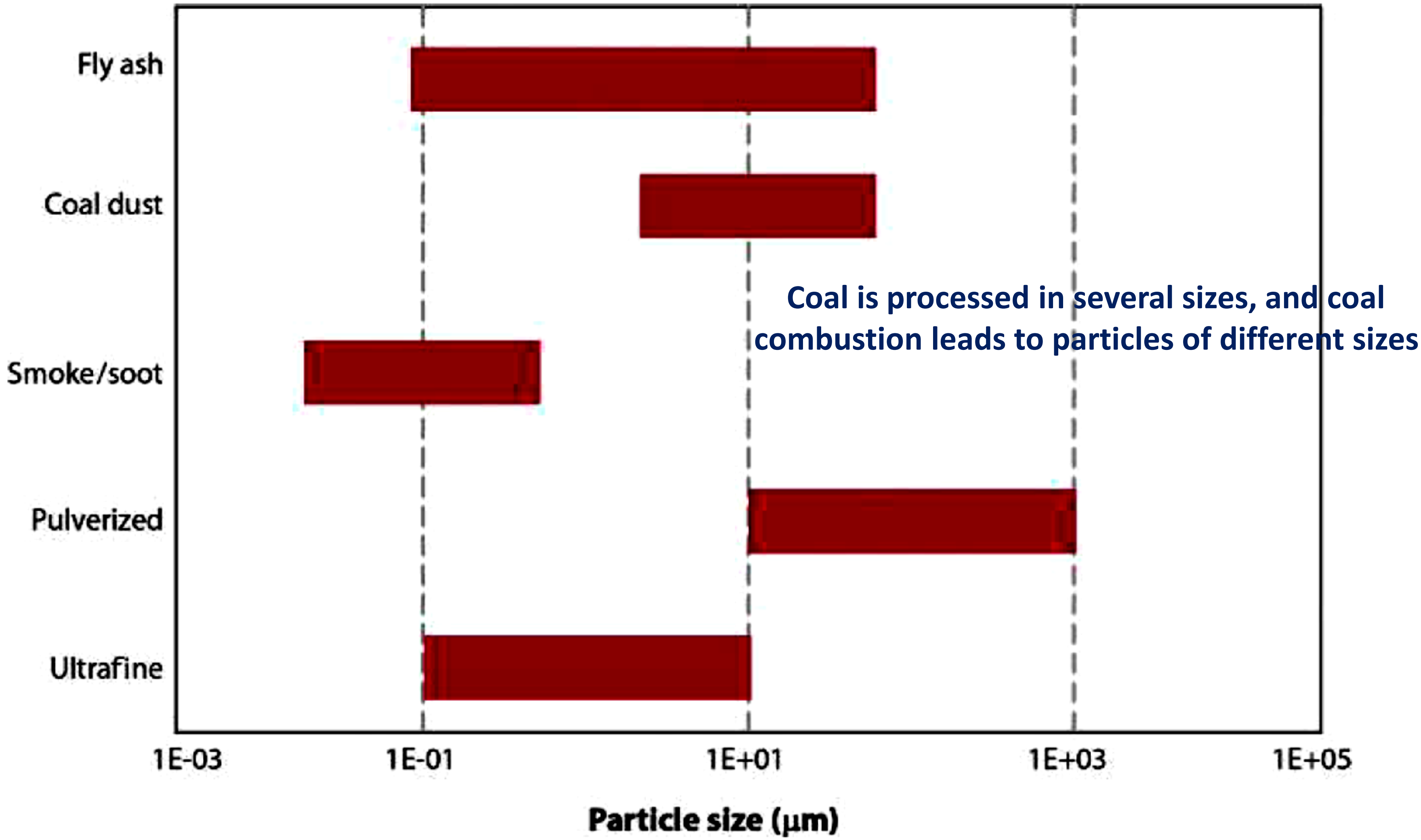
- Density and porosity
- Composition
- Thermal conductivity
- Particle size and shape
- Mechanical properties ‘Strength and hardness’

Definitions

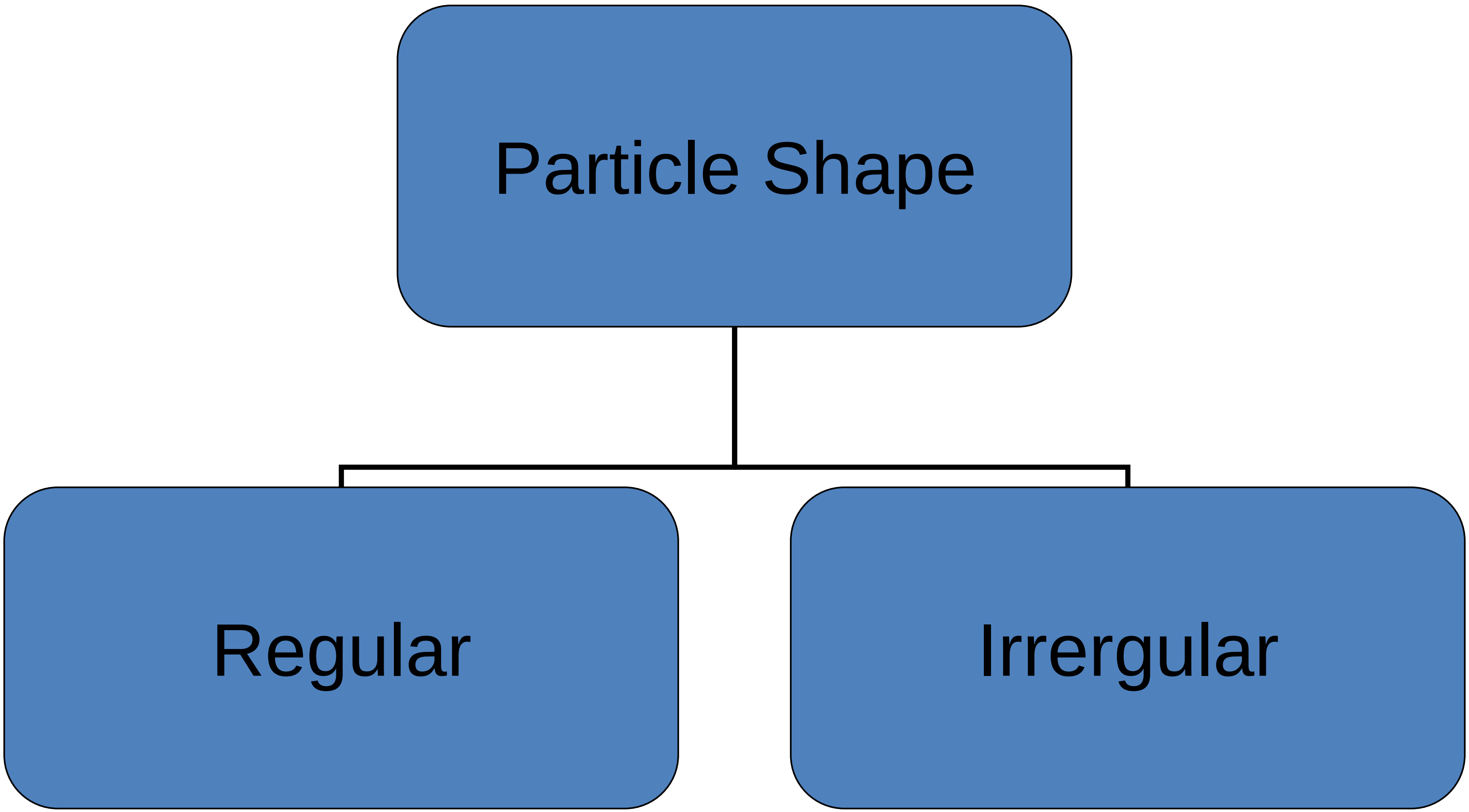
- A particle
- Particulate material

Dust	Shape is not visible by eye
Powder	Produced by comminution Shape is not visible by eye
Granules or Fibers	Shapes are visible to eye rounded or elongated
Lump	Manipulated by hand

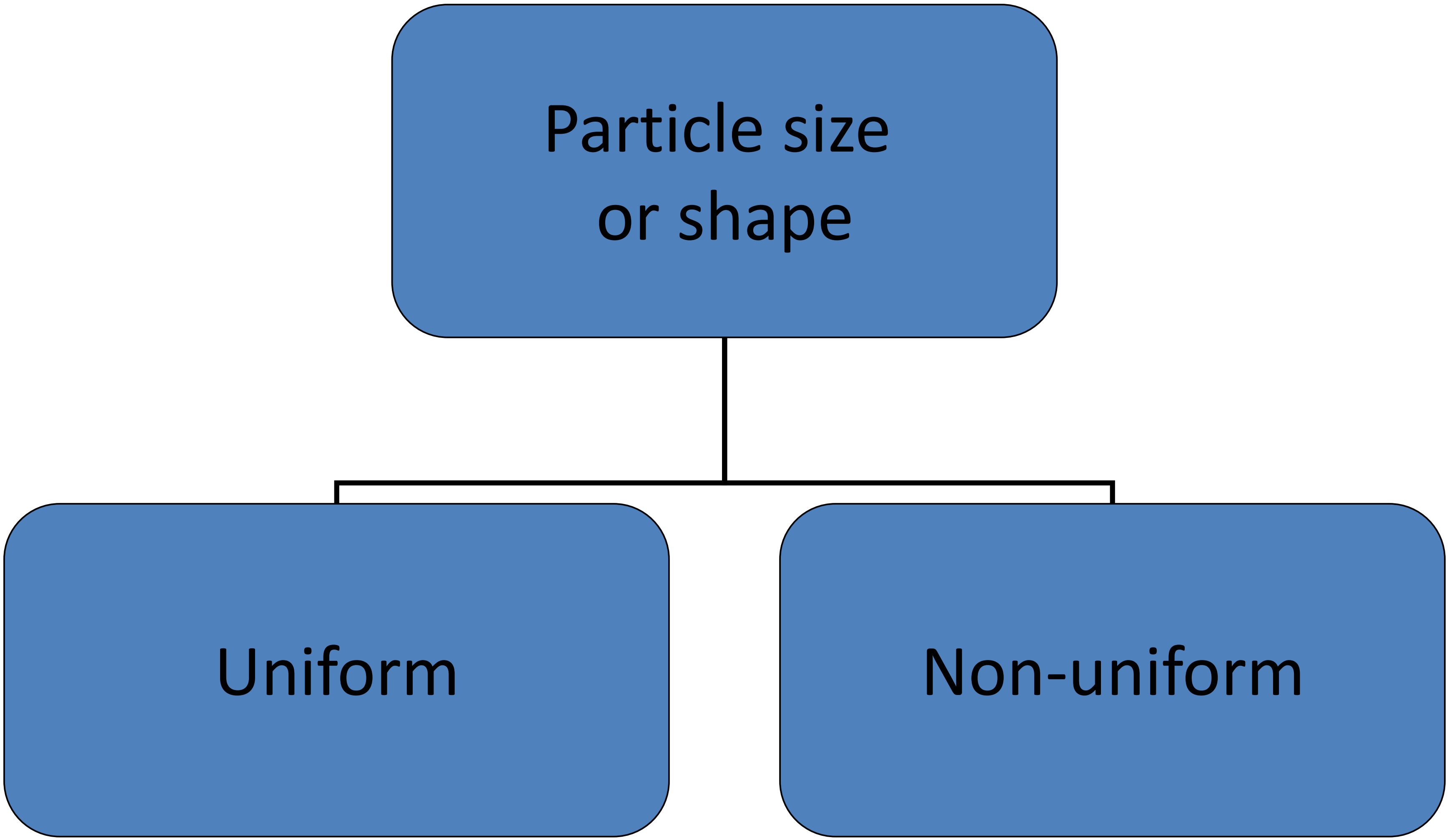
Example of material systems with the relevant particle size



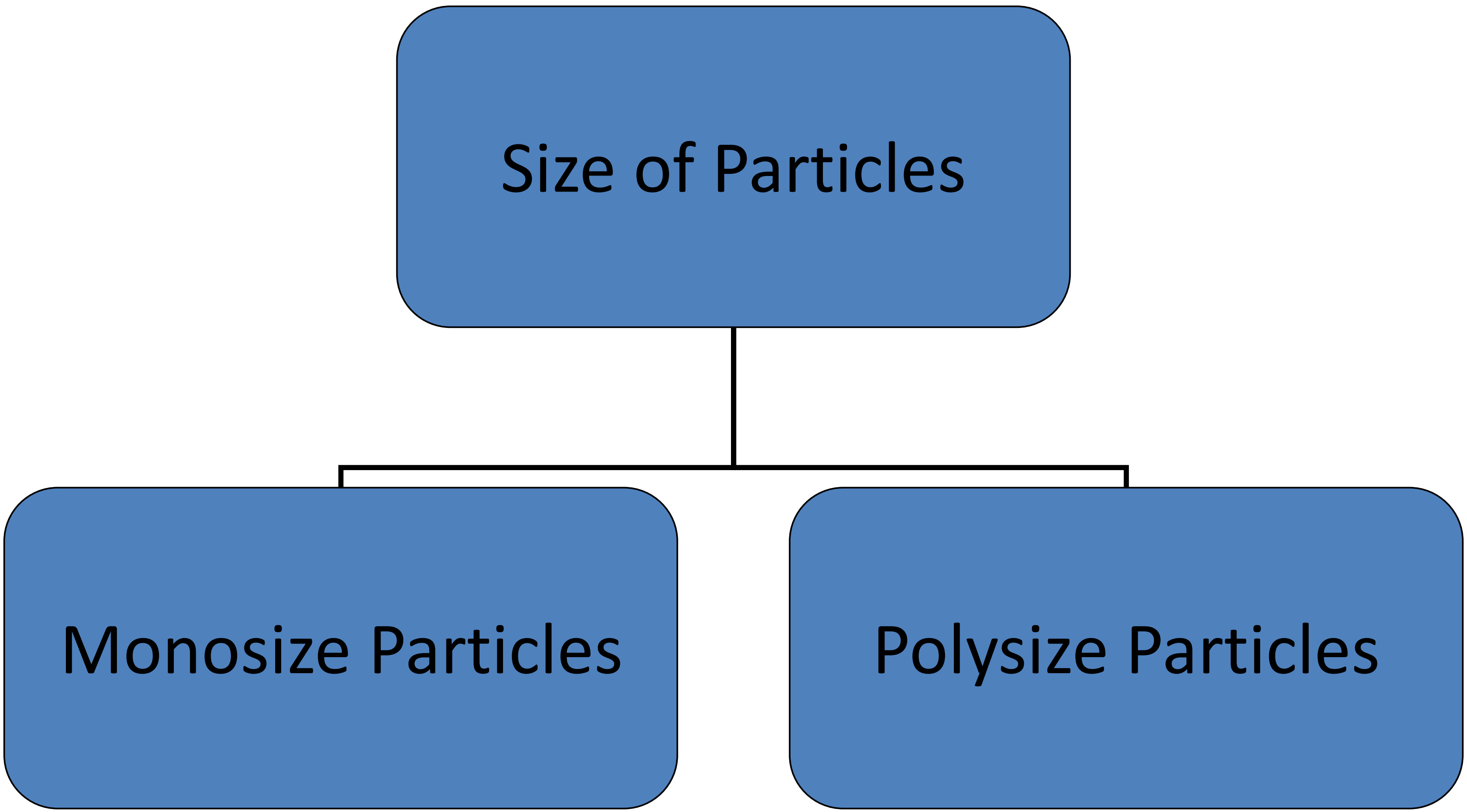
Definitions



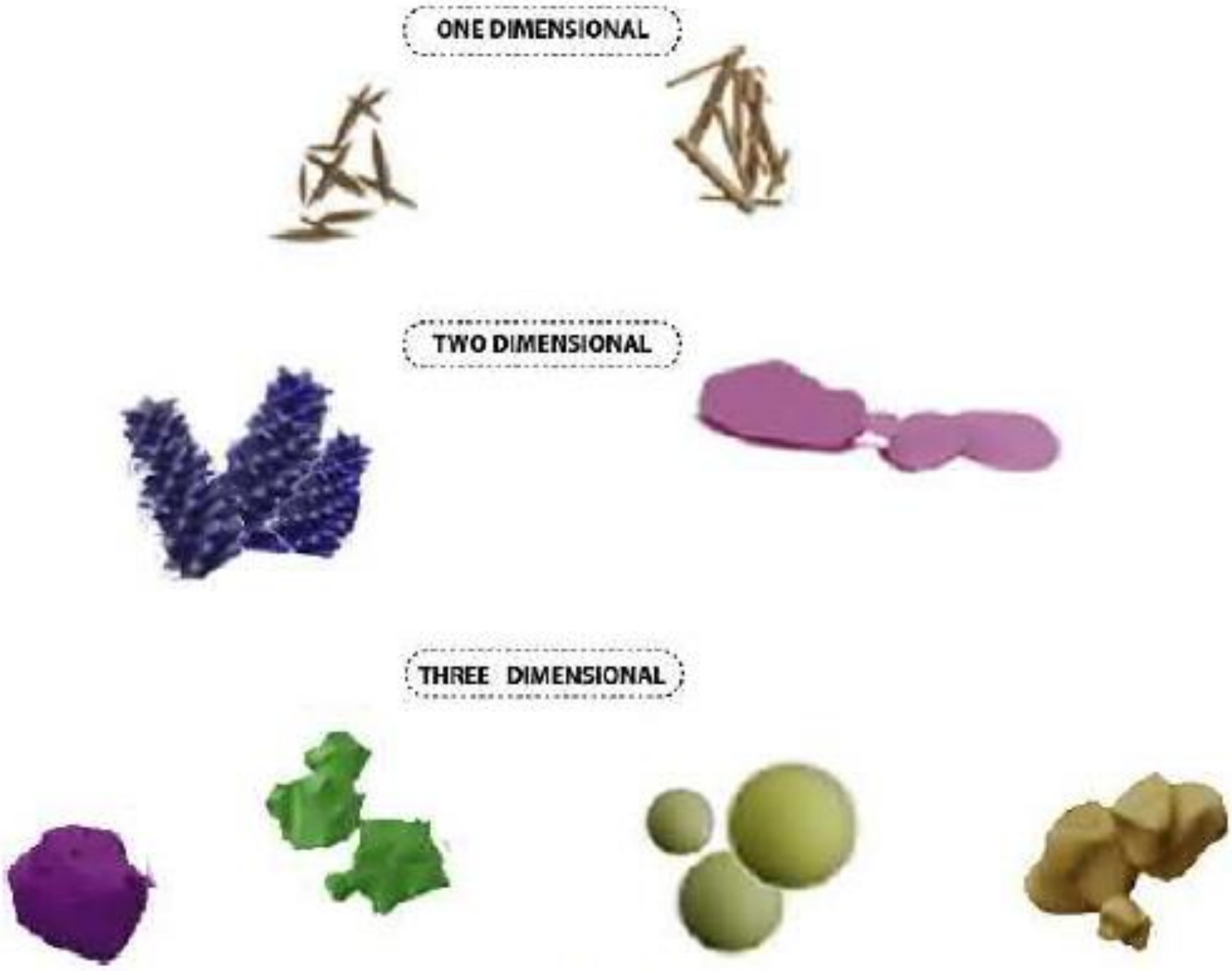
Definitions



Definitions



Typical shapes of particles



Definitions

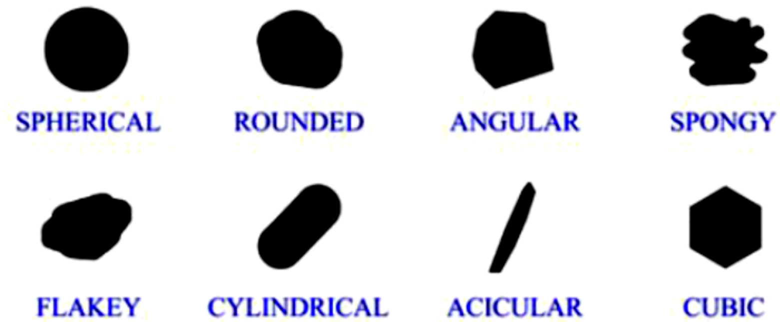
- A particulate system
- Porosity
- Specific surface area of particle
- Specific surface area of particulate system
- Fixed bed of solids
- Porous mass of solids
- Moving bed
- Fluidized bed

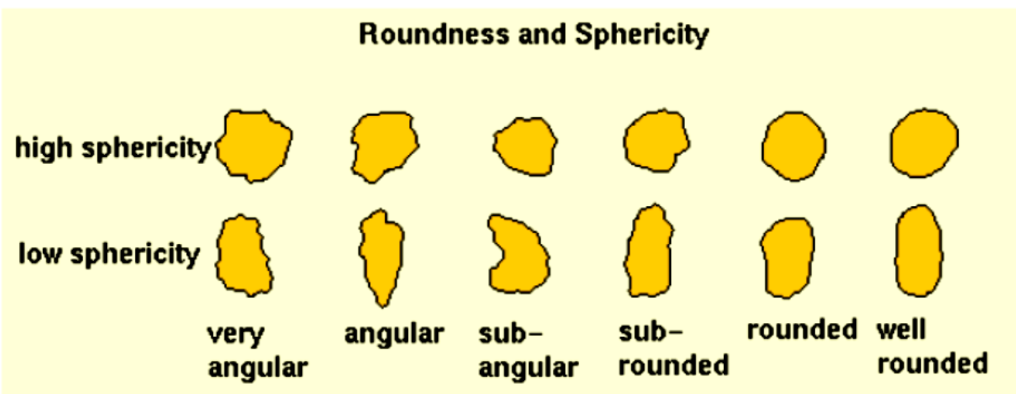
Characterization of Solid Particles

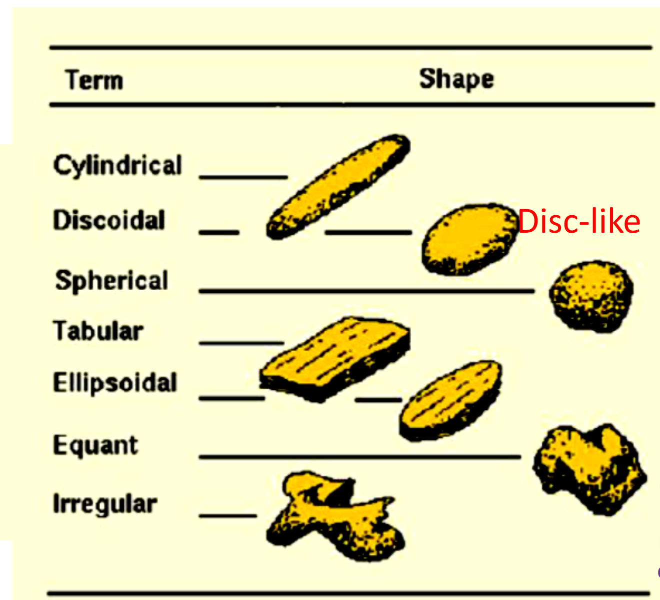
Particle Shapes and Measuring Techniques

CHARACTERIZATION OF SOLID PARTICLES

- SIZE
- SHAPE
- DENSITY







Different diameters ≈ equal like Cubes or spheres

Single Particles: Particle shape

- The shape of an individual particle is expressed in terms of the sphericity φ_s, which is independent of particle size. The sphericity of a particle is the ratio of the surface-volume ratio of a sphere (with equal volume as the particle) and the surface-volume ratio of the particle. For a spherical particle of diameter D_p, $\varphi_s = 1$; for a non-spherical particle, the sphericity is defined as

$$\boldsymbol{\varphi} = \frac{\textit{surface area of sphere of same volume as particle}}{\textit{surface area of particle}}$$

Sphericity can also be found from the following equation:

$$\varphi_s = \frac{6\mathrm{v}_p}{S_p D_p}$$

- D_p: equivalent volume diameter of particle (volume diam)
- S_p: surface area of one particle
- v_p: volume of one particle

For many crushed materials, φ_s is between 0.6 and 0.8. For particles rounded by abrasion, φ_s may be as high as 0.95.

Notes

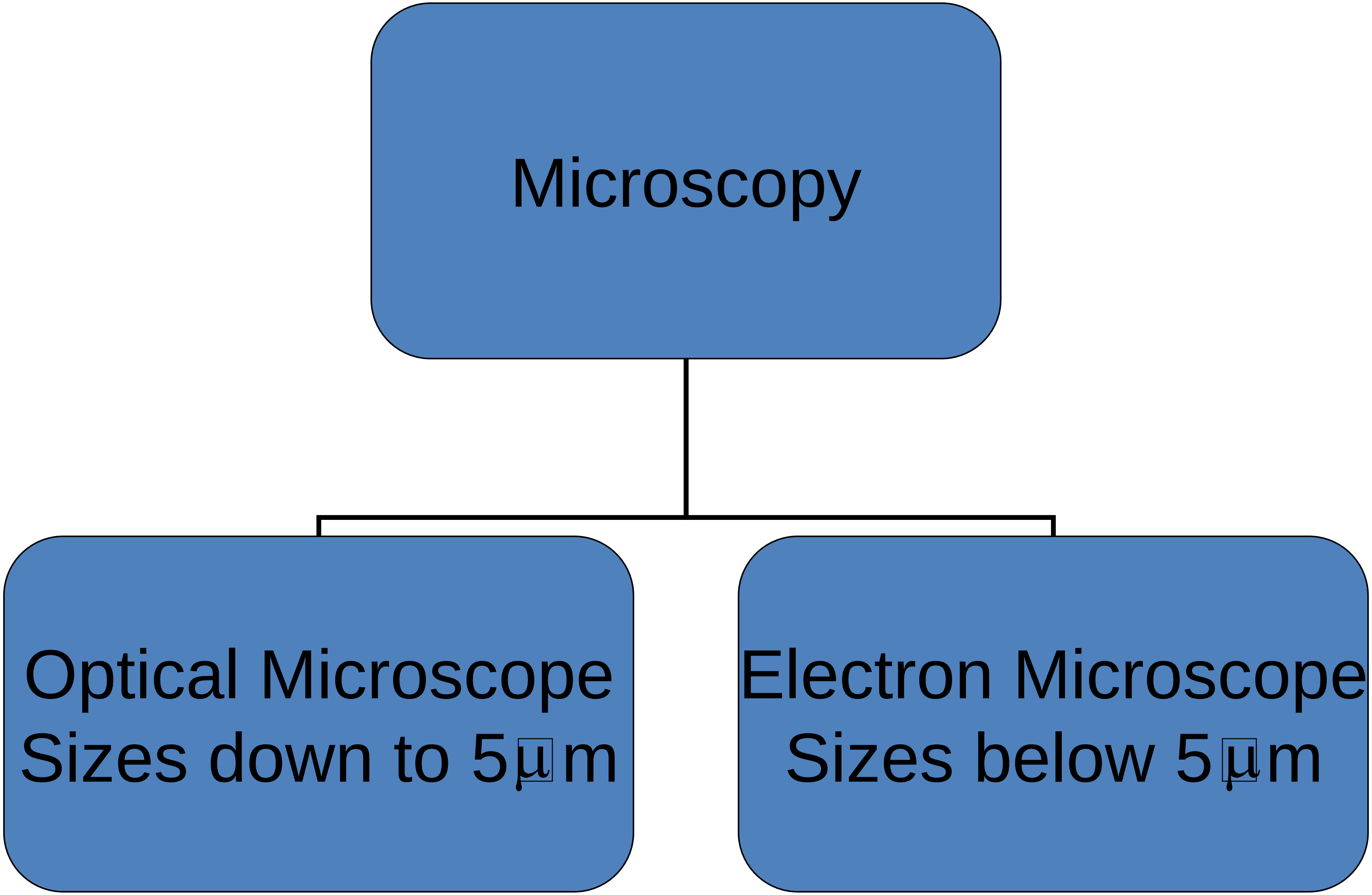
- The equivalent diameter is sometimes defined as the diameter of a sphere of equal volume.
- For fine particles, D_p is usually taken to be the nominal size based on screen analysis or microscopic analysis.
- The surface area is found from adsorption measurements or from the pressure drop in a bed of particles.

Particle size

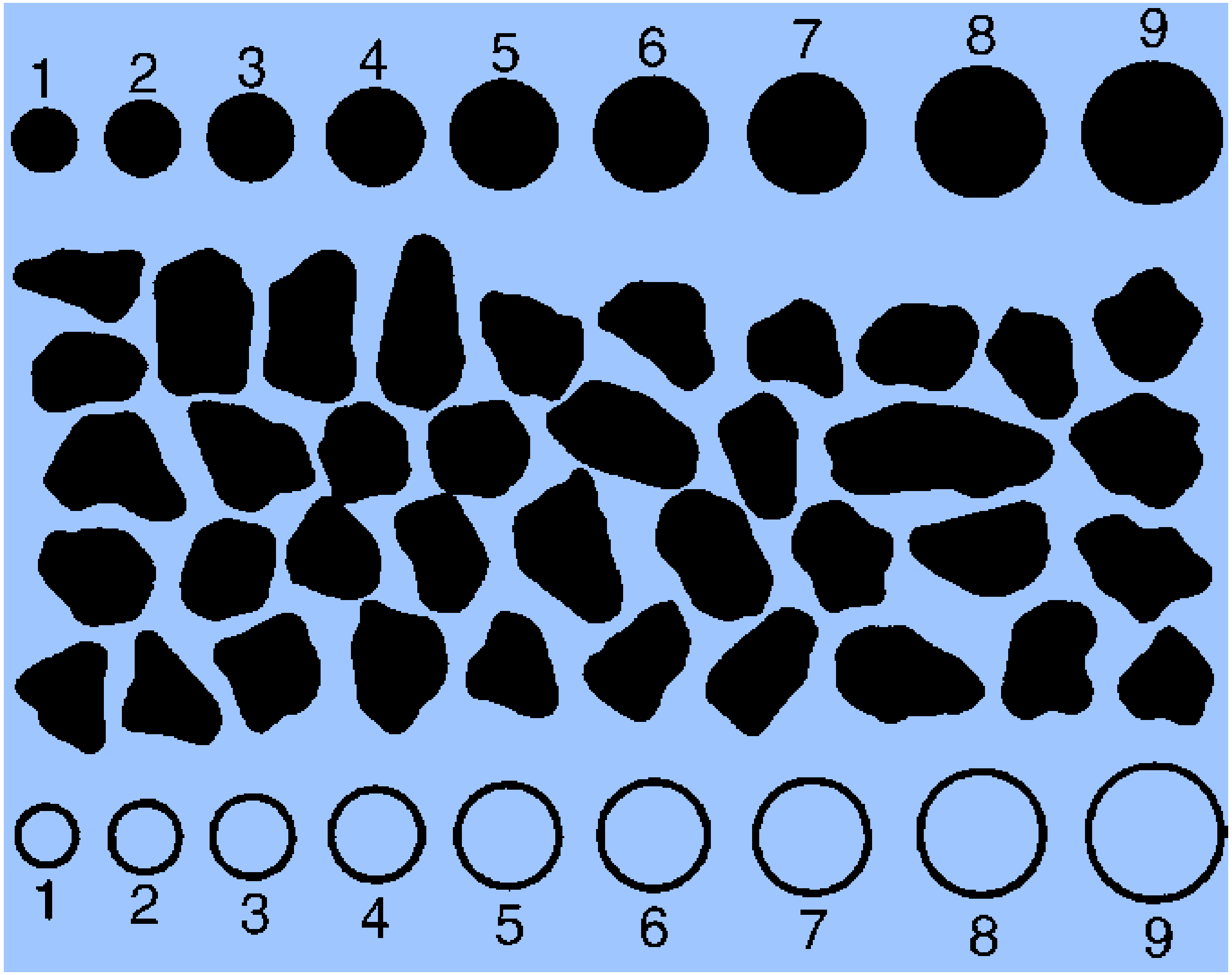
- In general "diameter" may be specified for any equidimensional particles.
- Particles that are not equidimensional, i.e. that are longer in one direction than in others, are often characterized by the second longest major dimension.
- For needle like particles, D_p would refer to the thickness of the particle, not their length

Measuring Techniques

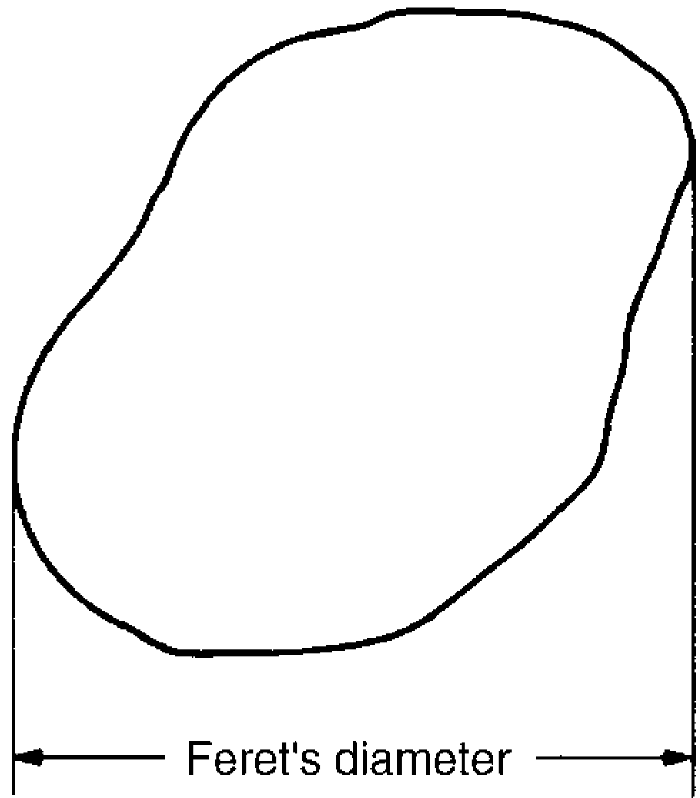
Microscopy


Microscopy
Optical Microscope
Sizes down to 5μm
Electron Microscope
Sizes below 5μm

Particle profiles and comparison circles

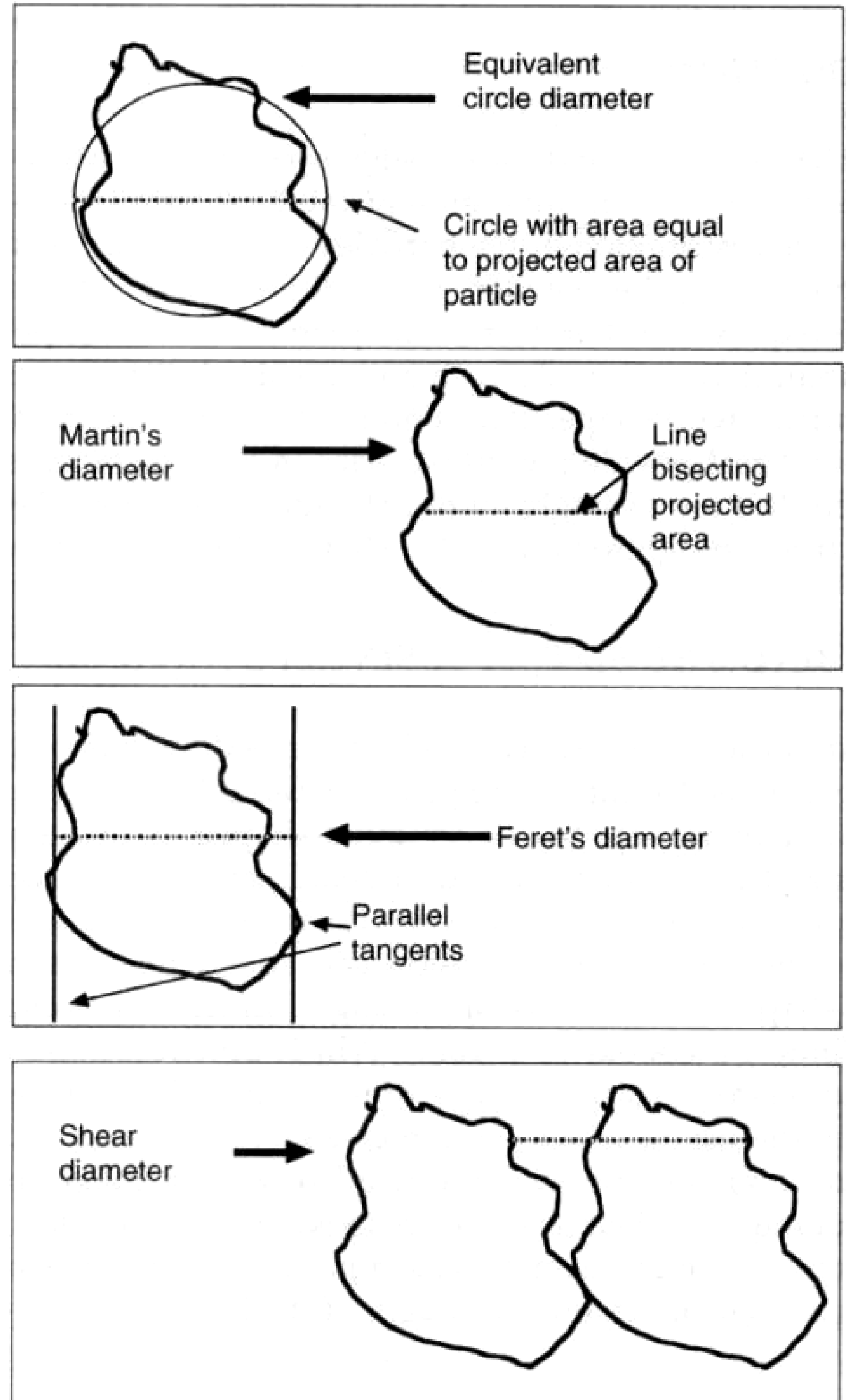


Feret’s diameter_ statistical diameter



The mean distance apart of two parallel lines which are tangential to the particle in an arbitrarily fixed direction. Changing orientation will lead to a large number of Feret’s diameter ≡ ‘Microscopy dimension’

- Describing the size of a single particle. Some terminology about diameters used in microscopy.
- Equivalent circle diameter.
- Martin’s diameter.
- Feret’s diameter.
- Shear diameter.

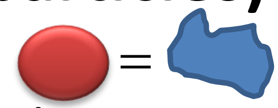


Particle Characterization

Size of equivalent sphere For Irregular Shape

PARTICLE CHARACTERISATION

Sizes of equivalent spheres (Single particles)

(a) The sphere of the same volume as the particle. 

(b) The sphere of the same surface area as the particle.

(c) The sphere of the same surface area per unit volume as the particle.

(d) The sphere of the same area as the particle when projected on to a plane perpendicular to its direction of motion.

(e) The sphere of the same projected area as the particle, as viewed from above, when lying in its position of maximum stability such as on a microscope slide for example.

(f) The sphere which will just pass through the same size of square aperture as the particle, such as on a screen for example.

(g) The sphere with the same settling velocity as the particle in a specified fluid.

Derived diameter

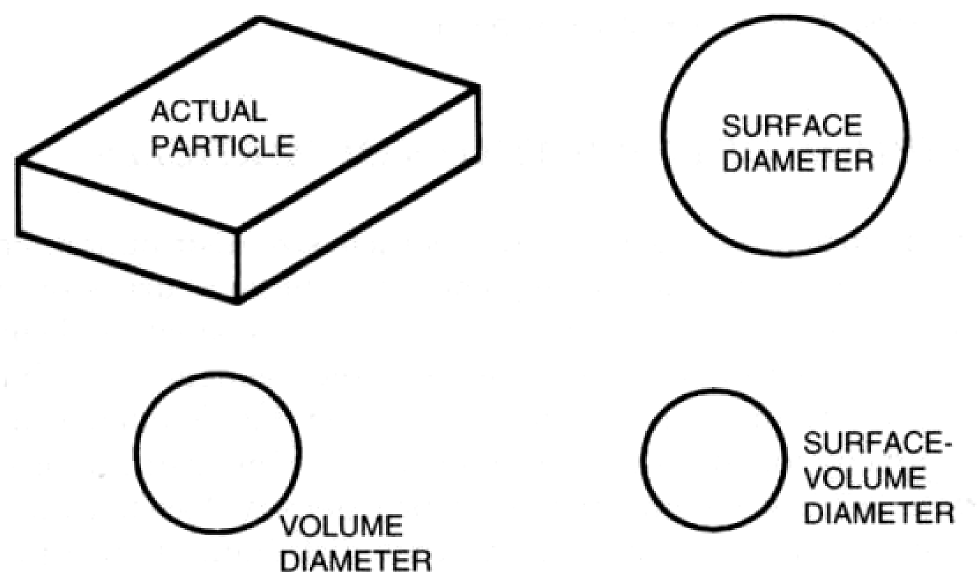
- Volume diameter, $\mathrm{d_v} = \sqrt[3]{\dfrac{6\mathrm{V_p}}{\pi}}$
- Surface diameter, $\mathrm{d_s} = \sqrt{\dfrac{S_p}{\pi}}$
- Surface volume diameter, $\mathrm{d_{sv}} = \mathrm{d_v^3} / \mathrm{d_s^2}$
- Free falling diameter
- Projected area diameter

Describing the size of a single particle

- Regular-shaped particles

Shape	Sphere	Cube	Cylinder	Cuboid	Cone
Dimensions	Radius	Side length	Radius and height	Three side lengths	Radius and height

- The orientation of the particle on the microscope slide will affect the projected image and consequently the measured equivalent sphere diameter.
- Sieve measurement: Diameter of a sphere passing through the same sieve aperture.
- Sedimentation measurement: Diameter of a sphere having the same sedimentation velocity under the same conditions.



Comparison of equivalent sphere diameters.

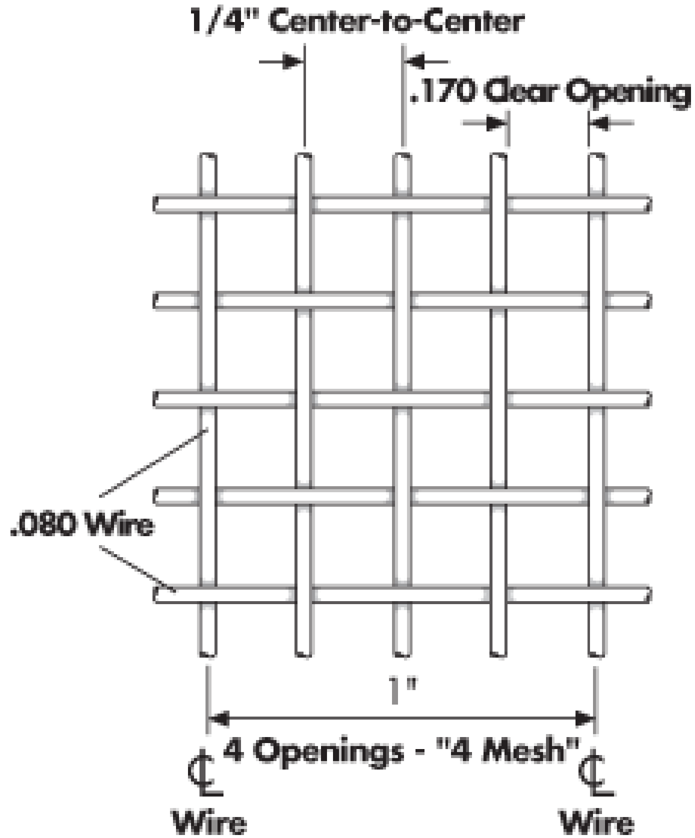
Comparison of equivalent diameters – See slid

- The volume equivalent sphere diameter is a commonly used equivalent sphere diameter.
- Example: Coulter counter size measurement. The diameter of a sphere having the same volume as the particle.
- Surface-volume diameter is the diameter of a sphere having the same surface to volume ratio as the particle.

Measuring techniques

Sieving > 50 μm

Particle size and shape



Mesh Number = Number of opening per linear inch

$$Clear\ opening = \frac{1}{mesh\ number} - Wire\ thickness$$

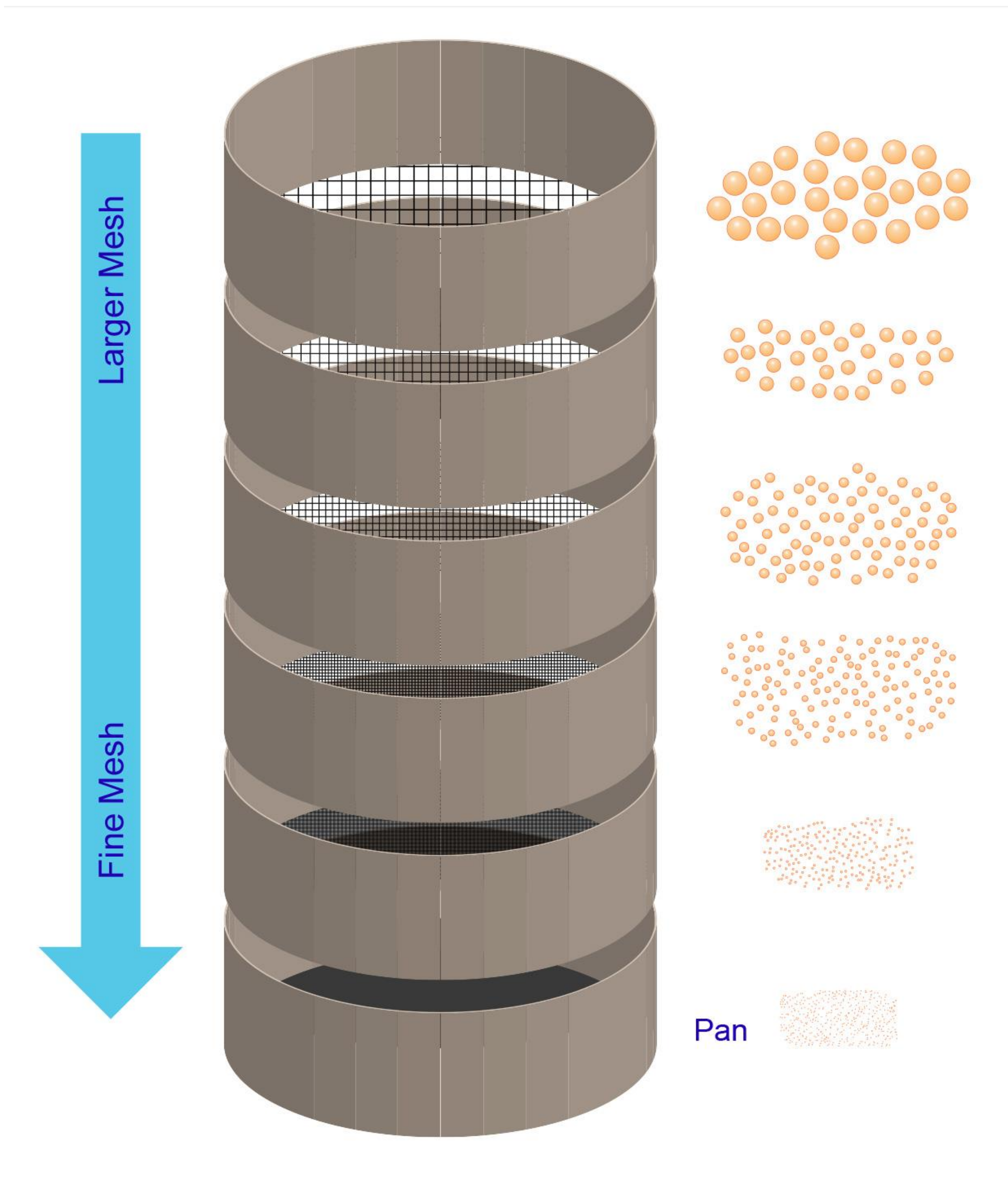
Table 1.1. Standard sieve sizes

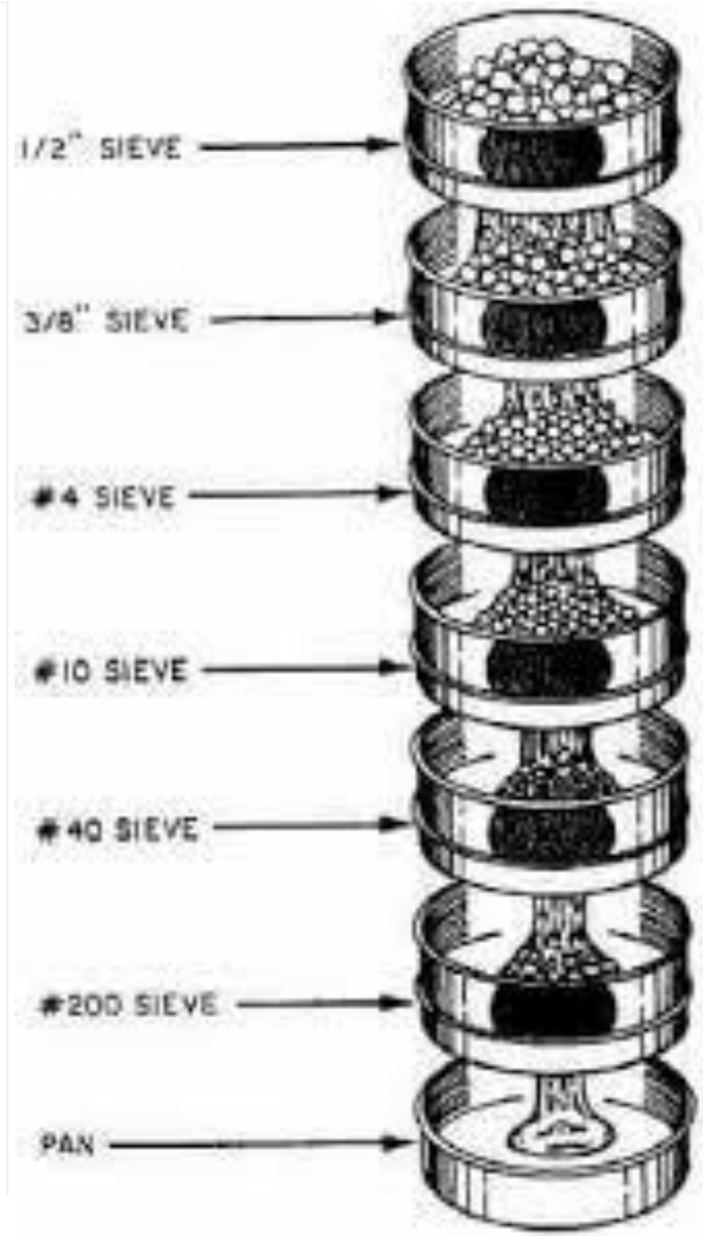
British fine mesh (B.S.S. 410)[3]			I.M.M.[4]			U.S. Tyler[5]			U.S. A.S.T.M.[5]		
Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture	
	in.	μm		in.	μm		in.	μm		in.	μm
						325	0.0017	43	325	0.0017	44
						270	0.0021	53	270	0.0021	53
300	0.0021	53				250	0.0024	61	230	0.0024	61
240	0.0026	66	200	0.0025	63	200	0.0029	74	200	0.0029	74
200	0.0030	76							170	0.0034	88
170	0.0035	89	150	0.0033	84	170	0.0035	89			
150	0.0041	104				150	0.0041	104	140	0.0041	104
120	0.0049	124	120	0.0042	107	115	0.0049	125	120	0.0049	125
100	0.0060	152	100	0.0050	127	100	0.0058	147	100	0.0059	150
			90	0.0055	139	80	0.0069	175	80	0.0070	177
85	0.0070	178	80	0.0062	157	65	0.0082	208	70	0.0083	210
			70	0.0071	180				60	0.0098	250
72	0.0083	211	60	0.0083	211	60	0.0097	246	50	0.0117	297
60	0.0099	251							45	0.0138	350
52	0.0116	295	50	0.0100	254	48	0.0116	295	40	0.0165	420
			40	0.0125	347	42	0.0133	351	35	0.0197	500

British fine mesh (B.S.S. 410)[3]			I.M.M.[4]			U.S. Tyler[5]			U.S. A.S.T.M.[5]		
Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture		Sieve no.	Nominal aperture	
	in.	μm		in.	μm		in.	μm		in.	μm
22	0.0275	699	20	0.0250	635	24	0.0276	701	25	0.0280	710
18	0.0336	853	16	0.0312	792	20	0.0328	833	20	0.0331	840
16	0.0395	1003				16	0.0390	991	18	0.0394	1000
14	0.0474	1204	12	0.0416	1056	14	0.0460	1168	16	0.0469	1190
12	0.0553	1405	10	0.0500	1270	12	0.0550	1397			
10	0.0660	1676	8	0.0620	1574	10	0.0650	1651	14	0.0555	1410
8	0.0810	2057				9	0.0780	1981	12	0.0661	1680
7	0.0949	2411				8	0.0930	2362	10	0.0787	2000
6	0.1107	2812	5	0.1000	2540	7	0.1100	2794	8	0.0937	2380
5	0.1320	3353				6	0.1310	3327			
						5	0.1560	3962	7	0.1110	2839
						4	0.1850	4699			
									6	0.1320	3360
									5	0.1570	4000
									4	0.1870	4760

Sieving >50 μm

- Set of sieves.
- Arrangement: ratio = $2 \text{ or } 2^{1/2} \text{ or } 2^{1/4}$
- Most common modern sieves are in sizes such that the ratio of adjacent sieve sizes is the fourth root of two (e.g. 45, 53, 63, 75, 90, 107 mm).
- Vibrator or shaker (vertical or horizontal vibration) .
- Fine particles may stick together (due to attractive forces) and block the screen.
- Standards: UK British standard, IMM standard, Tyler standard particle size scale


Larger Mesh
Fine Mesh
Pan


1/2" SIEVE
3/8" SIEVE
#4 SIEVE
#10 SIEVE
#40 SIEVE
#200 SIEVE
PAN

Particle Size Conversion Table

Sieve Designation		Nominal Sieve Opening		
Standard	Mesh	inches	mm	Microns
25.4 mm	1 in.	1.00	25.4	25400
22.6 mm	7/8 in.	0.875	22.6	22600
19.0 mm	3/4 in.	0.750	19.0	19000
16.0 mm	5/8 in.	0.625	16.0	16000
13.5 mm	0.530 in.	0.530	13.5	13500
12.7 mm	1/2 in.	0.500	12.7	12700
11.2 mm	7/16 in.	0.438	11.2	11200
9.51 mm	3/8 in.	0.375	9.51	9510
8.00 mm	5/16 in.	0.312	8.00	8000
6.73 mm	0.265 in.	0.265	6.73	6730
6.35 mm	1/4 in.	0.250	6.35	6350
5.66 mm	No.3 1/2	0.223	5.66	5660
4.76 mm	No. 4	0.187	4.76	4760
4.00 mm	No. 5	0.157	4.00	4000
3.36 mm	No. 6	0.132	3.36	3360
2.83 mm	No. 7	0.111	2.83	2830
2.38 mm	No. 8	0.0937	2.38	2380
2.00 mm	No. 10	0.0787	2.00	2000
1.68 mm	No. 12	0.0661	1.68	1680
1.41 mm	No. 14	0.0555	1.41	1410
1.19 mm	No. 16	0.0469	1.19	1190

Mesh	Micron	Inches
4	4760	0.185
6	3360	0.131
8	2380	0.093
12	1680	0.065
16	1190	0.046
20	840	0.0328
30	590	0.0232
40	420	0.0164
50	297	0.0116
60	250	0.0097
70	210	0.0082
80	177	0.0069
100	149	0.0058
140	105	0.0041
200	74	0.0029
230	62	0.0023
270	53	0.0021
325	44	0.0017
400	37	0.0015
625	20	0.0008
1250	10	0.0004
2500	5	0.0002

Reading Notes

- Larger sieve openings (1 in. to 1/4 in.) have been designated by a sieve "mesh" size that corresponds to the size of the opening in inches.
- Smaller sieve "mesh" sizes of 3 1/2 to 400 are designated by the number of openings per linear inch in the sieve.
- The following convention is used to characterize particle size by mesh designation:

  a "+" before the sieve mesh indicates the particles are retained by the sieve;

  a "-" before the sieve mesh indicates the particles pass through the sieve;

  typically 90% or more of the particles will lie within the indicated range.

For example, if the particle size of a material is described as -4 +40 mesh, then 90% or more of the material will pass through a 4-mesh sieve (particles smaller than 4.76 mm) and be retained by a 40-mesh sieve (particles larger than 0.420 mm). **If a material is described as -40 mesh, then 90% or more of the material will pass through a 40-mesh sieve (particles smaller than 0.420 mm.**

Sieving efficiency & rate of screening or passage of particles through sieve

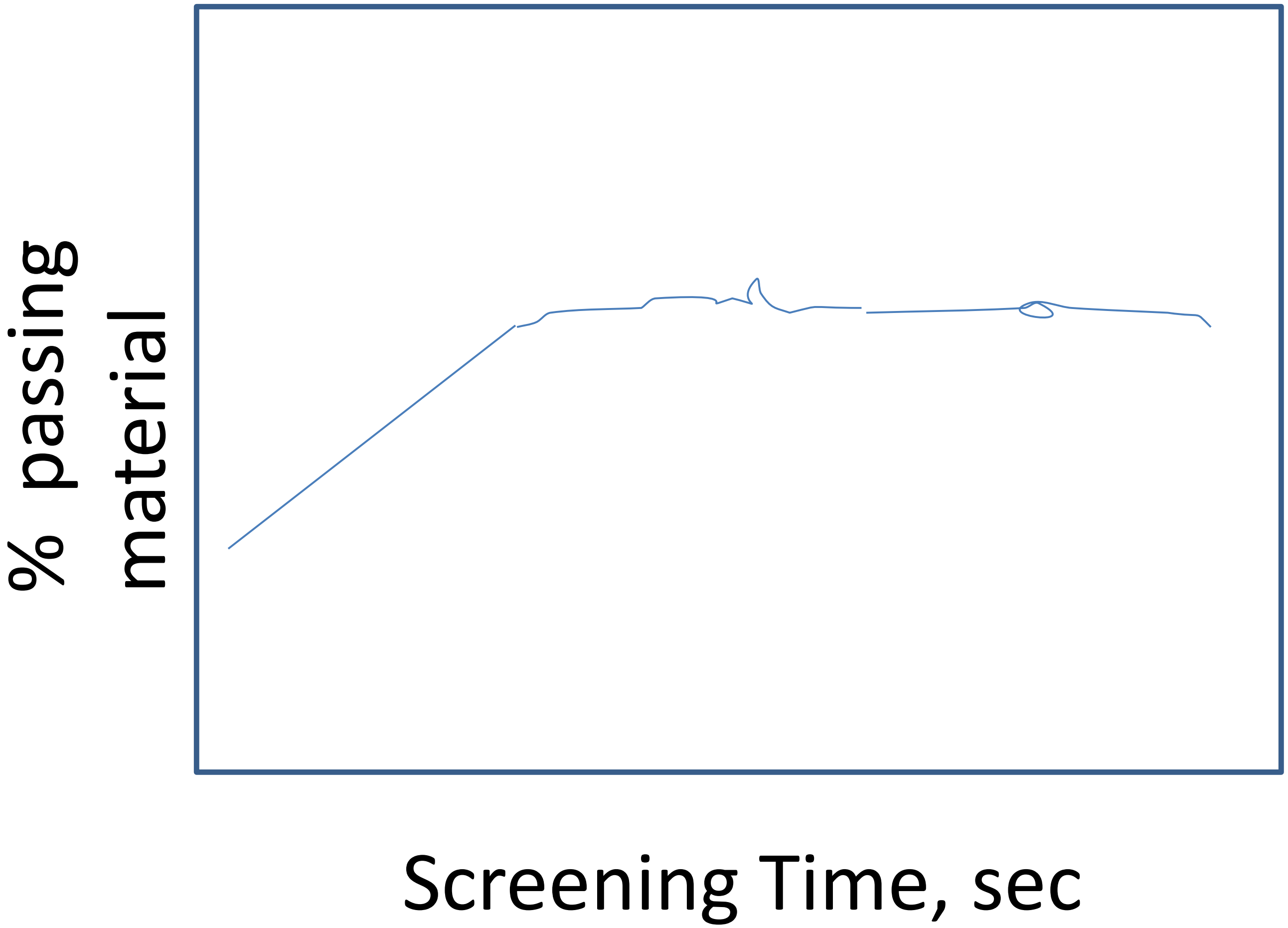
$$\eta_{sieving} = \frac{\text{wt of material which passes the screen}}{\text{wt of material which capable of passing}}$$

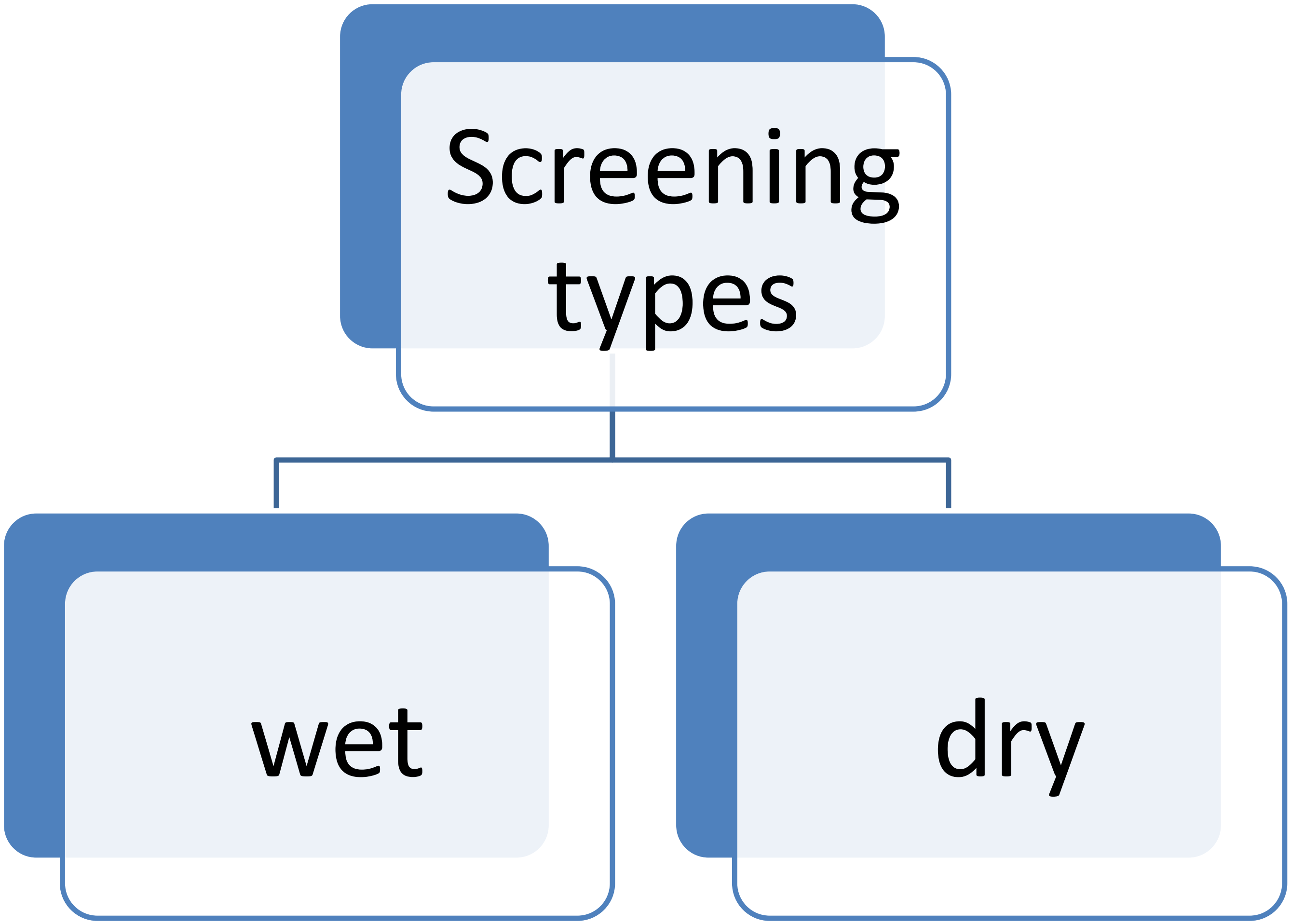
$$\text{rate of passage of particles} = \frac{dw}{dt} = -kw$$

where w is the mass of particles of a certain size on the screen.

mass of particles, $w_1 - w_2$, passing the screen in time t is $\ln\frac{w_2}{w_1} = -kt$;0r $w_2 = w_1 e^{-kt}$

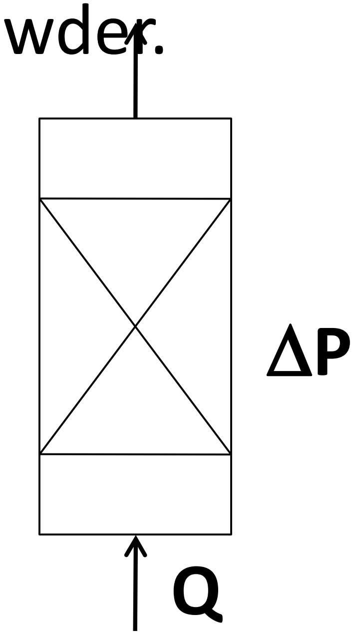
General trend of screening process




Screening types
wet
dry

Permeability methods (>1 μm)

•These methods depend on the fact that at low flow rates the flow through a packed bed is directly proportional to the pressure difference, the proportionality constant being proportional to the square of the specific surface of the powder.



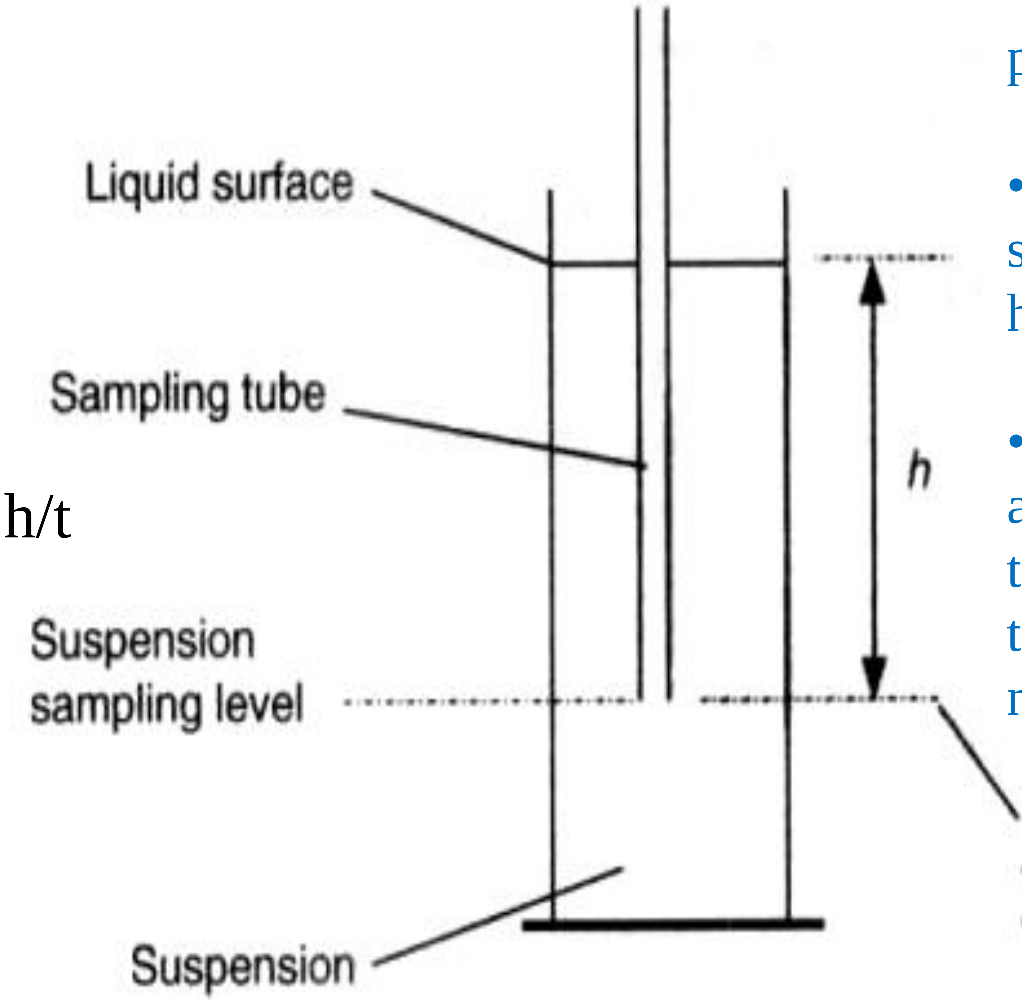
•From this method it is possible to obtain the diameter of the sphere with the same specific surface as the powder.

$$\boxed{\frac{(-\Delta p)}{H} = 180\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu U}{x^2}}$$

• Further details will be given in next chapters.

Size analysis by Sedimentation pipette method

Note:
U* = h/t



- Re_p <0.3. Motion of the particle obeys Stoke’s law.
- The suspension is sufficiently dilute (No hindered settling).
- Particles are assumed to accelerate rapidly to their terminal free fall velocity, time for acceleration is negligible.

C_o: original uniform suspension density.

Sampling point: C at time t after the start of settling.

Note: settling velocity
$u^* = h/t$

$$\boldsymbol{u^* = \frac{d_p^2(\rho_p - \rho_f)g}{18\mu}}$$

At time t all particles traveling faster than h/t will have fallen below the sampling point.

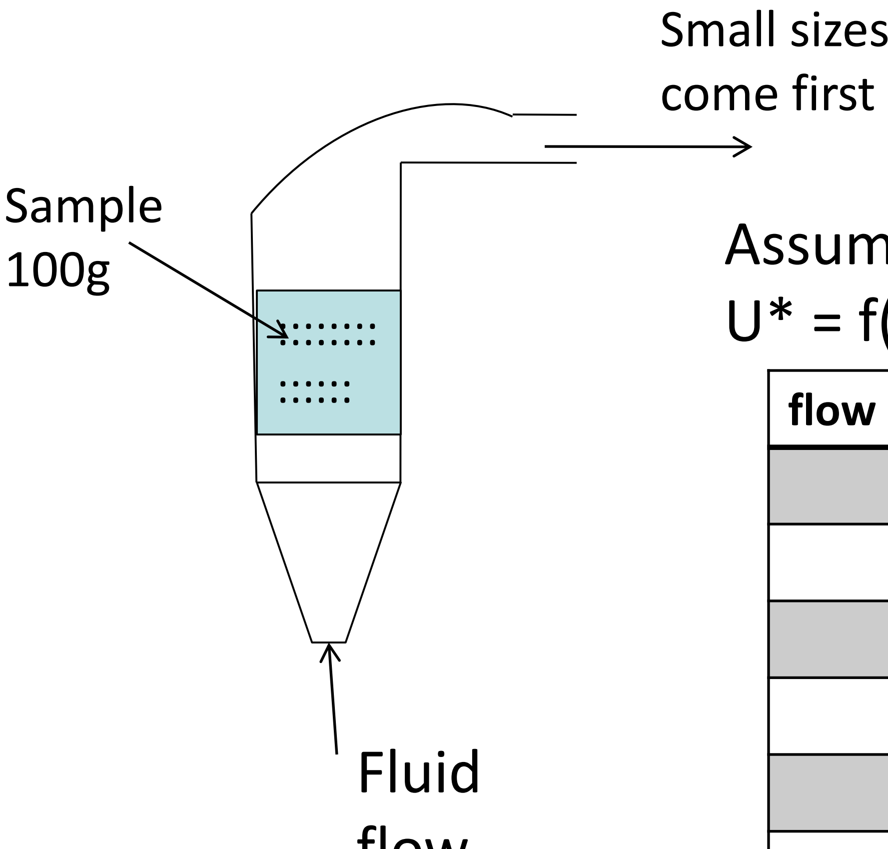
C represents the suspension density for all particles which travel at a velocity <= h/t.

$$\text{cumulative mass fraction} = \frac{C}{C_0}$$

Re <0.3
$d_p = [18\mu h/t(\rho_p - \rho_f)g]^{1/2}$ Cum.

mass fraction = C/C_0Type equation here.

Size analysis by Elutriation



Assume Stokes regime U_f = U^* = $f(d_p)$ undersize

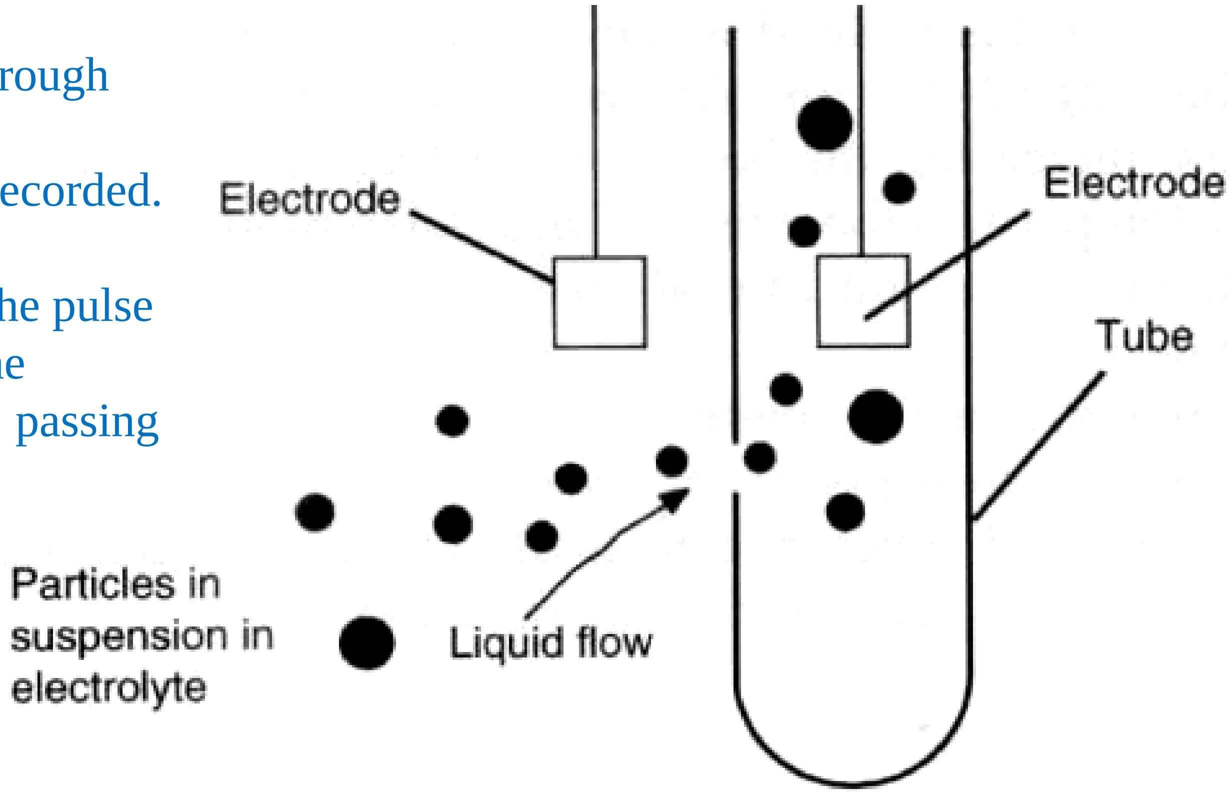
flow	mass	Uf
	100 g	

- Coulter Counter and Electrozone analyzer (1-1000μm)

As particle flow through the orifice,
a voltage pulse is recorded.

The amplitude of the pulse can be related to the volume of particle passing the orifice.

Particle range: 0.3-1000 μm.



- Schematic of electrozone sensing apparatus

Principle of Coulter Counter and Electrozone Analyser

- As particles enter the orifice they displace an equivalent volume of electrolyte, thereby producing a change in the electrical resistance of the circuit, the magnitude of which is related to the displaced volume.
- The consequent voltage pulse across the electrodes is fed to a multi-channel analyzer. The distribution of pulses arising from the passage of many thousands of particles is then processed to provide a particle (volume) size distribution.
- By using orifices of various diameters, different particle size ranges may be examined and the resulting data may then be combined to provide size distributions extending over a large proportion of the sub-millimetre size range.

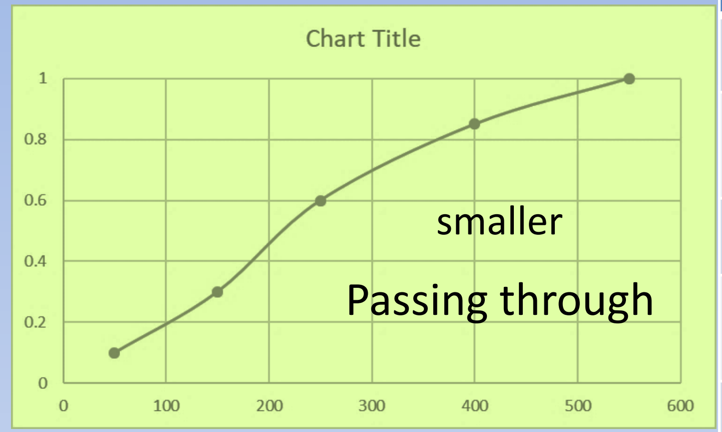
Mean Particle Size

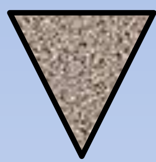
Cumulative screen analysis curve

Plot Cumulative mass fraction vs Average particle size

Example for Discussion

Size range	x	d	x_{cum}
0 - 100	0.1	50	0.1
100 – 200	0.2	150	0.3
200 – 300	0.3	250	0.6
300 – 500	0.25	400	0.85
500 - 600	0.15	550	1.00
total	1.00		

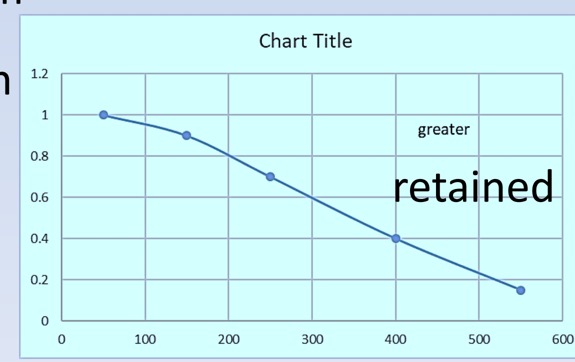


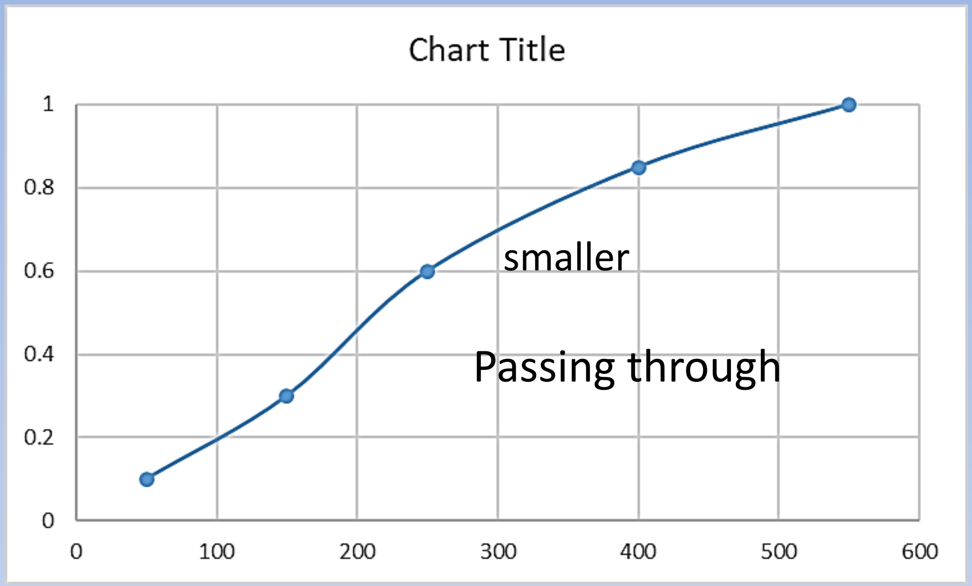


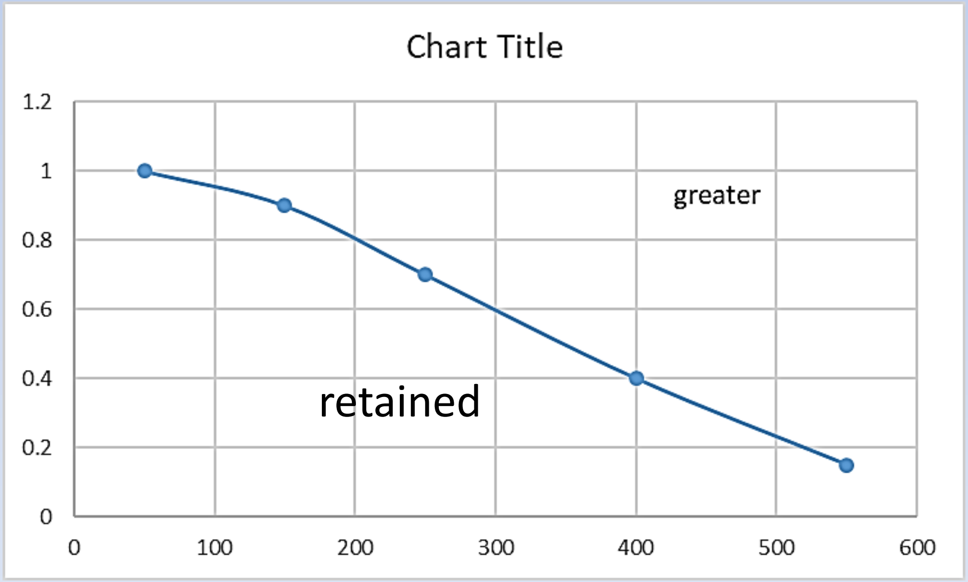
Sample
100 g

Size, μm	Wt retains on sieve	Fraction, x	x cum	d, avg
600	0 g	0.00 or 0%		
500	15 g	0.15 or 15%	15%	550 μm
300	25 g	0.25 or 25%	40%	400 μm
200	30 g	0.30 or 30%	70%	250 μm
100	20 g	0.20 or 20%	90%	150 μm
Pan	10 g	0.10 or 10%	100%	50 μm
Total	100 g	1.00 or 100%		

Arbitrary values







Plot Cumulative mass fraction vs Average particle size

Particle size distribution

cumulative mass fraction curve, in which the proportion of particles *(x) smaller* than a certain size *(d) is plotted against that size (d)*

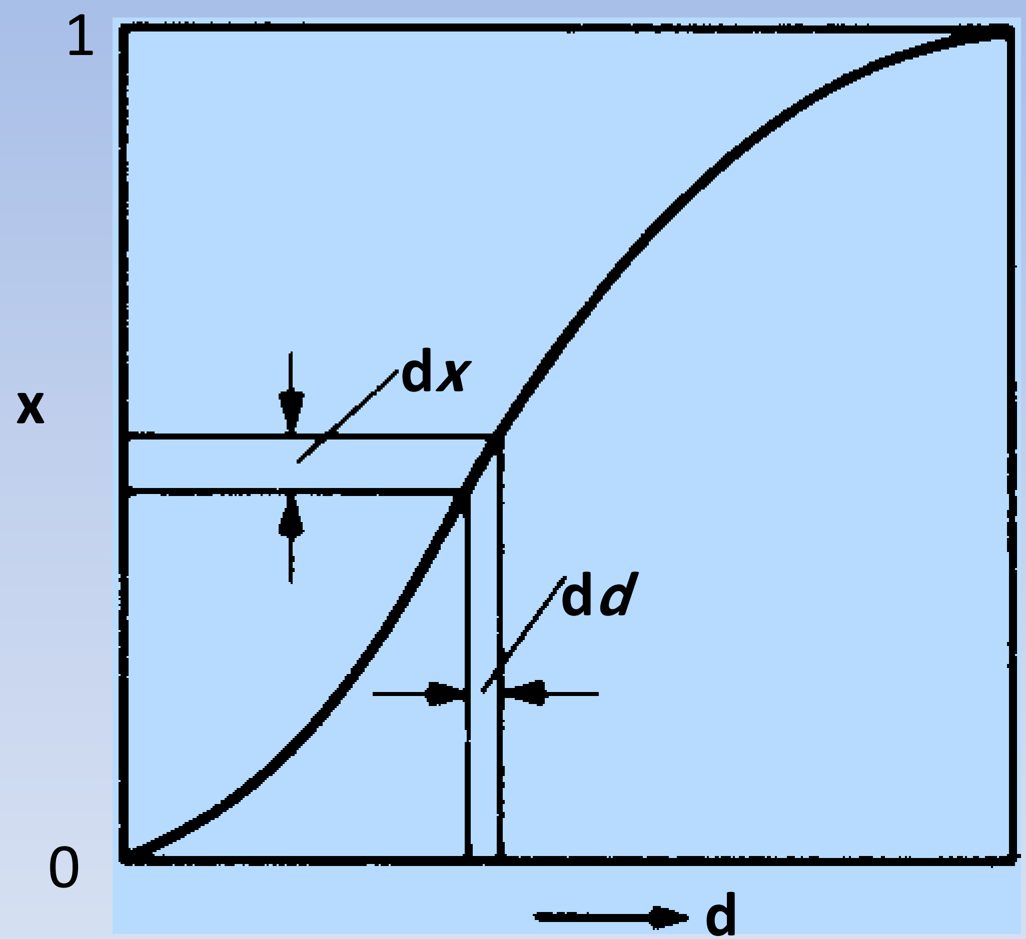


Figure 1.5. Size distribution curve—cumulative basis

Particle size distribution

the slope *(dx/dd) of the cumulative* curve (Figure 1.5) is plotted against particle size *(d).*

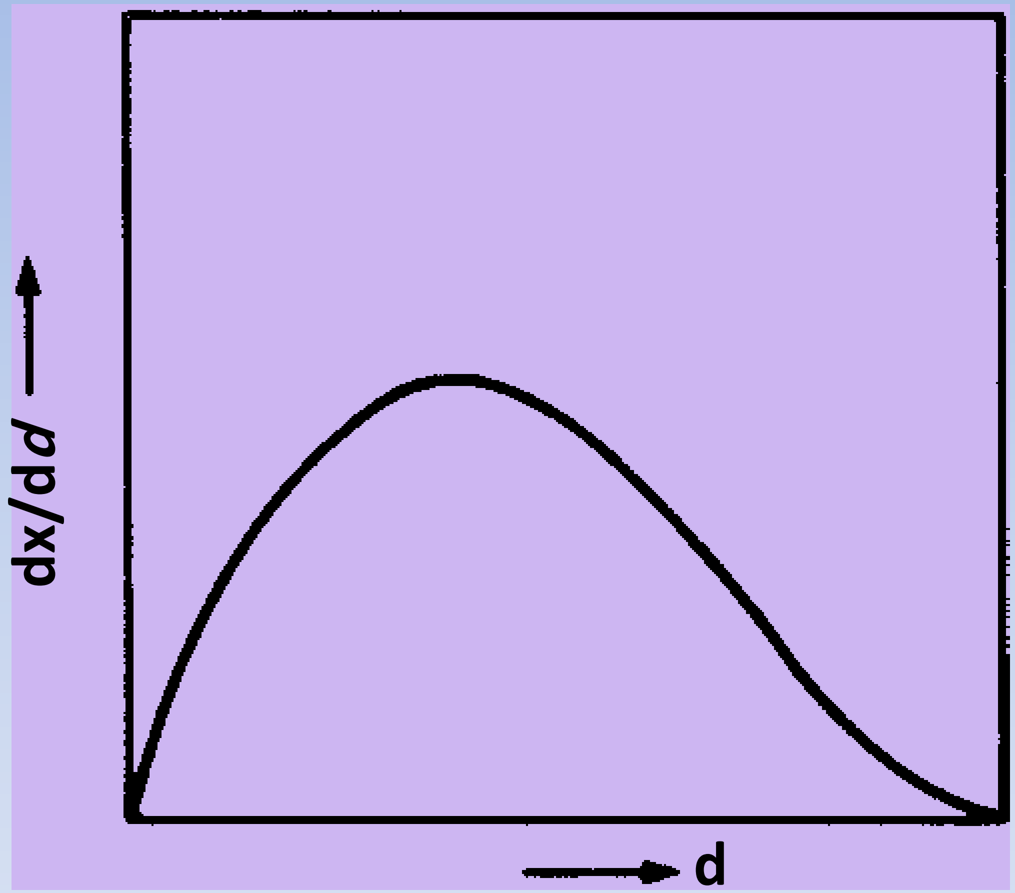


Figure 1.6. Size distribution curve—frequency basis

Three values are very important

I. Mode
II. Median
III. Mean

These values can be found directly from the frequency and cumulative curves.

Mean values can be found algebraically based on size intervals.

Consider a unit mass of n_1 particles of characteristic length d_1 with mass fraction x_1 and so on

$$\left\{ \begin{array}{ccc} n_1 & d_1 & x_1 \\ n_2 & d_2 & x_2 \\ . & . & . \\ . & . & . \\ n_x & d_x & x_x \end{array} \right\} \textbf{unit mass}$$

Note:

Aggregate length $n_i\, d_i$

Aggregate surface $n_i d_i^2$

Aggregate volume $n_i d_i^3$

Mean particle size

- Considering unit mass of particles consisting of n_1 *particles of characteristic dimension* d_1, *constituting a mass fraction* x_1, n_2 *particles of size* d_2, *and so on, then:*

$$x_i = n_i k_i d_i^3 \rho_s \qquad (1.4)$$

and: $$\Sigma x_i = 1 = \rho_s k \Sigma(n_i d_i^3) \qquad (1.5)$$

Thus: $$n_i = (1/\rho_s k_i)(x_i/d_i^3) \qquad (1.6)$$

- If the size distribution can be represented by a continuous function, then:

$dx = \rho_s k_i\, d^3\, dn$

or:

$$\frac{dx}{dn} = \rho_s k_1 d^3 \qquad (1.7)$$

And:

$$\int_0^1 dx = 1 = \rho_s k_1 \int d^3\, dn \qquad (1.8)$$

where ρ_s *is the density of the particles, and*

k_1 *is a constant whose value depends on the shape of the particle.*

Summary
Means based on volume

- Volume mean diameter, dv

$$d_v = \frac{\sum (n_i d_i) v_i}{\sum n_i v_i} = \frac{\sum n_i d_i^4}{\sum n_i d_i^3}$$

in terms of x_i :

$$d_v = \frac{\sum d_i x_i}{\sum x_i} = \sum d_i x_i$$

- Mean volume diameter dv'

$$d_{v'}^3 \sum n_i = \sum n_i d_i^3$$

$$d_{v'} = \sqrt[3]{\frac{\sum n_i d_i^3}{\sum n_i}} \quad , \text{ since } n_i = {x_i}\big/{\rho_s k_i d_i^3}$$

$$\therefore \qquad d_{v'} = \sqrt[3]{\frac{\sum x_i}{\sum (x_i / d_i^3)}}$$

Means based on surface

- Surface mean diam, ds

$$d_s = \frac{\sum (n_i d_i) s_i}{\sum n_i s_i} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2}$$

in terms of x_i :

$$d_s = \frac{\sum x_i}{\sum (x_i / d_i)} = \frac{1}{\sum (x_i / d_i)}$$

Sauter mean diameter

- Mean surface diam, ds'

$$d_{s'}^2 \sum n_i = \sum n_i s_i = \sum n_i d_i^2$$

$$d_{s'} = \sqrt{\frac{\sum n_i d_i^2}{\sum n_i}} \quad \text{, since} \quad n_i = \frac{x_i}{\rho_s k_i d_i^3}$$

$$\therefore \qquad d_{s'} = \sqrt{\frac{\sum (x_i / d_i)}{\sum (x_i / d_i^3)}}$$

Means based on length

- Length mean diam, d_l

$$d_l = \frac{\sum (n_i d_i) d_i}{\sum n_i d_i} = \frac{\sum n_i d_i^2}{\sum n_i d_i}$$

in terms of x_i :

$$d_l = \frac{\sum (x_i / d_i)}{\sum (x_i / d_i^2)}$$

- Mean length diam, d_l'

$$d_{l'} \sum n_i = \sum n_i d_i$$

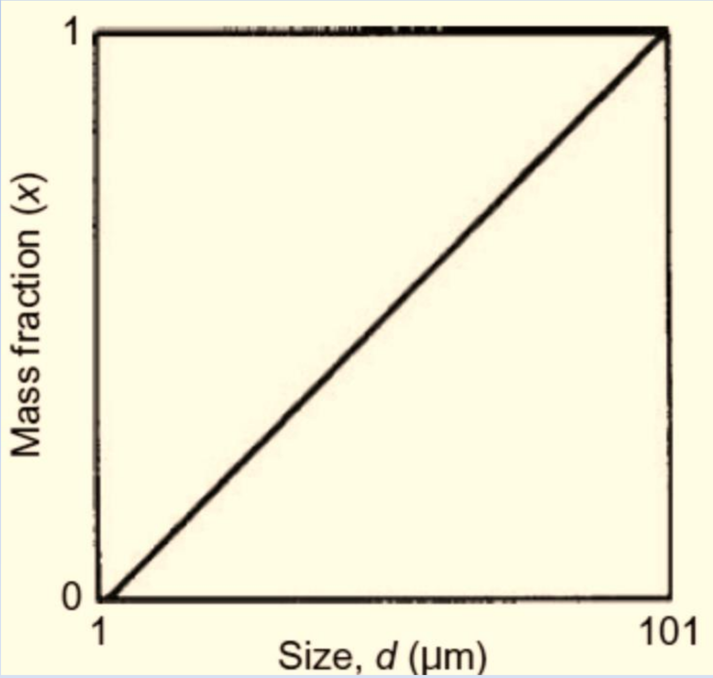
$$d_{l'} = \frac{\sum n_i d_i}{\sum n_i}$$

in terms x_i

$$\therefore \qquad d_{l'} = \frac{\sum (x_i / d_i^2)}{\sum (x_i / d_i^3)}$$

Example

The size analysis of a powdered material on a mass basis is represented by a straight line from 0 % mass at 1 μm particle size to 100% mass at 101 μm particle size as shown in the adjacent Figure. Calculate the surface mean diameter of the particles constituting the system.



Size analysis of powder

Solution

- The surface mean diameter is given by:

$$d_s = \frac{1}{\Sigma(x_1/d_1)}$$

- Because the size analysis is represented by the continuous curve: $d = 100x + 1$
- then

$$d_s = \frac{1}{\int_0^1 \frac{dx}{d}}$$

$$= \frac{1}{\int_0^1 \frac{dx}{100x + 1}}$$

$$= (100/\ln 101)$$

$$= 21.7\,\mu m$$

Exercise 1

Sl. No.	Mesh No.	Screen Opening D_{pi} (cm)	Mass retained on a screen m_i (gm)
	4		0
	6		25
	8		125
	10		325
	14		250
	20		160
	28		50
	35		20
	48		10
	65		8
	100		6
	150		4
	200		3
	pan		2

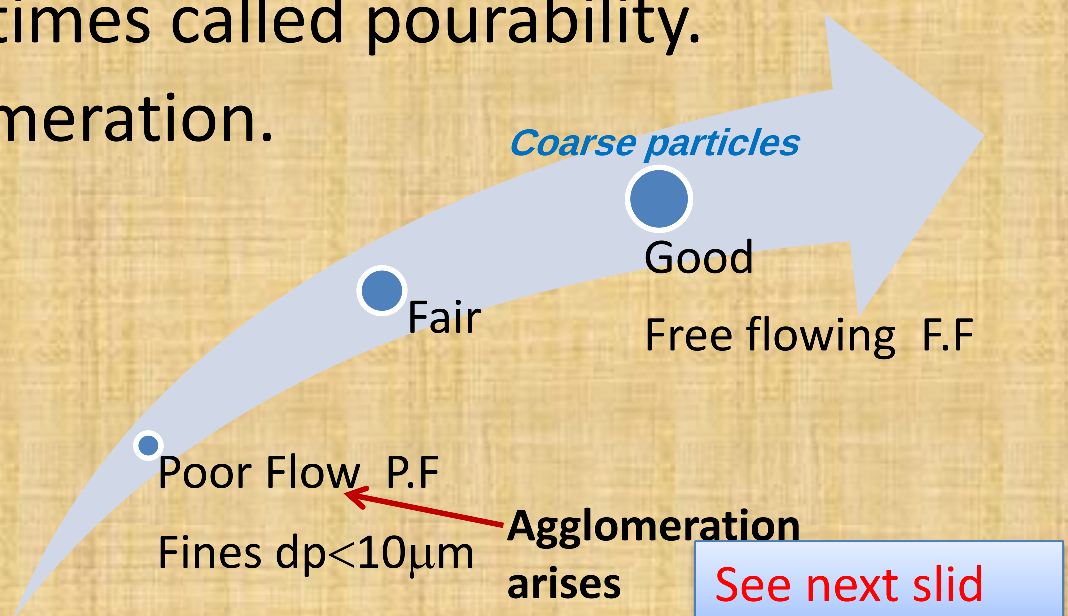
Plot the cumulative curve, and calculate the Sauter mean diameter. Use mat lab or excel programs

PARTICULATE SOLIDS IN BULK

Characteristics of solid Particulates & Hoppers & Conveyors & Classification of solid particles

PARTICULATE SOLIDS IN BULK

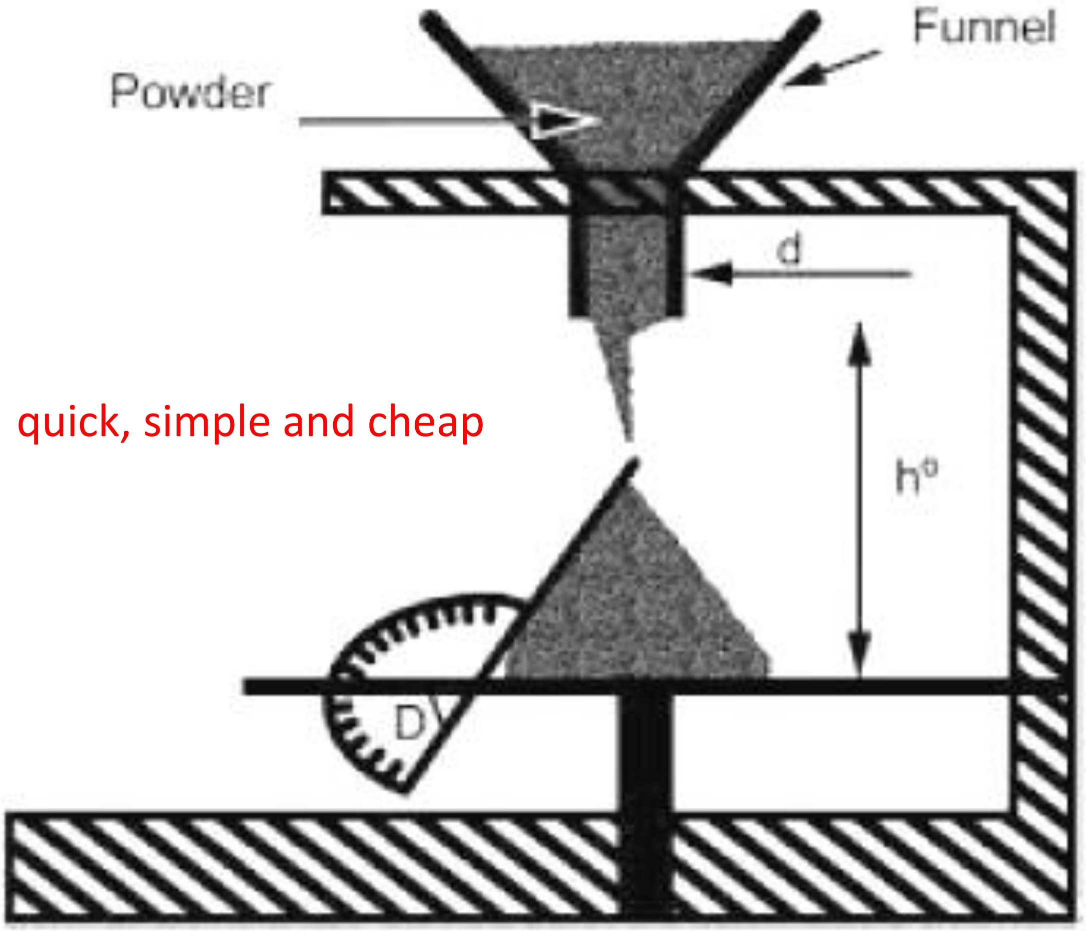
- fluid-solid & solid-solid interactions cause new behaviors and lead to new properties such as
  - Voidage & bulk density.
  - Flow properties “followability” ⇒ look to this flow chart. Followability shows the capability to flow. Sometimes called pourability.
  - Agglomeration.



Definition_ Pourability "Flowability"

- It is defined as a measure of the time required for a standard quantity of material to flow through a funnel of specified dimension.
- It characterizes the handling properties of fine particles.

Measuring Flowability



quick, simple and cheap

A schematic view of an angle of repose measurement with an open base and fixed height

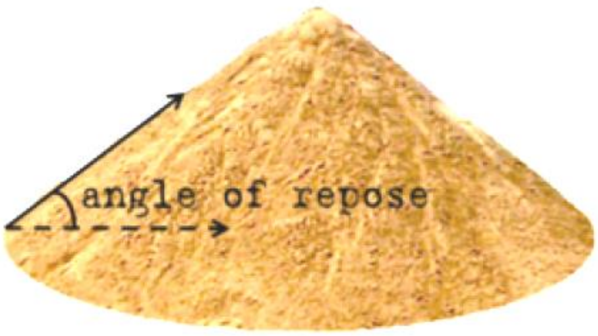
Methods:

1) Angle of repose
2) Flow through an orifice
3) Tapped density "Carr's Compressibility Index (CCI) and Hausner Ratio (HR)"

$$CCI = \frac{\rho_t - \rho_p}{\rho_t} x\ 100 \quad ; \quad HR = \frac{\rho_t}{\rho_p}$$

where ρ_t is tapped density and ρ_p is poured density

4) Shear cell analysis.



