



Electrical

First semester 2025-2026

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Madar Team

Topic 9:- 3-phase circuit Analysis.

a) Single phase system

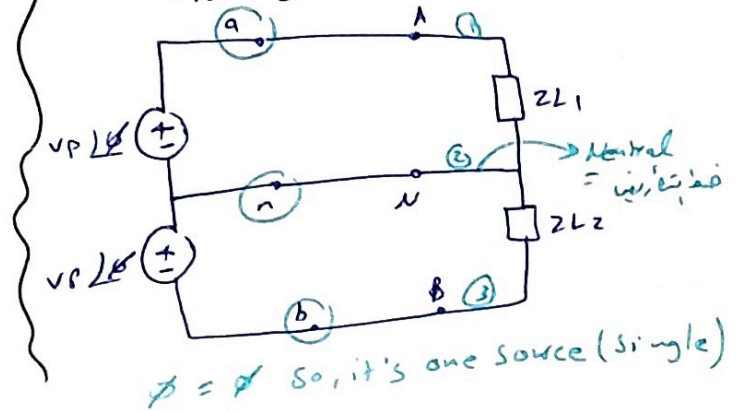


→ two-wire type

one source V_P .

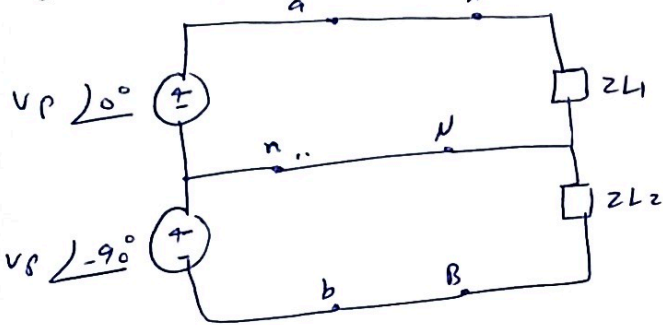
→ $V_P \cos(\omega t + \phi)$

b) Single phase system with 3 wire type.



$\phi = \phi$ So, it's one source (single)

c) Two phase three wire type.



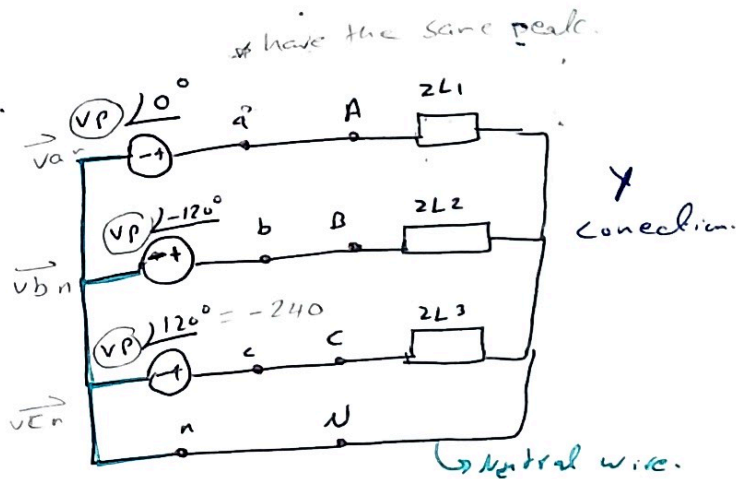
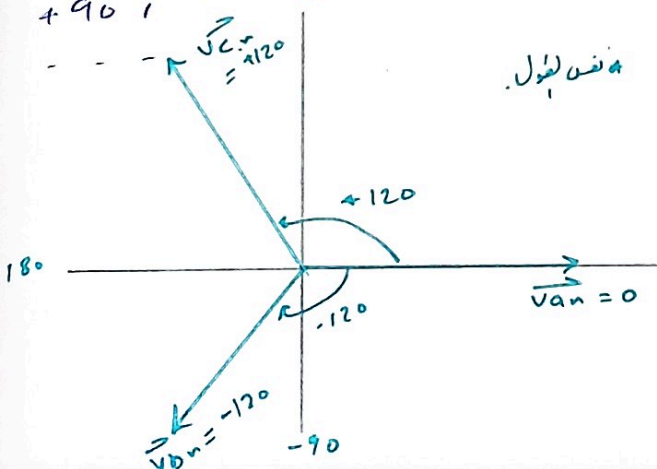
$\phi_1 = \phi_2 \rightarrow$ AC source operate at different phase.



→ Balanced Three-phase voltage.

Three-phase four-wire system.

الفولتية المتوازية لثلاث أطوار متساوية
 $+120^\circ, -120^\circ$
 $+240^\circ, -240^\circ$
 $+90^\circ, -90^\circ, 90^\circ$



$$v_{an}(t) = V_P \cos(\omega t)$$

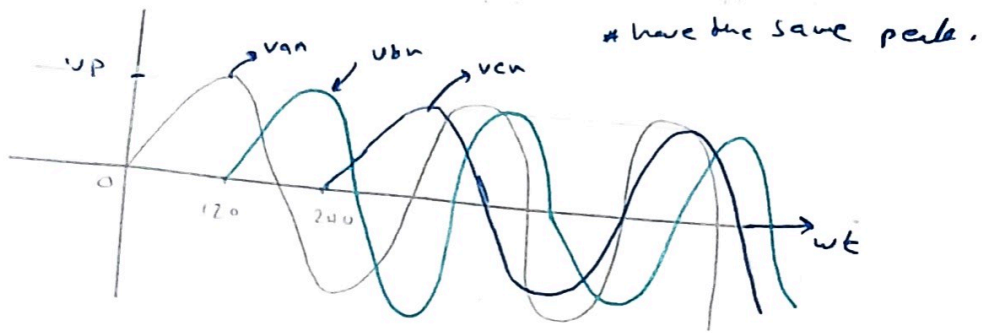
$$v_{bn}(t) = V_P \cos(\omega t - 120^\circ)$$

$$v_{cn}(t) = V_P \cos(\omega t - 240^\circ)$$

$$v_{an} = V_P \angle 0^\circ$$

$$v_{cn} = V_P \angle -240^\circ$$

$$v_{bn} = V_P \angle -120^\circ$$

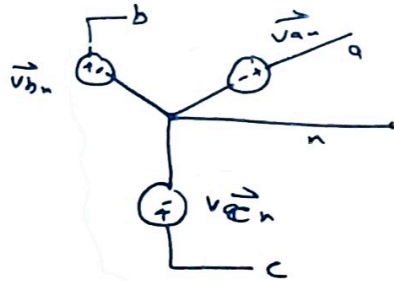


* V_{an} leads v_{bn} in 120°

* v_{bn} leads v_{cn} in 120°

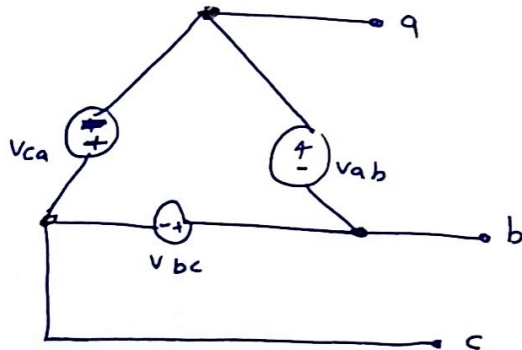
* V_{an} leads v_{cn} in 240°

* Y →
Connection



* I have neutral wire.

* Delta connection. (no neutral) * I don't have neutral wire.

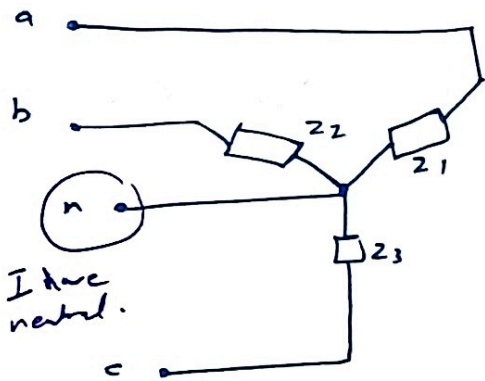


* connected between lines not nodes.

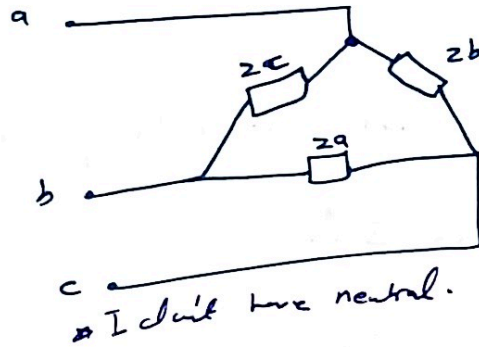
→ Balanced Three phase loads.

→ A balanced load has equal impedances on all three phases.

a) Wye-connection load.



b) Delta-connection load.



Balanced Impedance Conversions:-

* Conversion of Delta circuit to wye or wye to Delta.

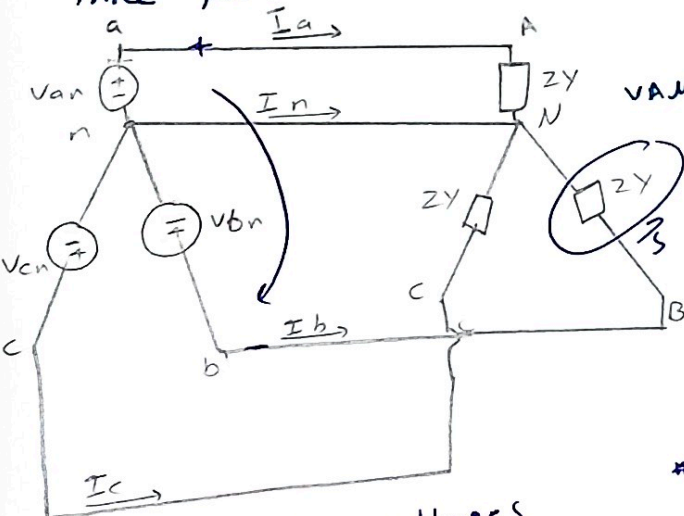
$$Z_y = Z_1 = Z_2 = Z_3$$

$$Z_\Delta = Z_a = Z_b = Z_c$$

$$Z_\Delta = 3 Z_y \quad , \quad Z_y = \frac{1}{3} Z_\Delta$$



Three phase connections Y-Y connection.



$$I_a \Rightarrow \frac{V_{an}}{Z_Y}$$

$$I_b \Rightarrow \frac{V_{bn}}{Z_Y} = \frac{V_{an} \angle -120^\circ}{Z_Y} = I_a \angle -120^\circ$$

$$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an} \angle -240^\circ}{Z_Y} = I_a \angle -240^\circ$$

* phase balance
و بفعل، باقی می ماند.

$$* I_n = I_{cd} = \sum I_a + I_b + I_c = 0$$

* line voltages

$$\rightarrow \vec{V}_{ab} = V_{an} - V_{bn}$$

$$V_{Line} = \sqrt{3} |V_{p}| = \text{voltage line. for } \phi = 0$$

$$* \text{ for any voltage } \Rightarrow \vec{V}_{ab} = \vec{V}_{an} (\sqrt{3} \angle 30^\circ) *$$

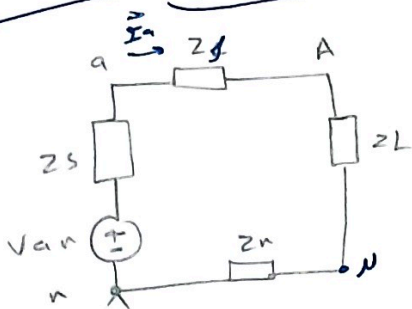
$$\vec{V}_{bc} = \vec{V}_{ab} (\angle 120^\circ)$$

$$\vec{V}_{ca} = \vec{V}_{ab} (\angle -240^\circ \text{ or } \angle 120^\circ)$$

So, when I have Y-Y connection.
I will calculate $\vec{V}_{an}$ & $\vec{I}_a$, $\vec{V}_{ab}$ first.