Scale of flowability

Flow character	Angle of repose (θ)	Compressibility index (%)	Hausner's ratio
Excellent	25–30	≤10	1.00–1.11
Good	31–35	11–15	1.12–1.18
Fair	36–40	16–20	1.19–1.25
Passable	41–45	21–25	1.26–1.34
Poor	46–55	26–31	1.35–1.45
Very poor	56–65	32–37	1.46–1.59
Very, very poor	>66	>38	>1.60

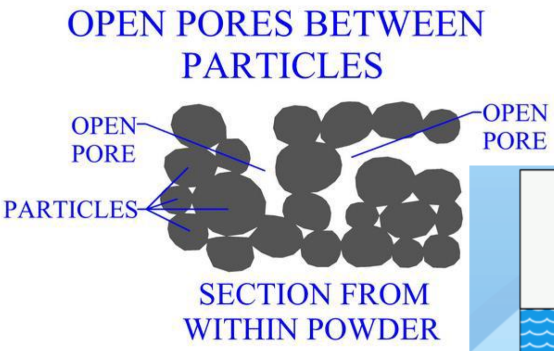
PROPTERTIES OF MASSES OF PARTICLES

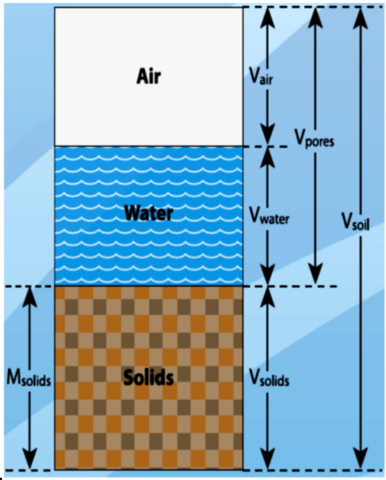
- Bulk density or apparent density, ρ_b, is defined as the weigh per unit volume of material including voids inherent in the material as tested. It is a measure of the fluffiness of the material.

$$\rho_b = \frac{m}{V_T} = \frac{m}{V_p+V_v} \text{ and } \rho_t = \frac{m}{V_p}$$

- Voidage (Porosity), It is the ratio of the void volume and bulk volume.

$$\varepsilon = \frac{V_T - V_p}{V_T} = 1 - \frac{V_p}{m}\frac{m}{V_T} = 1 - \frac{\rho_b}{\rho_t}$$





Where,
V_p: volume particles,
V_v: volume of voids,
V_T: total volume,
m: mass of material,
ρ_b: bulk density
ρ_t: true or particle density without pores,
ε: voidage

Agglomeration

(1) ***Mechanical interlocking****. This can occur particularly if the particles are long and* thin in shape, in which case large masses may become completely interlocked.

(2) ***Surface attraction.*** *Surface forces, including van der Waals' forces, may give rise* to substantial bonds between particles, particularly where particles are very fine (*<10 μm), with the result that their surface per unit volume is high.*

In general, freshly formed surface, such as that resulting from particle fracture, gives rise to high surface forces.

(3) ***Plastic welding****. When irregular particles are in contact, the forces between the particles will be born on extremely small surfaces and the very high pressures developed may give rise to plastic welding.*

(4) ***Electrostatic attraction****. Particles may become charged as they are fed into equipment and significant electrostatic charges may be built up, particularly on fine solids.*

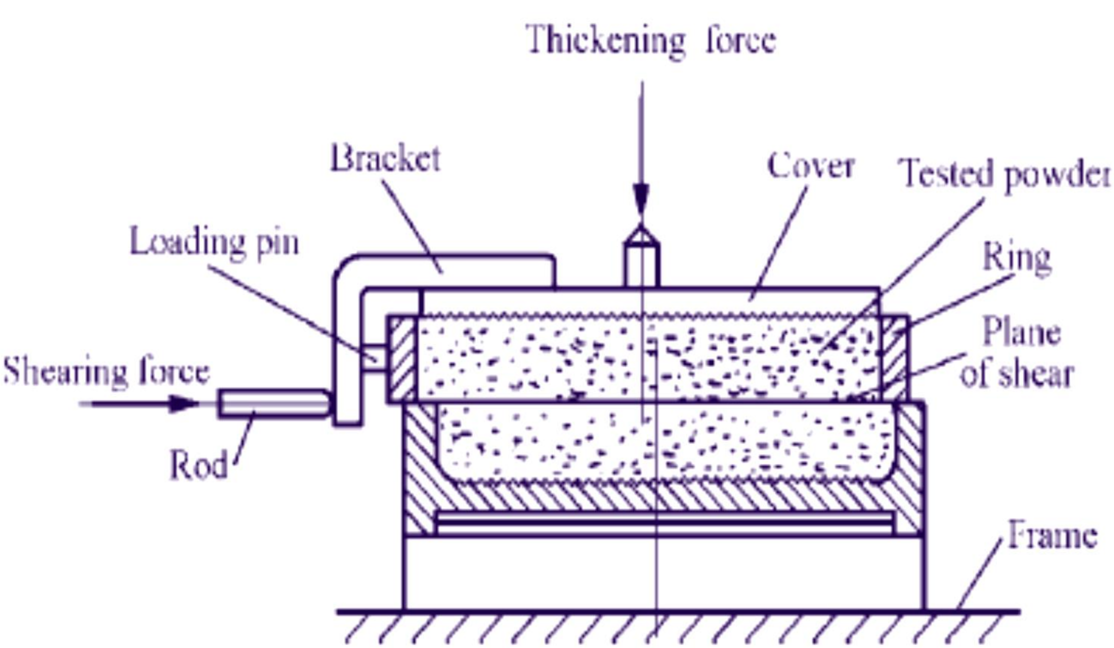
(5) ***Effect of moisture***. *Moisture may have two effects. Firstly, it will tend to collect* near the points of contact between particles and give rise to surface tension effects. *Secondly*, it may dissolve a little of the solid, which then acts as a bonding agent on subsequent evaporation.

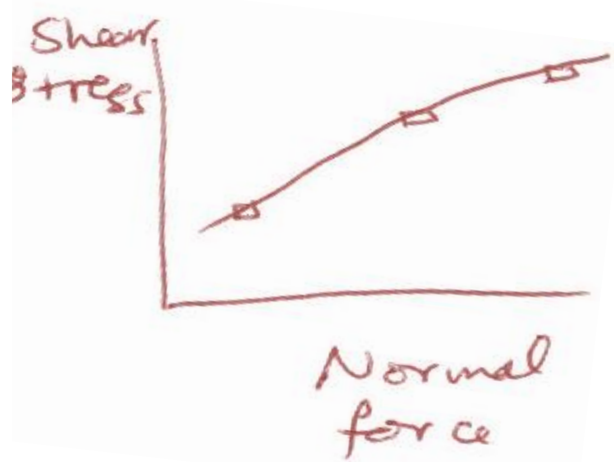
(6) ***Temperature fluctuations*** *give rise to changes in particle structure and to greater* cohesiveness.

- Resistance to shear and tensile forces

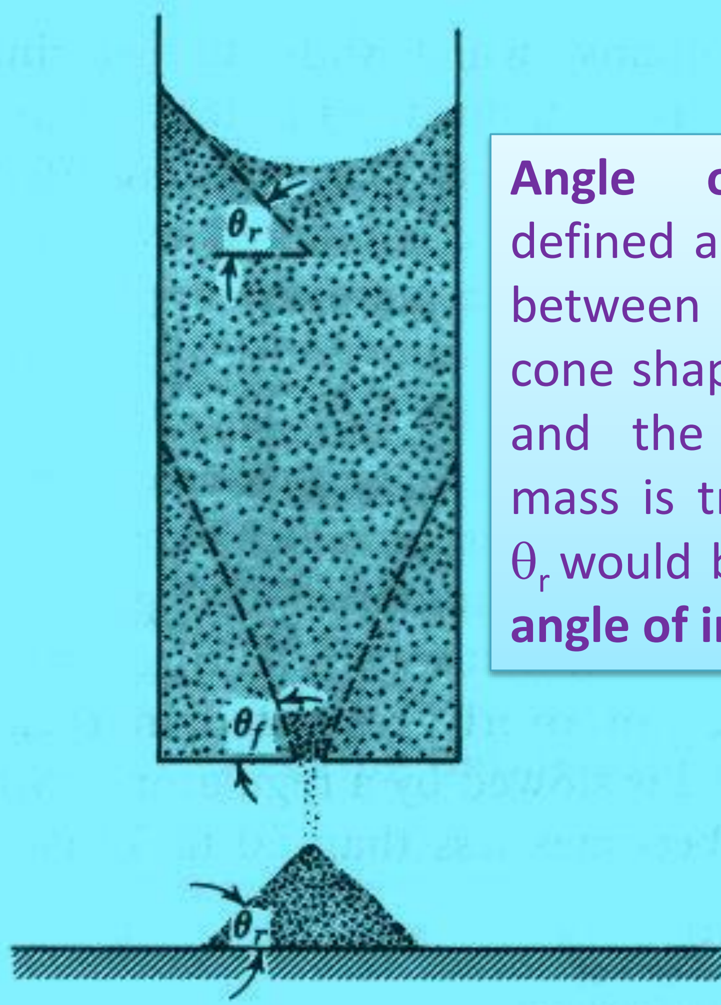
- Particulate material (agglomerated or non-agglomerated) shows a significant resistance to shear and tensile forces.
- This property is very important for flow characteristics of powder material. Moreover, it is very important for the hopper and silo designs.
- As the packing density material increases, the powder material shows a higher resistance to shear and tension.

- Shear forces are measured by using Jenike shear cell.



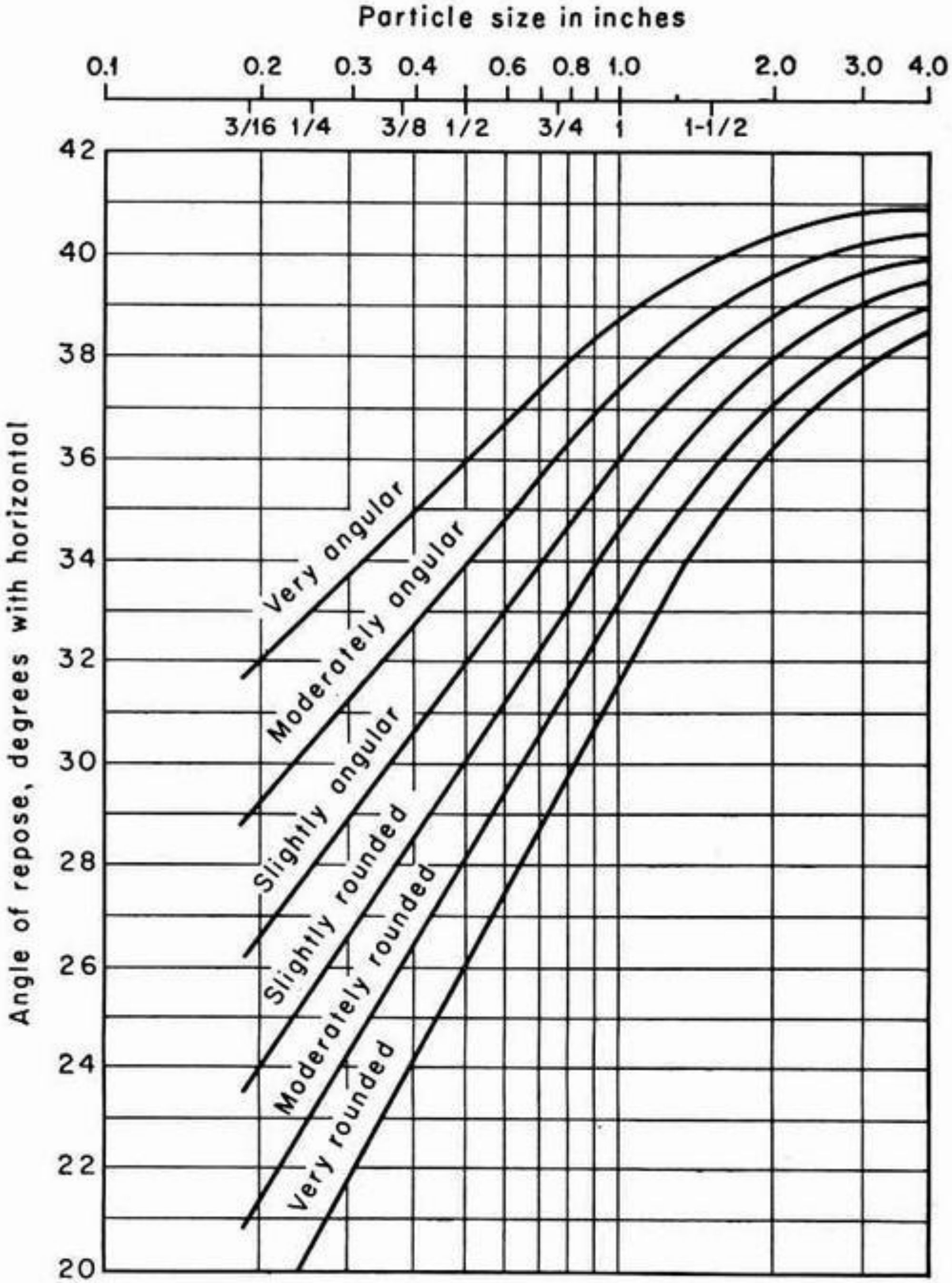


Angles of repose and of friction



Angle of repose, θ_r**,**is defined as the angle formed between sloping side of a cone shaped pile of material and the horizontal if the mass is truly homogeneous, θ_r would be equal to θ_f , the **angle of internal friction.**

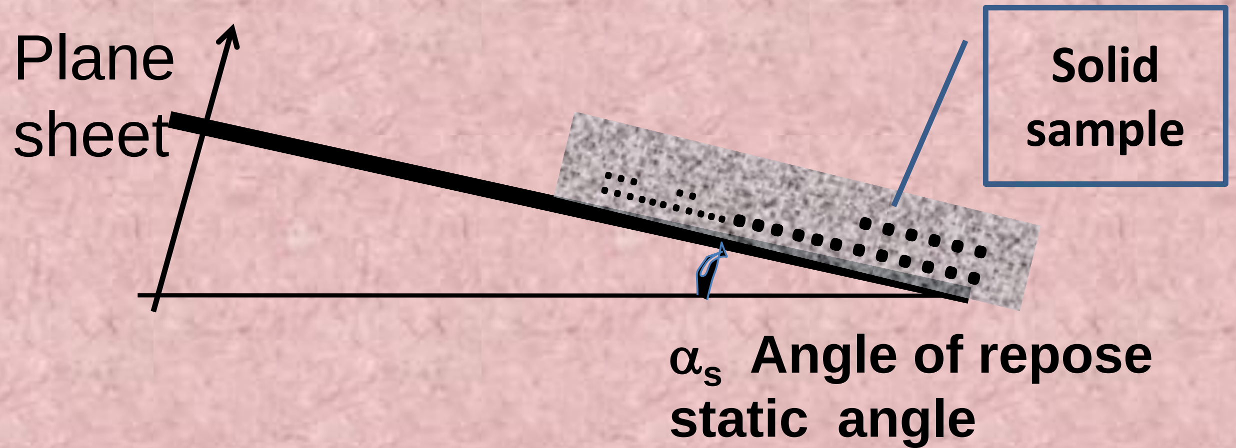
Repose angle θ_r and angle of internal friction θ_f for fine solids.



Angles of repose of noncohesive material. (*U.S. Bureau of Reclamation.*)

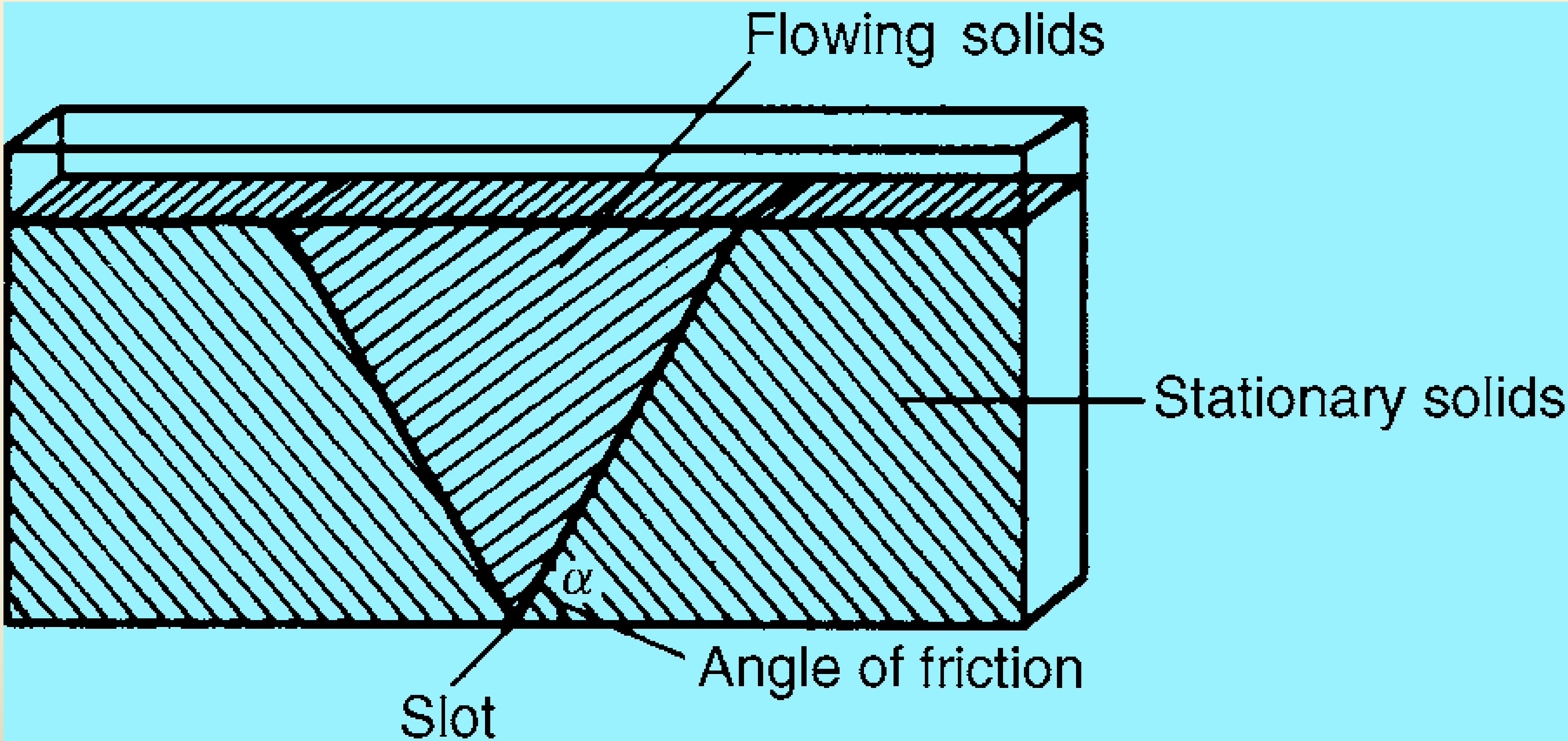
Fine sand assumes a shallower angle of repose than angular pebbles do

Angles of repose and of friction

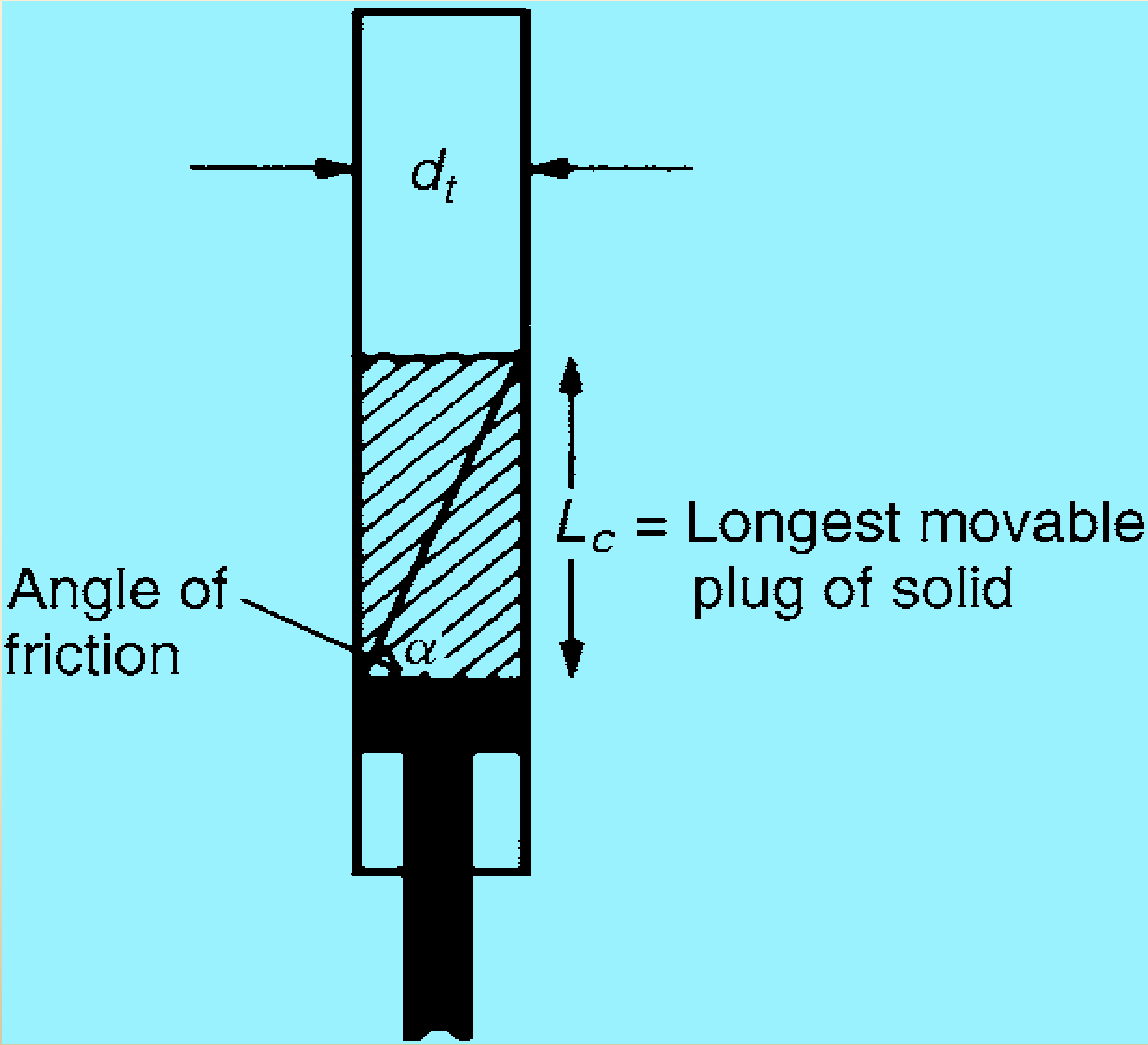


In general the range of repose angle is
20° - 60°
F.F P.F
90° highly agglomerated solid

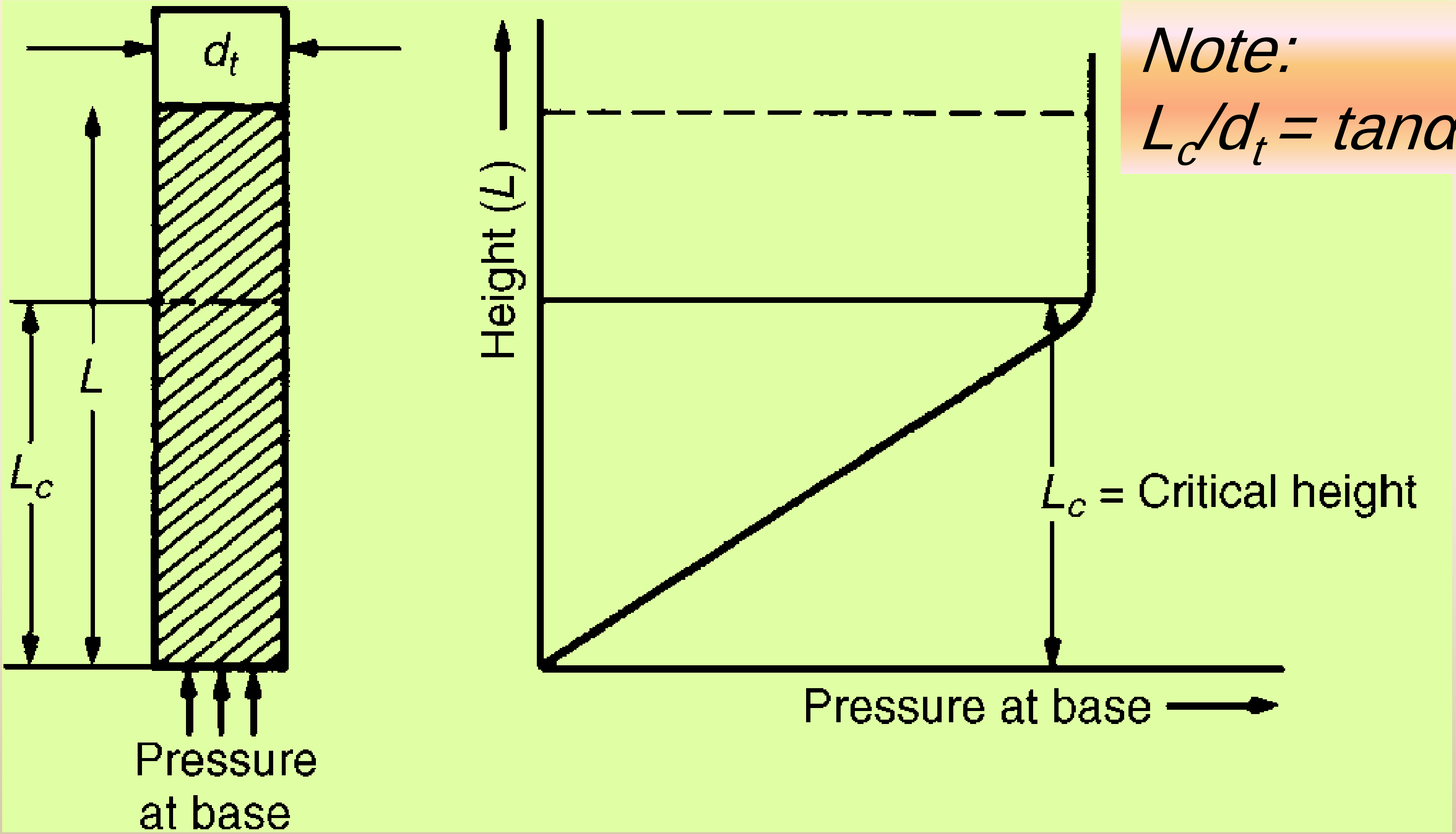
Angle of friction—flow through slot



Angle of friction—tube test



Angle of friction—pressure at base of column



Note:
$L_c/d_t = \tan\alpha$

Notes

Coefficient of internal friction is the measure of resistance present when one layer of solids over another layer of same particles. It is defined as:

Coeff. Of internal friction =**tan α_m**

For free flowing material α_m is between 15^0 to 30^0

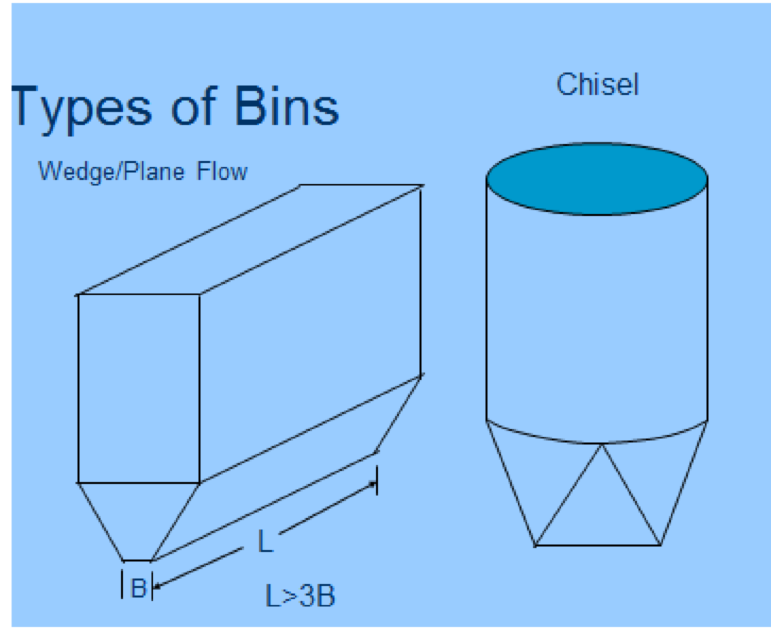
Coefficient of external friction is the measure of the resistance at an interface between particles and the wall of different material of construction.

Coefficient of External friction = tan α_s

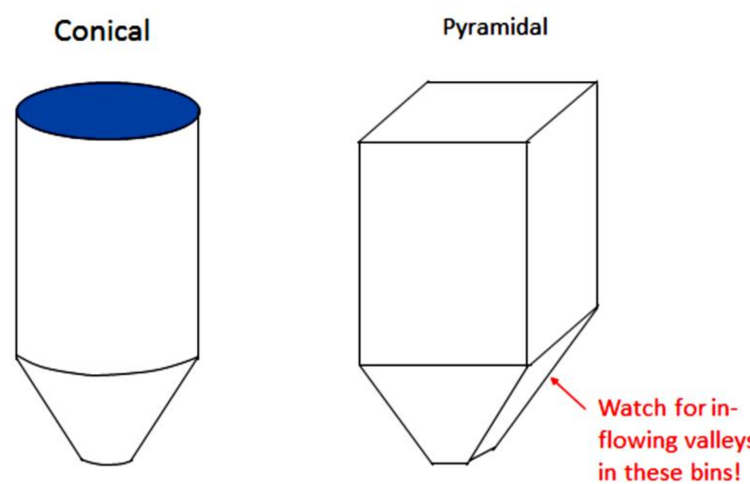
Where α_s is the angle of external friction of solid and material of construction "wall of vessel".

Storage of solid Particulates

- Bulk storage: coarse large quantity solids like gravel and coal outside in large piles.
- Protection storage:
  - Silos –tall and small diameter
  - Bins –fairly wide and Not tall
  - Hoppers –Small vessel with sloping bottom, generally temporary storage before feeding solids to a process.

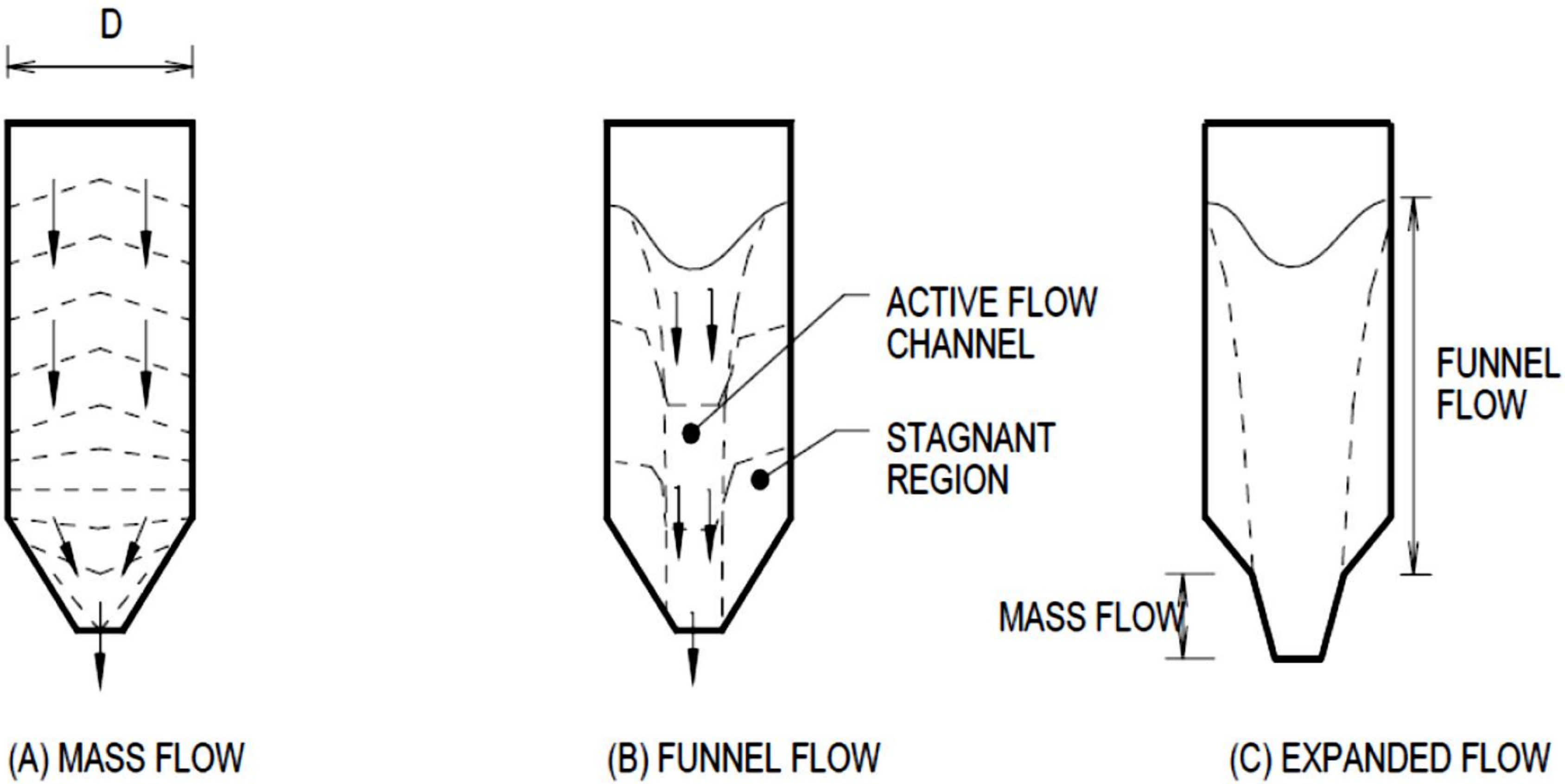


Types of Bins
Chisel
Wedge/Plane Flow
L
B
L>3B



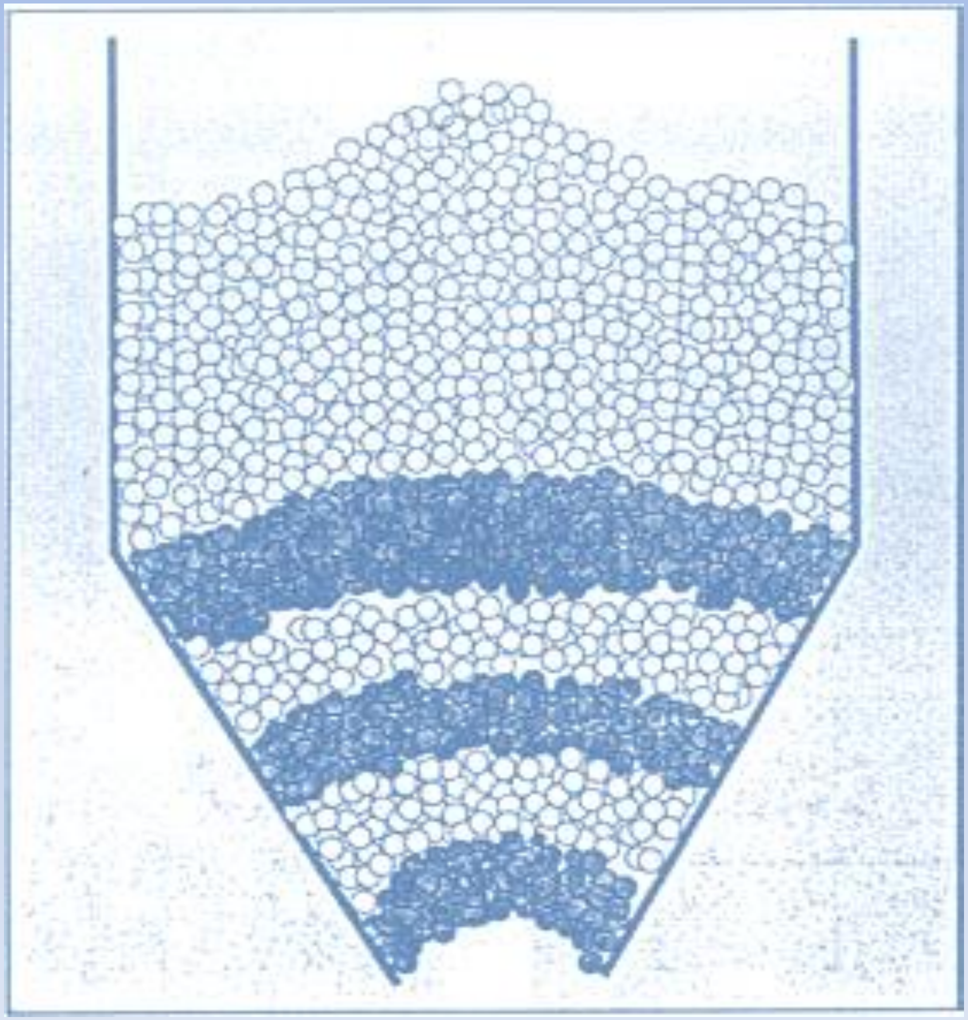
Conical
Pyramidal
Watch for in-flowing valleys in these bins!

Flow of solids in hoppers



In mass flow (A) all material moves in the bin including near the walls. In funnel flow (B) the material moves in a central core with stagnant material near the walls. Expanded flow (C) is a combination of mass flow in the hopper exit and funnel flow in the bin above the hopper

Arch formation
Flow of solids in hoppers

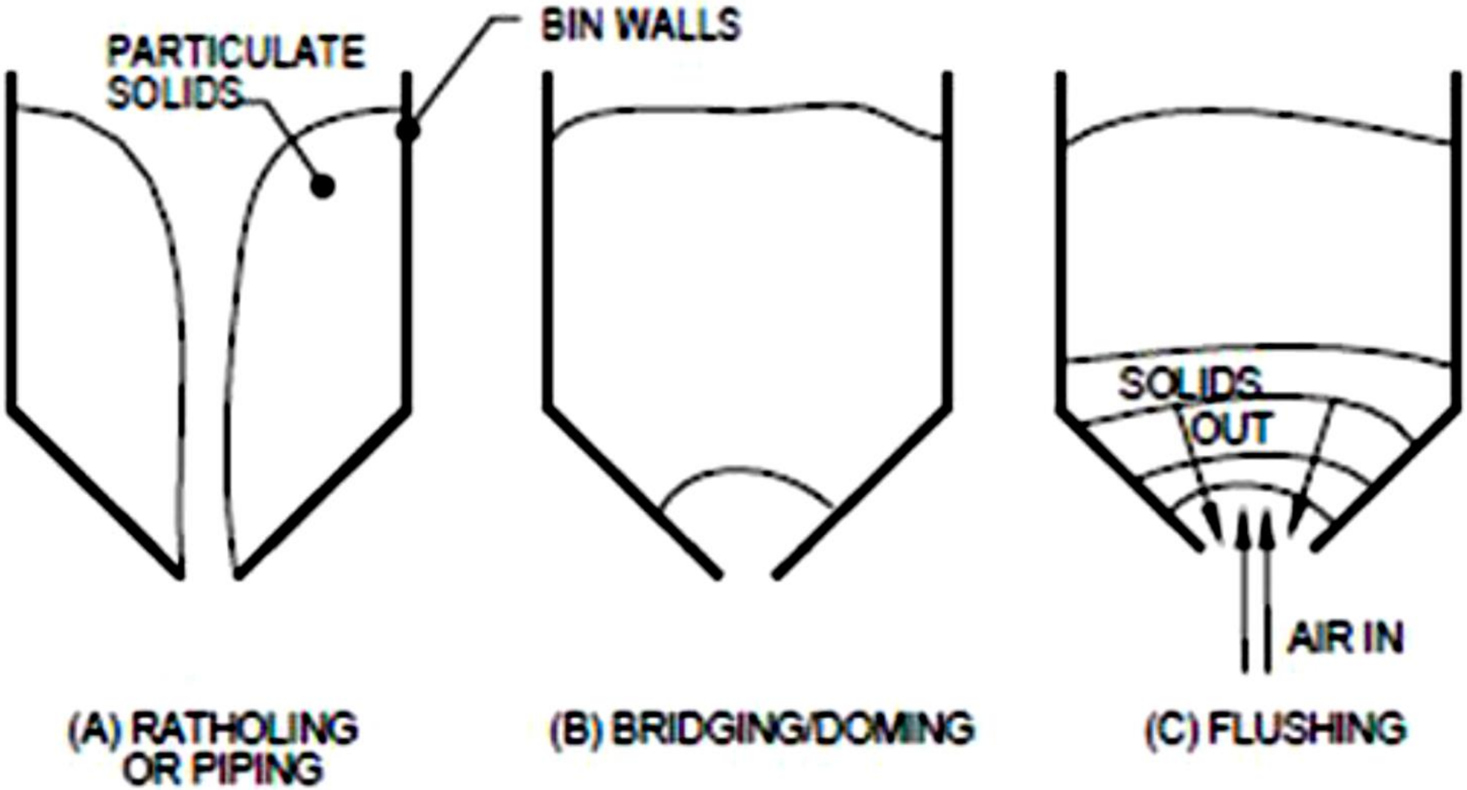


HOPPER DESIGN PROBLEMS

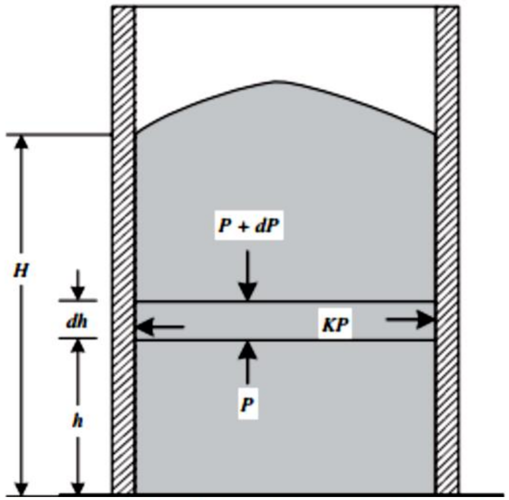
- ***RATHOLING/PIPING***
- ***FLOW IS TOO SLOW***
- ***NO FLOW DUE TO ARCHING OR DOMING***
- ***FLUSHING***. Flushing occurs when the material is not cohesive enough to form a stable dome, but strong enough that the material discharge rate slows down while air tries to penetrate into the packed material to loosen up some of the material. The resulting effect is a sluggish flow of solids as the air penetrates in a short distance freeing a layer of material and the process starts over with the air penetrating into the freshly exposed surface of material.

- I*NCOMPLETE EMPTYING*.
- ***SEGREGATION.*** Different size and density particles tend to segregate due to vibrations and a percolation action of the smaller particles moving through the void space between the larger particles.
- ***TIME CONSOLIDATION.*** For many materials, if allowed to sit in a hopper over a long period of time the particles tend to rearrange themselves so that they become more tightly packed together. The consolidated materials are more difficult to flow and tend to bridge or rat hole.
- ***CAKING****.* Caking refers to the physiochemical bonding between particles what occurs due to changes in humidity. Moisture in the air can react with or dissolve some solid materials such as cement and salt. When the air humidity changes the dissolved solids re-solidify and can cause particles to grow together.

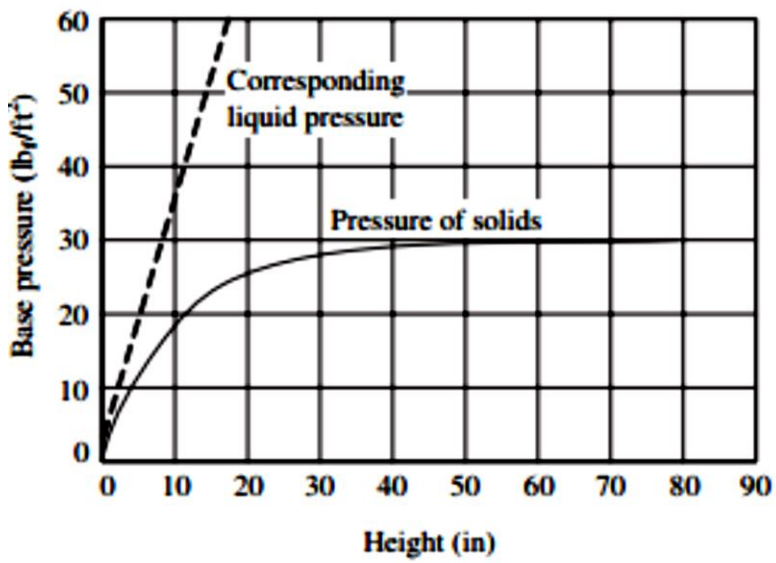
HOPPER DESIGN PROBLEMS



Pressure at the base of a vertical bin filled with particulate solids



A Vertical bin filled with particulate solids



The Janssen equation for the vertical pressure *P* on dependence of the depth *H* below the bulk solids top level reads as follows for a cylindrical silo:

$$P = \frac{g\rho_b D}{4\mu' K'}\left[1 - e^{-\left(\frac{4\mu' K' H}{D}\right)}\right] \quad \text{...............(a)}$$

$$K' = \frac{1-\sin\alpha_s}{1+\sin\alpha_s} \quad \text{...............(b)}$$

where g is the acceleration due to gravity, ρ_b is the bulk density of solids, D is the silo diameter, μ' is the sliding friction coefficient along the wall, and K' is the ratio of the horizontal to the vertical pressure, which can be expressed as given in eq. b above. where α_s is the angle of internal friction of solids.

Flow of solids through orifices

$$G = \frac{\pi}{4}\rho_s d_{eff}^{2.5} g^{0.5}\left(\frac{1-\cos\beta}{2\sin^3\beta}\right)^{0.5}$$

where: *G is the mass flow rate,*

ρ_s is the density of the solid particles,

*d_{eff} is the effective diameter of the orifice (orifice - 1.25*particle diameter),*

g is the acceleration due to gravity, and

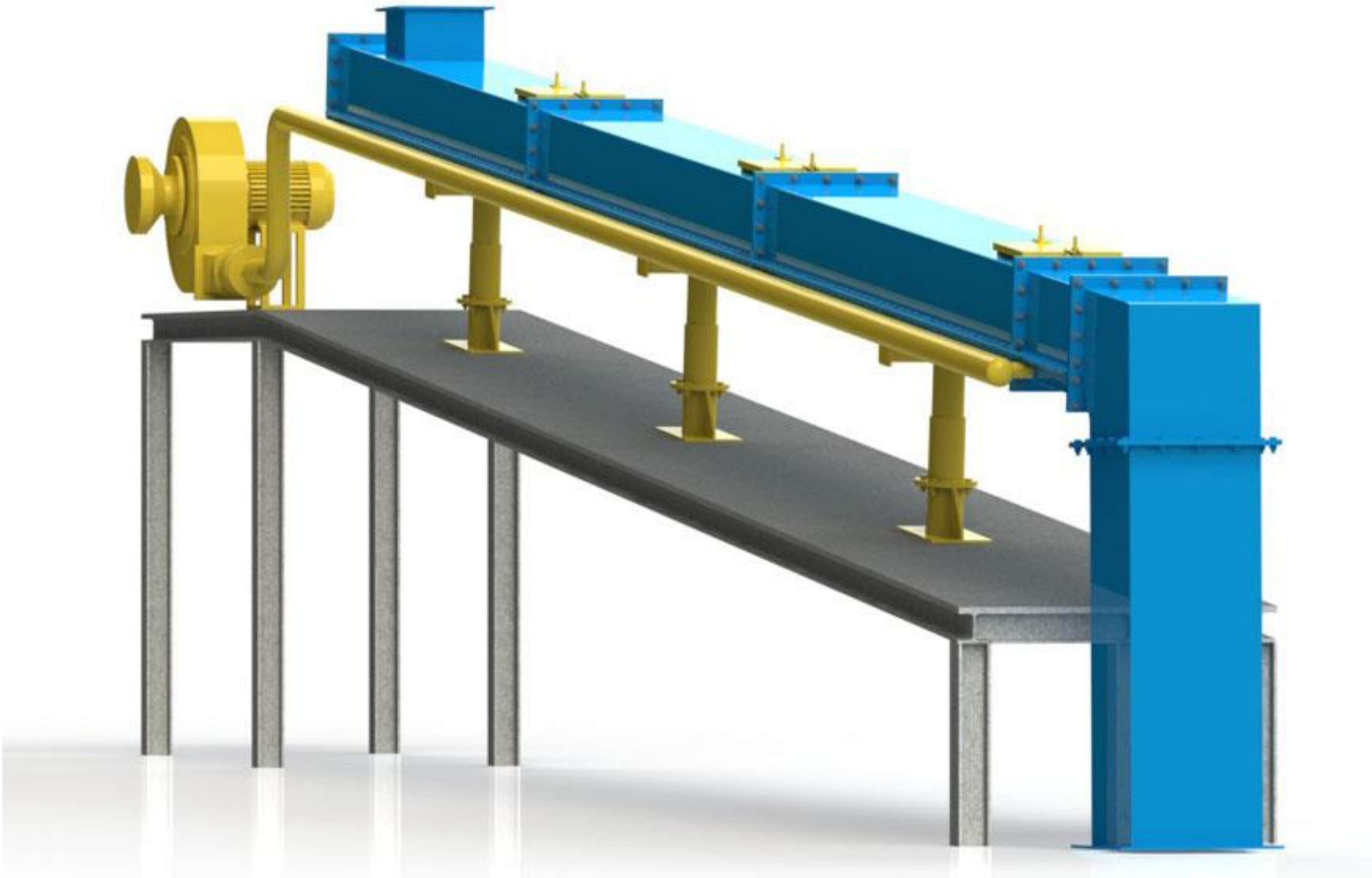
β is the acute angle between the cone wall and the horizontal.

Conveying of solids

- ***Gravity chutes*** *—down which the solids fall under the action of gravity.*
- ***Air slides*** *—where the particles, which are maintained partially suspended in a* channel by the upward flow of air through a porous distributor, flow at a small angle to the horizontal.

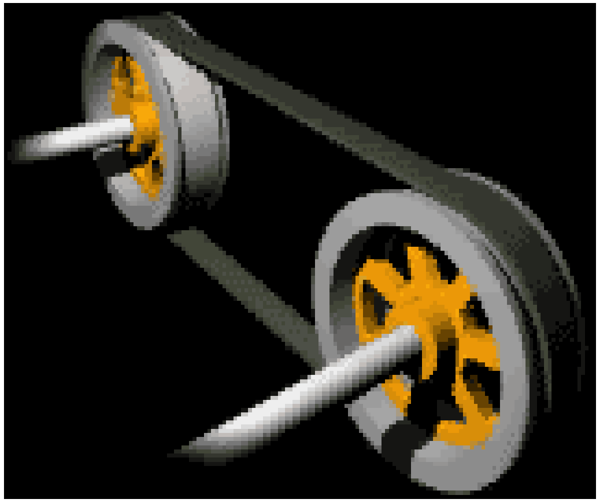
Gravity Chute

Air slide



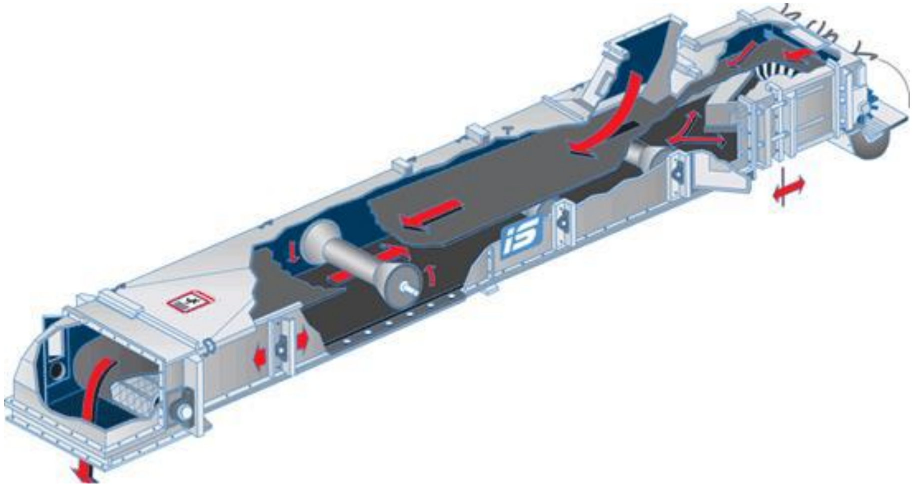
- ***Belt conveyors*** *—where the solids are conveyed horizontally, or at small angles to* the horizontal, on a continuous moving belt.
- ***Screw conveyors*** *—in which the solids are moved along a pipe or channel by a* rotating helical impeller, as in a screw lift elevator.
- ***Bucket elevators*** *—in which the particles are carried upwards in buckets attached* to a continuously moving vertical belt, as illustrated in the figures in the next slides.

Belt conveyor



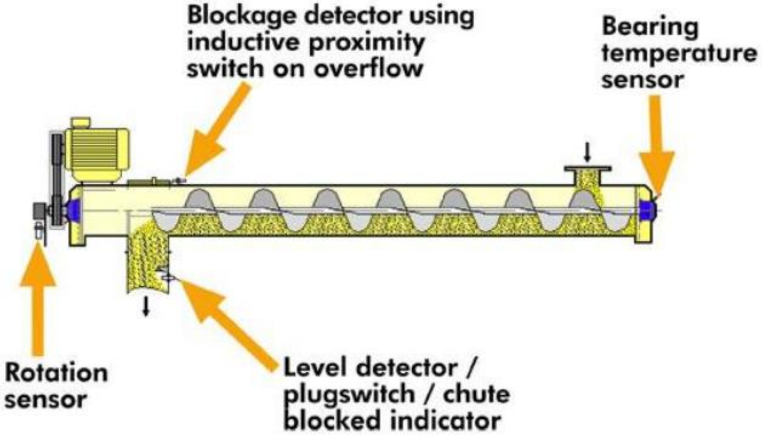
Continuous belt passing around two large pulleys at two ends, one drive pulley other tail pulley.

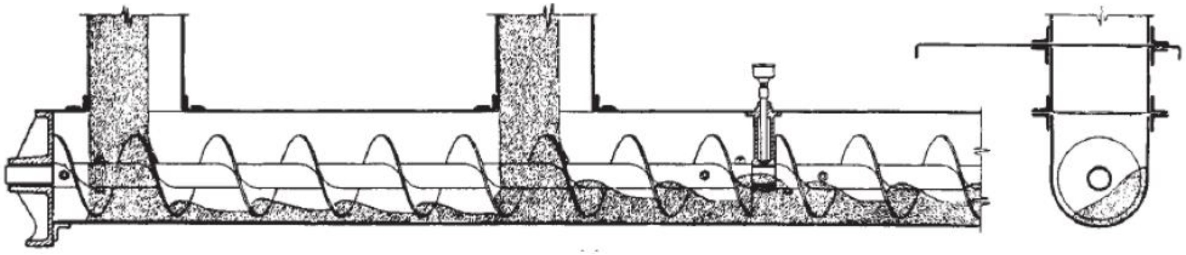
A conveyor belt uses a wide belt and pulleys and is supported by rollers or a flat pan along its path.



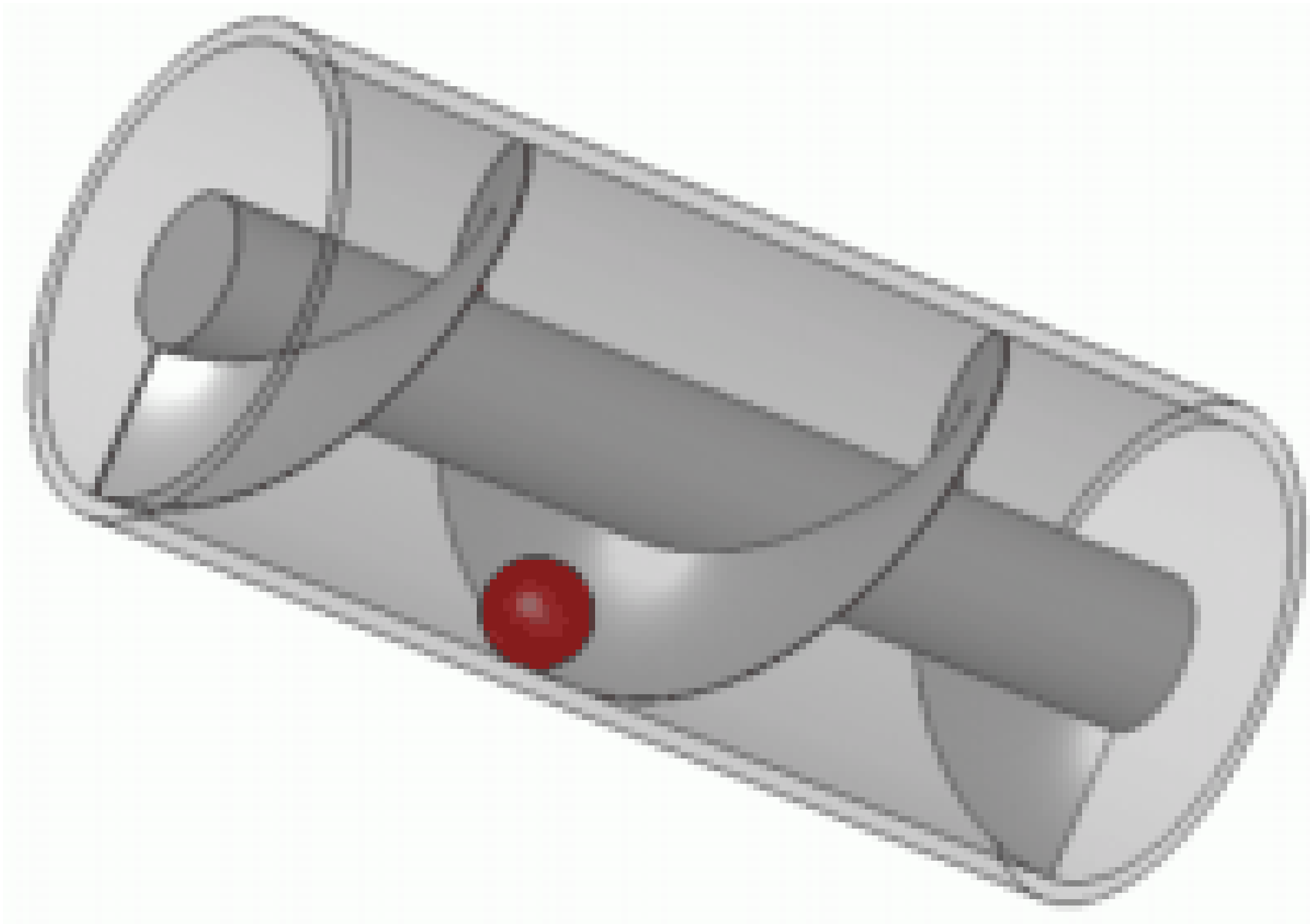
Screw conveyor

Helicoid (helix rolled from flat steel bar) or sectional flight mounted on a pipe or shaft rotating in a U shaped trough.





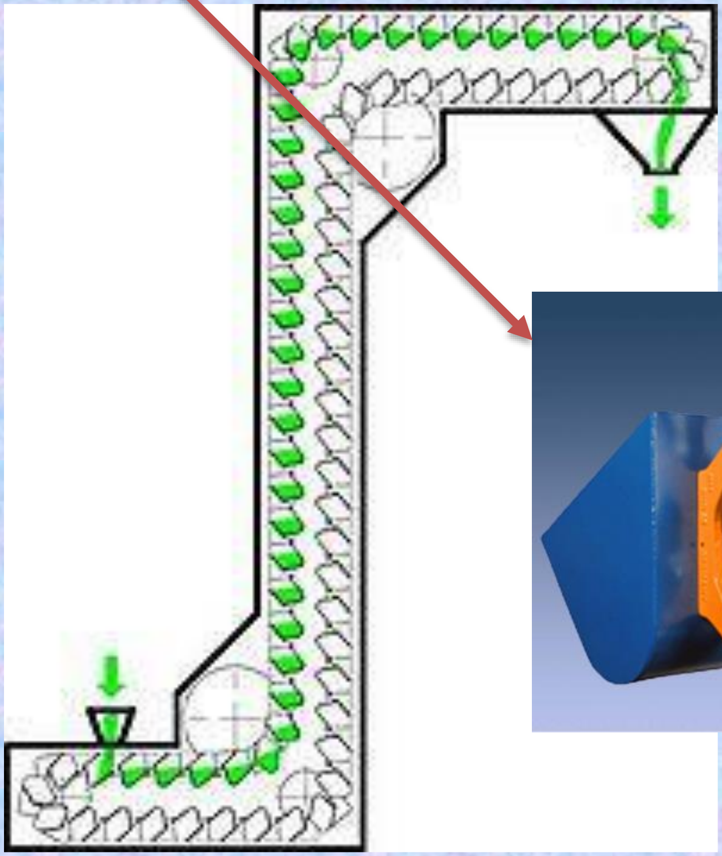
Screw conveyor

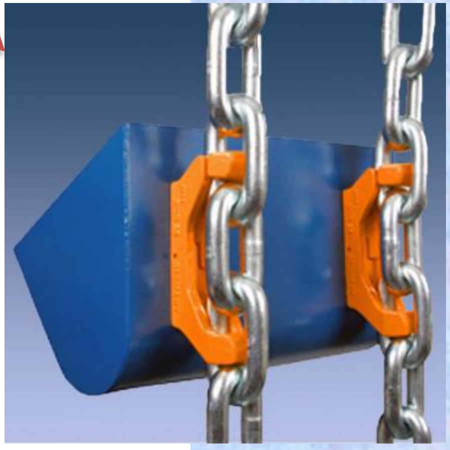


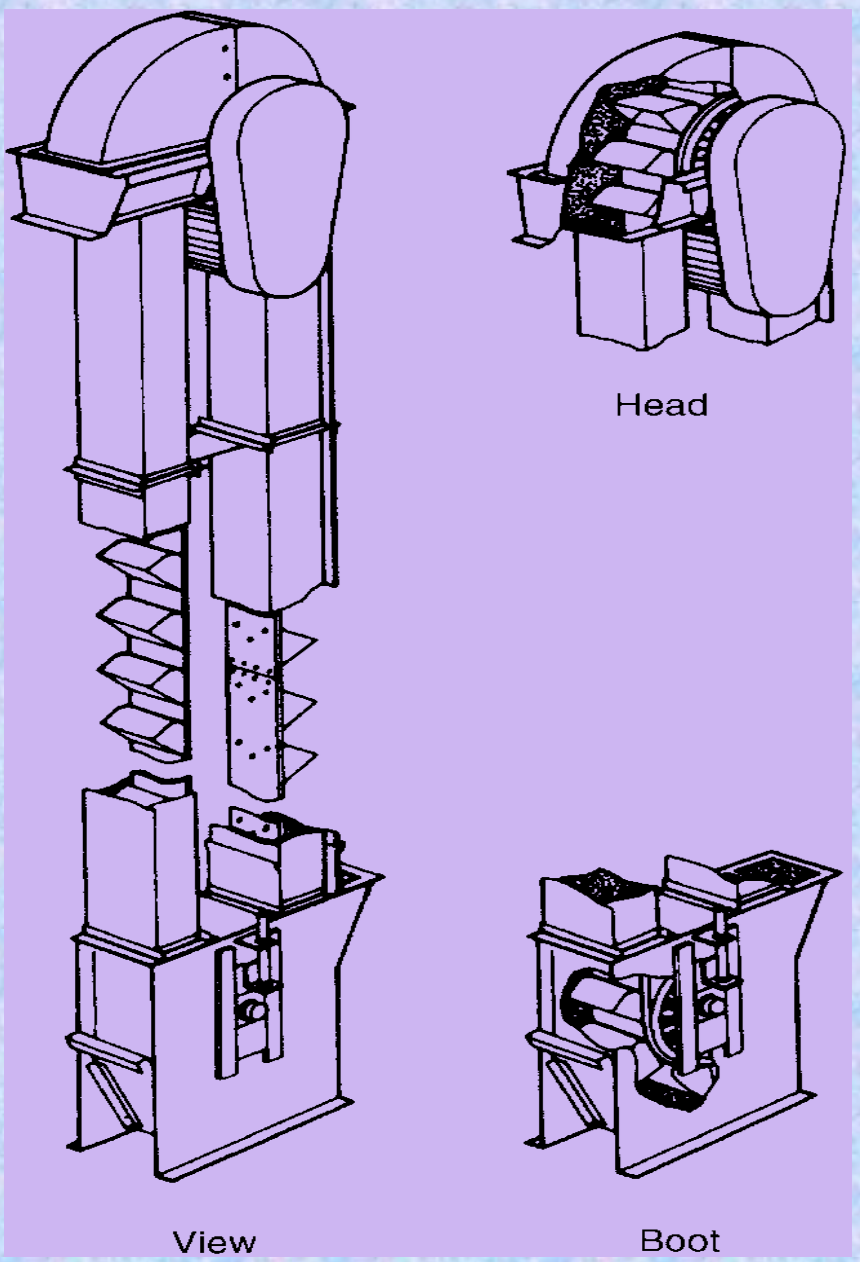
Screw conveyors

Bucket elevator

Bucket elevator consists of a number of buckets attached to a continuous double strand chain which passes over two pulleys







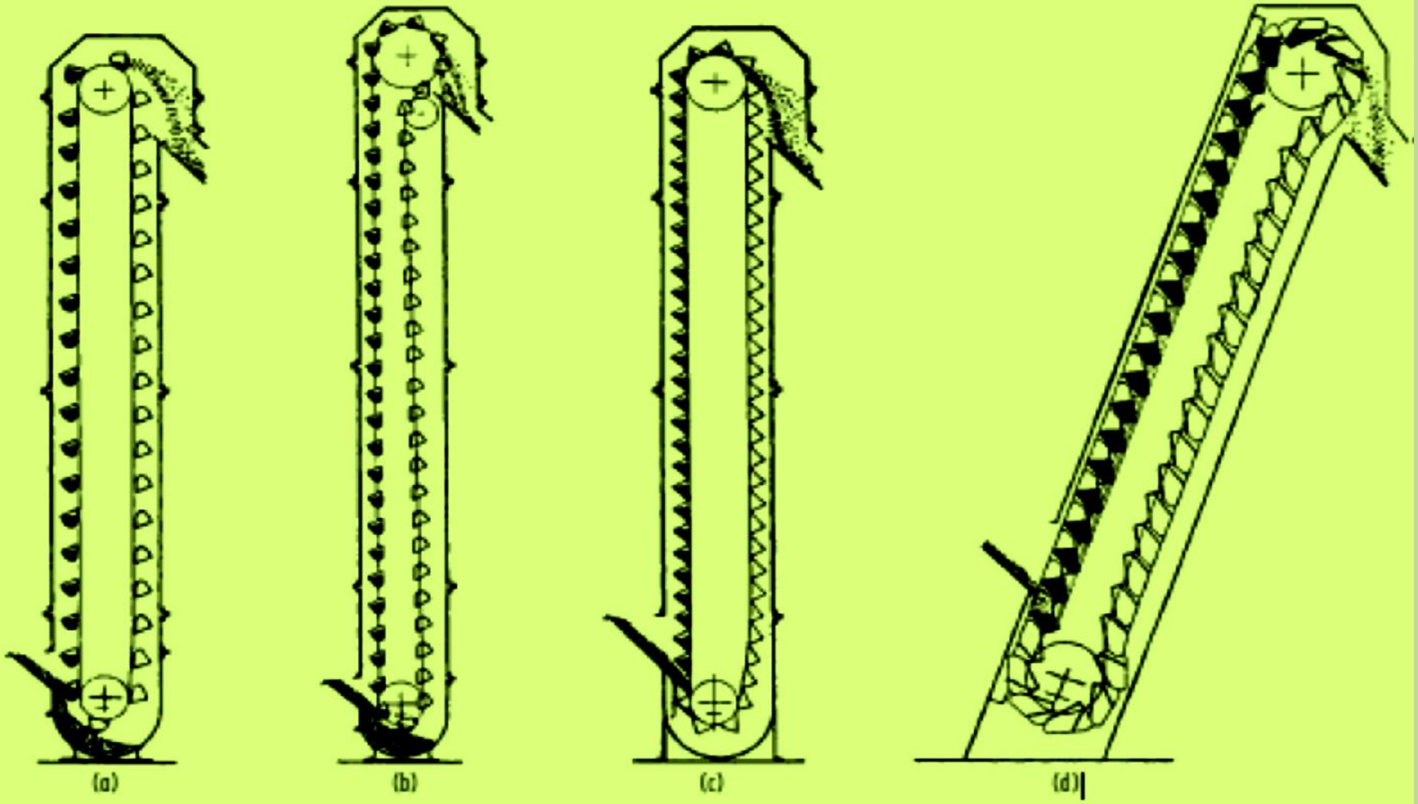
BUCKET ELEVATORS

(a)Spaced-Bucket Centrifugal-Discharge Elevators

(b)Spaced-Bucket Positive-Discharge Elevators

(c)Continuous-Bucket Elevators

(d)Supercapacity Continuous-Bucket Elevators



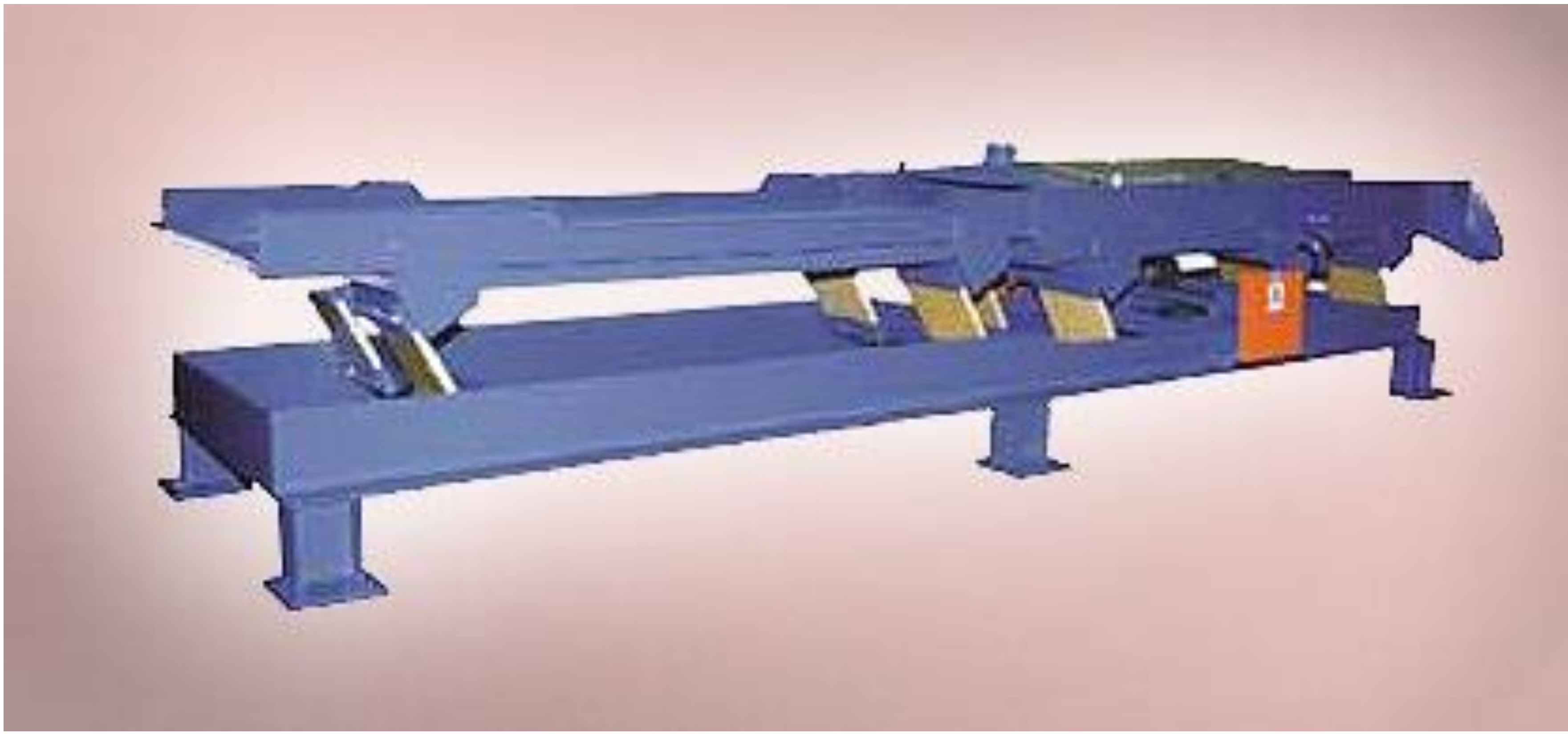
See next slide for definitions.

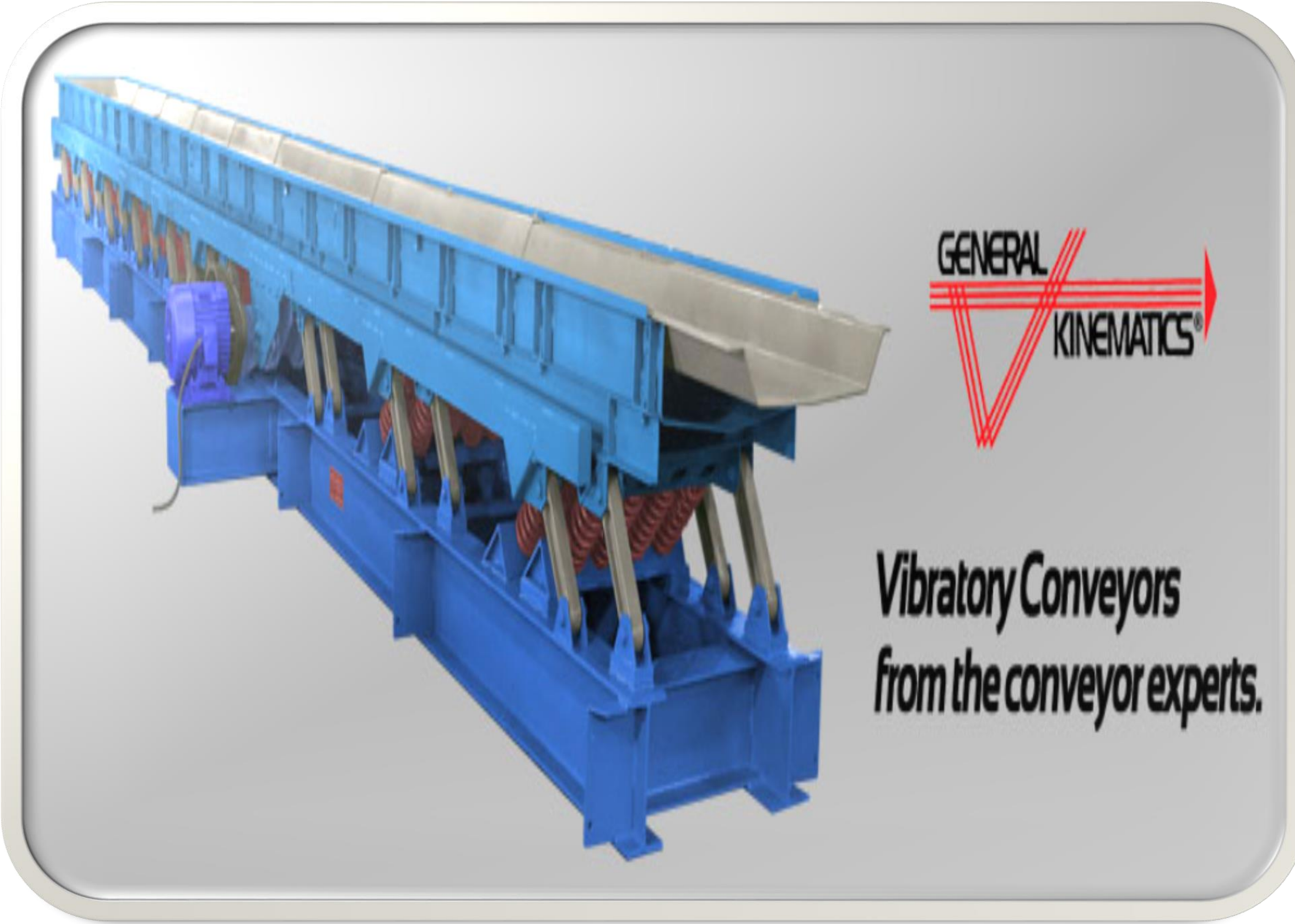
- Spaced bucket ; Centrifugal discharge –free flowing, fine or small lump material like grain, coal, sand or dry chemicals
- Spaced bucket positive discharge-buckets are for sticky materials which tend to lump, inverted for positive discharge, knockers can also be used
- Continuous –finely pulverized or fluffy materials, the back of the preceding bucket serves as a discharge chute for the bucket. Gentle movement Preventing degradation.
- Super capacity continuous bucket: Very high tonnage, big particles, Generally inclined

- ***Vibrating conveyors*** *—in which the particles are subjected to an asymmetric* vibration and travel in a series of steps over a table. During the forward stroke of the table the particles are carried forward in contact with it, but the acceleration in the reverse stroke is so high that the table slips under the particles. With fine powders, vibration of sufficient intensity results in a fluid-like behaviour.

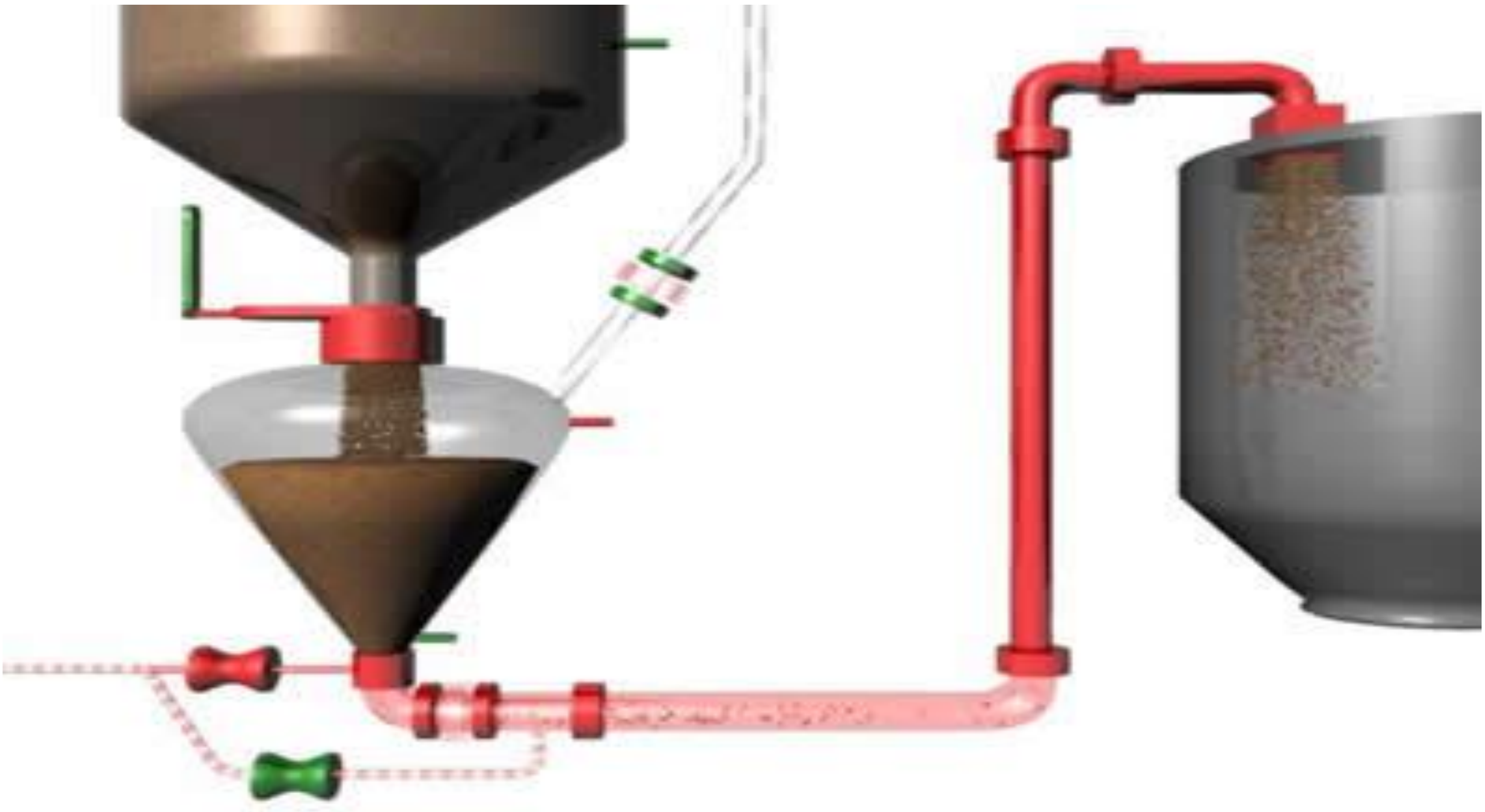
- ***Pneumatic/hydraulic*** *conveying installations—in which the particles are transported* in a stream of air/water.

Vibratory conveyor

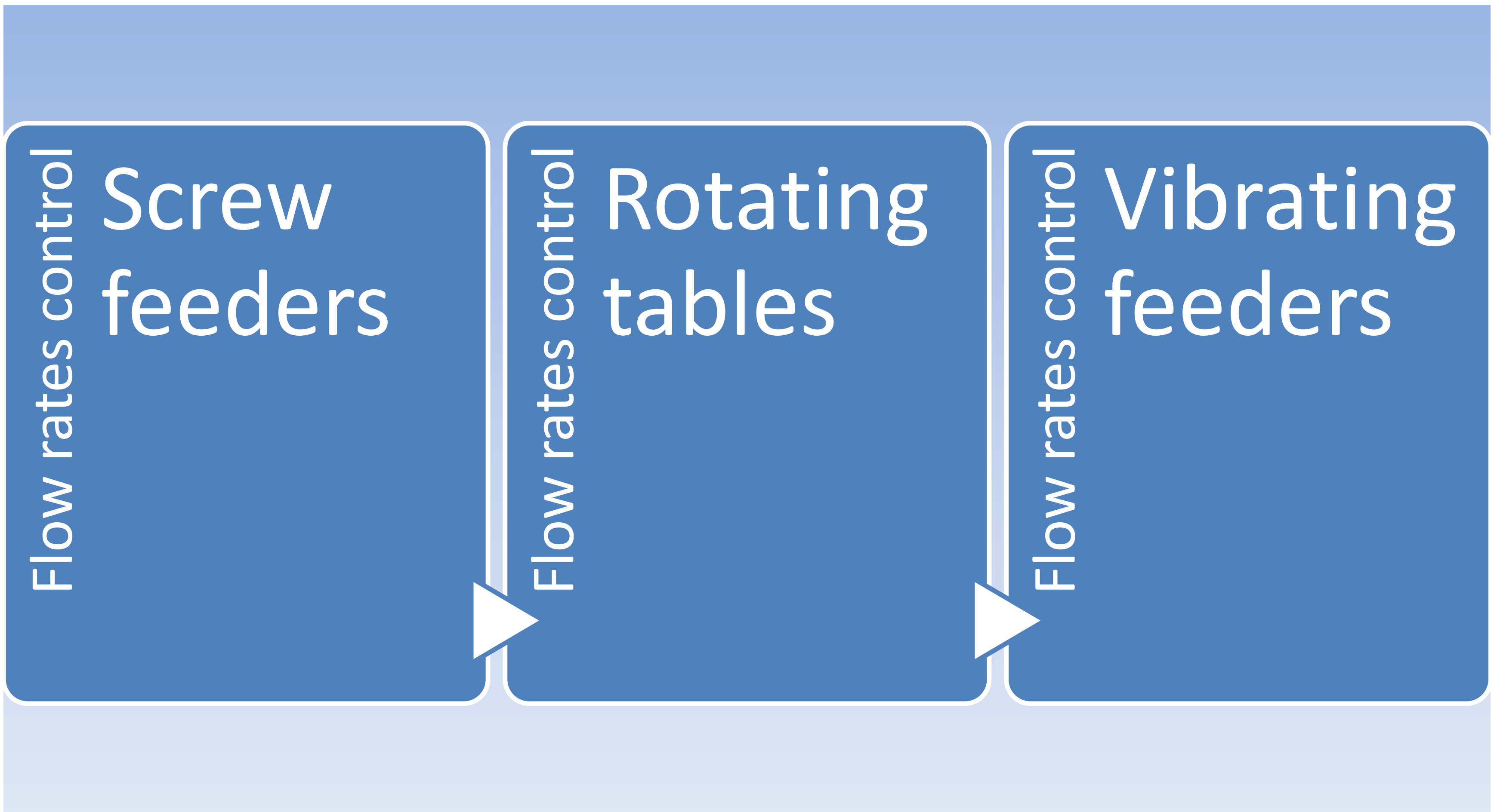



GENERAL KINEMATICS®
Vibratory Conveyors
from the conveyor experts.

Pneumatic or hydraulic conveyor



Flow rates control



How to measure the flow rates

Monitoring the solids leave the hopper via

Monitoring the solids come to the conveyor

Load cell

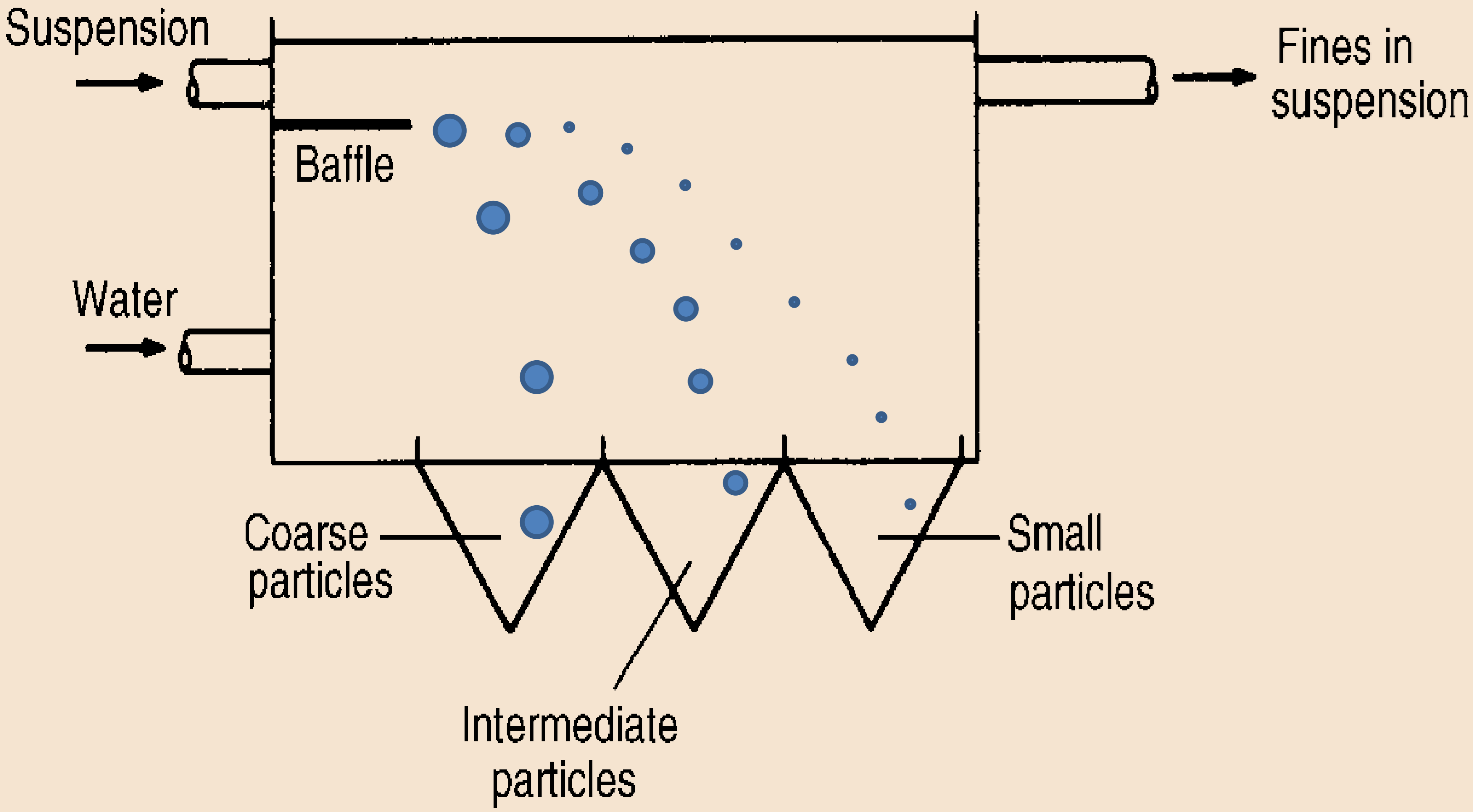
Recording the level

Continuous weighing

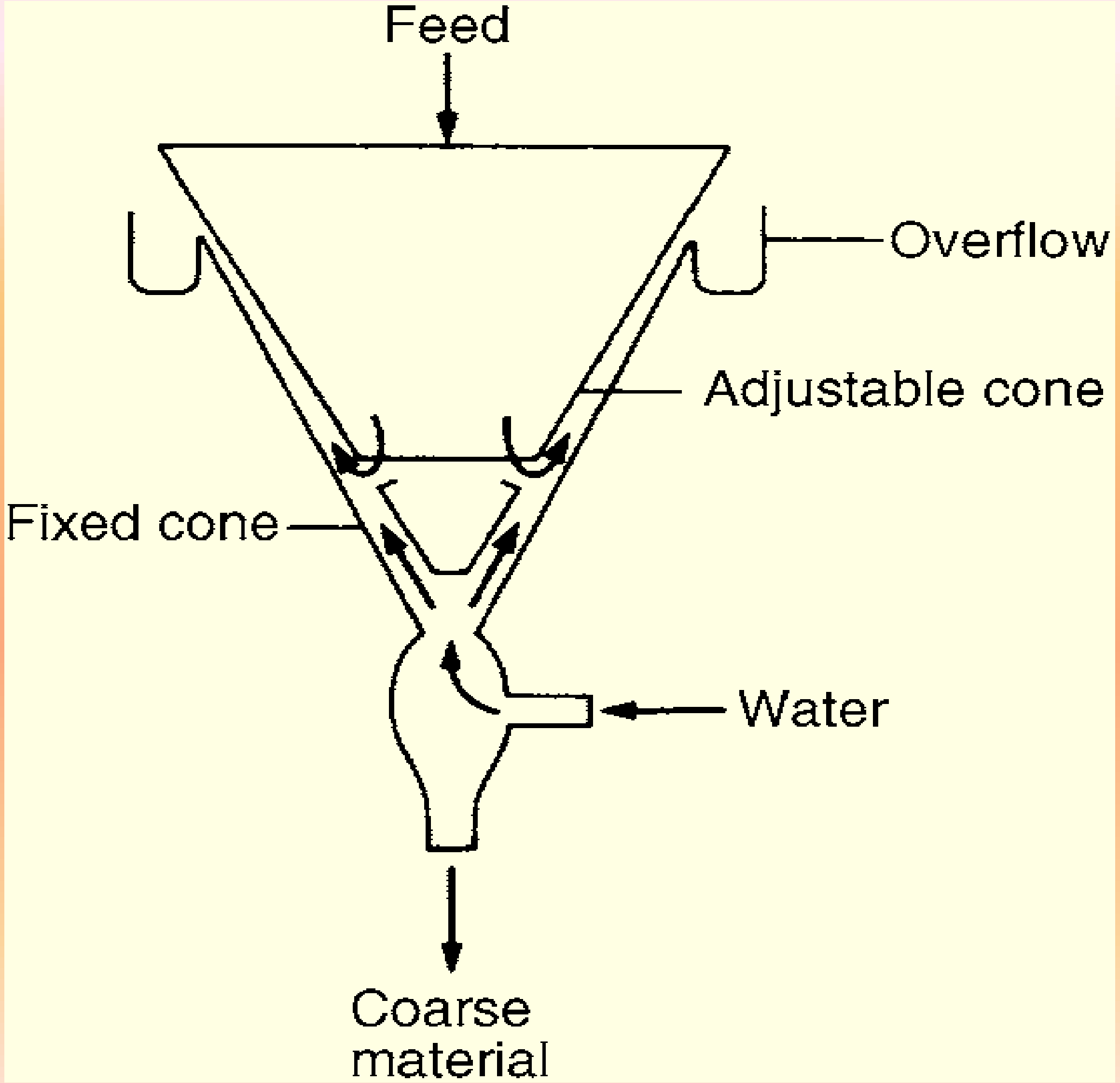
CLASSIFICATION OF SOLID PARTICLES

Fluid Separation

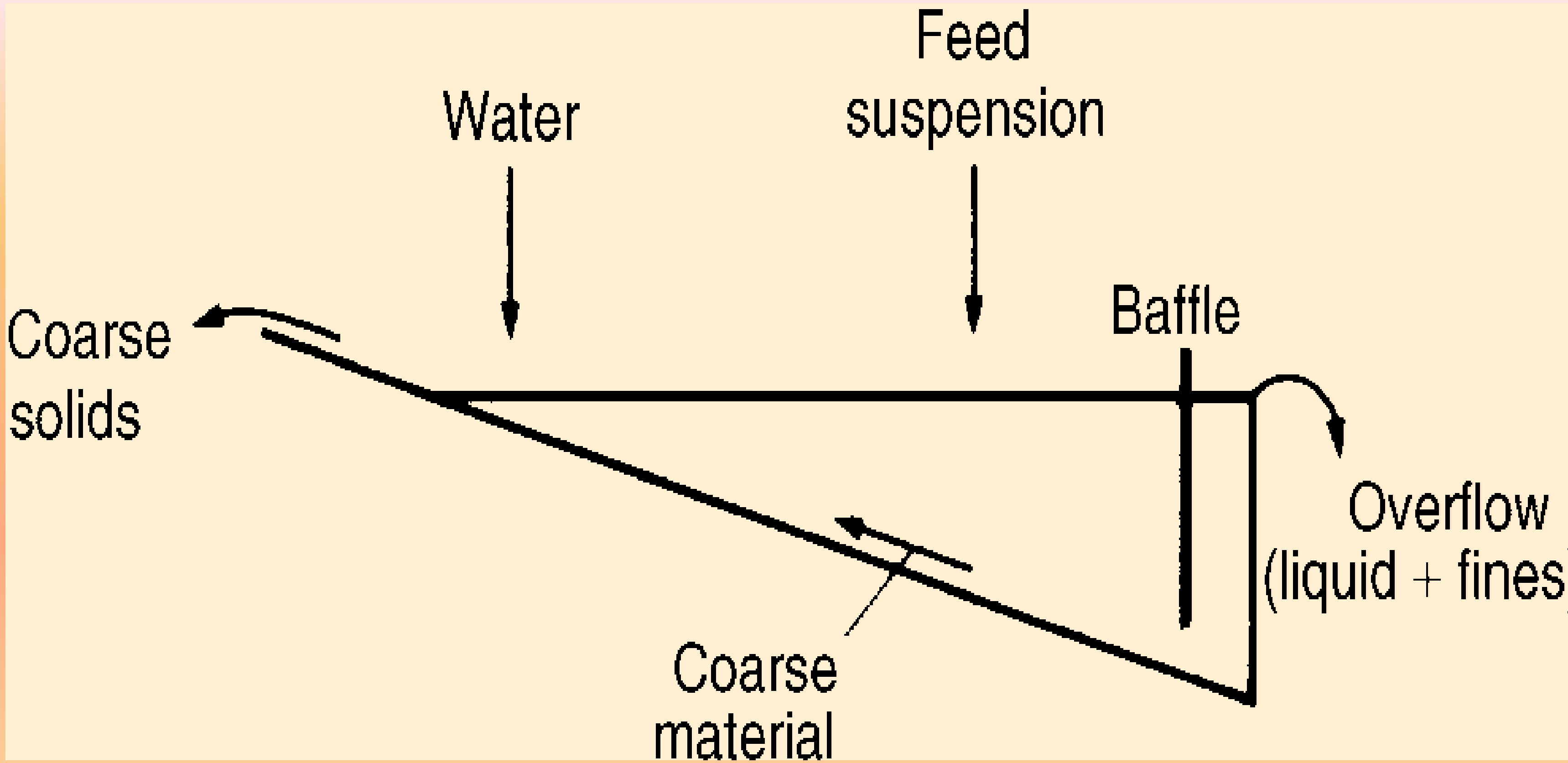
1.Gravity settling tank



2. Double cone classifier



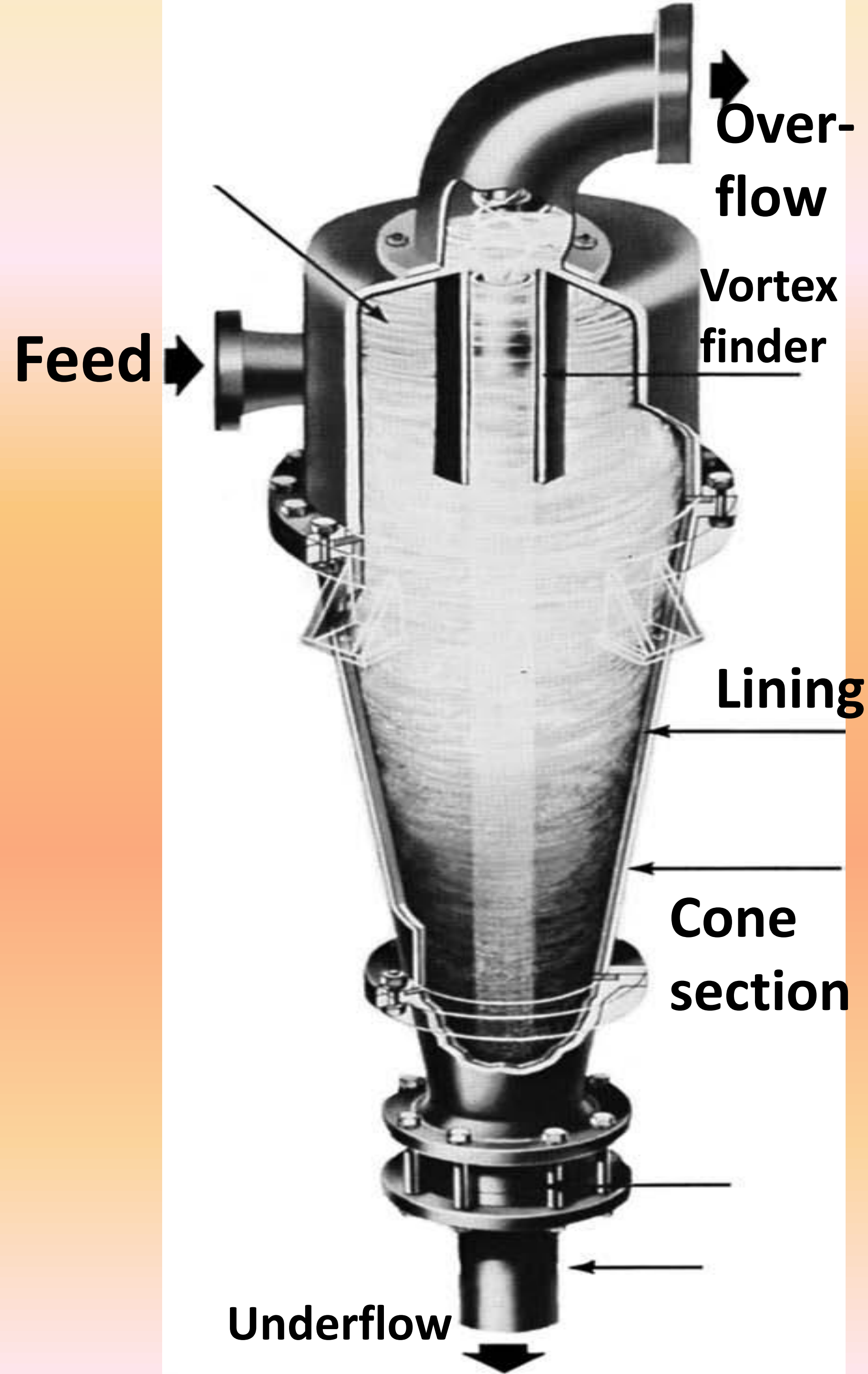
3. Mechanical classifier



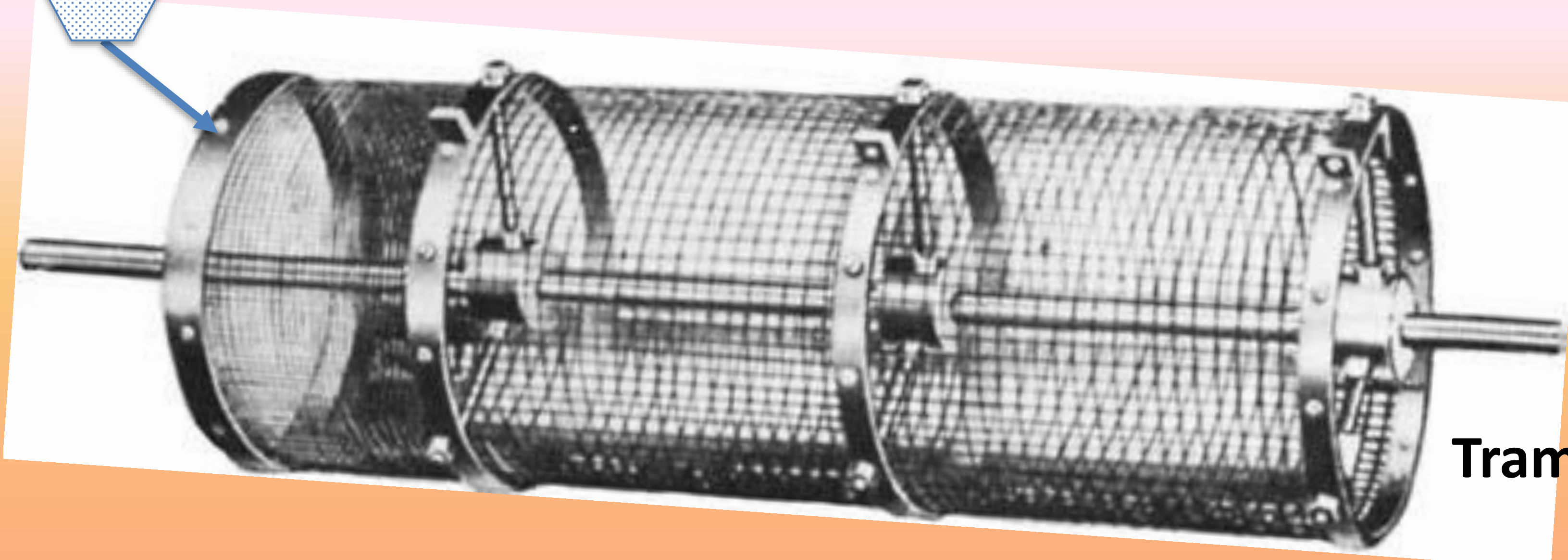
Removed by rake that moves mechanically

4. Centrifugal Separator

Liquid cyclone



5. Sieves or screens



Trammel or c

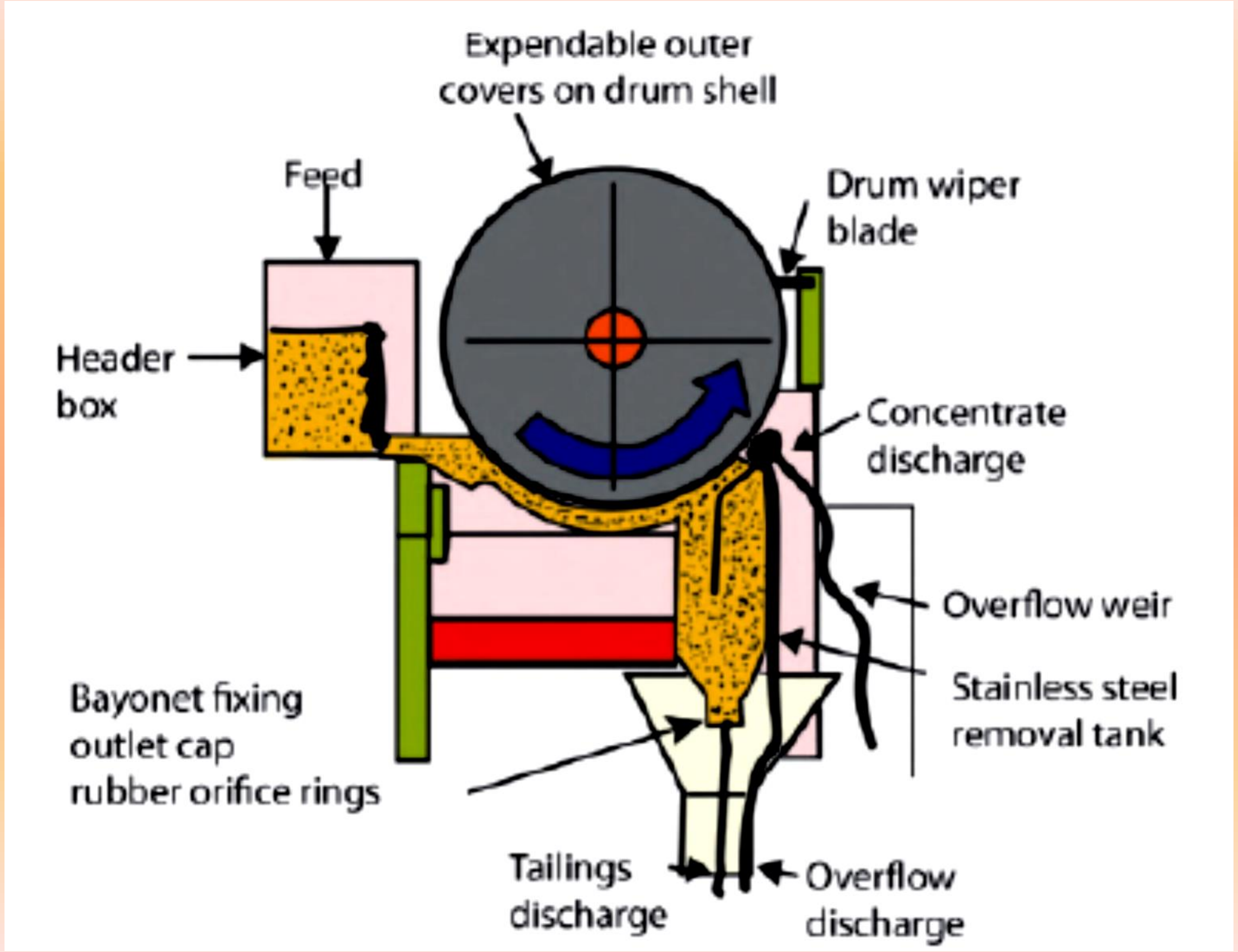
✓It consists of a slowly rotating perforated cylinder with its axis at a slight angle to the horizontal.