To find the complex power for $\square Z_Y$

$$\vec{S}_{\text{phase}} = \vec{V}_{an} \vec{I}_a^*$$



$$\rightarrow Z_Y = Z_s + Z_l + Z_n$$

$$\vec{I}_a = \frac{\vec{V}_{an}}{Z_Y}$$

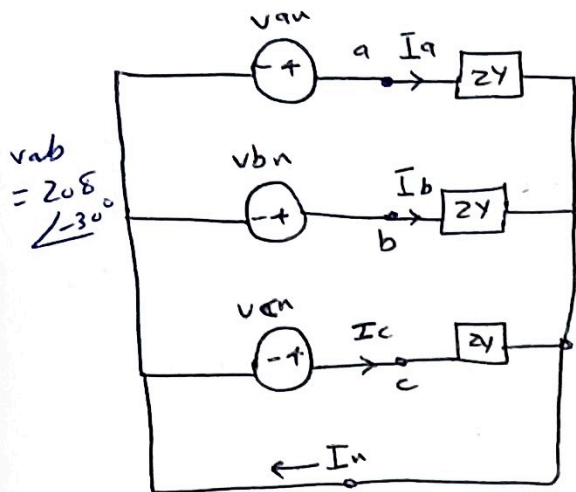
$$V_{AN} \neq V_{an}$$



∴ For an abc sequence, balance Y-Y three phase circuit.

$$V_{ab} = 208 \angle -30^\circ$$

Determine the phase voltages.



$$V_{an} \Rightarrow V_{ab} = V_{an} (\sqrt{3} \angle 30^\circ)$$

$$V_{an} = \frac{208 \angle -30^\circ}{\sqrt{3} \angle 30^\circ} = V_{an} = 120 \angle -60^\circ$$

$$V_{bn} = 120 \angle -60 - 120 = 120 \angle -180^\circ$$

$$V_{cn} = 120 \angle -60 + 120 = 120 \angle 60^\circ$$

$$V_{bc} = 208 \angle -30 - 120^\circ$$

$$V_{bc} = 208 \angle -150^\circ$$

$$V_{ca} = 208 \angle -30 - 240^\circ$$

$$V_{ca} = 208 \angle -270^\circ \text{ or } 208 \angle -30 + 120 = 208 \angle 90^\circ$$

*

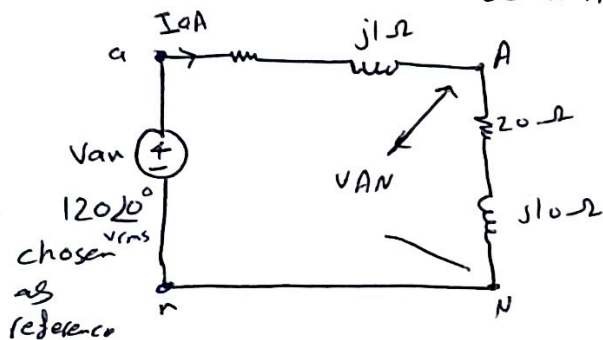


Example:-

For an abc sequence, balance Y-Y three phase circuit.

Source $|V_{\text{phase}}| = 120 \text{ (V)}_{\text{rms}}$, $Z_{\text{line}} = 1 + j1 \Omega$, $Z_{\text{phase}} = 20 + j10 \Omega$

Determine line currents and load voltages



one phase. $\angle 0^\circ, 120^\circ, 240^\circ$

$$I_{aA} = \frac{V_{an}}{Z_{\text{line}} + Z_{\text{phase}}} = \frac{120 \angle 0^\circ}{21 + j11} = \frac{120 \angle 0^\circ}{23.71 \angle 27.65^\circ}$$

$$= 5.06 \angle -27.65^\circ \text{ A (rms)}$$

line currents

$$I_{bB} = 5.06 \angle -27.65 - 120^\circ \text{ Arms}$$

$$I_{cC} = 5.06 \angle -27.65 + 120^\circ \text{ Arms}$$

$$\begin{bmatrix} V_{an} = 120 \angle 0^\circ \\ V_{bn} = 120 \angle -120^\circ \\ V_{cn} = 120 \angle 120^\circ \end{bmatrix}$$

phase voltages

$$\begin{bmatrix} V_{AN} = I_{aA} * (20 + j10) = 113.15 \angle -1.084^\circ \\ V_{BN} = 113.15 \angle -1.08 - 120 = 113.15 \angle -121.68^\circ \\ V_{CN} = 113.15 \angle -1.08 + 120 = 113.15 \angle 118.92^\circ \end{bmatrix}$$

load voltages

Ex:- For an abc sequence, balanced Y-Y three phase circuit.

$V_{an} = 120 \angle 90^\circ \text{ V (rms)}$. Find the line voltage.

if V_{ab} leads V_{an} by 30°

$$\vec{V}_{bn} = 120 \angle 90 - 120 = 120 \angle -30^\circ$$

$$\vec{V}_{cn} = 120 \angle 90 + 120 = 120 \angle 210^\circ \text{ or } 120 \angle -150^\circ$$

line voltages, V_{ab} , V_{bc} , V_{ca} .

$$V_{ab} = V_{an} * \sqrt{3} \angle 30^\circ$$

$$= 120 * \sqrt{3} \angle 30 = 207.8 \angle 120 \text{ vrms}$$

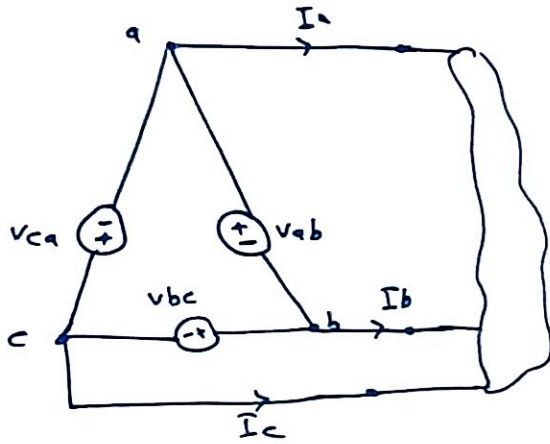
$$V_{bc} = V_{bn} * \sqrt{3} \angle 30^\circ = 120 * \sqrt{3} \angle -30 + 30 = 207.8 \angle 0^\circ \text{ vrms}$$

$$V_{ca} = V_{cn} * \sqrt{3} \angle 30^\circ = 120 * \sqrt{3} \angle 210 + 30 = 207.8 \angle -120^\circ$$

$$\text{or } 207.8 \angle 240^\circ$$



DELTA Connected Sources



Line voltages = phase voltages

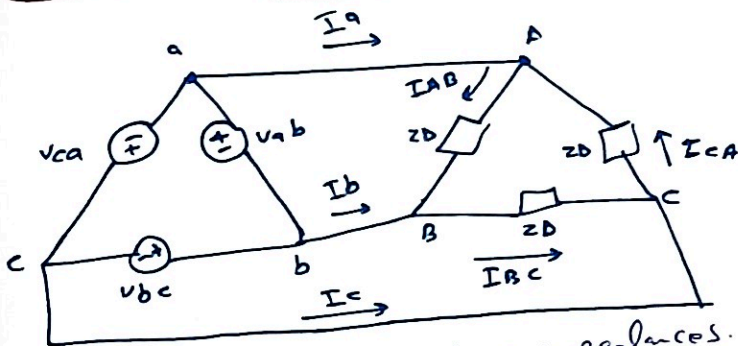
$$v_{ab} = V_L \angle 0^\circ$$

$$v_{bc} = V_L \angle 0 - 120^\circ$$

$$v_{ca} = V_L \angle 0 + 120^\circ$$

* In delta connected $\left\{ \begin{array}{l} \text{Voltage line} = \text{phase voltage} \\ \text{Current line} \neq \text{phase current} \end{array} \right.$

* In y connected $\left\{ \begin{array}{l} \text{Voltage line} \neq \text{phase voltage} \\ \text{Current line} = \text{phase current} \end{array} \right.$



assuming there is no line impedances.
 $v_{ab} = V_{AB}$, $v_{bc} = V_{BC}$, $v_{ca} = V_{CA}$.

→ line currents, $\vec{I}_a$, $\vec{I}_b$, $\vec{I}_c$

→ phase currents, $\vec{I}_{AB}$, $\vec{I}_{BC}$, $\vec{I}_{CA}$

* First write phase current then from it write line current.*

Line currents:-
 $\vec{I}_a = \vec{I}_{AB} - \vec{I}_{CA} = I_{AB} \sqrt{3} \angle -30^\circ$

$$I_b = I_{BC} - I_{AB} = I_a \angle -120^\circ$$

$$I_c = I_{AC} - I_{BC} = I_a \angle 120^\circ$$



phase current:-

$$I_{AB} = \frac{V_{AB}}{Z_D} = \frac{V_{ab}}{Z_D}$$

$$I_{BC} = \frac{V_{BC}}{Z_D} = \frac{V_{bc}}{Z_D} = I_{AB} \angle -120^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_D} = \frac{V_{ca}}{Z_D} = I_{AB} \angle +120^\circ$$

EX:- A balance Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected, positive-sequence generator having $V_{ab} = 330 \angle 0^\circ$ V. Calculate the phase currents of the load and the line currents. ~~vs~~ $I_a, I_b, I_c = ?$
 $I_{AB}, I_{BC}, I_{CA} = ?$

The load impedance per phase is

$$Z_D = 20 - j15 = 25 \angle -36.87^\circ \Omega$$

$V_{AB} = V_{ab}$ the phase current are.

$$I_{AB} = \frac{V_{AB}}{Z_D} = \frac{330 \angle 0^\circ}{25 \angle -36.87^\circ} = 13.2 \angle 36.87^\circ \text{ A.}$$

$$I_{BC} = I_{AB} \angle -120^\circ = 13.2 \angle -83.13^\circ \text{ A.}$$

$$I_{CA} = I_{AB} \angle +120^\circ = 13.2 \angle 156.87^\circ \text{ A.}$$

* For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current.

$$I_a = I_{AB} \sqrt{3} \angle -30^\circ = 13.2 \angle 36.87^\circ (\sqrt{3} \angle -30^\circ) = 22.86 \angle 6.87^\circ \text{ A.}$$

$$I_b = I_a \angle -120^\circ = 22.86 \angle -113.13^\circ \text{ A.}$$

$$I_c = I_a \angle +120^\circ = 22.86 \angle 126.87^\circ \text{ A.}$$



ch.1 units and power supply.

7. oct. 2025

* units :-

Time $\Rightarrow$ Sec.

Electrical current $\Rightarrow$ Amper (A)

Voltage $\Rightarrow$ Volt (V)

mass $\Rightarrow$ kg

length $\Rightarrow$ (m) or (cm)

power $\Rightarrow$ watt

charge $\Rightarrow$ C

* prefixes:-

pico $\rightarrow 10^{-12}$

nano $\rightarrow 10^{-9}$

Micro $\rightarrow 10^{-6}$

milli $\rightarrow 10^{-3}$

kilo $\rightarrow 10^3$

Mega $\rightarrow 10^6$

giga $\rightarrow 10^9$

* power supply :-

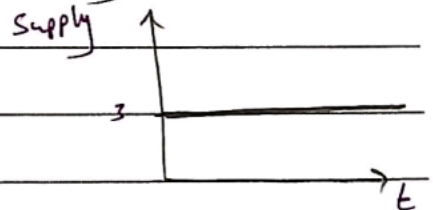
- $\rightarrow$ types $\Rightarrow$ 1) voltage source
2) current source



$\rightarrow$ output signal :-

- 1) D.C out put signal :- value of the supply is fixed with time.