✓The material to be screened is fed in at the top and gradually moves down the screen and passes over apertures of gradually increasing size, with the result that all the material has to pass over the finest screen.

6. Magnetic separators

- **Eliminators (little duty)**, which are used for the removal of small quantities of *magnetic material* from the charge to a plant. These are frequently employed, for example, for the removal of stray pieces of scrap iron from the feed to crushing equipment. A common type of eliminator is a *magnetic pulley incorporated in a belt conveyor* so that the non-magnetic material is discharged in the normal manner and the magnetic material adheres to the belt and falls off from the underside.

➢**Concentrators**, which are used for the separation of magnetic ores from the accompanying mineral matter.



Principle of operation of a wet drum separator

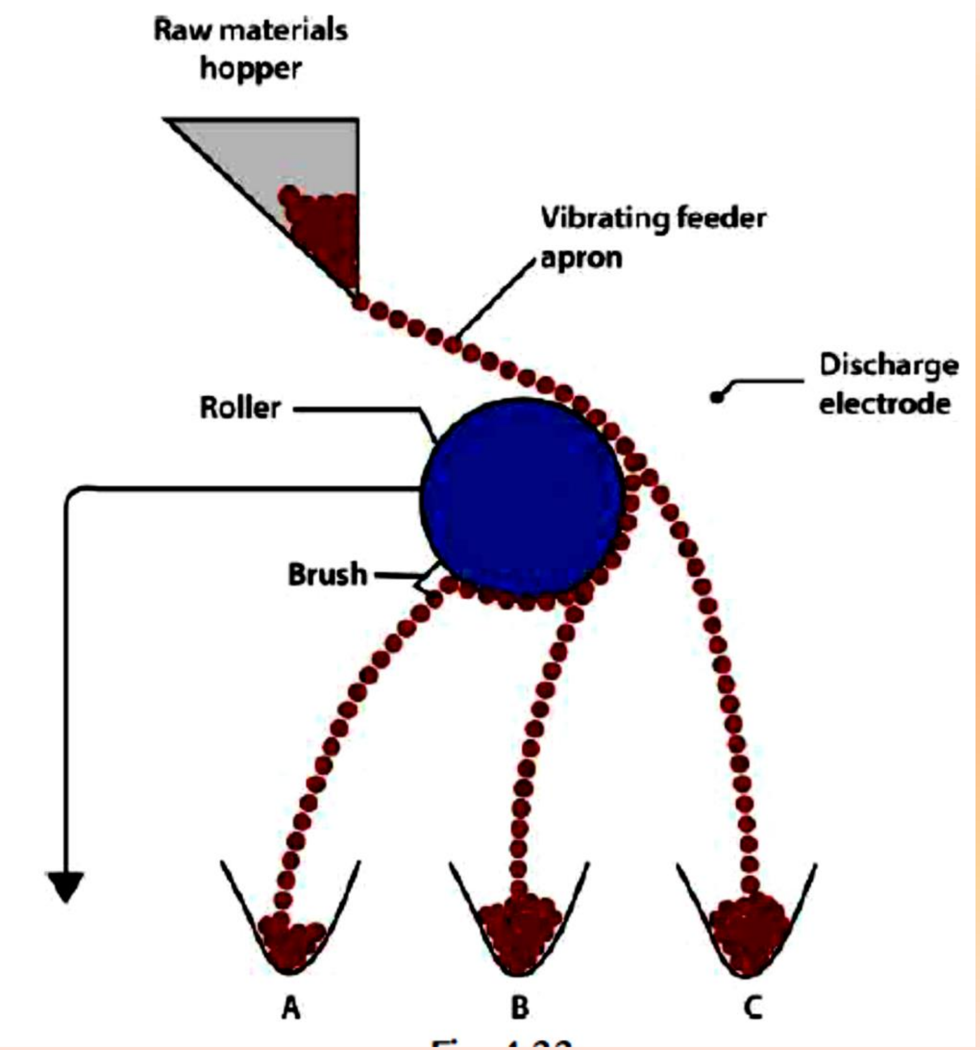
Dry or wet feeds

Electrostatic separator

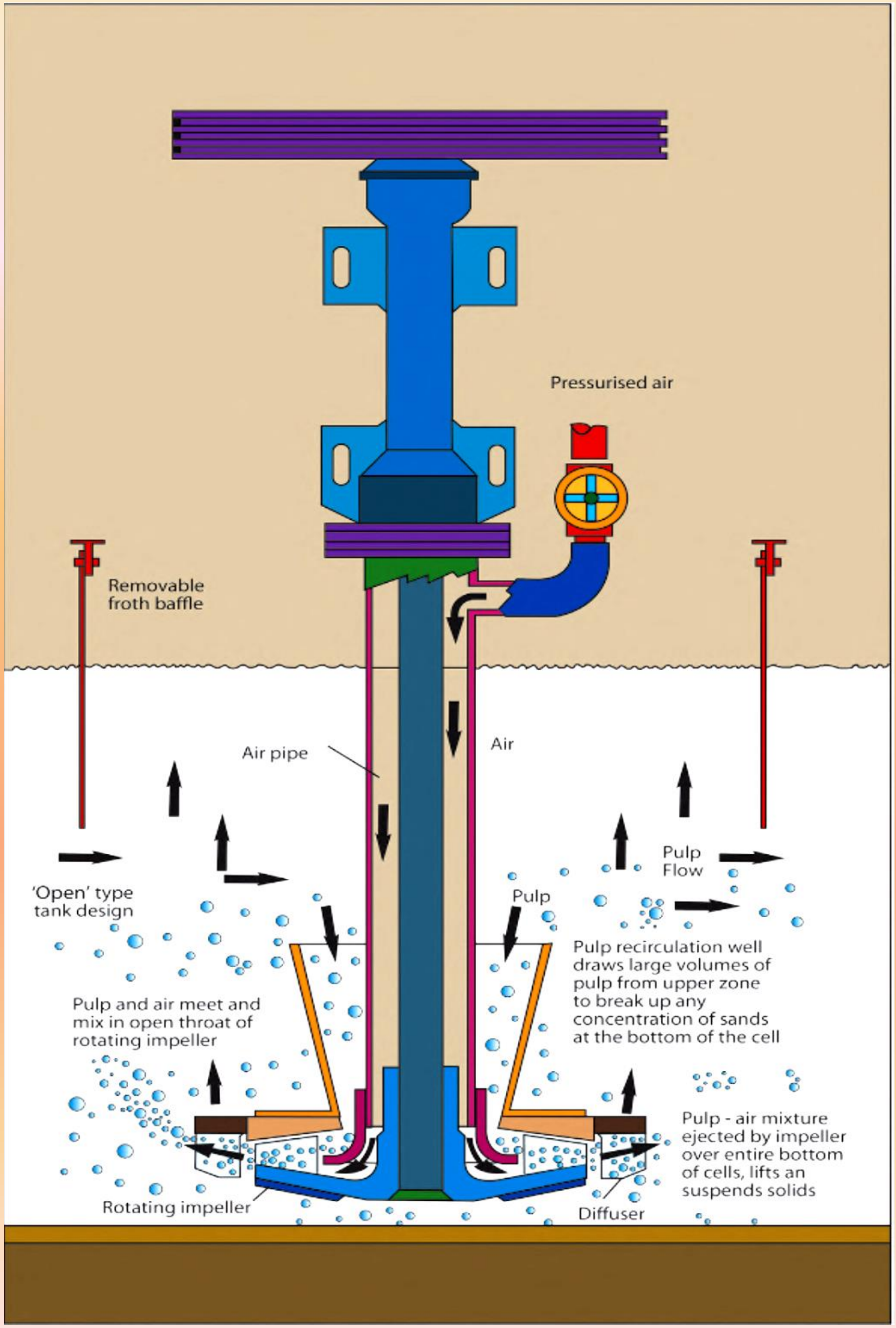
❖Differences in the electrical properties of the materials are exploited.

❖The solids are fed from a hopper on to a rotating drum, which is either charged or earthed, and an electrode bearing the opposite charge is situated at a small distance from the drum.

❖The point at which the material leaves the drum is determined by the charge it acquires.



7. Flotation



See the text for details. "self-reading".

Flotation - In Sum

- Flotation is a process in which suspensions, the particle phase of which has a specific gravity less than that of the suspending medium, are clarified by allowing the suspended material to float to the surface, where it is removed by skimming.
- In most applications the <u>effective specific gravity</u> of the suspended phase is artificially lowered by the attachment of gas bubbles. This enables the process to be used for a wide variety of suspended solids, the specific gravities of which are slightly greater than those of their suspending media.

- The **gas bubbles** required to effect flotation of solids may be generated in a number of ways, including by electrolytic means, by vacuum-activated release of dissolved gases, <u>by air injection through submerged diffusers</u> and by the dissolution of air at high pressure in part of the flow, with its subsequent release in fine bubble form on reduction of pressure to atmospheric level.
- FLOTATION AIDS. Oil emulsion; surfactants are often used (in metallurgical flotation processes) to depress the liquid surface tension and create a stable foam, which can retain particles lifted in it,

Mixing & Segregation

Mixing &Segregation

- **Introduction**
- **Objective ~ to obtain homogeneous mixture in terms: concentration or density, particle size distribution.**

  ~ To reduce differences in properties such as concentration and color, between different parts of a system
- **Mixing of solids is not an easy process.**

Example

- **Example: F.F Powders of different sizes and densities ~ lead to segregation by nature and due to differences in densities and sizes. Steal ball or disk immersed in sand particles.**

Exercise: Solid mixing against liquid mixing

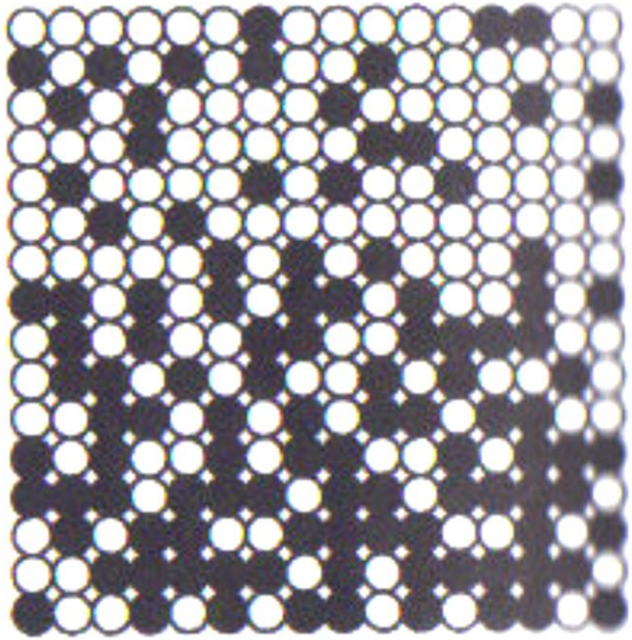
- Solid mixing is similar to liquid mixing. However, it shows some differences mainly come from that solid mixture after mixing (and sometimes during mixing) is subjected to demixing or segregation.
- The diverse characteristics of particles such as size, shape, volume, surface area, density, porosity, and flow charge contribute to the solid mixing.

Types of mixture

Perfect mixture

Random mixture

Types of mixture

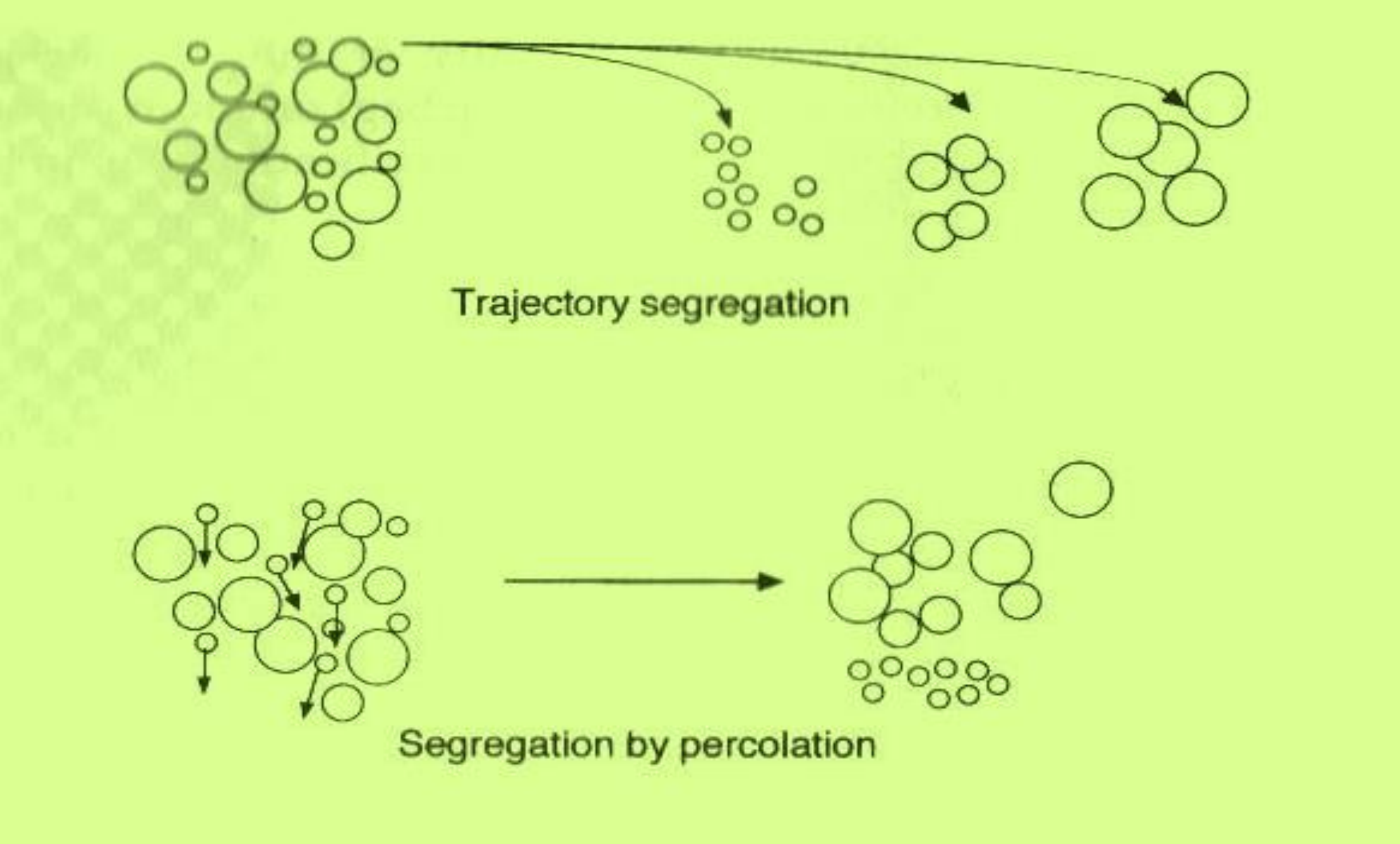


Segregating mixture

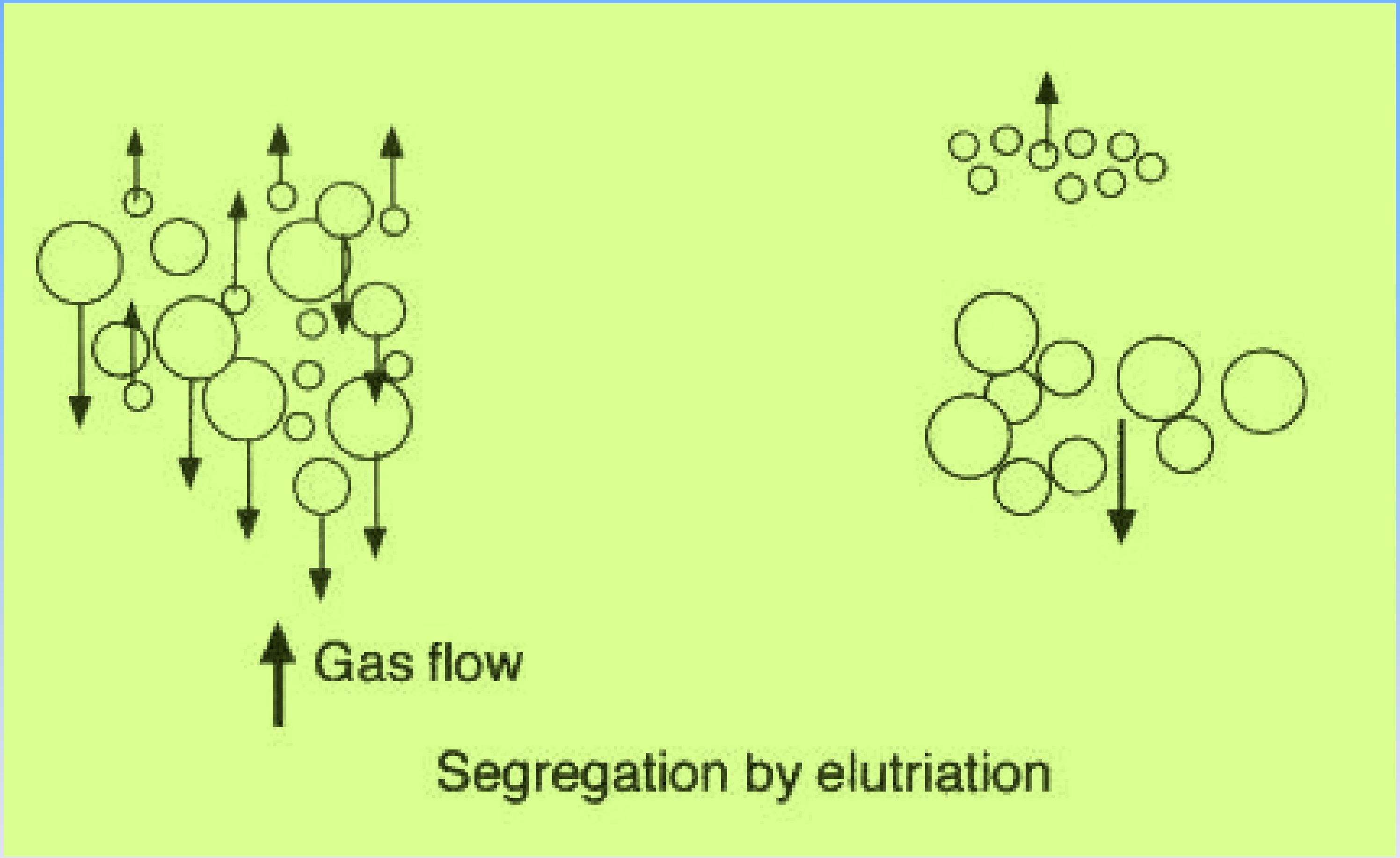
Conclusion

The best powder mixing process will result in a case of the random mix where the **probability (chance) of finding** one type of particle at any point in the mixture **is equal** to its proportion in the mixture.

Mechanism of Segregation



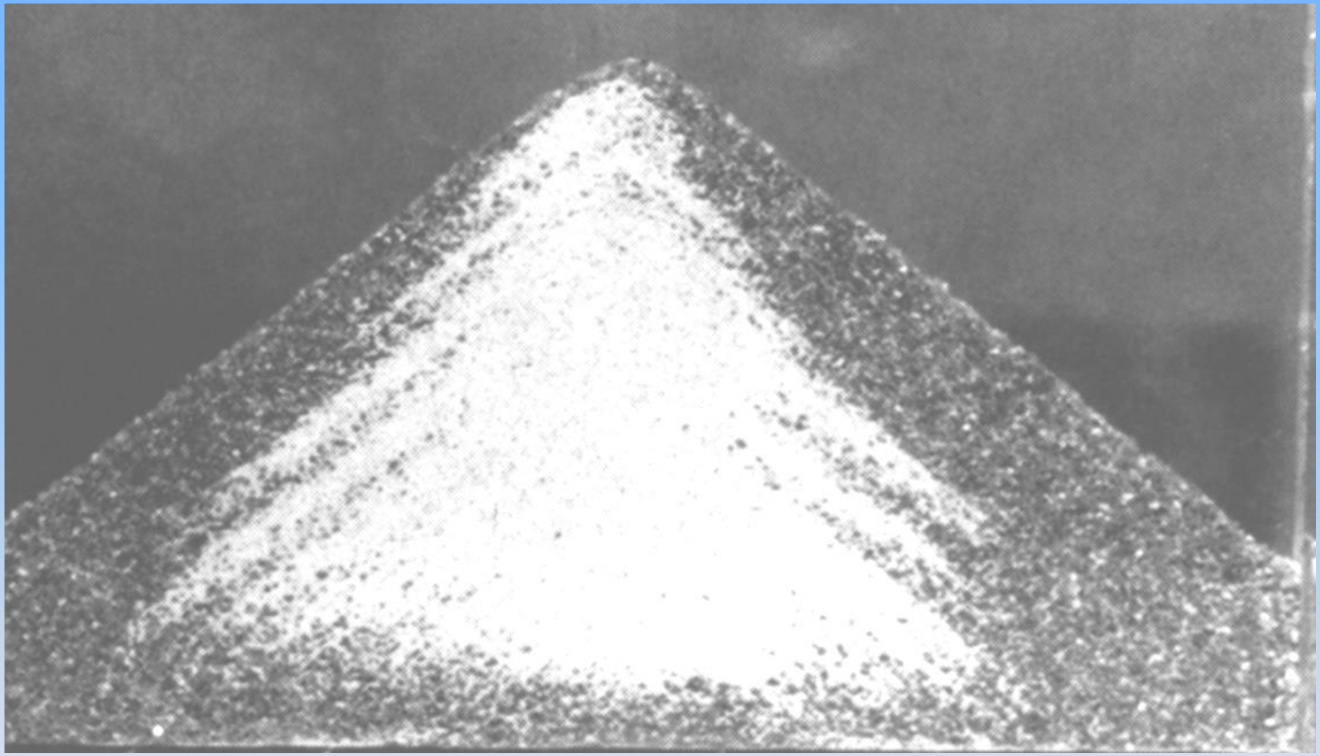
Mechanism of Segregation



Note _ Trajectory segregation

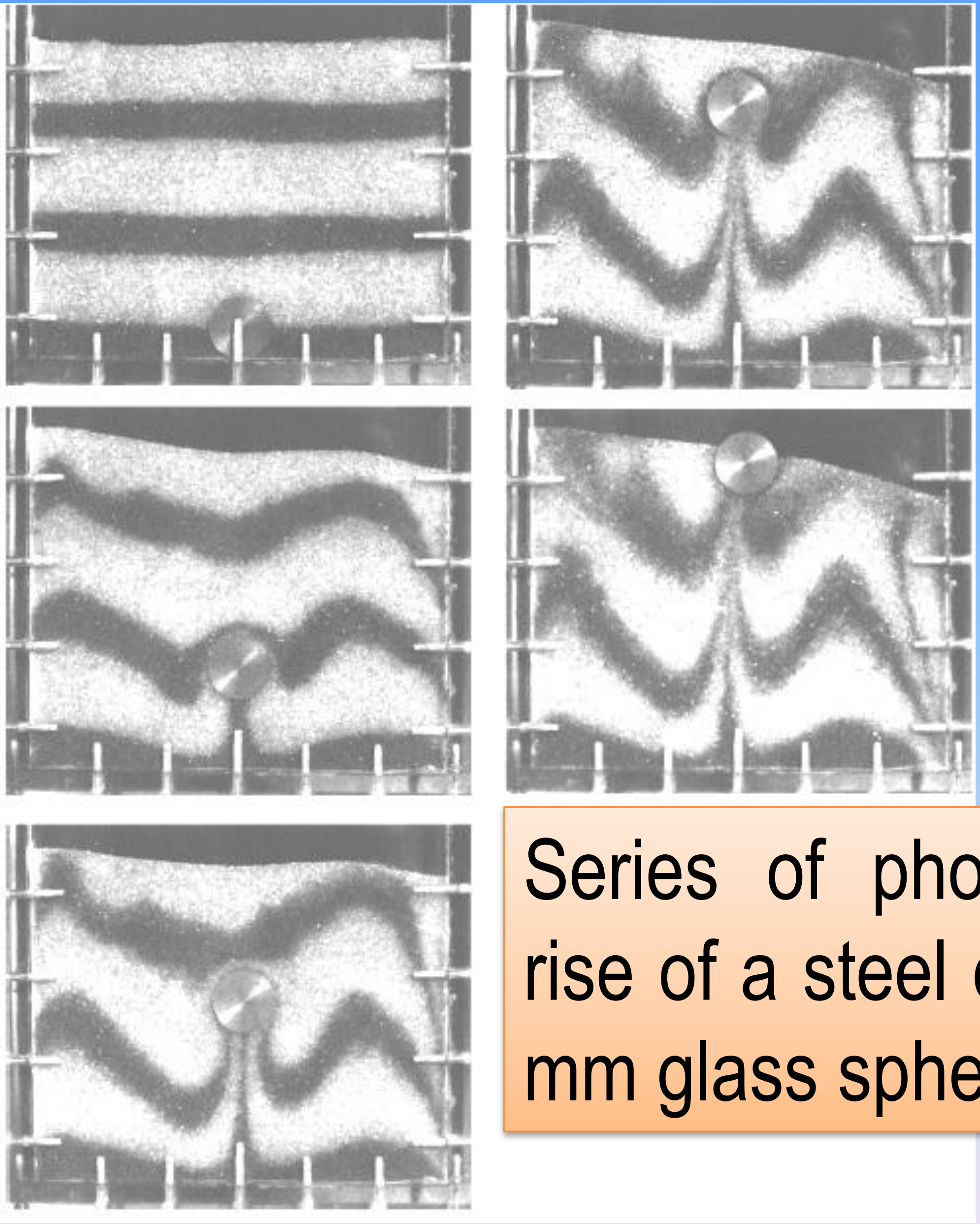
- Suppose a solid particle of size d and density ρ_p , is projected horizontally with velocity u into a fluid of density ρ_f and viscosity μ the limiting distance that it can travel horizontally is $[u \rho_p d^2/18\mu]$
- $\therefore$ Travelling dis $\propto$ size^2

Segregation pattern formed by pouring a free-flowing mixture of two sizes of particles into a heap



Mechanism of Segregation

Rise of coarse particles on Vibration



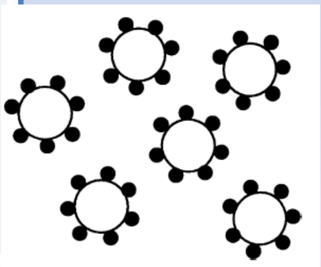
Series of photographs showing the rise of a steel disc through a bed of 2 mm glass spheres due to vibration.

Discussion

Suggest appropriate techniques for reduction of segregation

Reduction of Segregation

- Make the size of components as similar as possible
- Segregation is not serious problem when sizes <30 μm
- The mobility of F.F. Particles can be reduced by the addition of liquid
- Using ordered or interactive mixtures (small particles <5μm) adhered to the surface of a carrier particle in a control manner



Note

If it is not possible to alter the size of the components of the mixture or to add liquid, then in order to avoid serious segregation, care should be taken to avoid ***situations*** which are likely to promote segregation. In particular pouring operations and the formation of a moving sloping powder surface should be avoided.

Mechanisms of Mixing

In the mixing of solid particles, the following ***three mechanisms*** may be involved:

(a) ***Convective mixing***, in which groups of particles are moved from one position to another.

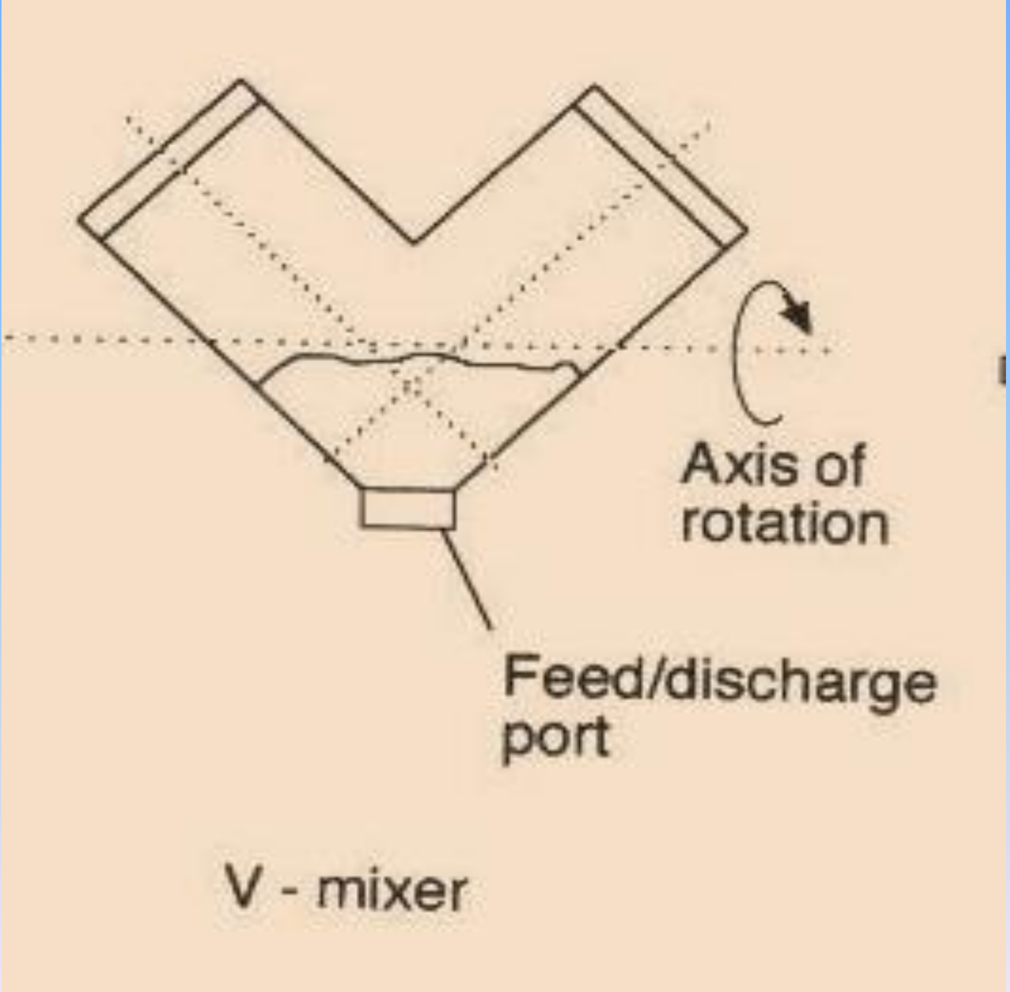
(b) ***Diffusion mixing***. This one takes place when particles roll down a sloping surface.

(c) ***Shear mixing.*** In this type, shear stresses give rise to slip zones and mixing takes place by interchange of particles between layers within the zone.

- These mechanisms operate to varying extents in different kinds of mixers and with different kinds of particles.

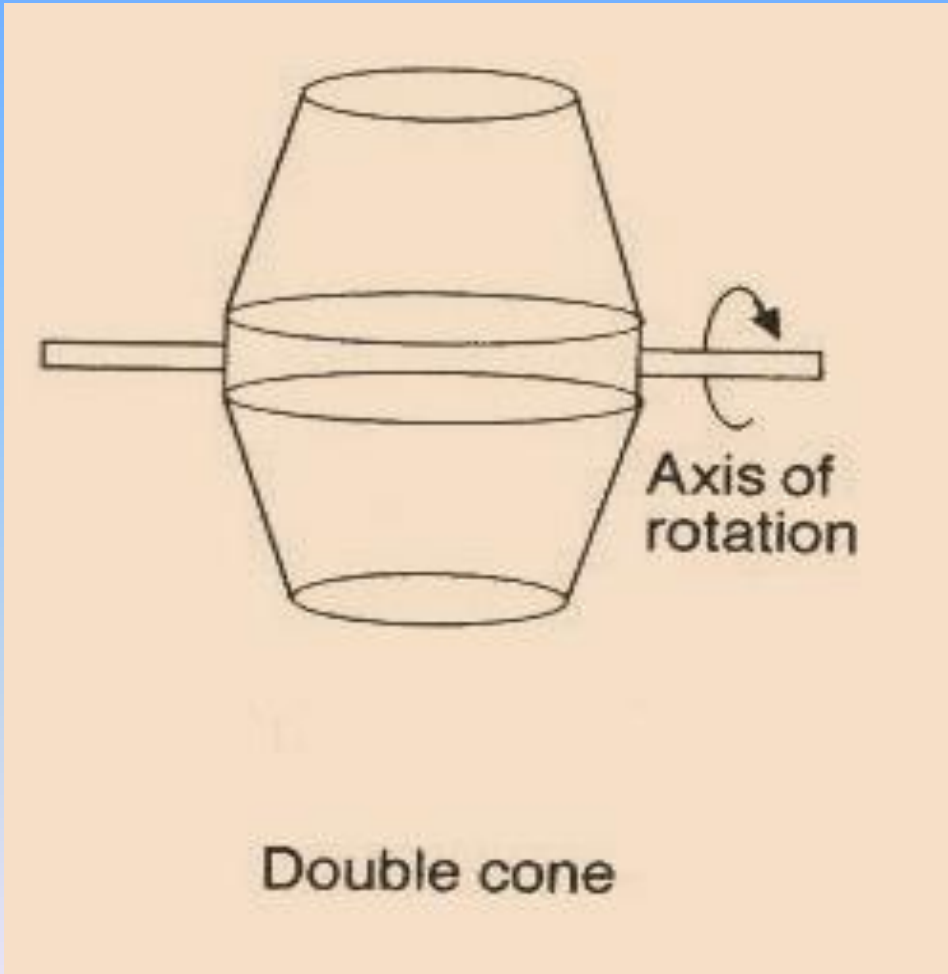
Type of mixers

1.Tumbling mixers 'V-mixer'



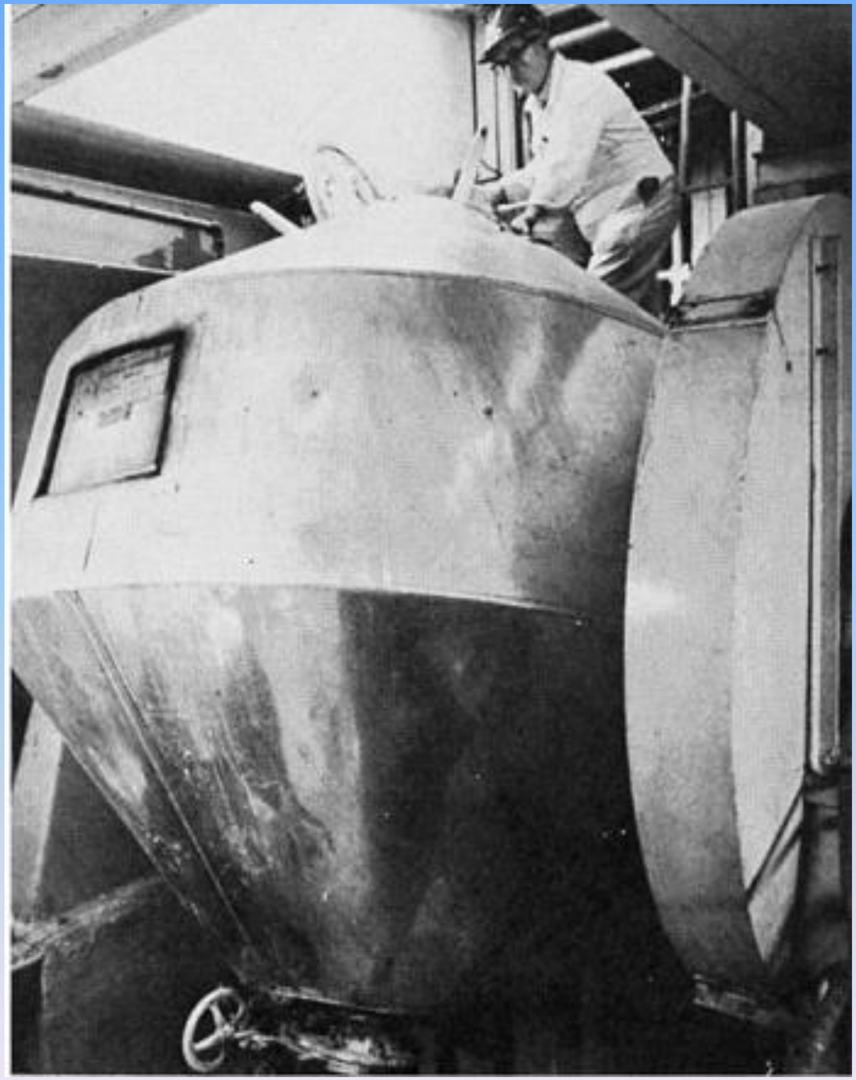
Dominant mechanism is diffusive mixing

1.Tumbling mixer 'Double cone'

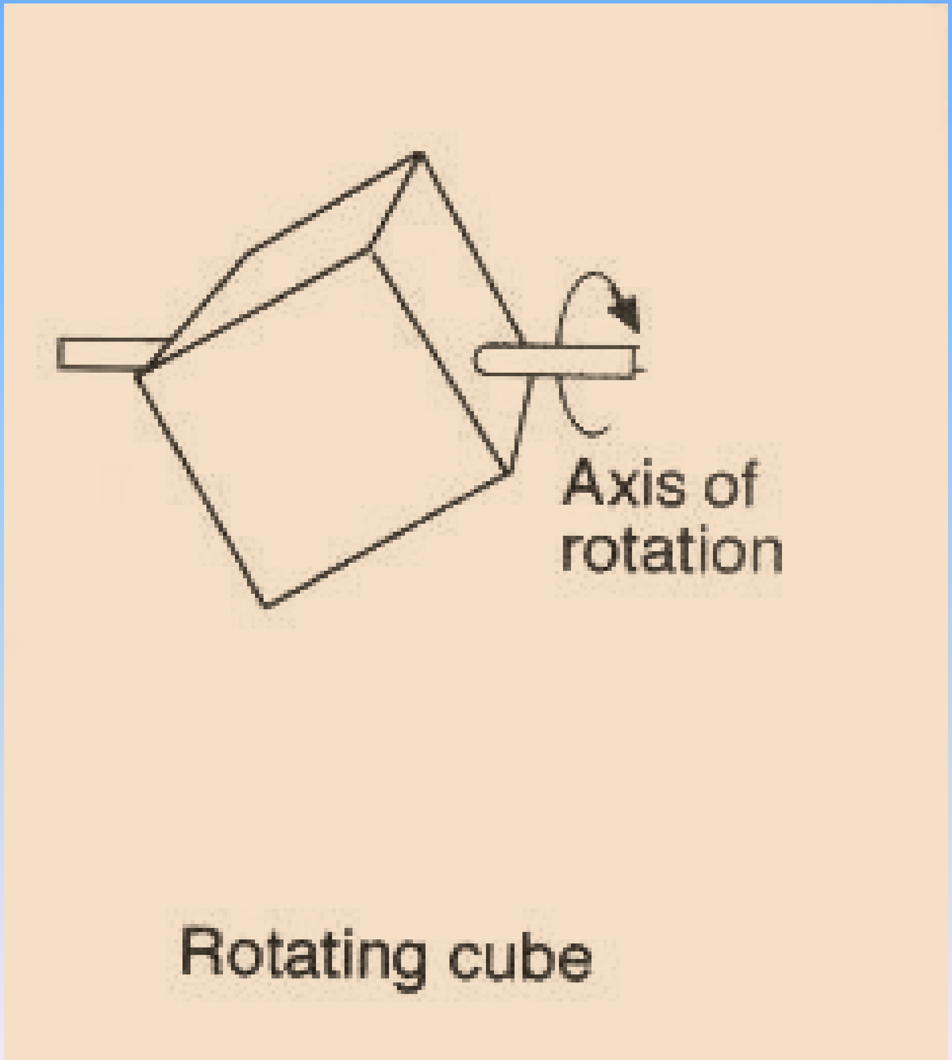


Dominant mechanism is diffusive mixing

Oblicone blender



1.Tumbling mixer 'cube mixer'



Dominant mechanism is diffusive mixing

Tumbler/ Blender (batch mixers)

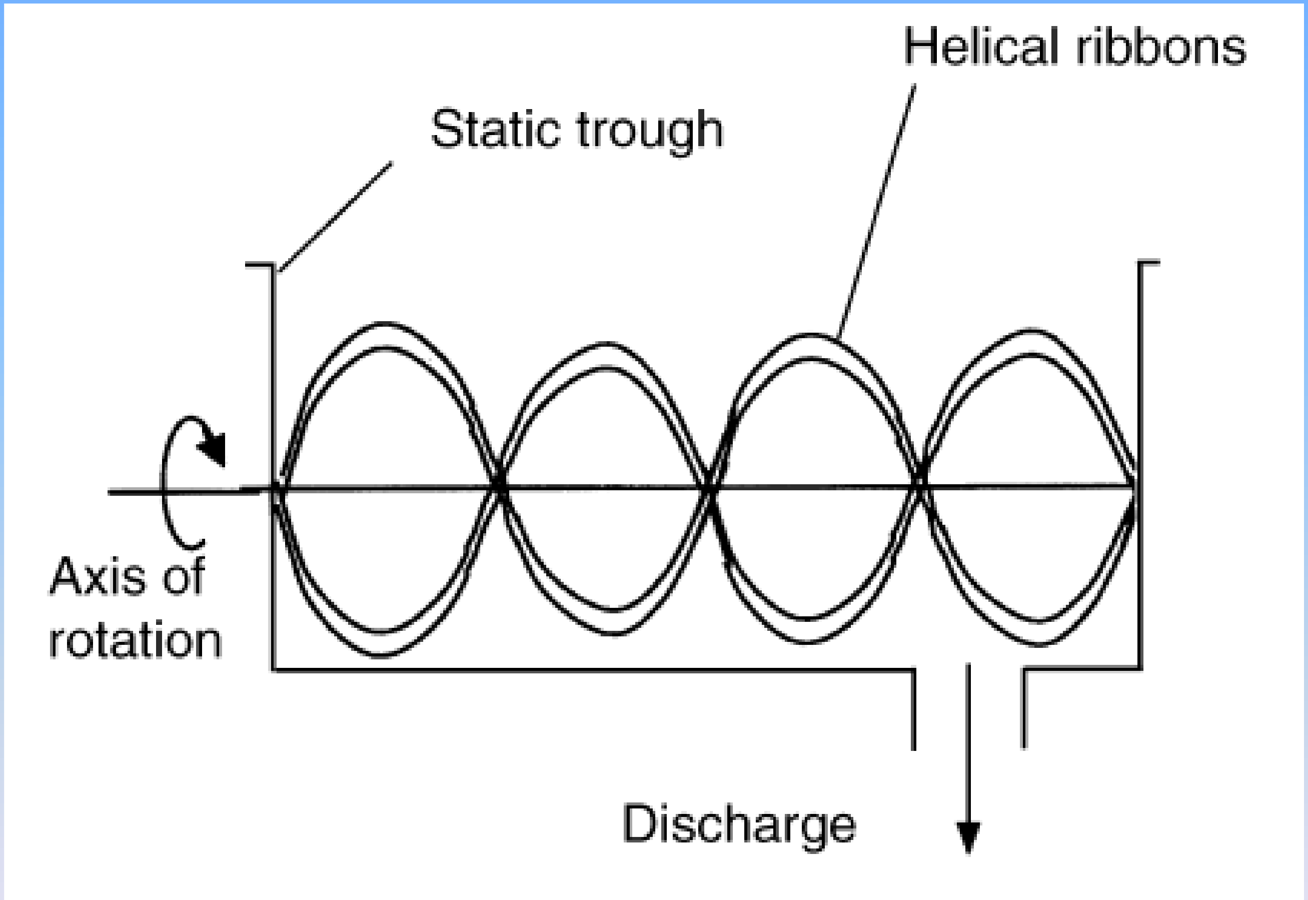
The **twin-shape mixer (V-shape** mixer) is the most preferred one, resulting in **satisfactory mixing in a reasonable time**.

In general, these types of solid mixers are:

- Efficient, **not** aggressive (good for friable powders),
- And **preferable** when mixing powders that have different particle sizes

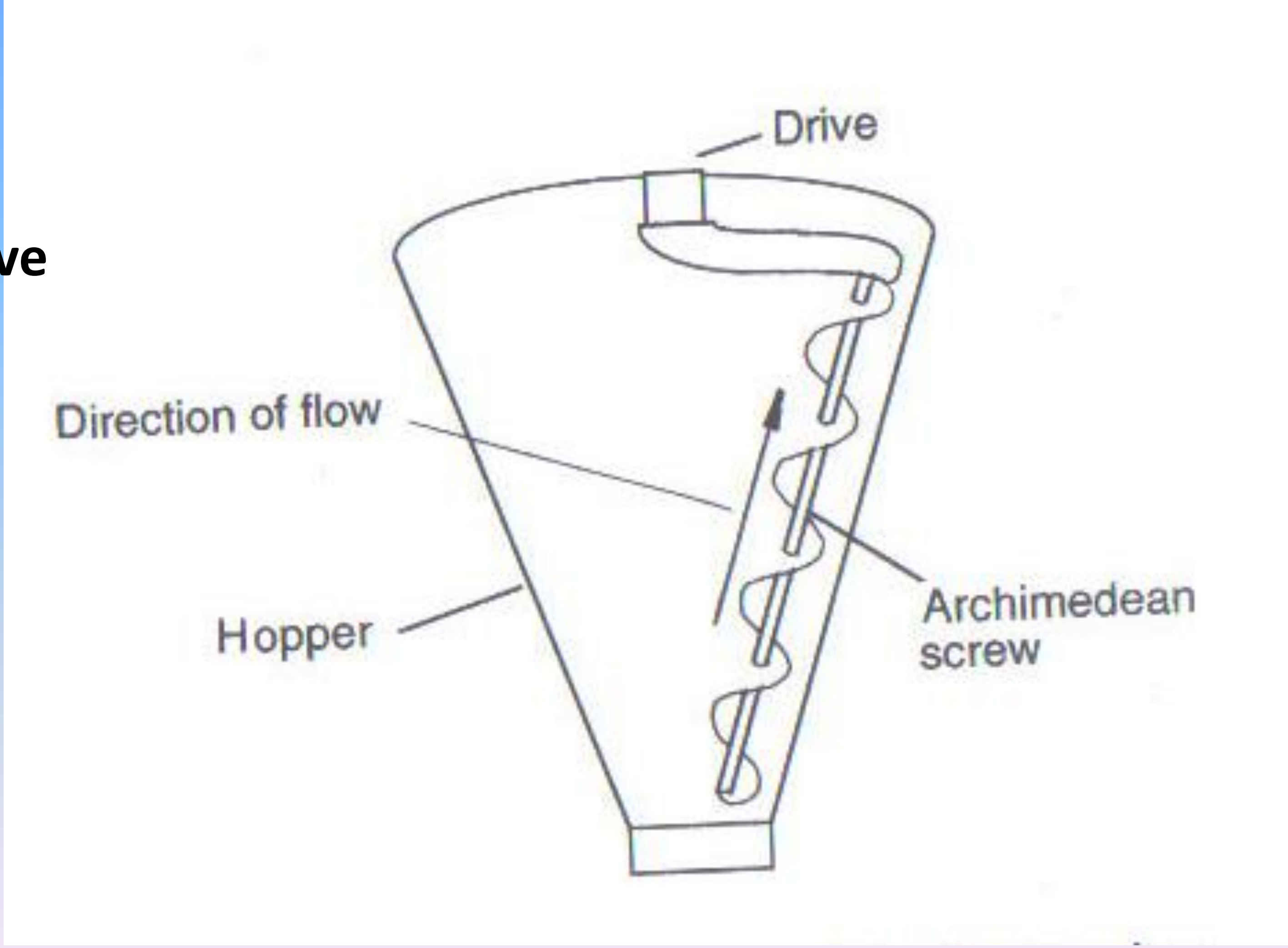
2.Convective mixers
'Ribbon blender'

Convective mixer



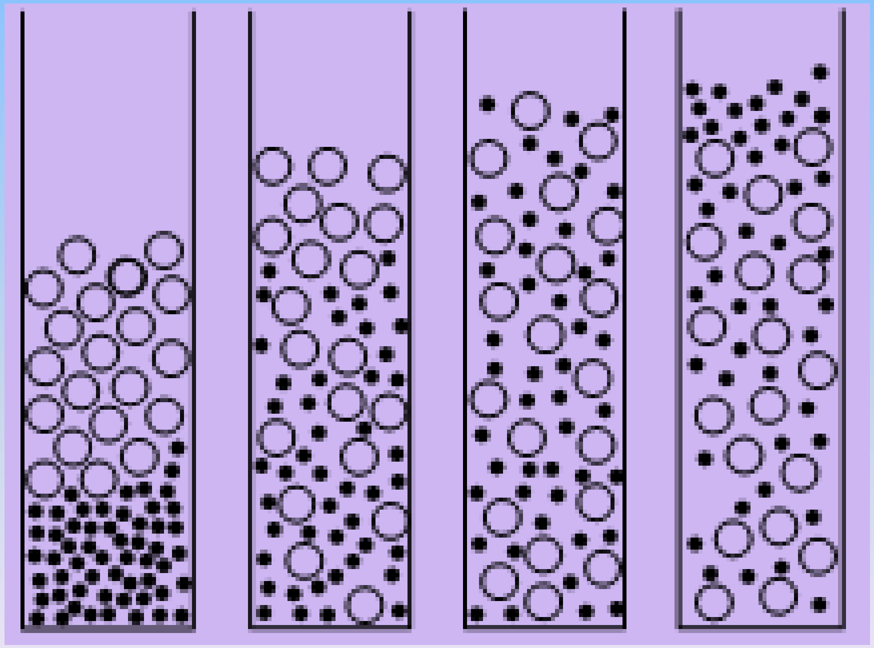
2.Convective mixers 'Nautamixer'

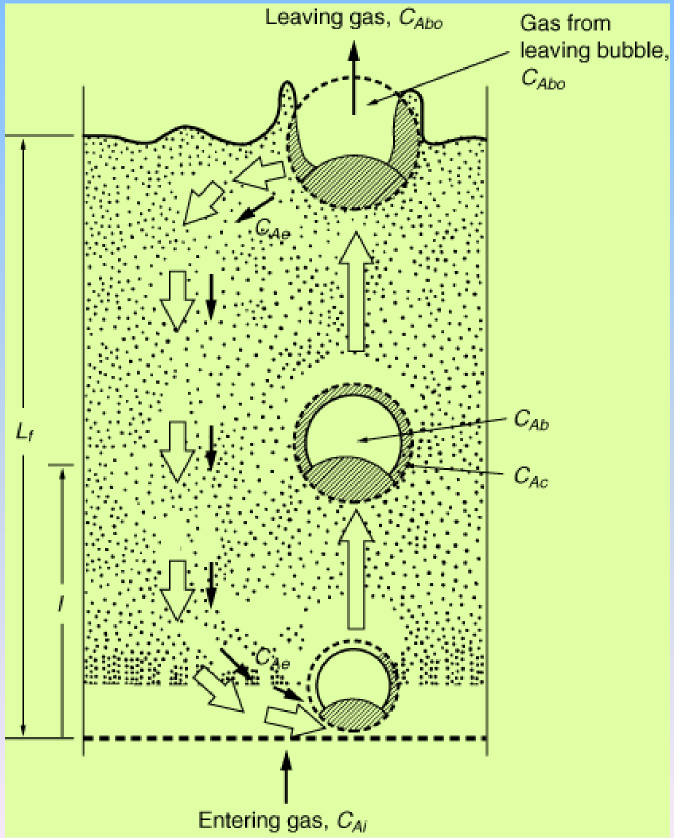
Convective mixer



Type of mixers

3. Fluidized bed mixer : convective mixing with circulating pattern of solids around bubbles.

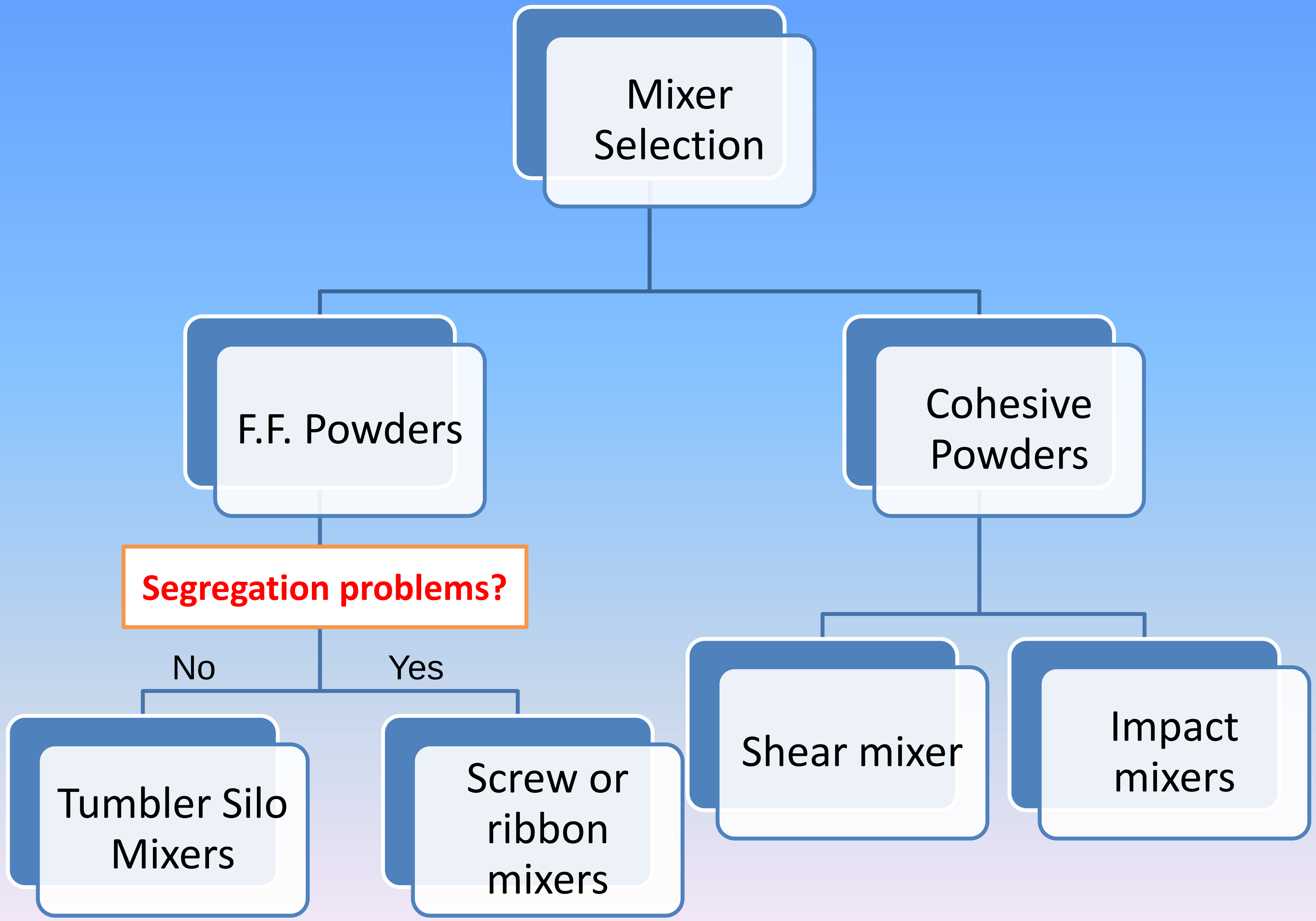




Type of mixers

High shear mixer

high shear impact mixer


Mixer Selection
F.F. Powders
Cohesive Powders
Segregation problems?
No
Yes
Tumbler Silo Mixers
Screw or ribbon mixers
Shear mixer
Impact mixers

The degree of mixing

- For solid particles, the **statistical variation in composition** among samples withdrawn at any time from a mix is commonly used as a measure of the degree of mixing.
- The standard deviation **s** (the square root of the mean of the squares of the individual deviations) or the variance s^2 is generally used.
- It should be noted that particulate material cannot attain the perfect mixing.

- The best that can be obtained is **a degree of randomness** in which two similar particles may well be side by side.
- For a ***completely* random mix** of uniform particles, it is suggested that:

$$s_r^2 = p(1 - p)/n \qquad (1.33)$$

Where *s_r^2 is the variance for the mixture, p is the overall proportion of particles of one* colour, and *n is the number of particles in each sample.*

- For ***<u>completely unmixed system</u>***, indicated by the suffix 0, it may be shown that:

$$s_0^2 = p(1 - p) \qquad (1.34)$$

which is independent of the number of particles in the sample.

- When a material is **<u>partly mixed</u>**, then **the degree of mixing** may be represented by some term ***b****, and several methods have been suggested for expressing* ***b*** *in terms of measurable* quantities.

- ***b*** *may be defined as being equal to* s_r/s*, or* $(s_0 - s)/(s_0 - s_r)$
- ***Note:***

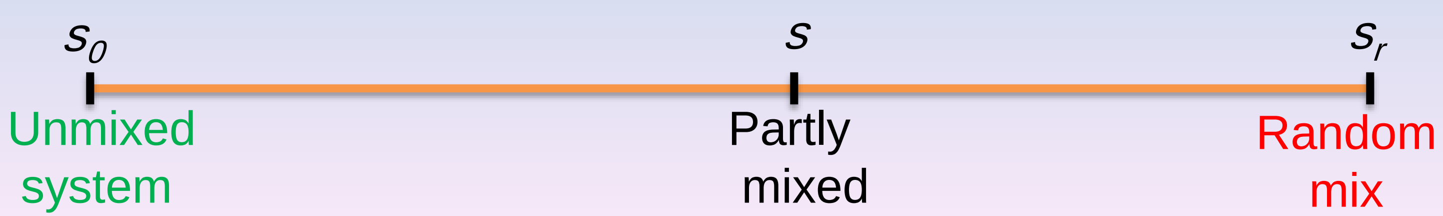
  $b = 0$ *for unmixed mixture*

  $b = 1$ *for a completely randomized material, where* $s = s_r$

- If s^2 is instead of s, then b could be written as:

$$\boldsymbol{b = (s_0^2 - s^2)/(s_0^2 - s_r^2)} \quad \textbf{or}$$

$$\boldsymbol{1\text{-}b = (s^2 - s_r^2)/(s_0^2 - s_r^2)} \qquad (1.35)$$



Discussion and useful Expressions

- When mixing continues over a long period demixing or segregation can occur, particularly if the materials are of different sizes or density.
- If the average number fraction of one type of particle is μ, consider N samples taken from the mixture.
- The measured number fraction of one type of particle in each of these N samples x_i (i = 1,2, ...,N) will differ from μ and from the measured values for the other samples.

- The mean, $\overline{X}$ of the x_i's will not be equal to μ unless a very large number of samples is taken.
- The standard deviation around is $\overline{X}$:

$$S = \left\{ \frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N-1} \right\}^{\frac{1}{2}}$$

- Even when mixing is complete (i.e random) the composition of successive samples will not be the same, unless the samples are very large. Otherwise, the standard deviation about μ, the true mean is

$$\sigma_e = \left\{ \frac{\mu(1-\mu)}{n} \right\}^{\frac{1}{2}}$$

'randomly mixed'

As given before

where n is the number of particles in each sample.

- The **Mixing Index**, *I*, is defined by

$$I = \frac{\sigma_e}{S} = \left\{ \frac{(N-1)\mu(1-\mu)}{n\sum_{i=1}^{N}(x_i - \overline{x})^2} \right\}^{\frac{1}{2}}$$

i.e the predicted standard deviation divided by the actual measured standard deviation.

- ***Note:*** S_e is always greater than S.

Mixing rate and mixing time

- **First Order Mixing Approximation**

$$\frac{dI}{dt} = k(1 - I)$$

where k is a constant characteristic of the system and the mixing unit used.

- This integrates to

$$t_{12} = \frac{1}{k}\int_{I_1}^{I_2} \frac{dI}{1 - I} = \frac{1}{k} ln\left(\frac{1 - I_1}{1 - I_2}\right)$$

- if the sample is unmixed I_0 is approximately equal to $\frac{1}{\sqrt{n}}$

Example

The performance of a solids mixer was assessed by calculating the variance occurring in the mass fraction of a component amongst a selection of samples withdrawn from the mixture. The quality was tested at intervals of 30 s and the data obtained are:

sample variance (–)	0.025	0.006	0.015	0.018	0.019
mixing time (s)	30	60	90	120	150

If the component analysed represents 20 per cent of the mixture by mass and each of the samples removed contains approximately 100 particles, comment on the quality of the mixture produced and present the data in graphical form showing the variation of the mixing index with time.

Solution

For a completely unmixed system:

$$s_0^2 = p(1-p) = 0.20(1-0.20) = 0.16 \qquad \text{(equation 1.34)}$$

For a completely random mixture:

$$s_r^2 = p(1-p)/n = 0.20(1-0.20)/100 = 0.0016 \qquad \text{(equation 1.33)}$$

The degree of mixing b is given by equation 1.35 as: $b = (s_0^2 - s^2)/(s_0^2 - s_r^2)$ In this case, $b = (0.16 - s^2)/(0.16 - 0.0016) = 1.01 - 6.313s^2$ The calculated data are therefore:

t(s)	30	60	90	120	150
s^2	0.025	0.006	0.015	0.018	0.019
b	0.852	0.972	0.915	0.896	0.890

These data are plotted in Figure 1.19 from which it is clear that the degree of mixing is a maximum at $\underline{\underline{t = 60\text{s}}}$.

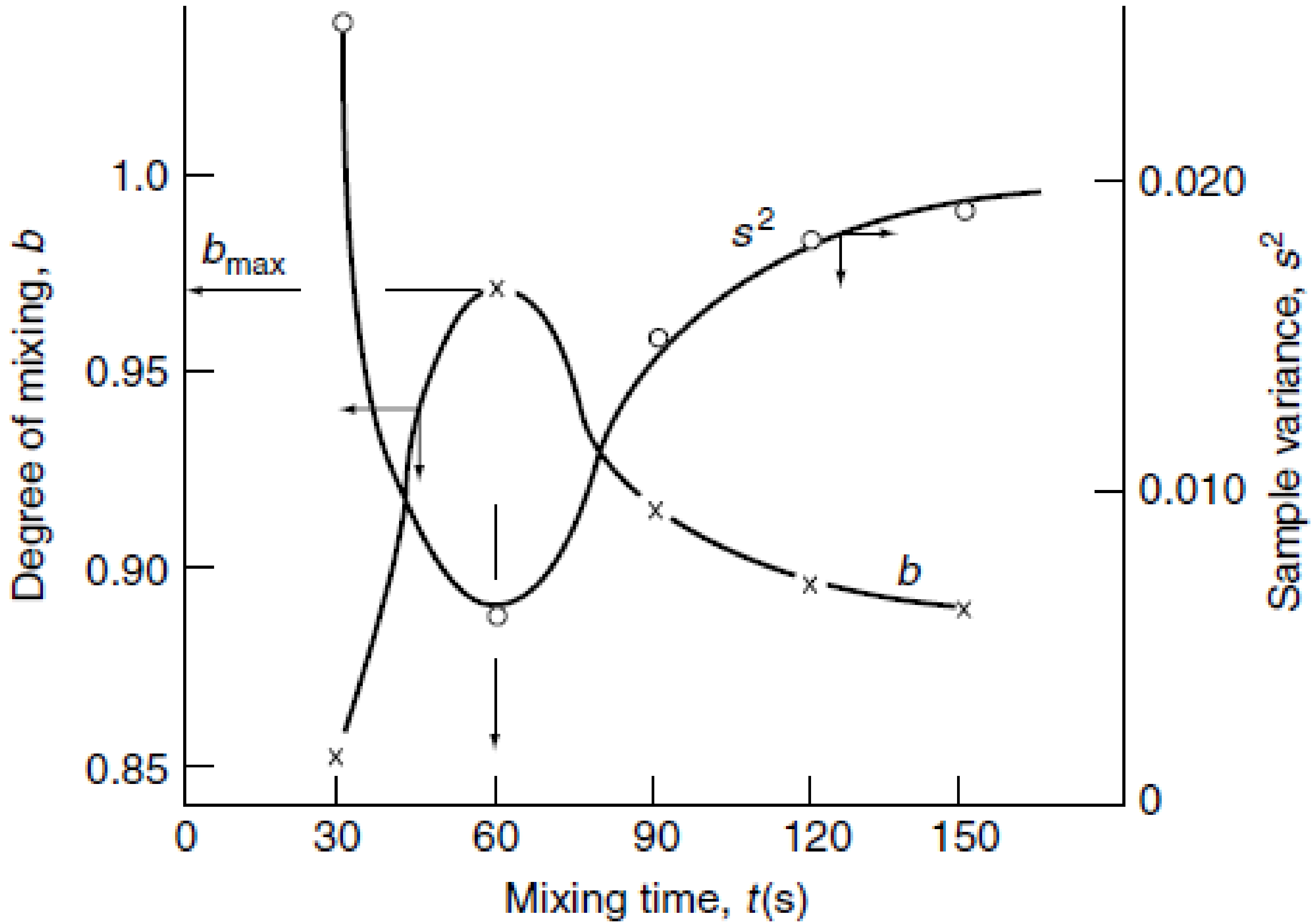


Figure 1.19. Example 1.3. Degree of mixing as a function of mixing time

PARTICLE SIZE REDUCTION

Theory and Practice

Introduction & Definitions

- **The objective of comminution is to produce small particles from larger ones. Smaller particles are the desired product either because of their large surface or because of their shape, size, and number.**
- **Size reduction may be a specific requirement of a process or may aid other operations such as extraction or reaction.**
- **Comminution is the generic term used for size reduction, although such a term includes operations such as crushing or pulverizing or grinding.**

Introduction

- The reduction mechanism consists of deforming the solid piece until it breaks or tears. Breaking of hard materials along cracks or defects in their structures is accomplished by applying diverse forces.
- The energy efficiency of the operation can be related to the new surface formed by the reduction in size.
- In an actual process, a given unit does not yield a uniform product, whether the feed is uniformly sized or not. The product normally consists of a mixture of particles, which may contain a wide variety of sizes and even shapes.

Introduction

- In comminuted products, the term “diameter” is generally used to describe a characteristic dimension related to particle size.
- As described in previous slides (mean sizes), the shape of an individual particle is conveniently expressed in terms of the sphericity φ_s, which is independent of particle size.
- For crushed materials, φ_s , its value lies between 0.6 and 0.7.

Summary

Objectives of Size Reduction

- To increase the **surface area**
- To get the desired **shape, size or size ranges** and **specific surface** particles
- To separate **unwanted particles** effectively
- To dispose **solid waste** easily
- To **mix** solid particles more **intimately** and
- To improve the **handling (storage and transportation)** characteristics

Solids can be broken by different types of forces:

- Compression
- Impact or striking
- Attrition or rubbing (shear)
- Cutting and tearing

<u>Note</u>: In a comminution operation, more than one type of force acts, usually.

In General

- **Compressive forces** are used for **coarse** crushing of hard materials. Coarse crushing implies reduction to a size of about 3 mm.
- **Impact forces** can be regarded as general purpose forces and may be associated with **coarse, medium, and fine** grinding of a variety of materials.
- **Shear or attrition forces** are applied in **fine** pulverization, when size of products can reach the micrometer range.
- **Cutting forces** produce exact and **defined sizes** and may, even, produce exact shapes

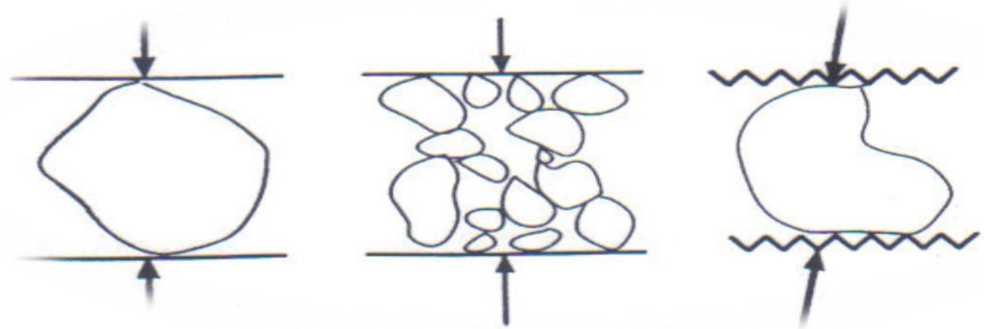
Types of Forces Used in Size Reduction Equipment

Force	Principle	Example of Equipment
Compressive	Nutcracker	Crushing rolls
Impact	Hammer	Hammer mill
Attrition	File	Disc attrition mill
Cut	Scissors	Rotary knife cutter

Stressing Mechanisms

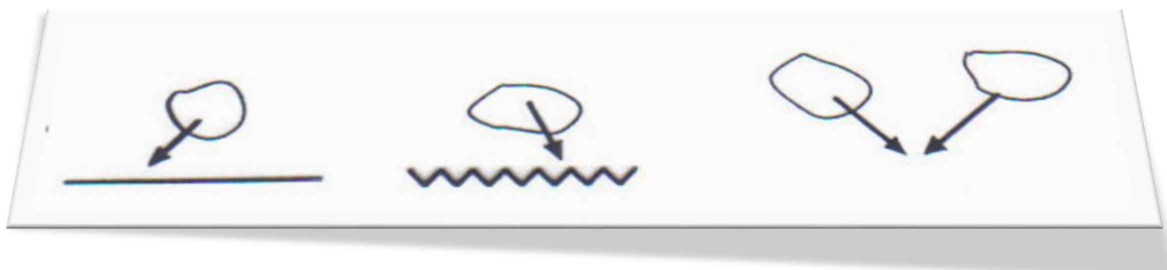
- Stress applied between two surfaces at low velocity

0.01-10 m/s
Crushing + attrition



- Stress applied at a single solid surface at high velocity

10-200 m/s
Impact fracture + attrition



Stressing Mechanisms

Stress applied by carrier medium*-usually in wet grinding to bring about dis-agglomeration

*** gas ~ air, inert gases or liquid ~ water, oil**

- **Aid to transport material**
- **Transmit forces to particles.**
- **Influence the friction and abrasion.**
- **Affect crack formation.**

How dose the crushing process take place?

- Cracks
- Opening cracks
- Applied force plays a significant role in cracking process
- Enough not enough force. Enough force yield crushing but no enough force will lead to material elastic deformation and then return to its original shape.
- Not enough force means energy losses.

Factors affecting the selection of crushing equipment

1. *Mechanism of the applied force.*
2. *Product and feed sizes.*
3. *Material properties (hardness, stickiness, …etc.)*
4. *Carrier medium.*
5. *Mode of operation*
6. *Capacity.*

Material Properties

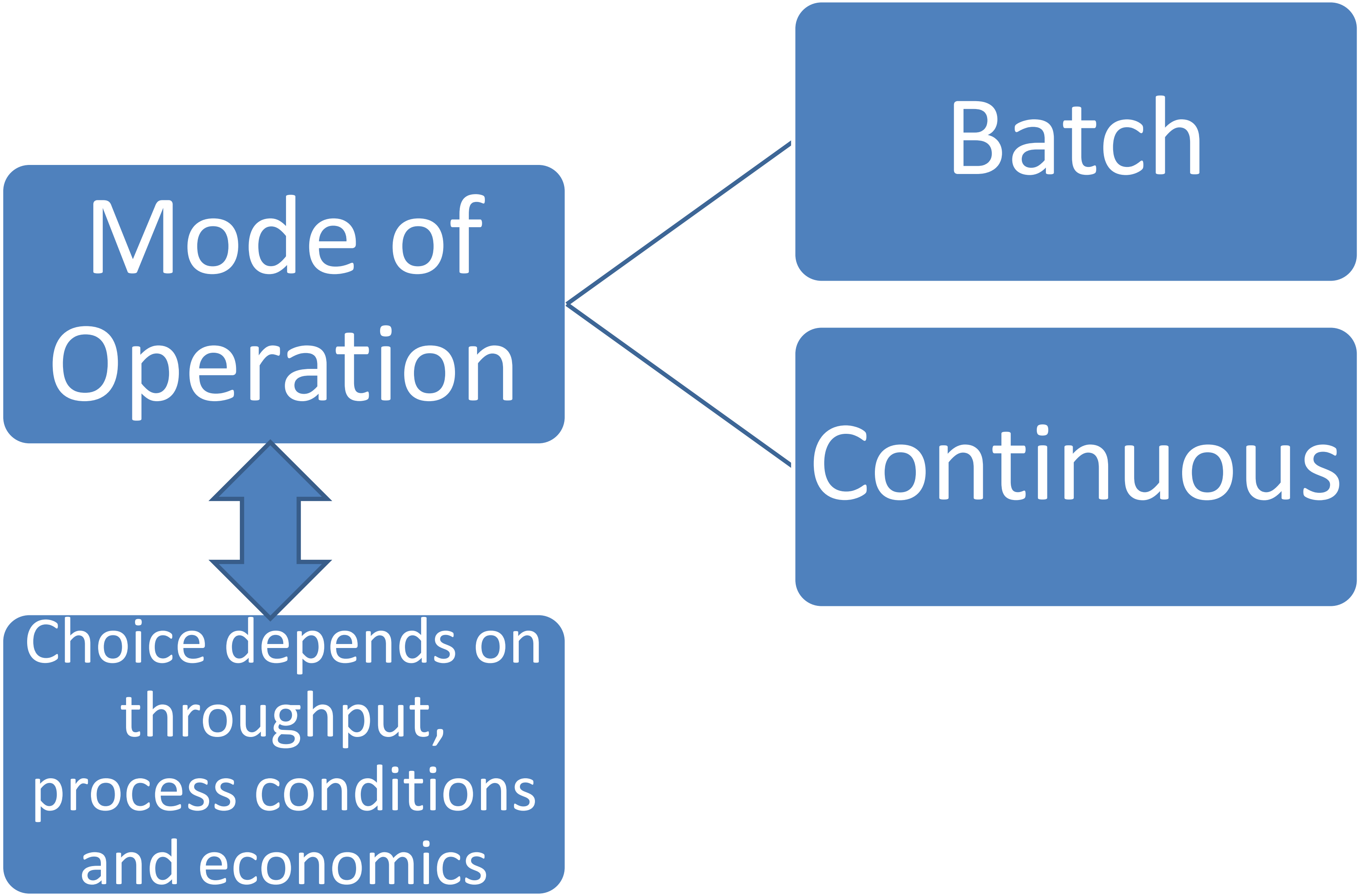
- **Hardness**
- **Abrasiveness**
- **Cohesivity/adhesivity ~ ‘stickiness’**
- **Moisture content**
- **Melting point**
- **Explosion limit ~ ‘Explosive material’**
- **Special properties such as toxicity or radioactivity’**

Low speed machine is recommended

The Mohs Scale of Hardness

1. Talc
2. Rock salt or gypsum
3. Calcite
4. Fluorspar
5. Apatite
6. Felspar
7. Quartz
8. Topaz (silicate mineral)
9. Carborundum (SiC)

   'Silicon Carbide'
10. Diamond.


Mode of Operation
Batch
Continuous
Choice depends on throughput, process conditions and economics

Conclusion

- Crushing Process is an ***inefficient operation.***
- about 10 per cent of the total power is usefully employed.
- Look to the energy utilization scheme in the next slide.

Energy utilization

(a) In producing **elastic deformation** of the particles before fracture occurs.

(b) In producing **inelastic deformation** which results in size reduction.

(c) In causing **elastic distortion** of the equipment.

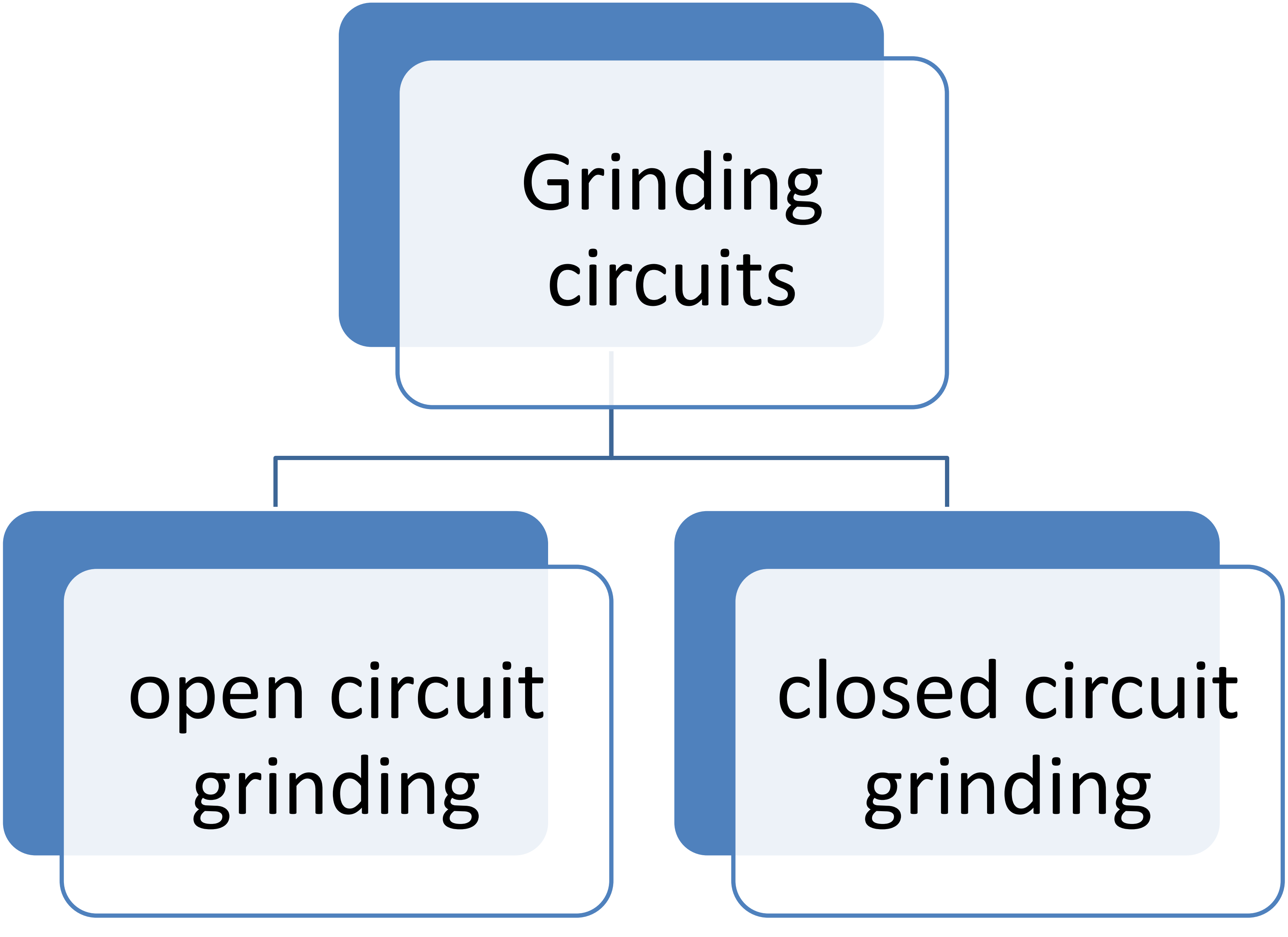
(d) In friction **between particles**, and **between particles and the machine.**

(e) In **noise, heat and vibration** in the plant, and

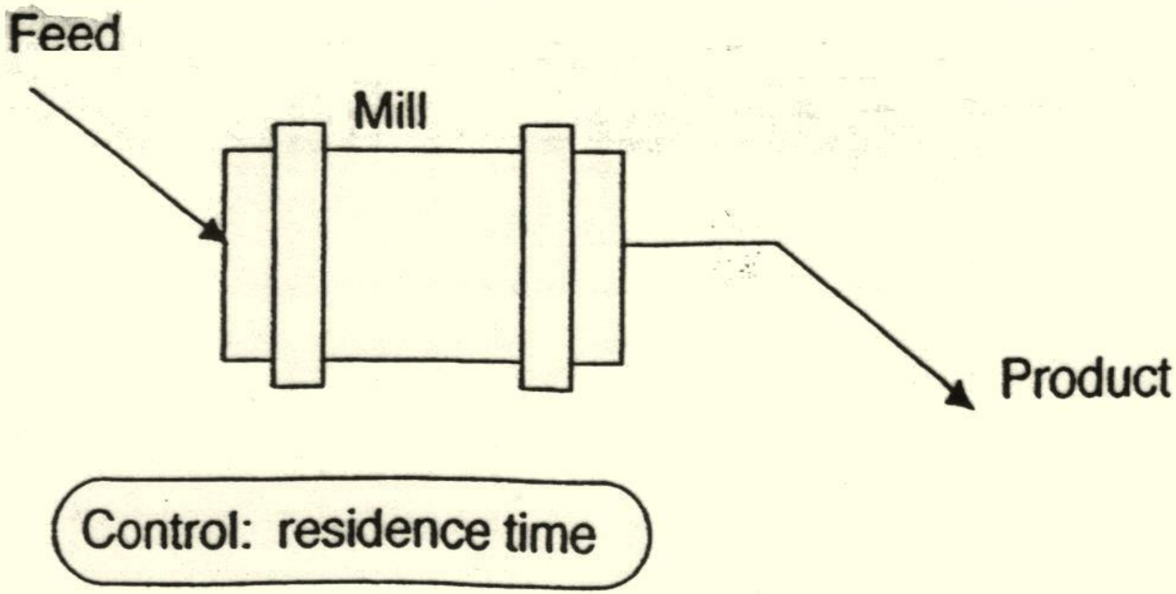
(f) In **friction losses in the plant** itself.

Methods of operating crushers

Free crushing	Choke feeding
✓ Continuous, low rate	✓ Batch
✓ Short residence time	✓ Long residence time
✓ Low degree of crushing	✓ Higher degree of crushing
✓ High capacity	✓ Low capacity
✓ Low energy consumption	✓ High energy consumption

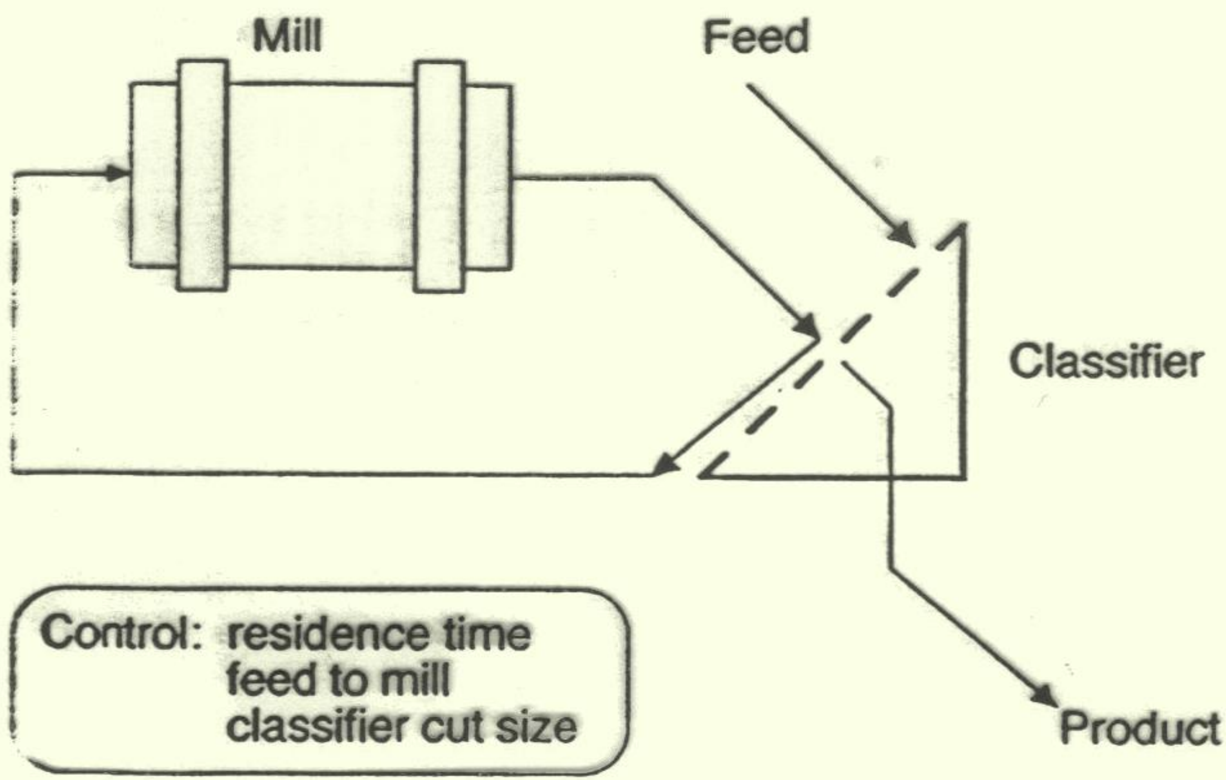

Grinding circuits
open circuit grinding
closed circuit grinding

Open circuit mill



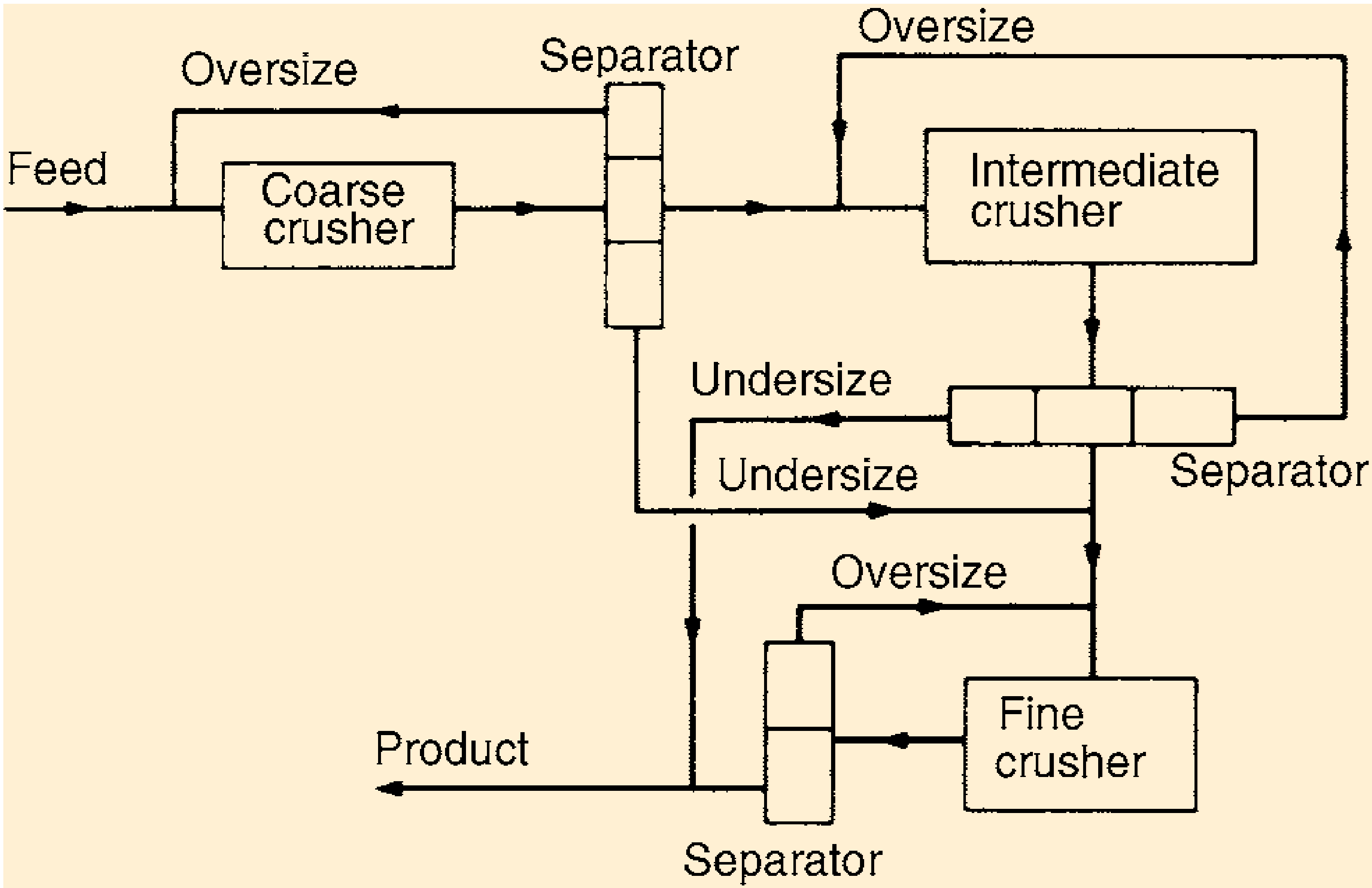
Open circuit milling

Closed circuit mill



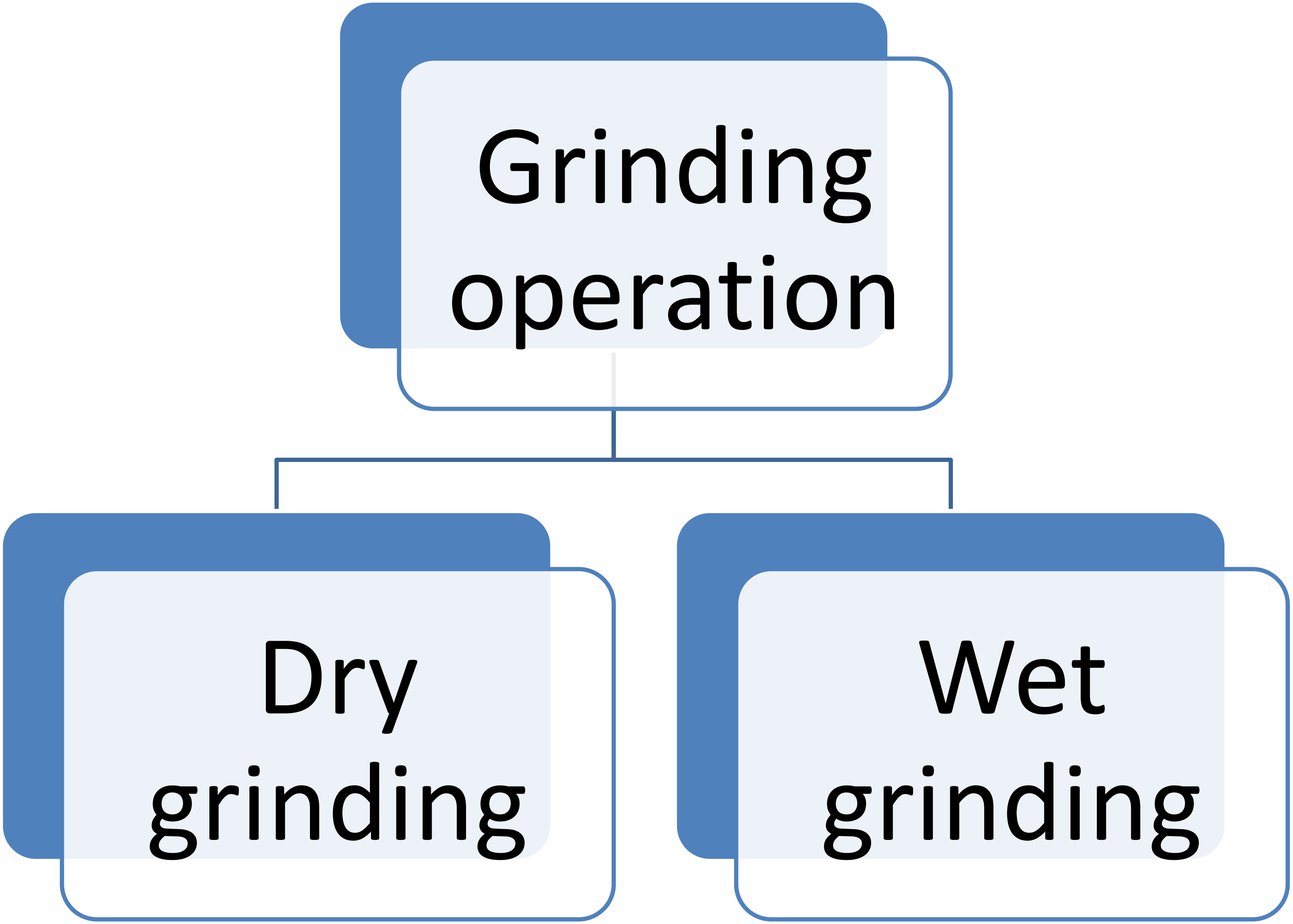
Closed circuit milling

Flow diagram for closed circuit grinding system

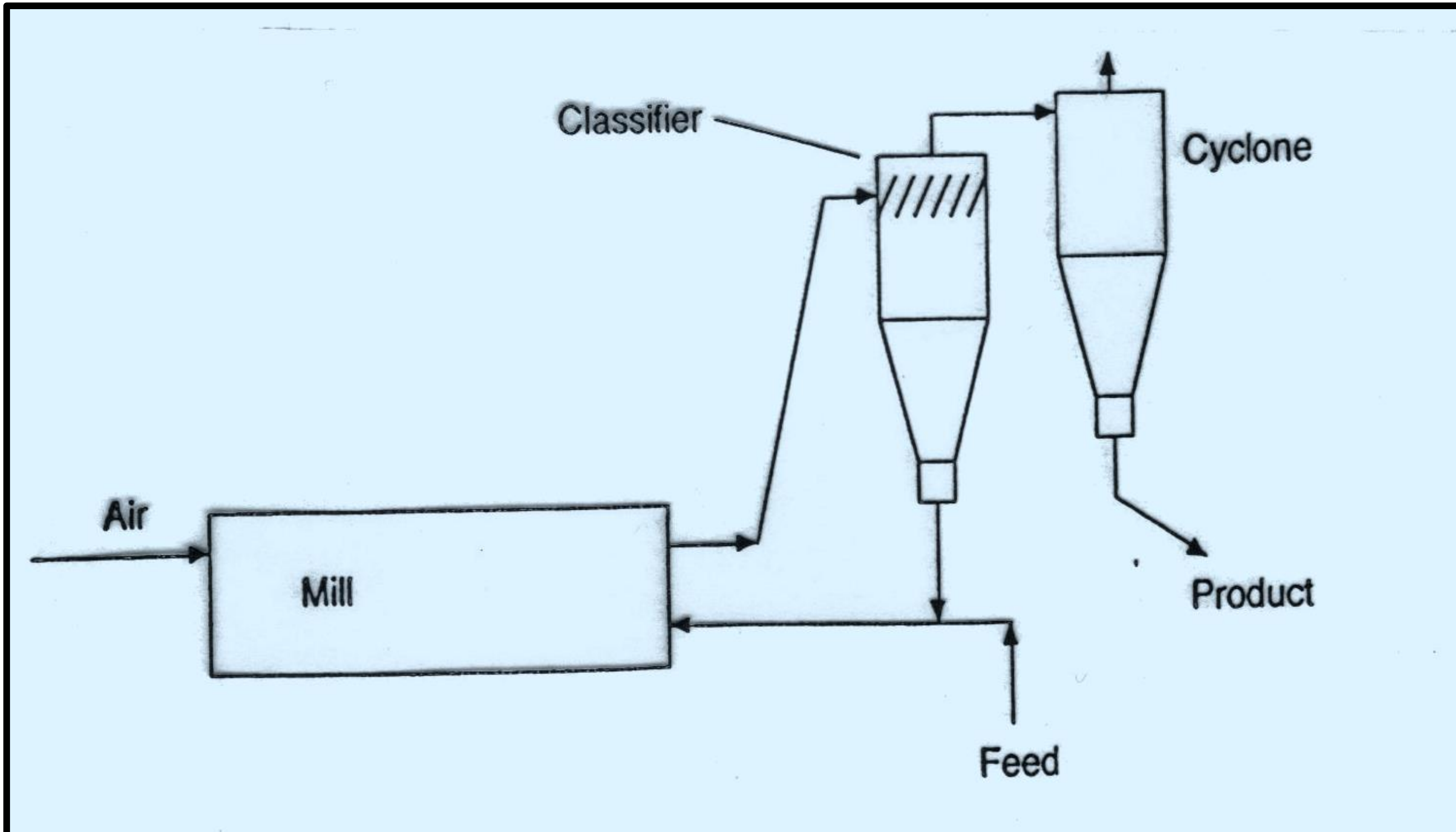


Classification of size reduction equipment

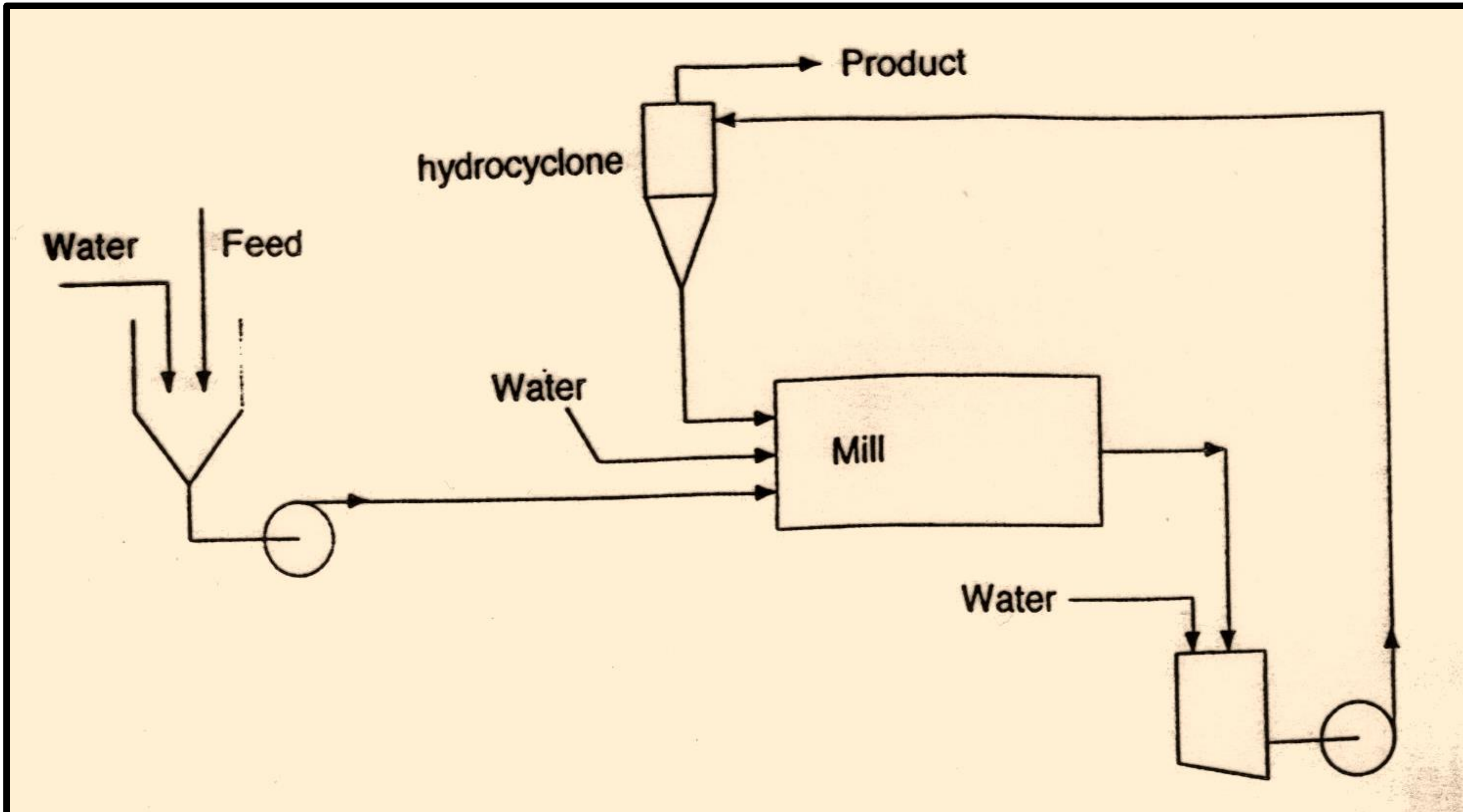
	Feed size	Product size
Coarse crushers	1500–40 mm	50–5 mm
Intermediate crushers	50–5 mm	5-0.1 mm
Fine crushers	5–2 mm	2-0.1 mm
Colloid mills	0.2 mm	down to 0.01 µm


Grinding operation
Dry grinding
Wet grinding

Dry milling; closed circuit operation



Wet milling; closed circuit operation



Advantages of wet grinding

(a) The power consumption is reduced by about 20–30 per cent.

(b) The capacity of the plant is increased.

(c) The removal of the product is facilitated and the amount of fines is reduced.

(d) Dust formation is eliminated.

(e) The solids are more easily handled.

Energy Input and Particle size

There are three postulates to predict the energy required for particle size reduction.
Most of these postulates depend on the following power formula:

$$dE = -C\left(\frac{dL}{L^{p}}\right) \qquad (1)$$

or

$$\frac{dE}{dL} = -C\,L^{-p} \qquad (2)$$

where
L : object's diameter
E : Energy Input
p, C : constants depend on the type and size of material and type of machine

If $p = 2$

equation (1) becomes

$$dE = -C\frac{dL}{L^2} \qquad (3)$$

or

$$E = C\left(\frac{1}{L_2} - \frac{1}{L_1}\right) \qquad (4)$$

equation 4 is called Rittinger's Law

Where

$C = K_R f_c$;

K_R: **Rittengir's constant m^4/kg**

f_c: **crushing strength of the material.**

If p=1 ; equation (1) after integration becomes;

$$E = C\ ln\left(\frac{L_1}{L_2}\right) \qquad (5)$$

Where

$C = K_K f_c$; K_K : Kick's constant ~ m^3/kg

L_1/L_2 = reduction ratio

Equation (5) is called Kick's Law, where the energy required is directly proportional to the reduction ratio.

However, the relation can be applied successfully to coarse crushing or sometimes to very fine grinding!

If we assume a value for the exponent, p, midway between the $p=2$ and $p=1$, say $p=3/2$. Upon Integration, we obtain the following relationship:

$$E = 2C\left(\frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}}\right) \qquad (6)$$

$$= 2C\sqrt{\frac{1}{L_2}}\left\{1 - \frac{1}{q^{1/2}}\right\} \qquad (7)$$

It is called Bond's Law

Where $q = L_1/L_2$ reduction ratio , $C = 5\ E_i$

Equation (7) can be rewritten as:

$$E = E_i \sqrt{\frac{100}{L_2}} \left\{ 1 - \frac{1}{q^{1/2}} \right\} \tag{8}$$

Where

E_i :work index “the amount of energy required to reduce the unit mass of the material from infinite large particle size to 80% passing a 100μm screen i.e with 80% of the particles passing”.

Equation 8 is better to be written like this

$$W = W_i \sqrt{\frac{100}{L_2}} \left\{ 1 - \frac{1}{q^{1/2}} \right\} = 10W_i \left\{ \frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}} \right\}$$

Where W_i : work index, kw.hm$^{1/2}$/ton

W : required work for crushing, kw.h/ton

L_1, L_2: size in m

Note: The Bond's work index is obtained from laboratory crushing tests on the feed material.

Bond's work index for some materials

Average indexes

Material	Sp. Gr.	Work index, W_i
Copper ore	3.2	12.7
Glass	2.53	12.3
Phosphate rock	2.74	9.9
Oil shale	1.84	15.8

Material	Work index
Bauxite	8.78
Cement clinker	13.45
Clay	6.30
Coal	13.00
Granite	15.13
Gravel	16.06
Limestone	12.74

Notes

1. The higher the value of the work index the harder it will be to grind the solid.
2. Units of Work index $kWh.m^{1/2}t^{-1}$

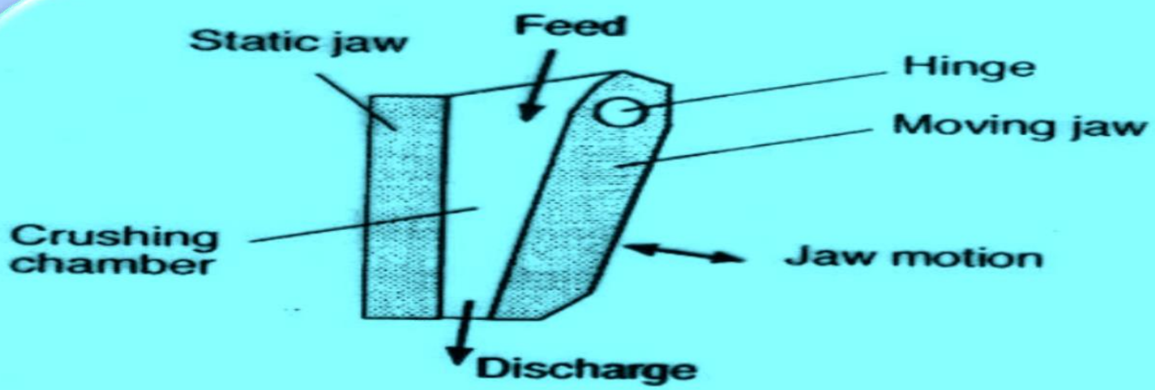
Summary

- Kick's law has been found to hold more accurately for coarser crushing where most of the energy is used in causing fracture along existing cracks.
- Rittinger's law has been found to hold better for fine grinding, where a large increase in surface results
- Bond's law holds reasonably well for a variety of materials undergoing coarse, medium, and fine size reduction.

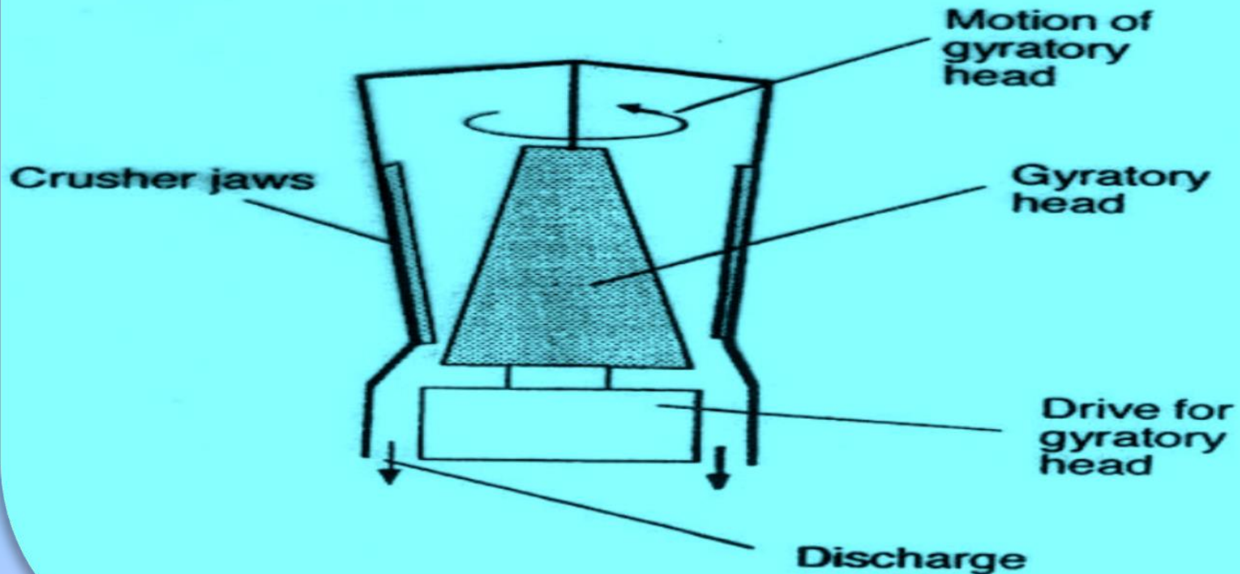
Crushing equipment

Coarse crushers	Intermediate crushers	Fine crushers
Jaw crusher	Crushing rolls	Roller mill
Gyratory crusher	Hammer mill	Ball mill
	Pin mill	Pendulum mill

Coarse crushers

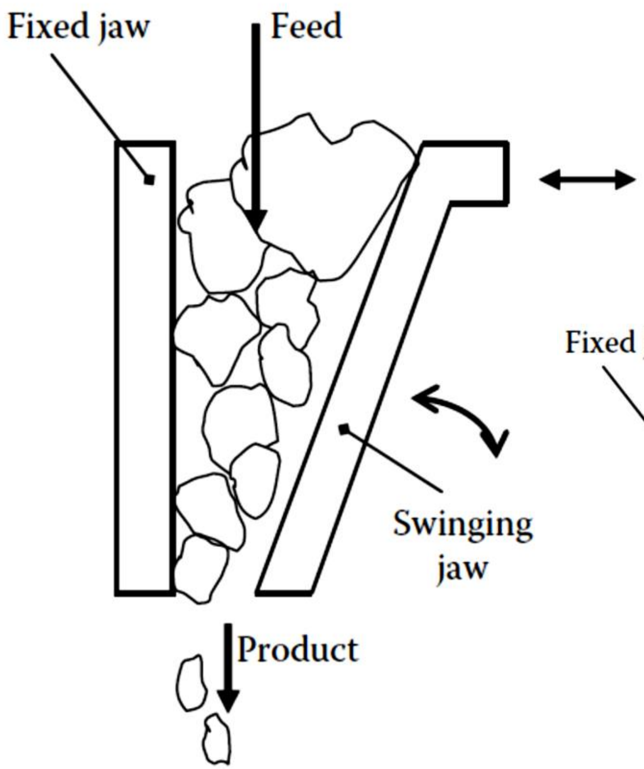


Schematic diagram of a jaw crusher

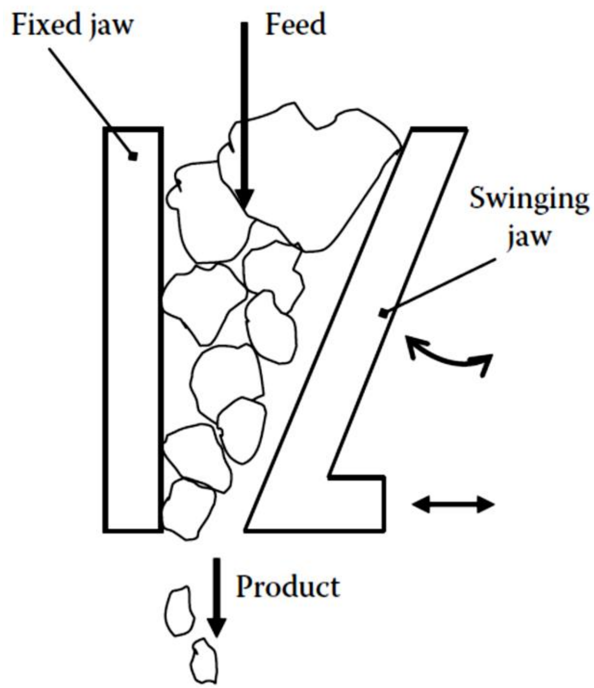


Schematic diagram of gyratory crusher

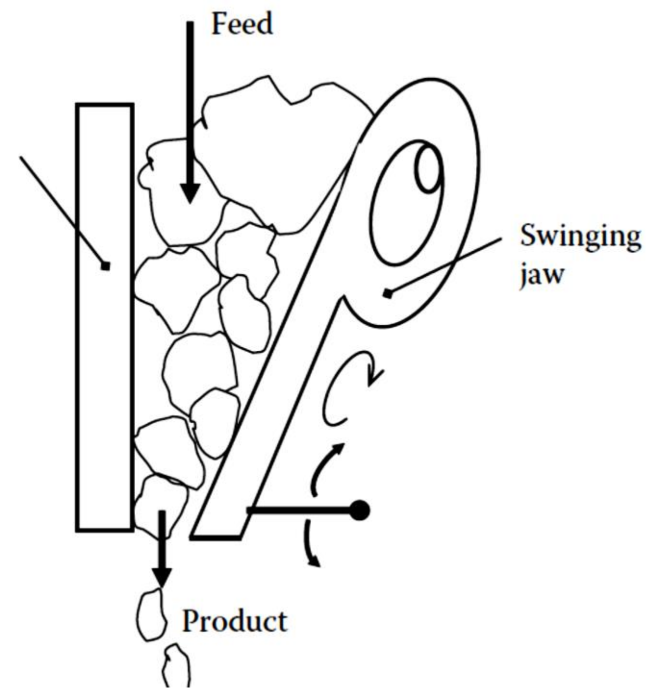
Jaw Crusher - Models



Dodge model

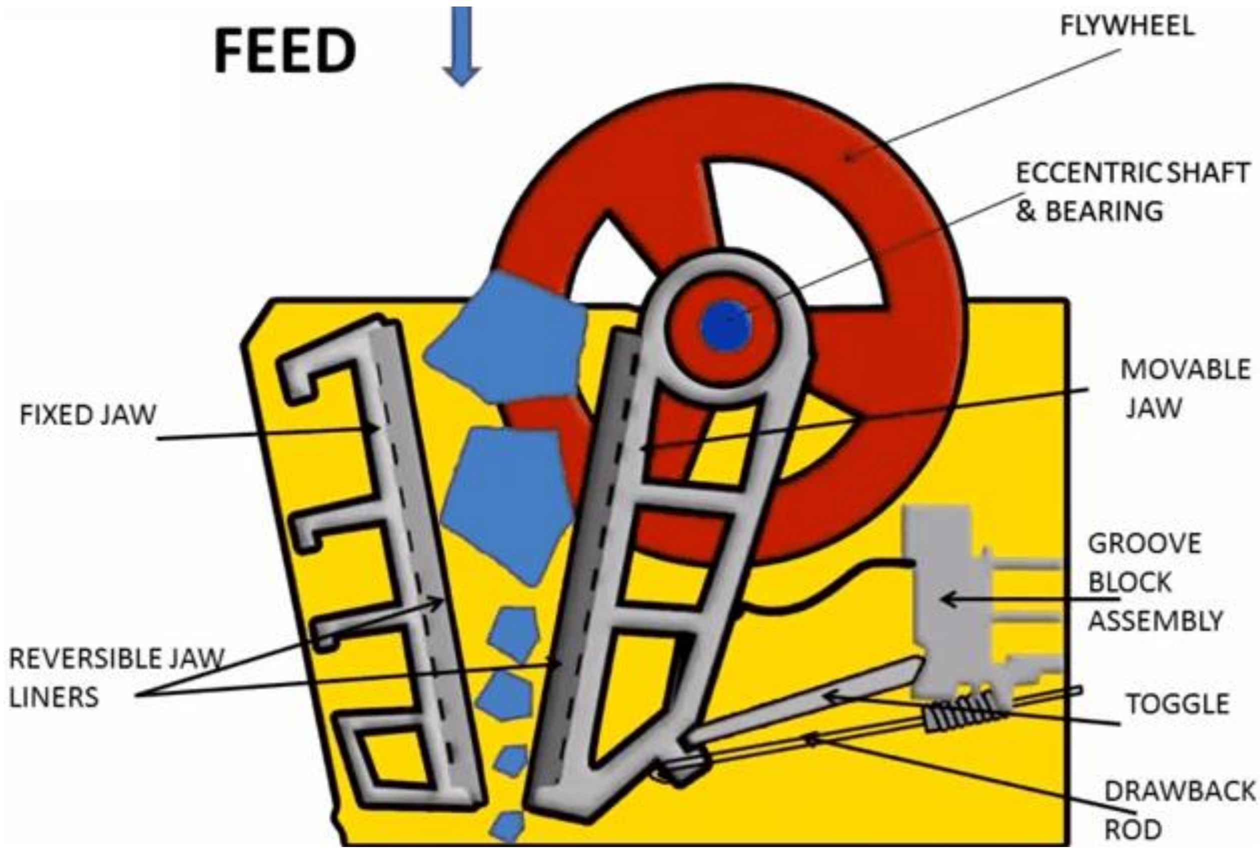


Blake model

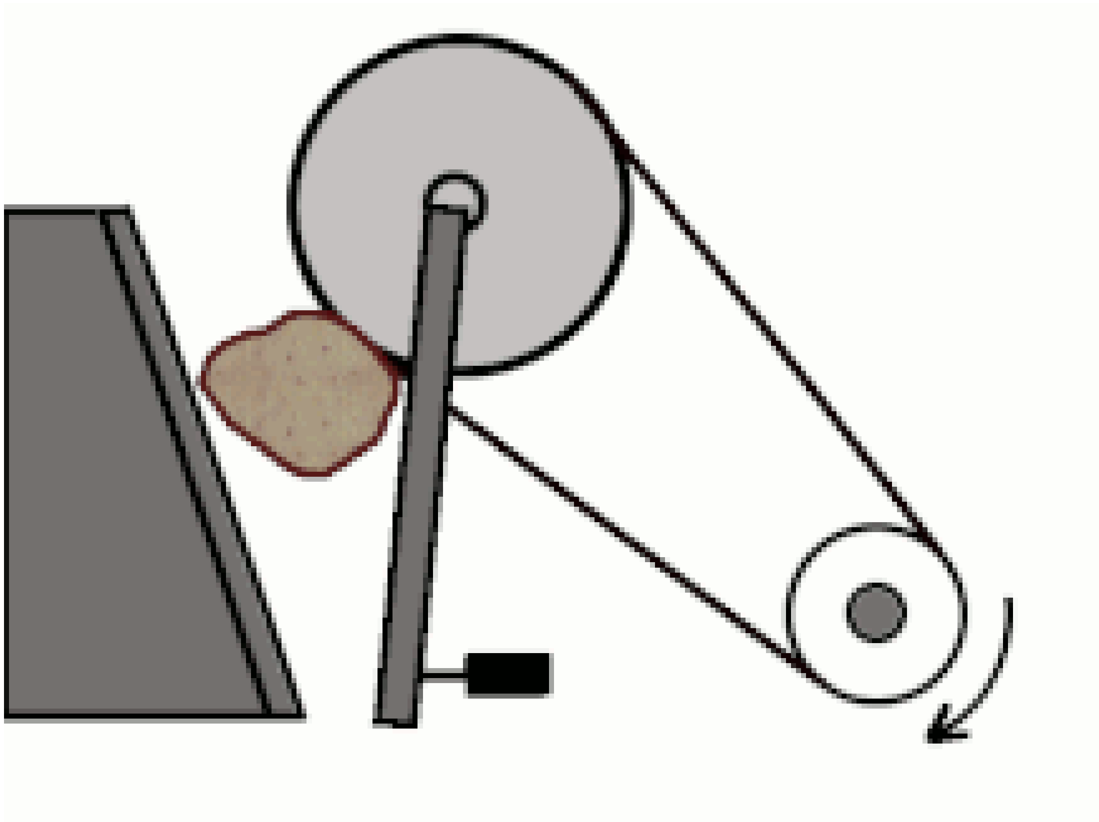


Denver model

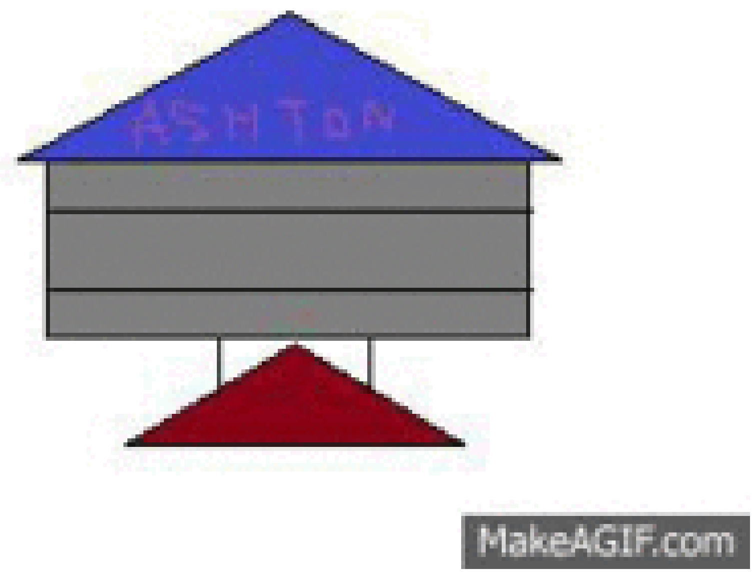
Jaw crusher

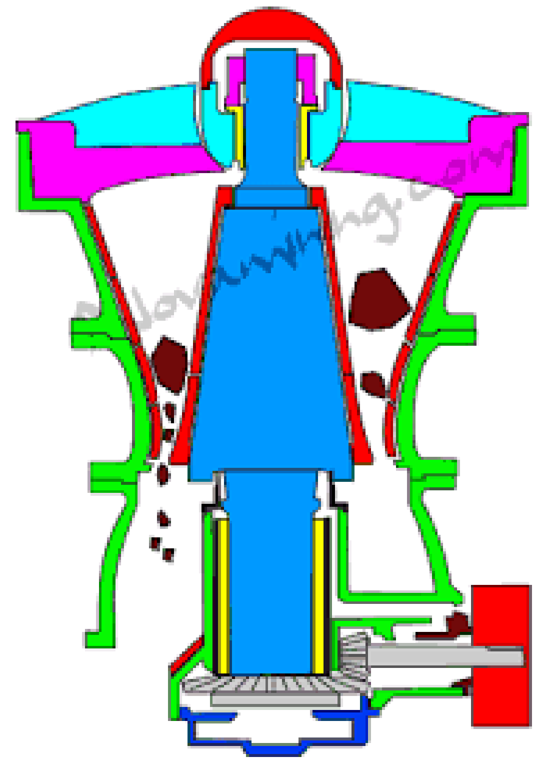


FEED
FLYWHEEL
ECCENTRIC SHAFT
& BEARING
MOVABLE
JAW
FIXED JAW
GROOVE
BLOCK
ASSEMBLY
REVERSIBLE JAW
LINERS
TOGGLE
DRAWBACK
ROD

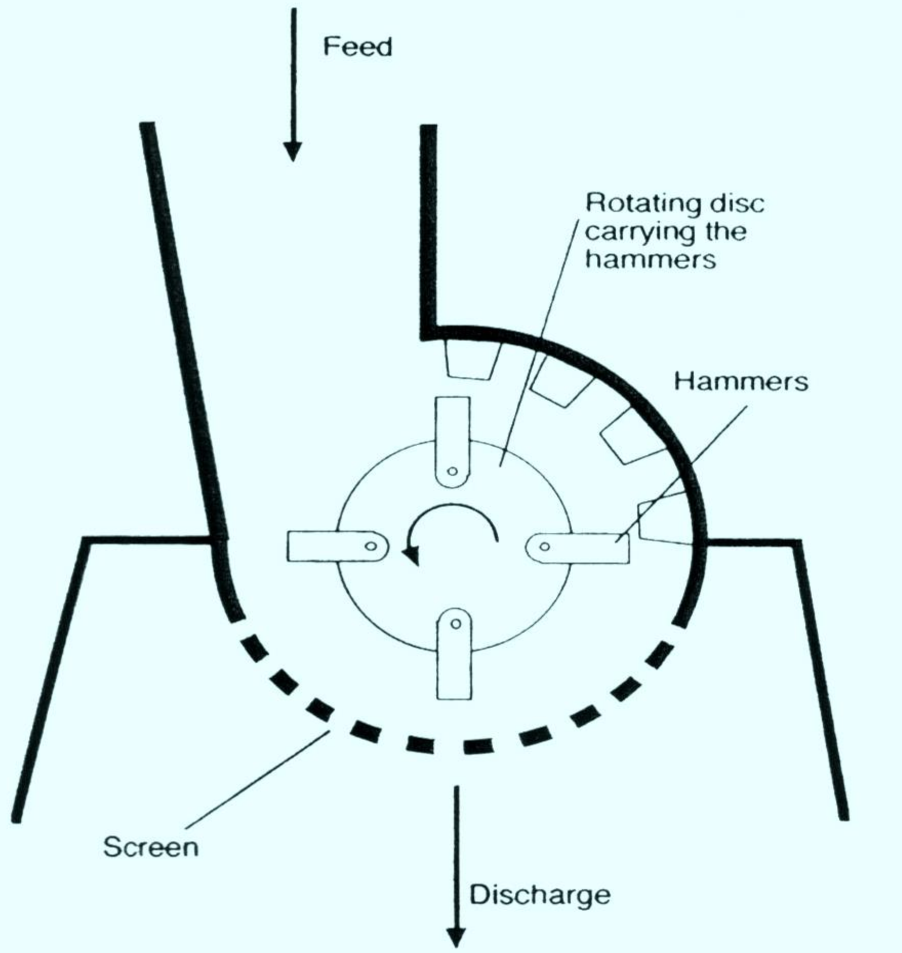


Gyratory Crusher





Hammer Mill

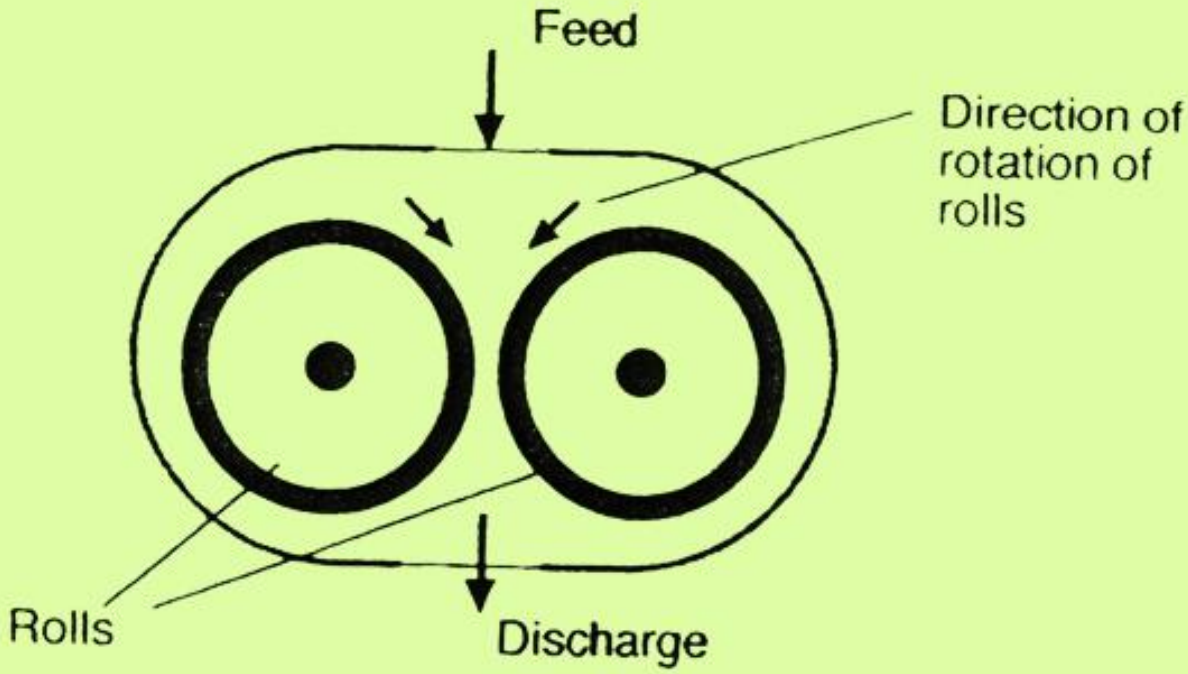


Schematic diagram of a hammer mill

Hammer mill

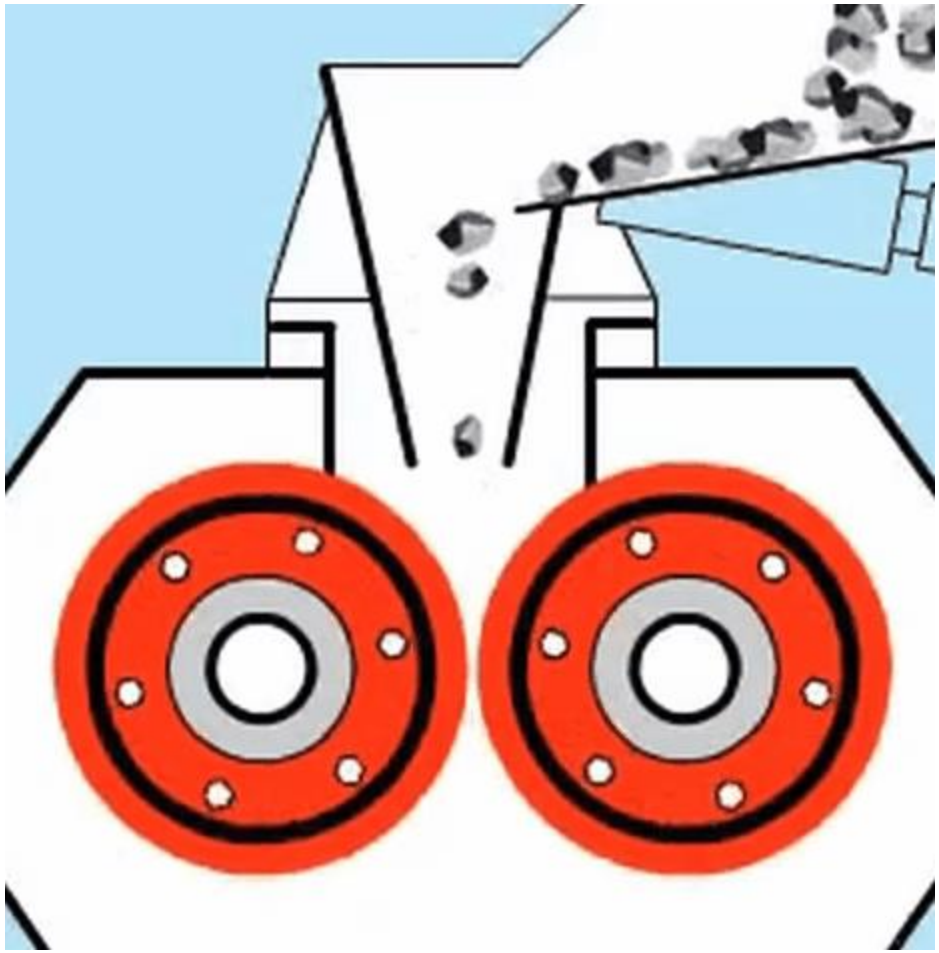
Hammer mill ‘High throughput’

Crushing rolls



Schematic diagram of crushing rolls

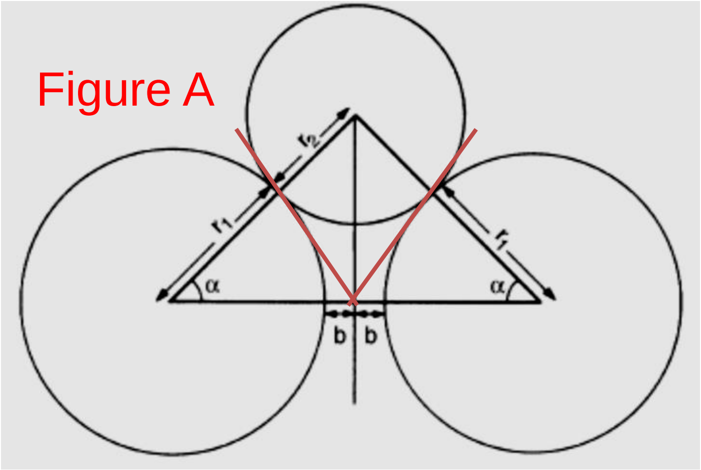
Crushing Rolls



Crushing Rolls

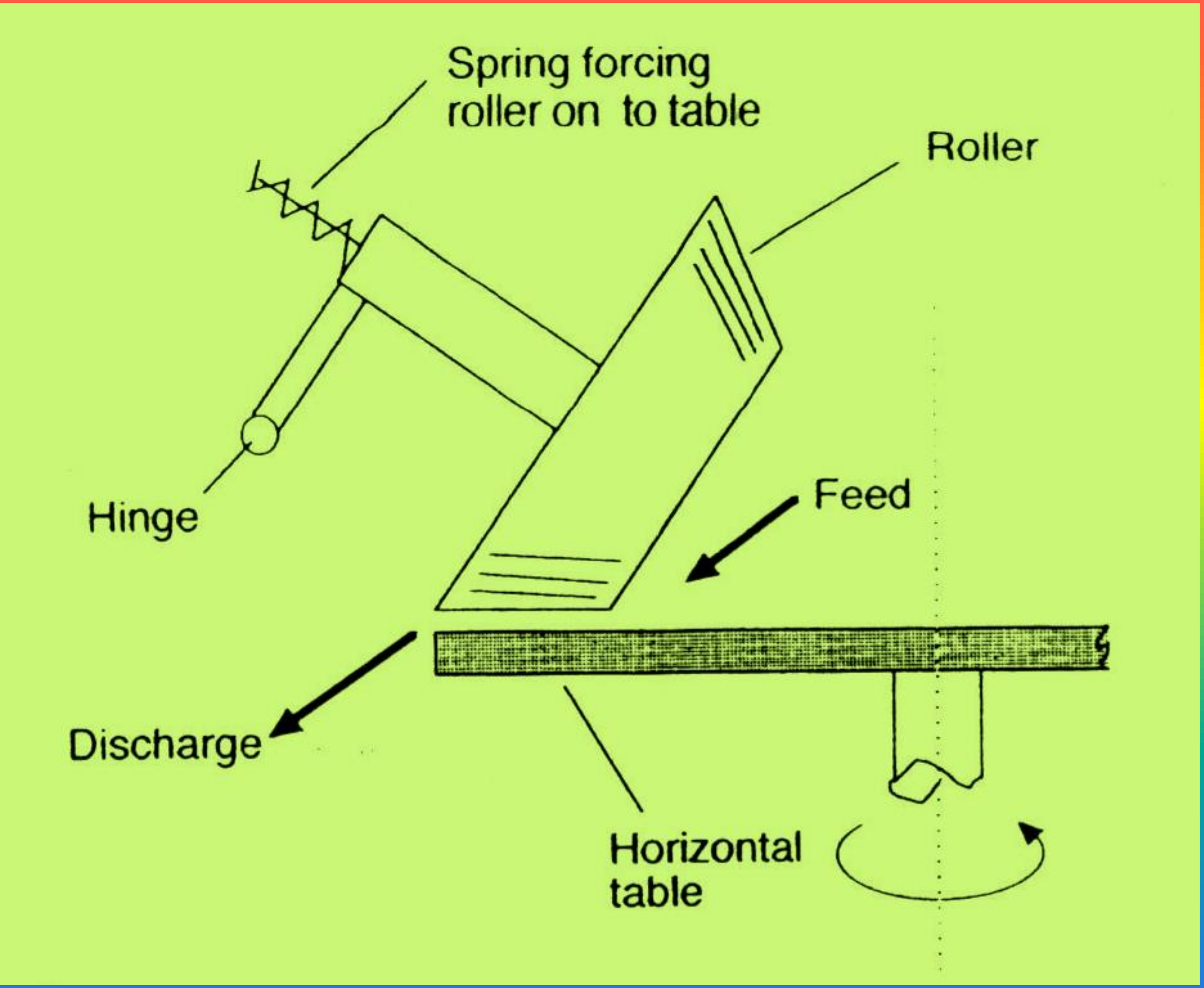
An idealized system where a spherical or cylindrical particle of radius r_2 is being fed to crushing rolls of radius r_1 is shown in Figure A. 2α is the angle of nip, the angle between the two common tangents to the particle and each of the rolls, and 2b is the distance between the rolls. It may be seen from the geometry of the system that the angle of nip is given by:

$$\cos\alpha = \frac{(r_1 + b)}{(r_2 + r_1)}$$

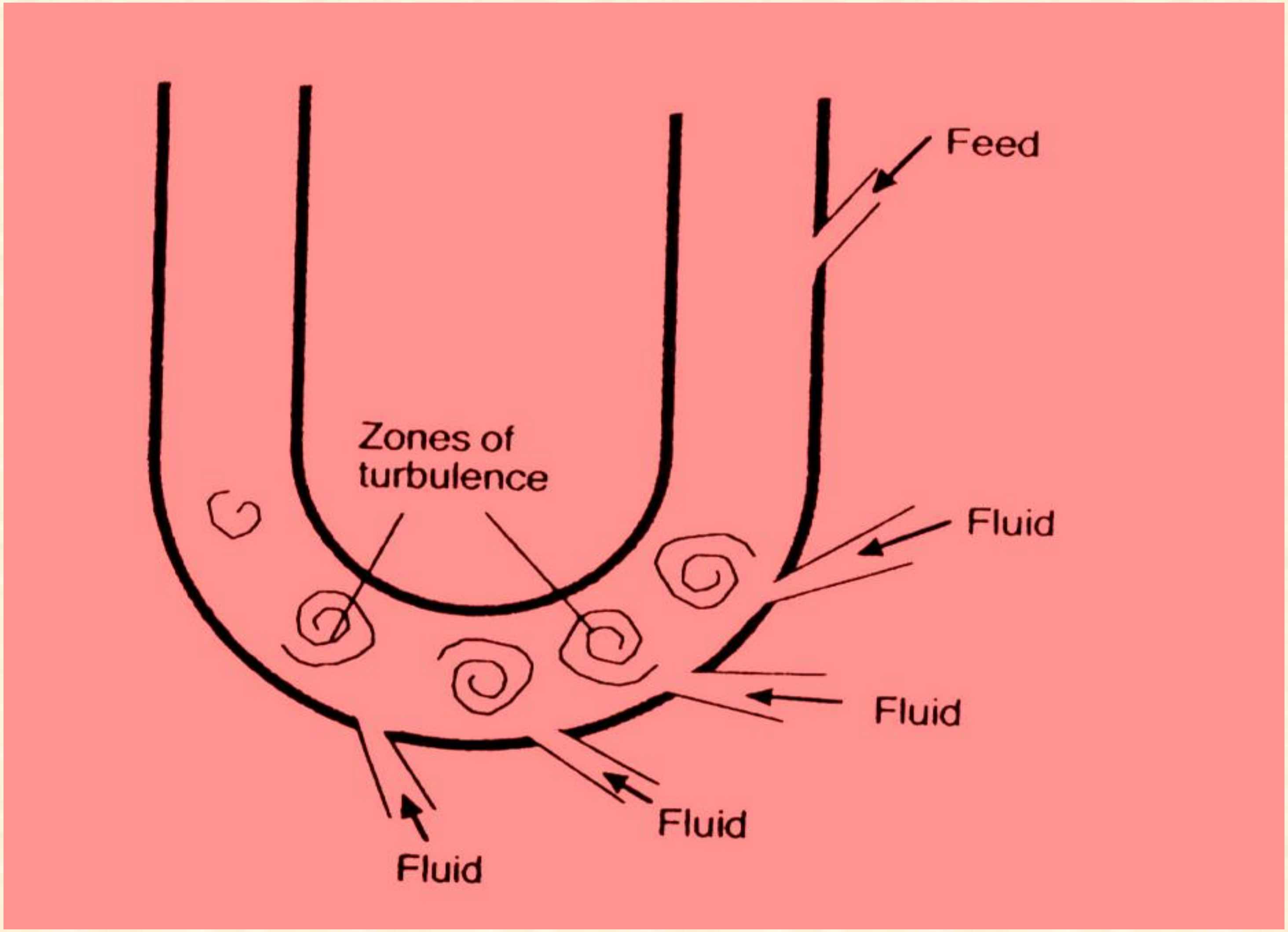


For steel rolls, the angle of nip is not greater than about 32°.
Crushing rolls are extensively used for crushing oil seeds and in the gunpowder industry and they are also suitable for abrasive materials. They are simple in construction and do not give a large percentage of fines.

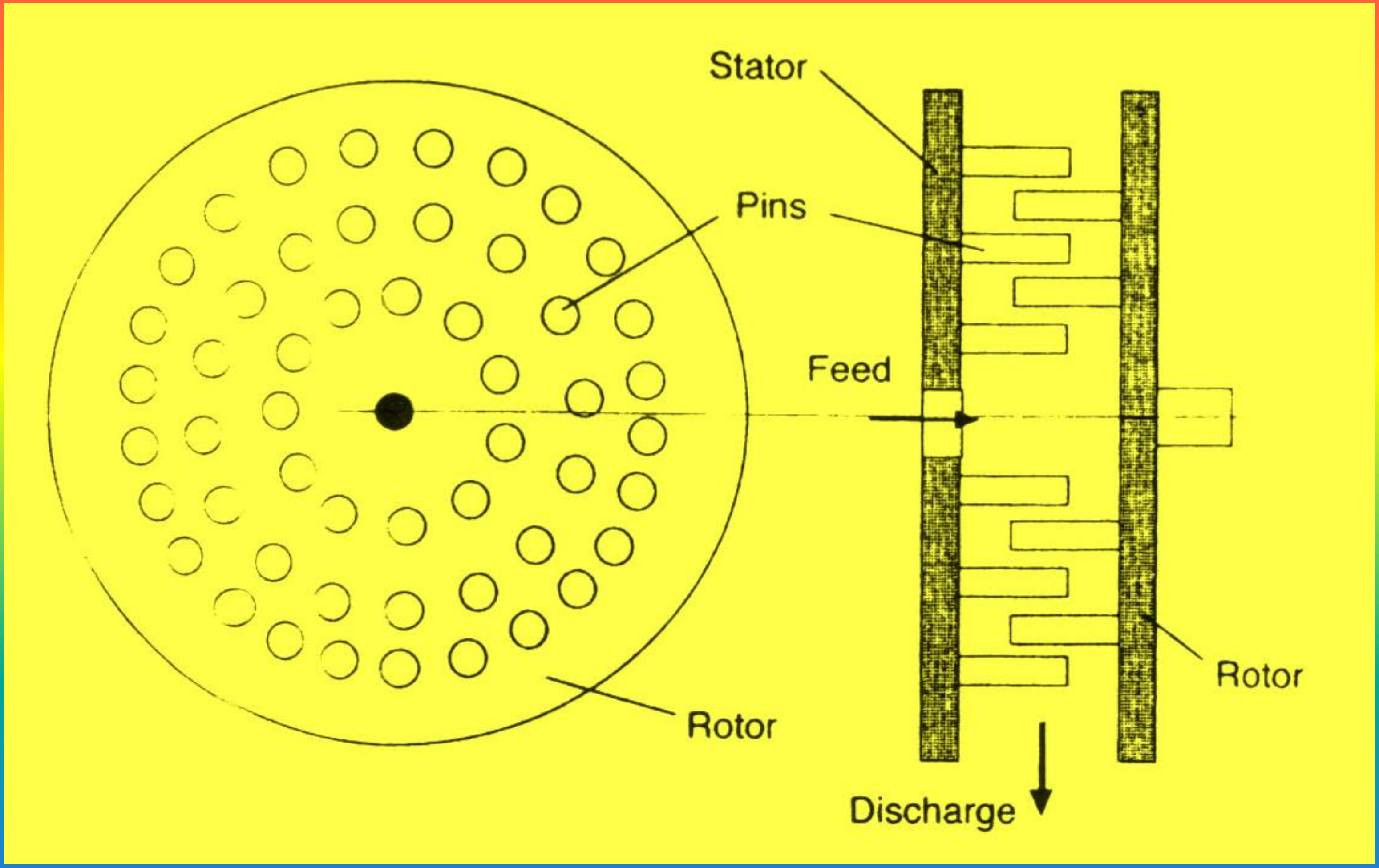
Horizontal table



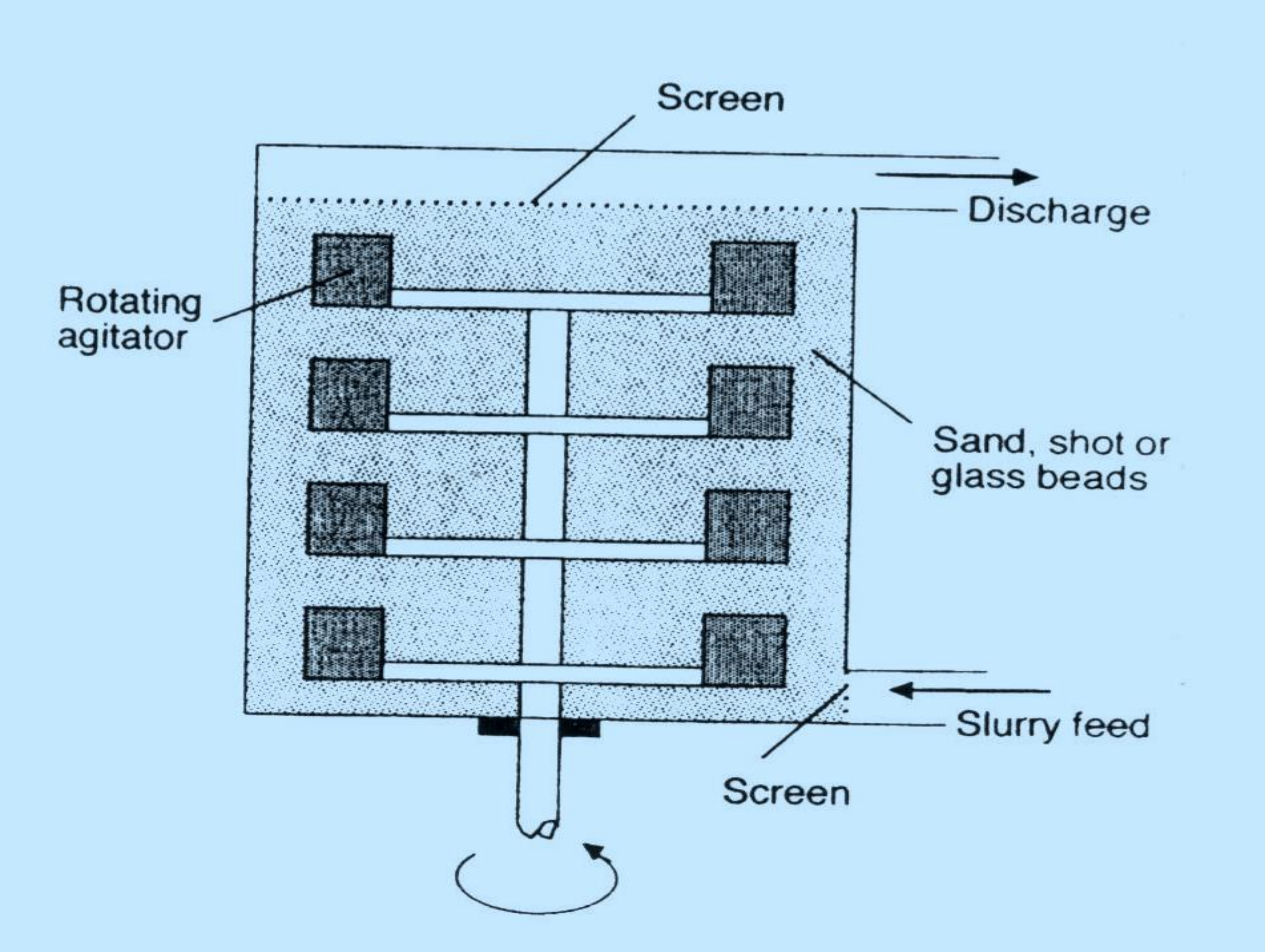
Fluid energy mill



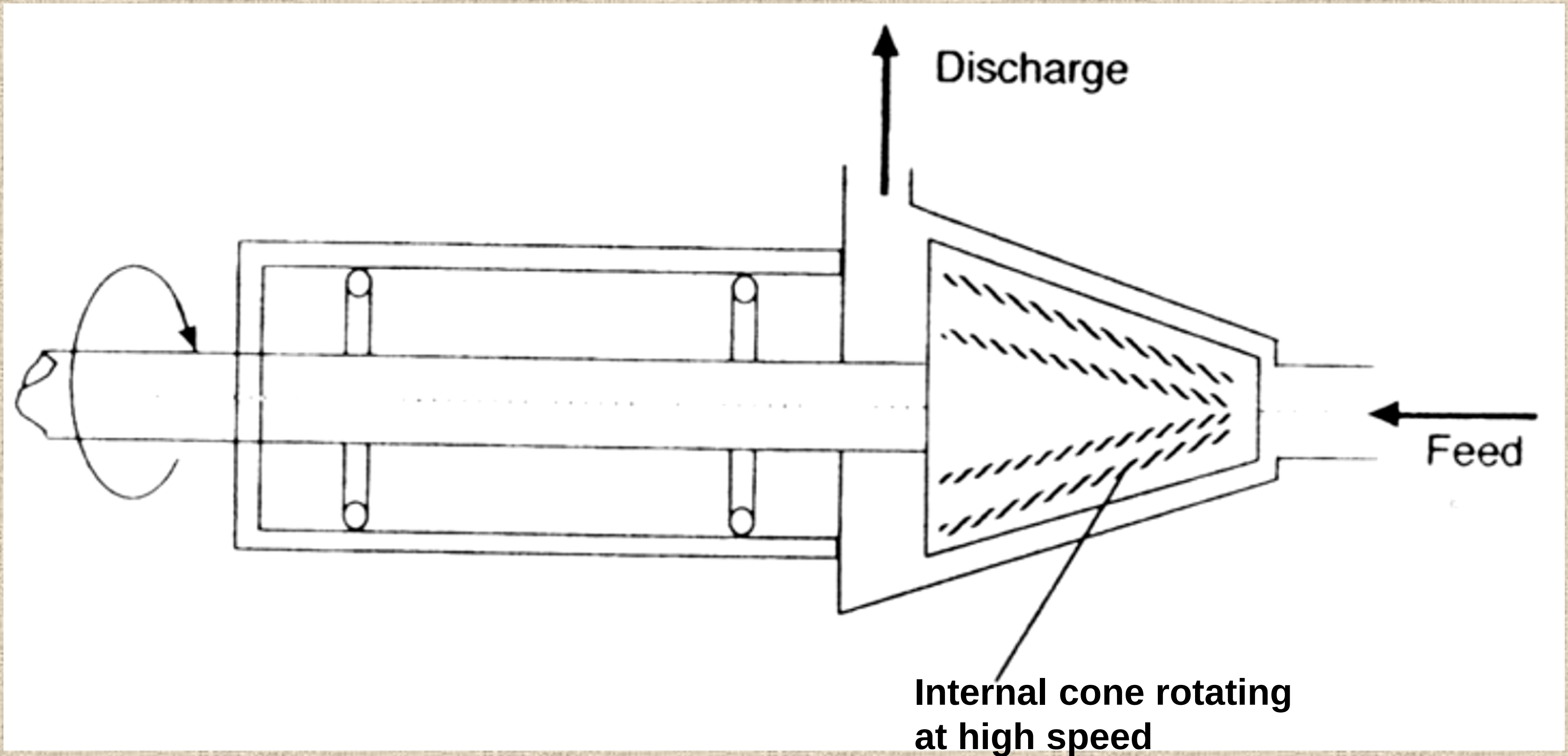
Pin Mill



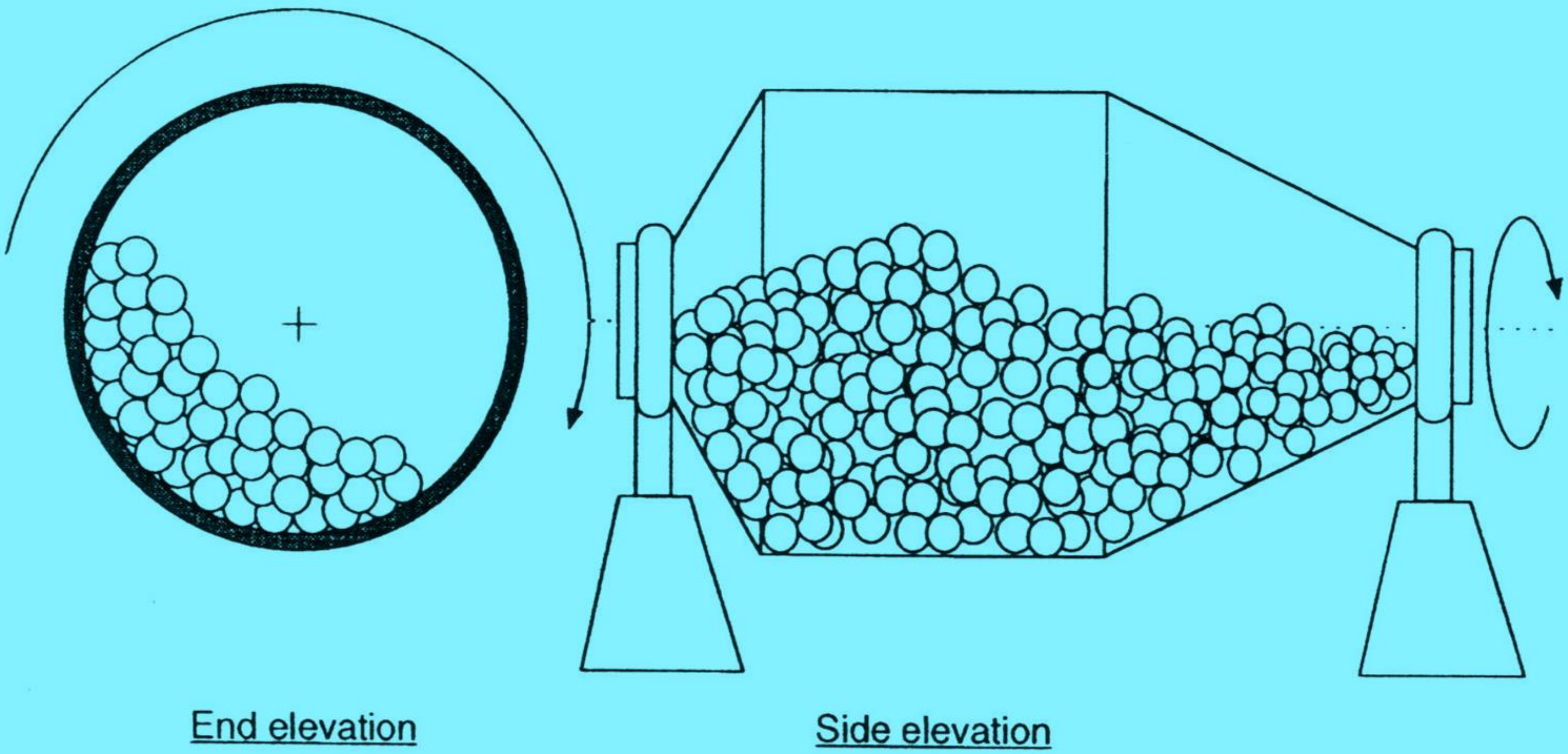
Sand mill


Screen
Discharge
Rotating agitator
Sand, shot or glass beads
Slurry feed
Screen

Colloid mill



Ball mill



Schematiac diagram of a ball mill

The speed of rotation of the mill

- ✓ At low speeds of rotation, the balls simply roll over one another and little crushing action is obtained.
- ✓ At slightly higher speeds, the balls are projected short distances across the mill, and at still higher speeds they are thrown greater distances and considerable wear of the lining of the mill takes place.
- ✓ At very high speeds, the balls are carried right round in contact with the sides of the mill and little relative movement or grinding takes place again.
- ✓ The minimum speed at which the balls are carried round in this manner is called the critical speed of the mill and, under these conditions, there will be no resultant force acting on the ball when it is situated in contact with the lining of the mill in the uppermost position, that is the **centrifugal force** will be exactly equal to **the weight of the ball**.
- ✓ If the mill is rotating at the critical angular velocity ω_c, then:

$$(R\text{-}r)\,\omega_c^2 = g$$

Or

$$\omega_c = \sqrt{\frac{g}{R-r}}$$

The corresponding critical rotational speed, N_c in revolutions per unit time, is given by:

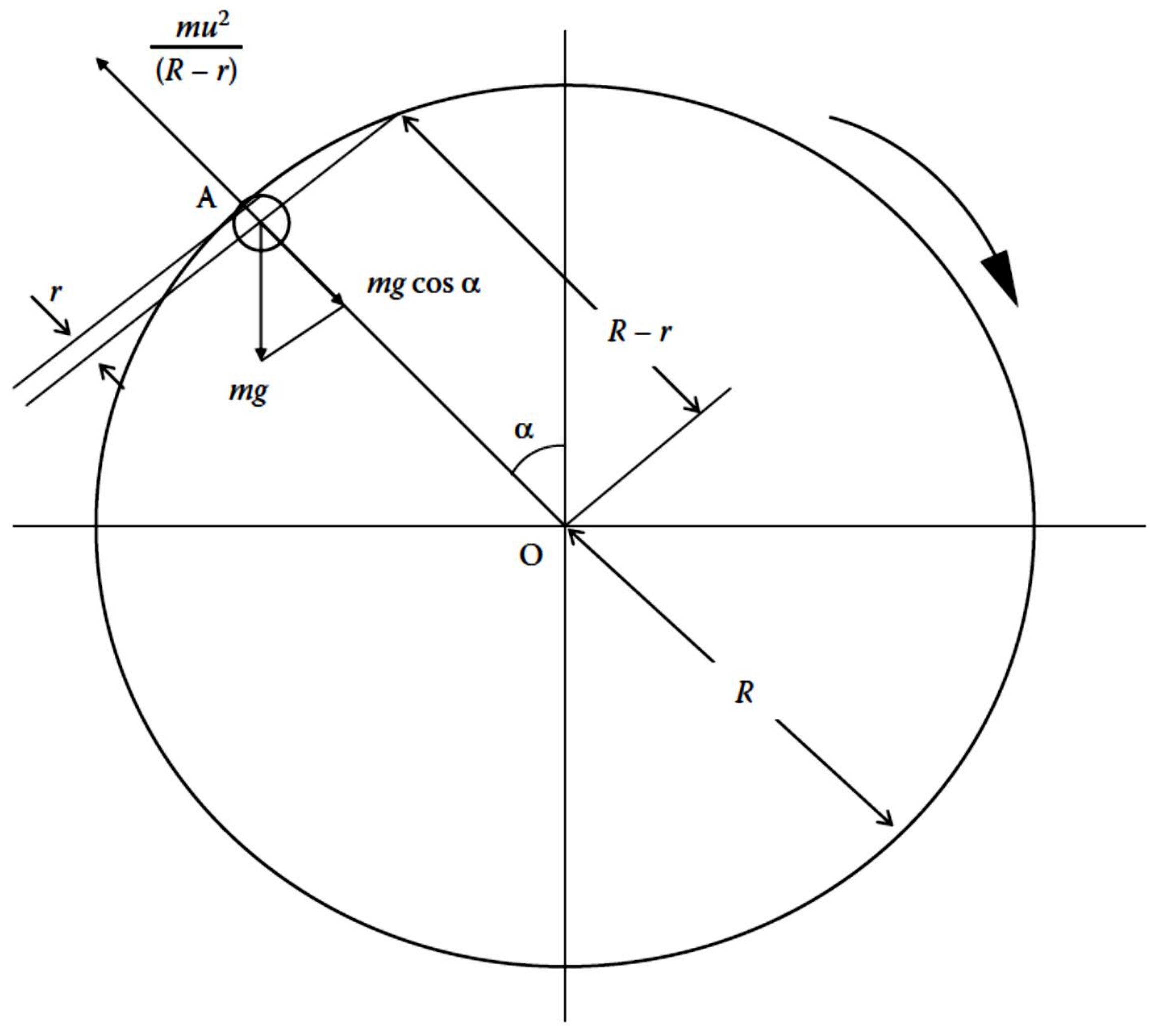
$$N_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{R-r}}$$

In this equation, ***R*** is the radius of the mill, and *r* is the particle or ball radius. It is found that the optimum speed is between one-half and three-quarters of the critical speed.

Note ω_c in units rad/time

$\omega_c = 2\pi N_c$, where N_c in rev/time

Derivation



Note

- Two forces act on the ball:
  - the force of gravity mg, where m is the mass of the ball, and
  - the centrifugal force $mu^2/(R - r)$, where u is the peripheral speed of the center of the ball.
- The centripetal component of the force of gravity is $mg(\cos \alpha)$, which opposes the centrifugal force.
- As long as the centrifugal force exceeds the centripetal one, the particle will not loss contact with the wall. As the angle α decreases, however, the centripetal force increases.

- If the speed does not exceed the critical value, a point is reached where the opposing forces are equal and the particle is nearly to fall.
- The angle at which this occurs is found by equating the centrifugal and centripetal forces, that is,

$$mg\cos\alpha = \frac{mu^2}{(R-r)}$$

hence,

$$\cos\alpha = \frac{u^2}{(R-r)g}$$

The speed u is related to the speed of rotation by the equation

$$u = 2\pi N(R-r)$$

$\therefore$ By substitution we obtain

$$\cos\alpha = \frac{4\pi^2 N^2 (R-r)}{g}$$

At the critical speed, $\alpha = 0$, consequently $\cos \alpha = 1$, and N becomes the critical speed N_c. With all these considerations, **the previous equation becomes**

$$N_c = \frac{1}{2\pi}\sqrt{\frac{g}{(R - r)}}$$

Example 1

A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50 to 10 mm with the consumption of energy of 13.0 kW/(kg/s). What would be the consumption of energy needed to crush the same material of an average size of 75 mm to an average size of 25 mm:

a) assuming Rittinger's law applies?
b) assuming Kick's law applies?

Which of these results would be regarded as being more reliable and why?

Solution

(a) *Rittinger's law.*

This is given by:

$$E = K_R f_c[(1/L_2) - (1/L_1)]$$

(b) *Kick's law.*

This is given by:

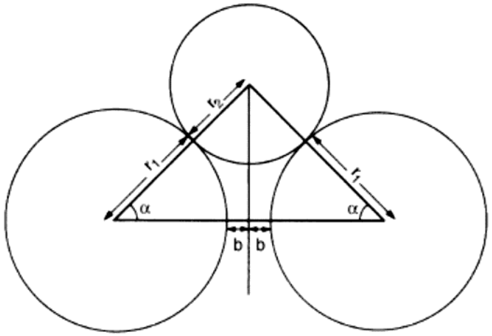
$$E = K_K f_c \ln(L_1/L_2)$$

The size range involved in this case should be considered as that for coarse crushing. Because Kick's law is more accurate for coarse crushing, this would be taken as given the more reliable result.

Example 2

If crushing rolls, 1 m in diameter, are set so that the crushing surfaces are 12.5 mm apart and the angle of nip is 31°, what is the maximum size of particle which should be fed to the rolls?
If the actual capacity of the machine is 12 per cent of the theoretical, calculate the throughput in kg/s when running at 4 rad/s (2.0 m/s) if the working face of the rolls is 0.4 m long and the bulk density of the feed is 2500 kg/m^3.

Solution



The particle size may be obtained from:

$$\cos\alpha = (r_1 + b)/(r_1 + r_2)$$

In this case: $2\alpha = 31°$ and $\cos\alpha = 0.964$, $b = (12.5/2) = 6.25$ mm or 0.00625 m and:

The cross sectional area for flow $= (0.0125 \times 0.4) = 0.005\ \text{m}^2$

and the volumetric flowrate $= (2.0 \times 0.005) = 0.010\ \text{m}^3/\text{s}$.
Thus, the actual throughput $= (0.010 \times 12)/100 = 0.0012\ \text{m}^3/\text{s}$

or: $$(0.0012 \times 2500) = \underline{\underline{3.0\ \text{kg/s}}}$$

Example 3

A ball mill, 1.2 m in diameter, is run at 0.80 Hz and it is found that the mill is not working properly. Should any modification in the conditions of operation be suggested?

Solution

The angular velocity is given by:

$$\omega_c = \sqrt{\frac{g}{R\text{-}r}}$$

In this equation, $R\text{-}r$ = (radius of the mill – radius of the particle). For small particles, $R\text{-}r \doteq 0.6$ m and hence:

$$\omega_c = \sqrt{(9.81/0.6)} = 4.04 \text{ rad/s}$$

The actual speed $= (2\pi \times 0.80) = 5.02$ rad/s and hence it may be concluded that the speed of rotation is too high and that the balls are being carried round in contact with the sides of the mill with little relative movement or grinding taking place.
The optimum speed of rotation lies in the range $(0.5–0.75)\omega_c$, say $0.6\omega_c$ or:

$$(0.6 \times 4.04) = 2.42 \text{ rad/s}$$

This is equivalent to: $(2.42/2\pi) = 0.39$ Hz, or, in simple terms:

the speed of rotation should be halved.

Note:
1 Hz 2π or 2 * 3.14 = 6.28 rad/s
2 Hz 4π or 4 * 3.14 = 12.56 rad/s

Radian speed to linear speed
$u = r * \omega$ if r in m and ω in rad/s hence, u in m/s

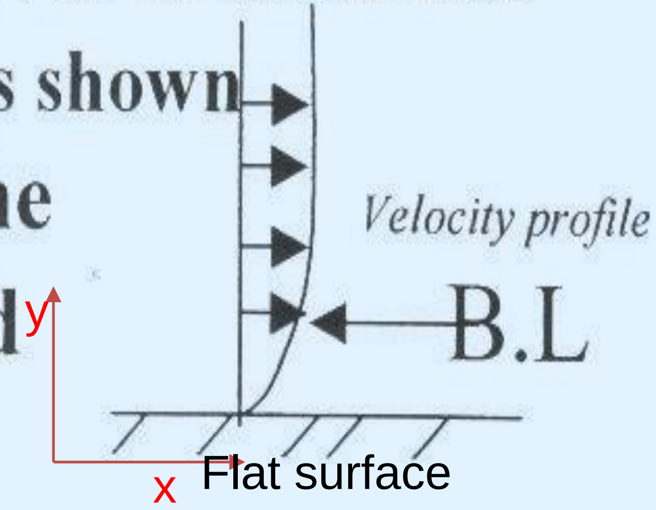
Motion of Particles in a Fluid

Motion of Particles in a Fluid

INTRODUCTION

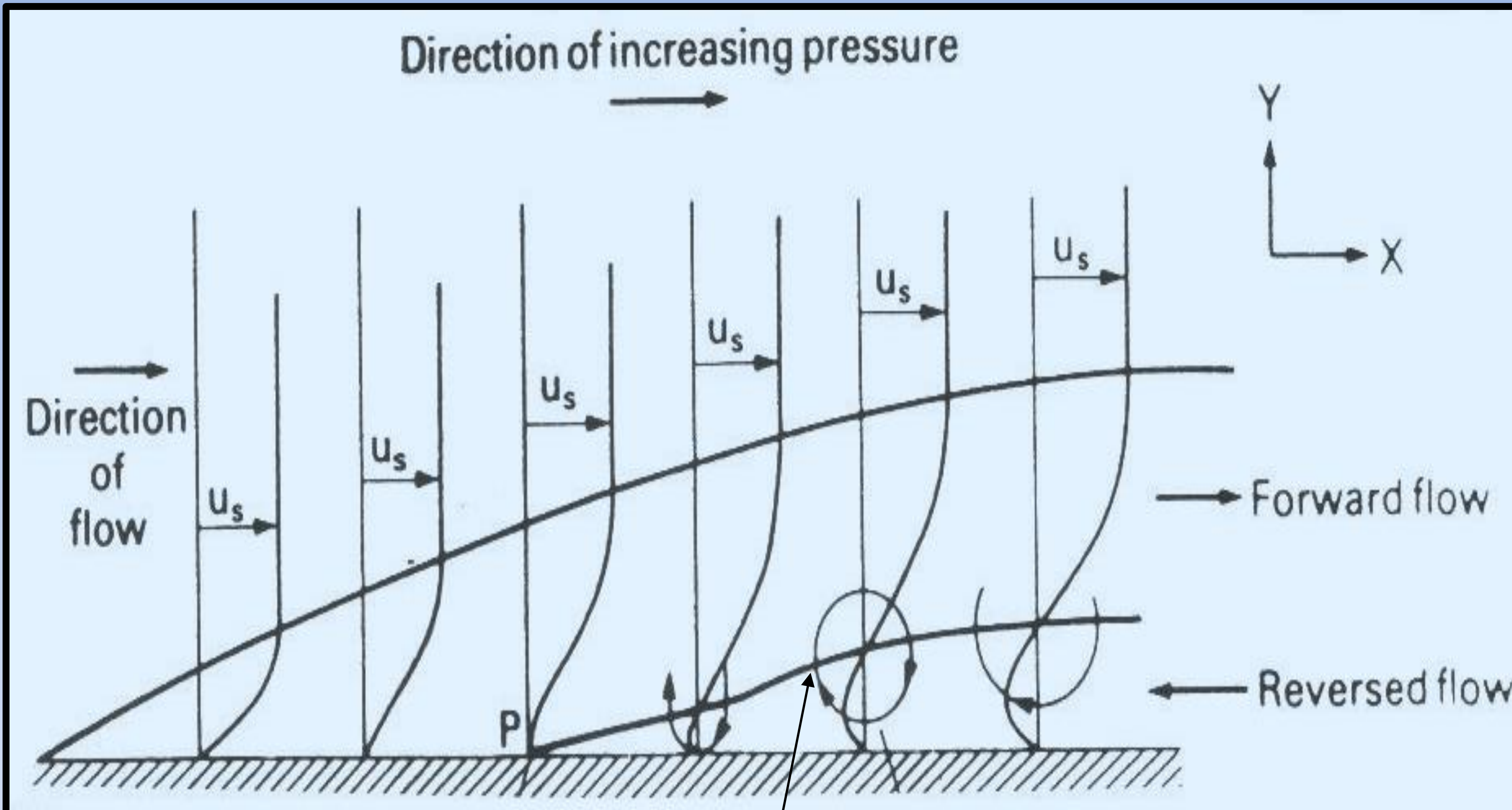
1. Flow over a flat surface

   a. Drag force ~ fluid flows over a solid flat surface. Velocity gradient as shown Drag force arises due to the effect Of retardation of fluid at the Surface



b. Boundary layer (B.L) ~ a specific region where the velocity profile is changed with distance.**(y-direction).**

Thickness of B.L = f (distance from the leading edge)



Eddy formation

NOTE: The force acting on the fluid at some point in the boundary layer may then be sufficient to bring it to rest or to cause flow in the reverse direction with the result that an eddy current is set up. A region of reverse flow then exists near the surface where the boundary layer has separated as shown in the above Figure.

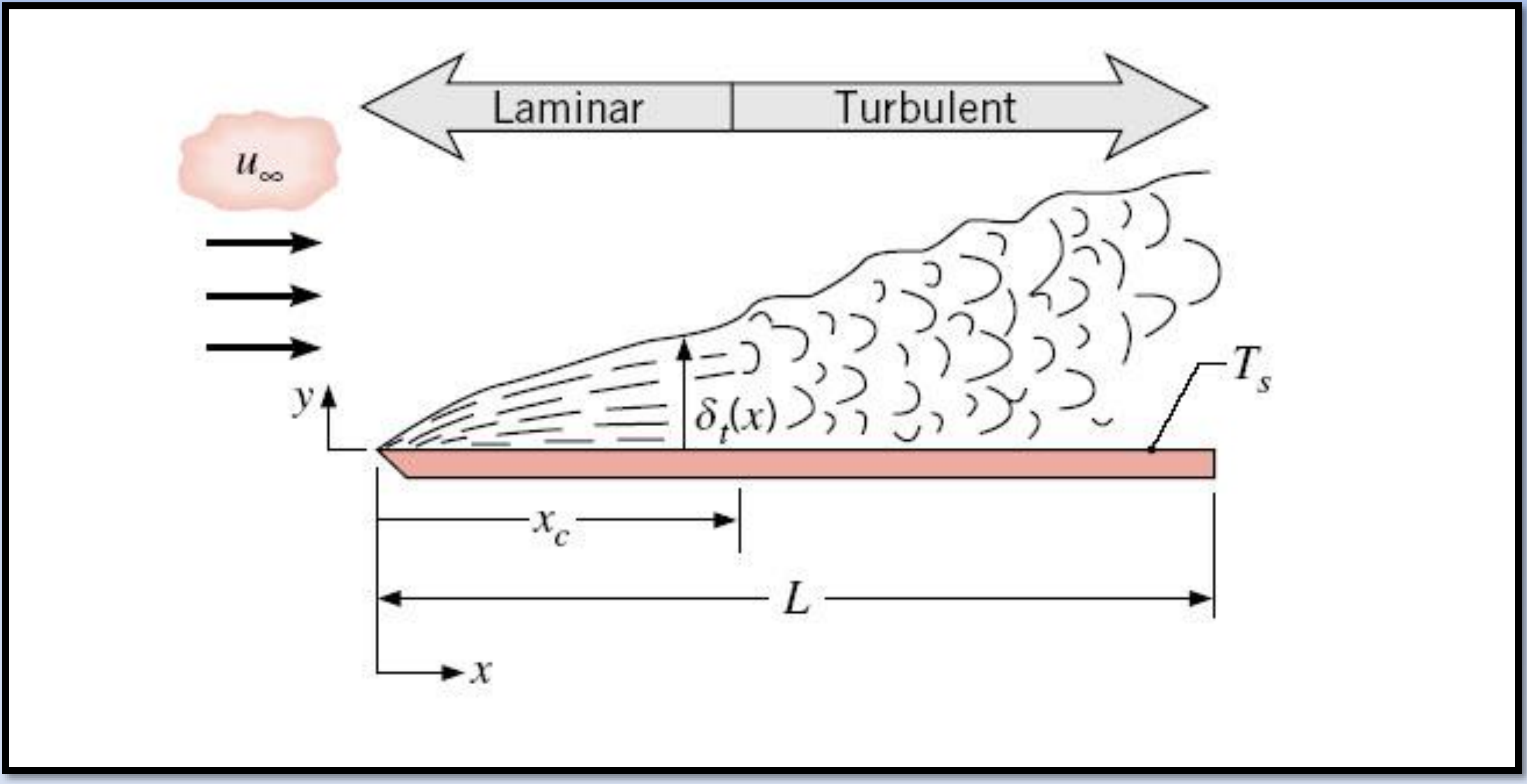
Notes

- The velocity at any pt. in the B.L varies from 0 at surface to the velocity of undisturbed stream, U_s.
- For a short distance on the surface, the flow is streamline.
- At a certain critical distance, x_c, the flow is changed for streamline to turbulent, except thin

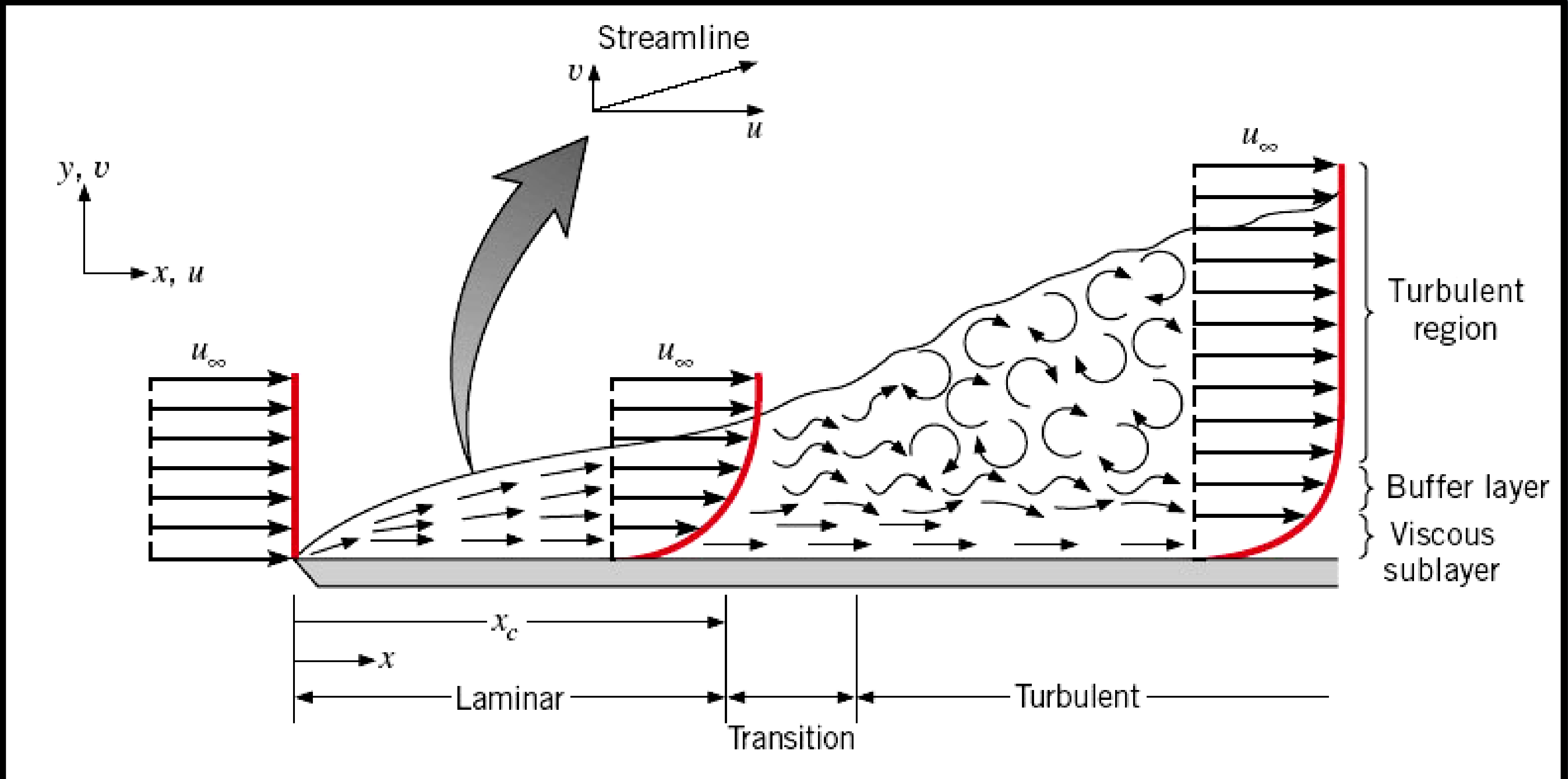
layer near the surface where it remains streamline ‘called laminar sub-layer’.

- There is a thin layer between the sub-layer and the turbulent , the regime is transient and the layer is called ‘buffer layer’!
- X_c = F (shape of leading edge, roughness of solid surface, properties of fluid, fluid velocity)
- Transition in regime can be examined via Re.

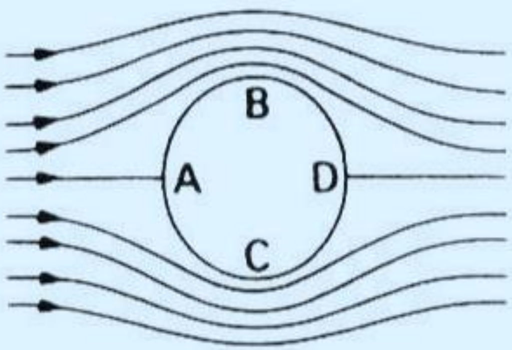
Laminar and turbulent flow over a flat surface

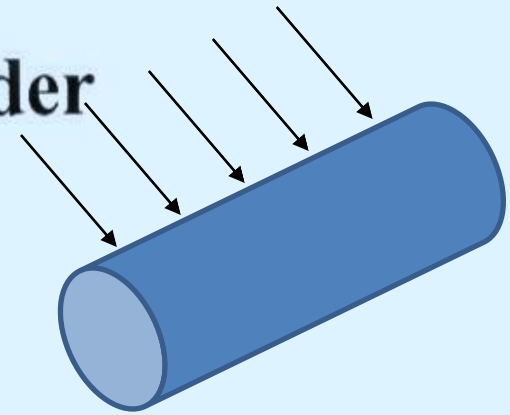


Velocity B.L developed on a flat surface



2. Flow round (past) a Cylinder





Flow round a·cylinder

- For viscous/non-viscous fluid, the velocity, u, varies round the wall of cylinder $\Rightarrow$ at A & D the fluid is stagnant i.e $u = 0$ $\Rightarrow$ whilst at B & C u is max. { K.E is max at B & C and zero at A & D

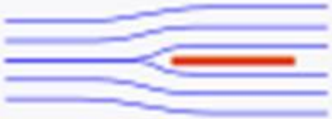
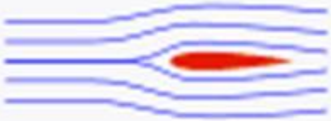
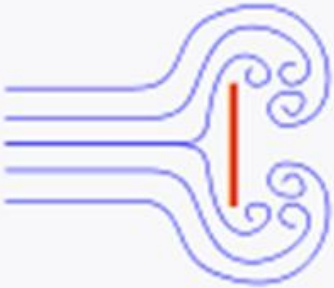
- Also the Pressure falls from A to B and rises from B to D
- $\sum$ {K.E + Pressure Energy} is constant at all the points on the surface.

Note 1: when the pressure falls in the direction of flow, the retardation of the fluid will be less and the B.L will be thinner and visa versa.

Note 2: Thin B.L is noted in the front of the cylinder, whilst a thick B.L takes place at the back or in the wake of the cylinder that tends to separate from the surface. If separation occurs, the eddy currents are builds up in the wake and drag force (form drag)* is made up.

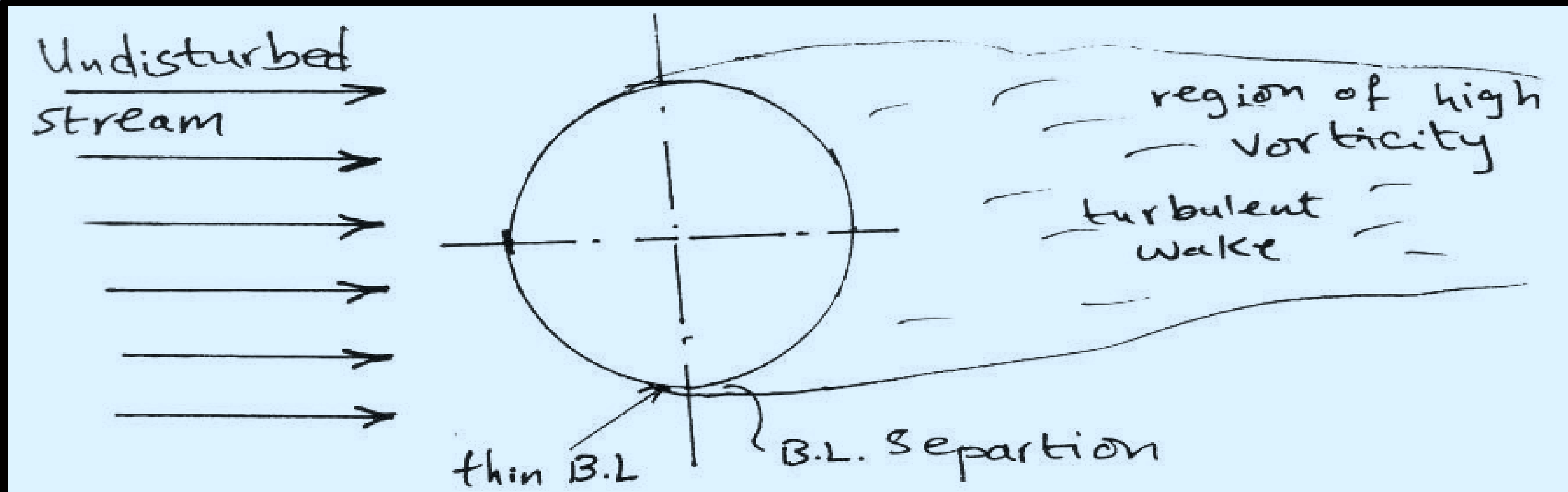
**** For details see "Transport phenomena 1 and fluid Mechanics courses"***

Note

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%

Wall drag

Flow past a sphere



- Similar observations as flow past a cylinder.
- At large values of Re number, we can see two main zones: (i) a region of high vorticity comprising a thin B.L over the front half of the sphere and a turbulent wake on the downstream side of the sphere, and (ii) an external irrotational flow region outside the B.L and wake.

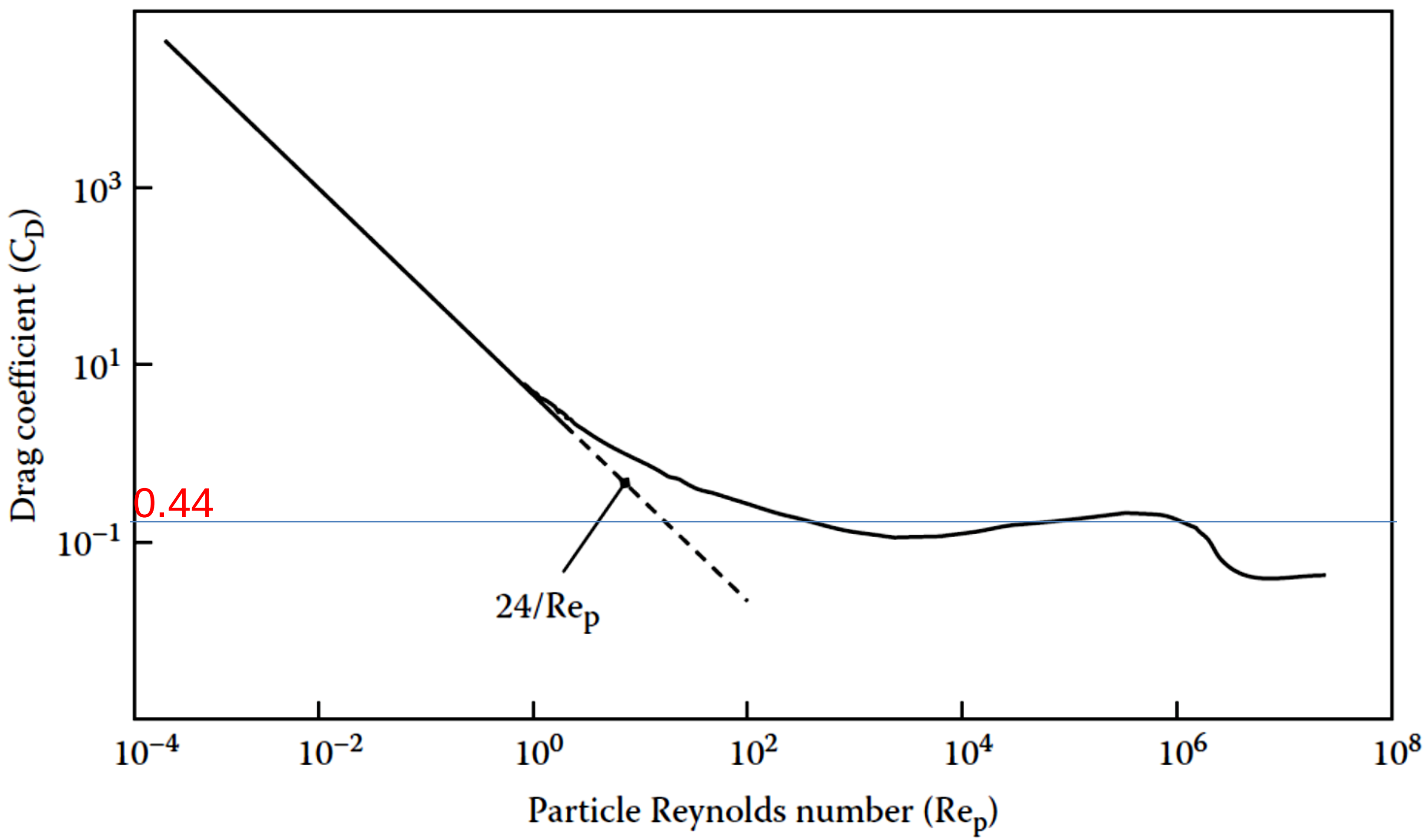
Drag force on a spherical particle

The drag force, F, on a sphere is usually given in terms of a drag coefficient, C_D.

$$C_D = \frac{F}{\frac{1}{2}\rho u^2 A}$$

Note: C_D is analogous to friction factor, f, for pipe flow 'funning friction factor'.

Drag coefficient versus particle Reynolds number for spherical particles



Stokes showed a formula for the drag force for a sphere moving at a low velocity (creeping motion) in a continuos fluid.

$$F = 3\pi\mu\, d\, u$$

Where

F: total drag-resisting motion

ρ, μ: density and viscosity of a fluid, respectively.

d: diameter of a spherical particle.

u: velocity of the fluid relative to the particle.

A: projected area

Note 1: F consists of two components; a pressure drag force, F_p, and a shear stress force, F_s.

$$F_p = 2\pi \mu \, d \, u \qquad\qquad F_s = \pi \mu \, d \, u$$

Note 2: Experimentally, Stoke's Law is found to hold almost exactly for single particle Reynolds number, Re≤0.1.

$$Particle \quad Re = ud\rho/\mu \quad and \quad C_D = f\,(Re)$$

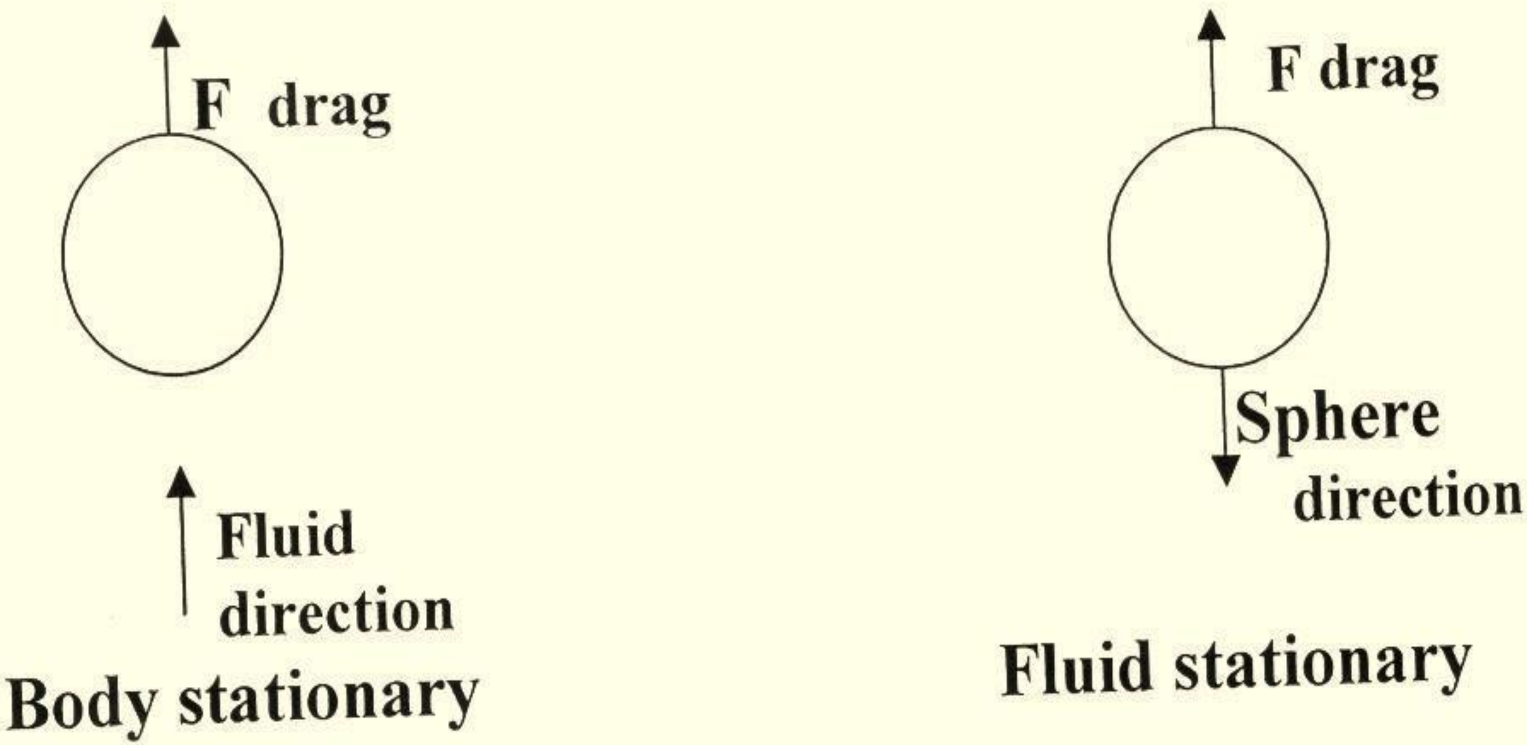
***Where ρ is the density of the fluid, μ is the viscosity* of the fluid, *d is the diameter of the sphere, and u is the velocity of the fluid relative to* the particle.**

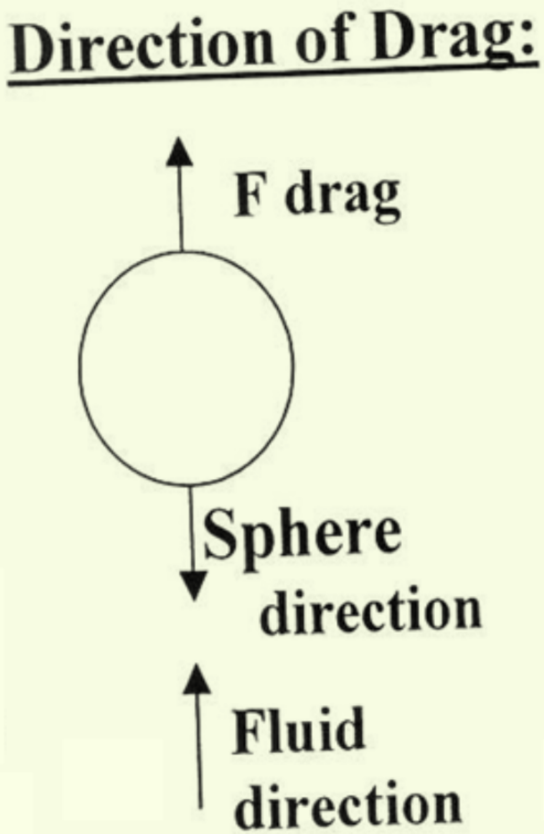
For spherical particle $A=(\pi/4)d^2$

$$\therefore C_D = \frac{F}{\frac{1}{2}\rho\, u^2 \left(\frac{\pi}{4}\right) d^2}$$

$$\therefore \quad F = \frac{\pi}{8} C_D \rho\, u^2\, d^2$$

Direction of Drag:





Both of Fluid and body are moving

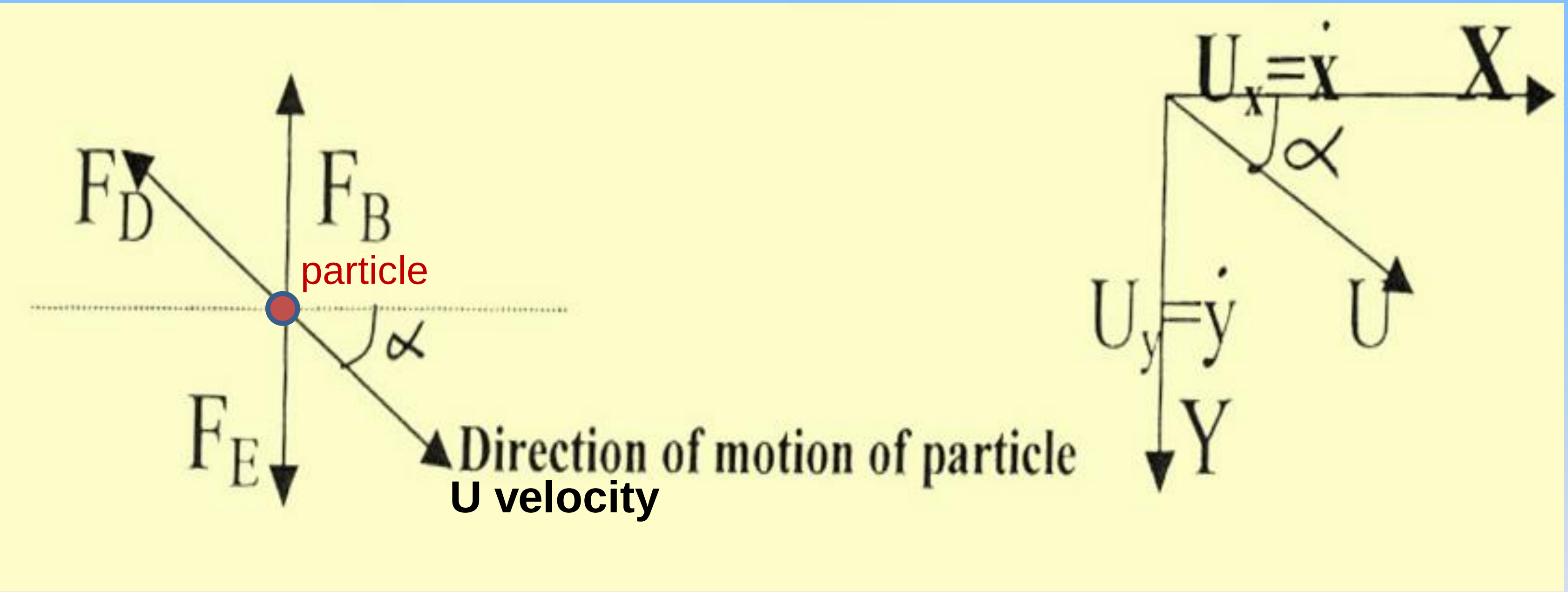
Equations of motion of particles:

Forces acting on particle

- ***External force*** $F_E \Rightarrow$ *gravity, centrifugal etc.*
- ***Buoyancy force*** $F_B \Rightarrow$ *if* $\rho_f \neq \rho_p$ *acts vertically upwards.*
- ***Drag force*** $F_D \Rightarrow$ *acts parallel to relative velocity in opposite direction.*

Motion of a particle in gravitational field

- 2 dimensional motion



$$\dot{x} = U \cos\alpha$$
$$\dot{y} = U \sin\alpha$$
$$U = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$F_E = m\ g$$
$$F_B = m\ \frac{\rho_f}{\rho_p}\ g$$
$$F_D = C_D \frac{1}{2} \rho_f\ U^2\ A$$

Force balance in X direction

Gravity – Buoyancy – drag = acceleration force

$$-F_D \cos\alpha = m\ddot{x}$$

$$-\frac{1}{2}\rho_f C_D A(\dot{x}^2 + \dot{y}^2)\frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = m\ddot{x}$$

$$-\frac{1}{2}\rho_f C_D A\dot{x}\sqrt{(\dot{x}^2 + \dot{y}^2)} = m\ddot{x}$$

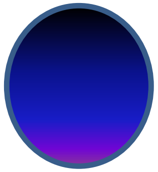
Force balance in Y direction

Gravity – Buoyancy – drag = acceleration force

$$mg\left(1-\frac{\rho_f}{\rho_p}\right)-\frac{1}{2}\rho_f C_D A\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}=m\ddot{y}$$

For spherical particle

For Sphere $\Rightarrow$ $A=(\pi/4)\,d^2$, $m=(\pi/6)\,d^3\,\rho_p$



$$\ddot{x} = -\frac{3}{4}\frac{C_D\,\rho_f}{d\,\rho_p}\,\dot{x}\,\sqrt{\dot{x}^2+\dot{y}^2} \qquad (1)$$

$$\ddot{y} = -\frac{3}{4}\frac{C_D\,\rho_f}{d\,\rho_p}\,\dot{y}\,\sqrt{\dot{x}^2+\dot{y}^2} + g\left(1-\frac{\rho_f}{\rho_p}\right) \qquad (2)$$

Note

<u>Special case</u>

Spherical particle in stokes' regime Re<0.1$\Rightarrow C_D=24/Re$

$$\ddot{x}=-\frac{3}{4}\left(\frac{24\mu}{\rho_f d u}\right)\left(\frac{\rho_f}{\rho_p d}\right)\dot{x}u$$

$$\ddot{x}=-\frac{18\mu}{d^2\rho_p}\dot{x}=-a\dot{x}$$

Note

Similarly

$$\ddot{y} = -a\dot{y} + g\left(1 - \frac{\rho_f}{\rho_p}\right) = -a\dot{y} + b$$

where a and b are constants.

In order to solve the previous equations, use the following B.C $\Rightarrow$at t=0 $\rightarrow$ x=0, y=0

$$\dot{x} = u_0, \quad \dot{y} = v_o$$

!!! Discussion | Discussion !!!

Equilibrium or terminal velocity

- Assume a particle that falls from rest in a fluid. The particle will initially accelerate as the shear stress drag (which increases with velocity) will be small. As the particle accelerates the drag force increases, causing the acceleration to reduce. Later on a force balance will be achieved where the acceleration is zero and a maximum or terminal velocity is reached.