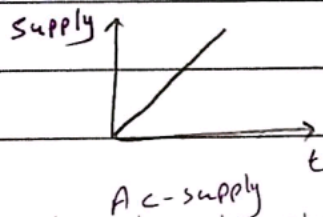
Ex:- Supply $V = 3$ volt or $I = 3$ A.



- 2) A.C out put signal :- its value vary with time.

Ex:- $V(t) = 3 \sin(t)$

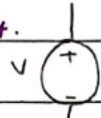
$I(t) = 2 e^{-t}$



c) Dependency :- الاعتمادية

1) independent power supply :- it's value don't depend on any element in circuit.

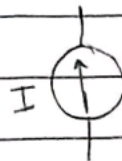
SymPol :- *



independent voltage

DC source.

*



independent DC current

Source.

لَمْ لَا تَفْتَقِدُ ثَابِتًا
لَا تَفْتَرِجُ لِنَفْسِكَ.



*



$v(t)$

independent AC voltage

Source.

*



independent AC

current Source

2) Dependent power supply :- it's value depend on or Function of voltage or current.

* قيمته تعتمد على فولت أو التيار موجود في الدائرة

* SymPol :-

* $I = 3 I_x$



dep. DC voltage

Source.

*



dep. DC current Source.

*



dep. AC voltage Source.

*



dep. AC current source

* Loads :- resistance R 
 inductive L 
 capacitive C 

* Charge :- Q

$$q_e = 1.6 \times 10^{-19} \text{ C}$$

* حركة الشحنات الكهربائية داخل موصل $\Rightarrow$ تيار كهربائي

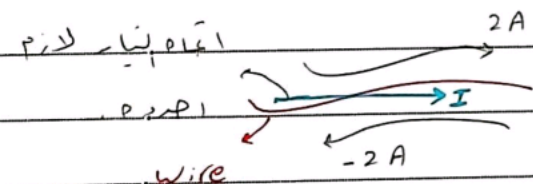
* Electrical current :- charges in motion in a wire for a closed loop.

* متى يتكون تيار كهربائي : (1) لازم يكون غشي حارة مغلقة (تيار بين نقطتين غير مغلقة لازم يرجع للبطارية)

(2) لازم يكون في موصل تتحرك فيه الشحنات في حارة مغلقة

$$I = \frac{dQ}{dt} \Rightarrow A$$

* التيارات في : لازم احدد مقدار واتجاه



if curve is like this

$I = \text{slope of curve}$

$$Q(t) = \int_{t_0}^t I(T) dT + \text{initial condition}$$

$\rightarrow$ or the area under the curve.



Ex: $\phi(t) = 3e^{-t}$ c, Find $I(t)$

Solution:- $I(t) = \frac{d\phi}{dt}$

$= -3e^{-t}$ A ~~##~~



Ex: Find $I(t)$

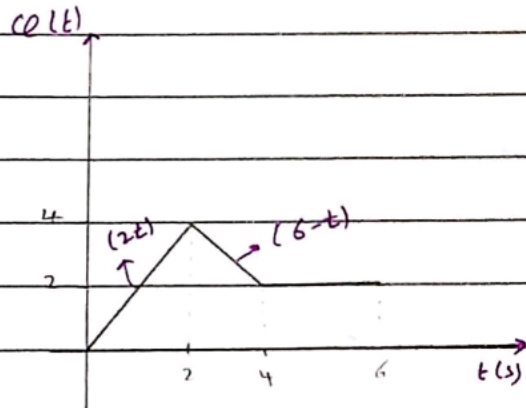
Solution:-

$I(t) = \frac{d\phi}{dt}$

① for $0 \leq t \leq 2$

$\phi(t) = 2t$

$I(t) = 2$ A ~~##~~ = slope.



② for $2 \leq t \leq 4$

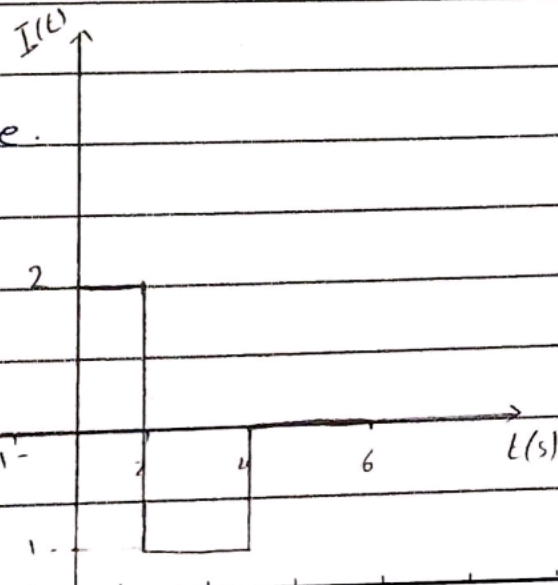
$\phi(t) = 6 - t$

$I(t) = -1$ A. = slope ~~##~~

③ for $4 \leq t \leq 6$

$\phi(t) = 2$

$I(t) = 0$ A. ~~##~~ has no slope.



initial بطل فترة بفرقة zero به ينسب اشارة

Ex:- Find the :-

- Total amount of charge $Q(t)$.
- Total amount of charge up to $t = 6$ sec
- Total amount of charge between $t = 2$ sec to $t = 8$ sec.

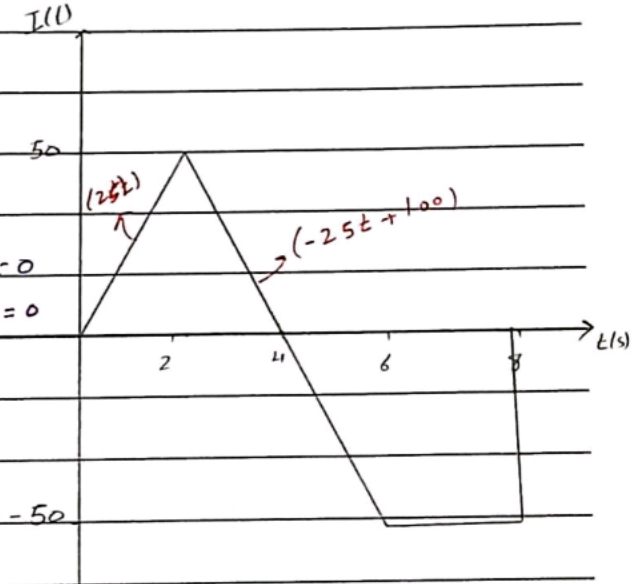
$$a) Q(t) = \int_{t_0}^t I(I) dI + \text{initial}$$

for $0 \leq t \leq 2$, $I(t) = 25t$, initial = 0
 $\hookrightarrow Q = 0$

$$Q(t) = \int_0^t 25I dI + Q(0)$$

$$= \frac{25I^2}{2} \Big|_0^t$$

$$= 12.5 t^2 \text{ C}$$



for $2 \leq t \leq 6 \Rightarrow$ initial at $t=2 \Rightarrow 12.5 \times 4 = 50 \text{ C}$

$$Q(t) = \int_2^t [-25I + 100] dI + Q(2)$$

$$= \left[\frac{-25I^2}{2} + 100I \right]_2^t + Q(2)$$

$$= (-12.5t^2 + 100t) - (-50 + 200) + 50$$

$$Q(t) = -12.5t^2 + 100t - 200$$



for $6 \leq t \leq 8$, initial at $t=6$ $Q(6) = -12.5(6)^2 + 100 \times 6 - 206$

$$Q(6) = -50$$

$$Q(t) = \int_6^t (-50) dT + Q(6)$$

$$= -50T \Big|_6^t + -50$$

$$Q(t) = -50(t - 6) + -50$$

b) amount charge up to 6 sec.

$$Q = Q(6) - Q(0)$$

$$= -50 - 0 = -50 \text{ C}$$

c) amount of charge between $t=2 \rightarrow 8$ sec

$$Q = Q(8) - Q(2)$$

$$= -150 - 50$$

$$= -200 \text{ C}$$

$$Q(8) = -50(t - 6) - 50$$

$$Q(8) = -350 \text{ C}$$

* voltage:- work needed to move unit charge in a wire

$$V = \frac{\text{Work}}{\text{charge}} \Rightarrow \text{volt}$$

A B
 $V_A = 5 \text{ volt}$ $V_B = 3 \text{ volt}$

* voltage at ground is zero

ie, relative to, volt

$$V_{AB} = V_A - V_B$$

$$V_{AB} = 5 - 3 = 2 \text{ V}$$

$$V_{BA} = -2 \text{ V}$$

$$V_{AB} = -V_{BA}$$

$$\begin{aligned} * V_{A-\text{ground}} &= V_A - V_{\text{ground}} \\ &= 5 - 0 = 5 \text{ volt} \end{aligned}$$



* power :- work needed per unit time.

$$P = \frac{\text{Work}}{\text{time}} \Rightarrow \text{Watt.}$$

$$P = I * V$$

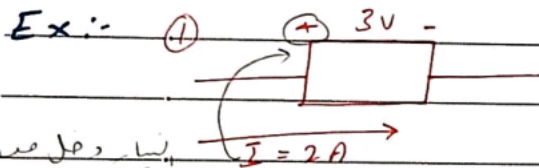
* if sign of power is positive $\Rightarrow$ power is dissipated (consumed)

* if sign of power is negative $\Rightarrow$ power is generated (supplied)

* Passive Sign Convention :-

* If current enter from the (+) sign $\Rightarrow P \Rightarrow (+)$

* If current enter from the (-) sign $\Rightarrow P \Rightarrow (-)$



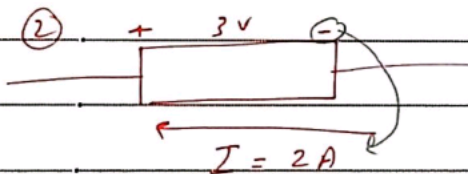
calculate the power ?

$$P = + (I \times V)$$

$$P = 3 \times 2 = 6 \text{ Watt}$$

So, it's dissipated.

or consumed.

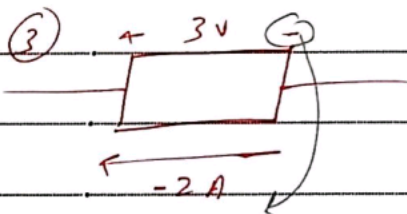


$$P = - (I \times V)$$

$$= - (2 \times 3) = -6 \text{ Watt}$$

So, it's generated

or supplied



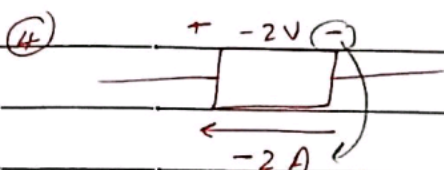
$$P = - (I \times V) = - (-2 \times 3)$$

$$P = + 6 \text{ Watt}$$

So, it's consumed or dissipated

$$P = (-)$$





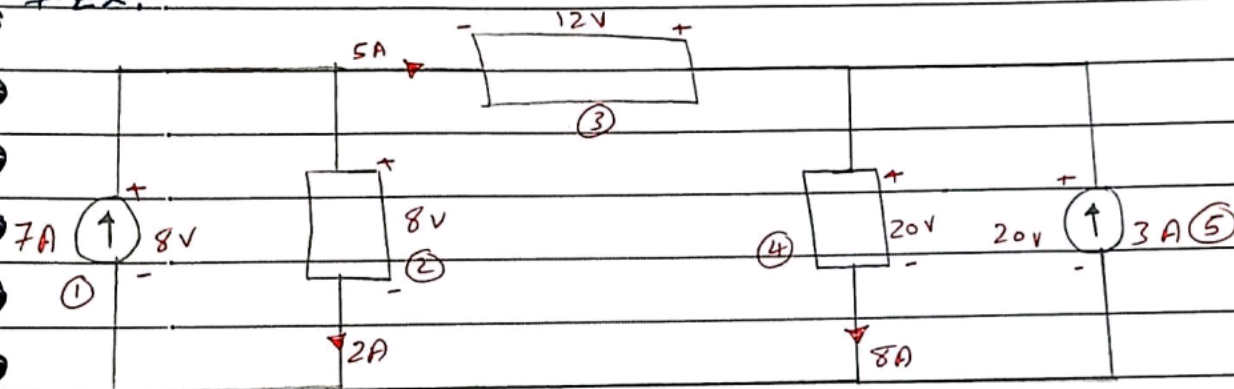
$$P = - (I \times V)$$

$$= - (-2 \times -2)$$

$$P = (-)$$

$P = -4$ watt so, it's generated or supplied.