Single Particle terminal velocity

- This is known as the single particle terminal velocity.

- For a spherical particle, the following equation becomes:

gravity –buoyancy-drag = acceleration force

$$F_E - F_B - F_D = 0$$

Or

Or

$$F_D = F_E - F_B = mg\,(1 - \rho_f/\rho_p)$$

$$F_D = (\pi/6)\, d^3\, \rho_p\, g\,(1 - \rho_f/\rho_p)$$

$$\therefore\ F_D = (\pi/6)\, d^3\, g\,(\rho_p - \rho_f)$$

for stokes' regime $F_D = 3\pi\mu u_o d$, where u_o is terminal velocity.

$$\therefore\ 3\pi\mu u_o d = (\pi/6)\, d^3\, g\,(\rho_p - \rho_f)$$

$$u_o = \frac{d^2 g}{18\mu}\left(\rho_p - \rho_f\right) = \frac{b}{a}$$

Note that in the stokes' law region the terminal velocity is proportional to the square of the particle diameter.

For the form drag regime (Newton's Law), C_D=0.44

$$F_D = (1/2)\,\rho_f\,u^2\,A\,C_D$$

At force balance

$$\frac{0.44}{8}\pi\,d^2\rho_f\,u_o^2 = \frac{d^3}{6}\pi\,g(\rho_p - \rho_f)$$

$$u_o = 1.75\left(d\,g\,\frac{(\rho_p - \rho_f)}{\rho_f}\right)^{1/2}$$

<u>Note</u> that in this region the terminal velocity is independent of the fluid viscosity and proportional to the square root of the particle size.

General Method to obtain Terminal Velocity

$$\because \frac{C_D}{8}\pi d^2 \rho_f u_o^2 = \frac{d^3\pi}{6} g(\rho_p - \rho_f)$$

$$\therefore C_D = \frac{4}{3}\frac{d\ g}{u_o^2}\left(\frac{\rho_p}{\rho_f} - 1\right) \text{..............................} (a)$$

$$Re = \frac{u_o\, d\ \rho_f}{\mu} \text{..} (b)$$

Using equations a and b, form the following dimensionless equations:

$$C_D Re^2 = \frac{4}{3}\frac{d^3 g \rho_f}{\mu^2}(\rho_p - \rho_f) = \frac{4}{3} Ga \quad \text{..........}(c)$$

$$\frac{C_D}{Re} = \frac{4}{3}\frac{\mu g}{u_0^3}\left(\frac{\rho_p - \rho_f}{\rho_f^2}\right) \quad \text{....................................}(d)$$

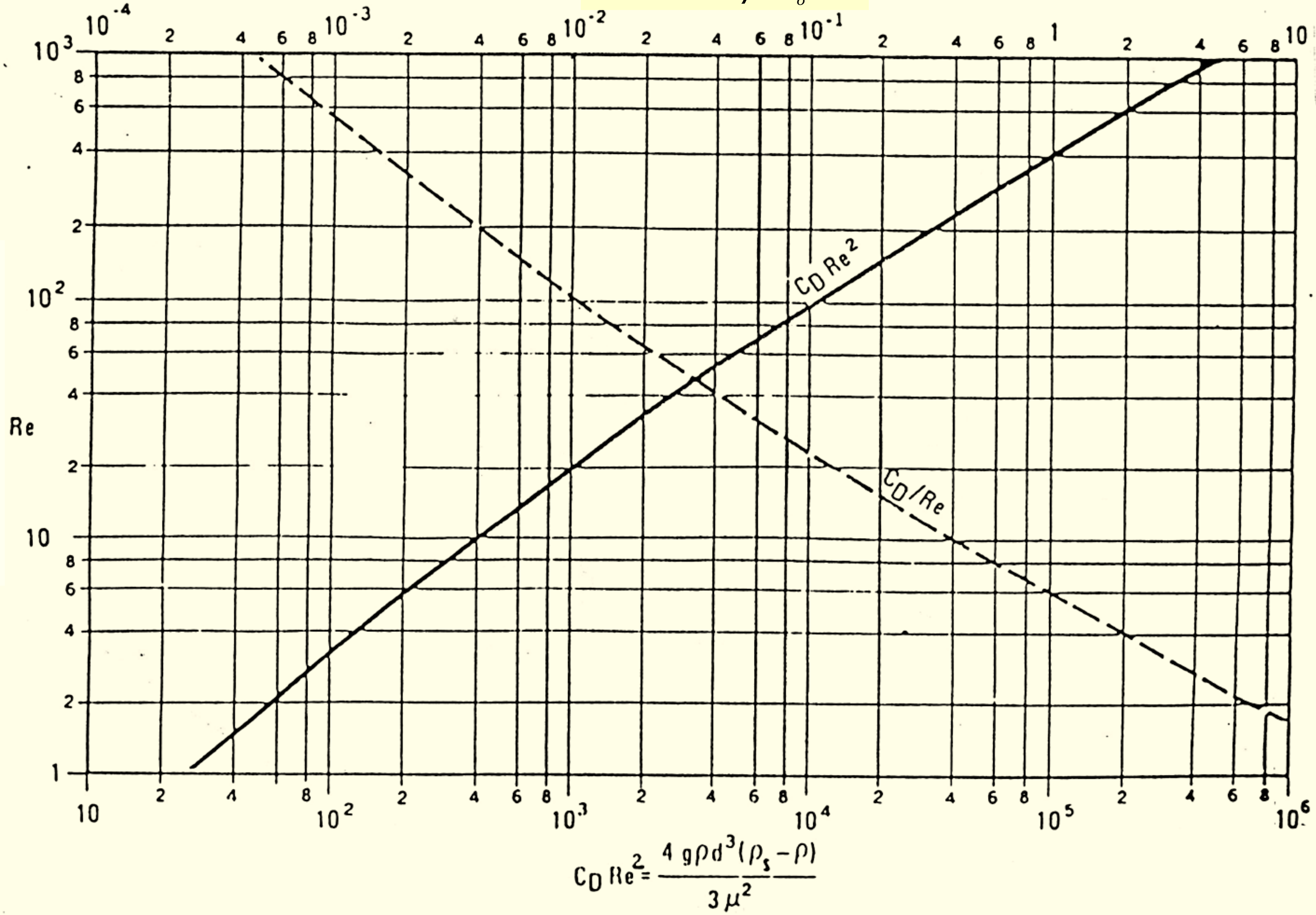
Procedure to find u_0

- Eq. (c) is free of u_0, which can therefore be calculated, if d specified.
- Eq. (d) is free of d, which can therefore be calculated, if u_0 specified.

- In order to obtain u_0 or d, use tables or charts given in the text.

- The method covers all the regimes; Stokes', Transition, and Form drag "Newton's".

$$\frac{C_D}{\mathrm{Re}} = \frac{4g(\rho_s - \rho)\mu}{3\rho^2 u_o^3}$$



$\mathrm{Re} = u_o\, d\, \rho/\mu$

Generally speaking, for free settling, the terminal velocity of a particle in a given fluid tends to be higher as both its particle size and density are increased.

Suppose we have two particles: particle A and particle B with densities and sizes: ρ_A, ρ_B , d_A , and d_B, respectively.

In case of Stokes' regime $\Rightarrow$ Terminal velocities are

$$u_{oA} = \frac{d_A^2 g}{18\mu}(\rho_A - \rho_f)$$

$$u_{oB} = \frac{d_B^2 g}{18\mu}(\rho_B - \rho_f)$$

If $u_{oA} = u_{oB}$

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]^{1/2}$$

In case Newton's regime:

$$u_{oA}^2 = \frac{d_A g}{0.33}\left(\frac{\rho_A - \rho_f}{\rho_f}\right)$$

$$u_{oB}^2 = \frac{d_B g}{0.33}\left(\frac{\rho_B - \rho_f}{\rho_f}\right)$$

For equal settling velocities

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]$$

In general, for equal settling velocities

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]^S$$

Where the index S= 1/2 for stokes' regime

Fine particle

S = 1 for Newton's regime

Coarse particle

1> S>1/2 for transient regime

Example 1

Oil drops 20 μm in diameter are to be settled in air at a temperature of 37.8°C and a pressure of 101.3 kPa. The density of the oil is 900 kg/m^3. Calculate the terminal sedimentation velocity of the oil drops.

Solution

- The density of the oil drops is very high compared to that of air, and so, they will settle in the same way a solid particle would do. At the given conditions of air, consulting values on any text book in thermodynamics, the density and viscosity of air are 1.137 kg/m^3, and 1.9 x 10^{-5} kg/m s, respectively. Assuming that settling would occur in the Stokes' region:

$$u_o = \frac{d^2 g}{18\mu}\left(\rho_p - \rho_f\right)$$

$$u_t = \frac{(2 \times 10^{-5})^2 \mathrm{m}^2 (900 - 1.137)\ \mathrm{kg/m^3} (9.81)\ \mathrm{m/s^2}}{18(0.000019)\ \mathrm{kg/ms}} = 0.0103\ \mathrm{m/s}$$

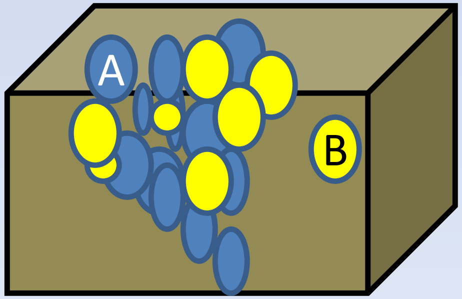
In order to verify whether the appropriate selling regime was chosen, the particle Reynolds number is determined as

$$\mathrm{Re_p} = \frac{(2 \times 10^{-5}) m (1.137)\ \mathrm{kg/m^3} (0.0103)\ \mathrm{m/s}}{(0.000019)\ \mathrm{kg/ms}} = 0.0123$$

The value of Rep is below the limiting figure of 0.2, and so this solution is correct and the terminal velocity of oil drops through air is u_t = 0.0103 m/s.

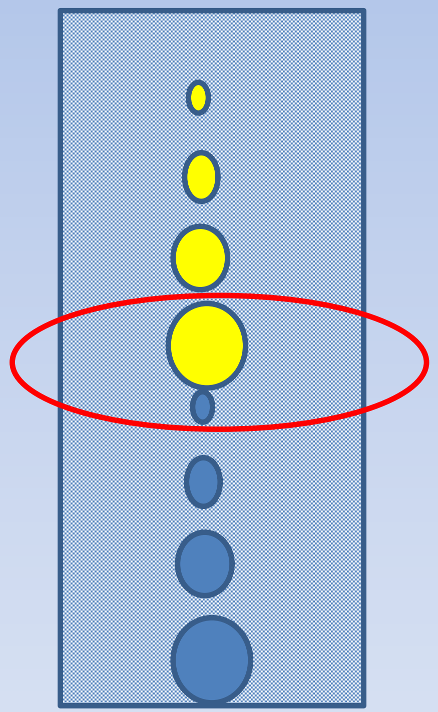
Note

- If, for example, it is desired to separate particles of a relatively dense material **A of** density ρ_A *from particles of a less dense material* ***B and the size range is large, the terminal*** falling velocities of the largest particles of **B of density** $\boldsymbol{\rho_B}$ ***may be greater than those of*** the smallest particles of **A, and therefore a complete separation will not be possible.** The maximum range of sizes that can be separated is calculated from the ratio of the sizes of the particles of the **two materials which have the same terminal falling velocities** as given by the previous equation.



Complete separation

Dense particles, ρ_D iron	Light particles, ρ_L sand
A	B



Example 2

A mixture of quartz and galena of a size range from 0.015 mm to 0.065 mm is to be separated into two pure fractions using a hindered settling process. What is the minimum apparent density of the fluid that will give this separation? The density of galena is 7500 kg/m^3 and the density of quartz is 2650 kg/m^3.

Solution

Assuming the galena and quartz particles are of similar shapes, then from previous equation , the required density of fluid when Stokes' law applies is given by:

$$\frac{0.065}{0.015} = \left(\frac{7500 - \rho}{2650 - \rho} \right)^{1/2}$$

$$\rho = 2377 \mathrm{kg/m}^3$$

The required density of fluid when Newton's law applies is given by:

$$\frac{0.065}{0.015} = \left(\frac{7500 - \rho}{2650 - \rho}\right)$$

$$\rho = 1196\,\mathrm{kg/m^3}$$

Thus, the required density of the fluid is between 1196 and 2377 kg/m^3.

Example 3

A finely ground mixture of galena and limestone in the proportion of 1 to 4 by mass is subjected to elutriation by an upward-flowing stream of water flowing at a velocity of 5 mm/s. Assuming that the size distribution for each material is the same, and is as shown in the following table, estimate the percentage of galena in the material carried away and in the material left behind. The viscosity of water is 1 mN s/m^2 and Stokes' equation may be used. The densities of galena and limestone are 7500 and 2700 kg/m^3, respectively.

Diameter (μm)	20	30	40	50	60	70	80	100
Undersize (per cent by mass)	15	28	48	54	64	72	78	88

Solution

The first step is to determine the size of a particle which has a settling velocity equal to that of the upward flow of fluid, that is 5 mm/s.

Firstly check the validity of Stoke's regime

Taking the largest particle, $d = (100 \times 10^{-6}) = 0.0001$ m

and: $$Re' = (5 \times 10^{-3} \times 0.0001 \times 1000)/(1 \times 10^{-3}) = 0.5$$

Thus, for the bulk of particles, the flow will be within region (a) in Figure 3.4 and the settling velocity is given by Stokes' equation:

$$u_0 = (d^2 g/18\mu)(\rho_s - \rho)$$

For a particle of galena settling in water at 5 mm/s:

$$(5 \times 10^{-3}) = ((d^2 \times 9.81)/(18 \times 10^{-3}))(7500 - 1000) = 3.54 \times 10^6 d^2$$

and: $$d = 3.76 \times 10^{-5} \text{ m or } 37.6\ \mu\text{m}$$

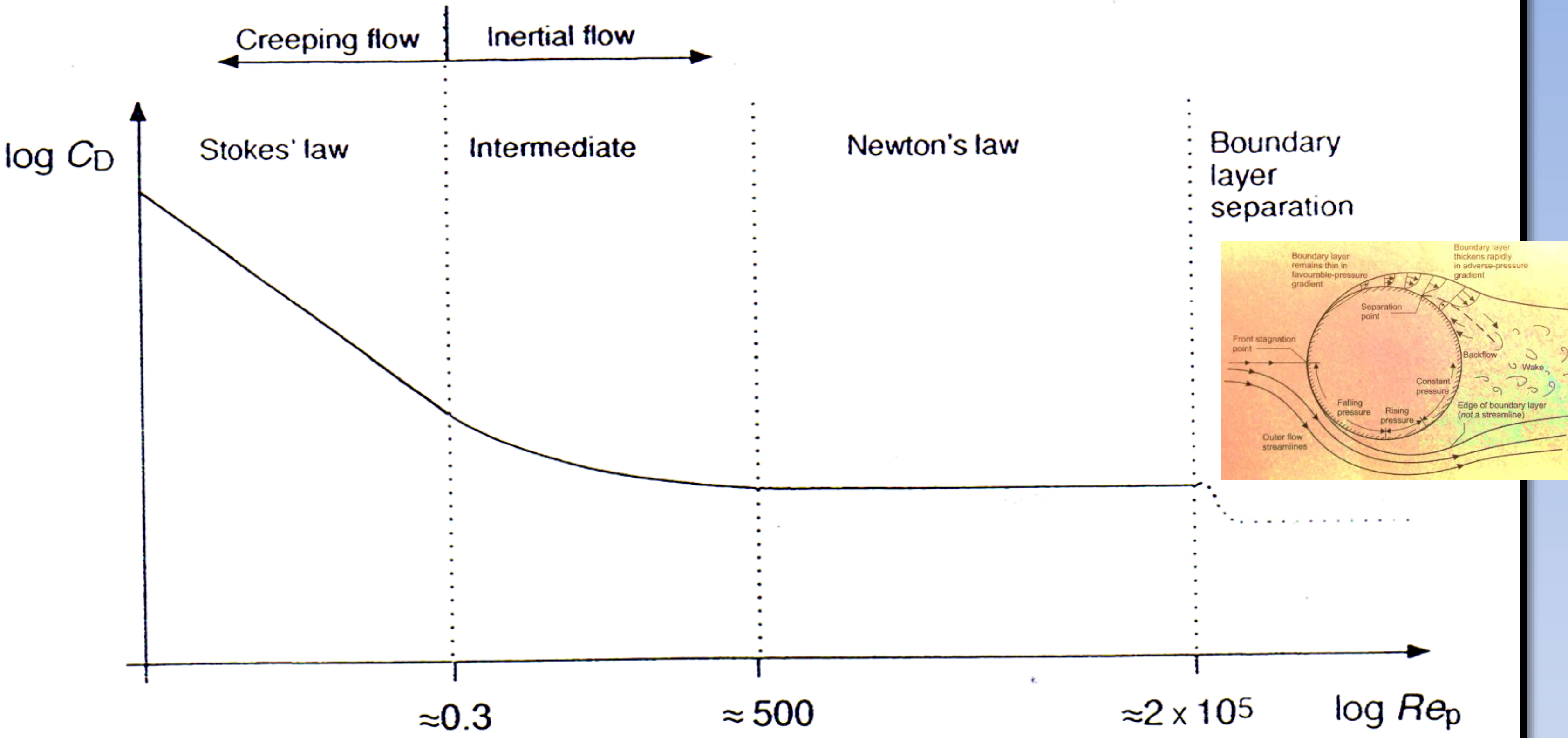
For a particle of limestone settling at 5 nmm/s:

$$(5 \times 10^{-3}) = ((d^2 \times 9.81)/(18 \times 10^{-3}))(2700 - 1000) = 9.27 \times 10^5 d^2$$

and: $$d = 7.35 \times 10^{-5} \text{ m or } 73.5\ \mu\text{m}$$

Hence in the *material removed*:

a



Standard drag curve for motion of a sphere in a fluid

Reynolds number ranges for single particle drag coefficient correlations

Region	Stokes	Intermediate	Newton's Law
Re_p range	< 0.3	$0.3 < Re_p < 500$	$500 < Re_p > 2 \times 10^5$
C_D	$24/Re_p$	$\approx 24/Re_p + 0.44$	≈ 0.44

Notes

$$\therefore \quad C_D = \frac{F_D}{\frac{1}{2}\rho u^2 A_p}$$

$$= \frac{F_D}{\frac{1}{2}\rho u^2 \frac{\pi}{4} d^2}$$

R'

$$= \frac{4 F_D}{\frac{1}{2}\rho u^2 \pi d^2}$$

Let $R' = \dfrac{4 f_D}{\pi d^2}$ *Drag force per projected area*

C_D'

$$\therefore \quad C_D = \frac{2 R'}{\rho u^2}$$

Note

$\therefore\ C_D = 2\,C_D'$

analogous to Fanning friction factor f

analogous to friction factor ϕ for pipe flow

For Stokes' Law $C_D = 24\,Re^{-1}$

Or $\dfrac{R'}{\rho u^2} = 12\,Re^{-1} = 12\dfrac{\mu}{u d \rho}$

C_D'

Note

$$\left(\frac{R'}{\rho u_o^2}\right)\mathrm{Re}^2 = \left(\frac{C_D}{2}\right)\mathrm{Re}^2 = \frac{2}{3}\frac{g\rho d^3(\rho_s - \rho)}{\mu^2}$$

$$\left(\frac{R'}{\rho u_o^2}\right)\frac{1}{Re} = \frac{2}{3}\frac{\mu g}{u_0^3}\left(\frac{\rho_p - \rho_f}{\rho_f^2}\right)$$

Figure 3.4. $R'/\rho u^2$ versus Re' for spherical particles

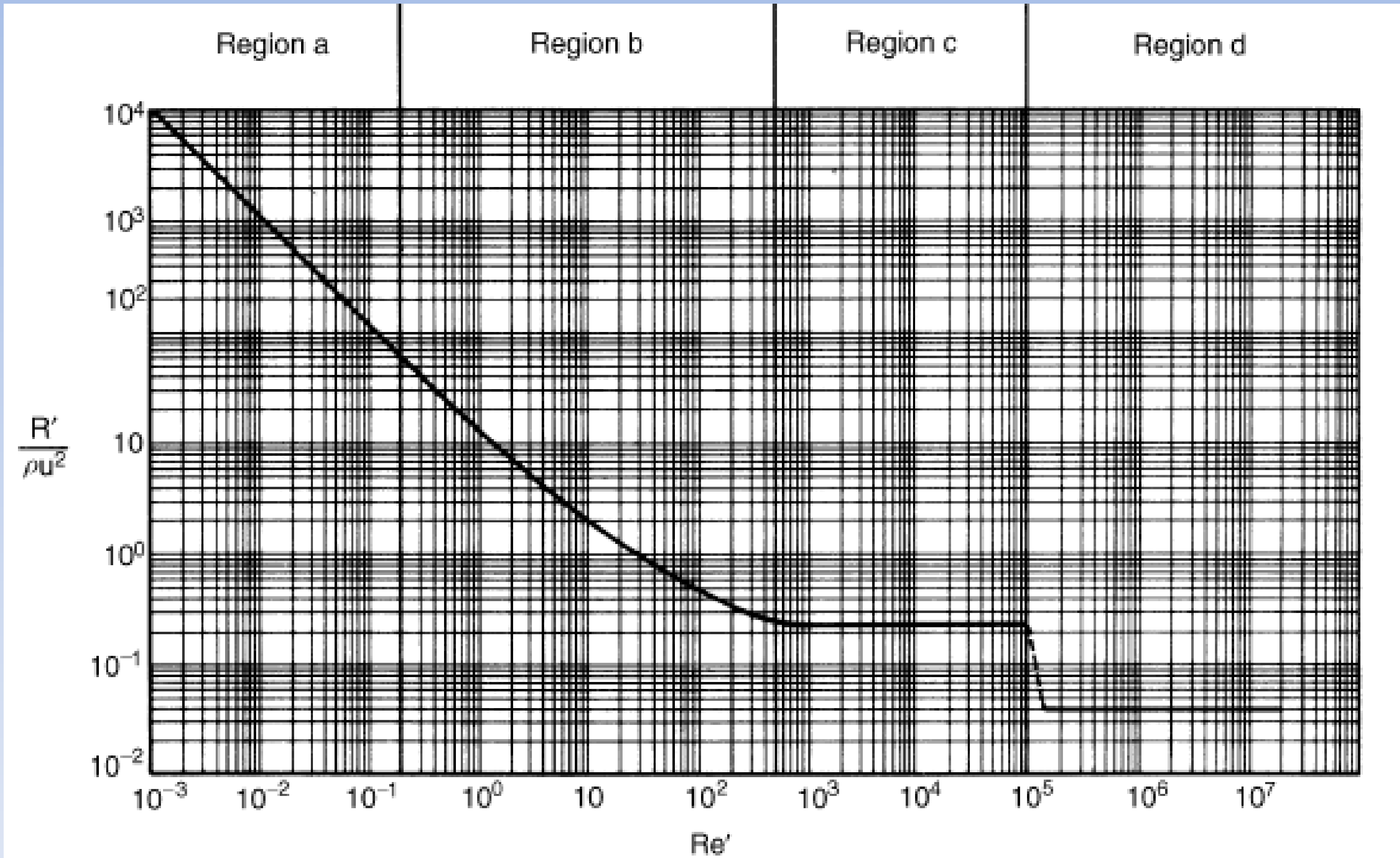
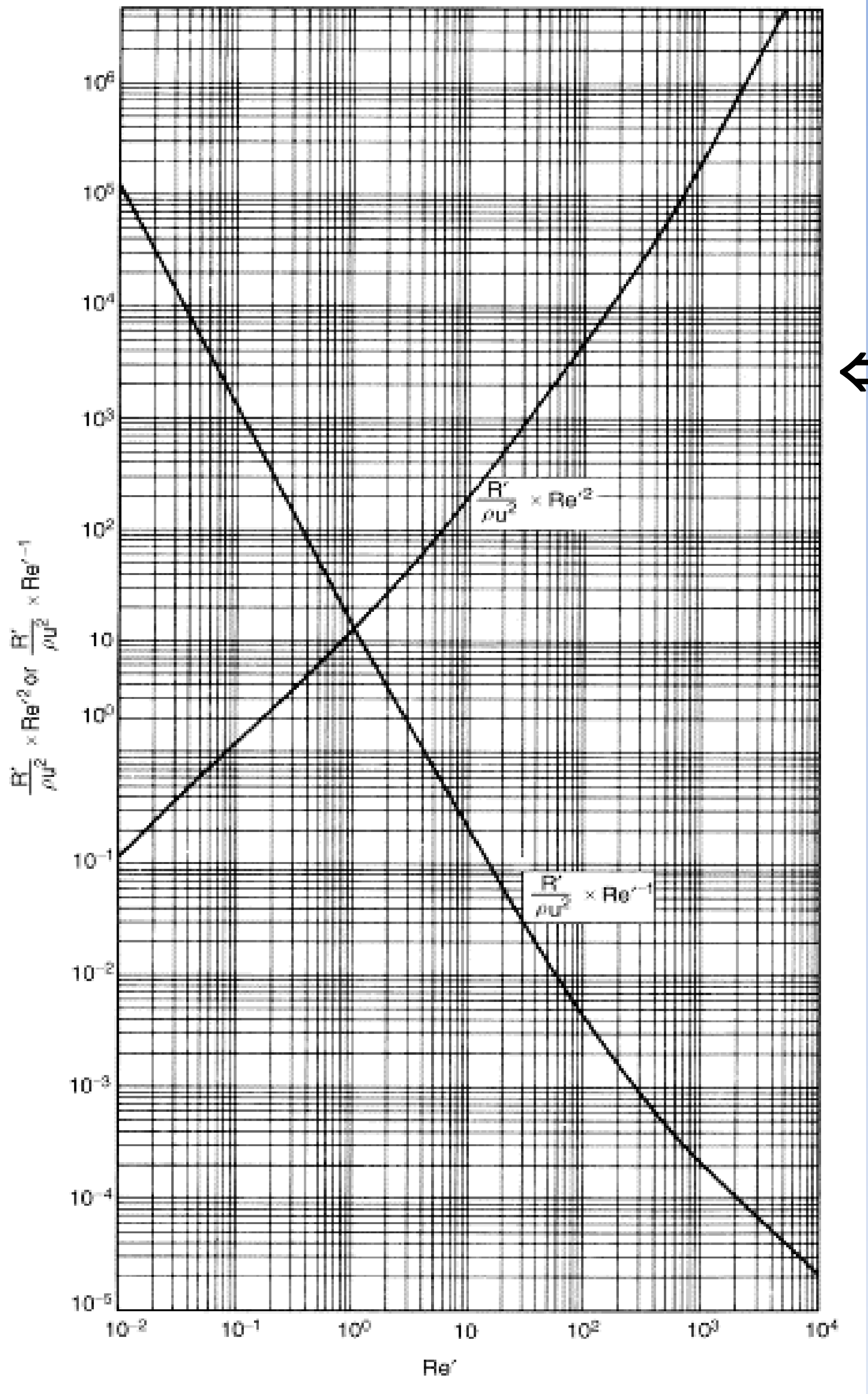


Table 3.2. $R'/\rho u^2$, $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ as a function of Re'

Re'	$R'/\rho u^2$	$(R'/\rho u^2)Re'^2$	$(R'/\rho u^2)Re'^{-1}$
10^{-3}	12,000		
2×10^{-3}	6000		
5×10^{-3}	2400		
10^{-2}	1200	1.20×10^{-1}	1.20×10^5
2×10^{-2}	600	2.40×10^{-1}	3.00×10^4
5×10^{-2}	240	6.00×10^{-1}	4.80×10^3
10^{-1}	124	1.24	1.24×10^3
2×10^{-1}	63	2.52	3.15×10^2
5×10^{-1}	26.3	6.4	5.26×10
10^0	13.8	1.38×10	1.38×10
2×10^0	7.45	2.98×10	3.73
5×10^0	3.49	8.73×10	7.00×10^{-1}
10	2.08	2.08×10^2	2.08×10^{-1}
2×10	1.30	5.20×10^2	6.50×10^{-2}
5×10	0.768	1.92×10^3	1.54×10^{-2}
10^2	0.547	5.47×10^3	5.47×10^{-3}
2×10^2	0.404	1.62×10^4	2.02×10^{-3}
5×10^2	0.283	7.08×10^4	5.70×10^{-4}
10^3	0.221	2.21×10^5	2.21×10^{-4}
2×10^3	0.22	8.8×10^5	1.1×10^{-4}
5×10^3	0.22	5.5×10^6	4.4×10^{-5}
10^4	0.22	2.2×10^7	2.2×10^{-5}
2×10^4	0.22		
5×10^4	0.22		
10^5	0.22		
2×10^5	0.05		
5×10^5	0.05		
10^6	0.05		
2×10^6	0.05		
5×10^6	0.05		
10^7	0.05		



⇐ C_D' C_D'

Figure 3.6. $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ versus Re' for spherical particles

Similar to ⇓

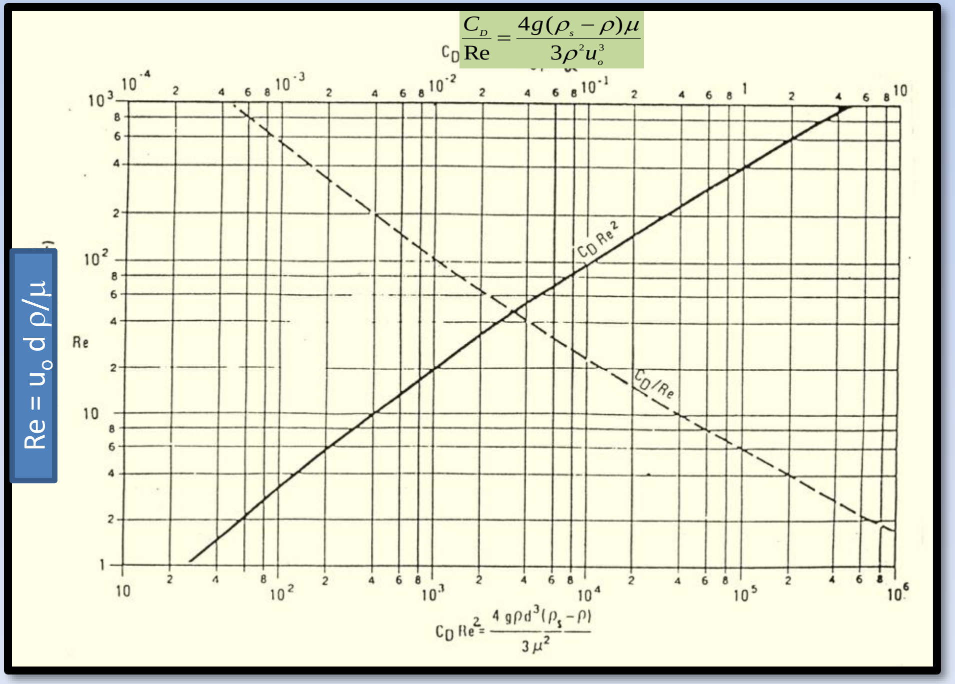


Table 3.4. Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

-ve values

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{2}$								$\bar{3}.620$	$\bar{3}.720$	$\bar{3}.819$
$\bar{1}$	$\bar{3}.919$	$\bar{2}.018$	$\bar{2}.117$	$\bar{2}.216$	$\bar{2}.315$	$\bar{2}.414$	$\bar{2}.513$	$\bar{2}.612$	$\bar{2}.711$	$\bar{2}.810$
0	$\bar{2}.908$	$\bar{1}.007$	$\bar{1}.105$	$\bar{1}.203$	$\bar{1}.301$	$\bar{1}.398$	$\bar{1}.495$	$\bar{1}.591$	$\bar{1}.686$	$\bar{1}.781$
1	$\bar{1}.874$	$\bar{1}.967$	0.008	0.148	0.236	0.324	0.410	0.495	0.577	0.659
2	0.738	0.817	0.895	0.972	1.048	1.124	1.199	1.273	1.346	1.419
3	1.491	1.562	1.632	1.702	1.771	1.839	1.907	1.974	2.040	2.106
4	2.171	2.236	2.300	2.363	2.425	2.487	2.548	2.608	2.667	2.725
5	2.783	2.841	2.899	2.956	3.013	3.070	3.127	3.183	3.239	3.295

Table 3.5. Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^{-1}\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^{-1}\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{5}$										3.401
$\bar{4}$	3.316	3.231	3.148	3.065	2.984	2.903	2.824	2.745	2.668	2.591
$\bar{3}$	2.517	2.443	2.372	2.300	2.231	2.162	2.095	2.027	1.961	1.894
$\bar{2}$	1.829	1.763	1.699	1.634	1.571	1.508	1.496	1.383	1.322	1.260
$\bar{1}$	1.200	1.140	1.081	1.022	0.963	0.904	0.846	0.788	0.730	0.672
0	0.616	0.560	0.505	0.449	0.394	0.339	0.286	0.232	0.178	0.125
1	0.072	0.019	$\bar{1}.969$	$\bar{1}.919$	$\bar{1}.865$	$\bar{1}.811$	$\bar{1}.760$	$\bar{1}.708$	$\bar{1}.656$	$\bar{1}.605$
2	$\bar{1}.554$	$\bar{1}.503$	$\bar{1}.452$	$\bar{1}.401$	$\bar{1}.350$	$\bar{1}.299$	$\bar{1}.249$	$\bar{1}.198$	$\bar{1}.148$	$\bar{1}.097$
3	$\bar{1}.047$	$\bar{2}.996$	$\bar{2}.946$	$\bar{2}.895$	$\bar{2}.845$	$\bar{2}.794$	$\bar{2}.744$	$\bar{2}.694$	$\bar{2}.644$	$\bar{2}.594$
4	$\bar{2}.544$	$\bar{2}.493$	$\bar{2}.443$	$\bar{2}.393$	$\bar{2}.343$	$\bar{2}.292$				

Example 3

What is the terminal velocity of a spherical steel particle, 0.40 mm in diameter, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of steel is 7870 kg/m^3.

Solution

For a sphere:

$$\frac{R'_0}{\rho u_0^2} Re_0'^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$

$$= \frac{2 \times 0.0004^3 \times 820(7870 - 820)9.81}{3(10 \times 10^{-3})^2} = 24.2$$

$$\log_{10} 24.2 = 1.384$$

From Table 3.4: $\log_{10} Re'_0 = 0.222$

Thus: $Re'_0 = 1.667$

and:

$$u_0 = \frac{1.667 \times 10 \times 10^{-3}}{820 \times 0.0004}$$

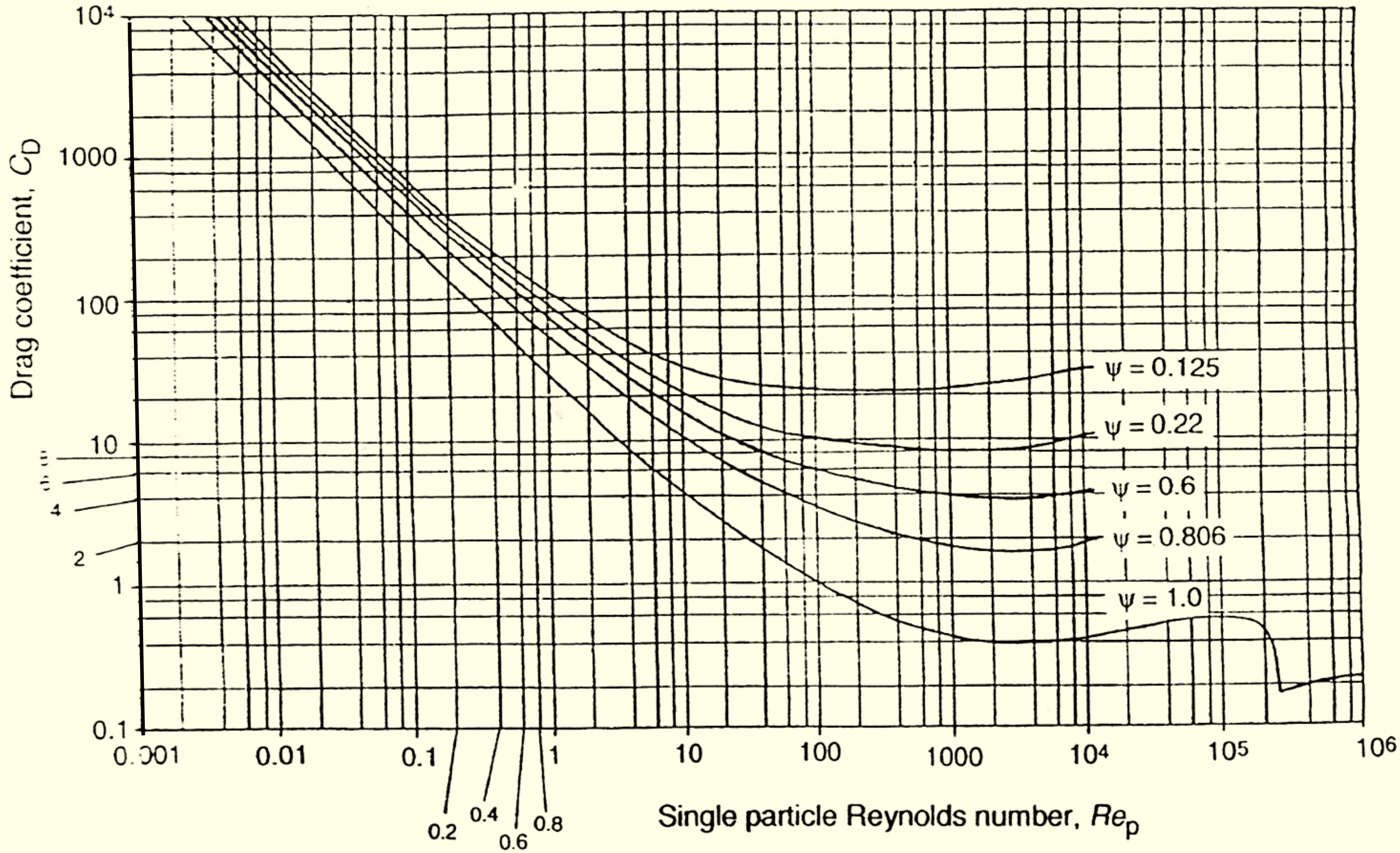
$$= 0.051 \text{ m/s or } 51 \text{ mm/s}$$

Alternative solution

- $C_D\ Re^2 = 48.4$
- from chart slid 37 Re = 1.7
- Since $Re = \rho\ u_o d/\mu$
- Thus $u_o = 0.05$ m/s

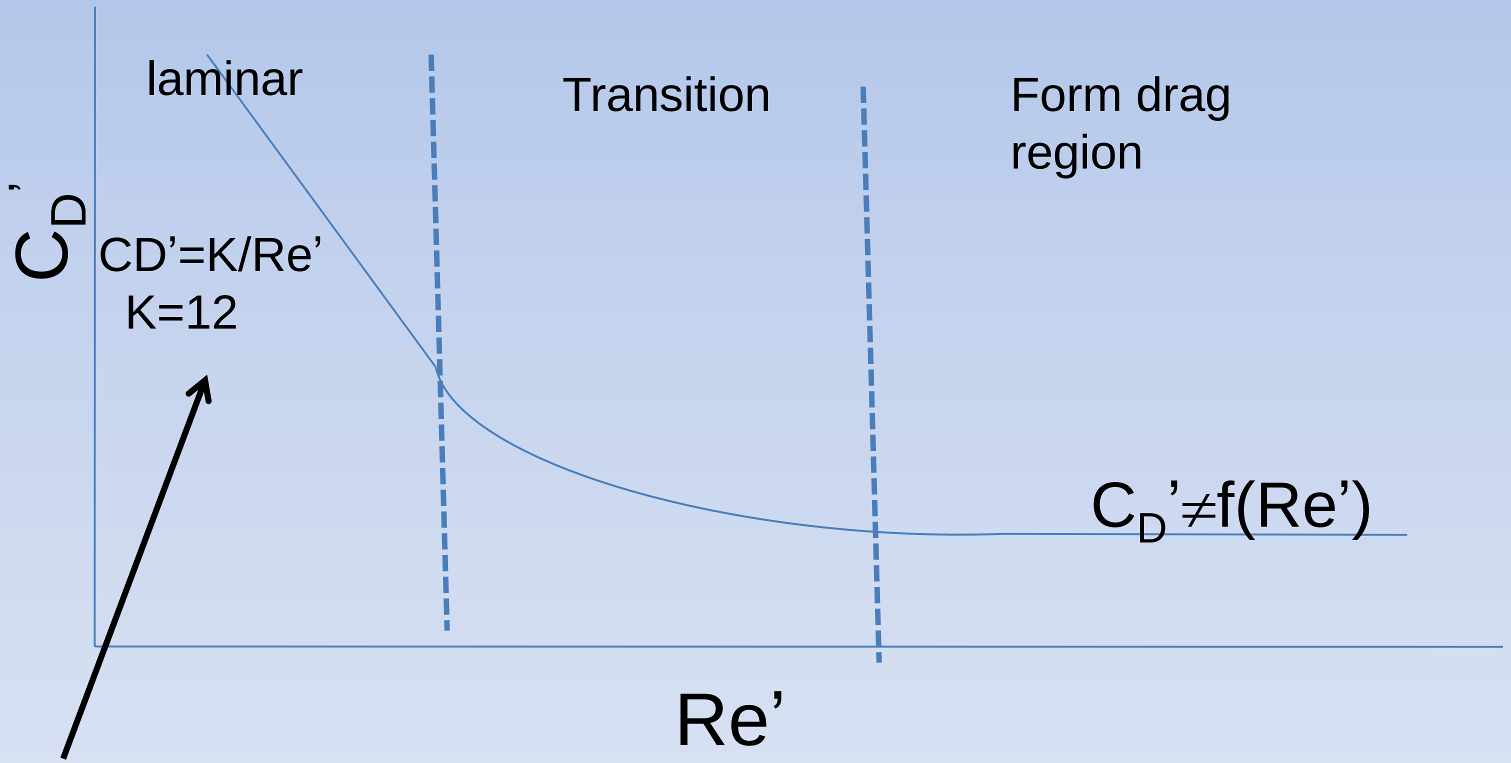
Non-spherical particles

- Since $C_D = f(Re', shape)$ See next slide
- There are a set of experimental curves (C_D against Re') for various shapes, similar to spherical particles.
- For non-spherical particles, the orientation must be specified before the drag force can be calculated.
- $Re' = ud'\rho/\mu$, *where* d' the diameter of the circle having the same area as the projected area of the particle



Drag coefficient C_D versus Reynolds number Re_p for particles of sphericity ψ ranging form 0.125 to 1.0 (note Re_p uses the equivalent volume diameter)

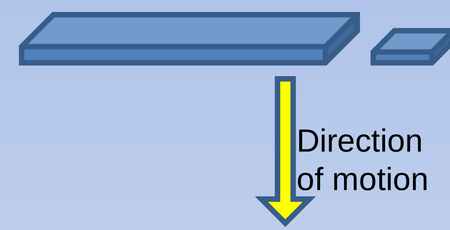
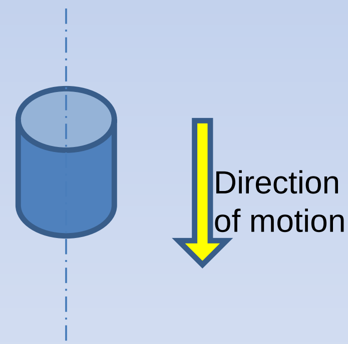
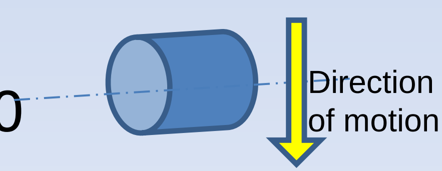
- The curve for (*R′/ρu²*) *against Re′ may be divided into three main regions, (a), (b), and (c) as before.*



In this region (a), a particle falling freely in the fluid under the action of gravity will normally move with its longest surface nearly parallel to the direction of motion.

- **Region *(b)*** *represents transition conditions and commences at a lower value* of *Re′, and a correspondingly higher value of $(R'/\rho u^2)$, than in the case of the sphere.* A freely falling particle will tend to change its orientation as the value of *Re′ changes* and some instability may be apparent.
- **In region *(c)*** *the particle tends to fall so that it is* presenting the maximum possible surface to the oncoming fluid (surface perpendicular to the direction of motion). Typical values of $(R'/\rho u^2)$ for non-spherical particles in region *(c) are given in Table 3.6*

Table 3.6. Drag coefficients C_D' for Non - Spherical Particles ‘region c’

Configuration	Length/breadth		$R'/\rho u^2$
Thin rectangular plates with their planes Perpendicular to the direction of motion	1–5		0.6
	20		0.75
	∞		0.95
Cylinders with axes parallel to the direction of motion	1		0.45
Cylinders with axes perpendicular to the Direction of motion	1		0.3
	20		0.45
	∞		0.6

Free falling velocity for Non-spherical particles

$$F_D = m g \left(1 - \frac{\rho_f}{\rho_p}\right)$$

$$= k d_p^3 \rho_p g \left(1 - \frac{\rho_f}{\rho_p}\right)$$

$$= k d_p^3 g (\rho_p - \rho_f)$$

$$C_D \cdot \frac{1}{2} \rho_f u_0^2 \cdot \frac{\pi}{4} d_p^2 = k d_p^3 g (\rho_p - \rho_f)$$

$$\frac{\cancel{2} R'}{\rho_f u_0^2} \frac{1}{\cancel{2}} \rho_f u_0^2 \frac{\pi}{4} d_p^2 = k d_p^3 g (\rho_p - \rho_f)$$

$$\therefore \frac{R_0'}{\rho_f u_0^2} = \frac{4 k d_p g}{\pi \rho_f u_0^2} (\rho_p - \rho_f)$$

Multiply by Re^2

$$C_D' \rightarrow \left(\frac{R_o'}{\rho u_o^2}\right) Re_o^2 = \frac{4 k \rho_f d_p^3 g}{\mu^2 \pi} (\rho_p - \rho_f)$$

free of u_o

Confusion

and divide by Re

$$C_D' \rightarrow \left(\frac{R_o'}{\rho u_o^2}\right) \frac{1}{Re} = \frac{4 k \mu g}{\pi \rho_f^2 u_o^3} (\rho_p - \rho_f)$$

free of particle size "d"

OR

$$C_D Re_o^2 = \frac{8 k \rho_f d_p^3 g}{\mu^2 \pi} (\rho_p - \rho_f)$$

$$\frac{C_D}{Re} = \frac{8 k \mu g}{\pi \rho_f^2 u_o^3} (\rho_p - \rho_f)$$

Example 4

What will be the terminal velocities of mica plates, 1 mm thick and ranging in area from 6 to 600 mm^2, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of mica is 3000 kg/m^3.

solution

	smallest particles	largest particles
A'	$6 \times 10^{-6}\ \text{m}^2$ $= \frac{\pi}{4} d_p^2$	$6 \times 10^{-4}\ \text{m}^2$
d_p	$\sqrt{(4 \times 6 \times 10^{-6}/\pi)} = 2.76 \times 10^{-3}\ \text{m}$	$\sqrt{(4 \times 6 \times 10^{-4}/\pi)} = 2.76 \times 10^{-2}\ \text{m}$
d_p^3	$2.103 \times 10^{-8}\ \text{m}^3$	$2.103 \times 10^{-5}\ \text{m}^3$
volume	$6 \times 10^{-9}\ \text{m}^3$ $= k' d_p^3$	$6 \times 10^{-7}\ \text{m}^3$
k'	0.285	0.0285

$$\left(\frac{R_0'}{\rho u^2}\right) Re_0'^2 = \frac{4k'}{\mu^2 \pi}(\rho_s - \rho)\rho d_p^3 g$$

$$= (4 \times 0.285/\pi \times 0.01^2)(3000 - 820)(820 \times 2.103 \times 10^{-8} \times 9.81)$$

$= 1340$ for the smallest particles and, similarly, 134,000 for the largest particles.

	smallest particles	largest particles
$\log\left(\frac{R_0'}{\rho u_0^2} Re_0'^2\right)$	3.127	5.127
$\log Re_0'$ (From table 3.4 as spherical)	1.581	2.857 (from Table 3.4)
Correction from Table 3.7	−0.038	−0.300 (estimated)
Corrected $\log Re_0'$	1.543	2.557
Re_0'	34.9	361
u_0 (From $Re_o' = u_0 d' \rho / \mu$)	0.154 m/s	0.159 m/s

Thus it is seen that all the mica particles settle at approximately the same velocity.

Table 3.4. Values of $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{2}$								$\bar{3}.620$	$\bar{3}.720$	$\bar{3}.819$
$\bar{1}$	$\bar{3}.919$	$\bar{2}.018$	$\bar{2}.117$	$\bar{2}.216$	$\bar{2}.315$	$\bar{2}.414$	$\bar{2}.513$	$\bar{2}.612$	$\bar{2}.711$	$\bar{2}.810$
0	$\bar{2}.908$	$\bar{1}.007$	$\bar{1}.105$	$\bar{1}.203$	$\bar{1}.301$	$\bar{1}.398$	$\bar{1}.495$	$\bar{1}.591$	$\bar{1}.686$	$\bar{1}.781$
1	$\bar{1}.874$	$\bar{1}.967$	0.008	0.148	0.236	0.324	0.410	0.495	0.577	0.659
2	0.738	0.817	0.895	0.972	1.048	1.124	1.199	1.273	1.346	1.419
3	1.491	1.562	1.632	1.702	1.771	1.839	1.907	1.974	2.040	2.106
4	2.171	2.236	2.300	2.363	2.425	2.487	2.548	2.608	2.667	2.725
5	2.783	2.841	2.899	2.956	3.013	3.070	3.127	3.183	3.239	3.295

Table 3.7. Corrections to $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{2}$	−0.022	−0.002	+0.032	+0.131
$\bar{1}$	−0.023	−0.003	+0.030	+0.131
0	−0.025	−0.005	+0.026	+0.129
1	−0.027	−0.010	+0.021	+0.122
2	−0.031	−0.016	+0.012	+0.111
2.5	−0.033	−0.020	0.000	+0.080
3	−0.038	−0.032	−0.022	+0.025
3.5	−0.051	−0.052	−0.056	−0.040
4	−0.068	−0.074	−0.089	−0.098
4.5	−0.083	−0.093	−0.114	−0.146
5	−0.097	−0.110	−0.135	−0.186
5.5	−0.109	−0.125	−0.154	−0.224
6	−0.120	−0.134	−0.172	−0.255

Motion of particles in a Centrifugal Field

Motion of particles in a centrifugal field

Examples : Cyclones , Settlers

In Case of a particle is moving in a fluid under the action of Centrifugal field, the effects of gravitational is comparatively Small and can be neglected.

Assume stokes' regime

For spherical particle and stokes' regime

$$\frac{\pi}{6} d^3 (\rho_p - \rho_f) r\omega^2 - 3\pi \mu d \frac{dr}{dt} = \frac{\pi}{6} d^3 \rho_p \frac{d^2 r}{dt^2} \quad \cdots (1)$$

instantaneous velocity

Inertial term

where

r : radius of rotation, m

w : angular velocity, rad/s

$r\omega^2$: Centrifugal acceleration

Note: As the particle moves outwards, the accelerating force increases and therefore it never acquires an equilibrium velocity in the fluid

$$\frac{d^2r}{dt^2} + \frac{18\mu}{d^2\rho_p} \cdot \frac{dr}{dt} - \frac{\rho_p - \rho_f}{\rho_p} \omega^2 r = 0 \quad \ldots\ldots (2)$$

OR

$$\frac{d^2r}{dt^2} + a\frac{dr}{dt} - nr = 0 \quad \ldots\ldots (3)$$

The Solution is

$$r = B_1 e^{-[a/2 + \sqrt{a^2/4 + n}]t} + B_2 e^{-[a/2 - \sqrt{a^2/4 + n}]t}$$

OR

$$\therefore r = e^{-at/2} \left\{ B_1 e^{-kt} + B_2 e^{kt} \right\} \quad \ldots\ldots (4)$$

where $a = 18\mu/d^2\rho_p$, $n = (1 - \rho_f/\rho_p)\omega^2$, $k = \sqrt{a^2/4 + n}$

$\underline{\underline{B.C^s}}$

$$t = 0\,,\quad r = r_1\,,\quad dr/dt = 0 \qquad \ldots\ldots(5)$$

$\therefore$ Eq.(4):

$$\frac{dr}{dt} = e^{-at/2}\left\{-kB_1 e^{-kt} + kB_2 e^{kt}\right\} - \frac{a}{2} e^{-\frac{at}{2}}\left\{B_1 e^{-kt} + B_2 e^{kt}\right\}$$

$$= e^{-\frac{at}{2}}\left\{\left(k - \frac{a}{2}\right) B_2 e^{kt} - \left(k + \frac{a}{2}\right) B_1 e^{-kt}\right\} \qquad \ldots\ldots(6)$$

Substituting the BC^s into eqs. (4) and (6)

$$r_1 = B_1 + B_2 \quad \text{- - - - - - - - (7)}$$

$$0 = (k - \frac{a}{2})B_2 - (k + \frac{a}{2})B_1 \quad \text{- - - - - (8)}$$

from (8) $\Longrightarrow$ $\frac{B_1}{B_2} = \frac{k - a/2}{k + a/2}$ ———— (9)

Sub. into (7)

$$r_1 = B_2\left(\frac{B_1}{B_2} + 1\right) = B_2\left(\frac{k - a/2}{k + a/2} + 1\right)$$

$$\therefore \quad r_1 = \frac{2k}{k + a/2} B_2 \quad \text{- - - - - - - - - (10)}$$

OR

$$B_2 = \frac{k + a/2}{2k} r_1 \quad \text{- - - - - - - - - (11)}$$

Sub. into (9)

$$\therefore \quad B_1 = \frac{k - a/2}{k + a/2} B_2 = \frac{k - a/2}{\cancel{k + a/2}} \cdot \frac{\cancel{(k + a/2)}}{2k} r_1$$

$$= \frac{k - a/2}{2k} r_1 \quad - - - - - - - (12)$$

Thus eq.(4) becomes

$$r = e^{-\frac{at}{2}} \left\{ \frac{k - a/2}{2k} r_1 e^{-kt} + \frac{k + a/2}{2k} r_1 e^{kt} \right\} \quad \ldots (13)$$

OR $$\frac{r}{r_1} = e^{-\frac{at}{2}} \left\{ \cosh kt + \frac{a}{2k} \sinh kt \right\} \quad \ldots\ldots (14)$$

$\therefore$ The ratio r/r_1 could be calculated at any time t, but numerical solution is required to obtain t for any particular r/r_1 value.

Suppose the effect of particle acceleration is neglected, then eq.(3) simplifies to:

$$a\,\frac{dr}{dt} - n\,r = 0 \Rightarrow \frac{dr}{dt} = \frac{n}{a}r$$

$$\int_{r_1}^{r} \frac{dr}{r} = \frac{n}{a}\int_{0}^{t} dt \Rightarrow \ln\frac{r}{r_1} = \frac{n}{a}t$$

$$\therefore\ \ln\frac{r}{r_1} = \frac{d^2(\rho_p - \rho_f)\,\omega^2}{18\mu}\,t$$

The time required for the particle to move to a radius r is

$$t = \frac{18\mu}{d^2\omega^2(\rho_p-\rho_f)} \ln\frac{r}{r_1} \quad \cdots\cdots (15)$$

If the particle is initially situated in the liquid surface ($r_1 = r_i$), the time taken to reach the wall of the basket ($r = R$).

$$\boxed{t = \frac{18\mu}{d^2\omega^2(\rho_p-\rho_f)} \ln\frac{R}{r_i}} \quad \cdots\cdots (16)$$

If h is the thickness of liquid layer at the wall: $h = R - r_i$

$$\therefore \ln \frac{R}{r_i} = \ln \frac{R}{R-h} = -\ln \frac{R-h}{R} = -\ln\left(1 - \frac{h}{R}\right) =$$

$$= \frac{h}{R} + \frac{1}{2}\left(\frac{h}{R}\right)^2 + \cdots$$

If h is small compared with R

$$\therefore \ln \frac{R}{r_i} = \frac{h}{R}$$

$\therefore$ Eq. (16) becomes

$$t = \frac{18 \mu h}{d^2 \omega^2 (\rho_p - \rho_f) R} \qquad ///$$

Form Drag regime "Newton's Law"

$$C_D = \text{cons} = 0.44$$

$$\frac{\pi}{6} d^3(\rho_p - \rho_f) r\omega^2 - \frac{1}{2}(0.44)\frac{\pi}{4} d^2 \rho_f \left(\frac{dr}{dt}\right)^2 = \frac{\pi}{6} d^3 \rho_p \frac{d^2r}{dt^2}$$

This equation can only be solved numerically

If the acceleration is neglected:

$$\left(\frac{dr}{dt}\right)^2 = 3 d \omega^2 \left(\frac{\rho_p - \rho_f}{\rho_f}\right) r$$

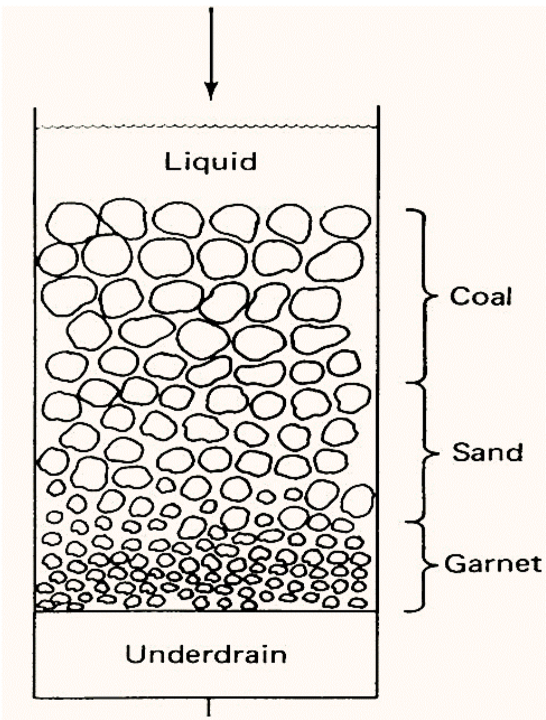
$$r^{-1/2}\frac{dr}{dt} = \left\{3 d \omega^2 \left(\frac{\rho_p - \rho_f}{\rho_f}\right)\right\}^{1/2}$$

on integration

$$2(r^{1/2} - r_1^{1/2}) = \left\{3 d \omega^2 \left(\frac{\rho_p - \rho_f}{\rho_f}\right)\right\}^{1/2} t \Rightarrow t = \left[\frac{\rho_f}{3 d \omega^2 (\rho_p - \rho_f)}\right] 2(r^{1/2} - r_1^{1/2})$$

Flow of Fluids Through Granular Beds and Packed Columns

Flow through Granular Bed & Packed Columns



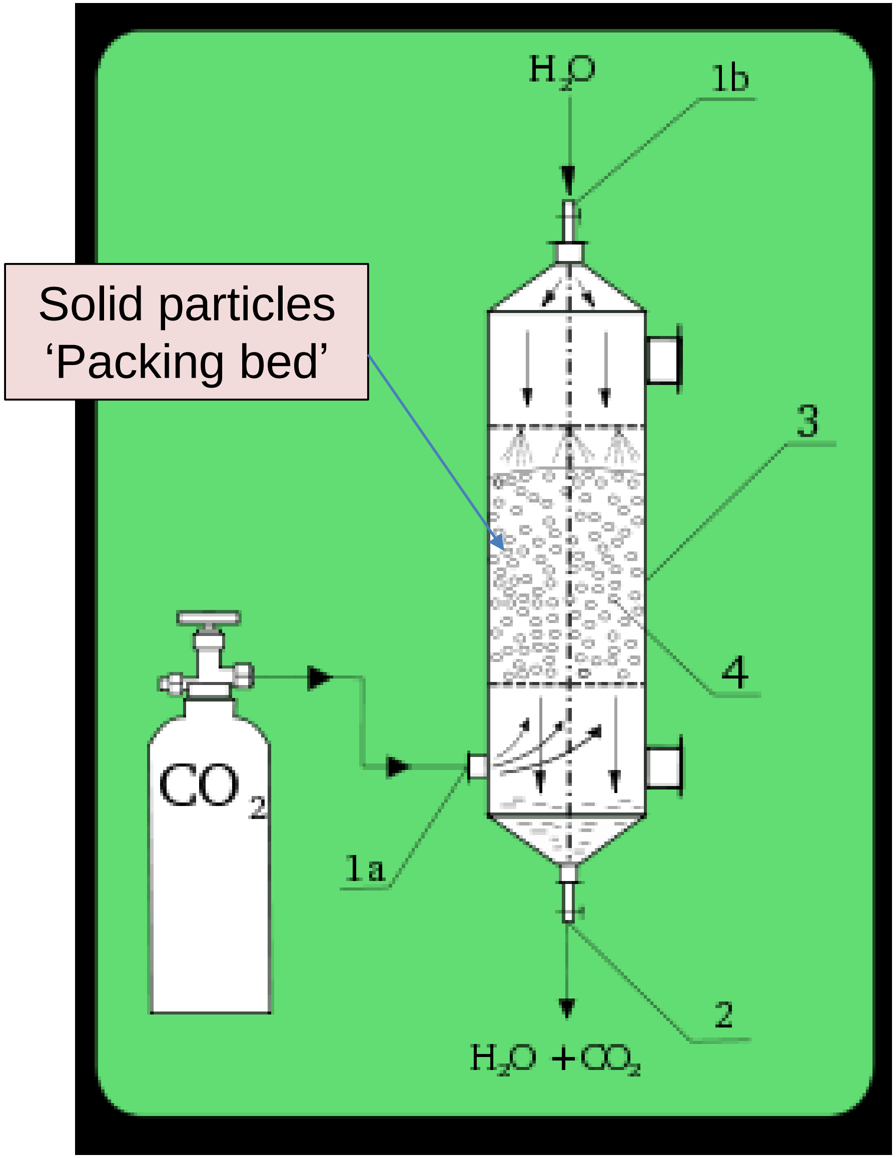
Absorption column "lab experiment"

Solid particles
'Packing bed'

Absorption column

Laboratory absorber

(1a): CO_2 inlet;
(1b): H_2O inlet;
(2):outlet;
(3): absorption column;
(4): **packing**.



Introduction

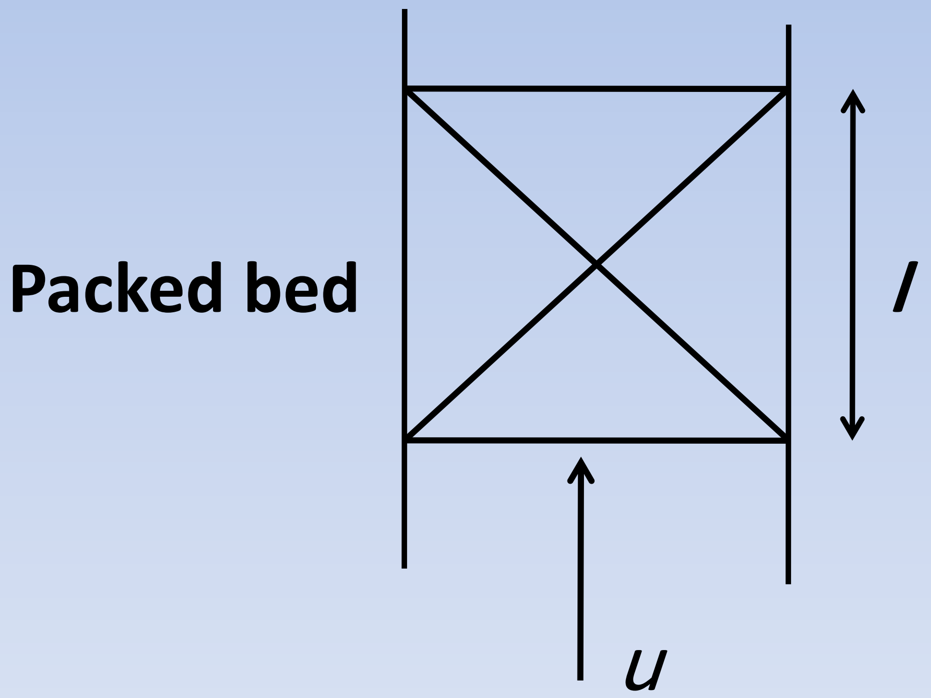
- **The presence of solid particles in a bed or a column will cause a pressure drop when the fluid passes through it.**
- **Examples: Dryers used solid material, Gas absorption columns using different types of packing, Sand filters, packed bed reactors.**
- **Formulating the pressure drop in terms of the controlling properties and parameters will lead to an important design formula.**

Introduction

Finding the pressure drop through a bed is also a one method used to measure the surface area and the particle size of a powder material as mentioned in the beginning of the course.

This is called Permeability cell technique for measuring the particle size of a powder material

Flow of a single fluid through a granular bed



Darcy's Law and Permeability

- Darcy observed that the flow of water through a packed bed of sand was governed by the relationship:

$$\begin{bmatrix} \textbf{\textit{pressure}} \\ \textbf{\textit{gradient}} \end{bmatrix} \alpha \begin{bmatrix} \textbf{\textit{liquid}} \\ \textbf{\textit{velocity}} \end{bmatrix} \textbf{\textit{or}} \ \frac{(-\Delta p)}{l} \alpha \ u$$

Where

-Δp : pressure drop across the bed

l : bed depth

u : superficial velocity= fluid volumetric flow rate/cross sectional area, ***(1/A) (dv/dt)***

A : cross sectional area of the bed.

$$\therefore \quad u = -K\left(\frac{\Delta P}{l}\right) = -\left(\frac{B}{\mu}\right)\left(\frac{\Delta P}{l}\right)$$

where

K : proportionality constant that depends on the properties of bed and fluid = B/μ.

B : permeability coeff. of the bed ‘depends on the properties of solid particles’.

μ : viscosity of the fluid.

Specific surface and voidage

(i) specific surface area of the bed, $S_B \Rightarrow$ 'surface area of the bed which is available to hold the fluid per unit volume of the bed' m^2/m^3 or $[length]^{-1}$.

(ii) fractional voidage of the bed, e $\Rightarrow$'fraction of volume of bed that not occupied by solid material'. (1-e) represents the fraction of bed volume that occupied by solid material.

Notes

- **S : sp. Surface area of particles. For a sphere; S=6/d**
- **$S \neq S_B$ 'due to voidage, e'**
- **$S_B = S(1\text{-}e)$**
- As **e** *is increased, flow through the bed becomes easier and so the permeability coefficient* ***B*** *increases.*
- If the particles are **randomly packed**, then **e** *should be approximately constant* throughout the bed and the resistance to flow is the same in all directions.

the properties of beds (S, e, B) of some common regular-shaped materials.

Solid Constituents		Porous Mass		
No.	Description	Specific Surface Area S(m^2/m^3)	Fractional Voidage, e (−)	Permeability Coefficient B (m^2)
		Spheres		
1	0.794 mm diam. ($\frac{1}{32}$ in.)	7600	0.393	6.2×10^{-10}
2	1.588 mm diam. ($\frac{1}{16}$ in.)	3759	0.405	2.8×10^{-9}
3	3.175 mm diam. ($\frac{1}{8}$ in.)	1895	0.393	9.4×10^{-9}
4	6.35 mm diam. ($\frac{1}{4}$ in.)	948	0.405	4.9×10^{-8}
5	7.94 mm diam. ($\frac{5}{16}$ in.)	756	0.416	9.4×10^{-8}
		Cubes		
6	3.175 mm ($\frac{1}{8}$ in.)	1860	0.190	4.6×10^{-10}
7	3.175 mm ($\frac{1}{8}$ in.)	1860	0.425	1.5×10^{-8}
8	6.35 mm ($\frac{1}{4}$ in.)	1078	0.318	1.4×10^{-8}
9	6.35 mm ($\frac{1}{4}$ in.)	1078	0.455	6.9×10^{-8}
		Hexagonal prisms		
10	4.76 mm × 4.76 mm thick ($\frac{3}{16}$ in. × $\frac{3}{16}$ in.)	1262	0.355	1.3×10^{-8}
11	4.76 mm × 4.76 mm thick ($\frac{3}{16}$ in. × $\frac{3}{16}$ in.)	1262	0.472	5.9×10^{-8}
		Triangular pyramids		
12	6.35 mm length × 2.87 mm ht. ($\frac{1}{4}$ in. × 0.113 in.)	2410	0.361	6.0×10^{-9}
13	6.35 mm length × 2.87 mm ht. ($\frac{1}{4}$ in. × 0.113 in.)	2410	0.518	1.9×10^{-8}
		Cylinders		
14	3.175 mm × 3.175 mm diam. ($\frac{1}{8}$ in. × $\frac{1}{8}$ in.)	1840	0.401	1.1×10^{-8}
15	3.175 mm × 6.35 mm diam. ($\frac{1}{8}$ in. × $\frac{1}{4}$ in.)	1585	0.397	1.2×10^{-8}
16	6.35 mm × 6.35 mm diam. ($\frac{1}{4}$ in. × $\frac{1}{4}$ in.)	945	0.410	4.6×10^{-8}

apply only to the laminar flow regime.

Properties of beds of some regular-shaped materials—Cont'd

Solid Constituents		Porous Mass		
No.	Description	Specific Surface Area S (m²/m³)	Fractional Voidage, e (−)	Permeability Coefficient B (m²)
		Plates		
17	6.35 mm × 6.35 mm × 0.794 mm ($\frac{1}{4}$ in. × $\frac{1}{4}$ in. × $\frac{1}{32}$ in.)	3033	0.410	5.0×10^{-9}
18	6.35 mm × 6.35 mm × 1.59 mm ($\frac{1}{4}$ in. × $\frac{1}{4}$ in. × $\frac{1}{16}$ in.)	1984	0.409	1.1×10^{-8}
		Discs		
19	3.175 mm diam. × 1.59 mm ($\frac{1}{8}$ in. × $\frac{1}{16}$ in.)	2540	0.398	6.3×10^{-9}
		Porcelain Berl saddles		
20	6 mm (0.236 in.)	2450	0.685	9.8×10^{-8}
21	6 mm (0.236 in.)	2450	0.750	1.73×10^{-7}
22	6 mm (0.236 in.)	2450	0.790	2.94×10^{-7}
23	6 mm (0.236 in.)	2450	0.832	3.94×10^{-7}
24	Lessing rings (6 mm)	5950	0.870	1.71×10^{-7}
25	Lessing rings (6 mm)	5950	0.889	2.79×10^{-7}

Table 4.3. Design data for various packings

	Size		Wall thickness		Number		Bed density		Contact surface S_B		Free space %	Packing factor F	
	(in.)	(mm)	(in.)	(mm)	$(/ft^3)$	$(/m^3)$	(lb/ft^3)	(kg/m^3)	(ft^2/ft^3)	(m^2/m^3)	(100 e)	(ft^2/ft^3)	(m^2/m^3)
Ceramic Raschig Rings	0.25	6	0.03	0.8	85,600	3,020,000	60	960	242	794	62	1600	5250
	0.38	9	0.05	1.3	24,700	872,000	61	970	157	575	67	1000	3280
	0.50	12	0.07	1.8	10,700	377,000	55	880	112	368	64	640	2100
	0.75	19	0.09	2.3	3090	109,000	50	800	73	240	72	255	840
	1.0	25	0.14	3.6	1350	47,600	42	670	58	190	71	160	525
	1.25	31			670	23,600	46	730			71	125	410
	1.5	38			387	13,600	43	680			73	95	310
	2.0	50	0.25	6.4	164	5790	41	650	29	95	74	65	210
	3.0	76			50	1765	35	560			78	36	120
Metal Raschig Rings	0.25	6	0.03	0.8	88,000	3,100,000	133	2130			72	700	2300
	0.38	9	0.03	0.8	27,000	953,000	94	1500			81	390	1280
	0.50	12	0.03	0.8	11,400	402,000	75	1200	127	417	85	300	980
	0.75	19	0.03	0.8	3340	117,000	52	830	84	276	89	185	605

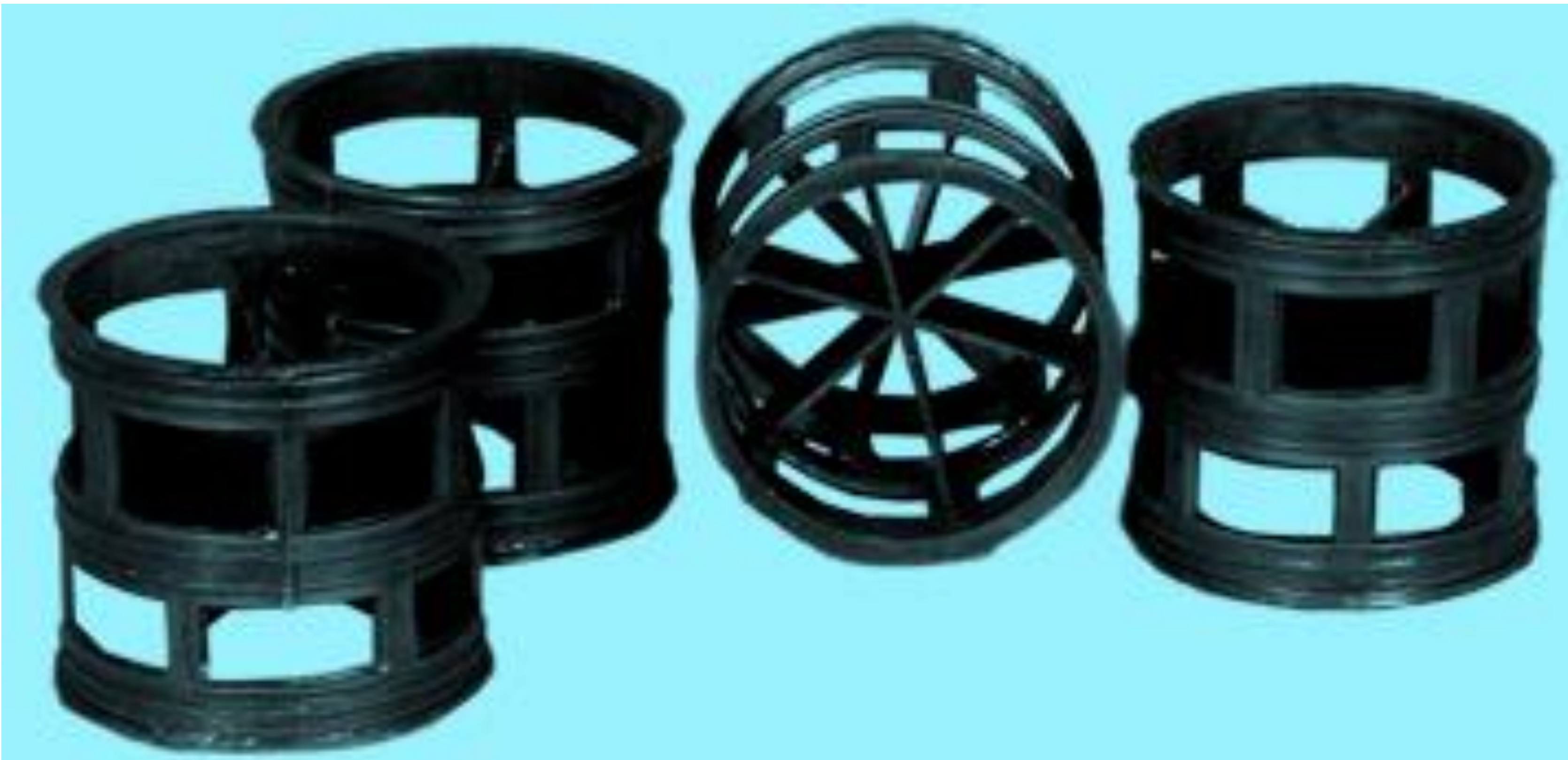
Raschig rings

Raschig rings are pieces of tube (approximately equal in length and diameter) used in large numbers as a packed bed within columns for distillations and other chemical engineering processes

Structured packing

Structured packing refers to a range of specially designed materials for use in absorption, distillation columns , chemical reactors and cooling towers.

Pall rings



Saddles

General Expressions for Flow Through Beds in Terms of Carman–Kozeny Equations

Streamline flow—Carman–Kozeny equation

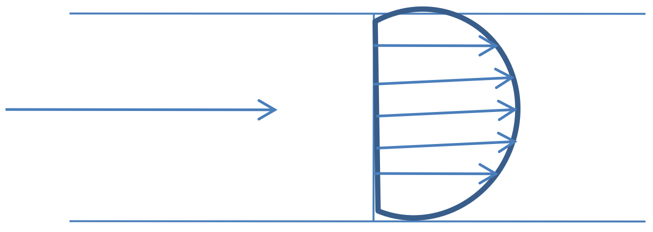
General expressions for flow through Beds

- **The flow of fluid through a bed can be analyzed in terms of the fluid flow through tubes.**

- ***Streamline flow***

  Start with the Hagen-Poiseuille equation for laminar flow through a circular tube

$$\frac{(-\Delta P)}{l} = \frac{32\mu\, u_t}{d_t^2} \quad \ldots\ldots\ldots\ldots\ldots (1)$$



OR

$$u_t = \frac{d_t^2}{32\mu} \frac{(-\Delta P)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

- **But in case of a packed bed, the free space can be assumed to form of a series of tortuous (twist) channels arranged in parallel, as shown schematically in fig. A and its idealization in fig. B . Therefore, the previous eq. can be modified as follows:**

$$u_1 = \frac{d_{m'}^2}{K'\mu}\frac{(-\Delta P)}{l'} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

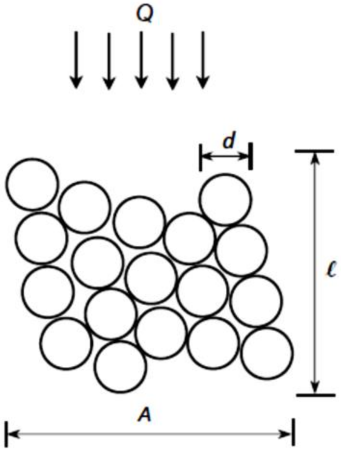


fig. A
Flow through a bed of uniform spheres

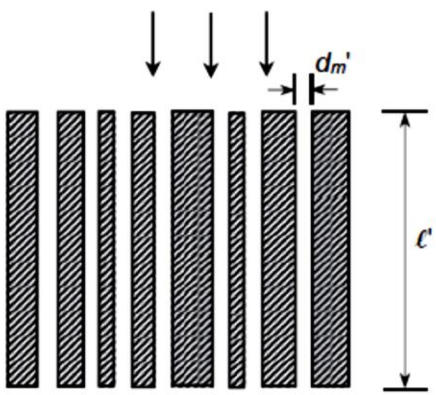
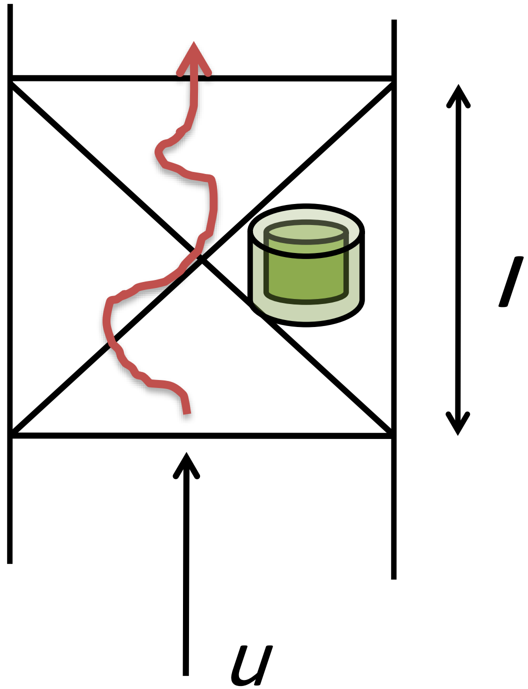


fig. B
The capillary model idealization



Where

$d_{m'}$:equivalent diam of the pore channels

K':dimensionless cons. depends on the bed structure.

l' :average length of Tortuous path of Capillaries.

u_1:average velocity through the pore channels.

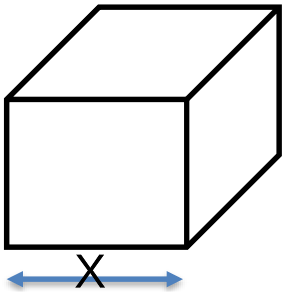
$u \equiv u_c$: superficial velocity based on empty cross sectional area

Note

Dupuit related u_c and u_1 by the following argument

In a cube of side X, the volume of free space is eX^3, so that the mean cross-sectional area for flow is the free volume divided by the height, or eX^2. The volume flow rate through this cube is u_cX^2, so that the average linear velocity through the pores, u_1, is given by:

$$u_1 = \frac{u_c X^2}{eX^2} = \frac{u_c}{e}$$



Since the equivalent diameter = flow area/wetted perimeter

For a packed bed; flow area *= e A*

Wetted perimeter = surface area of *bed/l*

$$= S_B A\, l / l = S_B A$$

$$\therefore dm' = \frac{\text{flow area}}{\text{wetted perimeter}} = \frac{e}{S_B} = \frac{e}{S(1-e)}$$

average velocity, $u_1 = u / e$

and, $l' \alpha\, l$

Equation (3) becomes

$$u = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This equation is called Carman-Kozeny equation.
The constant K'' depends on porosity, particle shape and other factors.
In general, K'' = 5.0 .

Note 1

Compare Carman-Kozeny equation with Darcy equation, you can deduce that

$$B = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2}$$

$$u = -\left(\frac{B}{\mu}\right)\left(\frac{\Delta P}{l}\right)$$

$$u = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l}$$

Note 2

For spherical particle ***S = 6/d***

$$u = \frac{1}{180} \frac{e^3 d^2}{(1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} \quad \text{..................}(5)$$

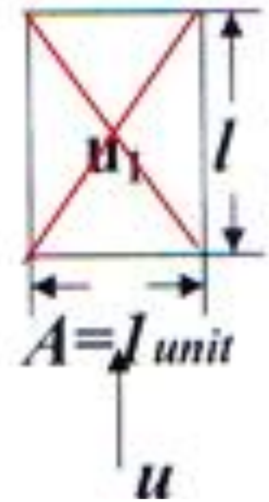
Note3

For non-spherical particle, the surface mean diameter, d_s, can be substituted instead of d

Streamline and Turbulent Flow

Modified Reynolds number, Re_1 :

$$Re_1 = \left(\frac{u}{e}\right)\frac{e}{S(1-e)}\frac{\rho}{\mu} = \frac{u\rho}{\mu(1-e)S} = B$$



Vol of particles in bed = $l\,A\,(1-e) = l\,(1-e)$

total surface = $S\,A\,l\,(1-e) = S\,l\,(1-e)$

the friction factor= $\dfrac{R_1}{\rho u_1^2}$

where R_1 is the component of the drag force per unit area of particle surface in the direction of motion

The resistance force or drag force = $R_1 S l (1-e)$

This force equals to the force resulting from a pressure difference ΔP across the bed.

$$(-\Delta P)(A)(e) = R_1 S l (1-e)$$

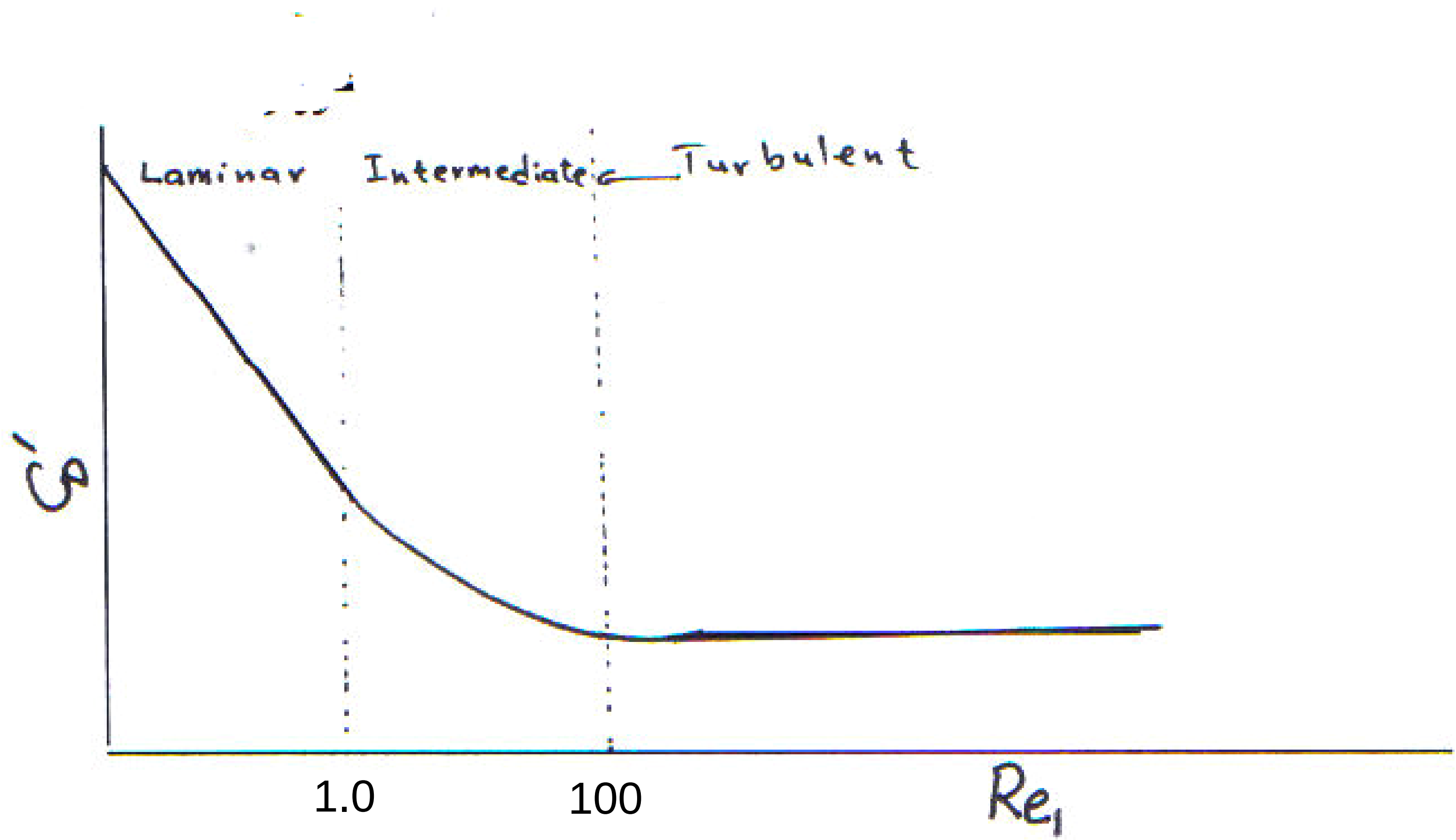
$$(-\Delta P)(e) = R_1 S l (1-e)$$

$$R_1 = e(-\Delta P)/ S(1-e)l$$

$$C_D{}' = \frac{R_1}{\rho u_1^2} = \frac{e}{S(1-e)} \cdot \frac{(-\Delta P)}{l} \cdot \frac{1}{\rho (\frac{u}{e})^2}$$

$$= \frac{e^3}{S(1-e)} \cdot \frac{(-\Delta P)}{l} \cdot \frac{1}{\rho u^2}$$

C_D against Re_l



Carman curve ***can be approximated by this eq.:***

$$\frac{R_1}{\rho u_1^2} = 5\,Re_1^{-1} + 0.4\,Re_1^{-0.1}$$

The 1st term 5/Re₁ represents friction coefficient For streamline flow Re₁<1.0.

For intermediate flow through a packed bed, the friction coefficient is the sum of the two terms $5/Re_1 + 0.4/Re_1^{0.1}$.

For turbulent regime $Re_1 > 100$, the friction coefficient is
$0.4/Re_1^{0.1}$

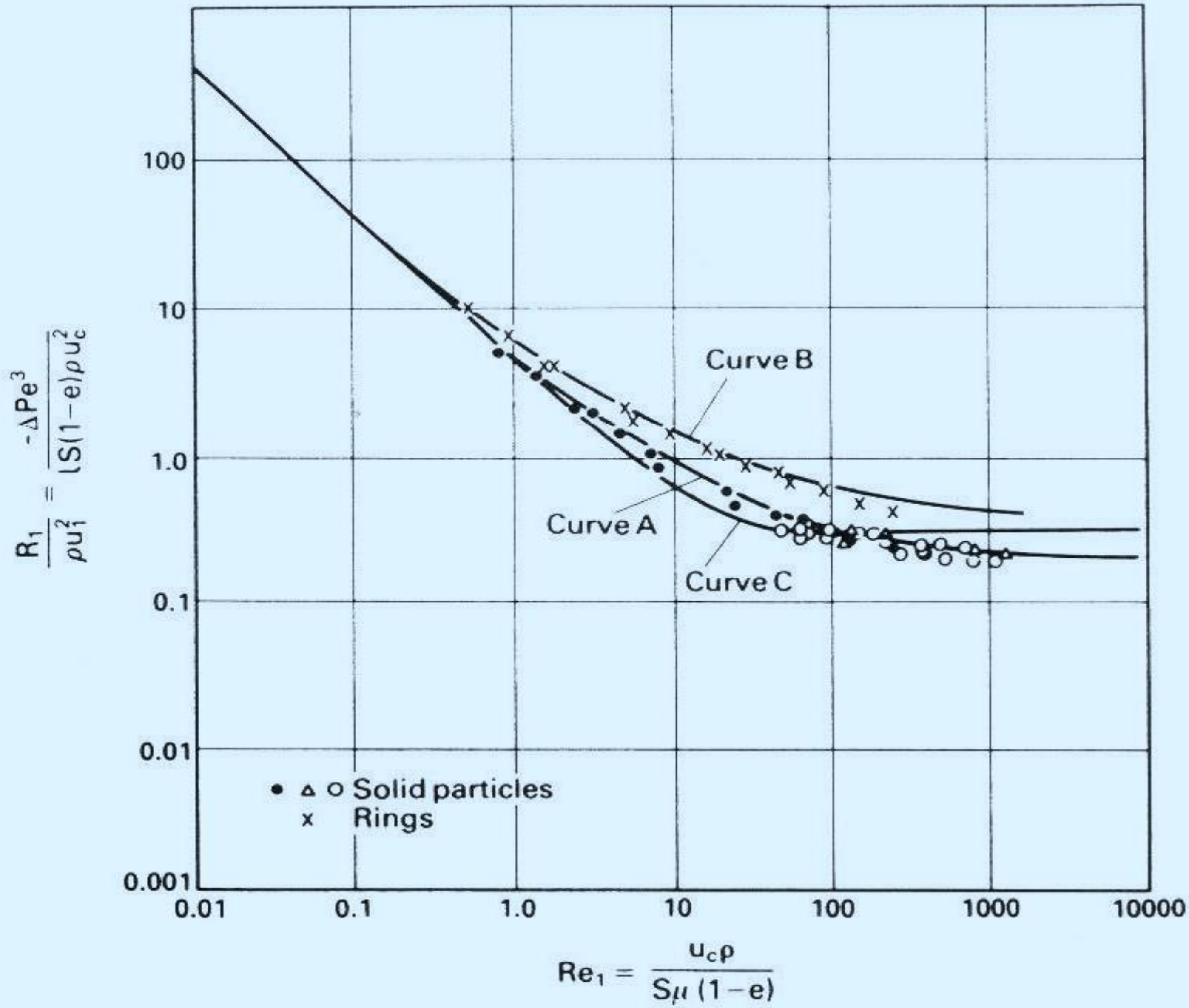


FIG. 4.1. Carman's graph of $R_1/\rho u_1^2$ against Re_1

Note:

- **The constant K'' is usually dependent on the structure of the bed, the shape of the cross-section of a channel, solid size and voidage. In general, $K''=(l'/l)^2 K_o$ 'see the text'**
- **Wall effect ' f_w'**

  $f_w = (1+0.5S_c/S)^2$

  where S_c is the surface of the container per unit volume of bed.

  This correction factor **' f_w' must be multiplied by Kozeny equation.**

where *(l'/l)* is the tortuosity and is a measure of the fluid path length through the bed compared with the actual depth of the bed,

K''_o is a factor which depends on the shape of the cross-section of a channel **through which fluid is passing.**

Ergun Equation

For flow through ring packings which as described later are often used in industrial packed columns, ERGUN[10] obtained a good semi-empirical correlation for pressure drop as follows:

$$\frac{-\Delta P}{l} = 150\frac{(1-e)^2}{e^3}\frac{\mu u_c}{d^2} + 1.75\frac{(1-e)}{e^3}\frac{\rho u_c^2}{d} \tag{4.20}$$

Writing $d = 6/S$ (from equation 4.3):

$$\frac{-\Delta P}{Sl\rho u_c^2}\frac{e^3}{1-e} = 4.17\frac{\mu S(1-e)}{\rho u_c} + 0.29$$

or:

$$\frac{R_1}{\rho u_1^2} = 4.17 Re_1^{-1} + 0.29 \tag{4.21}$$

Compare with
Carman Expression

$$\frac{R_1}{\rho u_1^2} = 5\, Re_1^{-1} + 0.4\, Re_1^{-0.1}$$

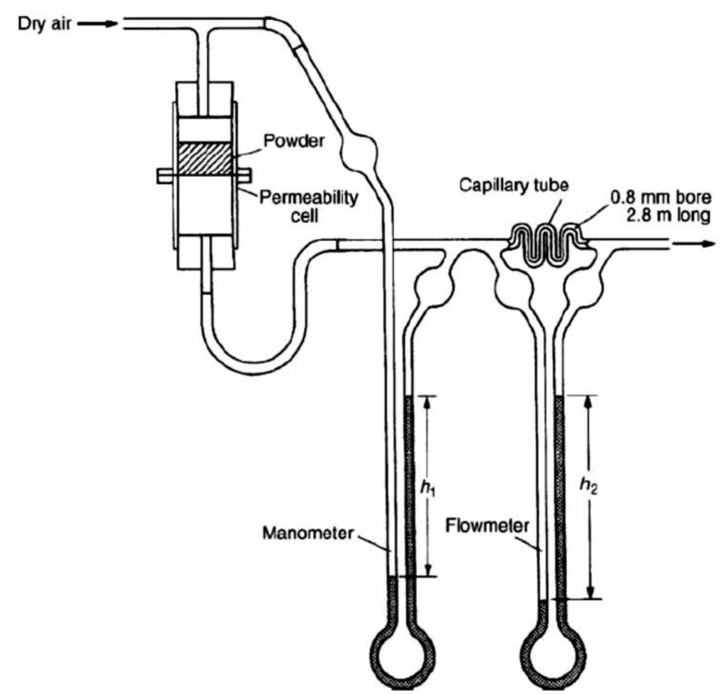
Use of Carman–Kozeny equation for measurement of particle surface area

The Carman–Kozeny equation relates the drop in pressure through a bed to the specific surface of the material and can, therefore, be used as a means of calculating S from measurements of the drop in pressure. This method is strictly only suitable for beds of uniformly packed particles, and it is not a suitable method for measuring the size distribution of particles in the sub sieve range.

The permeability apparatus

- In this apparatus, air or another suitable gas flows through the bed contained in a cell (25mm diameter, 87mm deep), and the pressure drop is obtained from h_1, and the gas flow rate from h_2.

- The method has been successfully developed for measurement of the surface area of cement and for such materials as pigments, fine metal powders, pulverized coal, and fine fibers



Example 1

- In a contact sulfuric acid plant the secondary converter is a tray type converter, 2.3 m in diameter with the catalyst arranged in three layers, each 0.45 m thick. The catalyst is in the form of cylindrical pellets 9.5 mm in diameter and 9.5 mm long. The void fraction is 0.35. The gas enters the converter at 675 K and leaves at 720 K. Its inlet composition is:

SO_3 6.6, SO_2 1.7, O_2 10.0, N_2 81.7 mole per cent and its exit composition is:

SO_3 8.2, SO_2 0.2, O_2 9.3, N_2 82.3 mole per cent

The gas flowrate is 0.68 kg/m^{2}s. Calculate the pressure drop through the converter. The viscosity of the gas is 0.032 mNs/m2.

Solution

- Find the density by assuming ideal gas, hence use ideal gas law to obtain the density.

$$\rho = PM/RT$$

- Find S   S= surface area of pellet/volume
- From the Carman equation:

$$\frac{R}{\rho u_1^2} = \frac{e^3}{S(1-e)} \frac{(-\Delta P)}{l} \frac{1}{\rho u_c^2}$$

$$\frac{R}{\rho u_1^2} = 5/Re_1 + 0.4/Re_1^{0.1}$$

Flow rate, kg/m^2.s = ρ u

and: $$Re_1 = \frac{G'}{S(1-e)\mu}$$

$$S = 6/d = 6/(9.5 \times 10^{-3}) = 631\ \mathrm{m^2/m^3}$$

Hence: $$Re_1 = 0.68/(631 \times 0.65 \times 0.032 \times 10^{-3}) = 51.8$$

and: $$\frac{R}{\rho u^2} = \left(\frac{5}{51.8}\right) + \left(\frac{0.4}{(51.8)^{0.1}}\right) = 0.366$$

From the Carman equation:

$$\frac{R}{\rho u_1^2} = \frac{e^3}{S(1-e)} \frac{(-\Delta}{l}$$

$$\frac{R}{\rho u_1^2} = 5/Re_1 + 0.4/$$

From equation 4.15:

Total length

$$-\Delta P = 0.366 \times 631 \times 0.65 \times (3 \times 0.45) \times 0.569 \times (1.20)^2/(0.35)^3$$

ρ

$$= 3.87 \times 10^3\ \mathrm{N/m^2} \text{ or } \underline{3.9\ \mathrm{kN/m^2}}$$

Flow rate, G' = ρ u

0.68 = 0.569 u

u = 0.68/0.569 = 1.2 m/s

Table A Comparison between packed bed & packed column

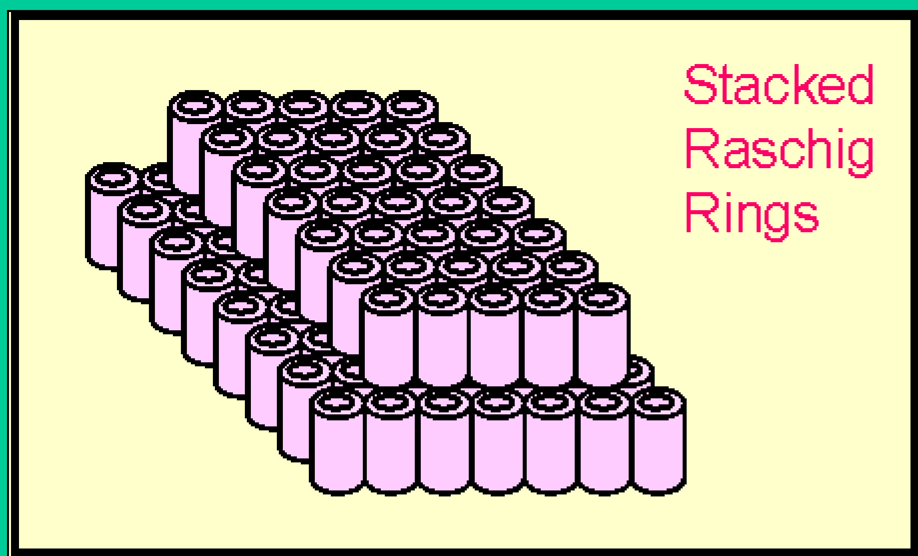
Packed bed	Packed column
Single flow through bed	Usually, two flows(liquid +gas)
Size of packing: small	Size of packing: large
Packing element: usually solid	Packing element: usually hollow with large internal surface area and small pressure gradient.
Regime of flow: Laminar	Regime of flow: Turbulent

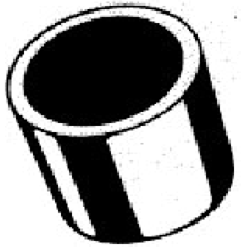
Types of Packing

1. Particulate: dumped or stacked Raschig Rings, Lessing rings, Berl Saddles, Stoneware, Porcelain, Carbon, Metal.
2. Grid Packings: wood, metal, carbon, plastic.
3. Wire-Mesh & Knit-mesh packing

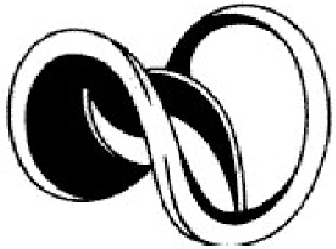
‘ See the Text for details’


Randomly-packed Raschig Rings


Stacked
Raschig
Rings

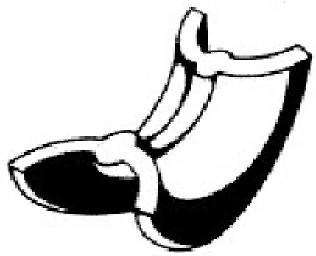


Raschig ring



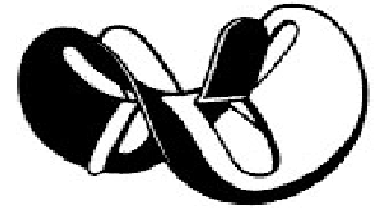
Berl saddle

Pall ring

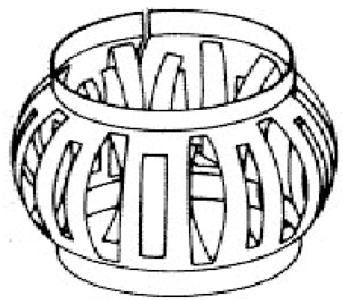


Intalox saddle

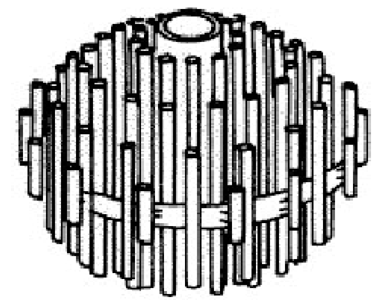
Interpak



Super saddle



Top-Pak



Hedgehog

Multi knit mesh columns packing

Stainless steel knitted wire mesh packing

TABLE 4.1. *Properties of Beds of Some Regular-shaped Materials*[2]

	Solid constituents		Porous mass	
No.	Description	Specific surface area S (m^2/m^3)	Fractional voidage, e	Permeability coefficient B (m^2)
	Spheres			
1	$\frac{1}{32}$ in. diam. (0.794 mm)	7600	0.393	6.2×10^{-10}
2	$\frac{1}{16}$ in. diam. (1.588 mm)	3759	0.405	2.8×10^{-9}
3	$\frac{1}{8}$ in. diam. (3.175 mm)	1895	0.393	9.4×10^{-9}
4	$\frac{1}{4}$ in. diam. (6.35 mm)	948	0.405	4.9×10^{-8}
5	$\frac{5}{16}$ in. diam. (7.94 mm)	756	0.416	9.4×10^{-8}
	Cubes			
6	$\frac{1}{8}$ in. (3.175 mm)	1860	0.190	4.6×10^{-10}
7	$\frac{1}{8}$ in. (3.175 mm)	1860	0.425	1.5×10^{-8}
8	$\frac{1}{4}$ in. (6.35 mm)	1078	0.318	1.4×10^{-8}
9	$\frac{1}{4}$ in. (6.35 mm)	1078	0.455	6.9×10^{-8}
	Hexagonal prisms			
10	$\frac{3}{16}$ in. $\times \frac{3}{16}$ in. thick (4.76 mm $\times$ 4.76 mm)	1262	0.355	1.3×10^{-8}
11	$\frac{3}{16}$ in. $\times \frac{3}{16}$ in. thick (4.76 mm $\times$ 4.76 mm)	1262	0.472	5.9×10^{-8}
	Triangular pyramids			
12	$\frac{1}{4}$ in. length $\times$ 0.113 in. ht. (6.35 mm $\times$ 2.87 mm)	2410	0.361	6.0×10^{-9}
13	$\frac{1}{4}$ in. length $\times$ 0.113 in. ht. (6.35 mm $\times$ 2.87 mm)	2410	0.518	1.9×10^{-8}
	Cylinders			
14	$\frac{1}{8}$ in. diam. $\times \frac{1}{8}$ in. (3.175 mm $\times$ 3.175 mm)	1840	0.401	1.1×10^{-8}
15	$\frac{1}{8}$ in. diam. $\times \frac{1}{4}$ in. (3.175 mm $\times$ 6.35 mm)	1585	0.397	1.2×10^{-8}
16	$\frac{1}{4}$ in. diam. $\times \frac{1}{4}$ in. (6.35 mm $\times$ 6.35 mm)	945	0.410	4.6×10^{-8}
	Plates			
17	$\frac{1}{4}$ in. $\times \frac{1}{4}$ in. $\times \frac{1}{32}$ in. (6.35 mm $\times$ 6.35 mm $\times$ 0.794 mm)	3033	0.410	5.0×10^{-9}
18	$\frac{1}{4}$ in. $\times \frac{1}{4}$ in. $\times \frac{1}{16}$ in. (6.35 mm $\times$ 6.35 mm $\times$ 1.59 mm)	1984	0.409	1.1×10^{-8}
	Discs			
19	$\frac{1}{8}$ in. diam. $\times \frac{1}{16}$ in. (3.175 mm $\times$ 1.59 mm)	2540	0.398	6.3×10^{-9}
	Porcelain Berl saddles			
20	0.236 in. (6 mm)	2450	0.685	9.8×10^{-8}
21	0.236 in. (6 mm)	2450	0.750	1.73×10^{-7}
22	0.236 in. (6 mm)	2450	0.790	2.94×10^{-7}
23	0.236 in. (6 mm)	2450	0.832	3.94×10^{-7}
24	Lessing rings (6 mm)	5950	0.870	1.71×10^{-7}
25	Lessing rings (6 mm)	5950	0.889	2.79×10^{-7}

Metal, Ceramic and carbon Raschig and Lessing rings

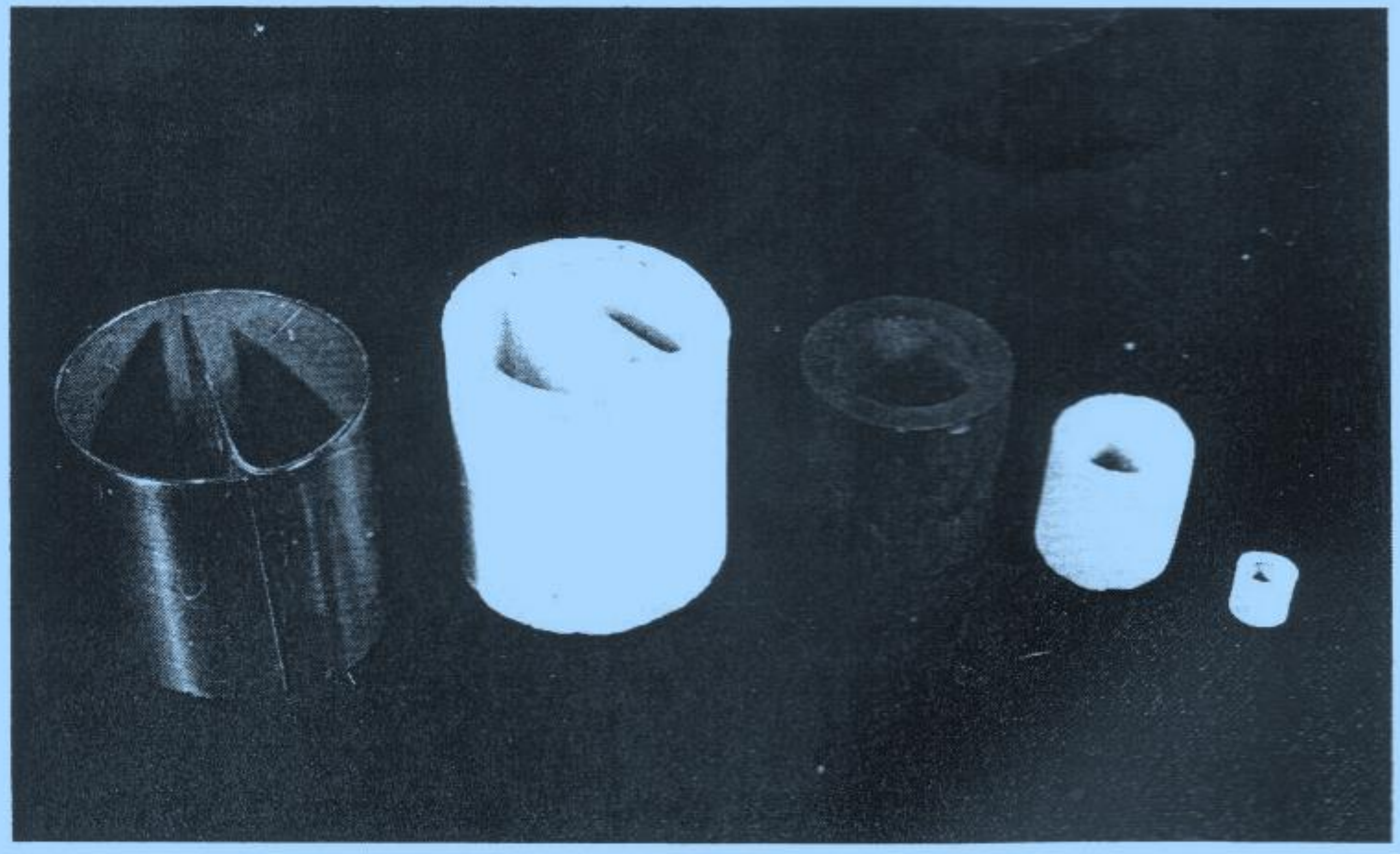


FIG. 4.13. Metal, ceramic and carbon Raschig and Lessing rings

Plastic, Ceramic and metal Pall rings

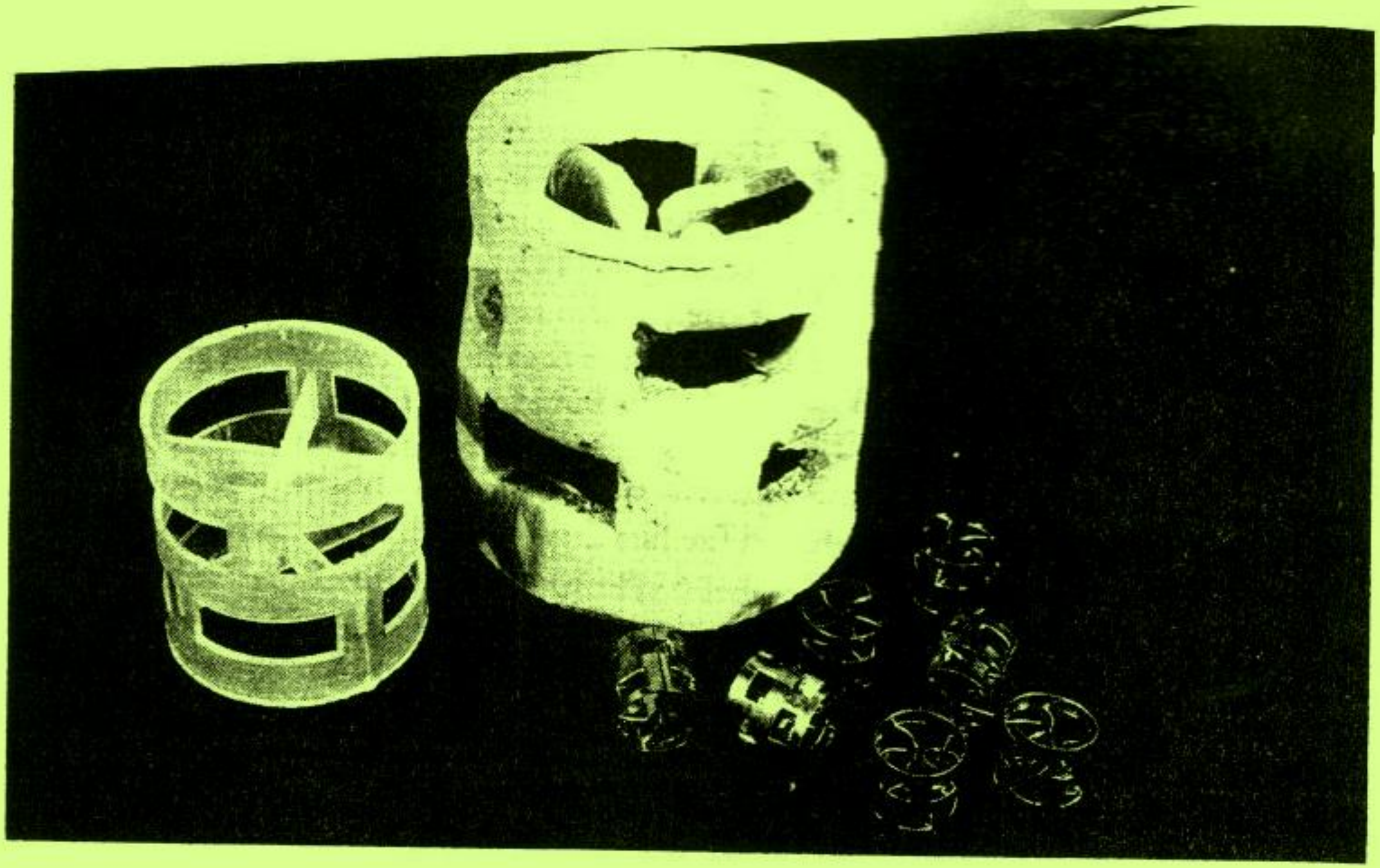


FIG. 4.14. Plastics, ceramic and metal Pall rings

Ceramic Intalox Berl Saddles and Intalox Metal

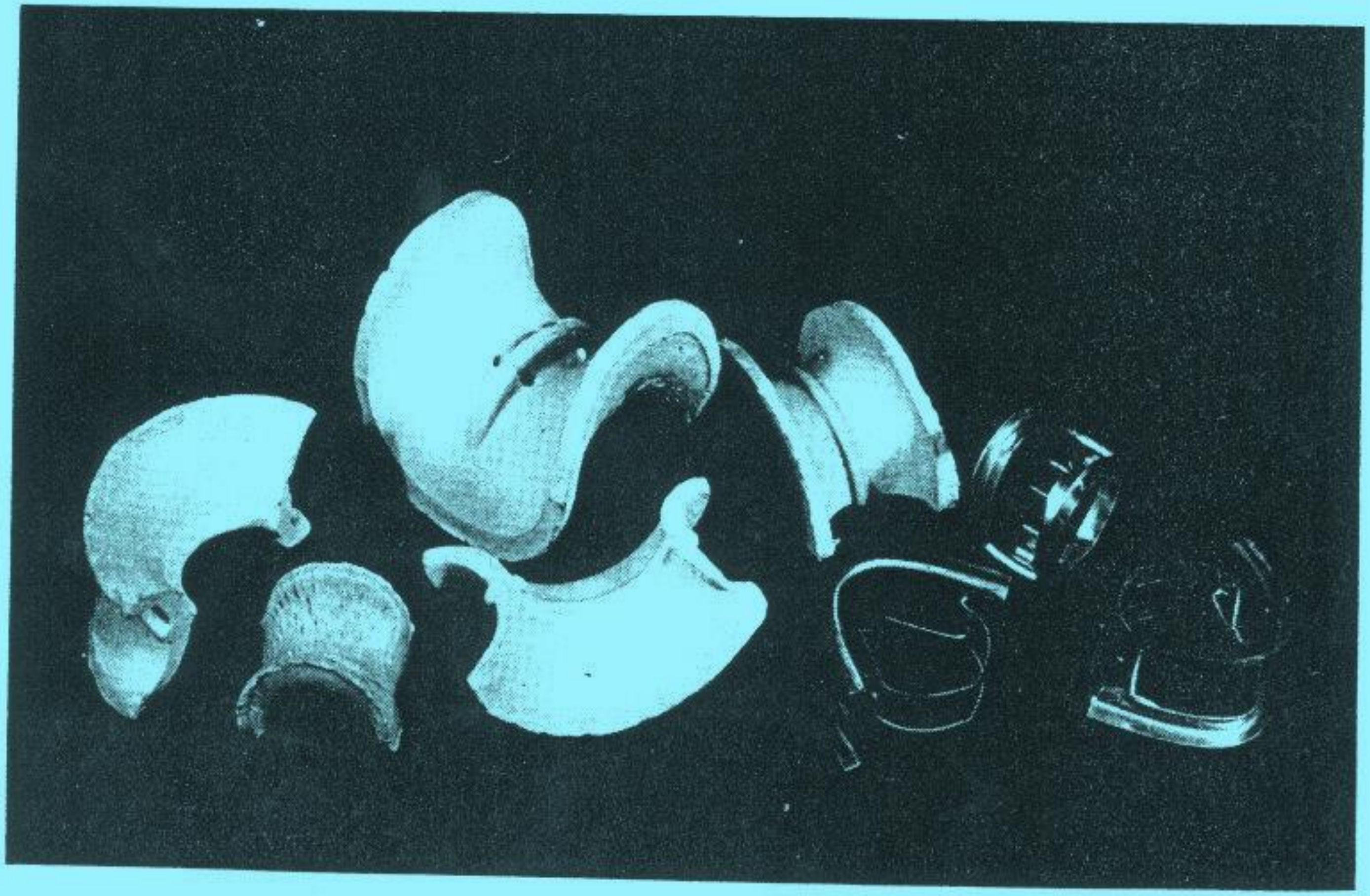


FIG. 4.15. Ceramic Intalox and Berl saddles and Intalox Metal

Design Data For Various Packings

	Size in.	Size mm	Wall thickness in.	Wall thickness mm	Number /ft^3	Number /m^3
Ceramic Raschig Rings	0.25	6	0.03	0.8	85,600	3,020,000
	0.38	9	0.05	1.3	24,700	872,000
	0.50	12	0.07	1.8	10,700	377,000
	0.75	19	0.09	2.3	3090	109,000
	1.0	25	0.14	3.6	1350	47,600
	1.25	31			670	23,600
	1.5	38			387	13,600
	2.0	50	0.25	6.4	164	5790
	3.0	76			50	1765
Metal Raschig Rings	0.25	6	0.03	0.8	88,000	3,100,000
	0.38	9	0.03	0.8	27,000	953,000
	0.50	12	0.03	0.8	11,400	402,000
	0.75	19	0.03	0.8	3340	117,000
(N.B. Bed densities	0.75	19	0.06	1.6	3140	110,000
are for mild	1.0	25	0.03	0.8	1430	50,000
steel; multiply	1.0	25	0.06	1.6	1310	46,200
by 1.105, 1.12, 1.37,	1.25	31	0.06	1.6	725	25,600
1.115 for stainless	1.5	38	0.06	1.6	400	14,100
steel, copper, aluminium,	2.0	50	0.06	1.6	168	5930
and monel respectively)	3.0	76	0.06	1.6	51	1800

	Bed density lb/ft³	kg/m³	Contact surface S_B ft²/ft³	m²/m³	Free space % (100 e)	Packing factor F ft²/ft³	m²/m³
Ceramic Raschig Rings	60	960	242	794	62	1600	5250
	61	970	157	575	67	1000	3280
	55	880	112	368	64	640	2100
	50	800	73	240	72	255	840
	42	670	58	190	71	160	525
	46	730			71	125	410
	43	680			73	95	310
	41	650	29	95	74	65	210
	35	560			78	36	120
Metal Raschig Rings	133	2130			72	700	2300
	94	1500			81	390	1280
	75	1200	127	417	85	300	980
	52	830	84	276	89	185	605
(N.B. Bed densities	94	1500			80	230	750
are for mild	39	620	63	207	92	115	375
steel; multiply	71	1130			86	137	450
by 1.105, 1.12, 1.37,	62	990			87	110	360
1.115 for stainless	49	780			90	83	270
steel, copper, aluminium,	37	590	31	102	92	57	190
and monel respectively)	25	400	22	72	95	32	105

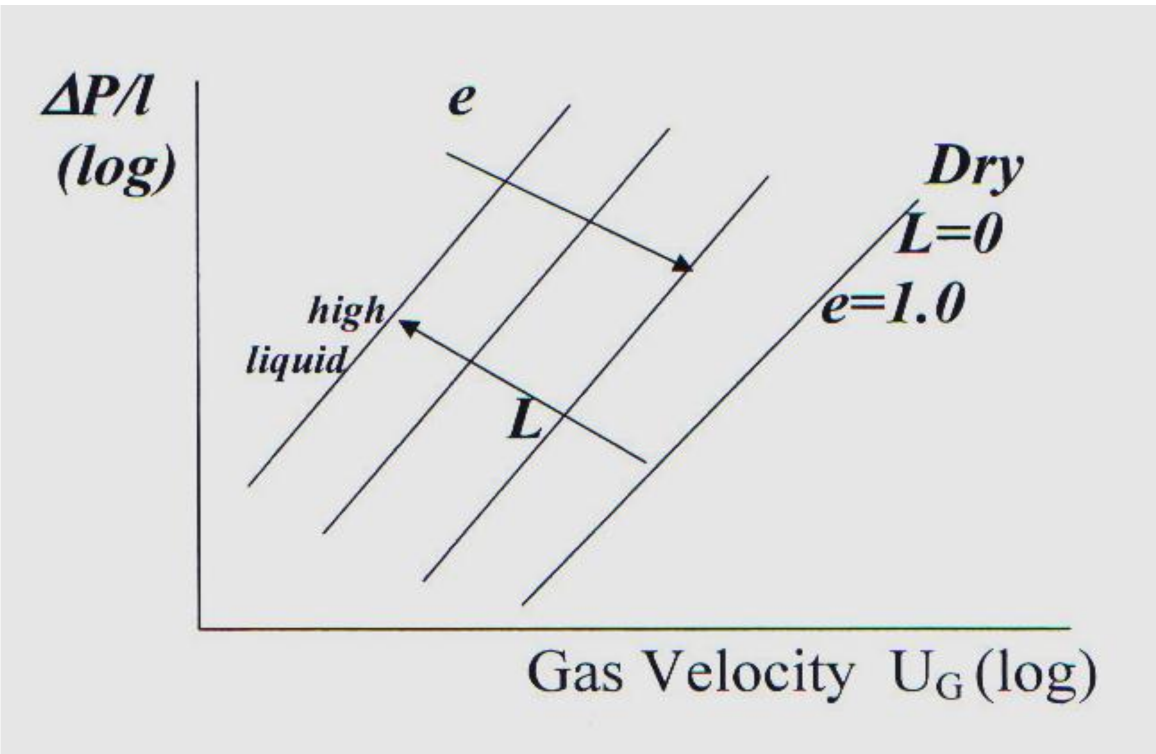
The packing factor F replaces the term S_B/e^3. Use of the given value of F in Figure 4.18 permits more predictable performance of designs incorporating packed beds since the values quoted are derived from operating characteristics of the packings rather than from their physical dimensions.

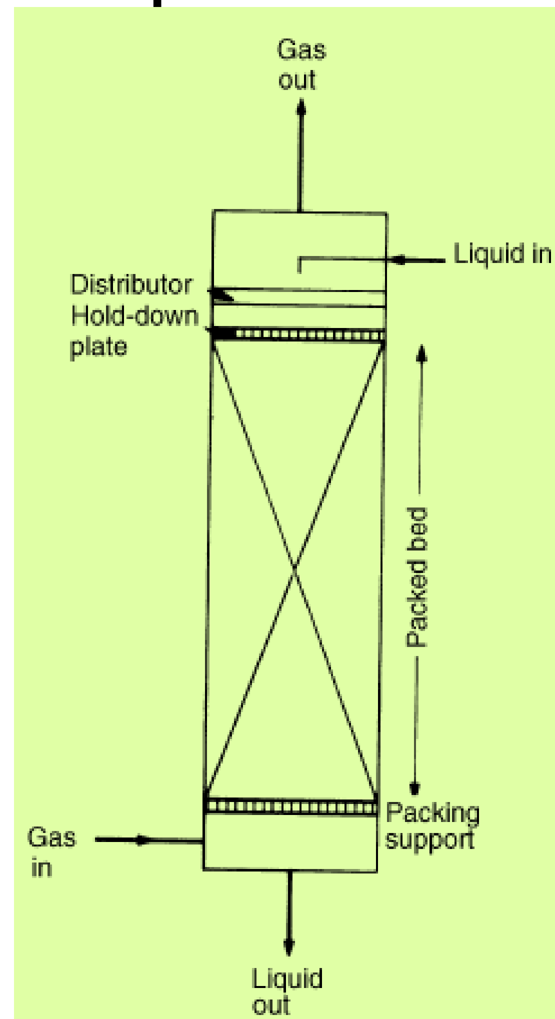
Table 7.3 Design data for various packings

	Size		Wall Thickness		Number Density		Bed Density		Contact Surface S_B		Free Space (%)	Packing Factor F	
	(in.)	(mm)	(in.)	(mm)	(/ft^3)	(/m^3)	(lb/ft^3)	(kg/m^3)	(ft^2/ft^3)	(m^2/m^3)	(100 e)	(ft^2/ft^3)	(m^2/m^3)
Ceramic Raschig	0.25	6	0.03	0.8	85,600	3,020,000	60	960	242	794	62	1600	5250
Rings	0.38	9	0.05	1.3	24,700	872,000	61	970	157	575	67	1000	3280
	0.50	12	0.07	1.8	10,700	377,000	55	880	112	368	64	640	2100
	0.75	19	0.09	2.3	3090	109,000	50	800	73	240	72	255	840
	1.0	25	0.14	3.6	1350	47,600	42	670	58	190	71	160	525
	1.25	31			670	23,600	46	730			71	125	410
	1.5	38			387	13,600	43	680			73	95	310
	2.0	50	0.25	6.4	164	5790	41	650	29	95	74	65	210
	3.0	76			50	1765	35	560			78	36	120
Metal Raschig	0.25	6	0.03	0.8	88,000	3,100,000	133	2130			72	700	2300
Rings	0.38	9	0.03	0.8	27,000	953,000	94	1500			81	390	1280
	0.50	12	0.03	0.8	11,400	402,000	75	1200	127	417	85	300	980
	0.75	19	0.03	0.8	3340	117,000	52	830	84	276	89	185	605
(Bed densities are	0.75	19	0.06	1.6	3140	110,000	94	1500			80	230	750
for mild steel;	1.0	25	0.03	0.8	1430	50,000	39	620	63	207	92	115	375
multiply by 1.105,	1.0	25	0.06	1.6	1310	46,200	71	1130			86	137	450
1.12, 1.37, 1.115	1.25	31	0.06	1.6	725	25,600	62	990			87	110	360
for stainless steel,	1.5	38	0.06	1.6	400	14,100	49	780			90	83	270
copper,	2.0	50	0.06	1.6	168	5930	37	590	31	102	92	57	190
aluminium, and	3.0	76	0.06	1.6	51	1800	25	400	22	72	95	32	105
monel													
respectively)													
Carbon Raschig	0.25	6	0.06	1.6	85,000	3,000,000	46	730	212	696	55	1600	5250
Rings	0.50	12	0.06	1.6	10,600	374,000	27	430	114	374	74	410	1350
	0.75	19	0.12	3.2	3140	110,000	34	540	75	246	67	280	920
	1.0	25	0.12	3.2	1325	46,000	27	430	57	187	74	160	525
	1.25	31			678	23,000	31	490			69	125	410
	1.5	38			392	13,800	34	540			67	130	425
	2.0	50	0.25	6.4	166	5860	27	430	29	95	74	65	210
	3.0	76	0.31	8.0	49	1730	23	370	19	62	78	36	120
Metal Pall Rings	0.62	15	0.02	0.5	5950	210,000	37	590	104	341	93	70	230
(Bed densities are	1.0	25	0.025	0.6	1400	49,000	30	480	64	210	94	48	160
for mild steel)	1.25	31	0.03	0.8	375	13,000	24	380	39	128	95	28	92
	2.0	50	0.035	0.9	170	6000	22	350	31	102	96	20	66
	3.5	76	0.05	1.2	33	1160	17	270	20	65	97	16	52

Flow Through Packed Towers

Gas flow through packed tower in presence of liquid flow





$(\Delta P/l)^{gas} \alpha\ U_G^n$

U_G superficial gas velocity $< 1 m/s$

l bed height

L mass flow rate of liquid 'L' liquid mass velocity'

In general, $L > 1\ kg/s.m^2$ tower

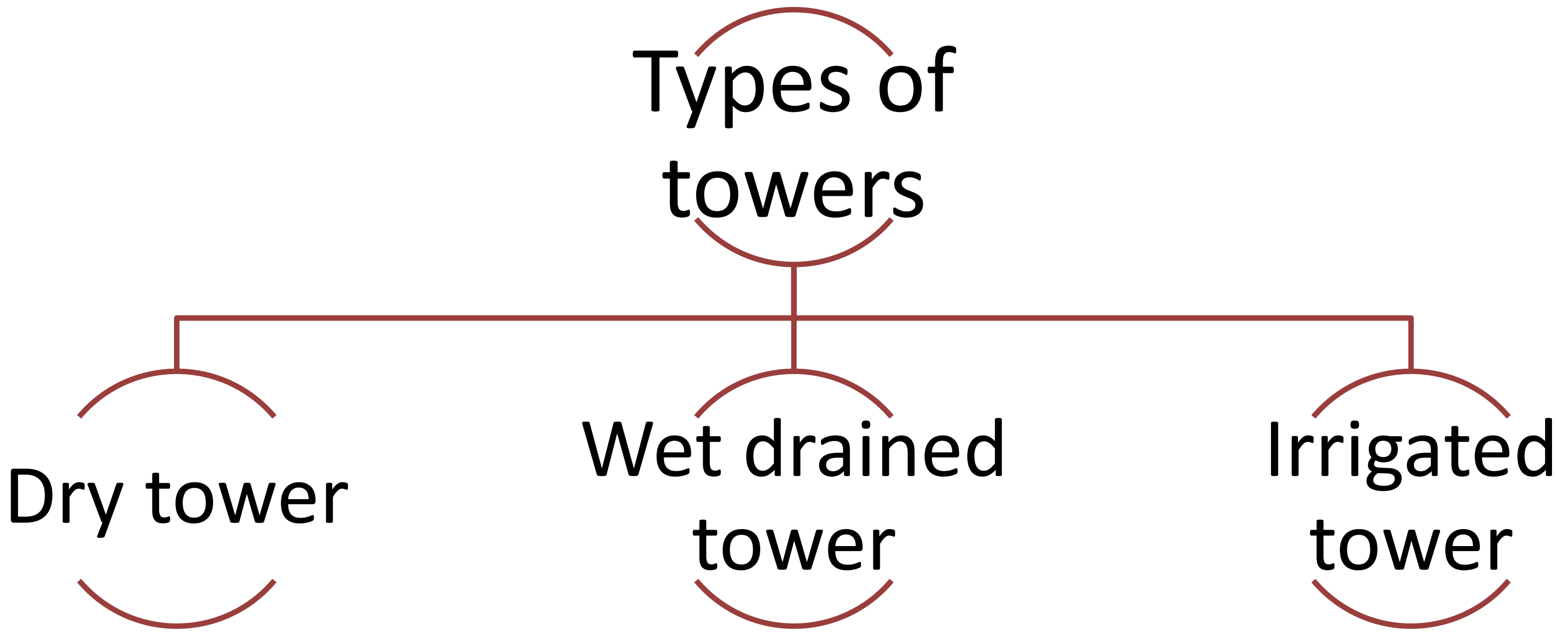
G mass flow rate of gas 'G′ Gas mass velocity'

e voidage

n index ; $1.6 < n > 2.0 \Rightarrow \therefore n \approx 1.8$

$\therefore$ ***In general,***

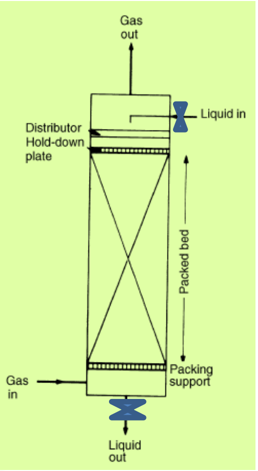
$$\frac{\Delta P}{l} \alpha U_G^{1.8}$$


Types of towers
Dry tower
Wet drained tower
Irrigated tower

Types of towers

- **Dry tower ⇒ Single gas flow $(-\Delta P/l)$; *use* modified Reynolds number, Re_1 . '$Re_1 > 30$'**
- **Wet Drained Tower L=0.0**

$$(-\Delta P_w) = (1 + k_1/d_n)\,\Delta P_d$$



Where

ΔP_w : pressure drop across the wet drained tower

ΔP_d : Pressure drop across the dry tower

d_n : size of solids 'normal size of element in mm'

k_1: constant, for broken solids: $k_1 = 5.5$, $d_n = 100mm$

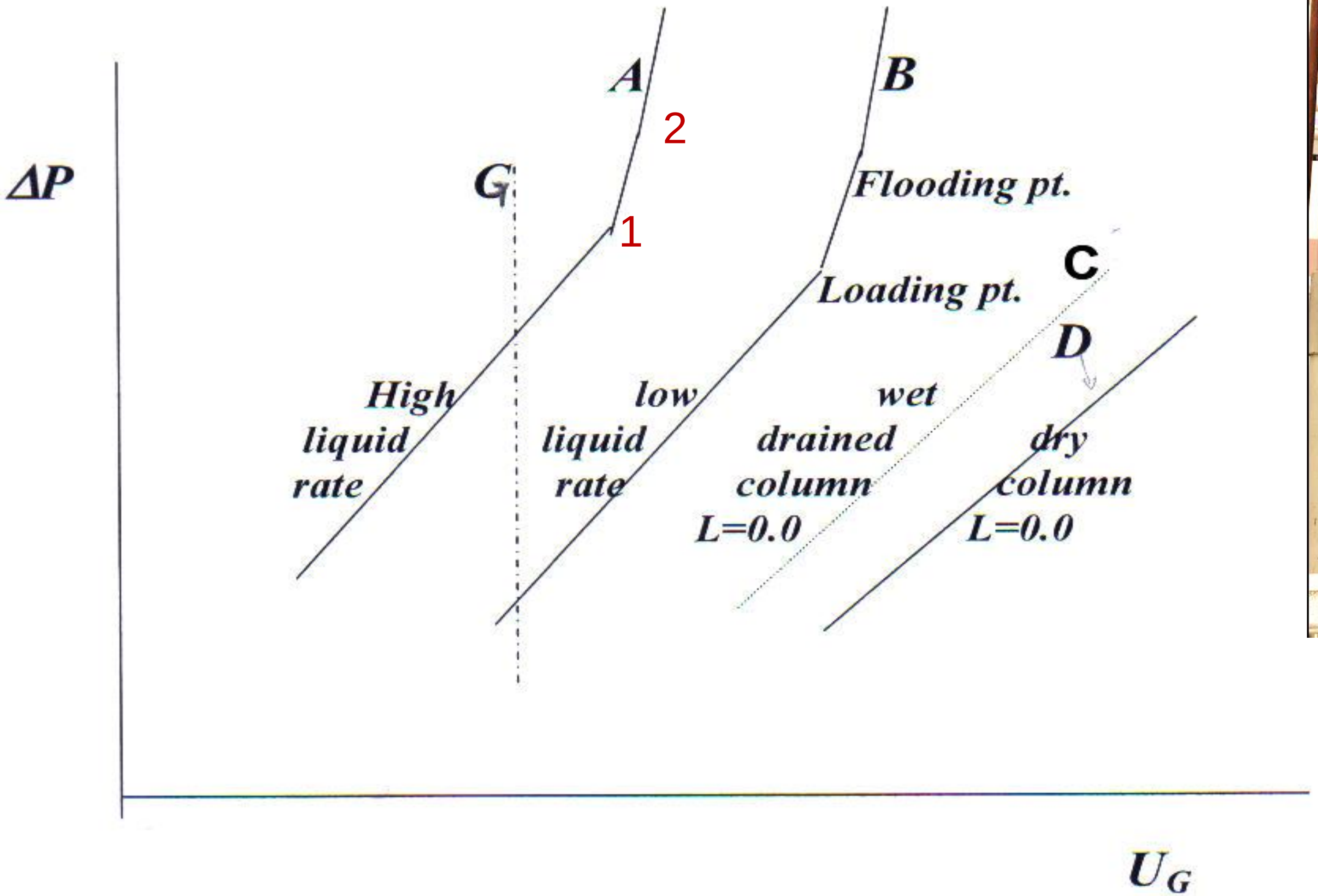
For Raschig rings : $k_1 = 3.3$

- **Irrigated Tower $L > 0.0$**

In this case we have different apprpaches:

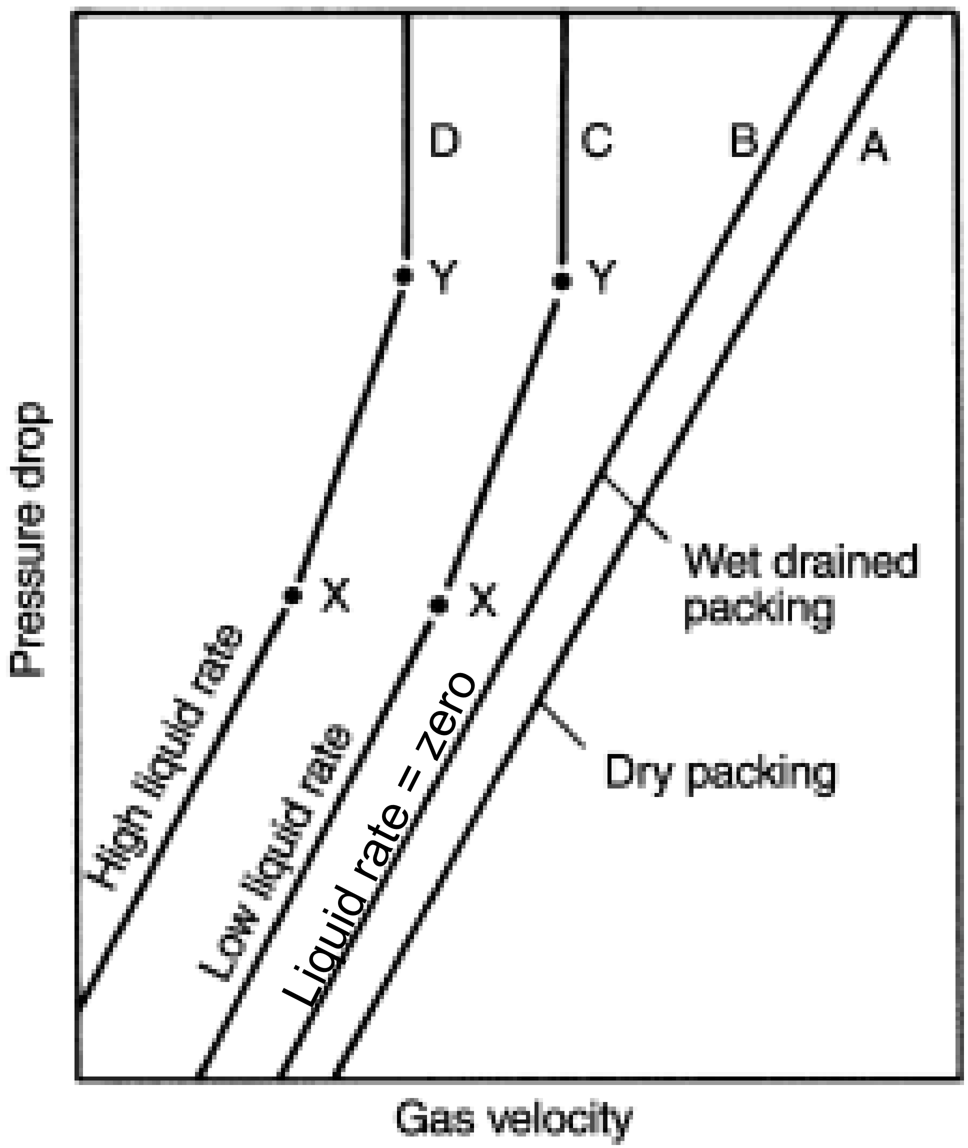
(a) $(-\Delta P_i) = (1 + k\,L/d_n)\,\Delta P_d$ 'See Perry'

Loading & Flooding Points



Transparent column

Pressure drops in wet packings (logarithmic axes



- ***In general, the pressure drop of a packed column is influenced by both gas and liquid flow rates as shown above.***

- ***For a constant gas velocity $\Delta P \uparrow$ as $L \uparrow$ line G.***

- ***Each type of packing has a certain void fraction available for liquid passage; as $L \uparrow$ the voids or $e \downarrow$ since part of it will be filled by liquid, hence reducing the cross-sectional area available for gas flow.***

- ***Consider transparent column and line A or B.***

Flooding Rates in Packed columns

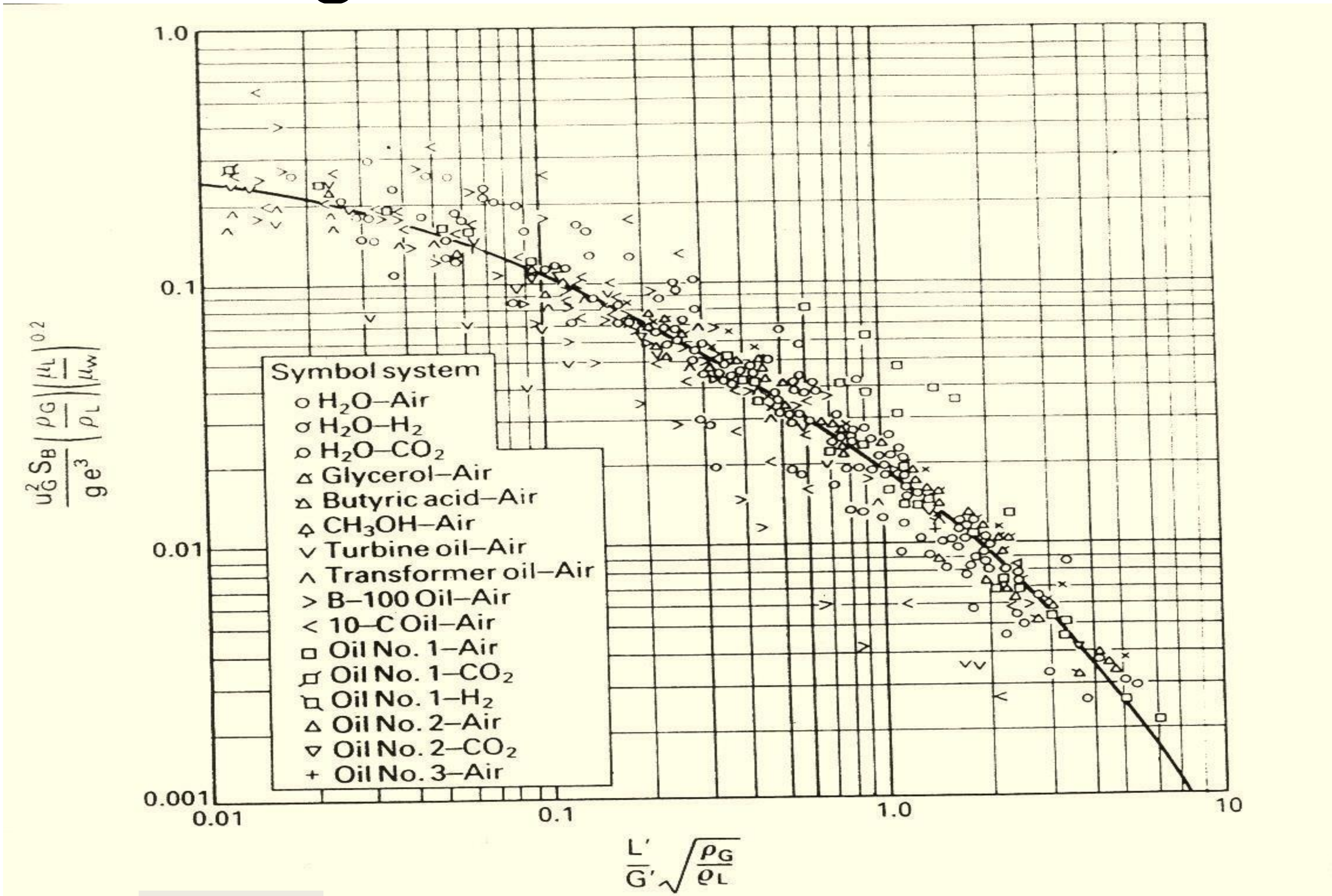
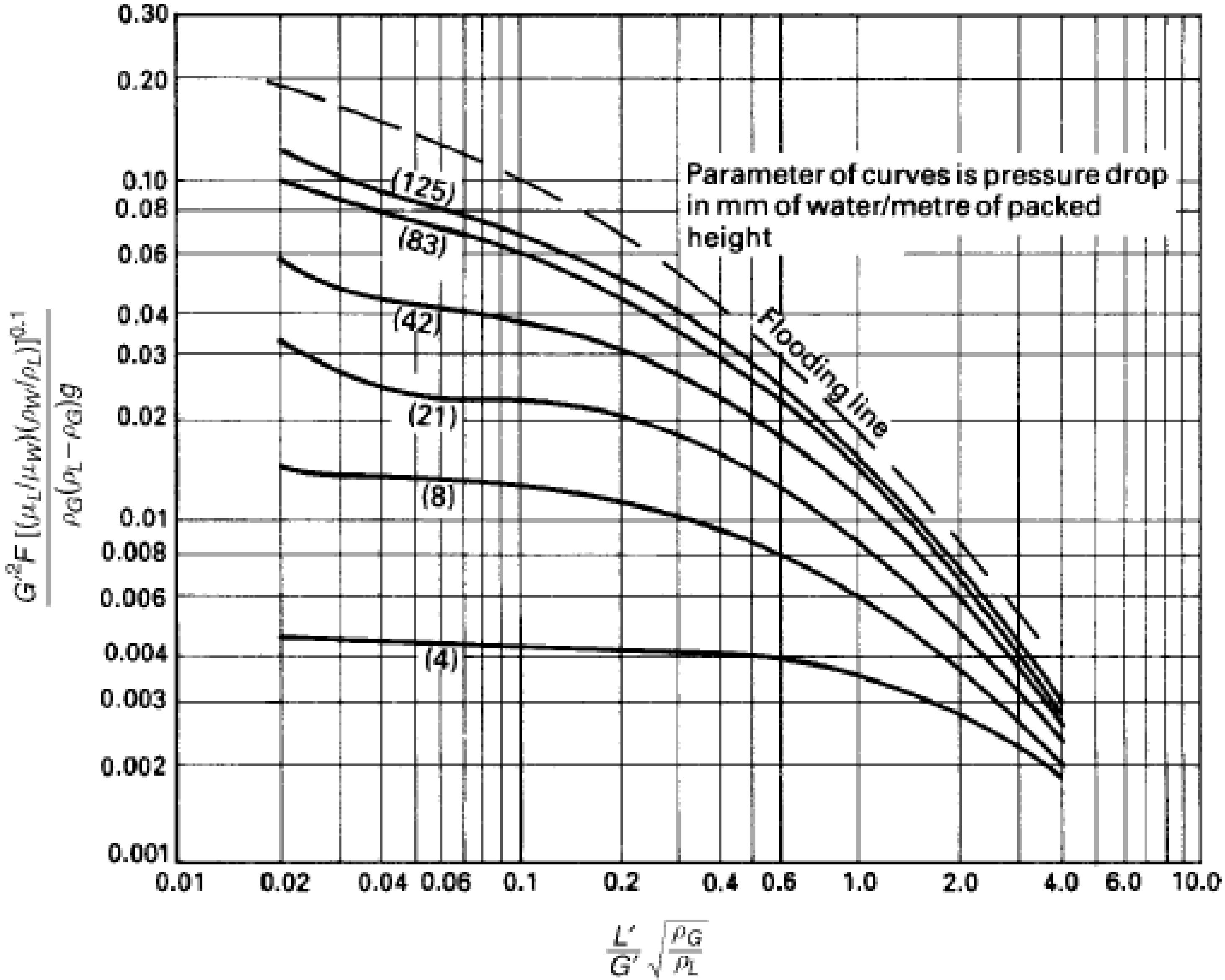


Fig 4.17

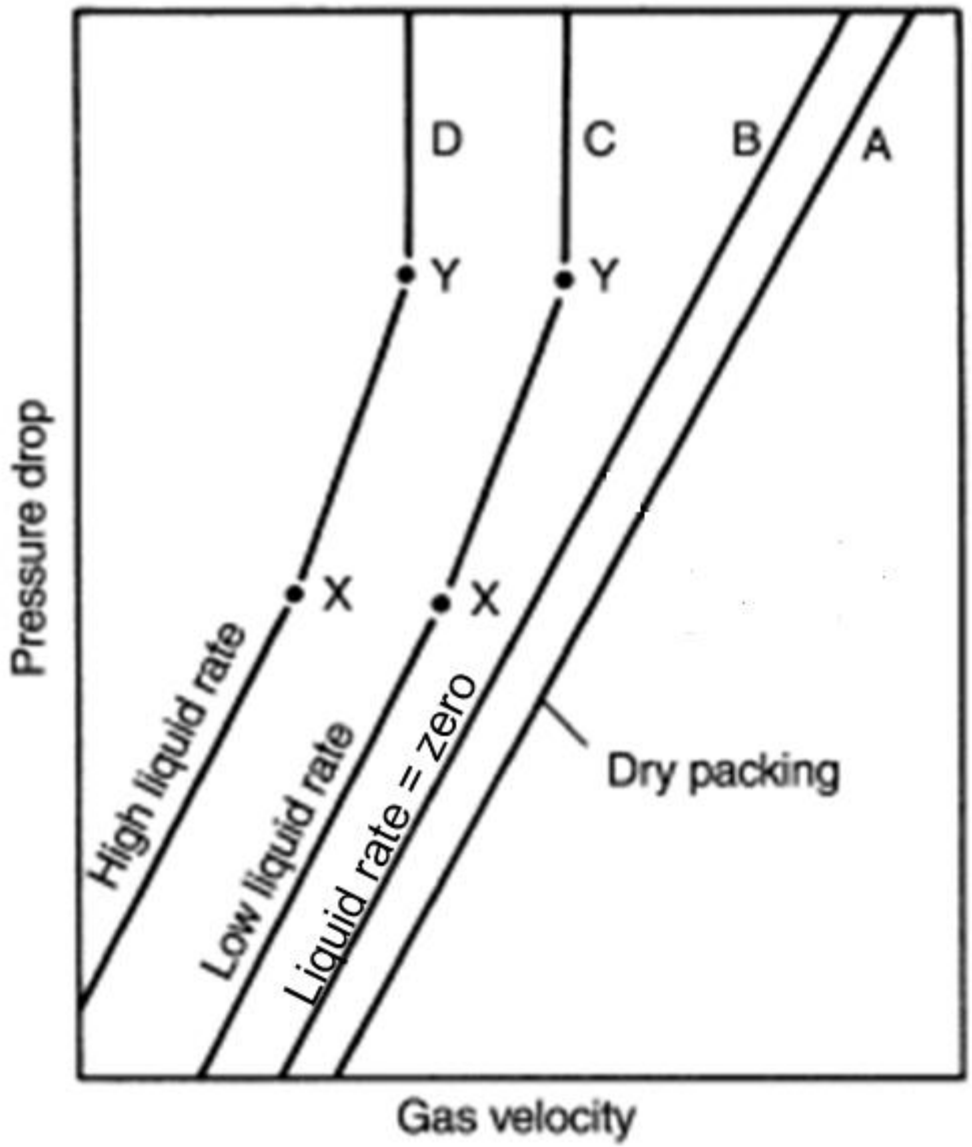
Generalised correlation for flooding rates in packed towers

Generalized pressure drop correlation Fig 4.18



Notes

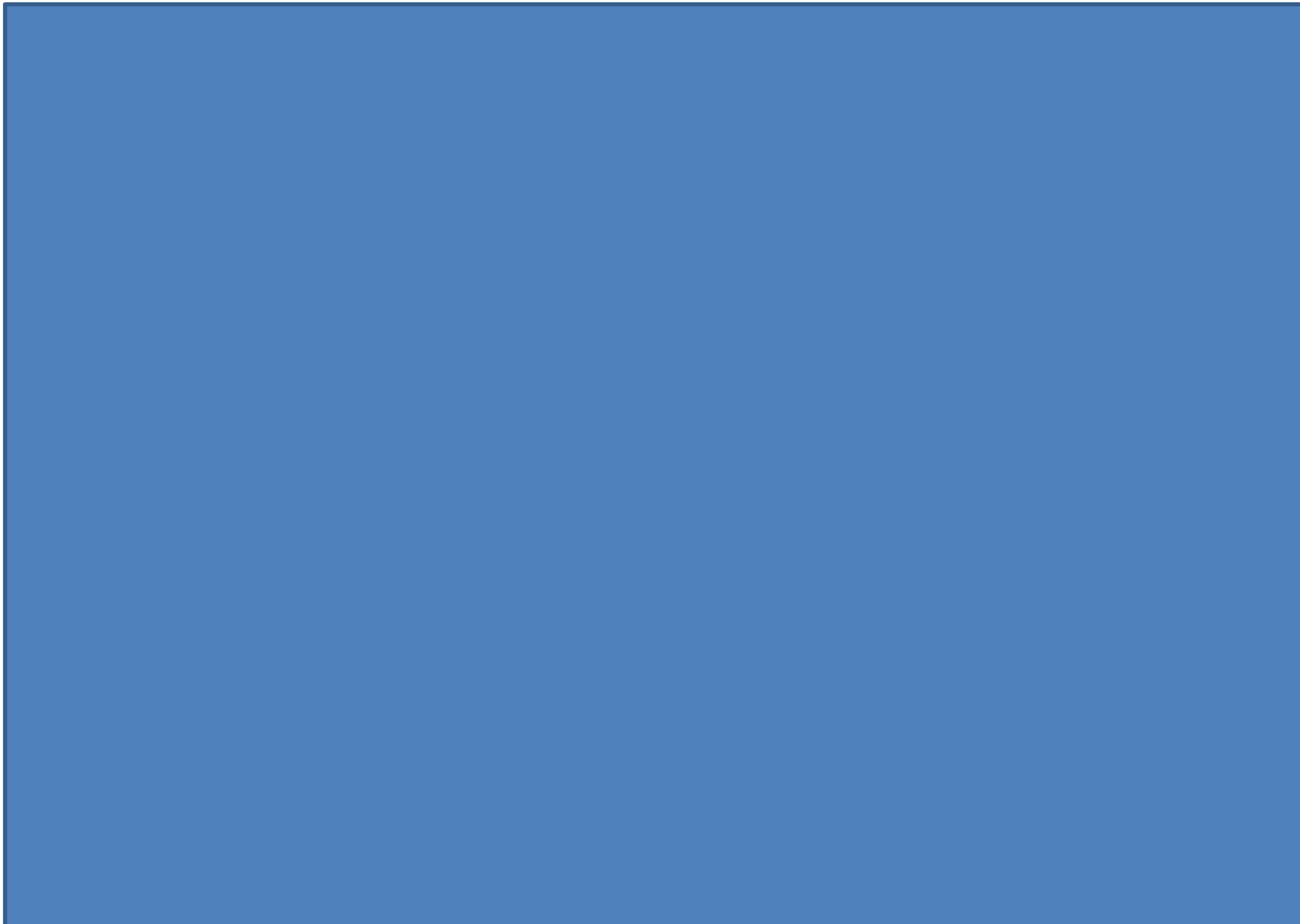
- Most of the data on which it is based are obtained for cases where the liquid is water and the correction factor $[(\mu_L/\mu_W)/(\rho_W/\rho_L)]^{0.1}$*, in which* μ_W *and* ρ_W *refer to* water at 293 K, is introduced to enable it to be used for other liquids.
- The packing factor *F which is employed in the correlation is a modification of the specific surface of the* packing S_B which is used in Figure 4.17.
- In practice, a pressure drop is selected for a given duty and use is made of the correlation to determine the gas flow rate per unit area G' from which the tower diameter may be calculated for the required flows.



D
C
B
A
Y
Y
X
X
High liquid rate
Low liquid rate
Liquid rate = zero
Dry packing
Pressure drop
Gas velocity

Example 2

A column 0.6 m diameter and 4 m high is, packed with 25 mm ceramic Raschig rings and used in a gas absorption process carried out at 101.3 kN/m^2 and 293 K. If the liquid and gas properties approximate to those of water and air respectively and their flow rates are 2.5 and 0.6 kg/m^2s, what is the pressure drop across the column? In making calculations, Carman's method should be used. By how much may the liquid flow rate be increased before the column floods?

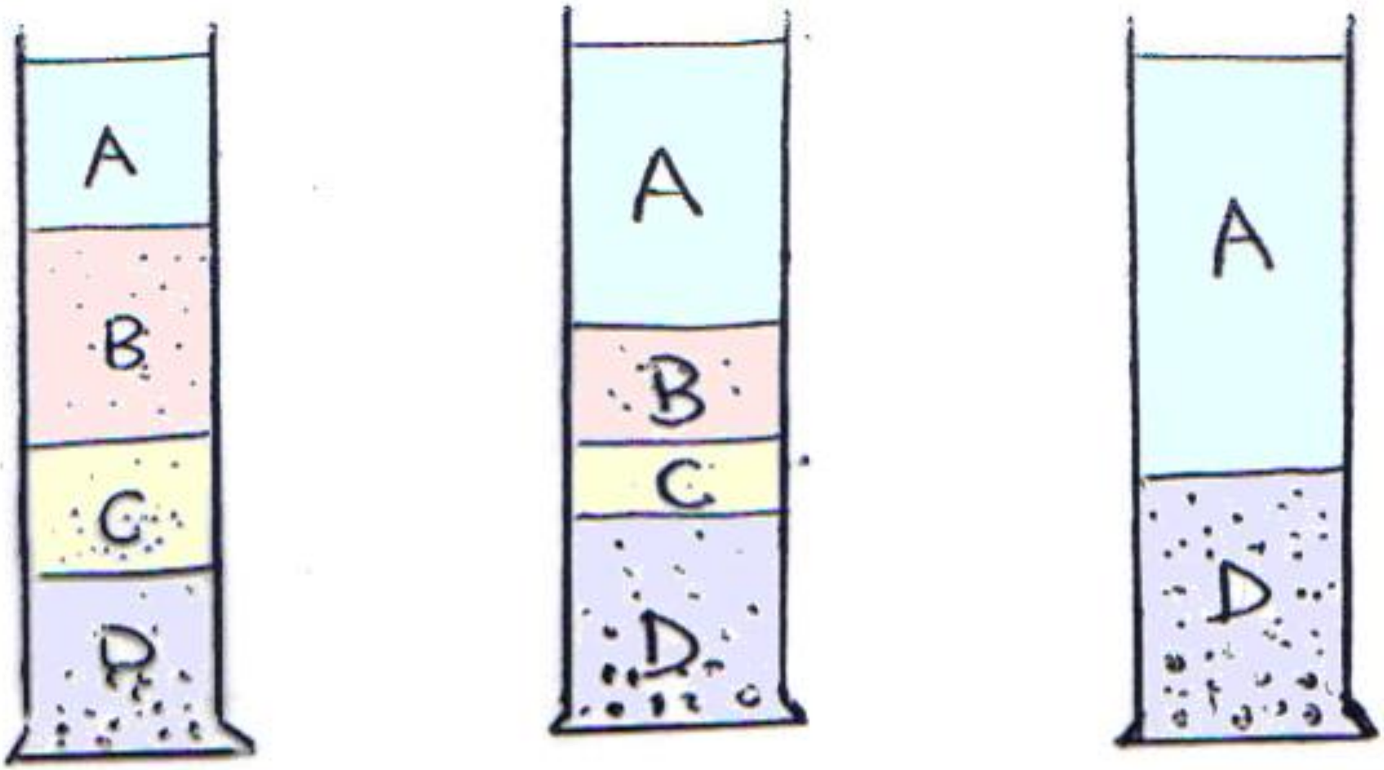


Sedimentation

Sedimentation

- Gravitational sedimentation
- Meaning
- The batch settling test 'Solids settle from a slurry in a glass cylinder'.
- 4 zones can be observed:
  clear liquid 'A'; uniform concentration 'B'; transition 'C'; coarse solids 'D'.
- Critical settling point $\Rightarrow$ only zones A and D.

Batch settling experiment



Fine particles

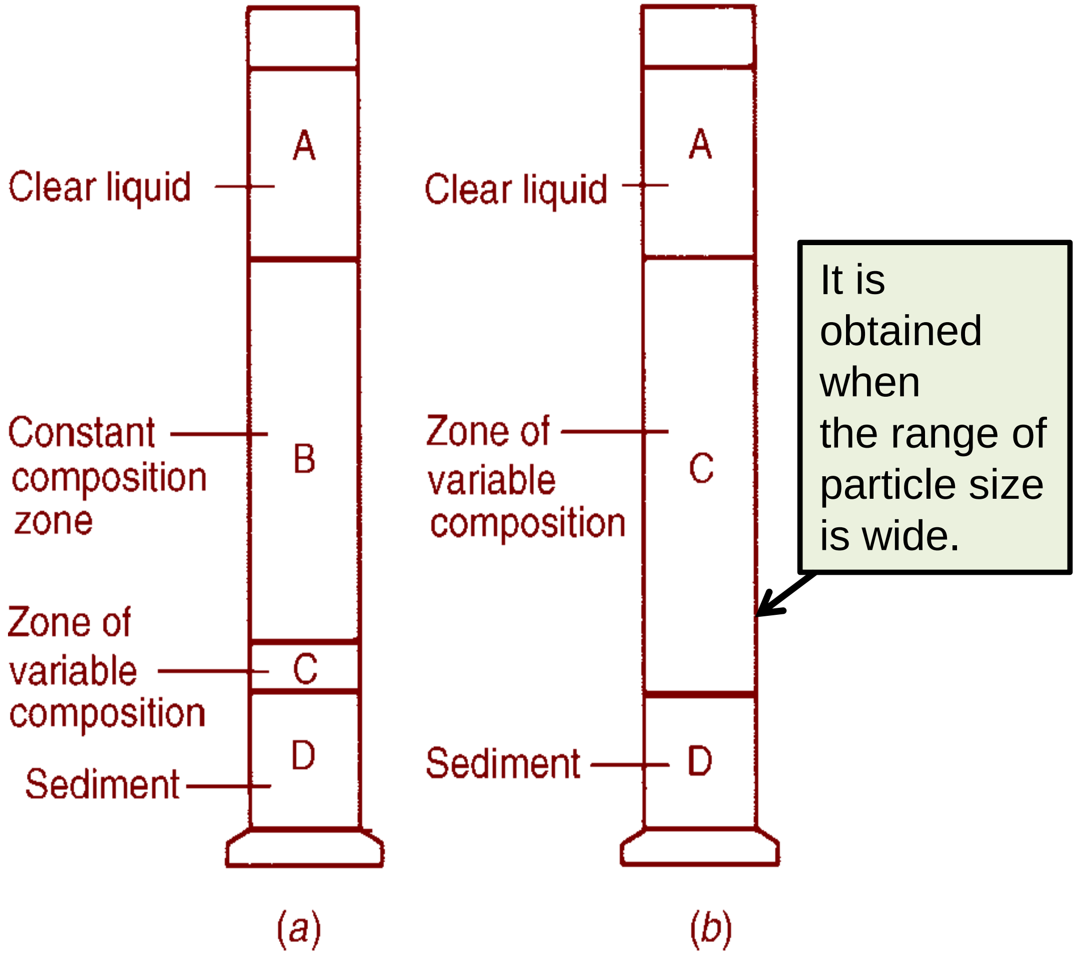
Time

Comparison between Type 1 and Type 2

Settling can be divided into two types according to the zones:

➢Type 1 contains zones A, B,C and D;

➢ Type 2 contains zones A, C and D.



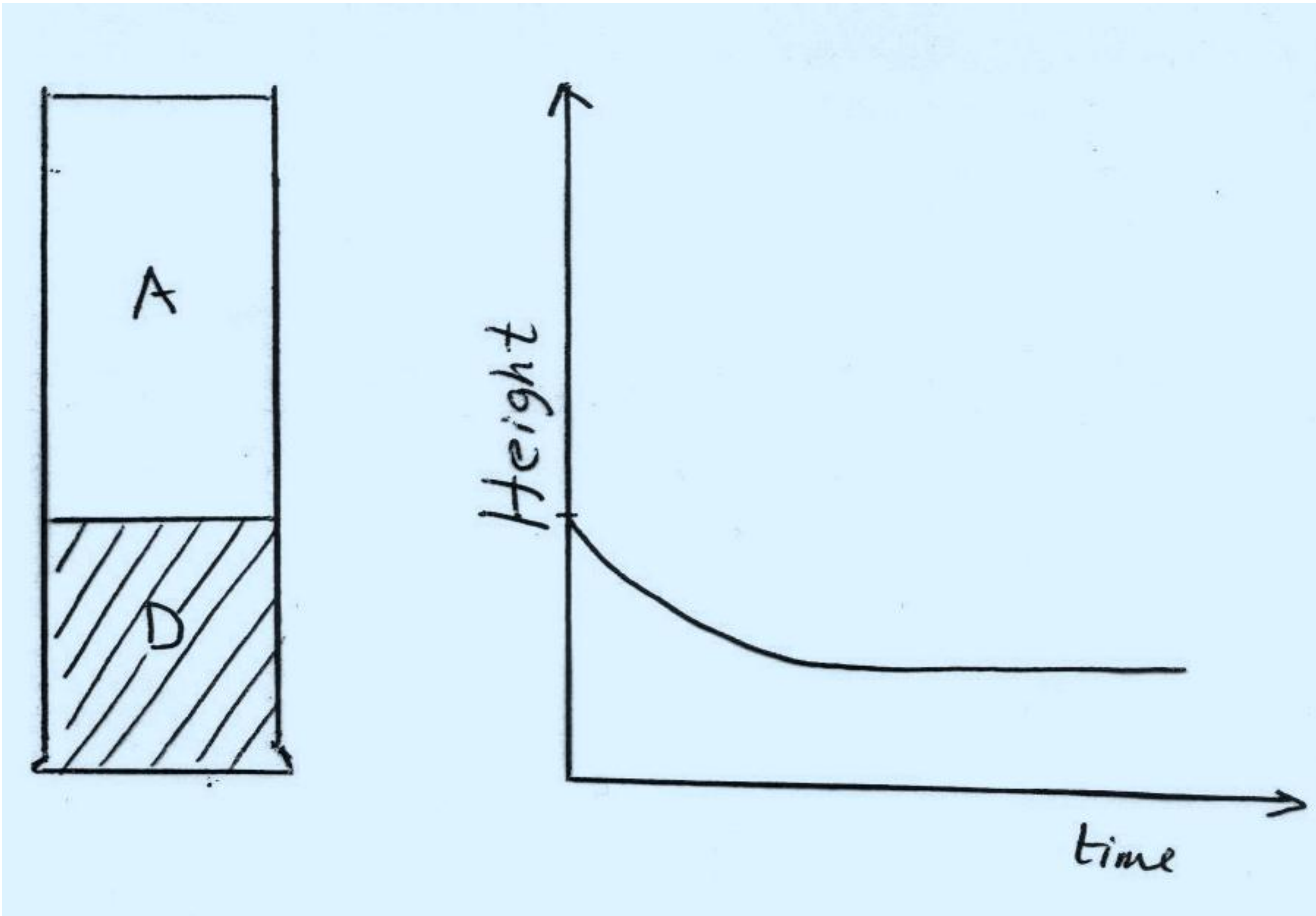
Type 1

Type 2

Gravity sedimentation

- Suspension can also be divided into concentrated suspension and dilute suspension. The first is called **'sludge line settling'** and the later **'selective settling'**.
- After critical settling pt., compression and consolidation of solid layer 'D' will take place with liquid being forced upwards into the clear liquid zone 'A'. Hence the height of layer D will be decreased.

Compression and consolidation of solid layer



Stokes' law

For a suspension of particles in a fluid, Stokes' law is assumed to apply but an effective viscosity and effective average density are used.

The modified settling or sedimentation velocity in suspension is:

$$u_C = K'' \frac{d^2(\rho_S - \rho_C)g}{\mu_C} \text{.......................................}(1)$$

where

K'' *: cons.*

$$(\rho_S - \rho_C) = \rho_S - \{\rho_S(1-e) + \rho e\} = e(\rho_S - \rho) \text{ ....}(2)$$

$$\mu_C = \mu(1 + k''c) \quad \leftrightarrow \quad c \text{ up to } 0.02 \quad \text{................}(3)$$

where

k'' *: cons. for a given shape of particles(2.5 for spheres)*

c *: volumetric conc. of particles.*

μ *: fluid viscosity*

But

$$\mu_C = \mu\, e^{k''c/(1+a'c)} \quad \leftrightarrow \quad c \succ 0.02 \quad \text{..................}(4)$$

where a' *: cons.* $= 0.609$ *for spheres*

It has been shown for a suspension of uniform particles, the velocity of particle relative to the fluid, u_p, is given by:

$$u_p = \frac{d^2(\rho_s - \rho_c)g}{18\mu} \mathrm{f}(e) \text{ ....(5)}$$

or

$$u_p = \frac{u_c}{e} \text{ ....(6)}$$

f(e) could be as

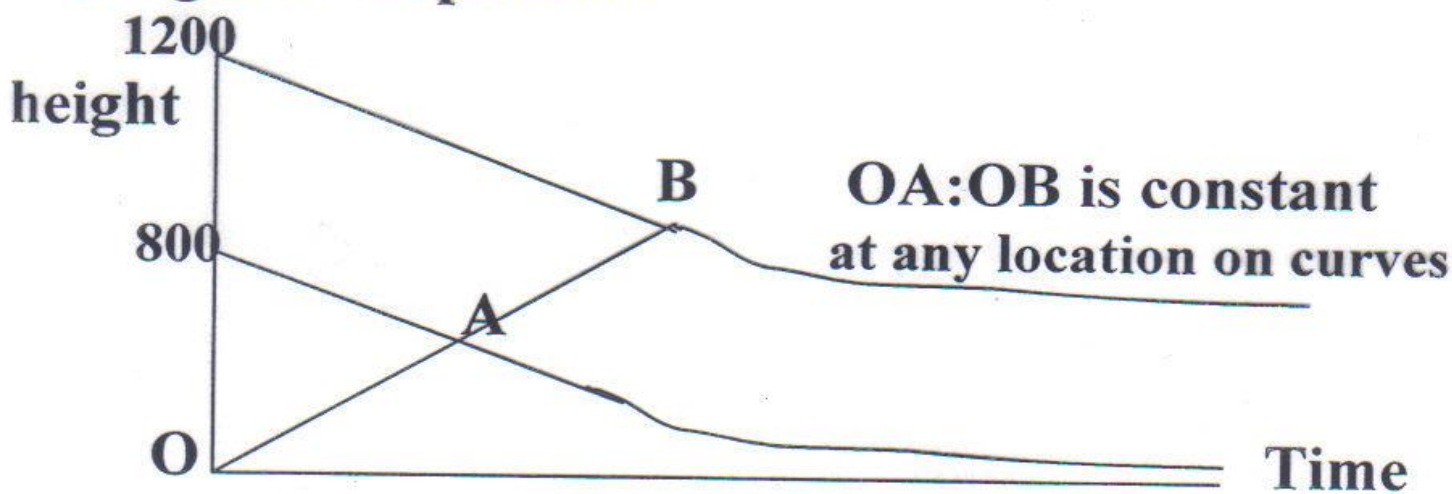
$$f(e) = 10^{-1.82(1-e)} \quad \text{.........................(7)}$$

Substituting 6,7,2, into 5

$$u_c = \frac{e^2 d^2 (\rho_s - \rho) g}{18\mu} 10^{-1.82(1-e)} \quad \text{...................(8)}$$

Main factors which affect sedimentation

1. **Height of suspension**



Suspension height has no effect on rate of sedimentation

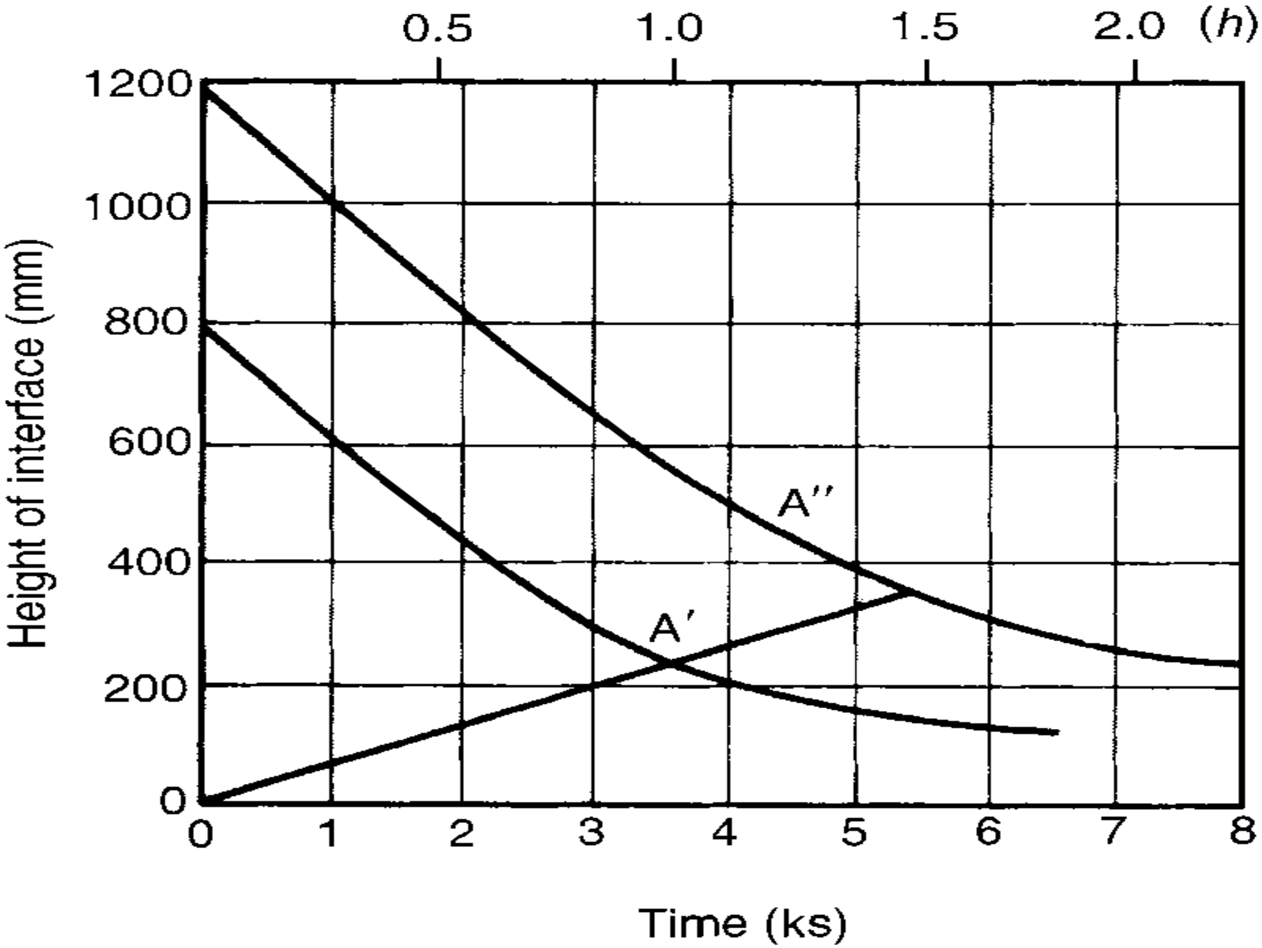
2. **Diameter of Vessel**

If $D_c/D_p > 100$, the walls have no effect on sedimentation rate.

If $D_c/D_p < 100$, walls reduce the sedimentation rate

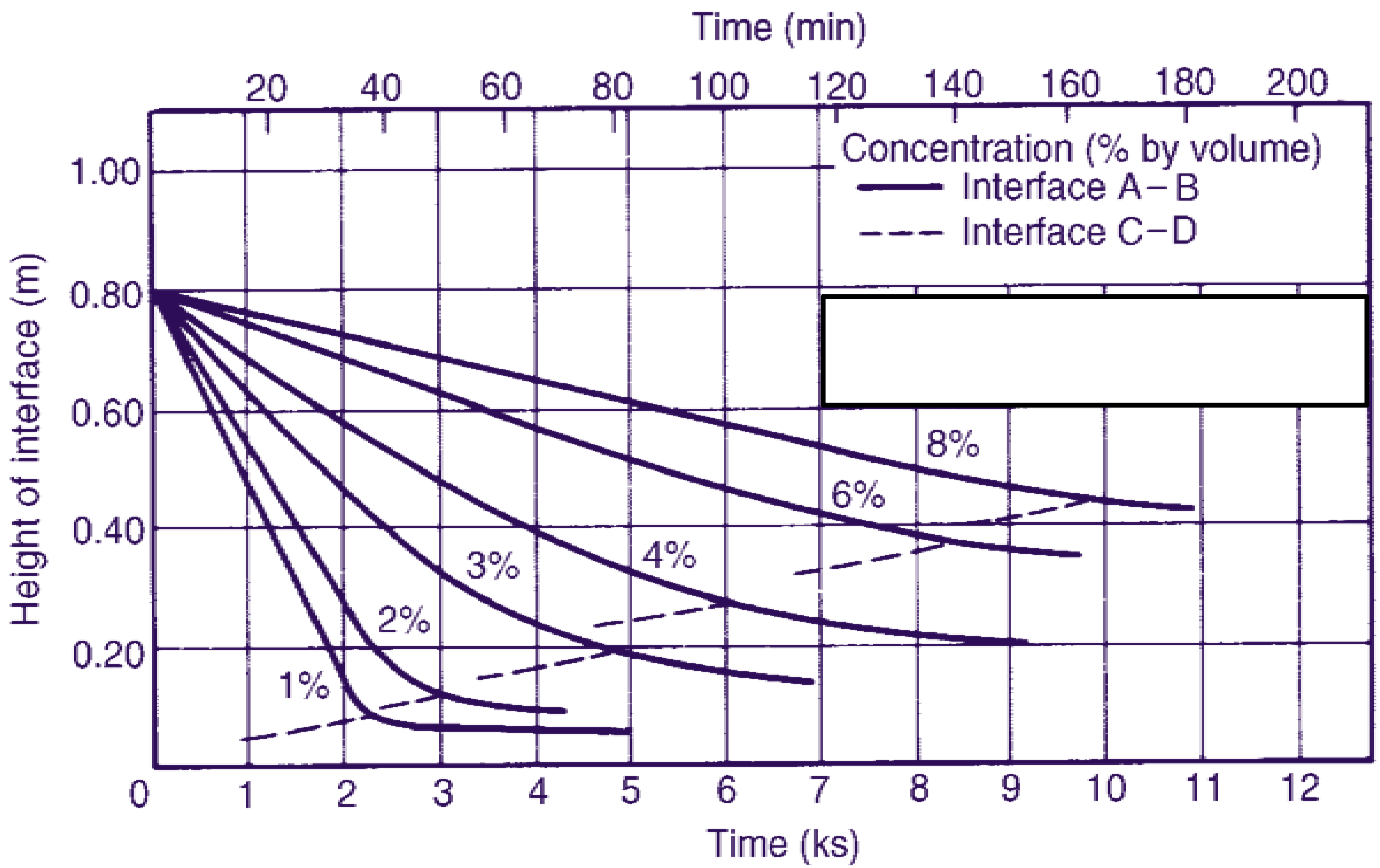
3. **Concentration of suspension**

Height of Suspension



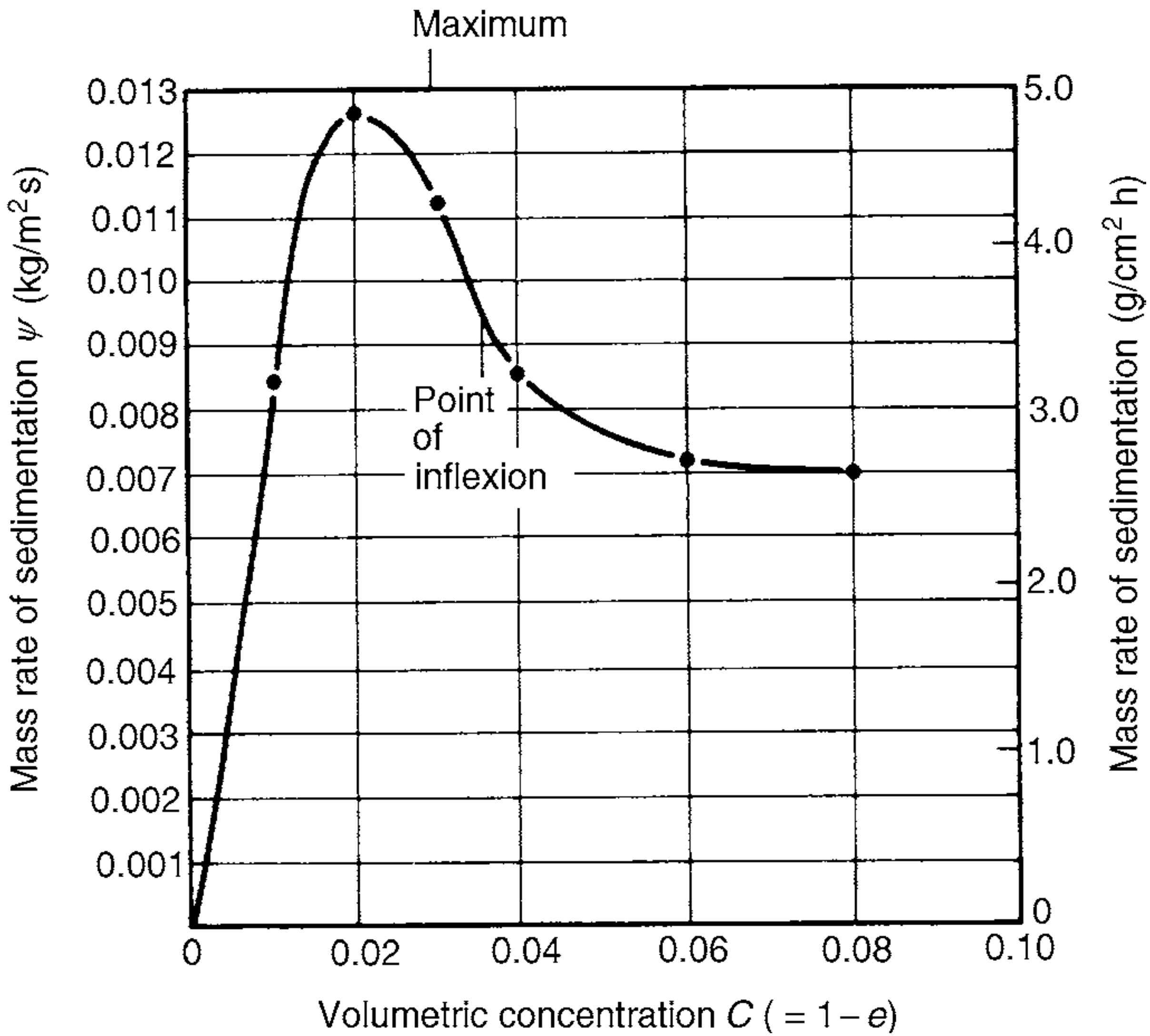
Effect of height on sedimentation of a 3 per cent (by volume) suspension of calcium carbonate

Effect of concentration on the sedimentation of calcium carbonate suspensions



∴ Conc.↑ Rate of sedimentation ↓

Effect of concentration on mass rate of sedimentation of calcium carbonate



Note

- **Rate of sedimentation during consolidation or solid-compression period is**

$$-\frac{dH}{dt} = b(H - H_{\infty})$$

$$On\ integration\ with\ B.C^{S}: t = o, H = H_{C}; t = t, H = H$$

$$\ln(H - H_{\infty}) - \ln(H_{C} - H_{\infty}) = -bt$$

Where H_C : the height of sluge at critical settling pt.

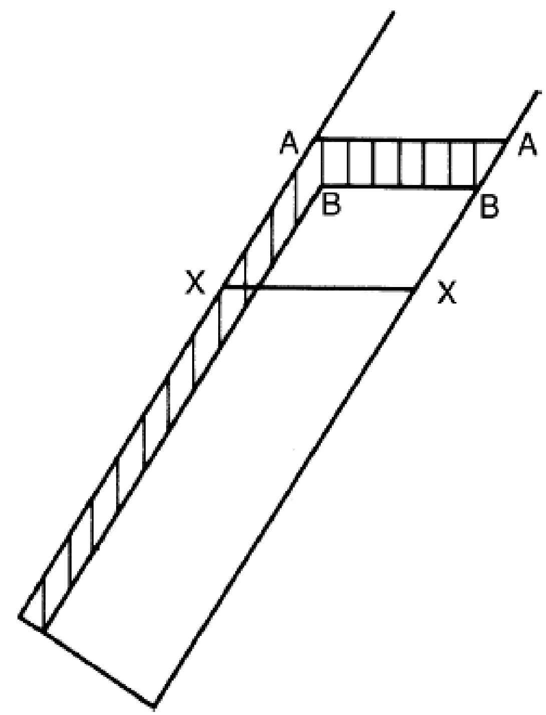
H_{∞}: the final height of the sediment

b : cons. For a given suspension

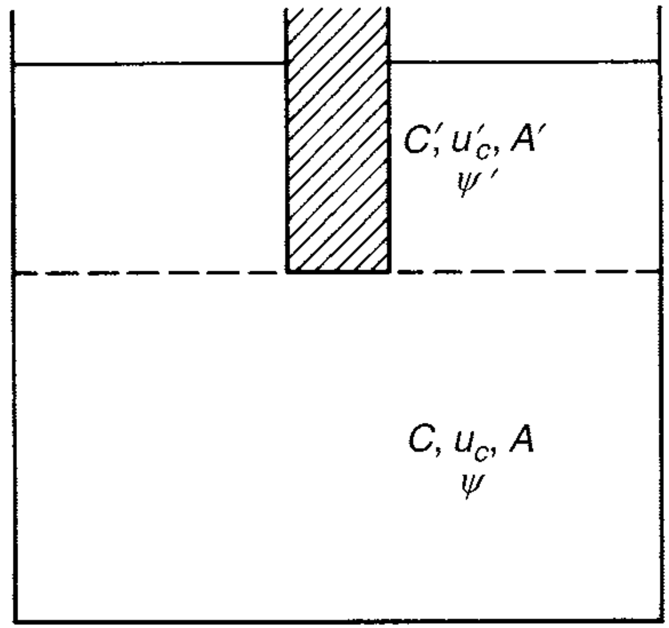
- **If $\ln(H-H_{\infty})$ is plotted against t, hence b can be found from the slope of the given line.**

4. Shape of vessel

- **It has little effect**
- **If the container has a slope with horizontal, after a short time interval, the system becomes to a stable condition ' XX state'. Hence, Area AAXX = shaded area**



- **Obstructed vessel**



Sedimentation in partially obstructed vessel

$$A\psi = Acu_c$$

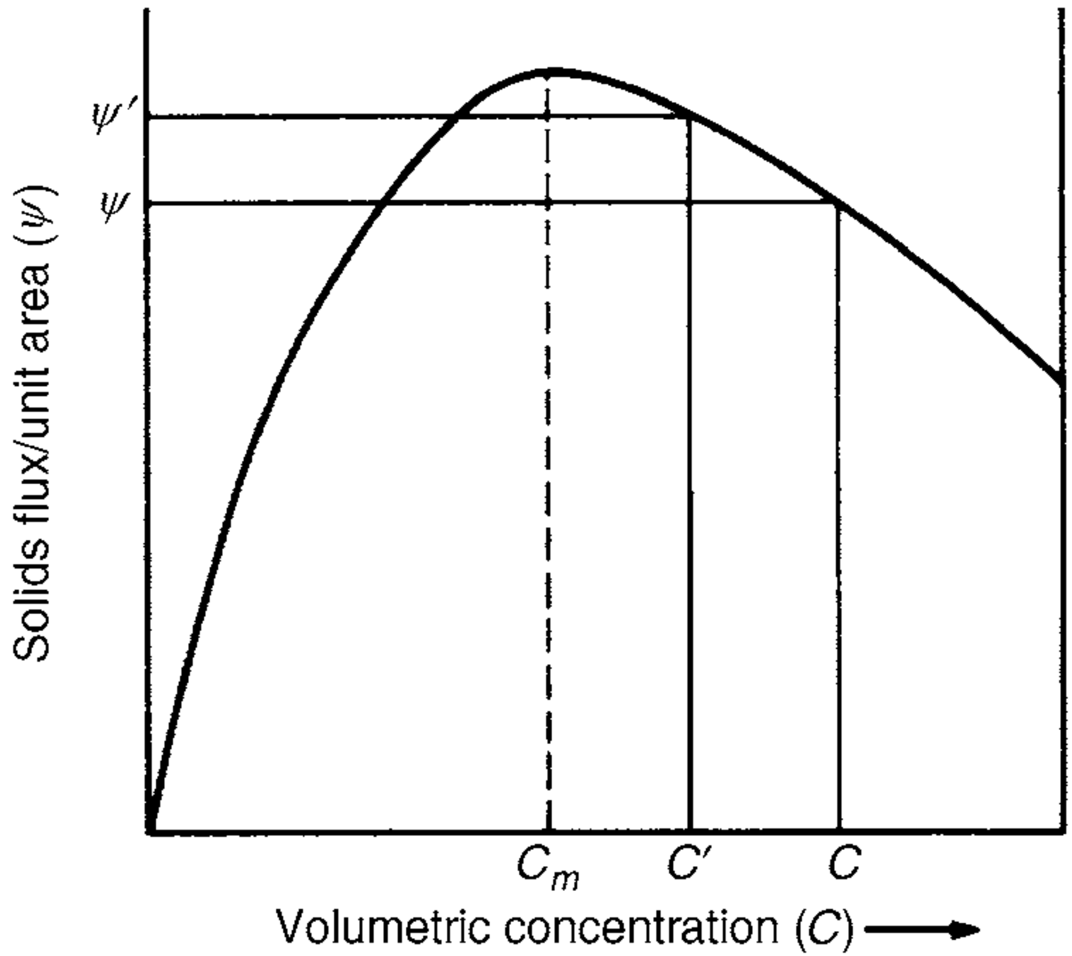
$$A'\psi' = A'c'u_c'$$

For contiuity at the bottom of construction

$$\psi' = \frac{A}{A'}\psi \quad \therefore \psi' \succ \psi \quad since\ A' \prec A$$

In sum as A $\downarrow$ $\psi \uparrow$

Solids flux per unit area as a function of volumetric concentration



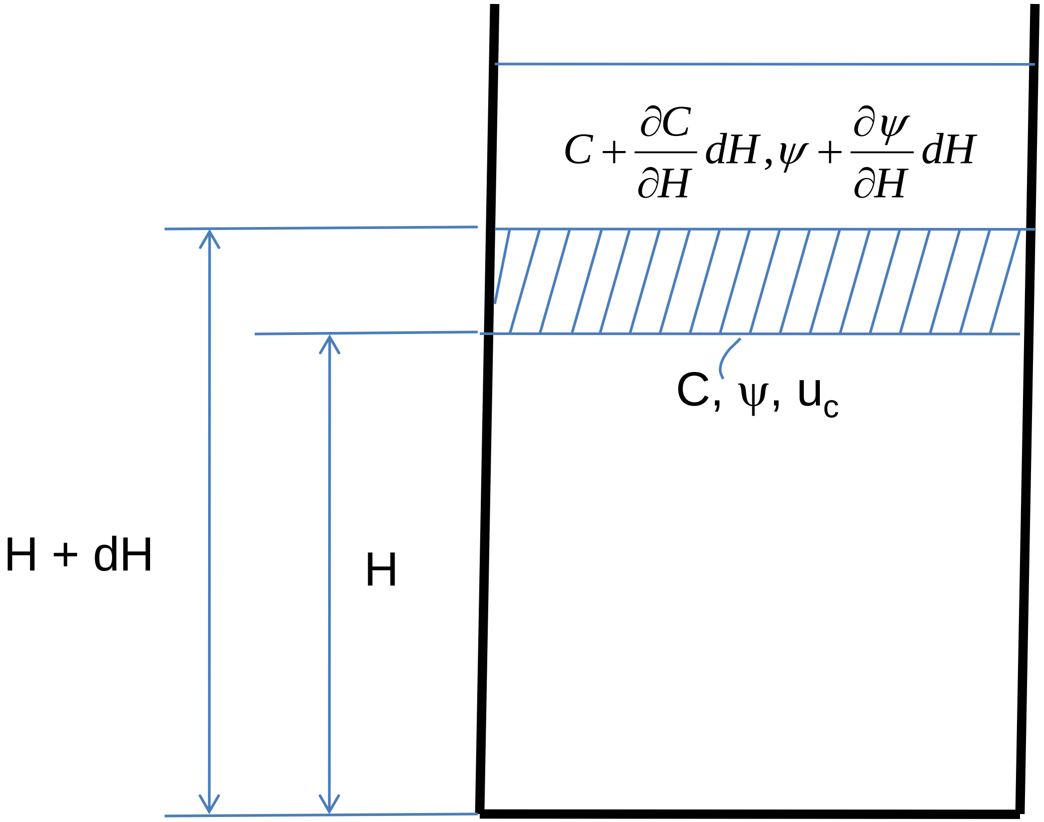
Kynch Theory

The mechanism of sedimentation of concentrated suspension can be studied via using the concept of continuity.

<u>Some basic assumption</u>:

- Uniform conc. at any level.
- No wall effects
- Velocity of fall of particles depends on the local conc. of particles.
- There is no differential settling
- Sedimentation velocity = 0.0 at the sediment layer at the bottom.

Kynch Theory



- **In general, $\psi = c\, u_c$**
  Making a material balance between the height h and the height H+dH

$$\left\{\left(\psi + \frac{\partial \psi}{\partial H} dH\right) - \psi\right\} dt = \frac{\partial}{\partial t}(c\,dH)\,dt$$

$$\frac{\partial \psi}{\partial H} = \frac{\partial c}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$note: \quad Right\ term \quad \frac{\partial dH}{\partial t} c + \frac{\partial c}{\partial t} dH$$

$$\frac{\partial \psi}{\partial H} = \frac{\partial \psi}{\partial c} \cdot \frac{\partial c}{\partial H} = \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} \quad since\ \psi\ depends\ only\ on\ c$$

Substituting in (1)

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$But\ c = f(H,t)$$

$$dc = \frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt$$

$$Assume\ constant\ conc.$$

$$\frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt = 0$$

$$\frac{\partial c}{\partial H} = - \frac{\partial c}{\partial t} \Big/ \frac{dH}{dt}$$

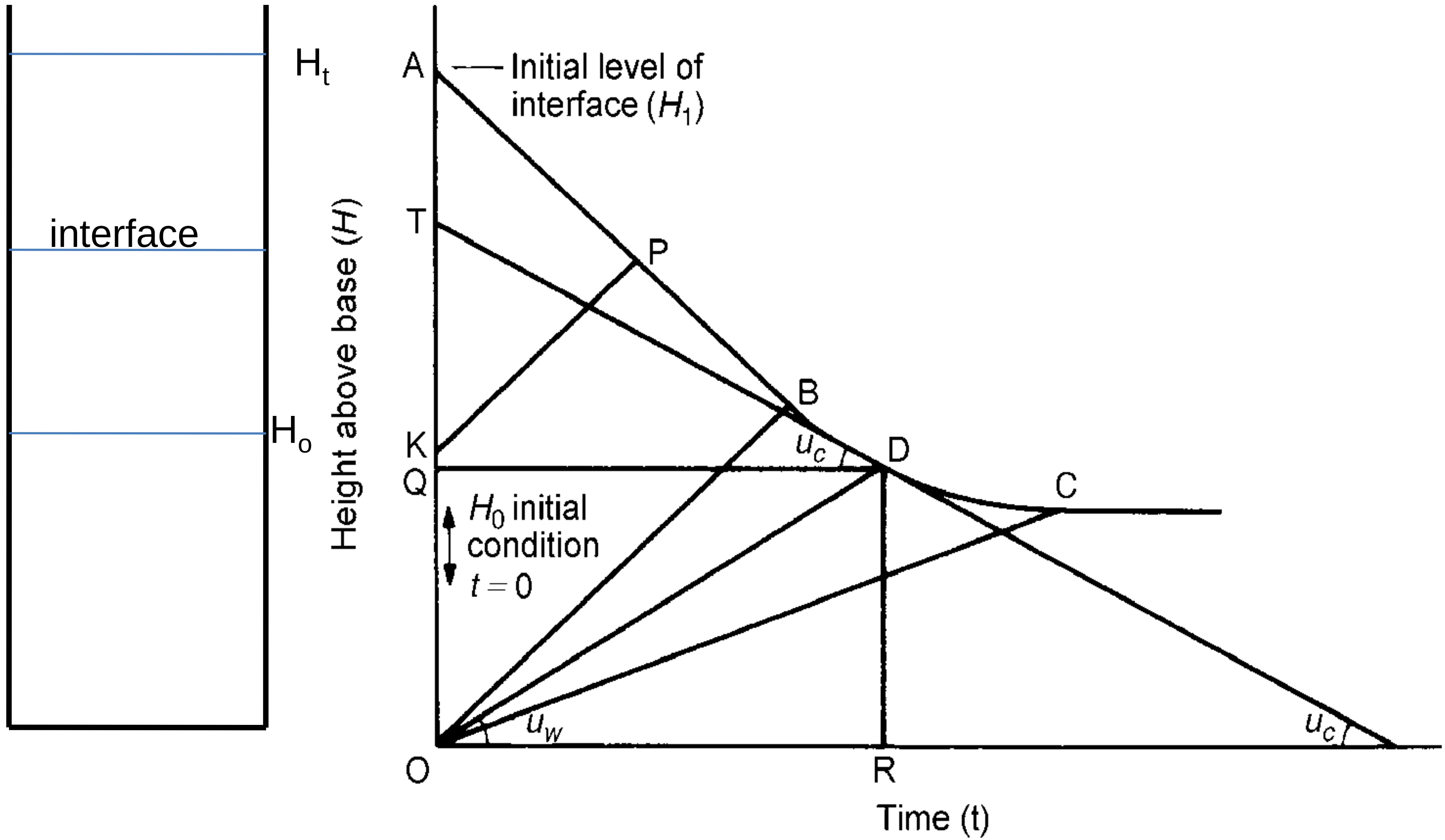
$$sub.\ in\ (2)$$

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc} \left\{ - \frac{\partial c}{\partial t} \Big/ \frac{dH}{dt} \right\} = 0$$

$$\therefore \quad - \frac{d\psi}{dc} = \frac{dH}{dt} = u_w \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where u_w velocity of propagation of a zone of constant concentration ‘velocity of propagation of concentration wave’. Note u_w is constant for any concentration.

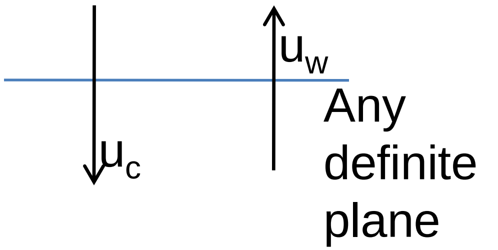
The relationship between flux ‘Ψ’ and concentration ‘c’



- lines such as KP and OB represent constant concentrations with slopes = $dH/dt = u_w$
- line AB represents the fall down of interface 'between clear liquid and suspension' with fall down velocity 'sedimentation velocity' ,u_c= slope of this line = - dH/dt
- the total volume of particles that pass through a certain plane in time

  't' per unit area is:

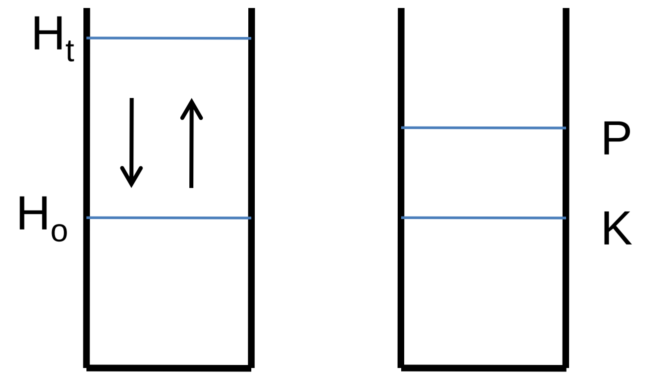
  $V = C_o (u_c + u_w) t$



- Total volume, V, must equal to the total volume of particles was originally above the K, for example.

$$C_o\,(u_c + u_w)\,t = C_o\,(H_t - H_o)$$

$$(u_c + u_w)\,t = (H_t - H_o)$$



- For intermediate concentration curve BC $C_o < C < C_{max}$
- Consider pt. D and apply the same previous approach:

- Here the line OD represents a wave starts from pt. O 'the base of the container' and terminates at pt. D ' the interface between clear liquid and suspension' on the sedimentation curve.

$C\,(u_c + u_w)\,t = C_o\,H_t$ ....(i)

But $H_t = OA$ 'see the fig.'