* Ex :-



Find the power for each element?

$$P_1 = - (I \times V) = - 8 \times 7 = -56 \text{ watt (generated)}$$

$$P_2 = (I \times V) = 2 \times 8 = 16 \text{ watt (consumed)}$$

$$P_3 = - (I \times V) = - 12 \times 5 = -60 \text{ watt (generated)}$$

$$P_4 = (I \times V) = 8 \times 20 = 160 \text{ watt (consumed)}$$

$$P_5 = - (I \times V) = - 3 \times 20 = -60 \text{ (generated)}$$

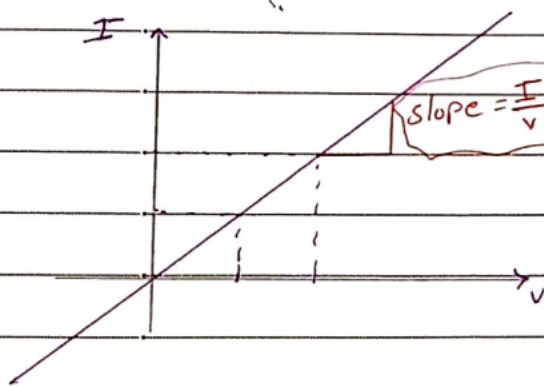
$$\sum P_{\text{Consumed}} = \sum P_{\text{generated}}$$

$$16 + 160 = 60 + 60 + 56$$

$$176 = 176 \checkmark$$

* ohms law:-

⇒ current that pass in a wire face a resistance.



$$\text{slope} = \frac{I}{V} = \frac{1}{R} = G$$

* $V \propto I$

voltage have directly relation with current (linear relation).

* $V = RI$

* $R = \frac{V}{I} \Rightarrow$ resistance (—)

ohm's law:- voltage across an element is directly proportional to the current.

* Conductance:- $G = \frac{1}{R}$ (S, Ω^{-1})
↓
Siminze moho

① * $P = IV$

= $I(IR) = I^2 R$ ②

* $P = IV$

= $\frac{V}{R} * V = \frac{V^2}{R}$ ③

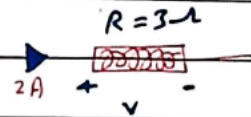


Ex:- wire has resistance of 3Ω , pass a current of $2A$. Find the voltage across the wire?

Solution:-

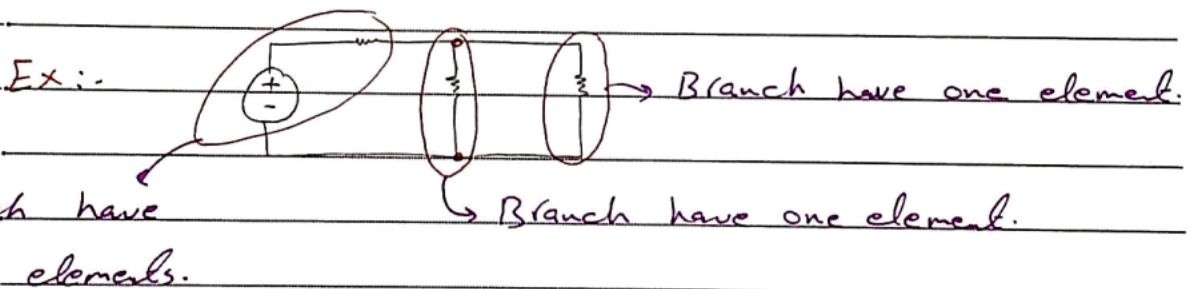
$$V = I \times R$$

$$= 2 \times 3 = 6 \text{ Volt.}$$



Definitions of electrical circuit networks:-

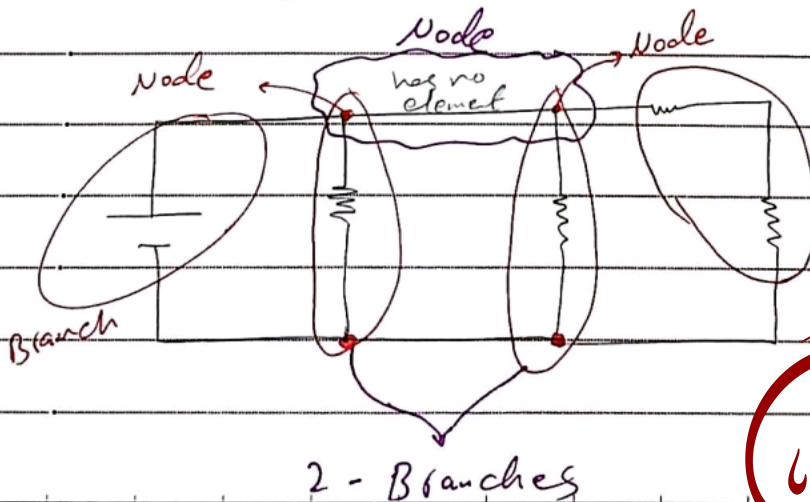
① **Branch**:- any part of the circuit that has one or more elements.



So, this circuit has 3- Branches.

(ملاحظة)

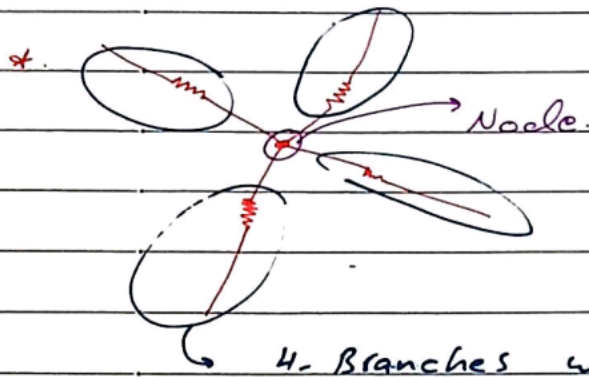
② **Node**:- junction for two or more Branches.



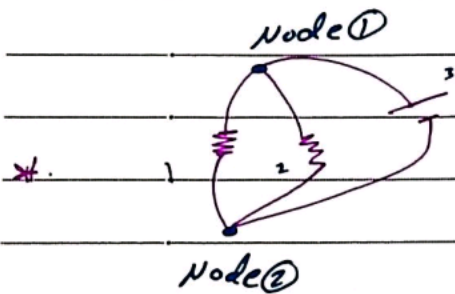
* Note:- if the connection between nodes has no element it's considered one node.



MADAR

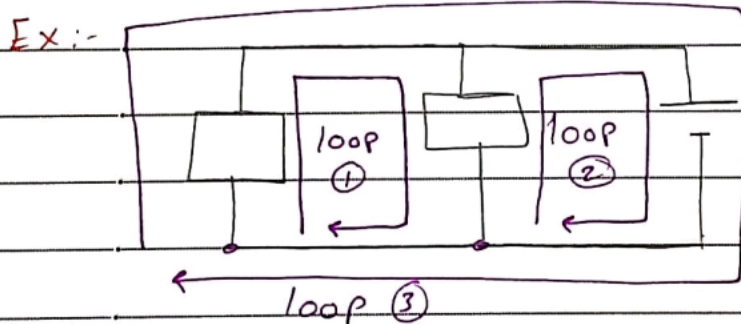


4. Branches with one Node.



3-Branches with 2-nodes.

③ Loop:- closed connection of Branches.

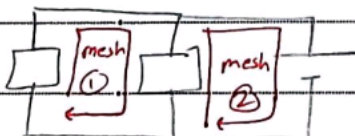


loop ① loop ② loop ③

④ mesh:- Loop that doesn't contain other loop.

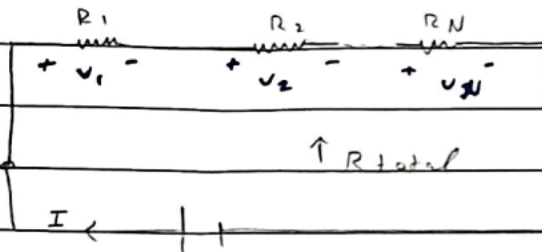
(Loop الذي لا يحتوي على حلقة أخرى)

من الممكن أن يكون loop ① loop ②



has two meshes

* Series Connection of Resistors :-



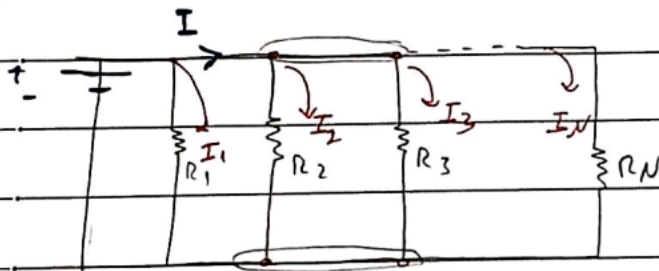
$$R_{total} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^N R_n$$

* In Series Connection $\Rightarrow$ same current pass through them.

* In Series Connection $\Rightarrow$ the voltage is different for each element.

and we use ohm's law to calculate the voltage for each other. $V_1 = I * R_1$, $V_2 = I * R_2$ - - -

* parallel connection of Resistors :-



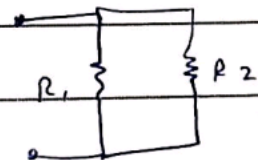
تفاوتی در ولتاژ نیست؟
بلکه در جریان است
در هر شاخه

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

* Special case :- If I have just two Resistance.

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{total}} = \frac{R_1 + R_2}{R_1 R_2} \Rightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$



* For parallel connection:-

* The current divided between Branches.

* The voltage is the same.

* Kirchhoff current law (KCL):-

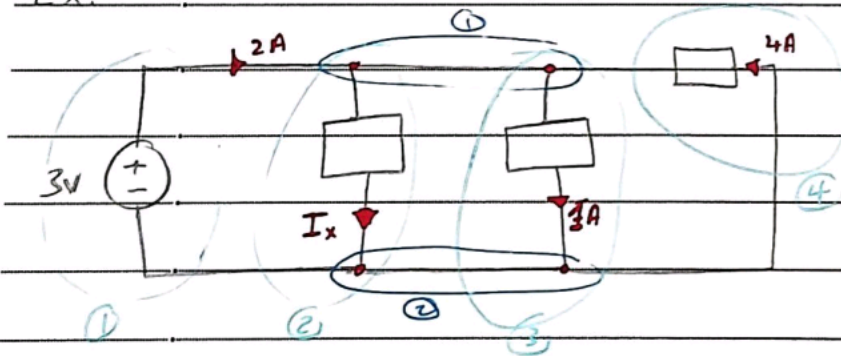
The sum of the current at a node must be equal to zero

$$\sum I_{\text{node}} = 0$$

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$



Ex:-



apply KCL
to find I_x ?