$u_c = QT/t$

or $u_c\,t = QT$ .... (ii)

and $u_w\,t = RD = OQ$ ....(iii)

add (iii) to (ii)

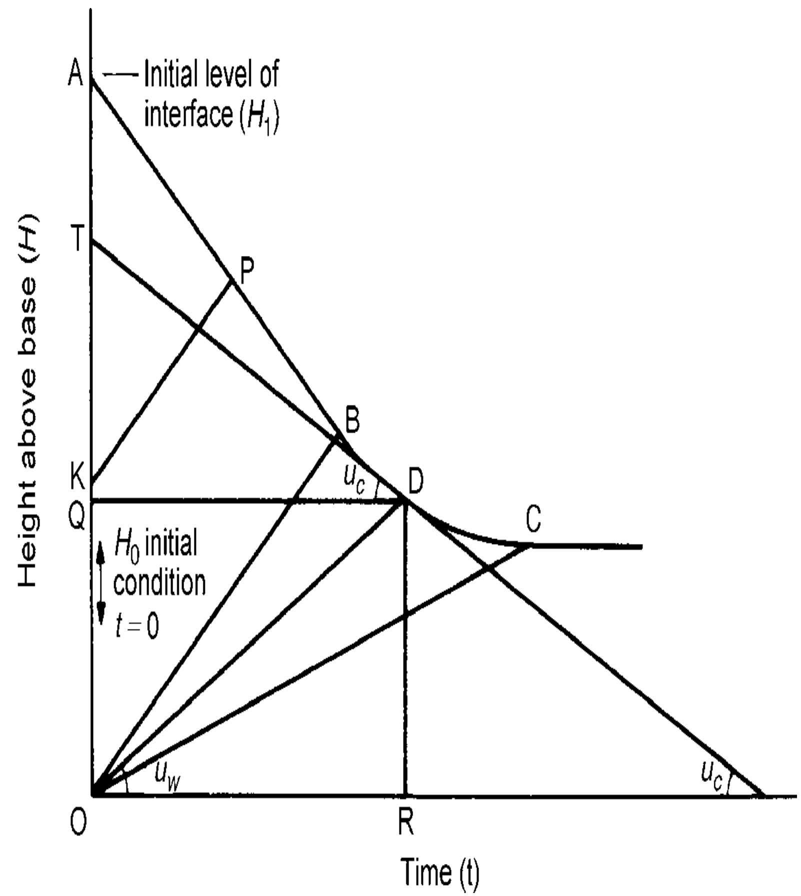
$(u_c + u_w)\,t = OT$ .... (iv)

divide (i) by (iv)

$C = C_o\,(OA/OT)$ ###

The corresponding Flux is

$\Psi = C\,u_c = C_o\,(OA/OT)\,u_c$ ###



Thickener design

Thickening zone

Thickening zone

- **Good design requires**:

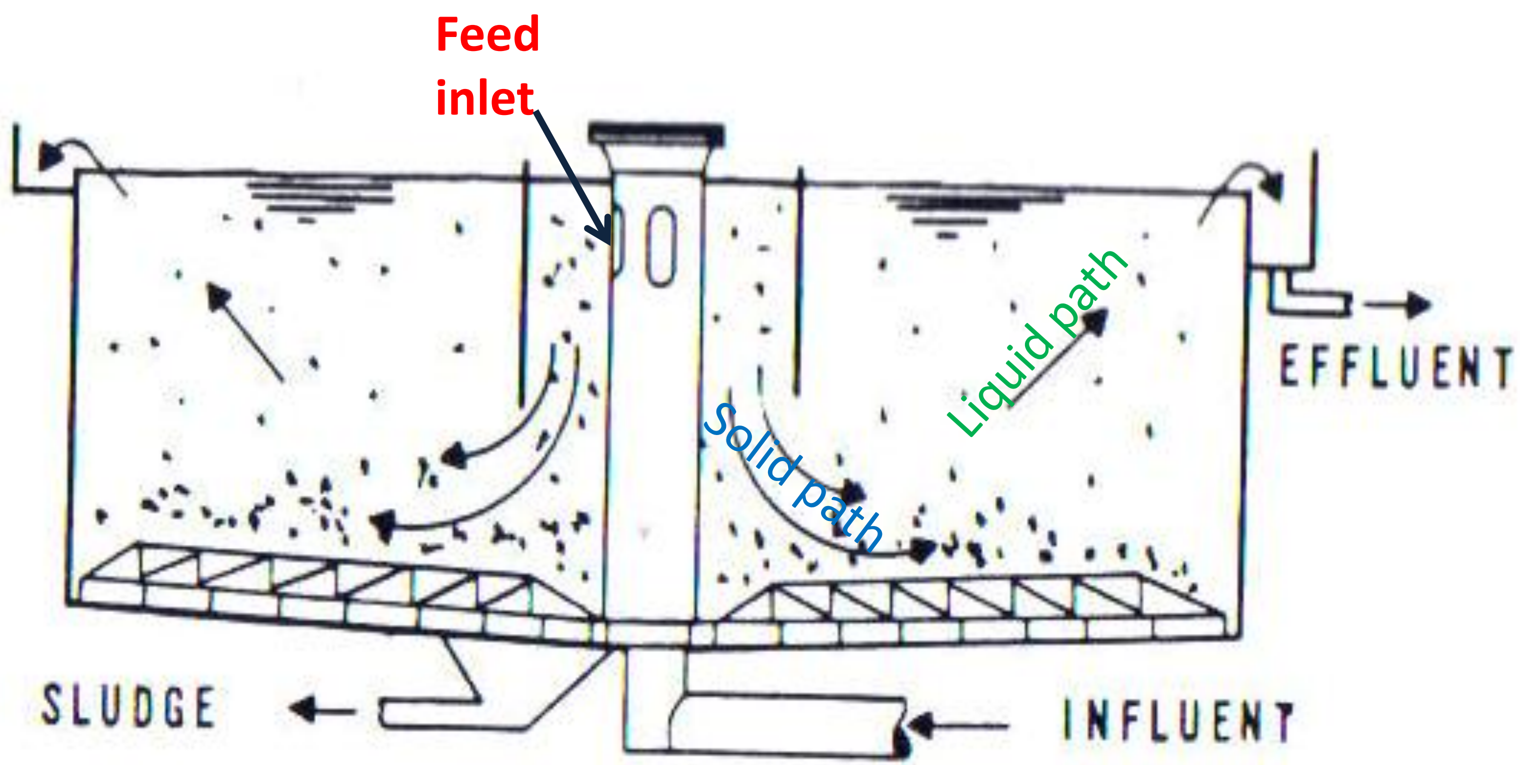
  1. adequate or suitable diameter to obtain reasonable clarification.

  2.Enough depth to obtain the required degree of thickening.

Constrain

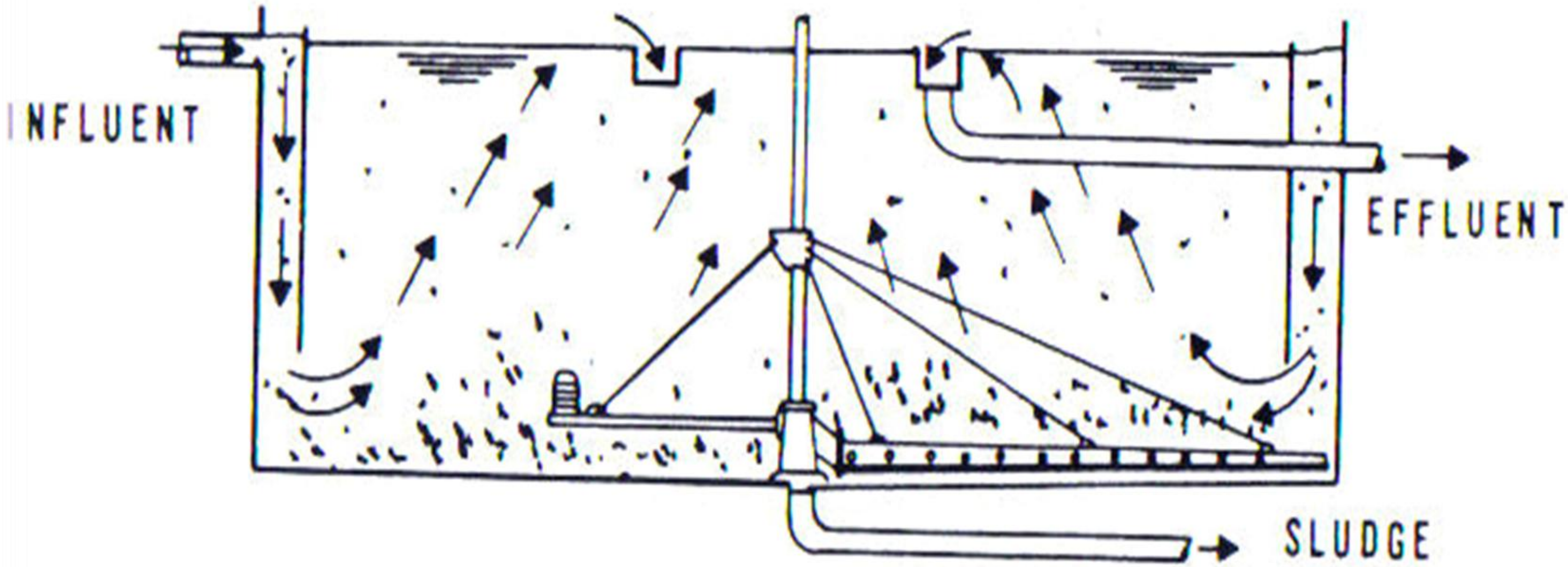
- In a continuous thickener, the area required for thickening must be such that the total solids flux (volumetric flow rate per unit area) at any level does not exceed the rate at which the solids can be transmitted downwards.
- If this condition is not met, solids will build up and unsteady-state operation will be possible.

Clarifier or thickener



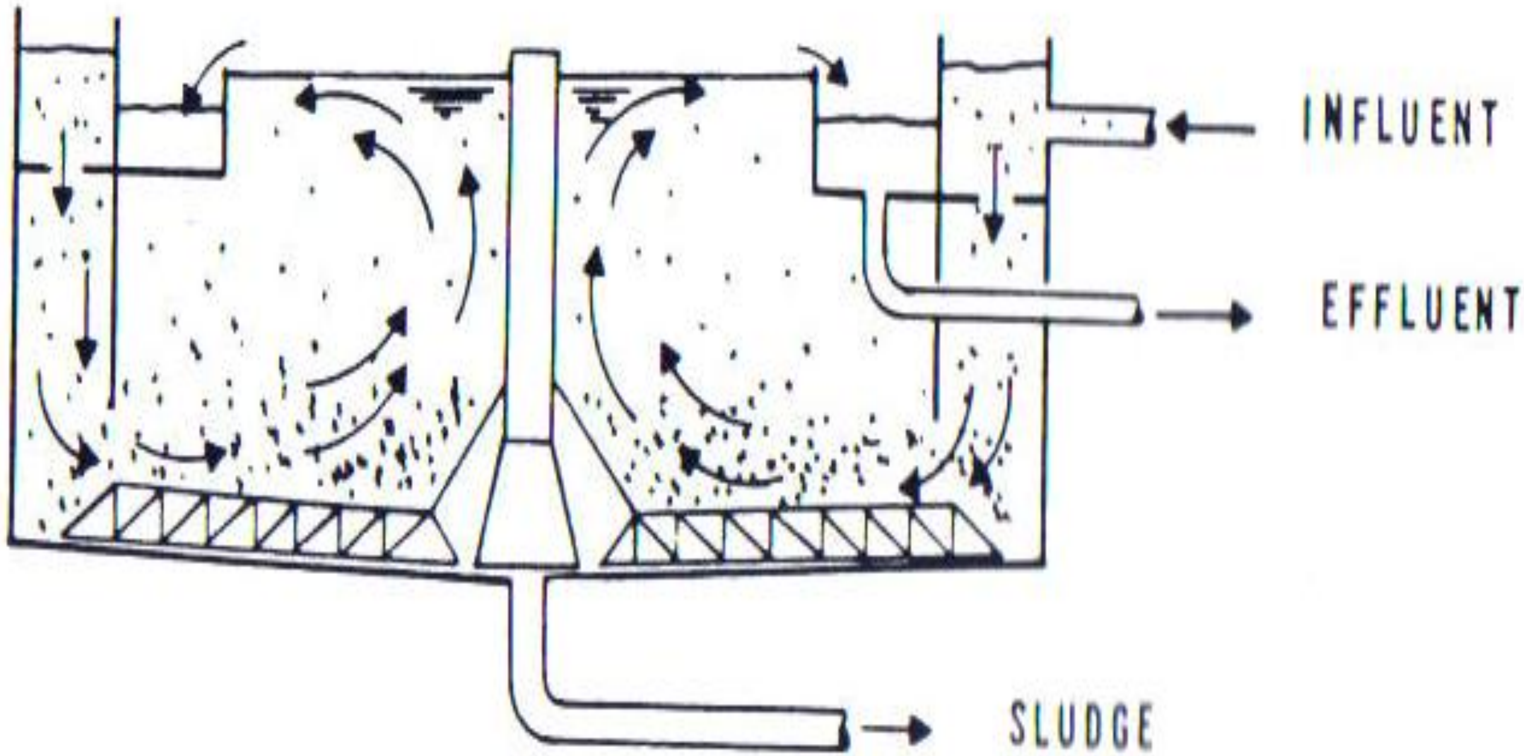
(a) CIRCULAR CENTER-FEED CLARIFIER WITH A SCRAPER SLUDGE REMOVAL SYSTEM

Clarifier or thickener



(b) CIRCULAR RIM-FEED, CENTER TAKE-OFF CLARIFIER WITH A HYDRAULIC SUCTION SLUDGE REMOVAL SYSTEM

Clarifier or thickener



(c) CIRCULAR RIM-FEED. RIM TAKE-0FF CLARIFIER WITH A SCRAPER SLUGE REMOVAL SYSTEM

Required Area

- To calculate the required area , it is very important to obtain the concentration at which the rate of sedimentation or the flux is a minimum.
- Total flux, ψ_t, consists of two componets:

  a. Due to the sedimentation of solid in the liquid (as measured by batch sedimentation experiment, $\psi = u_c$ C).

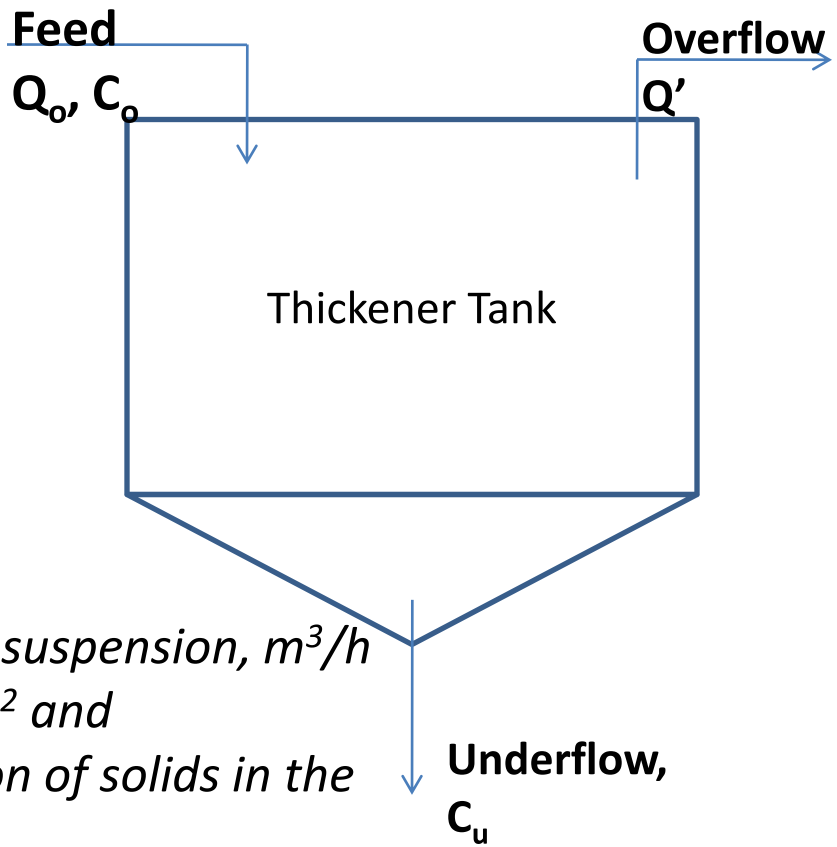
  b. Arising from the removal of the underflow at the base of the thickener, $\psi_u = u_u$ C.

$\therefore$ Total flux is,

$$\Psi_T = \Psi + \Psi_u$$
$$= \Psi + u_u C$$

This flux equals the volumetric flow rate per unit area at which the solids are fed to the thickener

$$\Psi_T = \left(\frac{Q_0}{A}\right) C_o$$



Where;

Q_0 is the volumetric feed rate of suspension, m^3/h

A is the area of the thickener, m^2 and

C_0 is the volumetric concentration of solids in the feed.

$$\frac{\partial \psi_T}{\partial C} = \frac{\partial \psi}{\partial C} + u_u$$

The minimum value of ψ_T $(=\psi_{TL})$ occurs when $\dfrac{\partial \psi_T}{\partial C} = 0$;

i.e. when:

$$\frac{\partial \psi}{\partial C} = -u_u$$

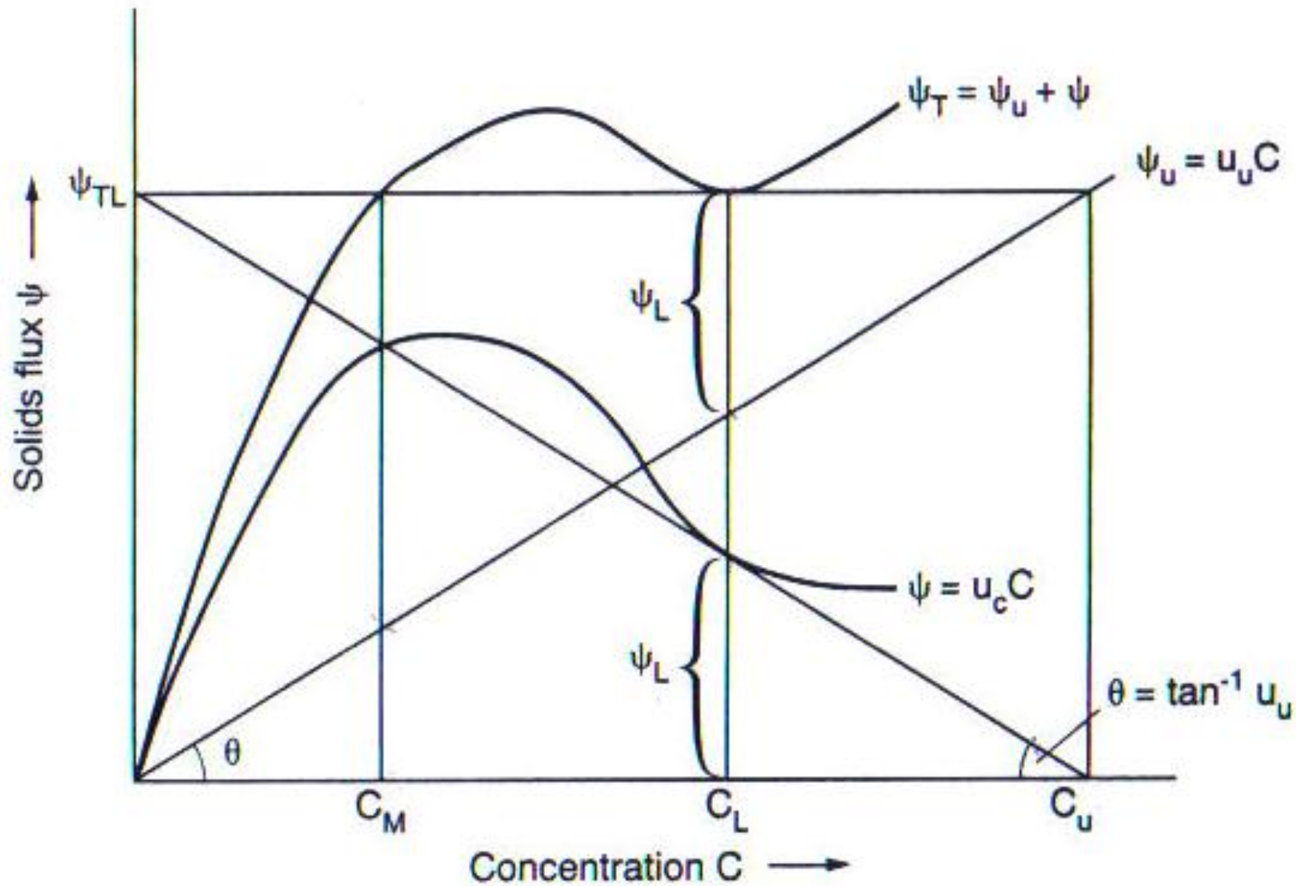


FIG. 5.13. Solids fluxes as functions of concentration and Yoshioka construction.

$$\frac{\psi_{TL}}{C_u} = \frac{\psi_L}{C_u - C_L}$$

where ψ_L is the value of ψ at the concentration C_L.

Since $\psi_L = u_{cL}\, C_L$

$$A = \frac{Q_0\, C_0}{\psi_{TL}}$$

$$= Q_0\, C_0 \left[\frac{\dfrac{1}{C_L} - \dfrac{1}{C_u}}{u_{cL}} \right]$$

where u_{cL} is the value of u_c at the concentration C_L. Thus, the minimum necessary area of the thickener may be obtained from the maximum value of

$$\left[\frac{\dfrac{1}{C} - \dfrac{1}{C_u}}{u_c} \right] \text{ which will be designated } \left[\frac{\dfrac{1}{C} - \dfrac{1}{C_u}}{u_c} \right]_{\max}.$$

Concentrations can also be expressed as mass per unit volume (using c in place of C) to give:

$$A = Q_0\, c_0 \left[\frac{[(1/c) - (1/c_u)]}{u_c} \right]_{\max}$$

Overflow

The liquid flowrate in the overflow (Q') is the difference between that in the feed and in the underflow.

Thus:

$$Q' = Q_0 (1 - C_0) - (Q_0 - Q')(1 - C_u)$$

or:

$$\frac{Q'}{Q_0} = 1 - \frac{C_0}{C_u} \tag{5.52}$$

At any depth below the feed point, the upward liquid velocity must not exceed the settling velocity of the particles. (u_c) Where the concentration is C, the required area is therefore given by:

$$A = Q_0 \frac{1}{u_c} \left[1 - \frac{C}{C_u} \right] \tag{5.53}$$

It is therefore necessary to calculate the *maximum* value of A for all the values of C which may be encountered.

Equation 5.53 can usefully be rearranged in terms of the mass ratio of liquid to solid in the feed (Y) and the corresponding value (U) in the underflow:

$$Y = \frac{1 - C}{C} \frac{\rho}{\rho_s} \quad \text{and} \quad U = \frac{1 - C_u}{C_u} \frac{\rho}{\rho_s}$$

Then:

$$C = \frac{1}{1 + Y(\rho_s/\rho)} \qquad C_u = \frac{1}{1 + U(\rho_s/\rho)}$$

$$A = \frac{Q_0}{u_c}\left\{1 - \frac{1 + U(\rho_s/\rho)}{1 + Y(\rho_s/\rho)}\right\}$$

$$= \frac{Q_0(Y - U)C\rho_s}{u_c\rho}$$

The values of *A* should be calculated for the whole range of concentrations present in the thickener, and the design should then be based on the maximum value so obtained.

Depth of the thickener Zone, H_t

The required depth of the thickening region is

$$H_t = \left\{ \frac{W t_R}{A \rho_s} + W \frac{t_R}{A \rho} X \right\} = \frac{W t_R}{A \rho_s} \left(1 + \frac{\rho_s}{\rho} X \right)$$

where:

t_R is the required time of retention of the solids, as determined experimentally,

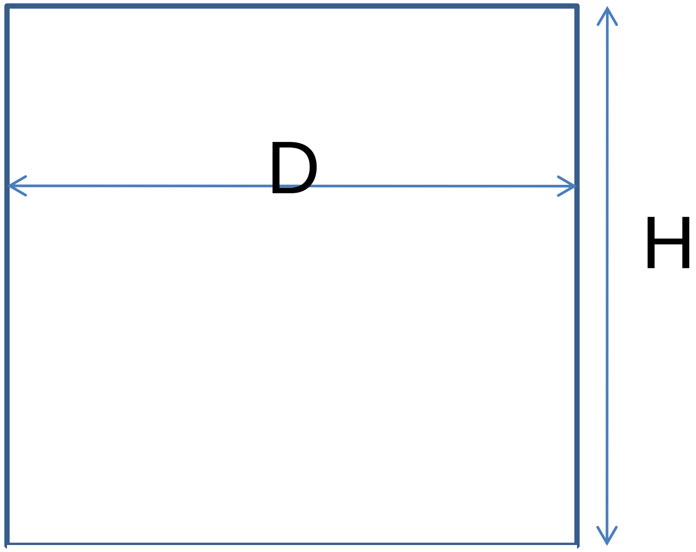
W is the mass rate of feed of solids to the thickener,

X is the average value of the mass ratio of liquid to solids in the thickening portion, and

ρ and ρ_s are the densities of the liquid and solid respectively.

NOTE

- Take the ratio H/D=1.5 for deep tank up to 19 m height.



Fluidization

Principles and definitions

Fluidization-Definition

Fluidization is defined as the operation in which the fine solid particles are transformed into a fluid-like state through contact with a gas or a liquid.

Fluidization

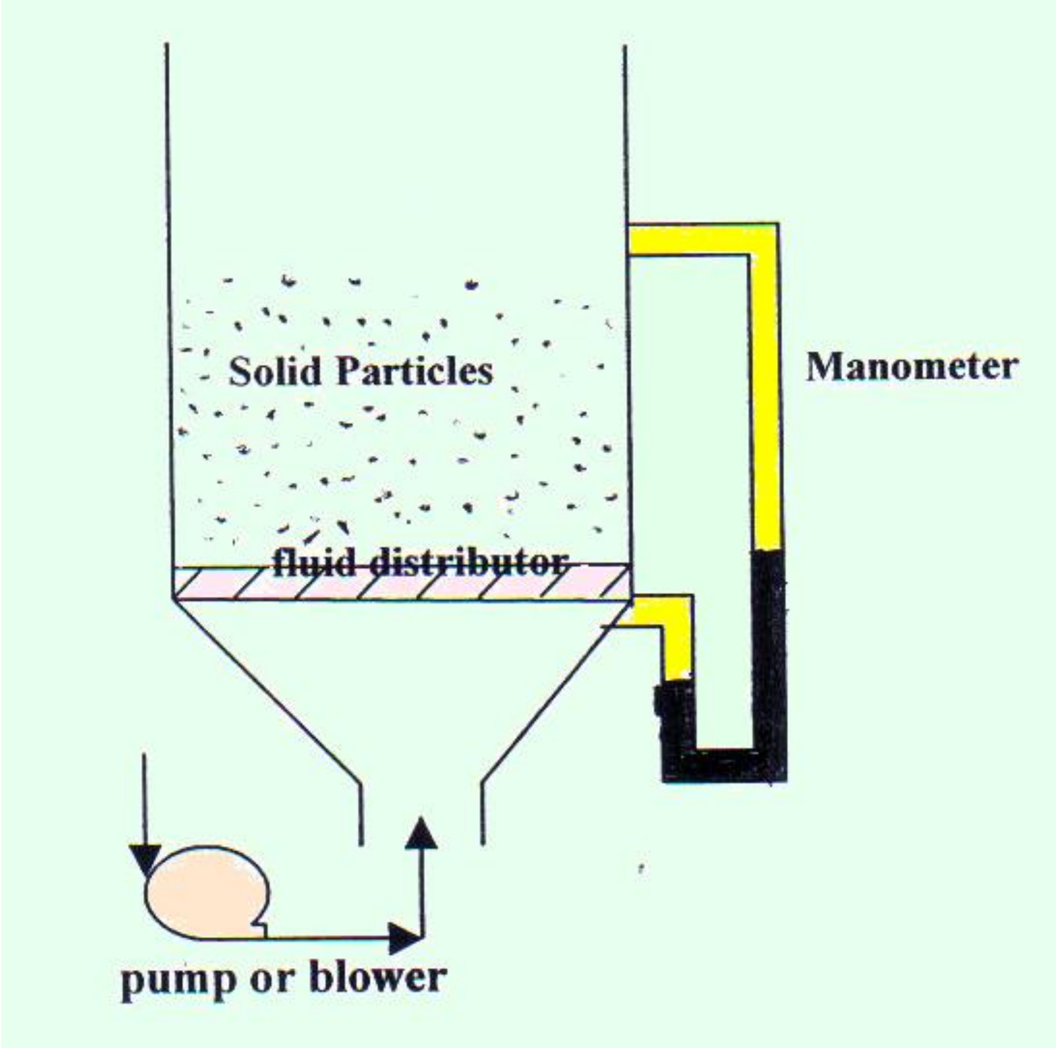
Advantages

- An extremely large area of contact between solid and fluid. So, high overall rates of heat and mass transfer.
- Ease of handling of solids when in the fluidized condition.
- High degree of solids mixing in gas-fluidized system.
- High rates of heat transfer between the fluidized solids and any immersed system.
- Suited to large-scale operations.

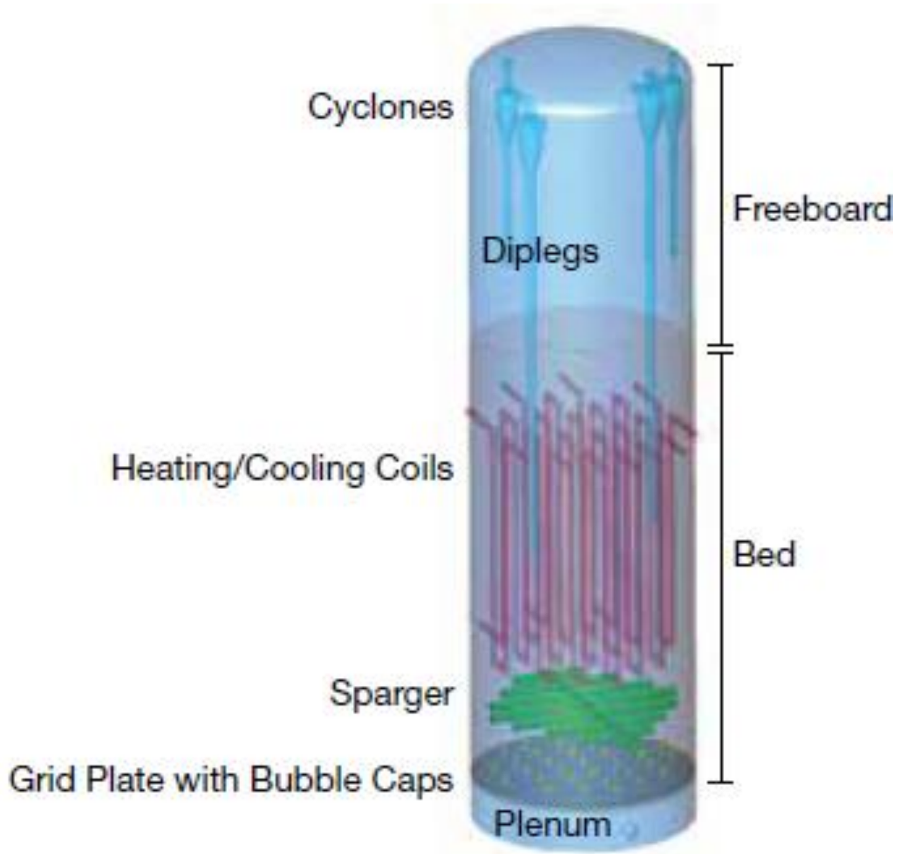
Disadvantages

- Some particles are difficult to fluidize because of i) an excessive tendency for particles for attrition , or ii) an excessive tendency to agglomeration.
- The process requires high power to achieve.
- Operation rates are limited to a relatively narrow range. Too low fluid velocity leads to bed settlement. Too high fluid velocity leads to excessive particle loss.
- The particle size range is normally limited to 1/4 in down to a few μm

Fundamentals

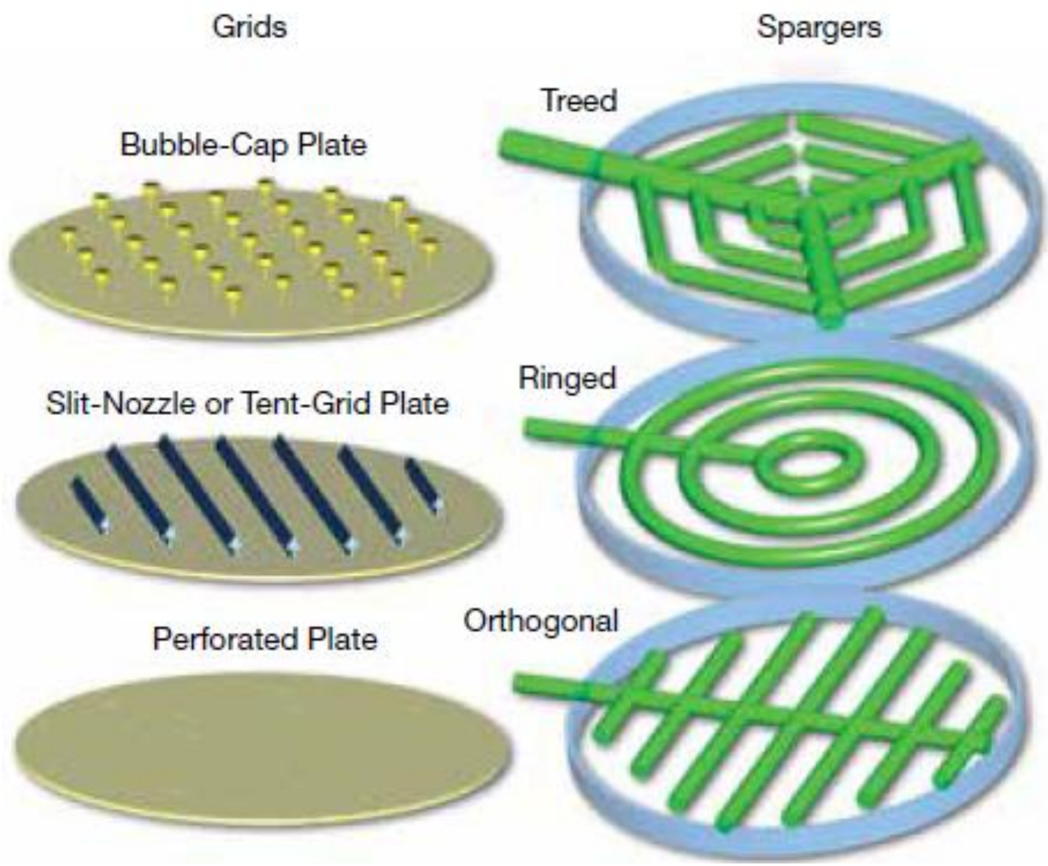


Typical Fluidized Bed



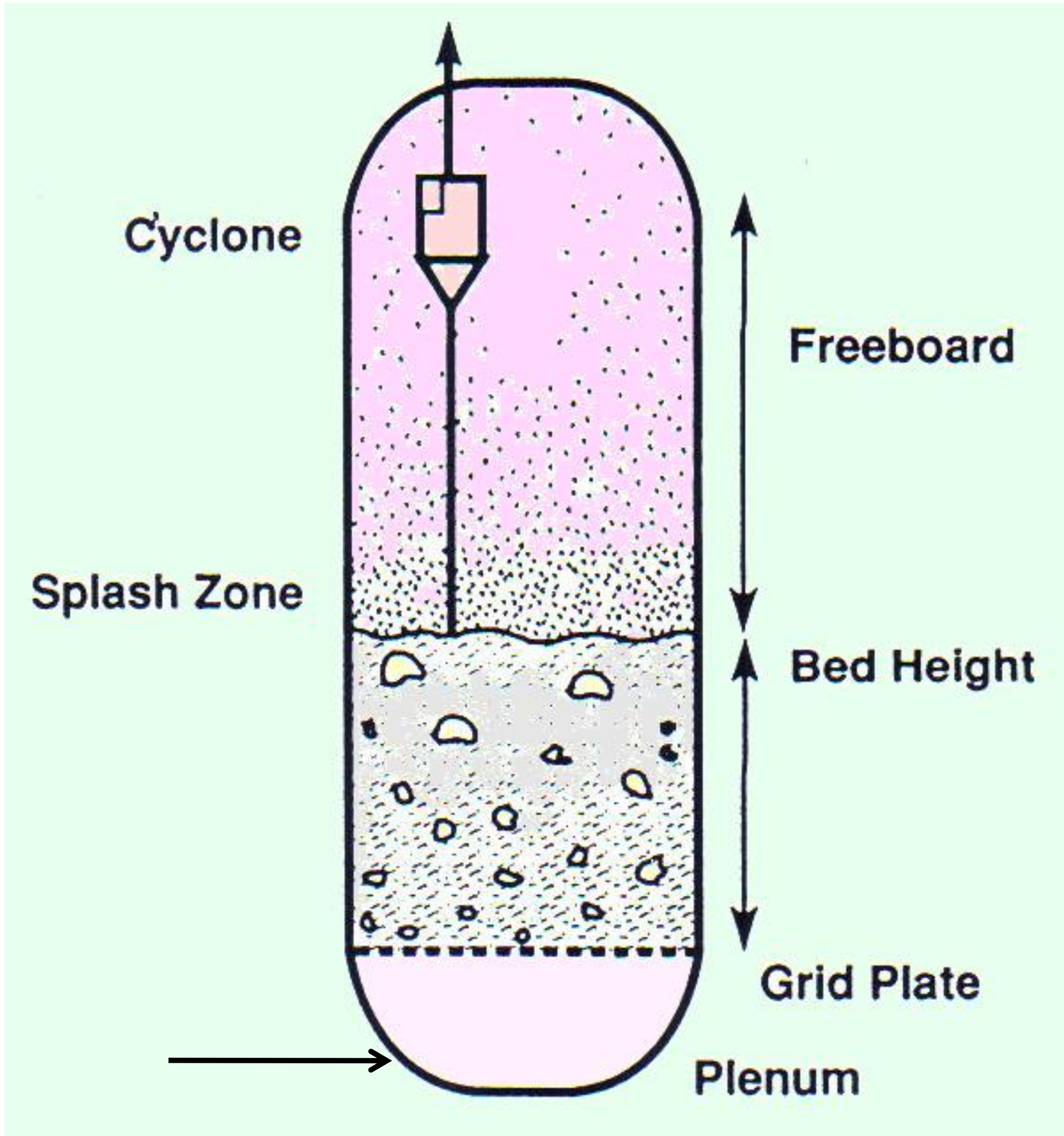
A typical fluidized bed has a plenum, a gas distributor, cyclones and diplegs, and heating/cooling coils. This fluidized bed has a dual feed system consisting of both a grid plate and a sparger.

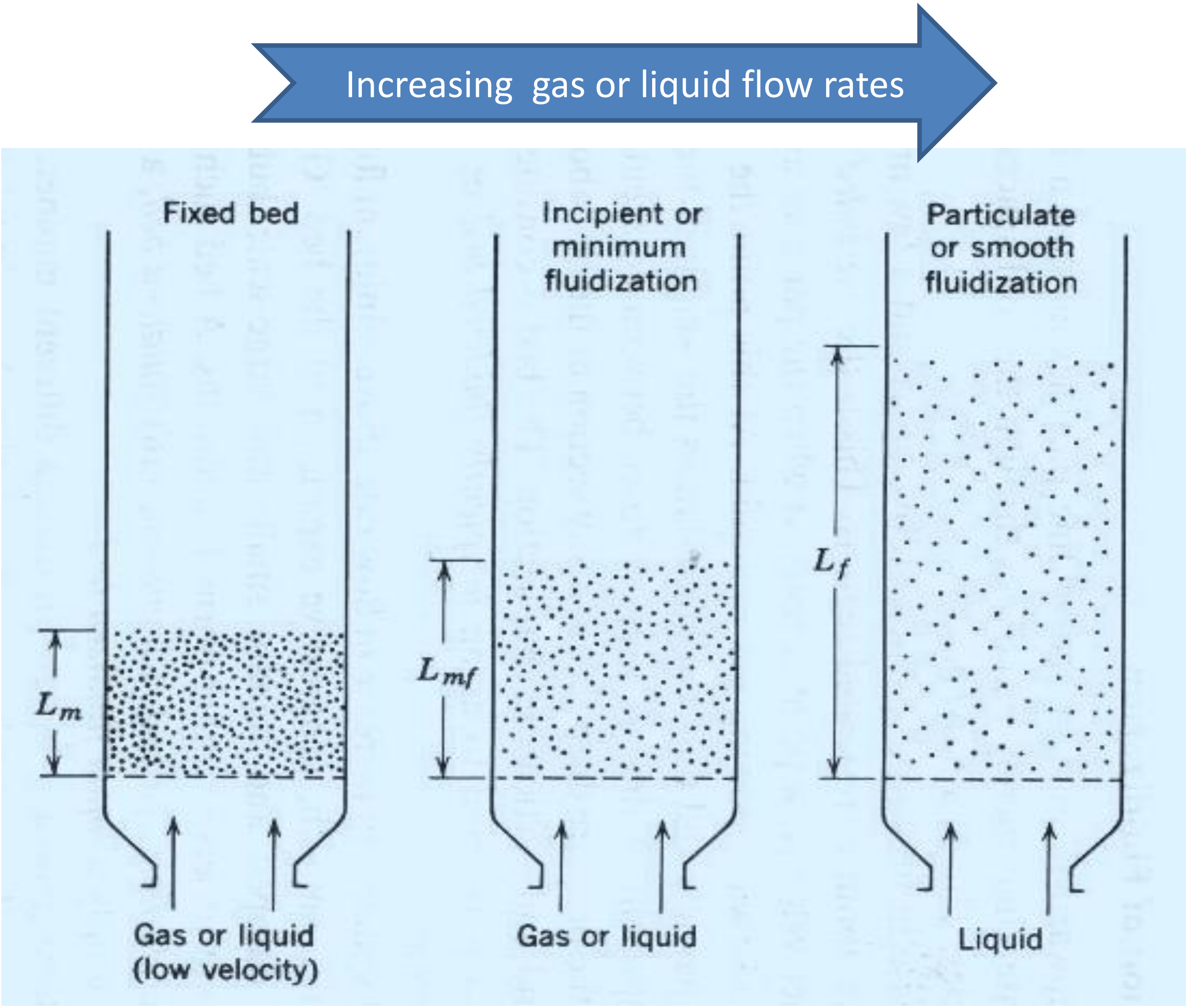
Grid plates & Spargers

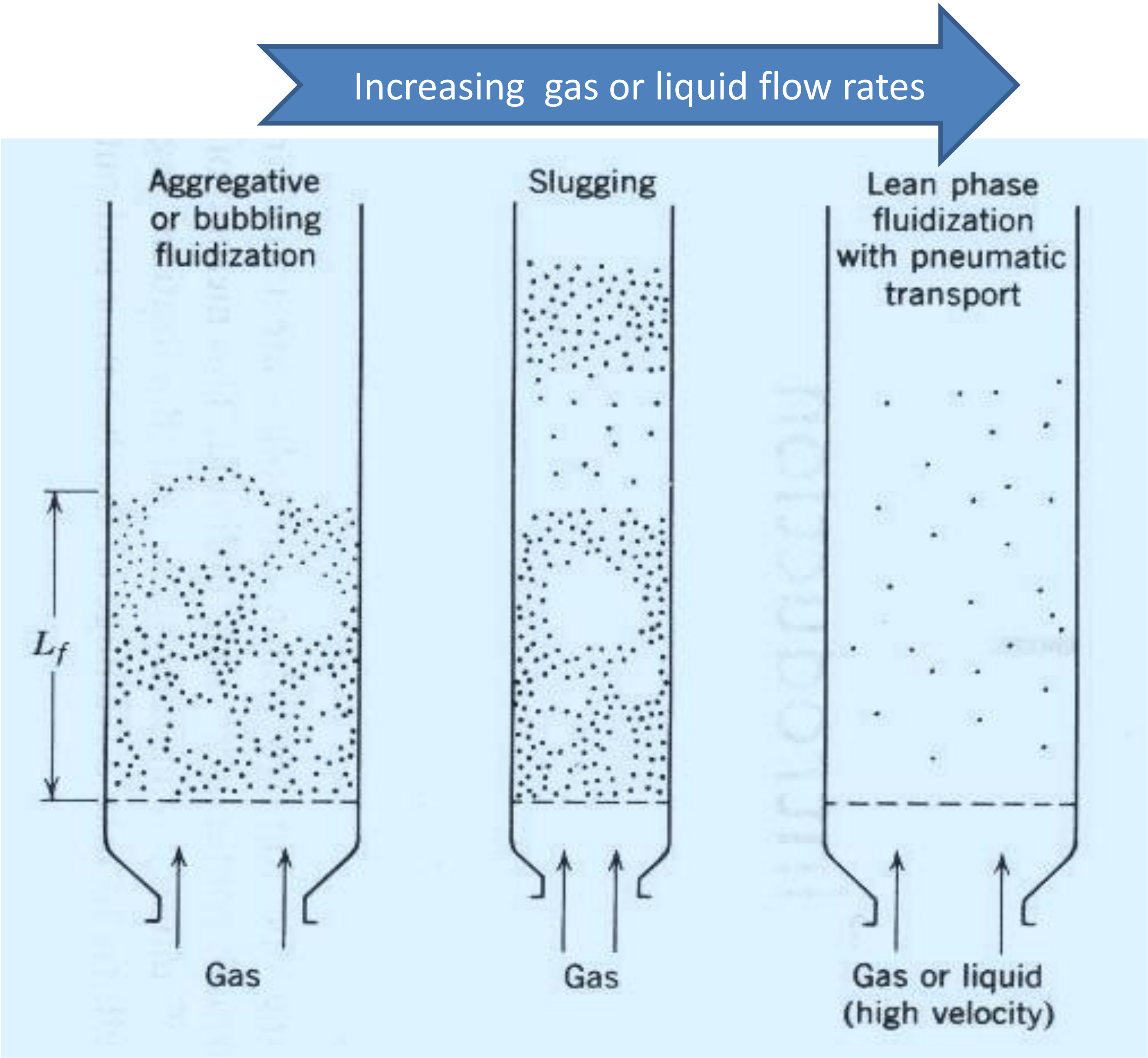


Fluidized beds employ a variety of grid plate and sparger designs. Grid plates with bubble caps help reduce particle weeping into the plenum. The most common sparger designs have a treed, ringed, or orthogonal pattern.

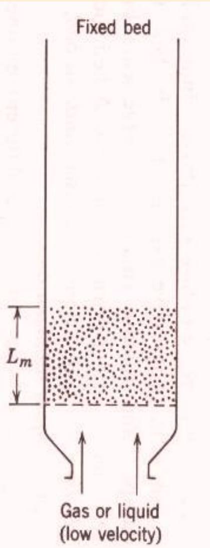
Fluid Bed Definition



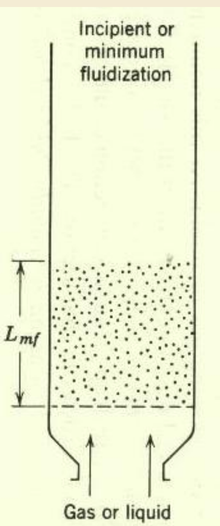

Increasing gas or liquid flow rates
Fixed bed
Incipient or minimum fluidization
Particulate or smooth fluidization
L_m
L_{mf}
L_f
Gas or liquid (low velocity)
Gas or liquid
Liquid


Increasing gas or liquid flow rates
Aggregative or bubbling fluidization
L_f
Gas
Slugging
Gas
Lean phase fluidization with pneumatic transport
Gas or liquid (high velocity)

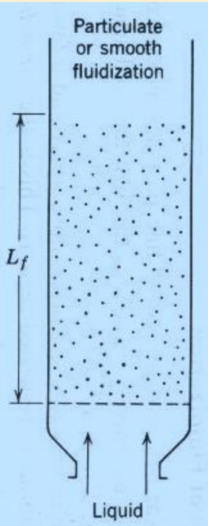
Fundamentals



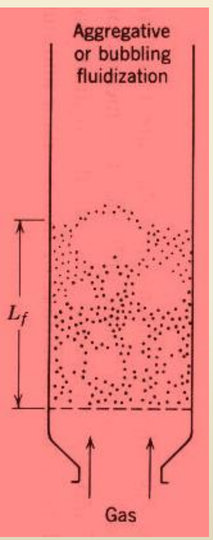
- **Fixed bed ~ no solid movement ~ Fluid: 'Gas or Liquid' ~ Fluid velocity: low. $\Delta P \uparrow$ $U_f \uparrow$ 'similar to fixed bed with single fluid flow'.**
- **Incipient fluidized bed ~ Fluid: 'gas or liquid'. If U_f is increased, at a certain value the solid particles begin to move or "fluidize". At this point the upward drag force exerted by the fluid on the particles = the apparent weight of particles. *The bed is known as incipient fluidized bed at minimum fluidization. $U \rightarrow U_{mf}$, "little expansion of solid-bed"***



Fundamentals



- **Particulate or Smooth Fluidized bed ~ "Fluid: liquid", $U_l > U_{mf}$. Smooth expansion of the bed takes place with little or negligible instabilities in gross flow of fluid.**

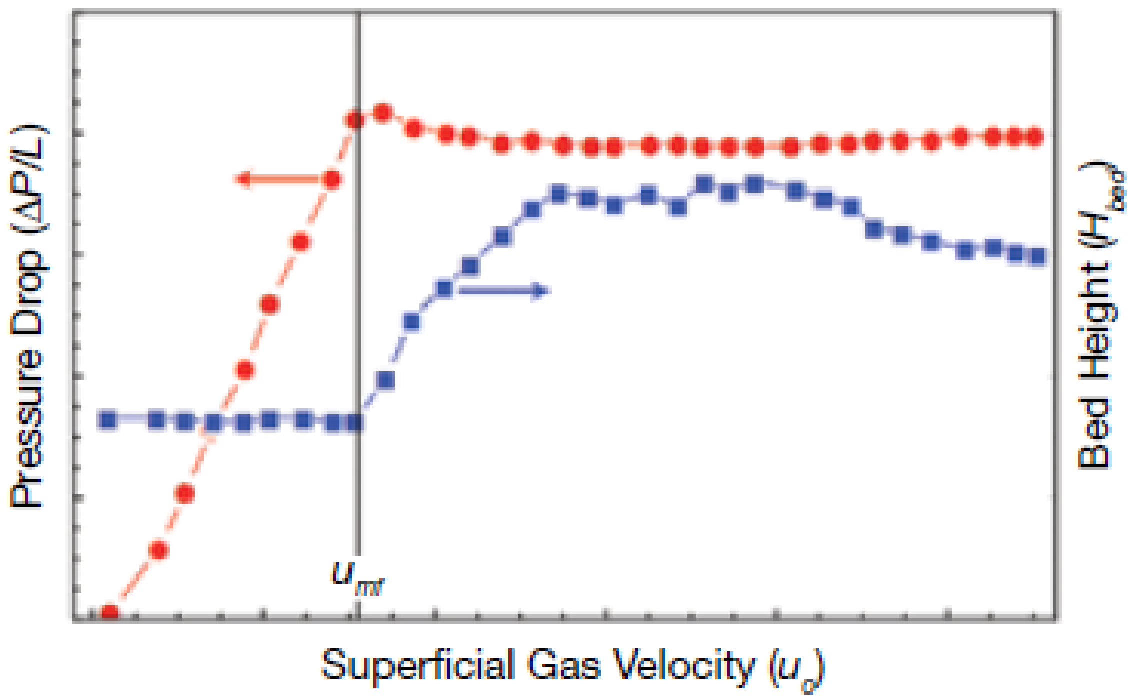


- **Aggregative or bubbling fluidized bed.**

  "Fluid: gas", $U_g > U_{mf}$.

  Here, we have large instabilities with bubbling and channeling of gas inside the bed. As $U_g \uparrow$ mixing and movement of solids becomes very high.

Pressure drop versus gas velocity



This typical minimum fluidization curve shows that the pressure drop (red line) increases with superficial gas velocity until the point at which the gas velocity is high enough that the drag force is equal to the weight of the particle. At this point, the minimum fluidization velocity, the bed becomes fluidized, and the bed height (blue line) increases.

Notes

Note 1:

- **The bed does not expand much beyond the min. fluidization.**

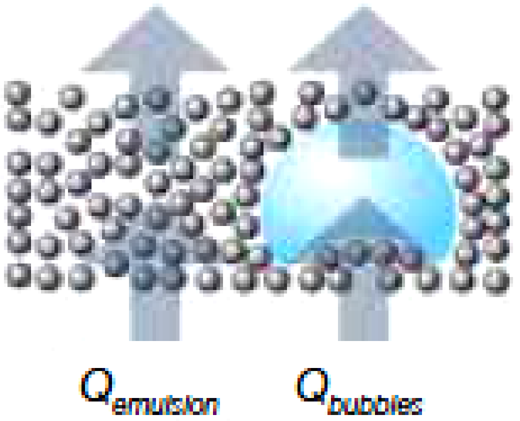
Note 2:

Two phases are obtained:

- **Continuous phase ~ dense or emulsion phase.**
- **Discontinuous phase ~ lean or bubble phase.**

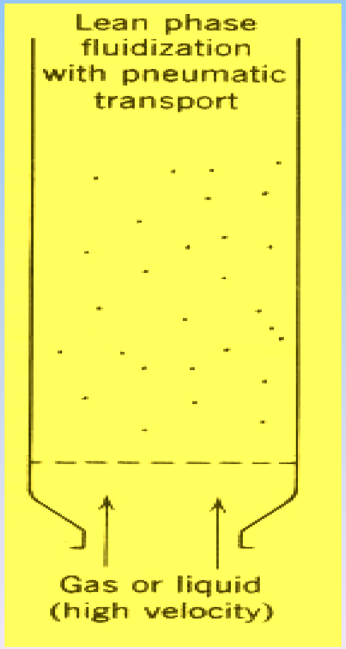
2-Phase fluidization

According to the theory of two-phase fluidization, gas moves through the particle bed as bubbles and as an emulsion (or dense phase).



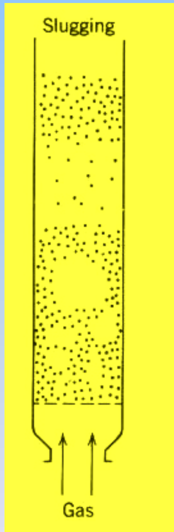
Fundamentals

- **Disperse, dilute or lean-phase fluidized bed. 'Fluid: gas or solid'. $U_f > U_t$ or U_o . Here, solids could be carried out of the bed with the fluid $\rightarrow$ pneumatic or hydraulic transport of solids.**



Notes

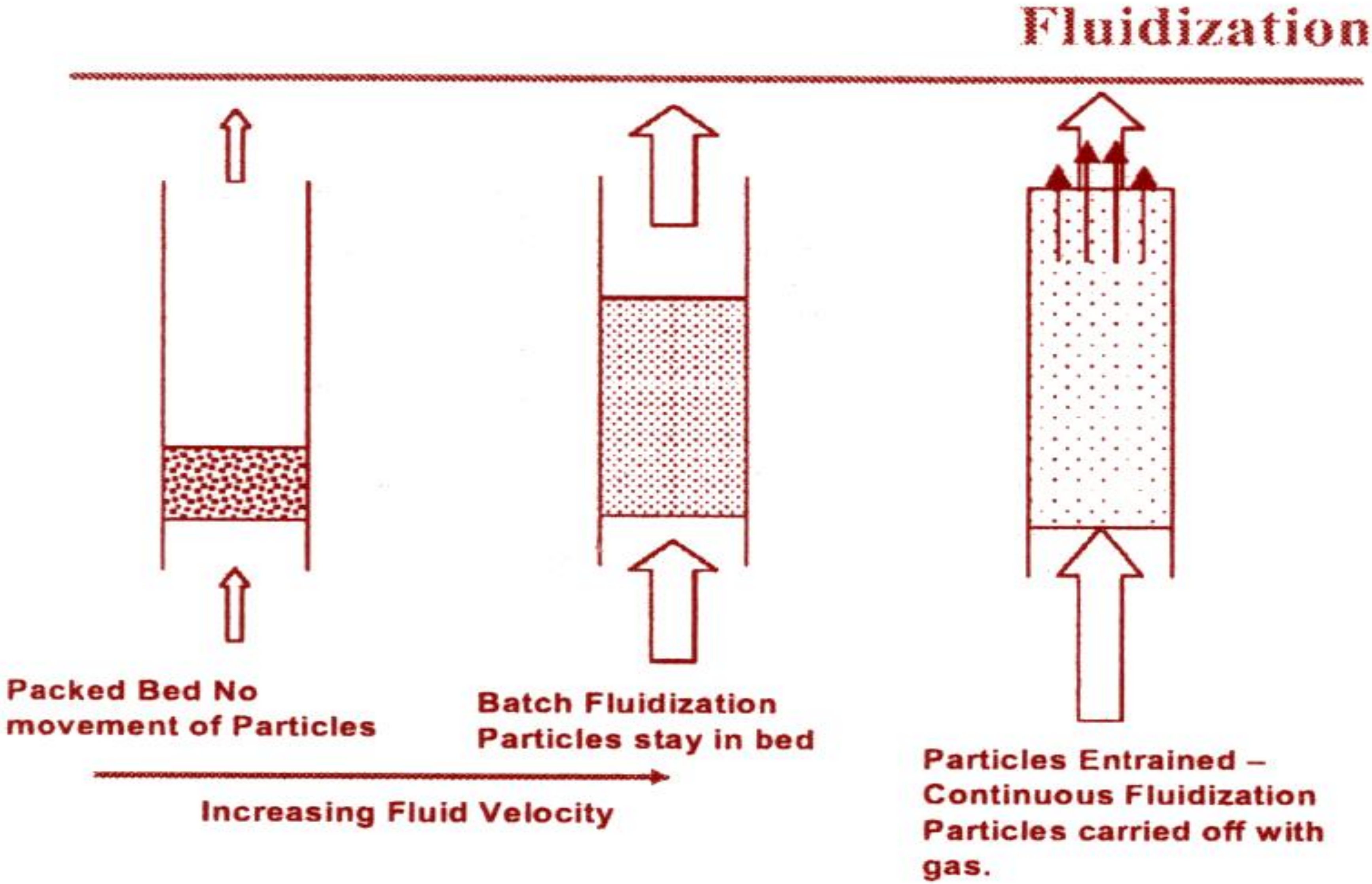
- Turbulent or fast fluidized bed, $U_g \cong 0.5\ U_o$
- Aggregative Fluidization $U_g > U_{mb}$
- Particulate Fluidization $U_f < U_{mb}$
- Slugging Bed $U_g >> U_{mf}$.
- Slugging phenomena takes place at high U_g and deep enough bed where the gas bubbles coalesce and grow as they rise and they may occupy the whole cross sectional area of the bed.



Notes

- ‘***Slug acts as piston; push the solids upward hence the particles rain down. This process will be repeated later on’.***

Summary


Fluidization
Packed Bed No
movement of Particles
Batch Fluidization
Particles stay in bed
Increasing Fluid Velocity
Particles Entrained –
Continuous Fluidization
Particles carried off with
gas.

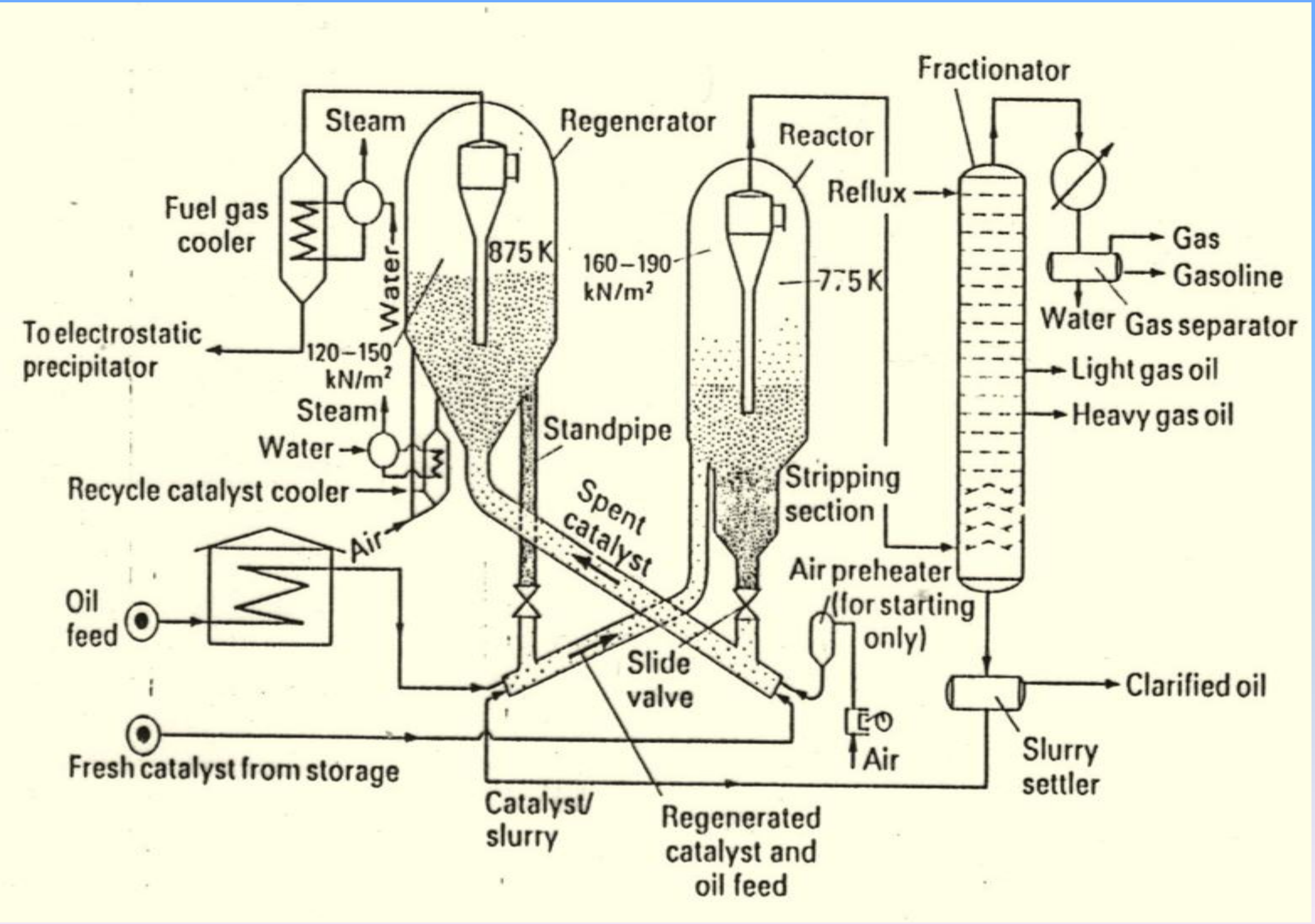
Factors influence the solid mixing, size of bubbles and degree of heterogeneity

- **Bed geometry**
- **Gas flow rate**
- **Type of gas distributor**
- **Vessel internals: screen, baffles and heat exchangers.**
- **Solid and fluid properties.**

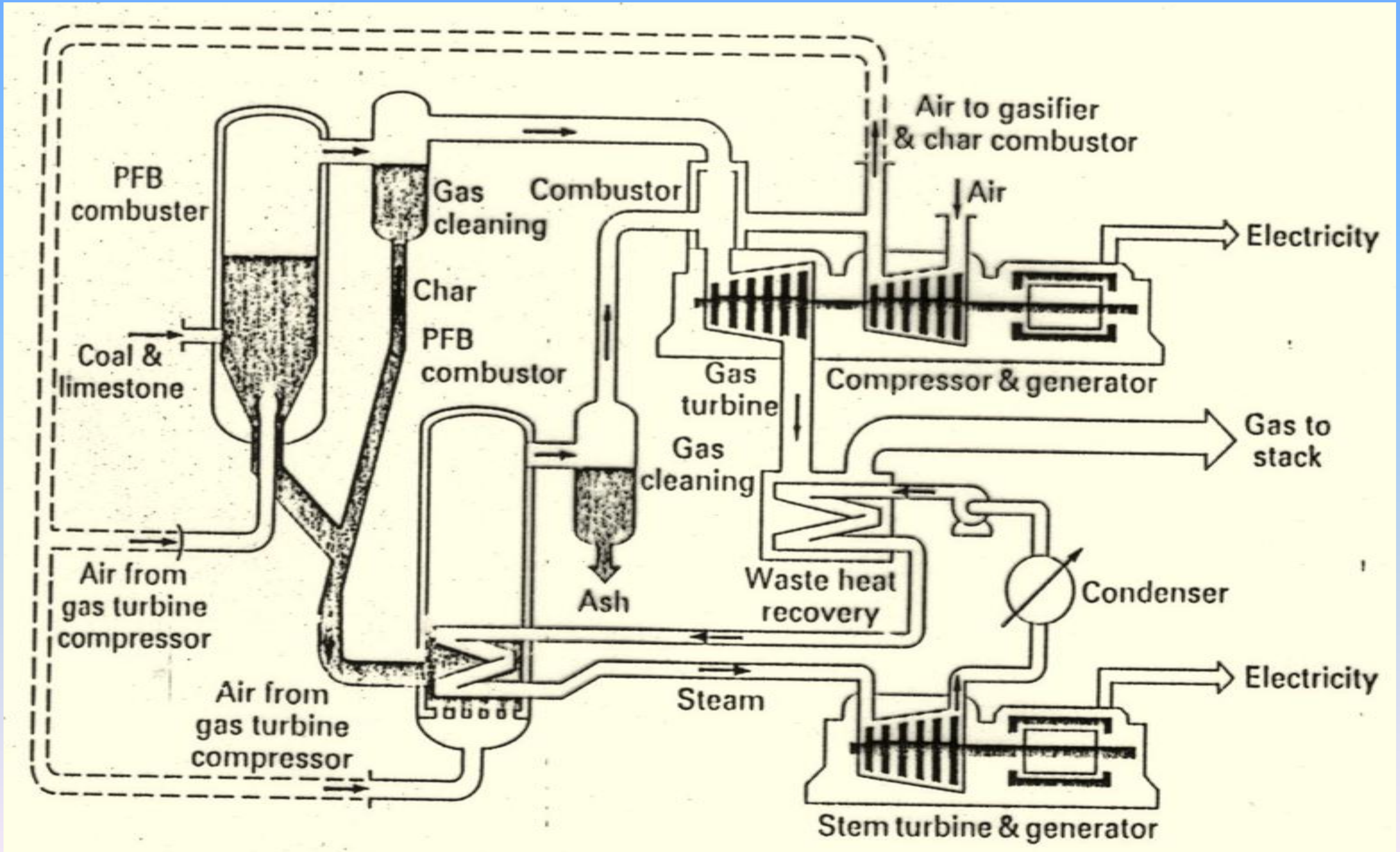
*********Applications*********
****

- Fluidized Bed Catalytic Cracking
- Fluidized Bed Combustion
- Drying of solid particles
- carbonization and gasification processes
- gas purification work; the removal of suspended dusts and mists from gases,
- lime burning and in the manufacture of phthalic anhydride

Fluid catalytic cracking plant



PFBC

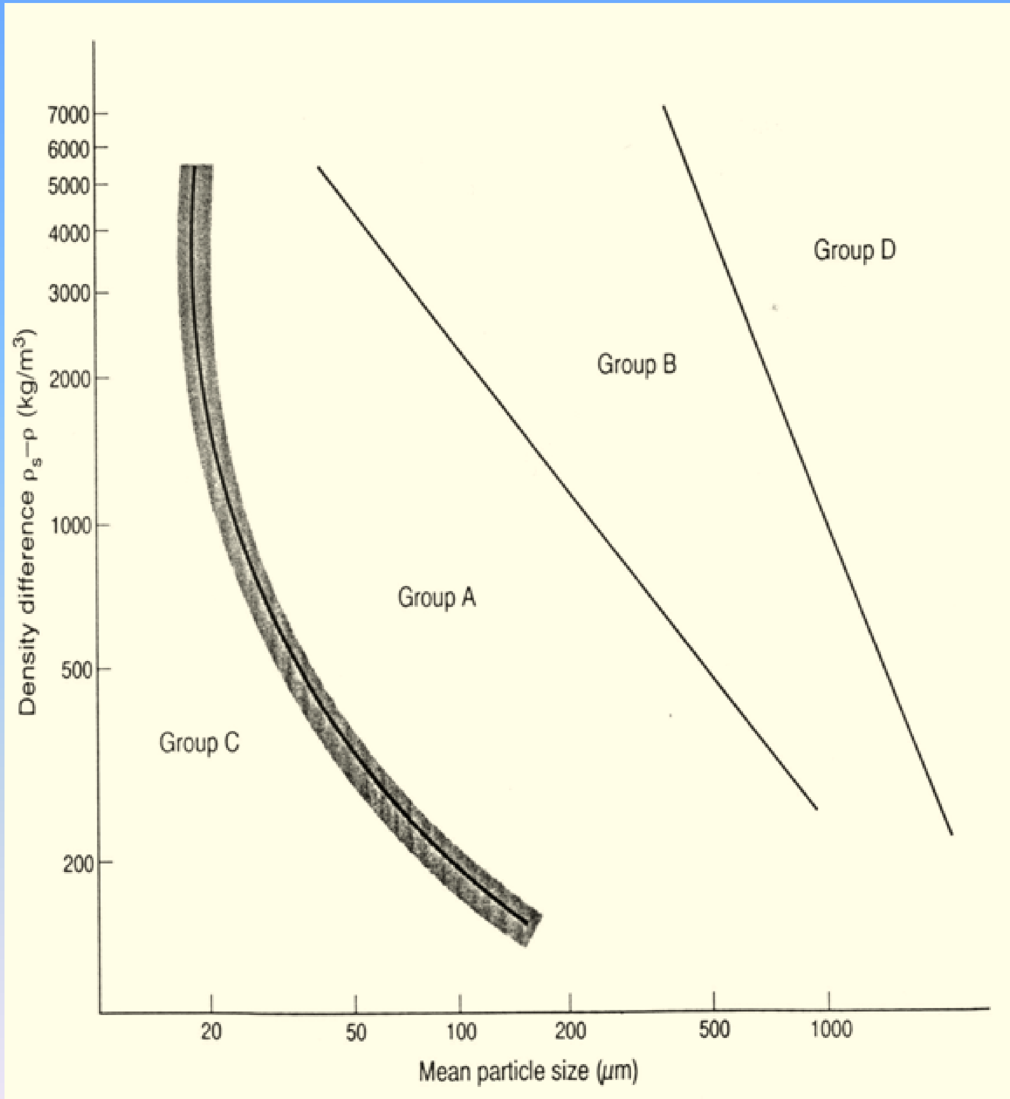


Air to gasifier
& char combustor
PFB
combuster
Gas
cleaning
Combustor
Air
Electricity
Char
PFB
combustor
Coal &
limestone
Gas
turbine
Compressor & generator
Gas
cleaning
Gas to
stack
Air from
gas turbine
compressor
Ash
Waste heat
recovery
Condenser
Air from
gas turbine
compressor
Steam
Electricity
Stem turbine & generator

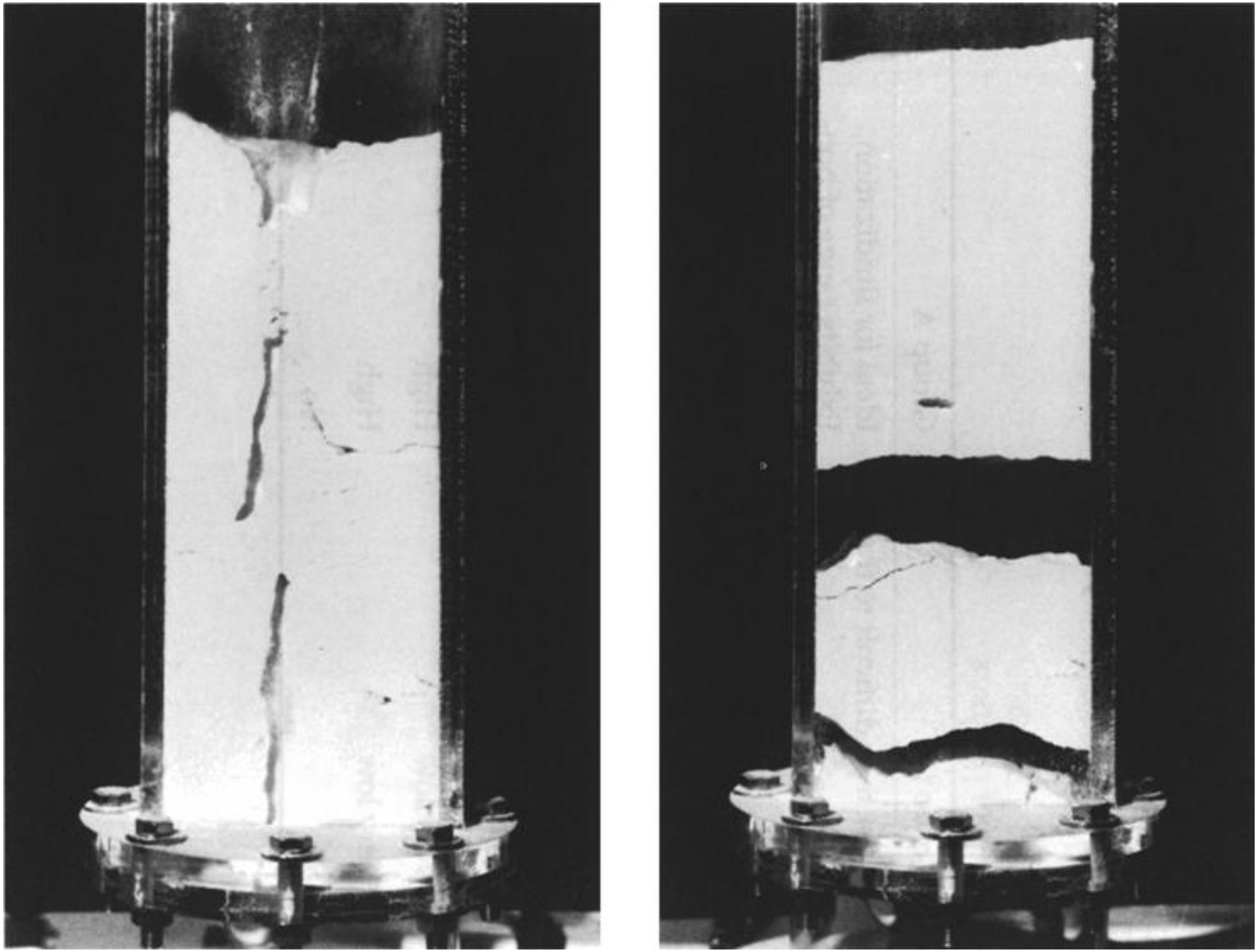
Powder Classification Diagram

There are 4 categories:

- Type A ρ_p <1400 kg/m^3, size range 20-100 μm, Particulate fluidization to 2 or 3 U_{mf} possible. Dense Phase expands slowly prior to bubbling.
- Type B 1400-4000 kg/m^3, 40-500 μm $U_{mB} \approx U_{mf}$, $U_B > U_{mf}/e_{mf}$ for most bubbles.

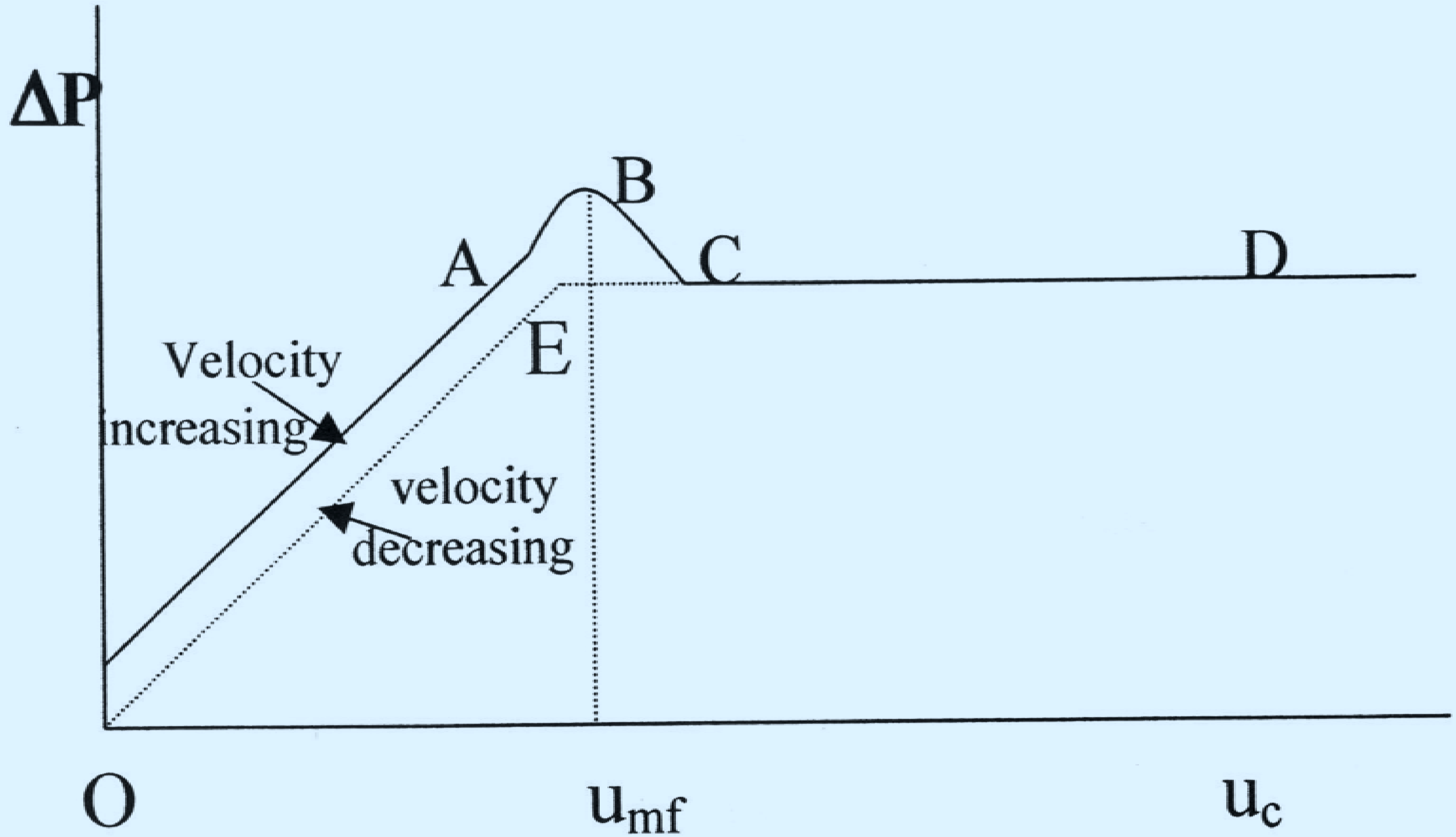


- **Type C small size $<30\ \mu m$, $\rho_p < 1400 kg/m^3$ inter-particle forces more important than gravity. Very difficult to fluidize. Channel instead.**
- **Type D size $> 600\ \mu m$, $\rho_p > 4000\ kg.m^{-3}$, $U_B < U_{mf}/e_{mf}$ for all but largest bubbles. Horizontal rather than vertical bubble coalescence. This type can be made to form spouted beds.**



Group C powder producing cracks and channels or discrete solid plugs

Relation between ΔP & U



Notes

The max. pt. at the curve 'B' is taken as u_{mf}

At min. fluidization:

The fluid pressure drop across the bed of particles is equal to the apparent weight of the particles per unit area of the bed;

$$\Delta P = \frac{Al(1-e)(\rho_S - \rho)g}{A}$$

$$\Delta P = l(1-e)(\rho_S - \rho)g \qquad (1)$$

- Line CD on the curve represents the fluidized region where the eq. (1) is applied.
- To obtain u_{mf} , use Carman-Kozeny eq. for Re_1<1.0 and spherical particles:

$$u = 0.0055 \frac{e^3}{(1-e)^2} \frac{\Delta P d^2}{\mu l} \qquad (2)$$

Eleminatin g ΔP using eq. (1)

$$u = 0.0055 \frac{e^3}{(1-e)} \frac{d^2(\rho_s - \rho) g}{\mu} \qquad (3)$$

- In general e at min. Fluidization = 0.4

$$\therefore \quad u_{mf} = 0.00059 \frac{d^2(\rho_s - \rho)g}{\mu} \qquad (4)$$

For large particles, turbulent regime is dominant, therefore use Ergun equation.

$$\frac{-\Delta P}{l} = 150 \frac{(1-e)^2}{e^3} \frac{\mu u_c}{d^2} + 1.75 \frac{(1-e)}{e^3} \frac{\rho u_c^2}{d} \qquad (5)$$

Substituting $e = e_{mf}$ at the incipient fluidization pt. and for ΔP from eq. 1 and then eq. 5 express in terms of the min. fluidization velocity u_{mf} :

$$(1 - e_{mf})(\rho_s - \rho)g = 150 \frac{(1 - e_{mf})^2}{e_{mf}^3} \frac{\mu u_{mf}}{d^2} + 1.75 \frac{(1 - e_{mf})}{e_{mf}^3} \frac{\rho u_{mf}^2}{d} \quad (6)$$

Multiplying both sides by $\dfrac{\rho d^3}{\mu^2(1 - e_{mf})}$:

$$\frac{\rho(\rho_s - \rho)gd^3}{\mu^2} = 150 \frac{1 - e_{mf}}{e_{mf}^3} \frac{u_{mf} d\rho}{\mu} + \frac{1.75}{e_{mf}^3} \left(\frac{u_{mf} d\rho}{\mu}\right)^2 \quad (7)$$

In equation 7:

$\dfrac{d^3\rho(\rho_s - \rho)g}{\mu^2}$ is the Galileo number Ga,

$\dfrac{u_{mf} d\rho}{\mu}$ is a form of Reynolds number and will be designated Re'_{mf}.

Equation 7 then becomes:

$$Ga = 150\frac{1-e_{mf}}{e_{mf}^3}Re'_{mf} + \frac{1.75}{e_{mf}^3}Re'^2_{mf} \tag{8}$$

For a typical value of $e_{mf} = 0.4$:

$$Ga = 1406\,Re'_{mf} + 27.3\,Re'^2_{mf} \tag{9}$$

Thus:

$$Re'^2_{mf} + 51.4Re'_{mf} - 0.0366\,Ga = 0 \tag{10}$$

and:

$$(Re'_{mf})_{e_{mf}=0.4} = 25.7\{\sqrt{1 + 5.53\times10^{-5}Ga} - 1\} \tag{11}$$

and, similarly for $e_{mf} = 0.45$:

$$(Re'_{mf})_{e_{mf}=0.45} = 23.6\{\sqrt{1 + 9.39\times10^{-5}\,Ga} - 1\} \tag{12}$$

By definition:

$$u_{mf} = \frac{\mu}{d\rho}Re'_{mf}$$

For non-spherical particles

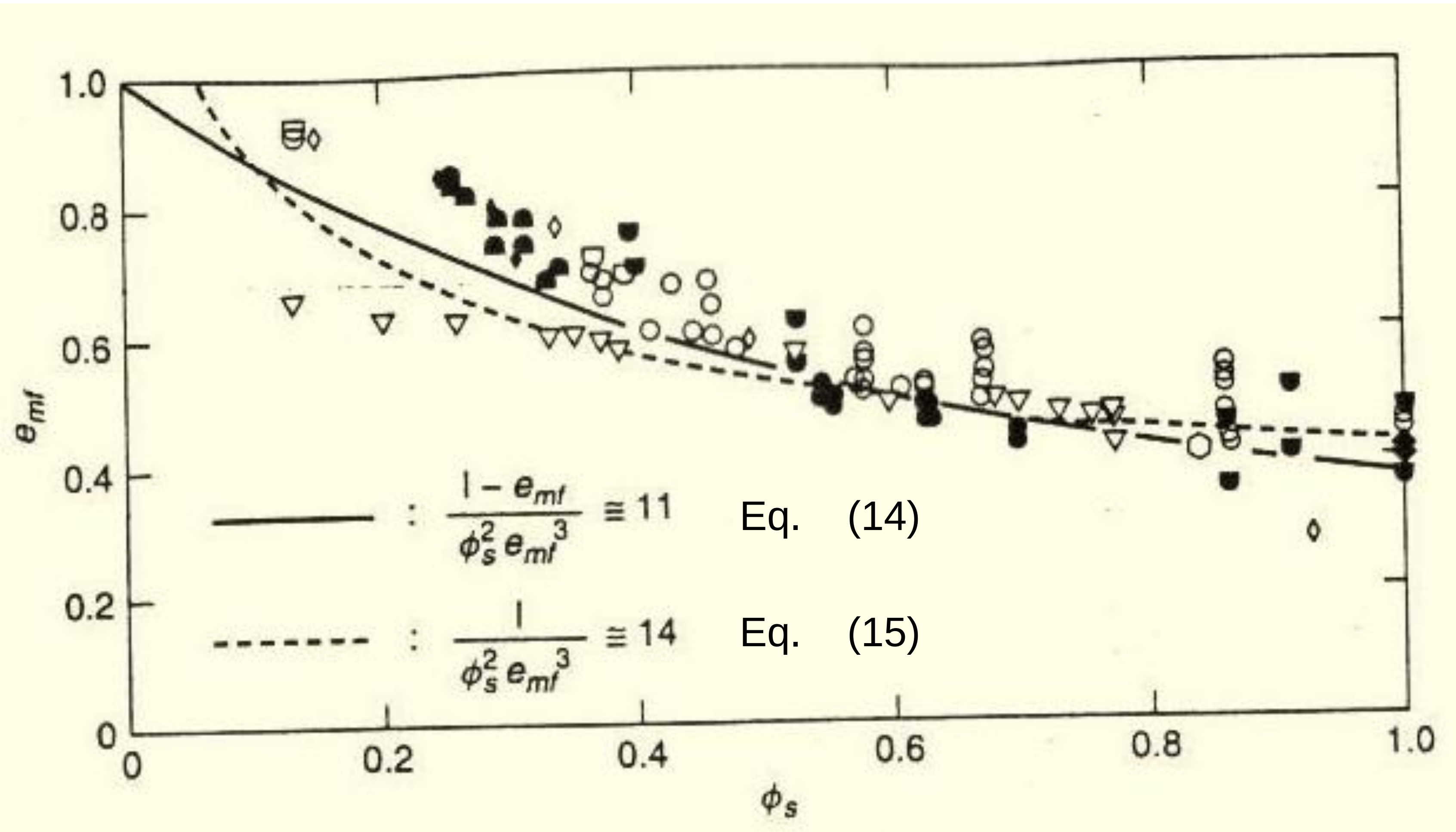
- Use the particle shape factor, φ_s
- $\varphi s = d/d_p$ *(13)*

where: $d = 6V_p/A_p$ *and* $d_p = (6Vp/\pi)^{1/3}$

<u>Note</u>

particle shape, φ_s *, which is defined as the ratio of the diameter* of the sphere of the same specific as the particle *d to the* diameter of the sphere with the same volume as the particle d_p.

Relation between e_{mf} and φ_s



$\psi_s d_p$

Using equation 13 to substitute for $\dfrac{\phi_s}{d_p}$ for d in equation 6 gives:

$$(1 - e_{mf})(\rho_s - \rho)g = 150\left(\frac{(1 - e_{mf})^2}{e_{mf}^3}\right)\left(\frac{\mu u_{mf}}{\phi_s^2 d_p{}^2}\right) + 1.75\left(\frac{1 - e_{mf}}{e_{mf}^3}\right)\frac{\rho u_{mf}{}^2}{\phi_s d_p}$$

Thus:

$$\frac{(\rho_s - \rho)\rho g d_p^3}{\mu^2} = 150\left(\frac{1 - e_{mf}}{e_{mf}^3}\right)\frac{1}{\phi_s^2}\left(\frac{\rho d_p u_{mf}}{\mu}\right) + 1.75\left(\frac{1}{e_{mf}^3\phi_s}\right)\left(\frac{\rho^2 d_p^2 u_{mf}^2}{\mu^2}\right)$$

Substituting from equations 14 and 15 :

$$Ga_p = (150 \times 11)Re'_{mfp} + (1.75 \times 14)Re'^2_{mfp}$$

where Ga_p and Re_{mfp} are the Galileo number and the particle Reynolds number at the point of incipient fluidisation, in both cases with the linear dimension of the particles expressed as d_p.

Thus:

$$Re'^2_{mfp} + 67.3Re'_{mfp} - 0.0408Ga_p = 0$$

giving:

$$Re'_{mfp} = 33.65[\sqrt{(1 + 6.18 \times 10^{-5} Ga_p)} - 1]$$

where:

$$u_{mf} = \left(\frac{\mu}{d_p \rho}\right) Re'_{mfp}$$

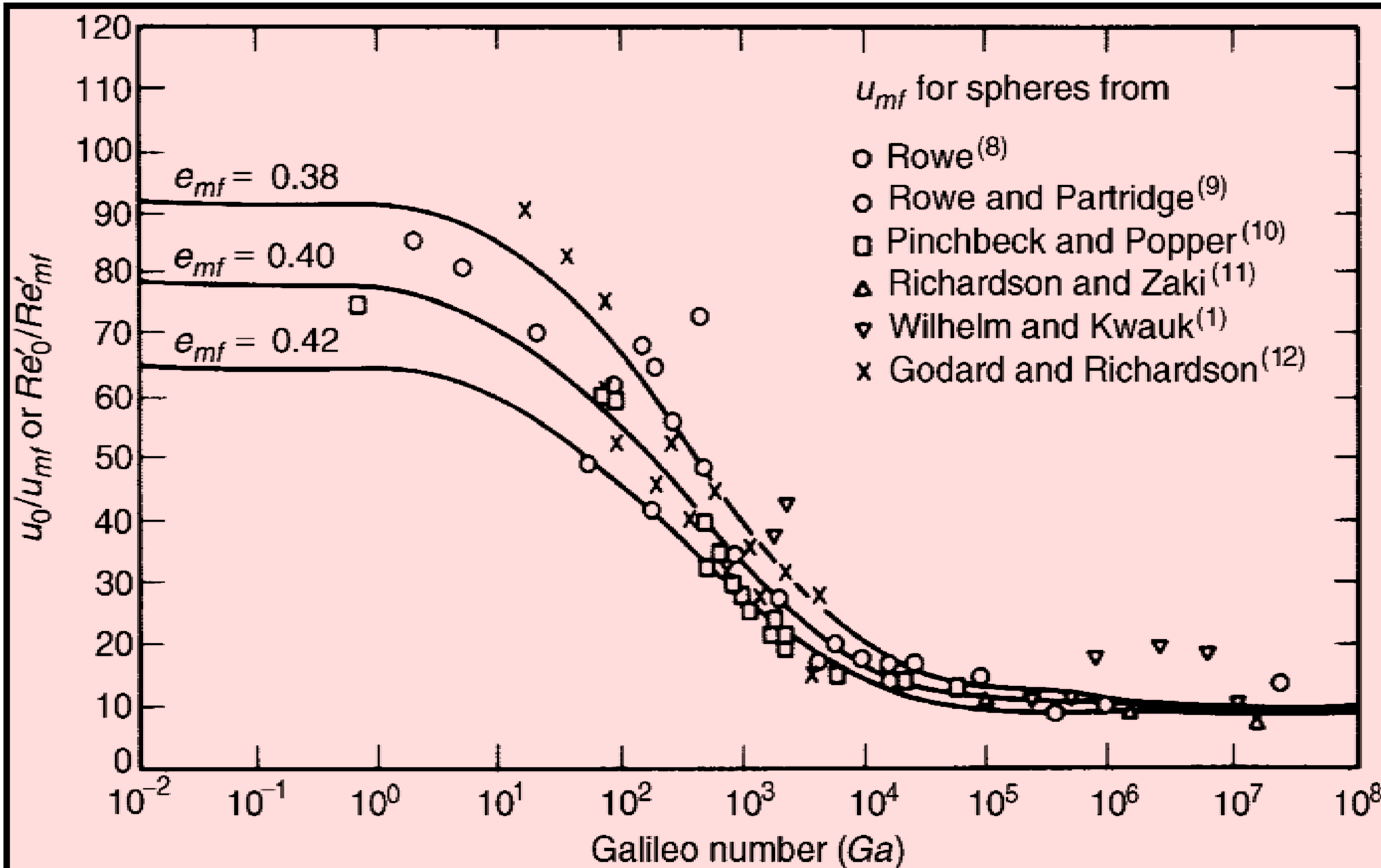
Minimum fluidizing velocity in terms of terminal falling velocity

- For a spherical particle the Reynolds number Re_o' is expressed in terms of the Galileo number *Ga by equation 3.40 which covers the whole range of values of* of interest. Re_0'
- This takes the form:

$$Re'_0 = (2.33Ga^{0.018} - 1.53Ga^{-0.016})^{13.3}$$

- Eq.s 8-10, given before, can be used to find Re_{mf}'
- The ratio Re'_0/Re'_{mf} *(=* u_0/u_{mf} *) may then be* plotted against *Ga with* e_{mf} *as the parameter.*

Ratio of terminal falling velocity to minimum fluidizing velocity, as a function of Galileo number



It is seen in Chapter 3 that it is also possible to express Re'_0 in terms of Ga by means of three simple equations, each covering a limited range of values of Ga (equations 3.37, 3.38 and 3.39) as follows:

$$Ga = 18Re'_0 \qquad (Ga < 3.6) \tag{6.22}$$

$$Ga = 18Re'_0 + 2.7Re_0'^{1.687} \qquad (3.6 < \mathrm{Ga} < 10^5) \tag{6.23}$$

$$Ga = \tfrac{1}{3}Re_0'^2 \qquad (Ga > 10^5) \tag{6.24}$$

It is convenient to use equations 6.22 and 6.24 as these enable very simple relations for Re'_0/Re'_{mf} to be obtained at both low and high values of Ga.

Taking a typical value of e_{mf} of 0.4, the relation between Re'_{mf} and Ga is given by equation 11

$$\cancel{Re_{mf}'^2} + 51.4Re'_{mf} - 0.0366\,Ga$$

For low values of Re'_{mf}(<0.003) and of Ga(<3.6), the first term may be neglected and:

$$Re'_{mf} = 0.000712Ga \tag{6.25}$$

Equation 6.22 gives:

$$Re'_0 = 0.0556Ga \tag{6.26}$$

Combining equations 6.25 and 6.26:

$$\frac{Re'_0}{Re'_{mf}} = \frac{u_0}{u_{mf}} = 78 \tag{6.27}$$

Again, for high values of $Re'_{mf}(>\sim 200)$ and $Ga(>10^5)$, equation 11 gives:

$$Re'^2_{mf} + 51.4\cancel{Re'_{mf}} - 0.0366\,Ga = 0$$

$$Re'_{mf} = 0.191Ga^{1/2} \tag{6.28}$$

Equation 6.24 gives:

$$Re'_0 = 1.732Ga^{1/2} \tag{6.29}$$

Thus :

$$\frac{Re'_0}{Re'_{mf}} = \frac{u_0}{u_{mf}} = 9.1 \tag{6.30}$$

Conclusion

This shows that u_0/u_{mf} is much larger for low values of Ga, generally obtained with small particles, than with high values. For particulate fluidisation with liquids, the theoretical range of fluidising velocities is from a minimum of u_{mf} to a maximum of u_0. It is thus seen that there is a far greater range of velocities possible in the streamline flow region. In practice, it is possible to achieve flow velocities greatly in excess of u_0 for gases, because a high proportion of the gas can pass through the bed as bubbles and effectively by-pass the particles.

Filtration

Principles, theory and operation

Filtration Process

Introduction:

- **Cake filtration is widely used in industry to separate solid particles from suspension in liquid.**
- **Cake filtration is a common application for the flow through packed beds of particles.**

- **Filtration process involves the build up of a bed or 'cake' of particles on a porous surface known as the filter medium, which commonly takes the form of a woven fabric. The pore size of the medium is less than the size of the particles to be filtered.**

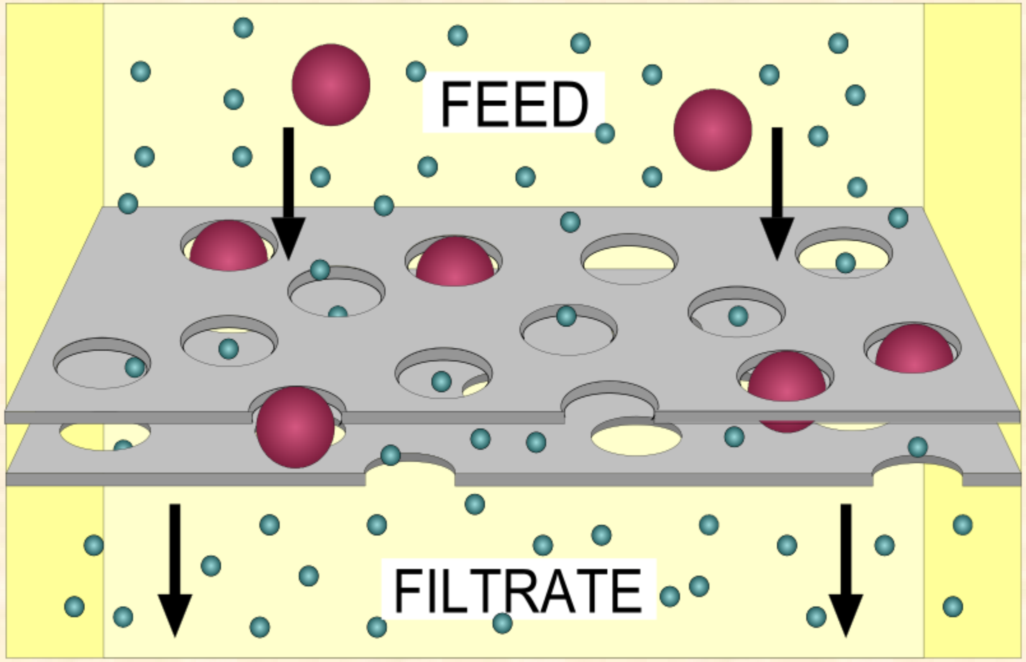
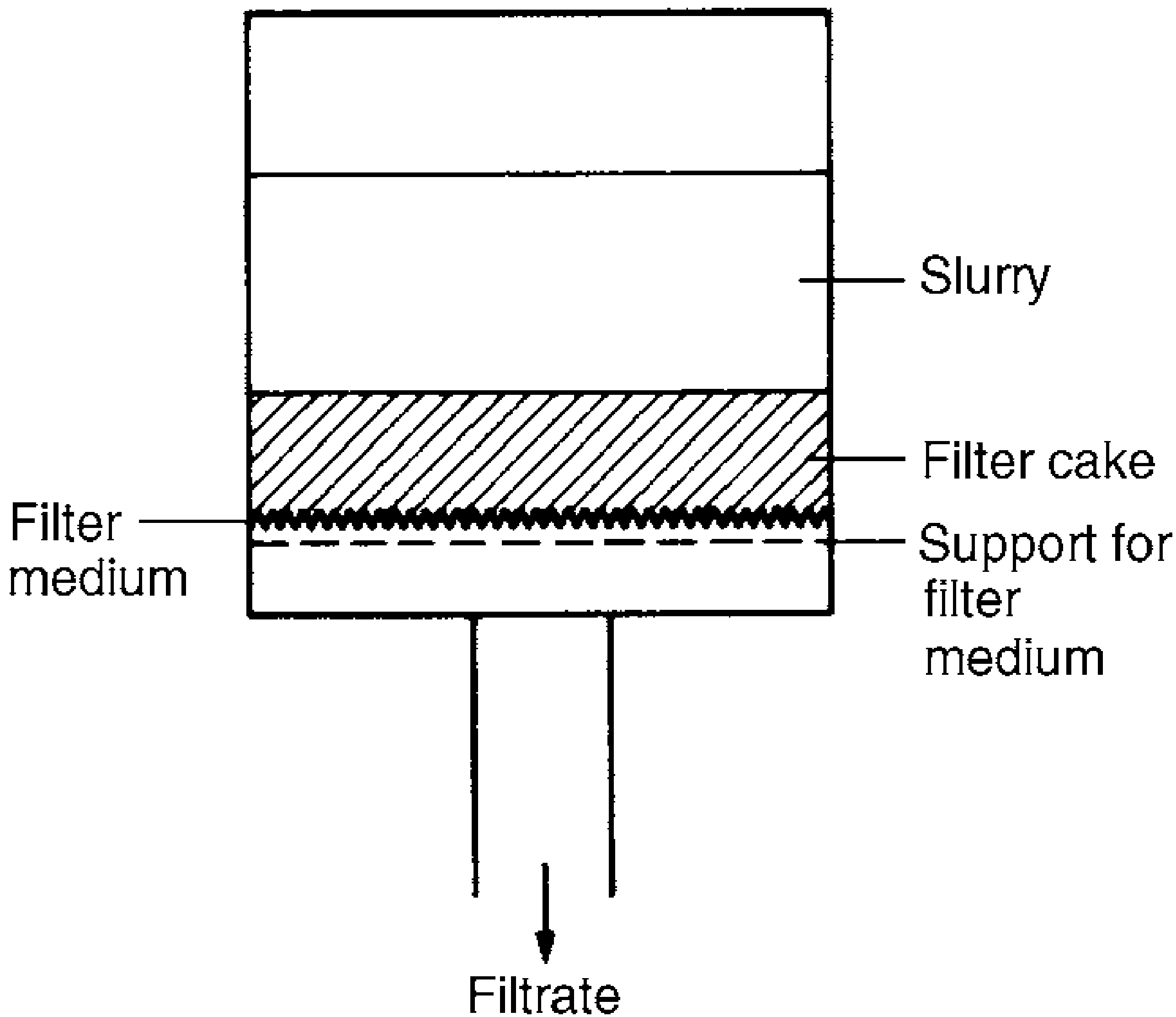


Diagram of simple filtration: oversize particles in the feed cannot pass through the lattice structure of the filter, while fluid and small particles pass through, becoming filtrate

- **In general, filtration process can be analyzed in terms of the flow of fluid through a packed bed of particles, the depth of which is increasing with time. The voidage of cake may be changed with time “ compressible cake” but we can assume, in some cases, the cake voidage is constant “ incompressible cake”.**

Principle of filtration



Summary & Definitions

- **Filtration is a separation of solid particles from liquid with the aid of a semi-permeable medium, which retains the solids and allows the liquid to pass through.**
- **Classification of equipment: gravity, pressure and vacuum filters.**
- **Filter cake: a bed of solid particles.**
- **Filtrate: liquids which pass through the filter medium.**

Cake

Compressible

Porosity varied

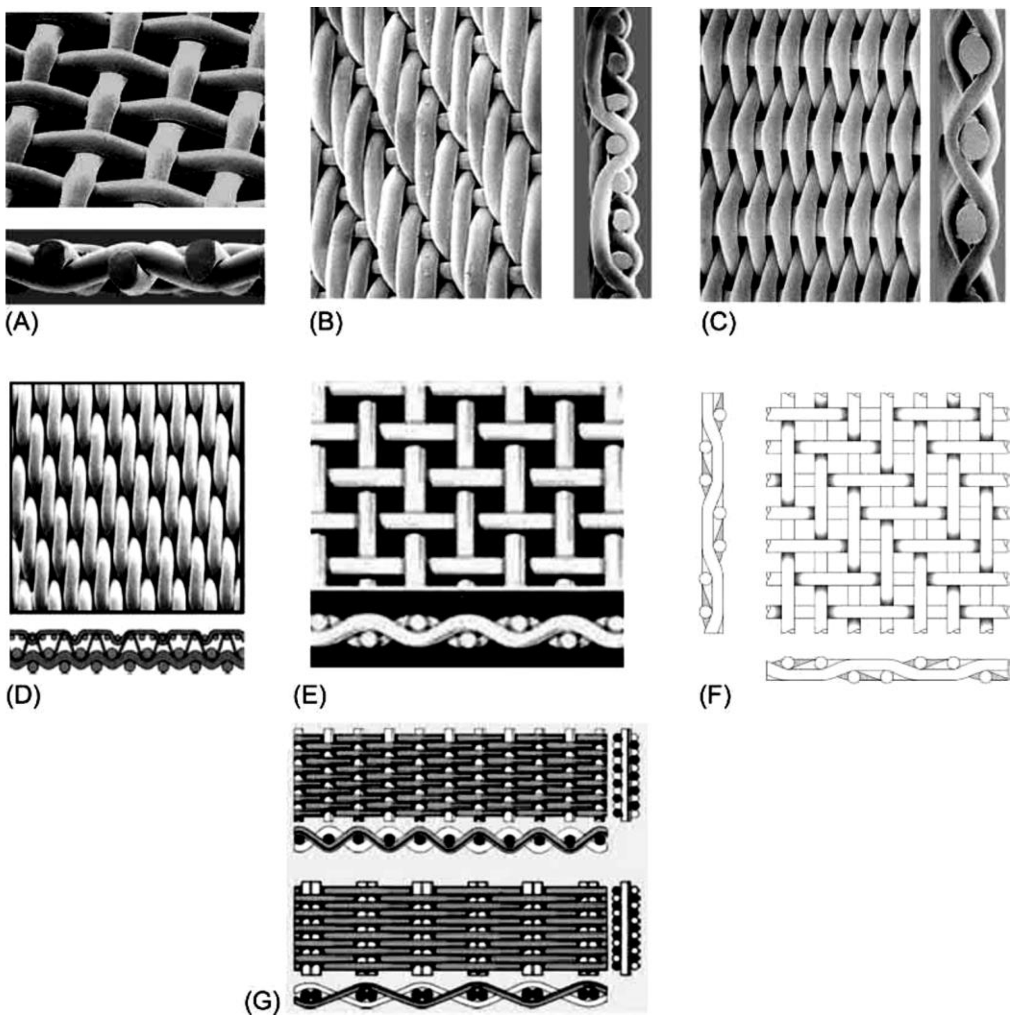
Incompressible

Porosity constant

Filter media $\equiv$ Septum

a) it must retain solids
b) it must not plug
c) it must be strong enough chemically and physically to withstand different operating conditions.
d) it must allow the cake to be removed cleanly and completely.
e) it must be cheap.