* Branches:- 4 Branches

* Nodes:- 2 Nodes

* loops:- 6 loops

* Meshes:- 3 Meshes

$$\sum I_{in} = \sum I_{out}$$

$$\begin{array}{rcl} 2A + 4A & = & 1A + I_x \\ -1A & -1A & \end{array}$$

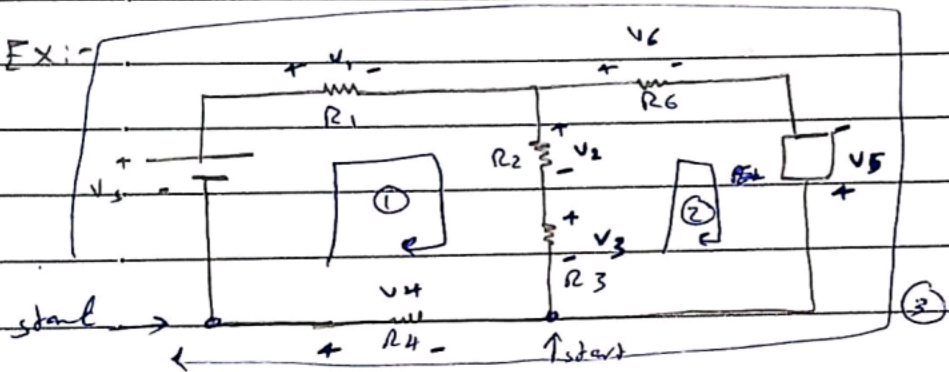
$$I_x = 5A$$



* kirshoff voltage law (KVL)

the net voltage around a closed circuit sum is zero

$$\sum V_{\text{closed loop}} = \text{Zero}$$



write kvl equations?

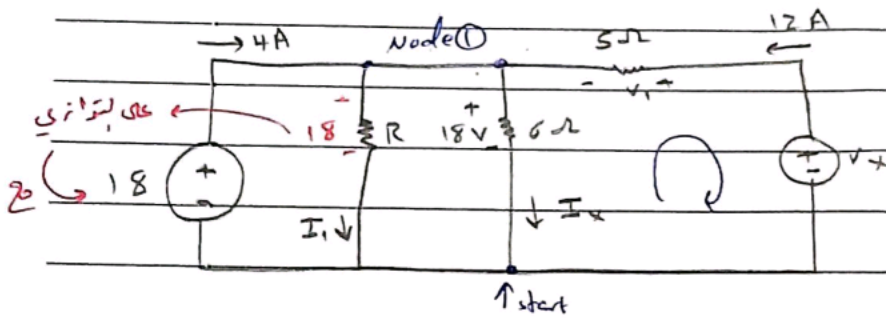
$$\sum V = 0$$

$$\textcircled{1} \rightarrow -V_s + V_1 + V_2 + V_3 - V_4 = 0$$

$$\textcircled{2} \rightarrow -V_3 - V_2 + V_6 - V_5 = 0$$

$$\textcircled{3} \rightarrow -V_s + V_1 + V_6 - V_5 - V_4 = 0$$

Ex:-



Final :- V_1, I_x, V_x, R, I_1

→ using ohm's law $I_x \Rightarrow I_x = \frac{V}{R} = \frac{18}{6} = 3 \text{ A}$

→ $V_x \Rightarrow V_1 = IR - 12 * 5 = 60 \text{ Volt}$

using kcl at Node ①:-

$$\sum I_{in} = \sum I_{out}$$

$$4 + 12 = I_x + I_1$$

$$16 = 3 + I_1$$

$$I_1 = 13 \text{ A}$$

using ohm's law $\Rightarrow R \Rightarrow V = IR$

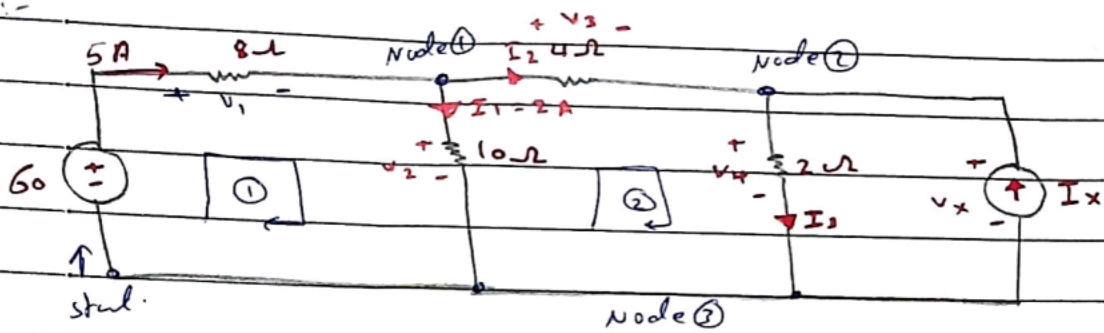
$$R = \frac{V}{I} = \frac{18}{13} \Omega$$

using kvl $\Rightarrow \sum V = 0$

$$-18 - 60 + V_x = 0$$

$$V_x = 78 \text{ Volt}$$

Ex:-



Final :- I_x , V_x , power of the current source.

$$\rightarrow V_1 = 2 \times R = 5 \times 8 = \boxed{40 \text{ volt}}$$

KVL At loop ①

$$\sum V = 0$$

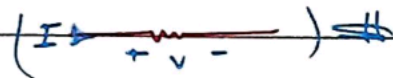
$$-60 + 40 + V_2 = 0$$

$$\boxed{V_2 = 20 \text{ volt}}$$

اتجاه polarity voltage

دائماً، التيار عندما يدخل إلى

مقاومة يدخل من (+) ويخرج من (-)



يعني إذا طالعنا الاتجاه افتراضه

إذا طلعنا ليعين (+) يكون الاتجاه مع

إذا طلعنا ليعين (-) يكون الاتجاه عكس.

apply ohm's law (I_1)

$$V = IR$$

$$I = \frac{V}{R} = \frac{20}{10} = \boxed{2A}$$



apply kcl at node ① $\Rightarrow \sum I_{in} = \sum I_{out}$

$$5 = 2 + I_2$$

$$\boxed{I_2 = 3A}$$

apply ohm's law $\Rightarrow \boxed{V_3 = 4 \times 3 = 12 \text{ volt}}$

Chapter three (3)

* Fundamental techniques for resistive Circuit Analysis:-

- ① Nodal Analysis.
- ② Mesh Analysis
- ③ Super position
- ④ Source transition
- ⑤ Thevenin & Norton.

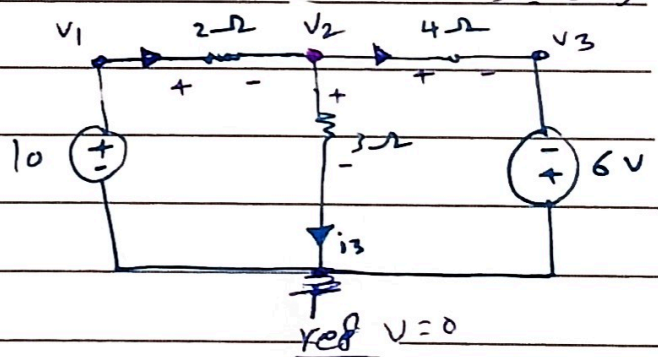
→ used to do analysis to any circuit.



① Nodal Analysis:- (step by step).

- ① Determine the nodes.
- ② assume one of the nodes as a reference. (أشكك على واحد من العقد)
- ③ name the nodes $\Rightarrow$ using $v_1, v_2, \dots$ من المقادير
- ④ if we have a voltage source between node & reference $\Rightarrow v_i$ at that node is known.
- ⑤ Apply KCL at each node.

Ex:- Find i_3 , $P(2\Omega) = ?$
Using Nodal Analysis.



v_1 related to reference = 10 V

v_3 related to ref = -6 V

apply KCL at node ②:-

$$\sum I_{in} = \sum I_{out}$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{4} + \frac{v_2 - 0}{3}$$

$$v_2 = 3.24 \text{ Volt}$$

$$\Rightarrow i_3 = \frac{v_2 - 0}{3} = 1.08 \text{ A}$$

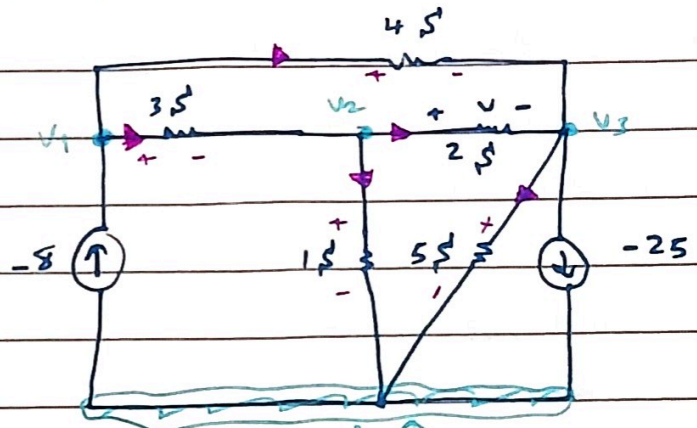
$$P = \frac{V^2}{R} = \frac{(v_1 - v_2)^2}{2} = \frac{(10 - 3.24)^2}{2}$$

$$P = 22.8 \text{ watt}$$

→ + (dissipation).

Ex:-

Find V using Nodal analysis?



Conductance $\Rightarrow G = \frac{1}{R}$ (Ω, V, A^{-1})

$$V = IR \Rightarrow I = \frac{V}{R} = V \times \frac{1}{R} = V \times G$$

at node ① apply KCL

$$-8 = (V_1 - V_2) \times 3 + (V_1 - V_3) \times 4$$

$$7V_1 - 3V_2 - 4V_3 = -8 \quad \dots (1)$$

at node ②

$$(V_1 - V_2) \times 3 = V_2 \times 1 + (V_2 - V_3) \times 2$$

$$-3V_1 + 6V_2 - 2V_3 = 0 \quad \dots (2)$$

at node ③

$$(V_2 - V_3) \times 2 + (V_1 - V_3) \times 4 = -25 + (V_3 - 0) \times 5$$

$$4V_1 + 2V_2 - 11V_3 = -25 \quad \dots (3)$$

$$V_1 = 1.33 \text{ volt}$$

$$V_2 = 1.69 \text{ volt}$$

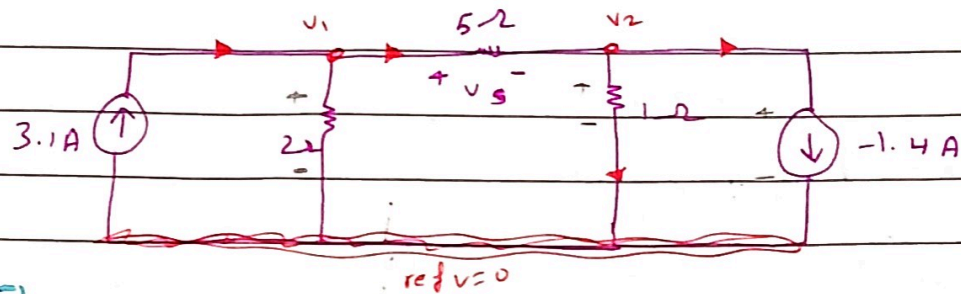
$$V_3 = 3.1 \text{ volt}$$

$$V = V_2 - V_3$$

$$= 1.69 - 3.1 = -1.41 \text{ volt}$$



Ex:-



Find ① v_5

② P_{gen} by (-1.4 A) source } Using Nodal Analysis.

at Node ① apply KCL:-

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$3.1 = \frac{(v_1 - 0)}{2} + \frac{(v_1 - v_2)}{5}$$

$$3.1 = 0.7v_1 - 0.2v_2 \quad \text{--- ①}$$

at Node ② apply KCL:-

$$\frac{v_1 - v_2}{5} = \frac{v_2 - 0}{1} + (-1.4)$$

$$0.2v_1 - 1.2v_2 = -1.4 \quad \text{--- ②}$$

$$v_1 = 5 \text{ V}, \quad v_2 = 2$$

$$\therefore v_5 = v_1 - v_2 = 5 - 2 = 3 \text{ Volt}$$

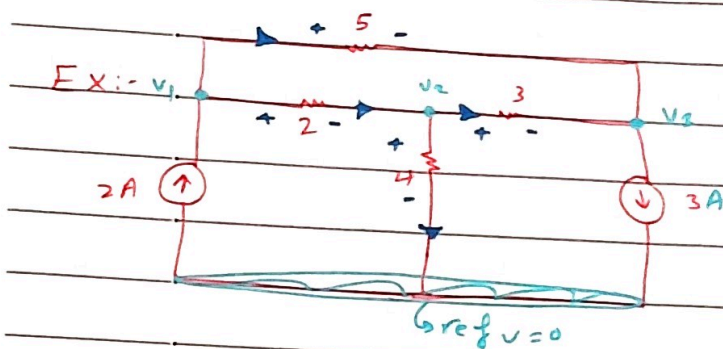


$$\therefore P = + I \times v_2 = + (-1.4) \times 2 = -2.8 \text{ watt}$$

↳ generated

$$\text{So, } P_{\text{gen}} = 2.8 \text{ watt}$$

* Super nodes:-



At Node ① $\Rightarrow \left(2 - \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{5} \right) \times 2$

$(4 = v_1 - v_2 + \frac{2v_1 - 2v_3}{5}) \times 5$

$20 = 5v_1 - 5v_2 + 2v_1 - 2v_3$

$7v_1 - 5v_2 - 2v_3 = 20 \dots \textcircled{1}$

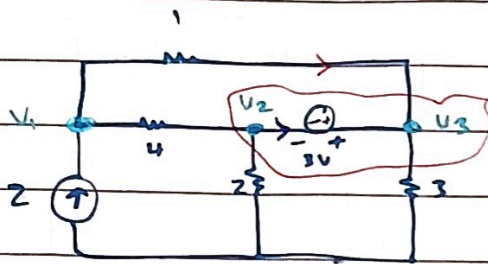
At node ② $\Rightarrow \left(\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{3} + \frac{v_2 - 0}{4} \right) \times 2$

$6v_1 - 13v_2 + 4v_3 = 0 \dots \textcircled{2}$

At node ③ $\Rightarrow \frac{v_2 - v_3}{3} + \frac{v_1 - v_3}{5} = 3$

$3v_1 + 5v_2 - 8v_3 = 40 \dots \textcircled{3}$

EX :-



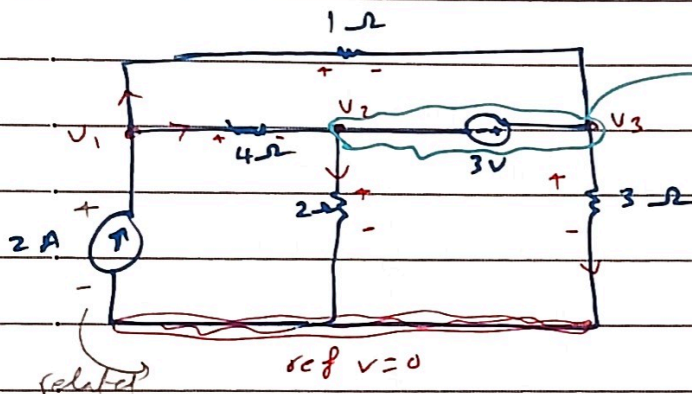
تغير ما ب Node و قوة

super node.

لما طابق الحرف فيتي بيار ب واحد branch

Voltage Source

EX :-



$V_2 \Rightarrow$ super node

at node ① $\Rightarrow 2 = \frac{V_1 - V_2}{4} + \frac{V_1 - V_3}{1} \dots \text{①}$

$$5V_1 - V_2 - 4V_3 = 8$$

at super node (node ②) $\Rightarrow \frac{V_1 - V_2}{4} + \frac{V_1 - V_3}{1} = \frac{V_2 - 0}{2} + \frac{V_3 - 0}{3}$

$$15V_1 - 9V_2 - 16V_3 = 0$$

at super node :- $V_3 - V_2 = 3V$

$$V_1 = 5.2, V_2 = 1.2, V_3 = 4.2$$

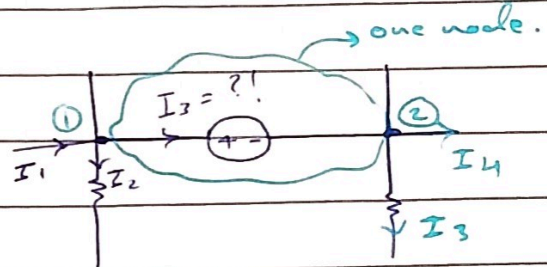
$P_{gen}(2A)$ $\leftarrow$ $\star$ $\leftarrow$ $\star$

$$P = - (2 \times V_1) = 2 \times 5.2 = 10.4 \text{ watt}$$

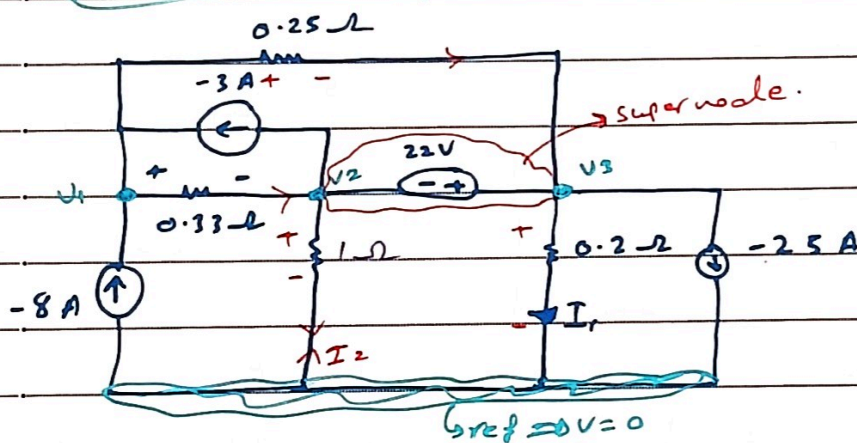


MADAR

* **Supernode**:- when there is a voltage source between nodes, we can't know the current as a function of voltage



Ex:-



use Nodal analysis to find I_1 ?!

$$\text{at node ①} \Rightarrow -8 - 3 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.25}$$

$$-0.91 = 0.58V_1 - 0.25V_2 - 0.33V_3 \quad \text{①}$$

$$\text{at node ②} \Rightarrow \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.25} = -3 + \frac{V_2 - 0}{1} + \frac{V_3 - 0}{0.2} - 2.5$$

$$0.58V_1 - 0.23V_2 - 0.25V_3 = -2.05 \quad \text{②}$$

$$\text{using supernode} \Rightarrow V_3 - V_2 = 22 \text{ V} \quad \text{③}$$

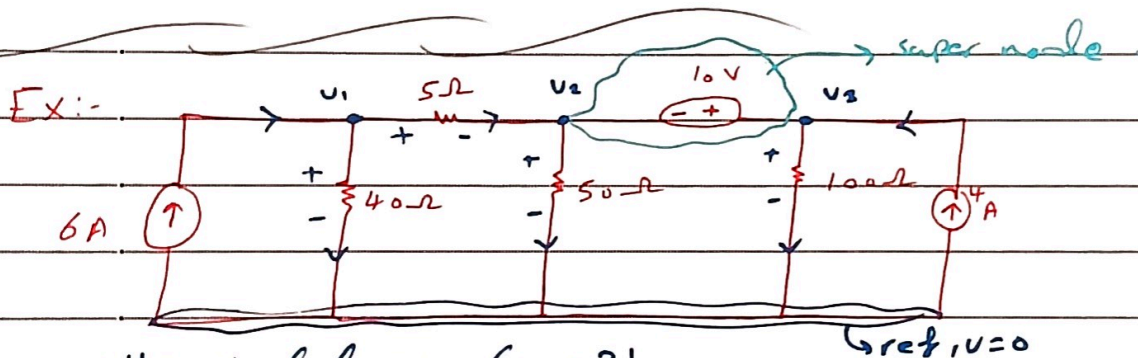


$$V_1 = 252.9, V_2 = 242, V_3 = 264$$

$$\text{So, } I_1 = \frac{V_3 - 0}{0.2} = \frac{264}{0.2} = 1320 \text{ A}$$

لحل المسألة باستخدام التيار I_2 في المقاومة 50Ω المتصلة بين العقدتين V_2 و V_3

$$I_2 = \frac{0 - V_2}{1} = \frac{0 - 242}{1} = -242 \text{ A}$$



write Node equations?!

at node ①: $6 = \frac{V_1 - 0}{40} + \frac{V_1 - V_2}{5}$

$$90V_1 - 40V_2 = 12000 \dots \text{①}$$

at node ②

$$\frac{V_1 - V_2}{5} + 4A = \frac{V_2 - 0}{50} + \frac{V_3 - 0}{100}$$

$$20V_1 - 30V_2 - V_3 = -4000 \dots \text{②}$$

super node $\Rightarrow V_3 - V_2 = 10 \dots \text{③}$



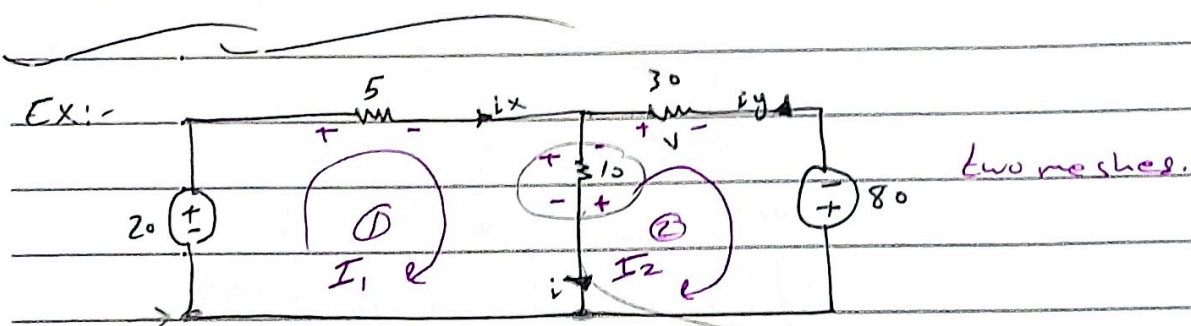
② Mesh analysis:-

Steps:- ① Define meshes

② assign mesh current I_1 for all meshes

③ Apply KVL for each mesh.

④ solve equations.



Find V using mesh Analysis:- $\rightarrow I_1$ and I_2 are the mesh currents

for mesh ① $\Rightarrow -20 + 5I_1 + 10I_1 - 10I_2 = 0 \quad \dots (1)$

for mesh ② $\Rightarrow 10I_2 - I_1 \cdot 10 + 30I_2 - 80 = 0 \quad \dots (2)$

$\Rightarrow V = I_2 \cdot 30 = 2.8 \cdot 30 = 84 \text{ Volt}$

$15I_1 - 10I_2 = 20 \quad \dots (1)$

$-10I_1 + 40I_2 = 80 \quad \dots (2)$

$I_1 = 3.2, I_2 = 2.8 \text{ A.}$

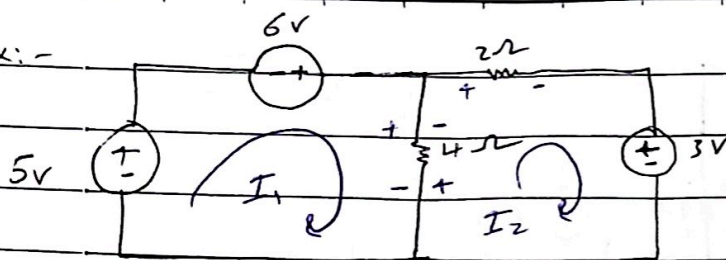
$i = ? \Rightarrow I_1 - I_2 = 3.2 - 2.8 = 0.4 \text{ A}$

$i_x = ? \Rightarrow i_x = I_1 = 3.2 \text{ A}$

$i_y = ? \Rightarrow i_y = -I_2 = -2.8 \text{ A}$



Ex: -



Q:- Find the absorbed power by the 6V supply using mesh analysis?