Typical filter media



(A) plain, square weave,
(B) twill weave,
(C) plain, reverse Dutch,
(D) Double layer weave,
(E) plain weave in metal media,
(F) twilled weave in metal media,
(G) twilled weave in metal media

Filter aids

Substances used to increase the porosity of solid cake in order to permit the liquid to pass at a reasonable rate. Filter aids such as diatomaceous earth, asbestos, purified wood cellulose or other inert porous solids.

<u>Note:</u> The support of media affects the way of the cake growth.

factors to be considered when selecting equipment and operating

- The properties of the fluid, particularly its viscosity, density, and corrosive properties.
- The nature of the solid—its particle size and shape, size distribution, and packing characteristics.
- The concentration of solids in suspension.
- The quantity of material to be handled, and its value.
- Whether the valuable product is the solid, the fluid, or both.
- Whether it is necessary to wash the filtered solids.
- Whether the feed liquor may be heated.

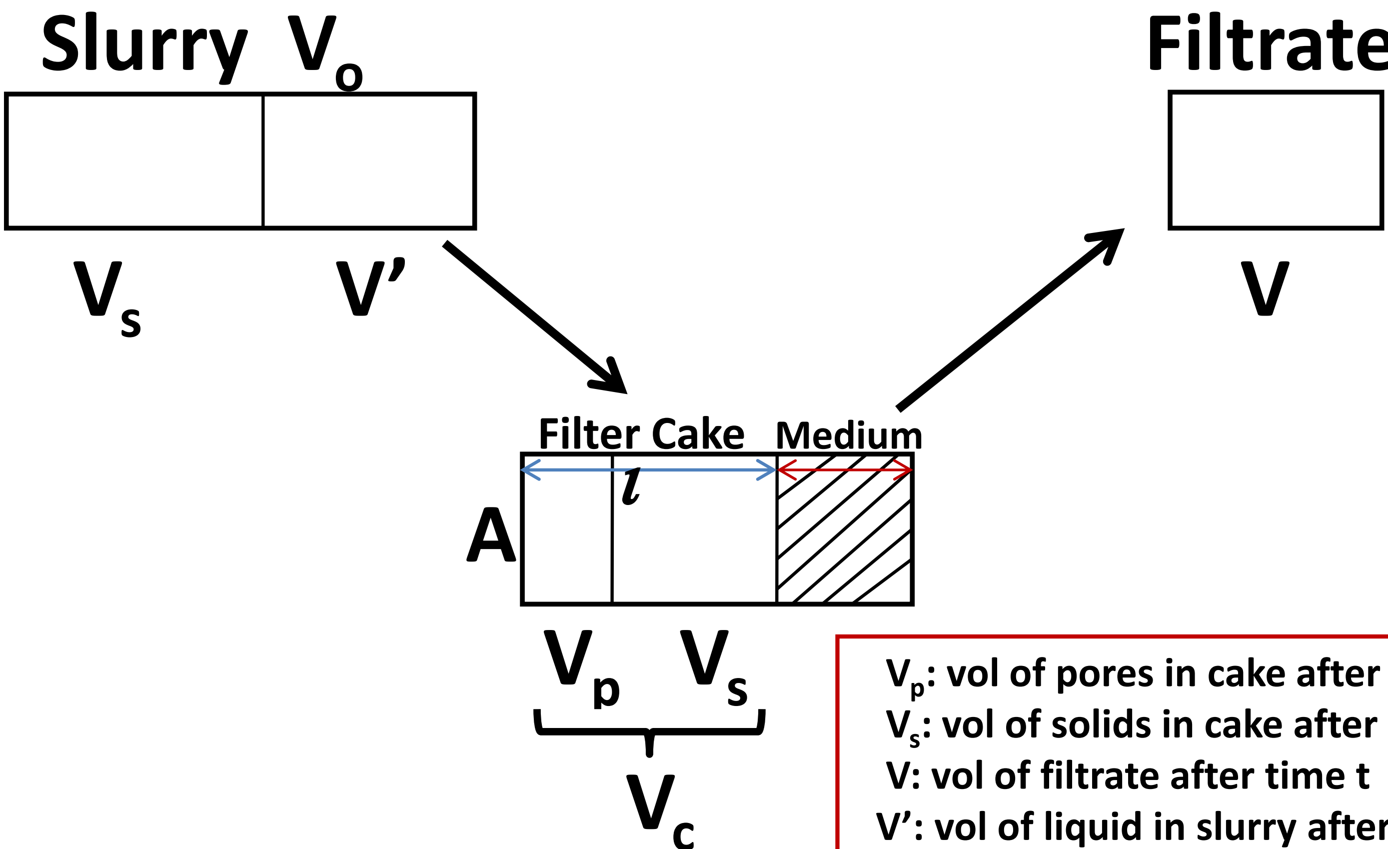
Theory of filtration

- Filtration is similar to the flow of a fluid through a granular bed. The only difference is that the thickness of cake is growing with time.
- There are two operating conditions:

  ~ filtration pressure is constant whilst the flow of filtrate disappears with time.

  ~ the flow of filtrate is constant whilst the pressure increases with time.

Process of filtration



V_p: vol of pores in cake after time t
V_s: vol of solids in cake after time t
V: vol of filtrate after time t
V': vol of liquid in slurry after time t
V_c: vol of cake formed after time t

l: thickness of filter cake

- **Since the particles forming the filter cake are very small, as well as the flow of filtrate through the cake is very slow, laminar constrains are dominant. Hence, Carman-Kozeny equation can be applied:**

$$u = \frac{1}{A}\frac{\mathrm{dv}}{\mathrm{dt}} = \frac{1}{5}\frac{e^3}{(1-e)^2 S^2}\frac{(-\Delta P)}{\mu\, l} \qquad (1)$$

Where

***ΔP*: applied pressure difference.**

***S*: sp. surface area of particles.**

Assume e = constant 'incompressible cake'

$$\therefore \quad {e^3}\Big/{5(1-e)^2 S^2} \equiv \text{Constant} = 1/r$$

$$\therefore \quad \frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\,\mu\,l} \qquad (2)$$

r represents a property of particles.

r is called *specific resistance* of cake which depends on e and S. It is constant for incompressible cake, but it depends on the rate of deposition, nature of particles and the force between the particles.

Typical values of specific resistance

Material	Upstream Filtration Pressure (kN/m^2)	r(m^{-2})
High-grade kieselguhr	–	2×10^{12}
Ordinary kieselguhr	270	1.6×10^{14}
	780	2.0×10^{14}
Carboraffin charcoal	110	4×10^{13}
	170	8×10^{13}
Calcium carbonate (precipitated)	270	3.5×10^{14}
	780	4.0×10^{14}
Ferric oxide (pigment)	270	2.5×10^{15}
	780	4.2×10^{15}
Mica clay	270	7.5×10^{14}
	780	13×10^{14}
Colloidal clay	270	8×10^{15}
	780	10×10^{15}
Gelatinous magnesium hydroxide	270	5×10^{15}
	780	11×10^{15}
Gelatinous aluminium hydroxide	270	3.5×10^{16}
	780	6.0×10^{16}
Gelatinous ferric hydroxide	270	3.0×10^{16}
	780	9.0×10^{16}
Thixotropic mud	650	2.3×10^{17}

<u>Note</u>

r sometimes referred to as the ‘permeability of cake’.

Material balance over the solid in cake and slurry

Mass of solids in cake = (1-e) Al ρ_s

Mass of liquid in cake = e A lρ

Assume J is the mass fraction of solids in suspension, then J/(1-J) is the solid to liquid ratio .

$$\therefore \quad (1-e)\,Al\,\rho_S = (V + eAl)\rho\frac{J}{1-J} \qquad (4)$$

$$\therefore \quad (1-J)(1-e)Al\rho_S = J\rho V + eAl\rho J \qquad (5)$$

$$\therefore \quad l = \frac{JV\rho}{A\{(1-J)(1-e)\rho_S - Je\rho\}} \qquad (6)$$

OR

$$V = \frac{Al\{(1-J)(1-e)\rho_S - Je\rho\}}{\rho J} \qquad (7)$$

Assume *v* is the vol of cake per unit vol of filtrate

$$v = lA/V \quad or \quad l = vV/A \qquad (8)$$

Combining equations 6 and 8

$$\therefore \quad v = \frac{J\rho}{(1-J)(1-e)\rho_S - Je\rho} \qquad (9)$$

Substituting for / in eq. 2

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\,\mu}\frac{A}{vV} \qquad (10)$$

OR

$$\frac{dV}{dt} = \frac{(-\Delta P)}{r\,\mu}\frac{A^2}{vV} \qquad (11)$$

For constant rate of filtration

$$\frac{dV}{dt} = \text{constant} = \frac{V}{t} \qquad (12)$$

$$\therefore \qquad \frac{V}{t} = \frac{A^2(-\Delta P)}{r\mu\, vV} \qquad (13)$$

OR

$$\frac{t}{V} = \frac{r\mu\, vV}{A^2(-\Delta P)} \qquad (14)$$

Where (-ΔP) is directly proportional to V.

For a filtration at constant pressure difference

$$\therefore \qquad \frac{dV}{dt} = \frac{(-\Delta P)}{r\,\mu}\frac{A^2}{vV}$$

$$\int_0^V V\,dV = \frac{(-\Delta P)}{r\,\mu}\frac{A^2}{v}\int_0^t dt$$

$$\frac{V^2}{2} = \frac{(-\Delta P)}{r\,\mu}\frac{A^2}{v}\,t \qquad (15)$$

$$OR$$

$$\frac{t}{V} = \frac{r\ \mu\ v}{2(-\Delta P)A^2}V \qquad (16)$$

For constant pressure filtration, there is a linear relation between V^2 and t or t/V and V.

Note

The pressure difference, ($-\Delta$P), is initially increased until a certain limit. Suppose it takes time t_1 with vol of filtrate V_1 to reach this limit. Then the integration of eq. 15 becomes:

$$\frac{1}{2}(V^2 - V_1^2) = \frac{(-\Delta P)}{r\mu}\frac{A^2}{v}(t - t_1) \qquad (17)$$

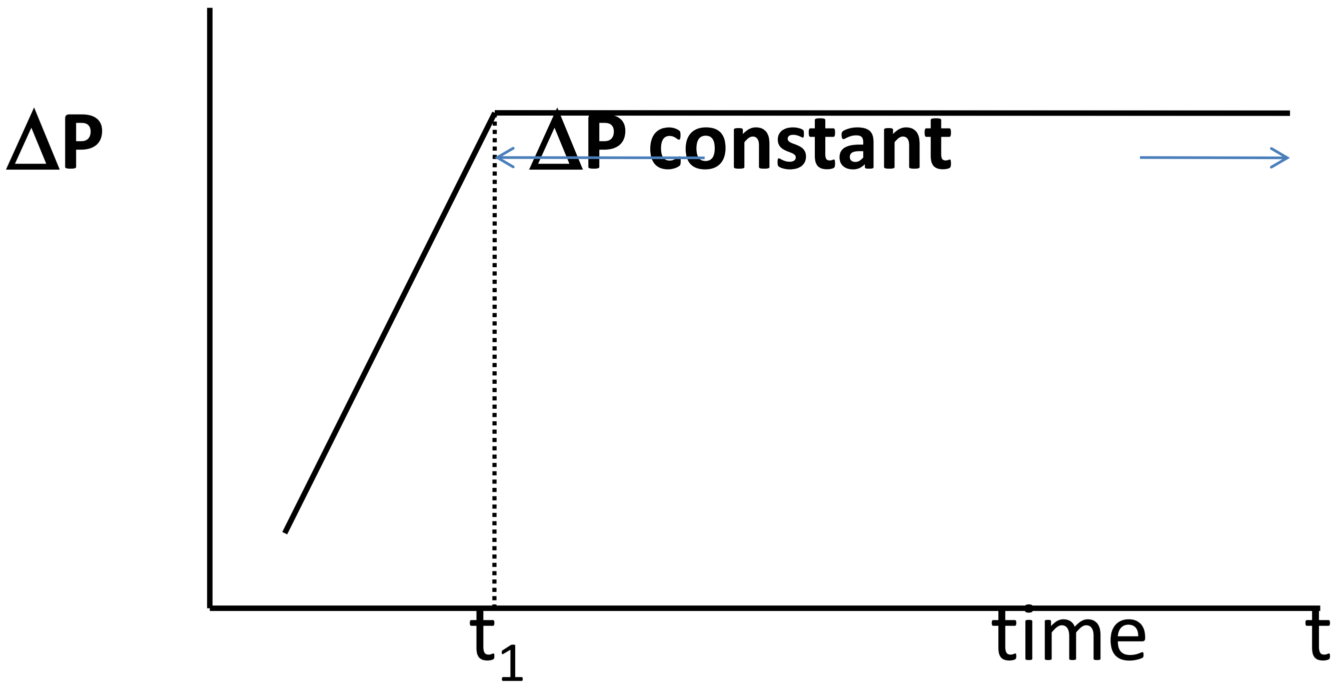
$$OR$$

$$\frac{t - t_1}{V - V_1} = \frac{r v \mu}{2A^2(-\Delta P)}(V - V_1) + \frac{r v \mu V_1}{A^2(-\Delta P)} \qquad (18)$$

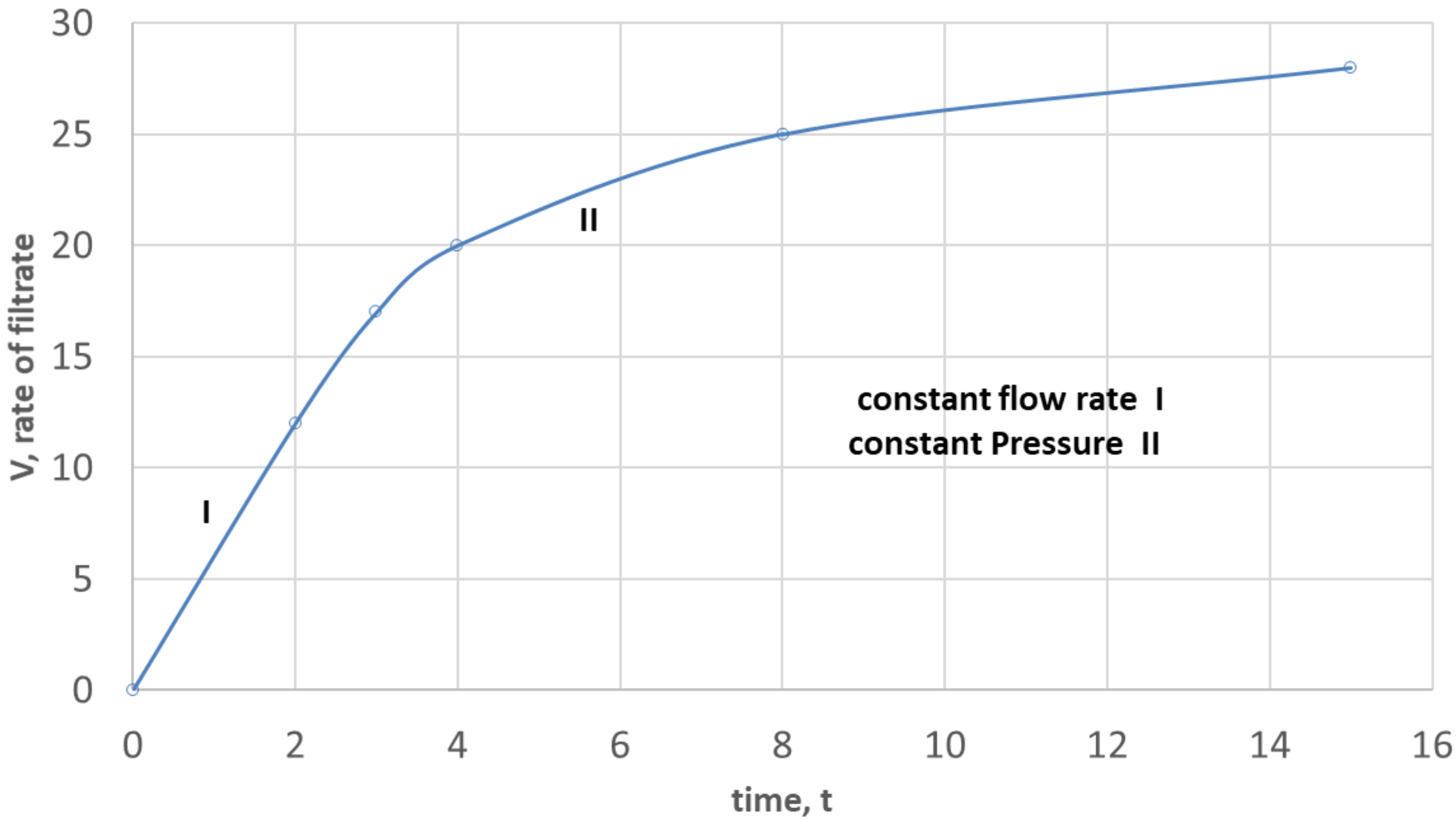
Where $t-t_1$: time of cons. Pressure filtration

$V-V_1$: volume of filtrate at cons. Pressure filtration

There is a linear relation between $(t-t_1)/(V-V_1)$ and $V-V_1$



2 Filtration stages I & II



Flow of liquid through the cloth

$$\therefore \quad \frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\,\mu\,l} \qquad (19)$$

There are two resistances;

- Resistance due to cloth and
- Resistance due to initial layer of cake

Assume the thickness of filter cloth and the initial layer of cake to be equivalent to thickness 'L' of cake as deposited at a later stage in the operation. Hence eq. 1 becomes:

$$\frac{1}{A}\frac{dV}{dt}=\frac{(-\Delta P)}{r\,\mu(l+L)} \qquad (20)$$

$where\ (-\Delta P)\ is\ the\ pressure\ drop\ across\ the\ cake\ plus\ the\ cloth.$ *Substituting eq.(8) into eq. (20)*

l=vV/A

$$\therefore\ \frac{dV}{dt}=\frac{A(-\Delta P)}{r\,\mu(\frac{Vv}{A}+L)}=\frac{A^2(-\Delta P)}{vr\mu(V+\frac{LA}{v})} \qquad (21)$$

<u>For constant rate filtration</u>

Equation 3 can be integrated.

$$\frac{dV}{dt} = \text{constant} = \frac{A^2(-\Delta P)}{vr\mu(V + \frac{LA}{v})}$$

$$\int_0^{V_1} dV = C \int_0^{t_1} dt \quad \Rightarrow \frac{V_1}{t_1} = C$$

OR

$$\frac{V_1}{t_1} = \frac{A^2(-\Delta P)}{vr\mu(V_1 + \frac{LA}{v})} \qquad (22)$$

OR

$$\frac{t_1}{V_1} = \frac{vr\mu}{A^2(-\Delta P)}(V_1 + \frac{LA}{v}) = \frac{vr\mu}{A^2(-\Delta P)}V_1 + \frac{r\mu L}{A(-\Delta P)} \qquad (23)$$

OR

$$V_1^2 + \frac{LA}{v}V_1 = \frac{A^2(-\Delta P)}{vr\mu}t_1 \qquad (24)$$

For constant pressure filtration

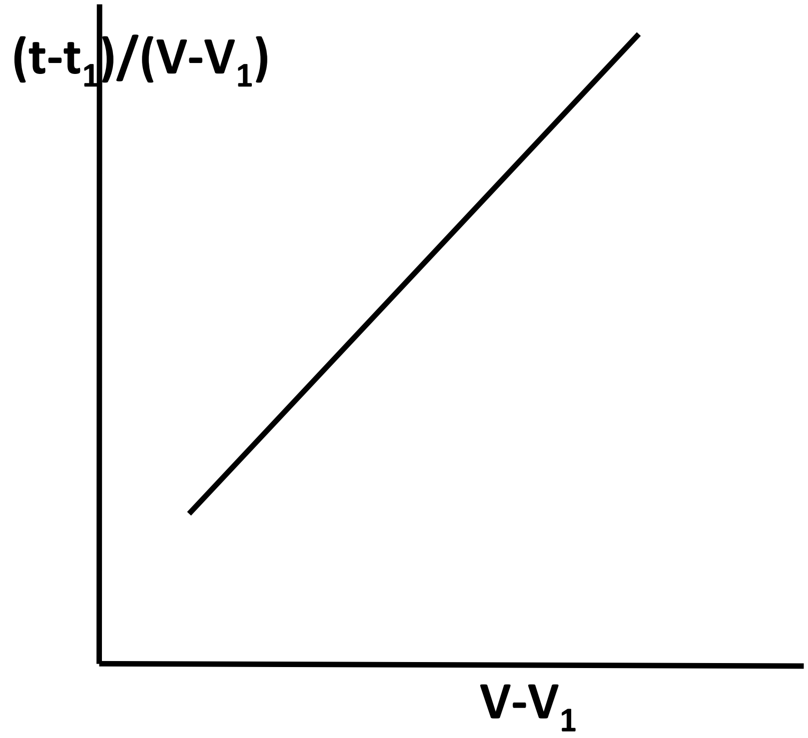
Integration between $t = t_1$, $V = V_1$ and $t = t$, $V = V$

$$\frac{1}{2}(V^2 - V_1^2) + \frac{LA}{v}(V - V_1) = \frac{A^2(-\Delta P)}{r\mu v}(t - t_1) \qquad (25)$$

OR

$$\frac{t - t_1}{V - V_1} = \frac{r\mu v}{2A^2(-\Delta P)}(V - V_1) + \frac{r\mu v}{A^2(-\Delta P)}V_1 + \frac{r\mu L}{A(-\Delta P)} \qquad (26)$$

Plot $(t-t_1)/(V-V_1)$ against $(V-V_1)$



<u>Note :</u>

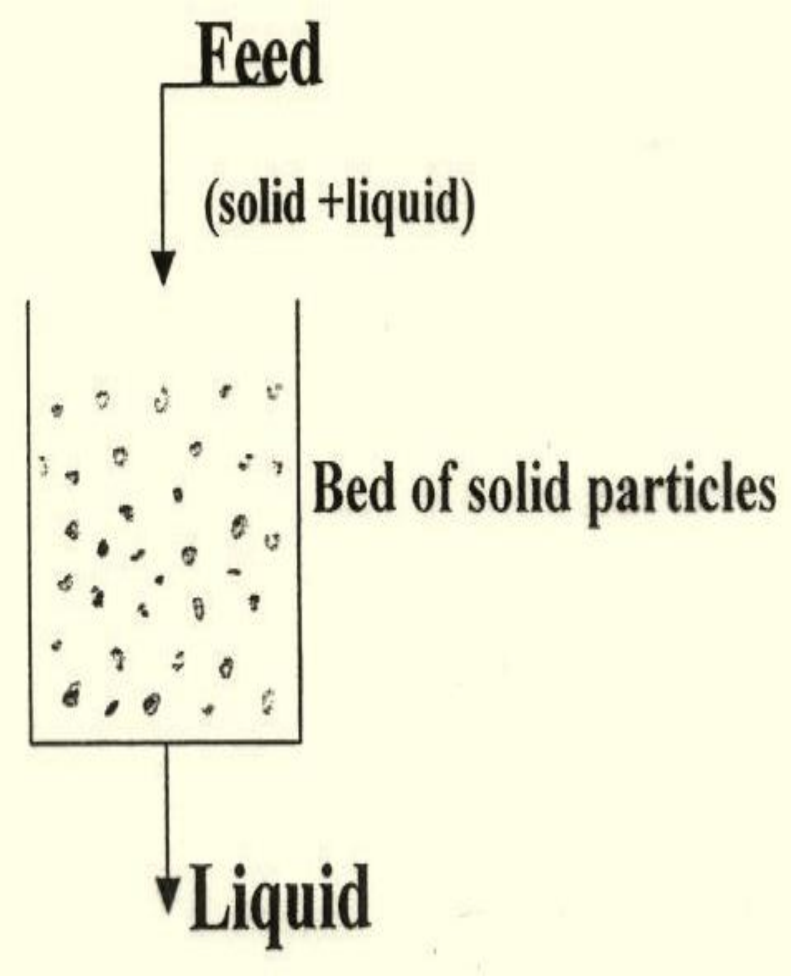
Slope gives sp. resistance r

Intercept yields L the equivalent thickness of cake

Filter types

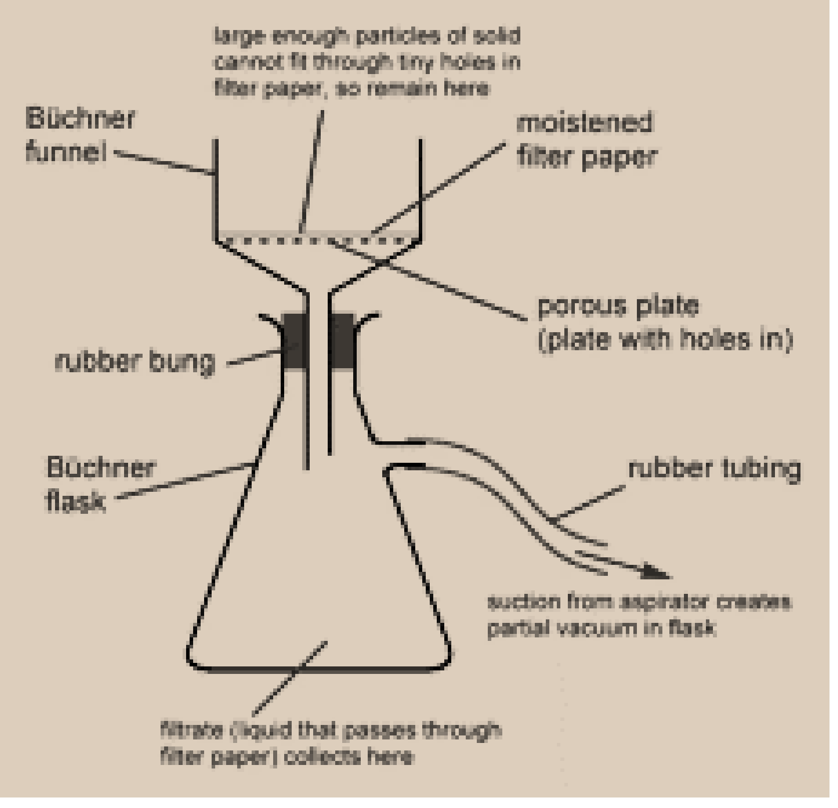
- **Laboratory test filter**

- **Bed filters**



Laboratory test filter

- Filter flask (suction flask, with sintered glass filter containing sample).



Bed filters

Applications**:** Granular bed filters, Sand filters

Purification of water; Waste water Treatment ~ Solid content:10g/m^3 or less

Specifications of bed**:** granular solids 0.6-1.2mm size, depth: 0.6-1.8m {not always}

The very fine particles of solids are removed by mechanical action although the particles finally adhere as a result of surface electric forces or adsorption.

Sand filter

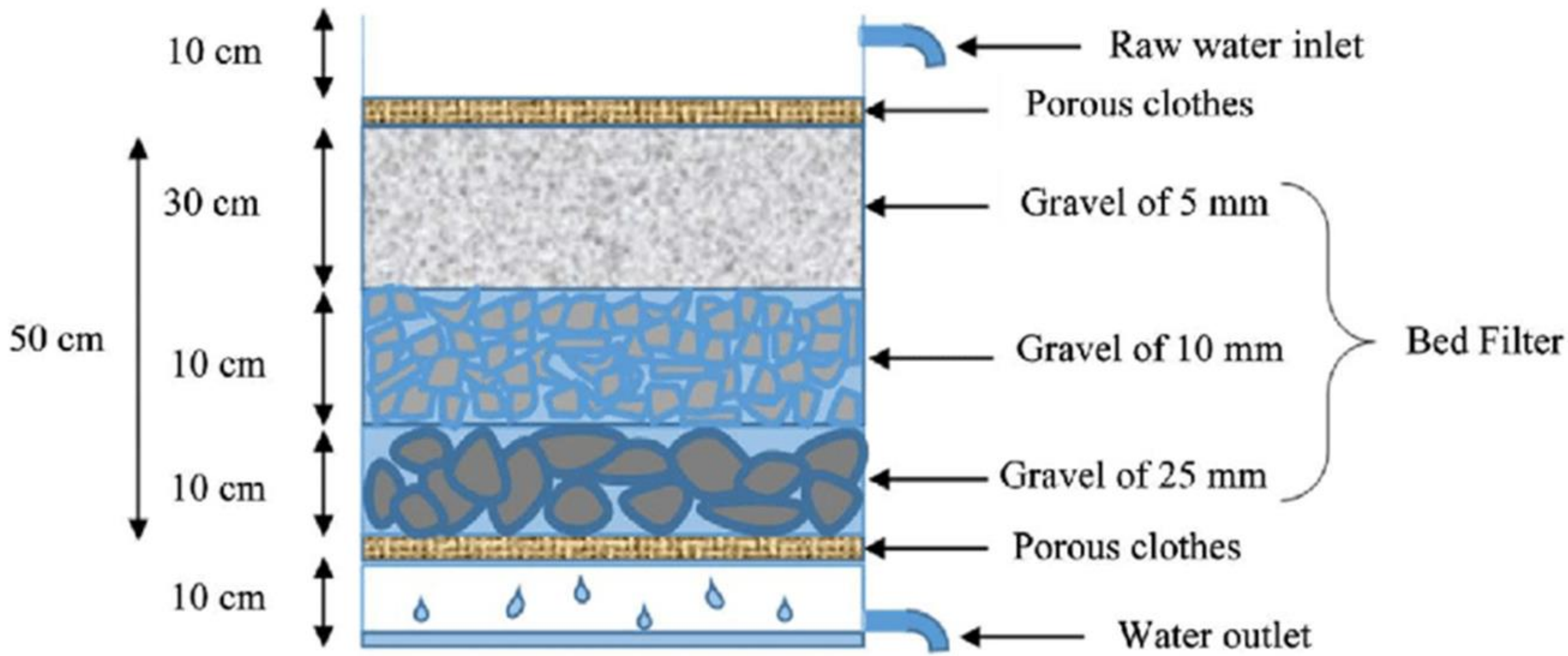
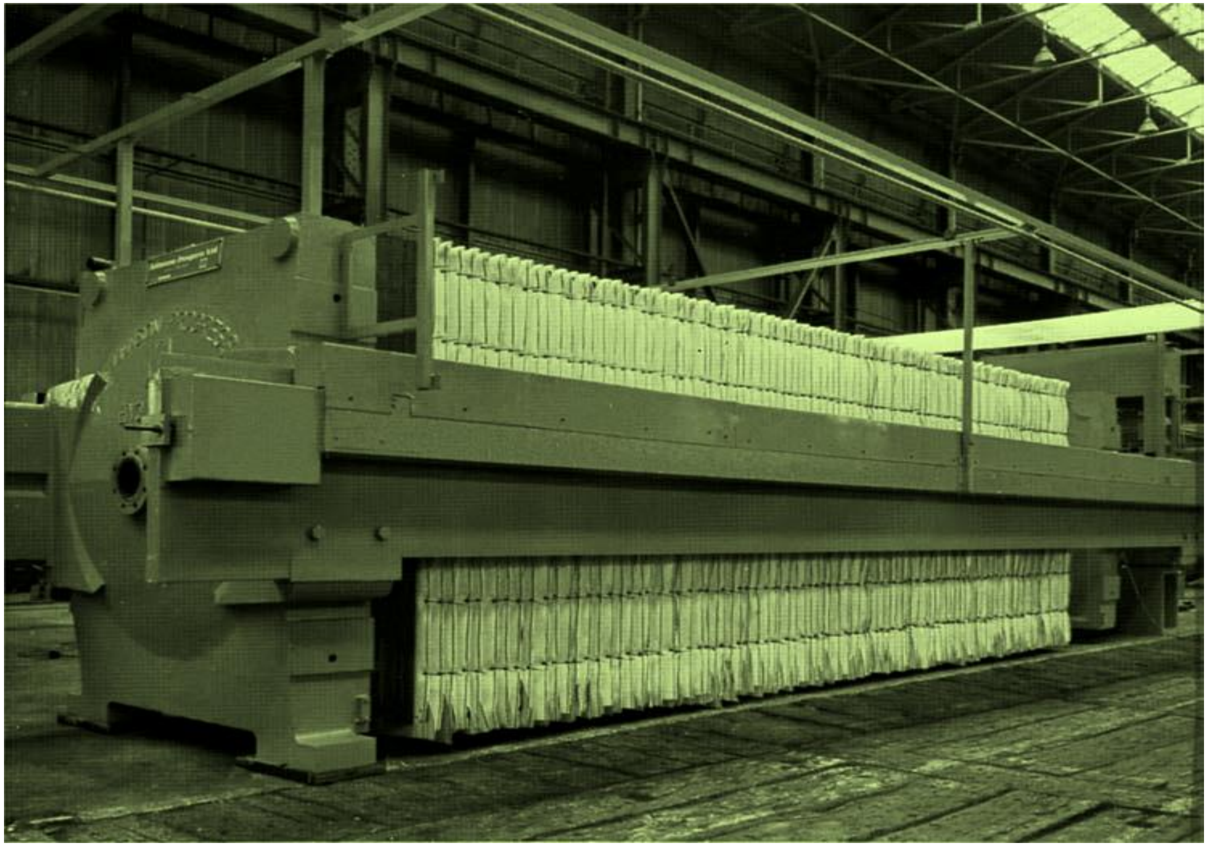

10 cm
Raw water inlet
Porous clothes
30 cm
Gravel of 5 mm
50 cm
10 cm
Gravel of 10 mm
Bed Filter
10 cm
Gravel of 25 mm
Porous clothes
10 cm
Water outlet

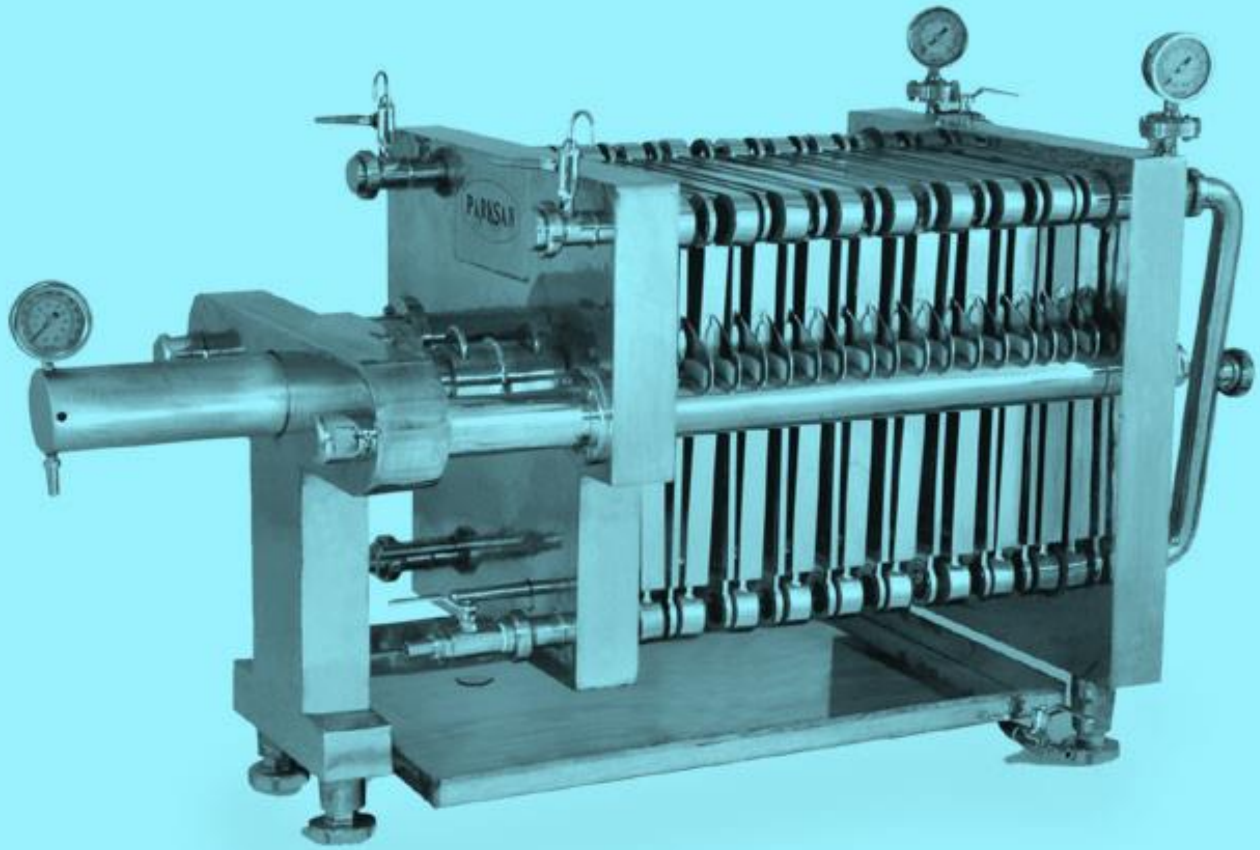
Plate and frame filter press

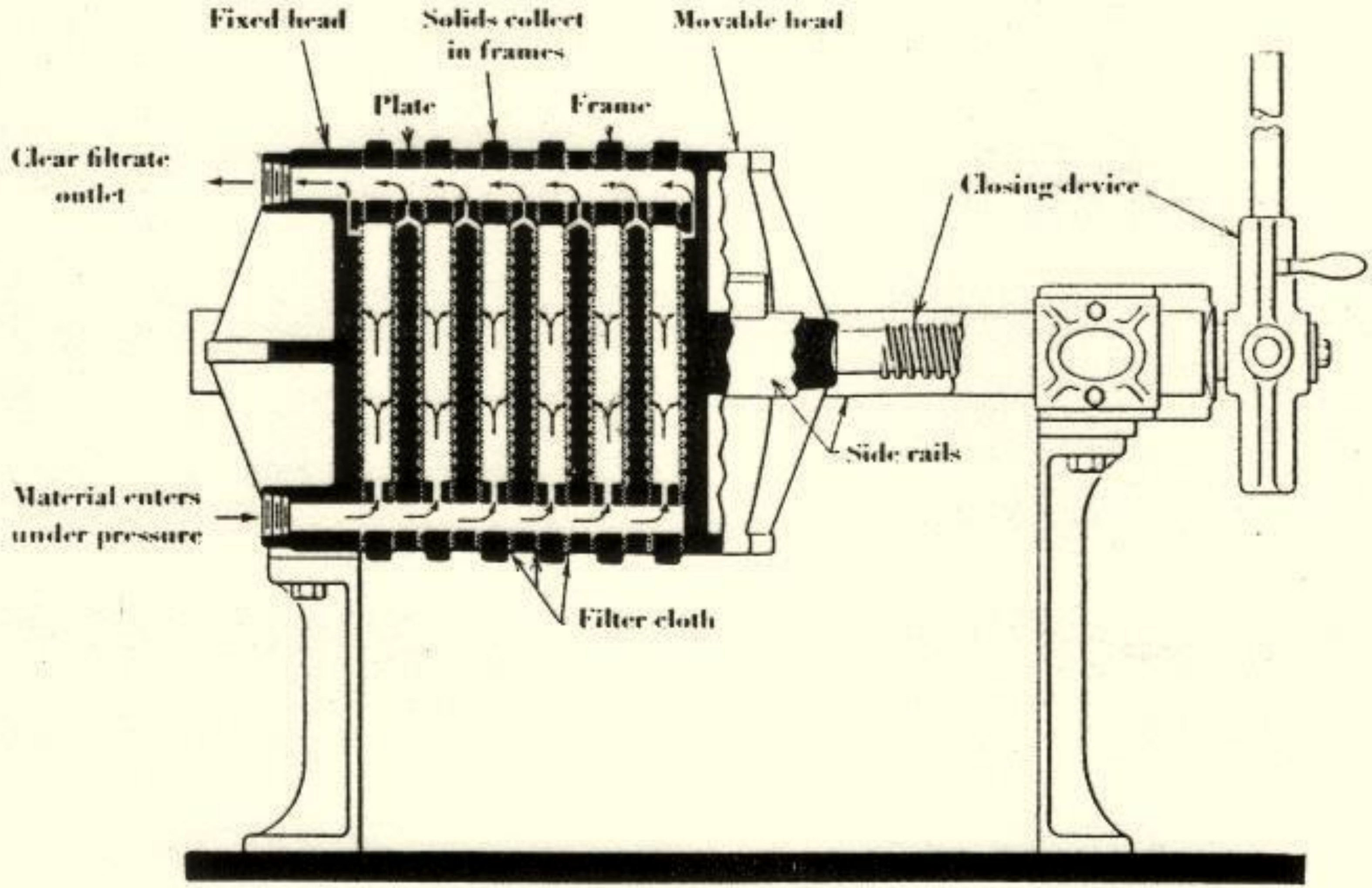
Here the plates, frames and cloths are pressed together as shown, and the entering slurry passes through lines leading to the frames.



SEPRA

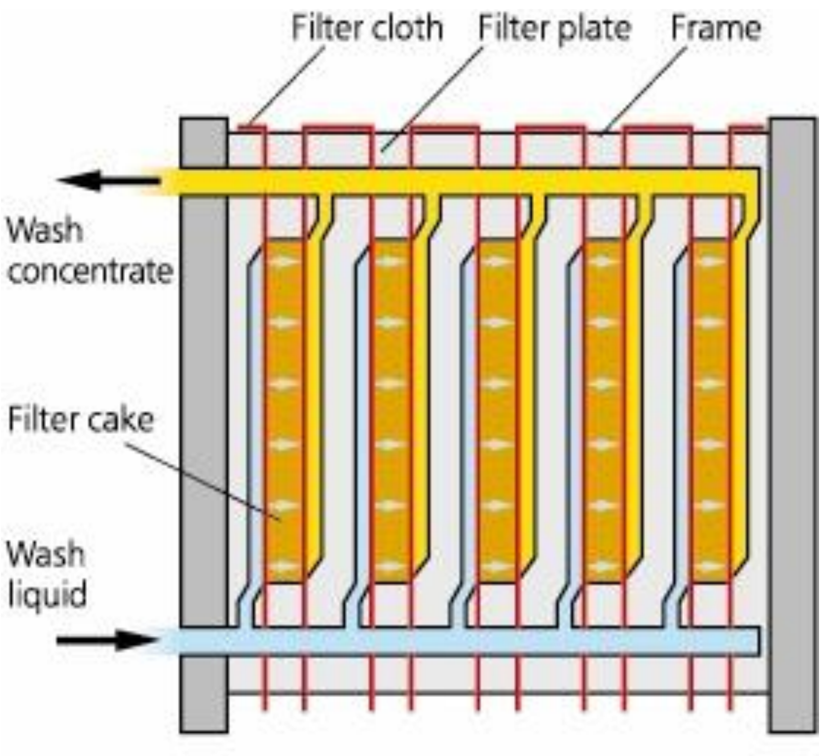
Plate and Frame Type Filter Press



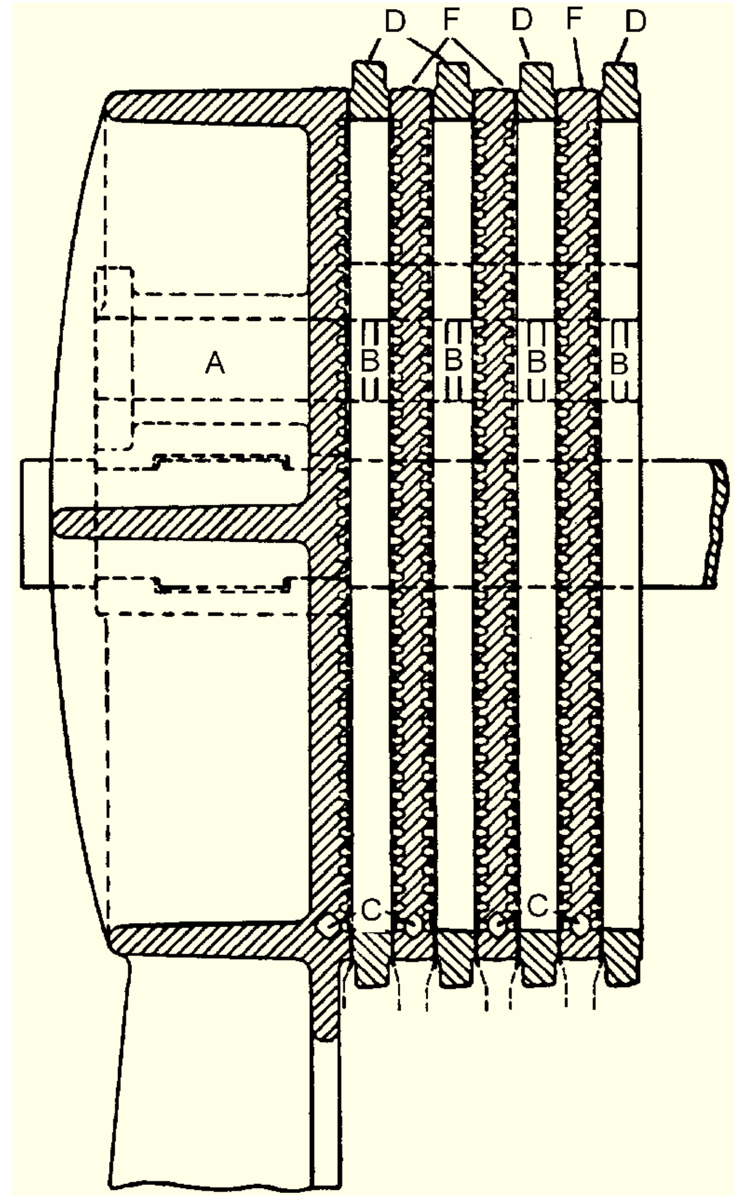


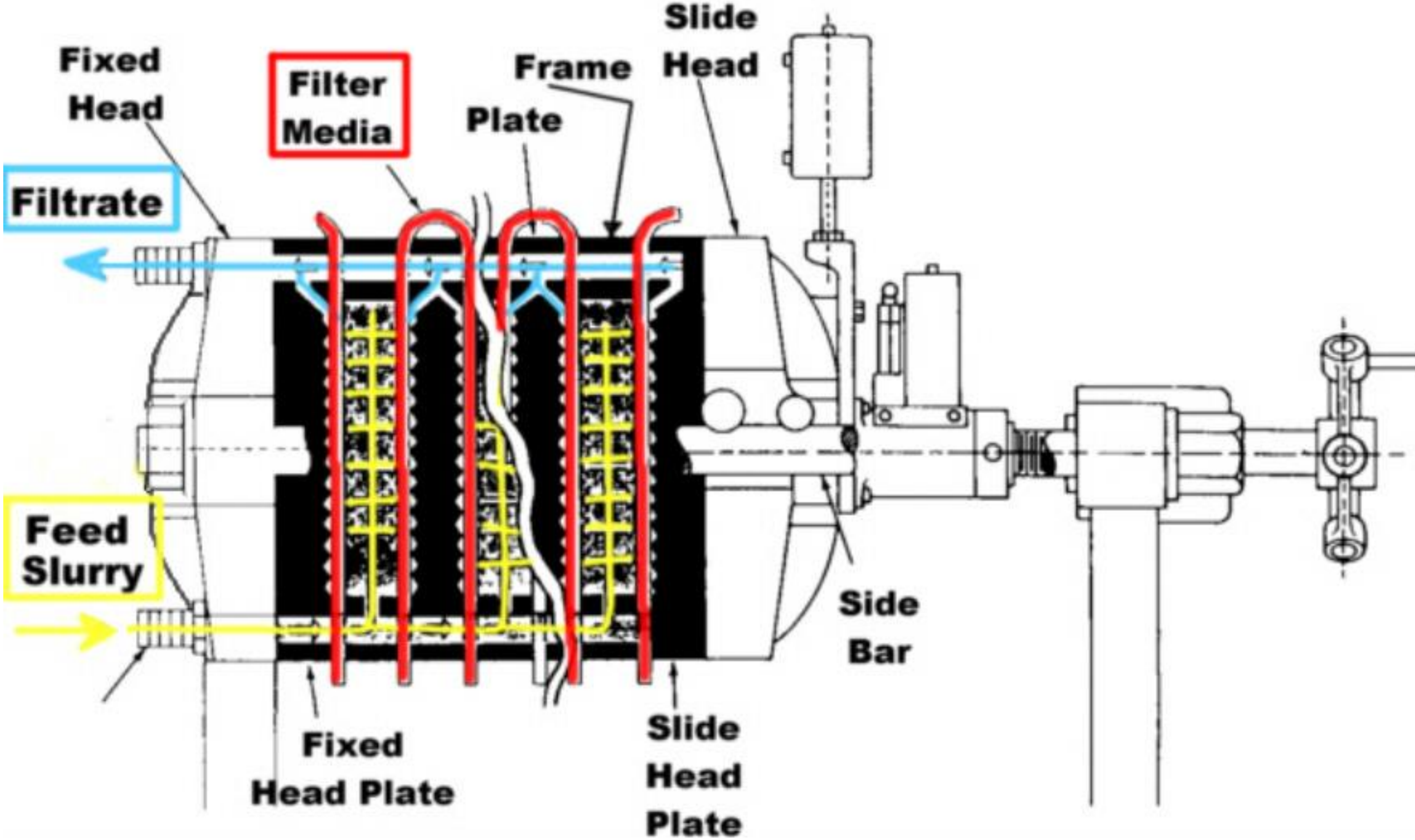
Fixed head
Solids collect in frames
Movable head
Plate
Frame
Clear filtrate outlet
Closing device
Side rails
Material enters under pressure
Filter cloth

Plate and frame press. A–inlet passage. B–feed ports. C–filtrate outlet. D–frames. F–plates

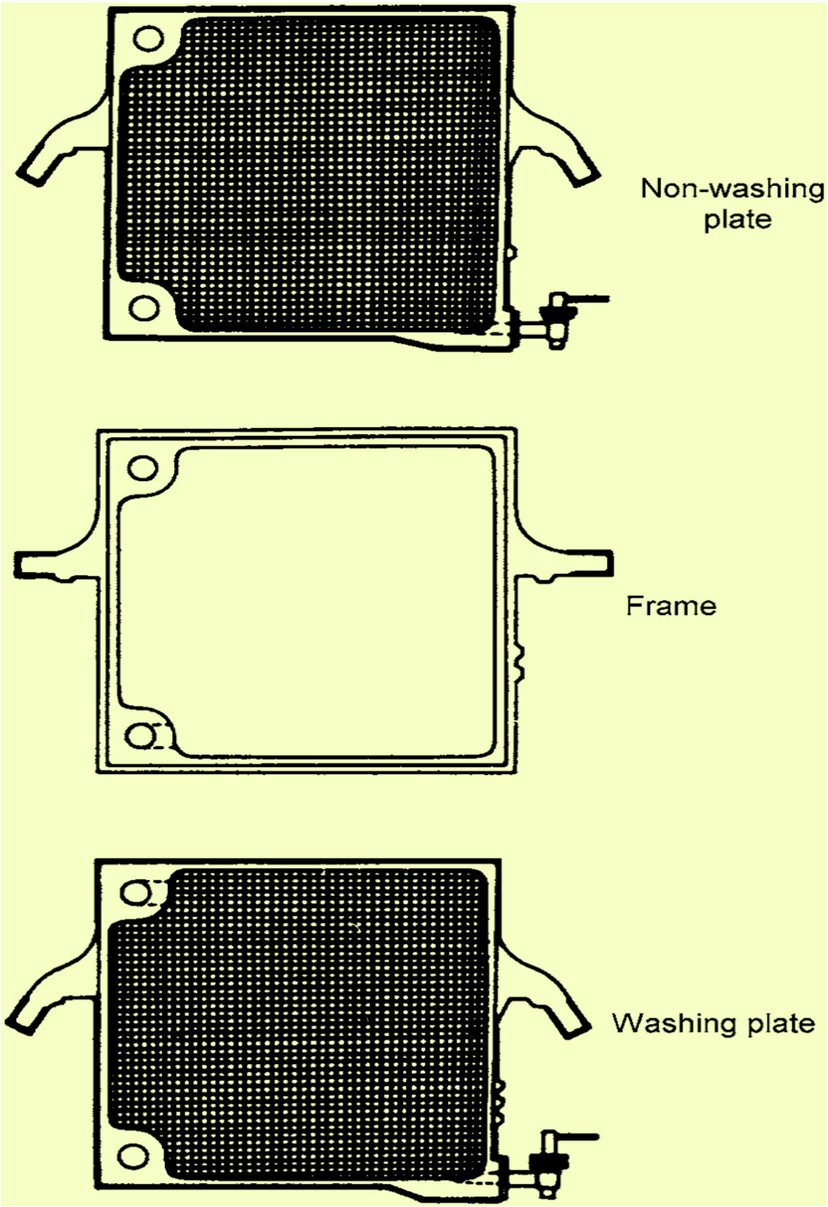


Filter cake washing in a plate and frame filter press

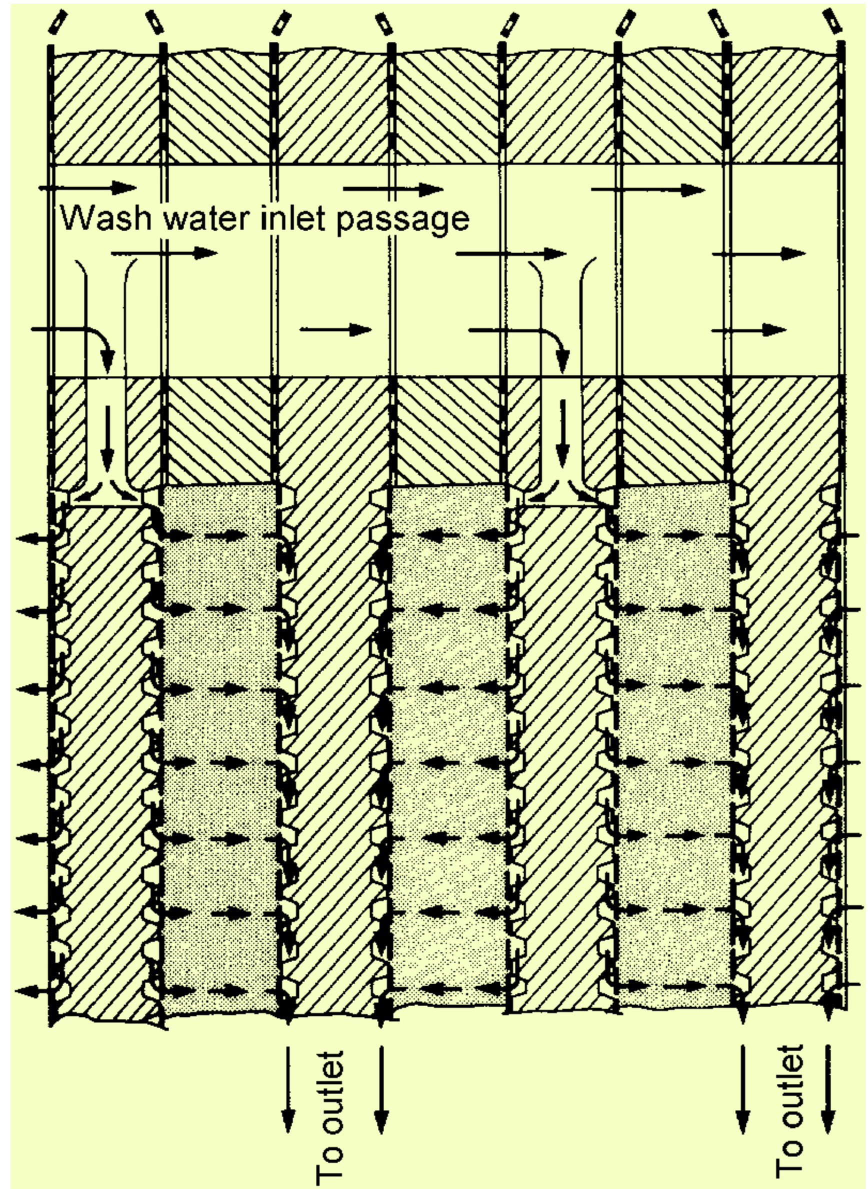



Slide
Head
Fixed
Head
Filter
Media
Frame
Plate
Filtrate
Feed
Slurry
Side
Bar
Fixed
Head Plate
Slide
Head
Plate

Plates and frames



Thorough washing



- **The cake is deposited against the filter cloths, which form the faces of the frames. The filtrate flows through the grooved channel on the faces of the plates and out through the openings at the tops of the plates.**
- **When the frames are filled with cake, washing can be done by introducing the washing liquid through washing plates. It flows through the entire cake thickness. The press is then opened, the solid removed, and another cycle of filtration started.**

Notes

This type of equipment suffers from the following disadvantages:

- **Process ~ batch**
- **Cost of labor ~ high.**
- **Capacity ~ Low.**
- **Sometimes it requires large quantity of washing liquid.**

Advantages of the filter press

(a) Because of its basic simplicity the filter press is versatile and may be used for a wide range of materials under varying operating conditions of cake thickness and pressure.

(b) Maintenance cost is low.

(c) It provides a large filtering area on a small floor space and few additional associated units are needed.

(d) Most joints are external and leakage is easily detected.

Advantages of the filter press

(e) High pressure operation is usually possible.

(f) It is equally suitable whether the cake or the liquid is the main product.

<u>Optimum time cycle</u>

- *The optimum thickness of cake to be formed in a filter press* **<u>depends</u>** **on** the resistance offered by the filter cake and **on** the time taken to dismantle and refit the press.

- Although the production of a thin filter cake results in a high average rate of filtration, it is **necessary to dismantle** the press more often and a **greater time** is therefore spent on this operation.
- For a filtration carried out entirely at constant pressure, a rearrangement equation 7.26 gives:

$$\frac{t}{V} = \frac{r\mu v}{2A^2(-\Delta P)} V + \frac{r\mu L}{A(-\Delta P)} \qquad (27)$$

$$= B_1 V + B_2 \qquad (28)$$

where B_1 *and* B_2 *are constants.*

Thus the time of filtration t *is given by:*

$$t = B_1 V^2 + B_2 V \qquad (10)$$

- The time of dismantling and assembling the press, say t', is substantially independent of the thickness of cake produced. The total time of a cycle in which a volume V *of filtrate* is collected is then $(t + t')$ and the overall rate of filtration, W, is given by:

$$W = V/\,(t + t')$$

$$W = V / (B_1 V^2 + B_2 V + t') \qquad (11)$$

W is a maximum when dW/dV = 0.

Differentiating *W with respect to V and equating to zero:*

$$B_1 V^2 + B_2 V + t' - V(2B_1 V + B_2) = 0$$

$$\therefore \; t' = B_1 V^2 \qquad (12)$$

$$\text{Or} \quad V = \sqrt{\frac{t'}{B_1}} \qquad (13)$$

Notes

- If the resistance of the filter medium is neglected, *$t = B_1V^2$ and the time during which* filtration is carried out is exactly equal to the time the press is out of service.
- In practice, in order to obtain the maximum overall rate of filtration, the filtration time must always be somewhat greater in order to allow for the resistance of the cloth, represented by the term B_2V.
- *In general, the lower the specific resistance of the cake, the greater will be the* economic thickness of the frame.

Washing Process

Washing Stages

- Displacement washing (90% removal of filtrate)
- Diffusion washing (washing material diffuses in the pores of the cake)

Drying Process

Air is used to dry the filter cake.

Note

The rate of washing is usually taken at the same pressure difference and the same final rate of filtration.

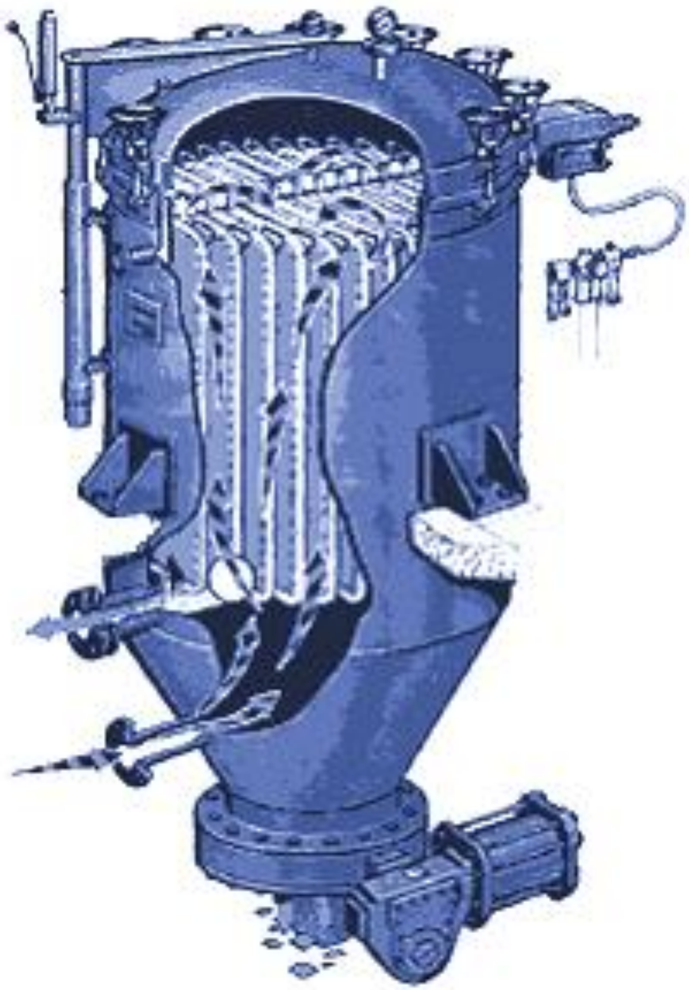
Methods of washing

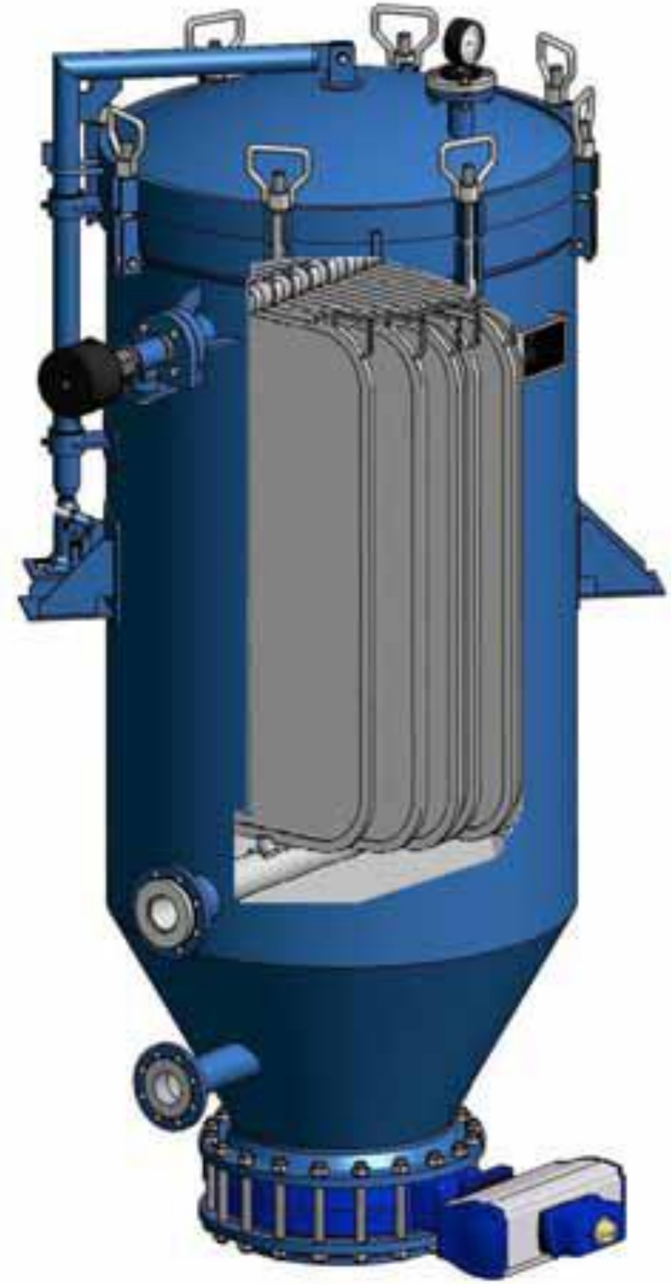
1) Simple washing : ‘the washing liquid passes through the same passage of slurry’.
2) Thorough washing : ‘washing liquid introduces through a specific plates called washing plates’. In general, the liquid passes through the whole thickness of the cake. Therefore, the area during washing is ½ the area during the filtration and the wash liquid flows through twice the thickness of cake so the washing rate is about ¼ of the final rate of the filtration. “***<u>See slide 41 and examples</u>***”

Leaf Filter

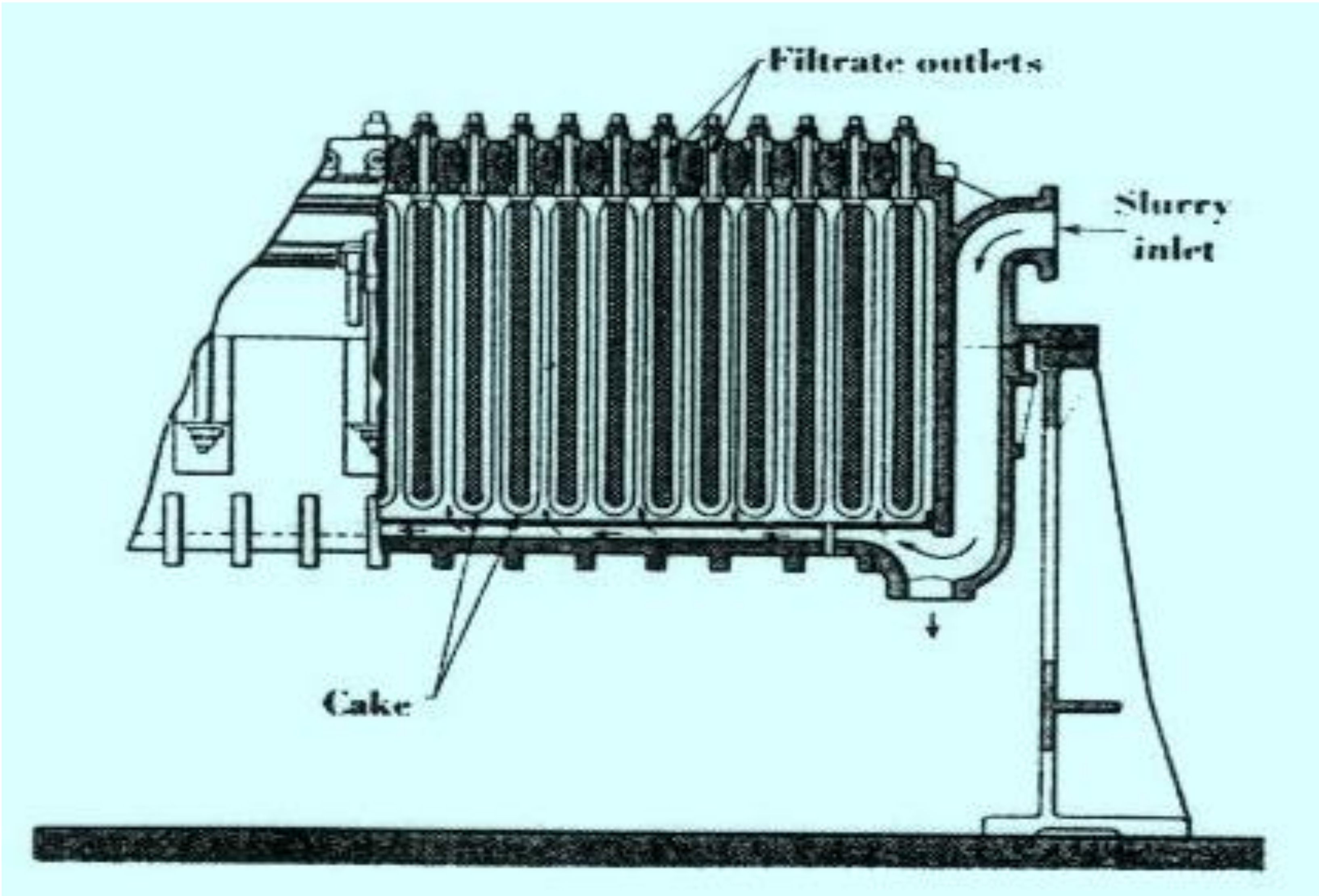
- This filter was developed for large volume of slurry and more efficient washing.
- Each leaf is a hollow wire framework covered by a sack (bag) of filter cloth.
- Shape: rectangular leaves
- ***Arranged longitudinally in side a cylindrical shell.***
- **The feed is pumped under pressure (>150psi) into a shell (it can be maintained by using a compressed air).**

Leaf filter

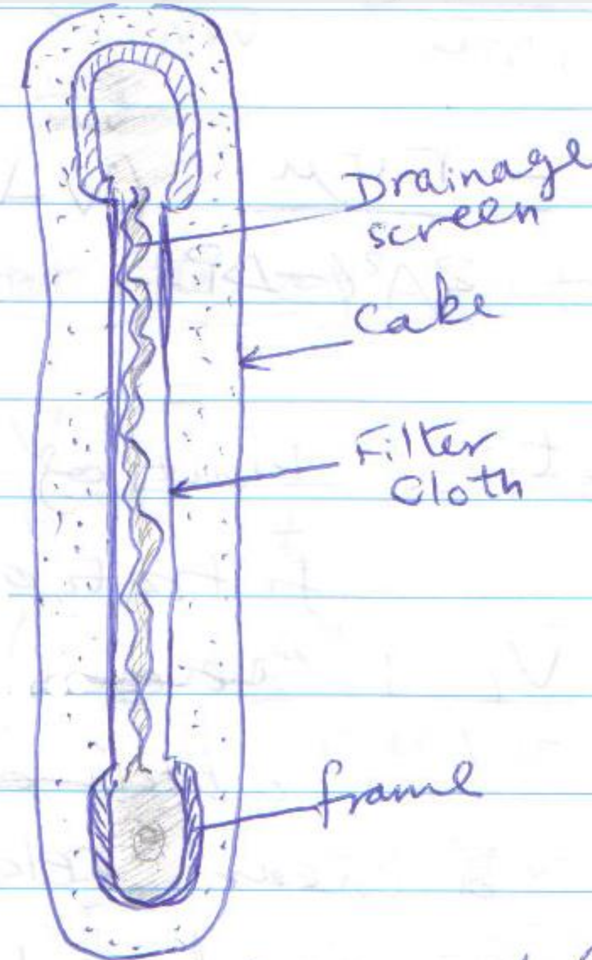




Leaf Filter



leaf Filters — vacuum operation eg Moore filter
— pressure operation eg Kelly filter



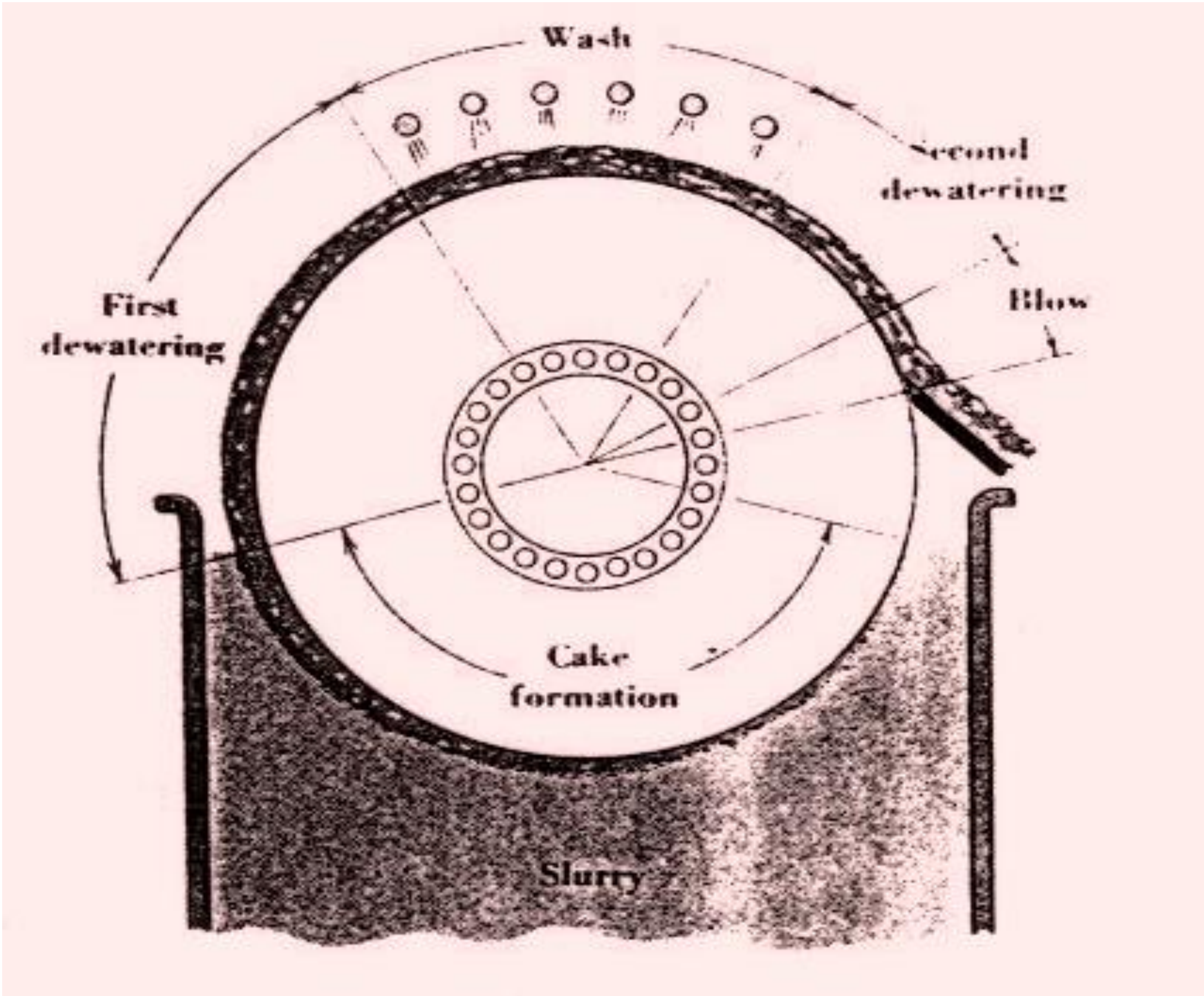
Filter leaf from a kelly filter

Leaf Filter

- The filtrate is removed from inside of the leaves and the cake forms on the outside.
- **The wash liquid follows the same path as the slurry.**
- When sufficient cake is deposited, the shell is opened and the cake removed.
- The filter suffers from the disadvantages common to batch operations.

A continuous rotary filter

- **The advantages of continuous operation are achieved by the use of a rotary vacuum filter of the type shown in the following figure.**



Wash
Second dewatering
Blow
First dewatering
Cake formation
Slurry

- **A vacuum is maintained on the interior of the rotating drum, which is covered on its curved surface by the filter medium.**
- **The drum dips into a tank of slurry, and the filtrate passes through the medium and removed through the axle of the filter.**
- **The cake is continuously washed and removed using a knife scraper.**

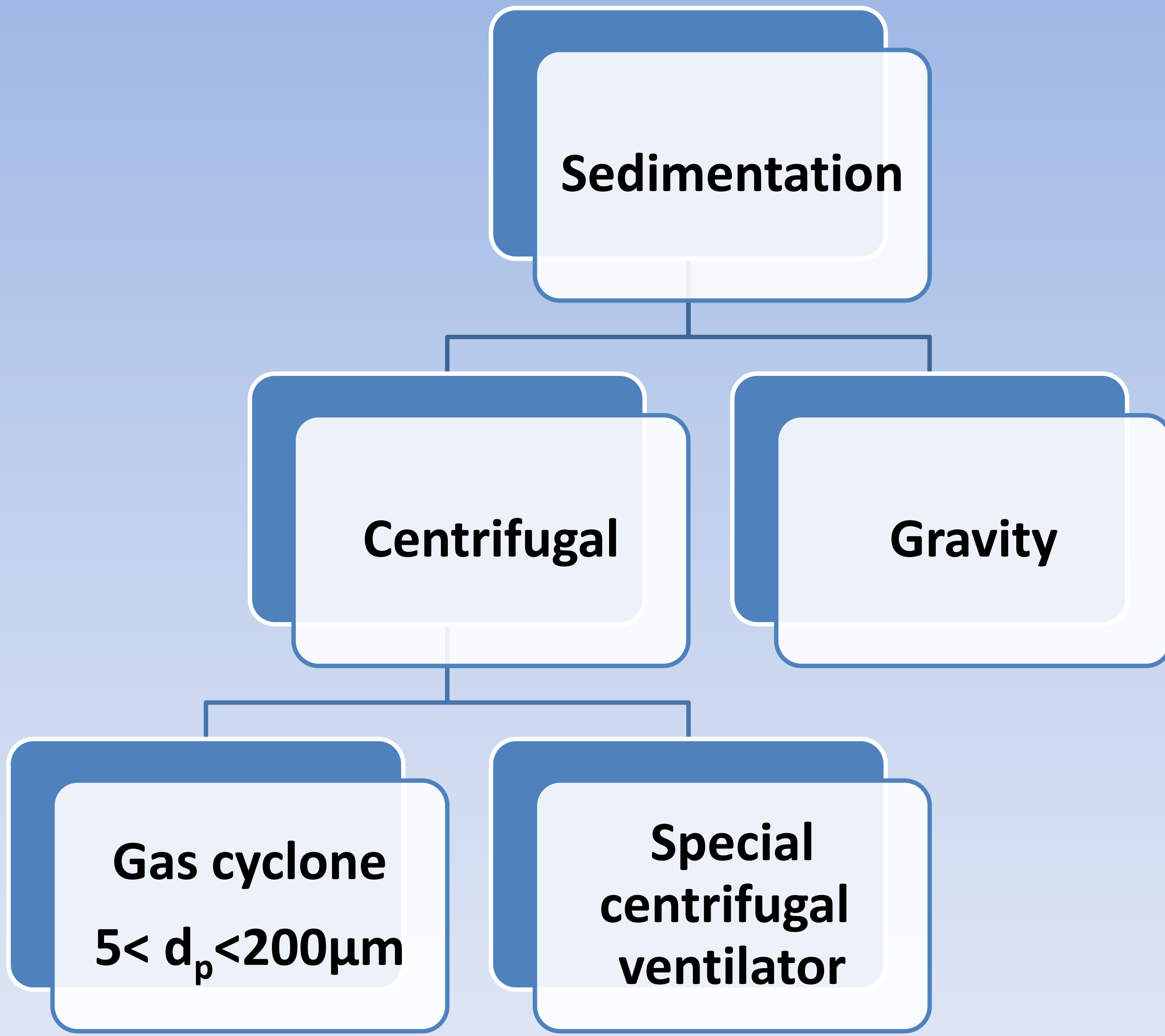
- The max. pressure differential for the vacuum filter is only 1 atm. Therefore, this type is not suitable for viscous liquids or for liquids which cannot be exposed to the atmosphere. These problems can be overcome by enclosing the filter in a shell, which is maintained above atmospheric pressure.

Gas-Solid Separation

Gas cyclones

Separation of solid from gas

- **Degassing**
- **dust separation**
  - Sedimentation
  - Collision
  - Filtration
  - Electrical
  - Combined methods


Sedimentation
Centrifugal
Gravity
Gas cyclone
5< d_p<200µm
Special centrifugal ventilator

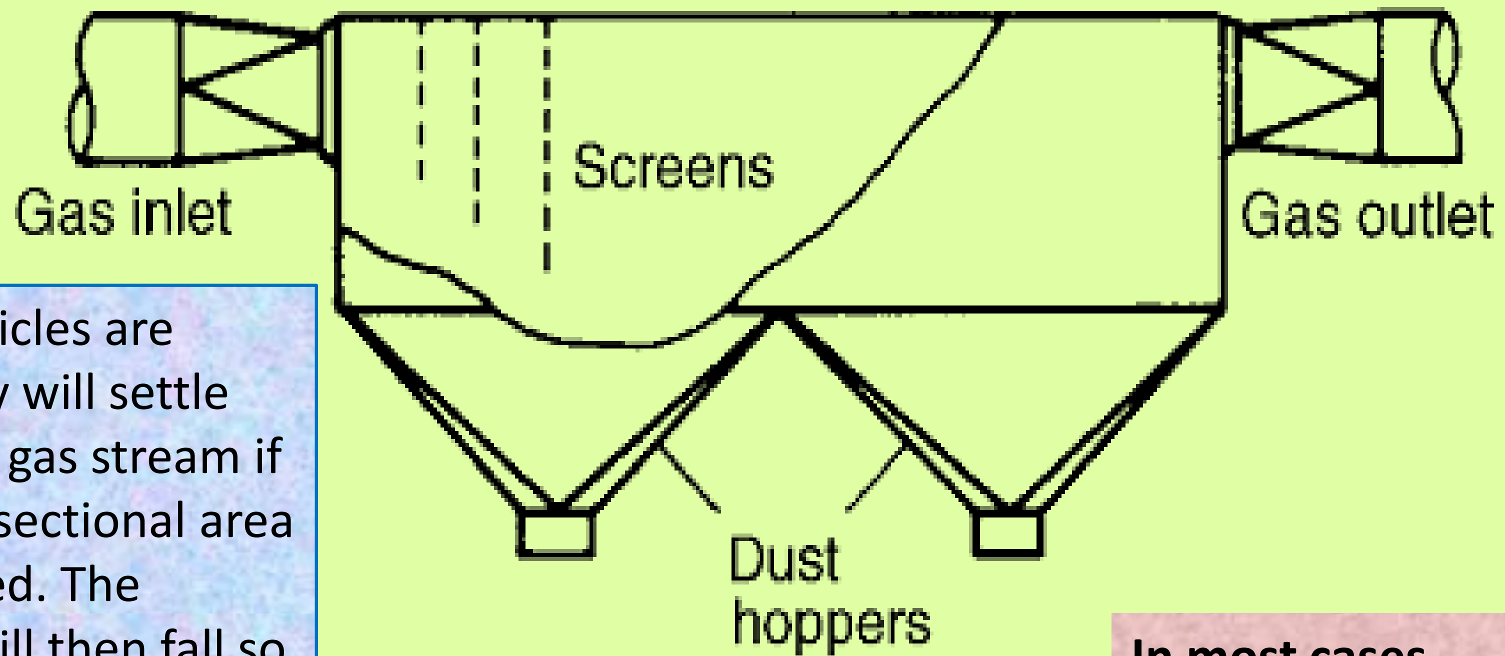
Note

- **It is recommended to separate the coarsest particles firstly with a simple device**

Settling by gravity
(Gravity separators)

- **Applicable for 100µm or more~ called dust separator**
- **Performance can be improved by inserting horizontal partitions.**
- **Disadvantage: separators required very large volume to decrease the gas velocity to low value to allow fine particles to settle.**

Settling chamber

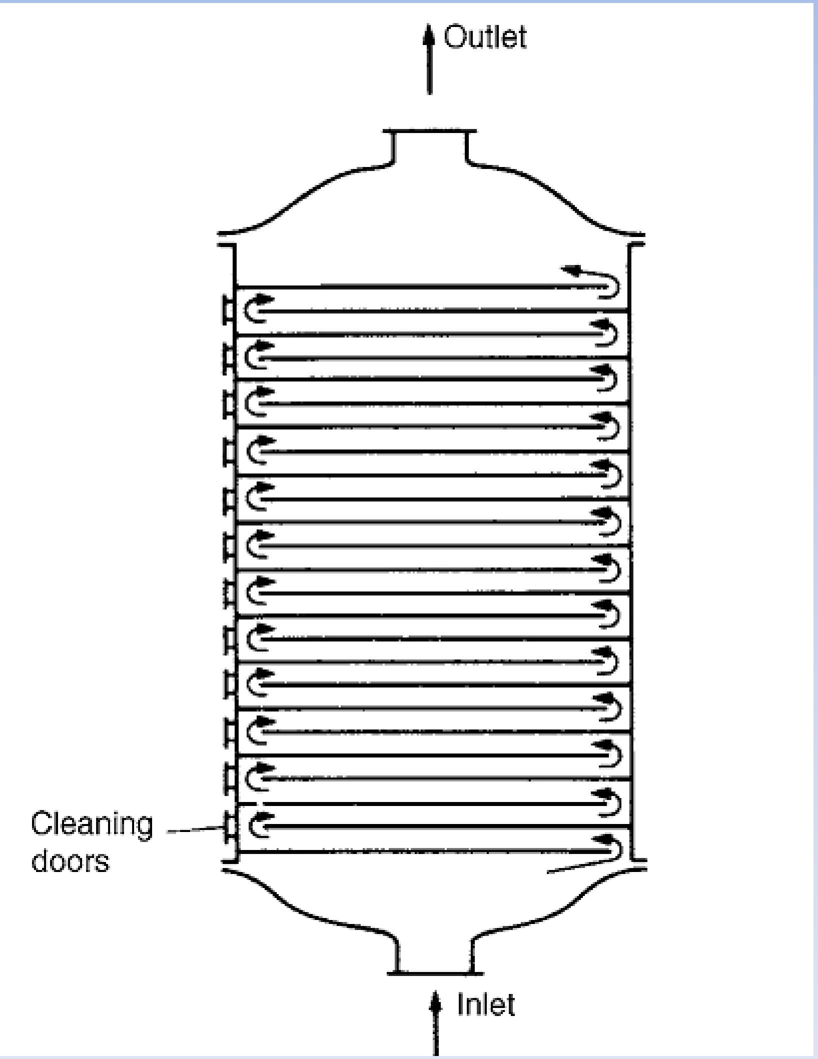


If the particles are large, they will settle out of the gas stream if the cross-sectional area is increased. The velocity will then fall so that the eddy currents which are maintaining the particles in suspension are suppressed.

In most cases baffles or screens are introduced to enhance particles separation.

Tray separator

❖Dust removal from sulphur dioxide produced by the combustion of pyrites.
❖Gas stream is forced through a series of trays in which the solid particles will settle over the trays.



CENTRIFUGE SEPARATION

How to create centrifuge separation

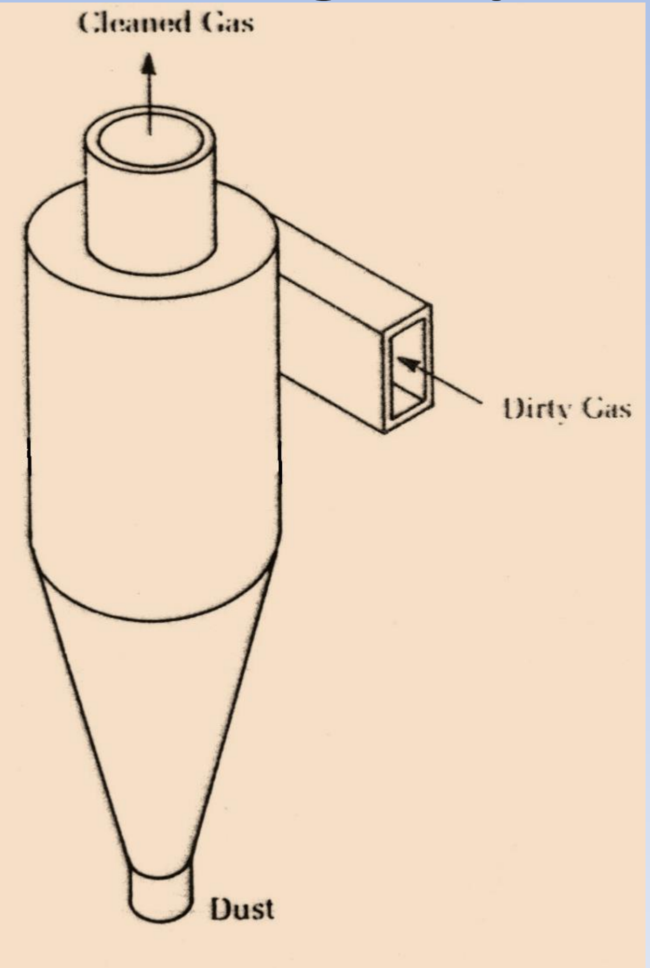
1. **Entering the gas tangentially with high velocity into a cylindrical space or**
2. **Bringing the gas into rotation via using a revolving impeller.**

NB the first way leads to the use of cyclones whilst the second to centrifugal ventilators of special construction.

Gas cyclones

The body of cyclone takes the following shape

- **Completely cylindrical**
- **Entirely conical**
- **Cylindrical + conical**



Note

The main principle of solid separation depends on the transformation of the linear gas velocity into vortex where the solid particles are thrown to the periphery of the cyclone while the gas ejected from the central region.

There are three openings:

1. An axial gas outlet.
2. A particulate discharge port.
3. A gas-solid inlet (usually rectangular in shape).

Sizing gas cyclones

Based on the theory given in chapter 3, there are two opposed forces:

1. **Centrifugal force ~ causes the particles to move in the direction of the cyclone wall; the gas flowing towards the core of the cyclone (*the gas tends to carry fine particles to the center and remove them through the overflow*).**

2. **Inward radial drag force ~ depends on the quantity of air fed to the volume of the cyclone. This ratio also determines the residence time in the cyclone.**

The equation of sedimentation in a centrifugal field developed in ch. 3 will be used. This eq. is:

$$\frac{dr}{dt} = \frac{d^2\omega^2(\rho_s - \rho)r}{18\mu} \qquad (1)$$

Where r is the instantaneous distance of the particle from the vertical axis of the cyclone. Multiplying both sides by r and ignore ρ; the density of gas because it is very small compared with the solid density ρ_s .

Hence eq. 1 becomes

$$r\frac{dr}{dt} = \frac{d^2(\omega r)^2 \rho_s}{18\mu} \qquad (2)$$

Now replacing *ωr term by its equivalent U_t, which is the tangential velocity of particle.*

$$r\frac{dr}{dt} = \frac{d^2 U_t^{\,2} \rho_s}{18\mu} \qquad (3)$$

Assume U_t is constant.

$$\int_{r_i}^{r_o} r\,dr = \frac{d^2 U_t^{\,2} \rho_s}{18\mu} \int_0^t dt \qquad (4)$$

$$\frac{r_o^{\,2} - r_i^{\,2}}{2} = \frac{d^2 U_t^{\,2} \rho_s t}{18\mu} \qquad (5)$$

where r_i : inner radius of rotating stream

r_o: radius of cylinder

Assume n is the number of turns required for a particle to travel the distance across the gas stream and so separate from it. The required time for these turns can be determined from angular velocity, ω, via this relation

$$t = \frac{2\pi n}{\omega} \qquad (6)$$

Eq. (6) can be written in the following form

$$t = \frac{2\pi n r_o}{(r_o\, \omega)} \qquad (7)$$

The denominator of this eq. represents the tangential velocity at its highest value when the particle reaches the wall of cyclone.

$$\therefore \qquad t = \frac{2\pi n r_o}{U_t} \qquad (8)$$

Substituting t into eq. (5)

$$r_o^{\ 2} - r_i^{\ 2} = \frac{2\pi d^2 U_t\, r_o\, n\, \rho_s}{9\mu} \qquad (9)$$

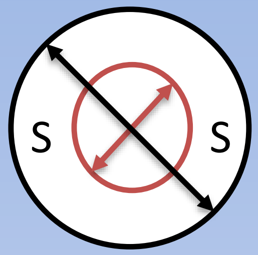
- **If the tangential velocity of the particle has a value similar to the average velocity of the gas stream, u, just before it enters the cyclone, then eq. 9 becomes**

$$r_o^{\,2} - r_i^{\,2} = \frac{2\pi d^2 u\, r_o\, n\, \rho_s}{9\mu} \qquad (10)$$

- **Let S is the width of the gas rotation; $S = r_o - r_i$ then**

$$r_0^{\,2} - r_i^{\,2} = (r_0 - r_i)(r_0 + r_i) = S(r_0 + r_i) \qquad (11)$$

- *Or in terms diameter*



$$r_o^{\,2} - r_i^{\,2} = S\,\frac{(D_o + D_i)}{2}$$

∴ Do – 2S =Di
Do + Di = Do +Do – 2S
=2Do – 2S = 2(Do-S)

but

$$D_o + D_i = 2(D_o - S)$$

$$\therefore \quad r_o^{\,2} - r_i^{\,2} = S\,(D_o - S) \qquad (12)$$

*Substituting for this eq, in eq.*10 *and taking* $r_o = D_o / 2$

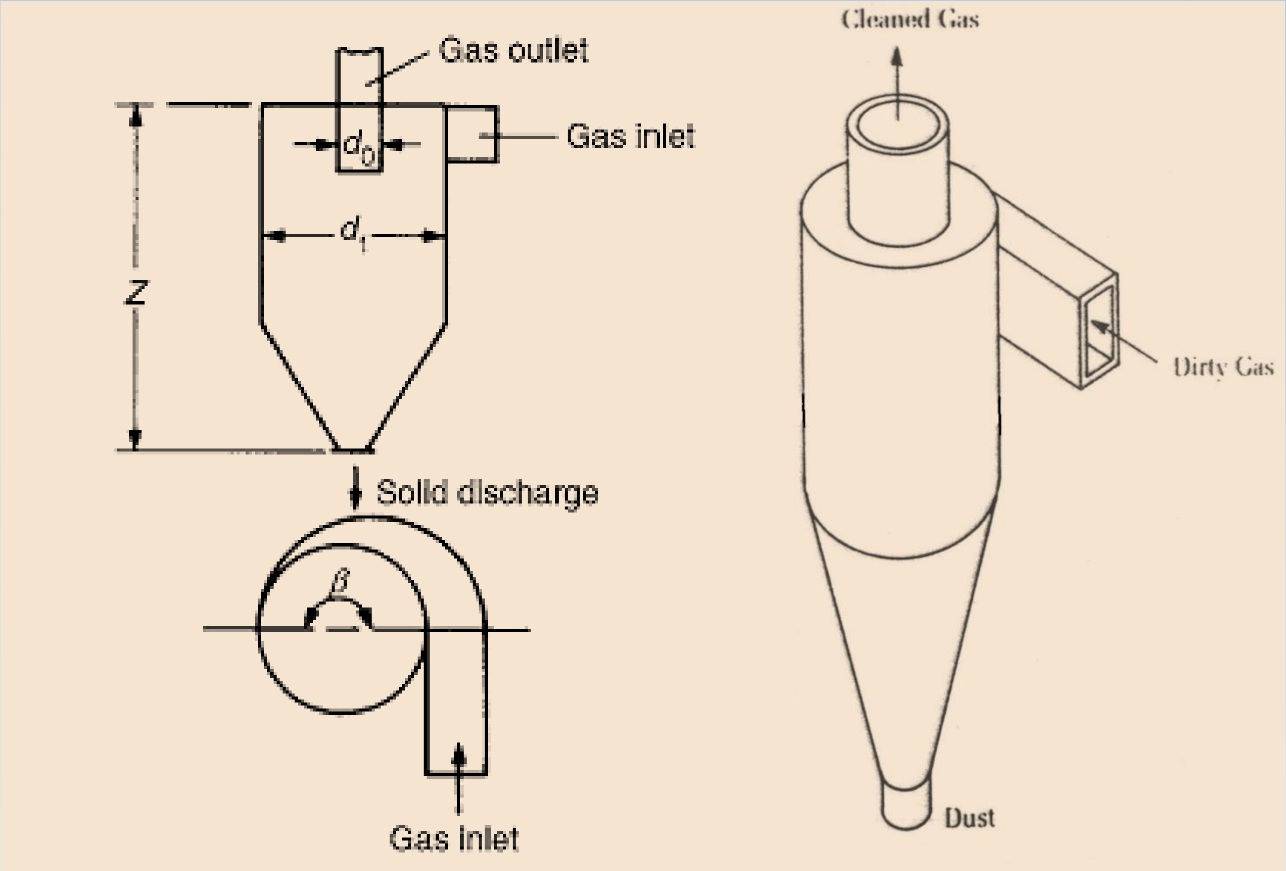
$$\therefore \quad S\,(D_o - S) = \frac{\pi\, d^2 u\, D_o\, n\, \rho_s}{9\,\mu} \qquad (13)$$

Note

Experimental studies have shown that the number of turns, n, which the gas stream makes before it leaves the cyclone range between 0.5 and 5.

Terminal velocity of a particle, fluid properties and the geometry of cyclone separator

Consider the two principal forces acting in a cyclone, namely the centrifugal force acting outwards and the frictional drag force of the gas acting inwards.



- The centrifugal force for a spherical particle rotating at radius r is:

$$\frac{m u_t^2}{r} = \frac{1}{6}\pi d^3 \rho_s \frac{u_t^2}{r} \qquad (14)$$

The inward radial force due to friction is

$$3\pi \mu \, \mathrm{d} \, \mathrm{u}_\mathrm{r}$$

where u_r is the radial component of the velocity of the gas.

At equilibriu m

$$\frac{\pi d^3 \rho_s u_t^2}{6 r} = 3\pi \mu d u_r \qquad (15)$$

Or

$$\frac{u_t^2}{r} = \frac{18 \mu u_r}{d^2 \rho_s} \qquad (16)$$

- **But the free falling velocity or terminal velocity, u_o, is ($\rho_s >> \rho$):**

$$u_o = \frac{d^2 g \rho_s}{18\mu} \qquad (17)$$

- Substituting for u_o in eq. (16)

$$\frac{u_t^2}{r} = \frac{u_r}{u_o} g$$

OR

$$u_o = \frac{u_r}{u_t^2} r g \qquad (18)$$

- **As u_o becomes higher, the radius of rotation becomes also greater.**
- **If it is assumed that a particle will be separated provided it tends to rotate outside the central core of diameter $0.4d_o$, the terminal falling velocity of the smallest particle which will be retained is found by substituting $r = 0.2d_o$ in equation (see figure; d_o diameter of gas core outlet)**

$$r = 0.2\, d_o \qquad (19)$$

- **Substituting in previous equation**

$$u_o = \frac{u_r}{u_t^{\,2}} 0.2 d_o\, g \qquad (20)$$

- **It is found that $u_r \approx$ constant at a given radius. However, it can be found from volumetric flow rate of gas per cylindrical surface area of flow at radius r:**

$$u_r = \frac{G}{2\pi\, r\, z\, \rho} \qquad (21)$$

- **u_t can be found from the following experimental relation:**

$$u_t = u_{to} \sqrt{\frac{d_t}{2\,r}} \qquad (22)$$

- where

  u_{to}: tangential velocity at circumference

  u_t : tangential velocity at radius r

  d_t: diameter of cyclone

- Moreover, it is found that u_{to}= the velocity of the gas which enters the cyclone.
- Now equation 20 becomes

$$u_o = \frac{0.2\,G\,d_o\,g}{\pi\,\rho\,z\,d_t\,u_{to}^{\ 2}} \qquad (23)$$

- If A_i is the cross-sectional area of the gas inlet
$G = A_i \rho u_{to} \Rightarrow u_{to} = G / A_i \rho$, by substitution

$$u_o = \frac{0.2 A_i^2 d_o \rho g}{\pi z d_t G} \qquad (24)$$

- See text for The effect of the arrangement and size of the gas inlet and outlet.
- If there is a large proportion of fine material present, a bag filter may be attached to the clean gas outlet.
- Alternatively, the smaller particles may be removed by means of a spray of water which is injected into the separator.
- For more details see the text.

Example

A cyclone separator, 0.3 m in diameter and 1.2 m long, has a circular inlet 75 mm in diameter and an outlet of the same size. If the gas enters at a velocity of 1.5 m/s, at what particle size will the theoretical cut occur?

The viscosity of air is 0.018 mN s/m^2, the density of air is 1.3 kg/m^3 and the density of the particles is 2700 kg/m^3.

Solution

Using the data provided:

cross-sectional area at the gas inlet, $A_i = (\pi/4)(0.075)^2 = 4.42 \times 10^{-3}\ \text{m}^2$

gas outlet diameter, $d_0 = 0.075$ m

gas density, $\rho = 1.30\ \text{kg/m}^3$

height of separator, $Z = 1.2$ m, separator diameter, $d_t = 0.3$ m.

Thus: mass flow of gas, $G = (1.5 \times 4.42 \times 10^{-3} \times 1.30) = 8.62 \times 10^{-3}$ kg/s

The terminal velocity of the smallest particle retained by the separator,

$$u_0 = 0.2{A_i}^2 d_0 \rho g/(\pi Z d_t G)$$

Eq. 24

$$u_0 = [0.2 \times (4.42 \times 10^{-3})^2 \times 0.075 \times 1.3 \times 9.81]/[\pi \times 1.2 \times 0.3 \times 8.62 \times 10^{-3}]$$

$$= 3.83 \times 10^{-4}\ \text{m/s}$$

- Use is now made of Stokes' law to find the particle diameter, as follows:

$$u_0 = d^2 g(\rho_s - \rho)/18\mu$$

$$d = [u_0 \times 18\mu/g(\rho_s - \rho)]^{0.5}$$

$$= [(3.83 \times 10^{-4} \times 18 \times 0.018 \times 10^{-3})/(9.81(2700 - 1.30))]^{0.5}$$

$$= 2.17 \times 10^{-6} \text{ m or } \underline{\underline{2.17\ \mu\text{m}}}$$