Two meshes.

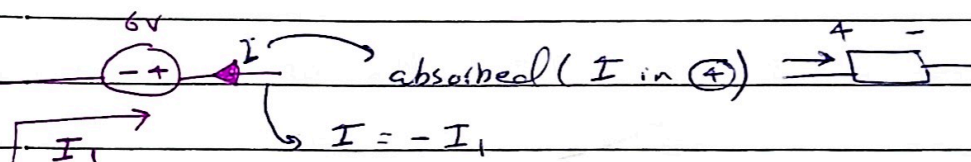
at mesh ① $\rightarrow -5 - 6 + 4I_1 - 4I_2 = 0$

$$4I_1 - 4I_2 = 11 \quad \dots \text{①}$$

at mesh ② $\rightarrow 4I_2 - 4I_1 + 2I_2 + 3 = 0$

$$-4I_1 + 6I_2 = -3 \quad \dots \text{②}$$

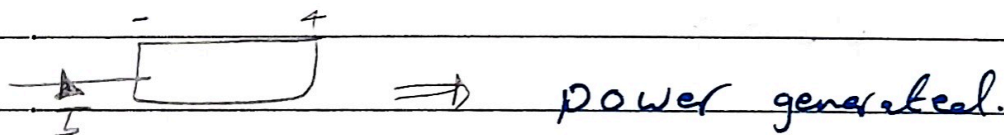
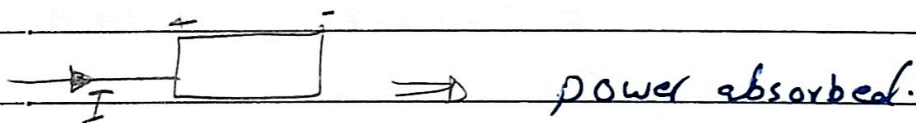
$$I_1 = 6.75, \quad I_2 = 4.$$



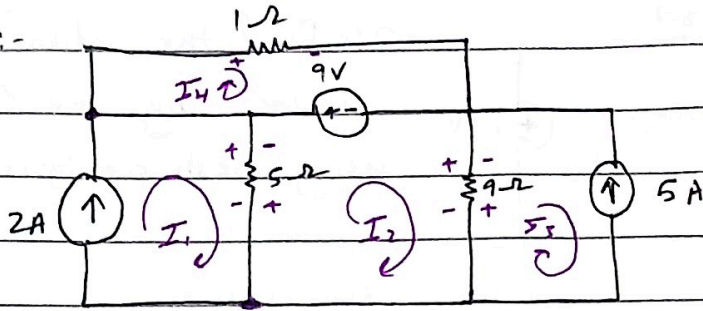
So, $P_{abs} = (-I_1) \times 6$

$$= -6.75 \times 6 = -40.5 \text{ watt.}$$

Note !!



Ex:-



Find $P_{\text{generated}}$ by (9V) supply using mesh analysis?

at mesh ① $\Rightarrow I_1 = 3A$.

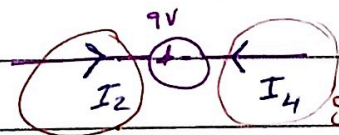
at mesh ③ $\Rightarrow I_3 = -5A$.

at mesh ② $\Rightarrow +5I_2 + 9 + 9I_2 - (9 \times -5) - (5 \times 3) = 0$

$$\frac{14}{14} I_2 = \frac{-39}{14} \Rightarrow I_2 = -2.79 A$$

at mesh ④ $\Rightarrow 1 \times I_4 - 9 = 0$

So, $I_4 = 9A$



abs

gen $\Rightarrow$ So $I = I_4 - I_2$

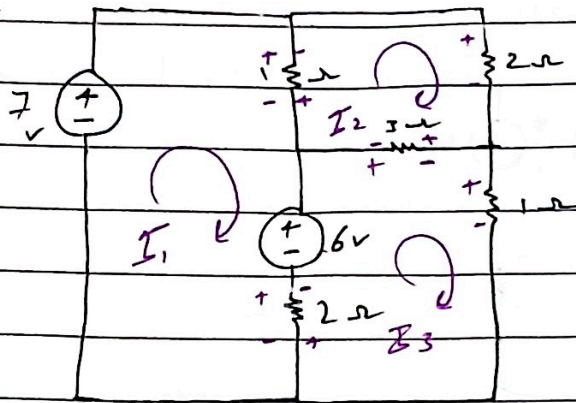
$$= 9 - (-2.79) = 11.79 A$$

$$P_{\text{gen}} = I \times 9$$

$$= 11.79 \times 9 = 106.11 \text{ Watt}$$



Ex:-

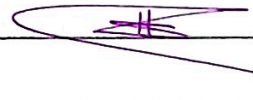


Q) Using Mesh analysis write mesh equation?

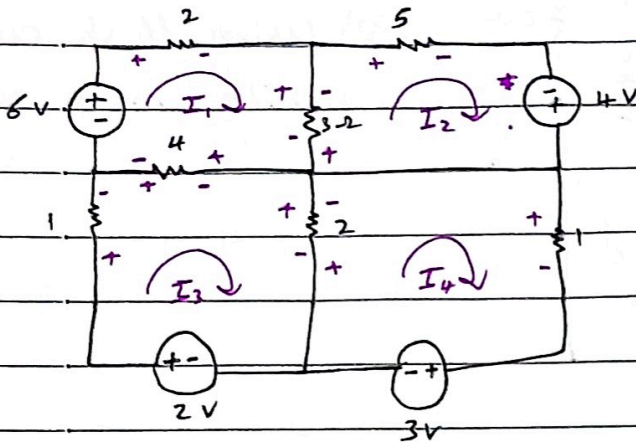
at mesh ① $\Rightarrow -7 + (1 \times I_1) + 6 + (2I_1) - (1 \times I_2) - (2 \times I_3) = 0$
 $3I_1 - I_2 - 2I_3 = 1 \dots \text{①}$

at mesh ② $\Rightarrow 1 \times I_2 + 2I_2 + 3I_2 - I_1 - 3I_3 = 0$
 $6I_2 - I_1 - 3I_3 = 0 \dots \text{②}$

at mesh ③ $\Rightarrow -6 + 3I_3 + I_3 + 2I_3 - 2I_1 - 3I_2 = 0$
 $6I_3 - 2I_1 - 3I_2 = 6 \dots \text{③}$



Ex:- Write mesh equations:-



at mesh ① $\Rightarrow \sum V = 0$

$$-6 + 2I_1 + 3I_1 + 4I_1 - 3I_2 - 4I_3 = 0$$

$$9I_1 - 3I_2 - 4I_3 = 6 \dots \textcircled{1}$$

at mesh ② $\Rightarrow -4 + 3I_2 + 5I_2 - 3I_1 = 0$

$$8I_2 - 3I_1 = 4 \dots \textcircled{2}$$

at mesh ③ $\Rightarrow -2 + I_3 + 4I_3 + 2I_3 - 4I_1 - 2I_4 = 0$

$$7I_3 - 4I_1 - 2I_4 = 2 \dots \textcircled{3}$$

at mesh ④ $\Rightarrow 3 + 2I_4 + I_4 - 2I_3 = 0$

$$3I_4 - 2I_3 = -3 \dots \textcircled{4}$$

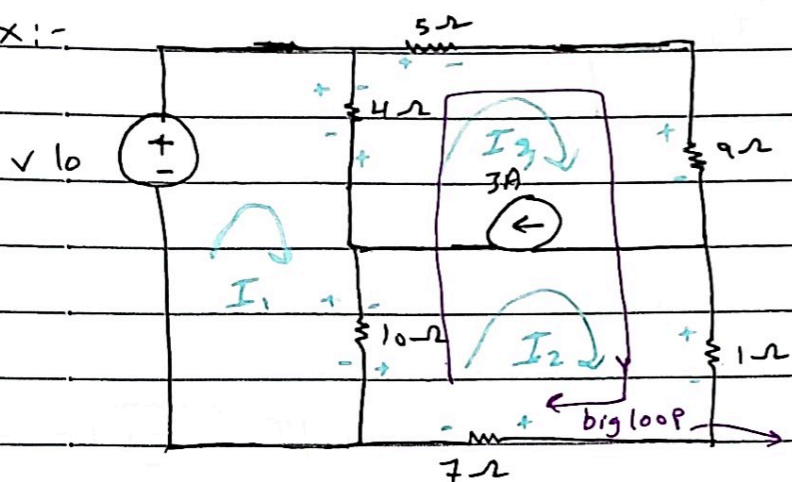


* Super mesh:-

* We make super mesh when there is a current source between two meshes.

⇒ So we take large loop that has both meshes

Ex:-



(p) Find the P_{gen} by the 10V supply using Mesh analysis?

$$P = - (I_1)(10) \checkmark$$

$$P_{ge}(10) = I_1 \times 10 = 1.16 \text{ Watt}$$

$$P_{abs}(10V) = -P_{ge}(10V) = -1.16 \text{ Watt}$$

at mesh ① $\sum V = 0$

$$-10 + 4I_1 + 10I_1 - 4I_3 - 10I_2 = 0$$

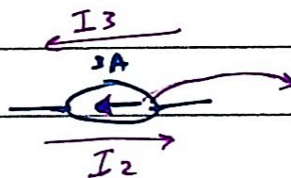
$$14I_1 - 10I_2 - 4I_3 = 10 \quad \dots (1)$$

at super mesh $\Rightarrow \sum V = 0$

$$10I_2 + 4I_3 + 5I_3 + 9I_3 + I_2 + 7I_2 - 4I_1 - 10I_1 = 0$$

$$-14I_1 + 18I_2 + 18I_3 = 0 \quad \dots (2)$$

from current source (3A):-



التيار في المقاومة

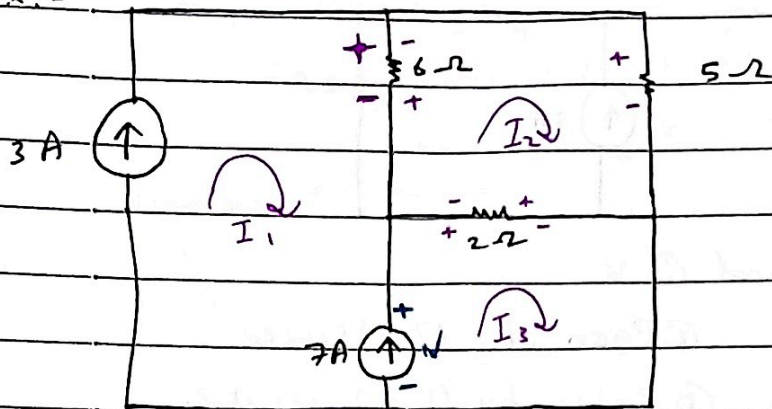
3A بيني صوب اليمين

$$\text{so, } I_3 - I_2 = 3A \quad \dots (3)$$

$$I_1 = 0.116, \quad I_2 = -1.45, \quad I_3 = 1.55$$



Ex:-

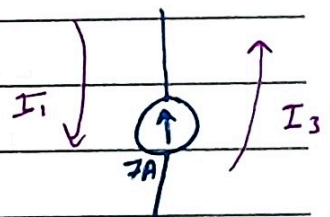


Q) Find generated power by (7A) source using mesh analysis?

of mesh ① $\Rightarrow I_1 = 3A$

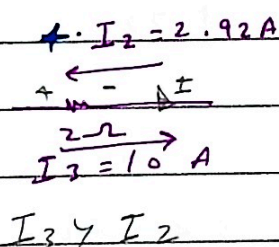
of mesh ② $\Rightarrow 5I_2 + 2I_2 + 6I_2 - 6I_1 - 2I_3 = 0$
 $13I_2 - 2I_3 = 18 \dots \text{①} \Rightarrow I_2 = 2.92A$

of mesh ③ $\Rightarrow I_3 = 10A$



* power gen $\rightarrow$

$P_{gen} = I \cdot V$
 $= 7 \cdot 14.16$
 $= 99.12 \text{ watt}$



* باتجاه I_3 اذا وصل لا غير

so, $7 = I_3 - I_1$

$7 = I_3 - 3$

$I_3 = 10A$

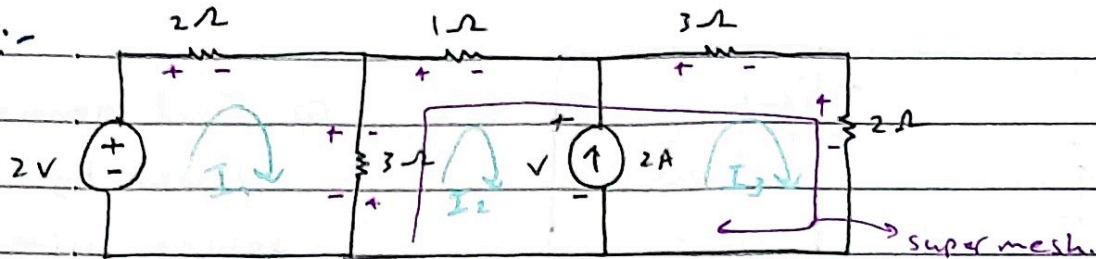
So, $I = 10 - 2.92 = 7.08A$

$\rightarrow$ loop ③ KVL $\Rightarrow -V + 2 \cdot 7.08 = 0$

$V = 14.16 \text{ volt}$



Ex:-



using Mesh Analysis Find ① V

② P_{gen} by (2A) source.

③ P_{abs} by (1Ω) resistor.

④ P_{gen} by (2V) source.

for mesh ① $\Rightarrow -2 + 2I_1 + 3I_1 - 3I_2 = 0$

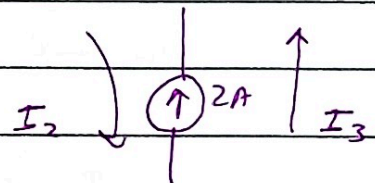
$$5I_1 - 3I_2 = 2 \quad \dots \textcircled{1}$$

for mesh ② $\Rightarrow \Sigma V = 0$

$$3I_2 + I_2 + 2I_3 + 3I_3 - 3I_1 = 0$$

$$-3I_1 + 4I_2 + 5I_3 = 0 \quad \dots \textcircled{2}$$

for $I_3 - I_2 = 2 \quad \dots \textcircled{3}$



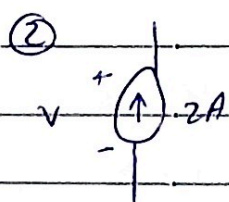
$$I_1 = -0.33, I_2 = -1.22$$

$$I_3 = 0.78$$

إشارة I_2 إذا صارت سالبة

① use loop ③ $\Rightarrow -V + 3 \times 0.78 + 2 \times 0.78 = 0$

$$(V = 3.9 \text{ volt})$$

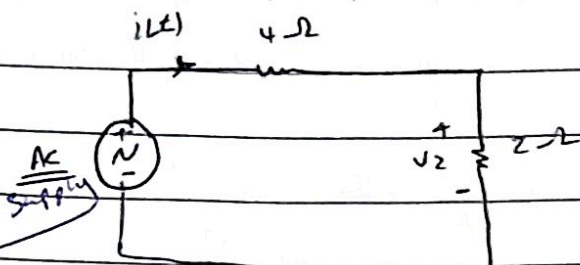


$$P = (-)(2)(3.9) = -7.8 \text{ W}$$

→ gen

$$\text{So, } P_{gen} = 7.8 \text{ watt}$$

Ex:-



$$v(t) = 12 \cos(377t + 40^\circ) \text{ V}$$

Find $i(t)$ & $v_2(t)$



$$V(j\omega) = 12 \angle 40^\circ$$

$$V_2(j\omega) = \frac{12 \angle 40^\circ \times 2}{2 + 4} \rightarrow \text{by volt division}$$

$$V_2(j\omega) = 4 \angle 40^\circ$$

$$v_2(t) = 4 \cos(377t + 40^\circ)$$

$$I(j\omega) = \frac{V_2(j\omega)}{2} = \frac{4 \angle 40^\circ}{2} = 2 \angle 40^\circ \text{ A}$$

$$i(t) = 2 \cos(377t + 40^\circ) \text{ A}$$

① Impedance:- ($Z(j\omega)$)

$\Rightarrow$ we will describe the circuit elements (R, L, C) in AC-circuit they will have impedances.

① Resistor $i(t) \rightarrow R \rightarrow v(t)$

$$\text{if } v(t) = V_m \cos(\omega t + \theta)$$

$$\Rightarrow V(j\omega) = V_m \angle \theta$$



* Sinusoidal Signals in Frequency domain:-

Time domain $\Rightarrow x(t) = A \cos(\omega t + \theta)$

Freq. domain $\Rightarrow X(j\omega) = A \angle \theta$ $\rightarrow$ phase shift refers to peak value. cosine signal.

(cos) ، علامه کنونی (cos)

Ex:- $v_1(t) = 15 \cos(377t + \frac{\pi}{4}) \text{ V}$

$v_2(t) = 15 \cos(377t + \frac{\pi}{12}) \text{ V}$

Find $v_s(t) = v_1(t) + v_2(t)$.

$V_1(j\omega) = 15 \angle \frac{\pi}{4} = 10.61 + j 10.61 \text{ V}$

$V_2(j\omega) = 15 \angle \pi/12^{(15)} = 14.49 + j 3.88 \text{ V}$

$V_s(j\omega) = V_1(j\omega) + V_2(j\omega)$

$= (10.61 + j 14.49) + j (10.61 + 3.88)$

$= 25.1 + j (14.49)$

$= 28.98 \angle 29.9^\circ = 28.98 \angle \frac{\pi}{6}$

$v_s(t) = 28.98 \cos(377t + \frac{\pi}{6})$ اذا كانت (freq) متساوية

فيها نرى خاصية

* اذا كانت متساوية متساوية متساوية



$$x_1 \pm x_2 = r_1 \angle \theta_1 \pm r_2 \angle \theta_2$$

Convert to Rectangular then compute
or using the calculator.

* Using calculator to solve complex numbers:-

① mode ② 2 (Complex)

③ shift $\rightarrow$ $\angle$ ENG

$$x = 3 + 2j$$

$$y = 5 + j$$

$$\Rightarrow x \cdot y = (3 + 2j)(5 + j) \\ = 13 + 13j \\ = 13\sqrt{2} \angle 45^\circ = 18.38 \angle 45^\circ$$

④ $r \angle \theta$

shift $\rightarrow 2 \rightarrow 3$

arg (Ans) = θ

polar $\rightarrow$ rectangular

$$\Rightarrow x \div y = (3 + 2j) \div (5 + j) \\ = \frac{17}{26} + \frac{7}{26}j$$

shift $\rightarrow 2 \leftarrow$ rec $\rightarrow$ ④

$$= \frac{\sqrt{2}}{2} \angle 22.4^\circ = 0.7 \angle 22.4^\circ$$

$$\text{Conj} \rightarrow \frac{17}{26} - \frac{7}{26}j$$

$$3 \angle 20^\circ + 4 \angle -15^\circ = 6.68 - 9.2 \times 10^{-3}j \quad \#$$

$$\text{shift} \rightarrow 2 \rightarrow 3 \Rightarrow 6.68 \angle -0.08^\circ \quad \#$$

$$\frac{3 \angle 20^\circ}{4 \angle -15^\circ} = 0.6144 \angle 0.43j \Rightarrow 0.75 \angle 35^\circ$$



(*) Review:-

$Z_R(j\omega) = R \rightarrow$ fixed with frequency variation.

$$Z_L(j\omega) = j\omega L = \omega L \angle 90^\circ$$

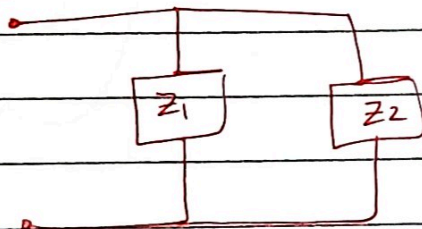
$$Z_C(j\omega) = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

Series



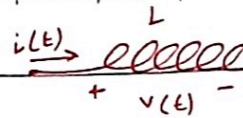
$$Z_{\text{total}} = Z_1 + Z_2$$

Parallel:-



$$\frac{1}{Z_{\text{Total}}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

② Inductor :-



$$v(t) = L \frac{di(t)}{dt}$$

if $i_L(t) = I_m \cos(\omega t + \theta)$

$$I_L(j\omega) = I_m \angle \theta$$

$$v(t) = I_m(\omega L) (-\sin(\omega t + \theta)) \overset{\text{لنحولها لـ cos}}{\cos} \rightarrow$$

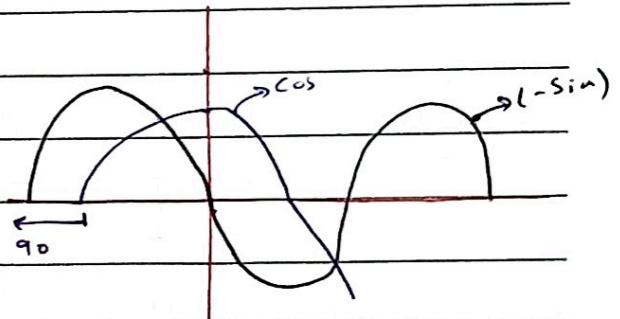
$$= I_m(\omega L) \cos(\omega t + \theta + 90^\circ)$$

$$V(j\omega) = I_m(\omega L) \angle \theta + 90^\circ$$

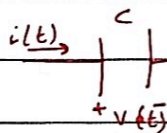
$$Z_L(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{I_m(\omega L) \angle \theta + 90^\circ}{I_m \angle \theta}$$

$$Z_L(j\omega) = \omega L \angle \theta + 90^\circ - \theta$$

$$Z_L(j\omega) = (\omega L) \angle 90^\circ \Rightarrow j\omega L \quad \text{ohm.}$$



③ capacitor :-



$$i_C(t) = C \frac{dv_C(t)}{dt}$$

if $v_C(t) = V_m \cos(\omega t + \theta) \Rightarrow v_C(j\omega) = V_m \angle \theta$

$$i_C(t) = V_m(C\omega) (-\sin(\omega t + \theta))$$

$$= (C\omega) V_m \cos(\omega t + \theta + 90^\circ)$$

$$I_C(j\omega) = (C\omega) V_m \angle \theta + 90^\circ$$

$$Z_C(j\omega) = \frac{V_C(j\omega)}{I_C(j\omega)} = \frac{V_m \angle \theta}{C\omega V_m \angle \theta + 90^\circ} = \frac{1}{C\omega} \angle \theta - \theta - 90^\circ$$

$$Z_C(j\omega) = \frac{1}{C\omega} \angle -90^\circ = \frac{-j}{\omega C}$$

To Find $I(j\omega) = \frac{V(j\omega)}{R}$

$V = IR$ (ohm's law).

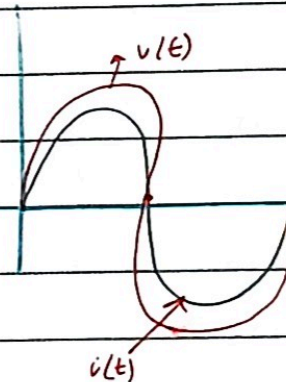
$$= \frac{V_m \angle 0}{R} = \frac{V_m}{R} \angle 0$$

$$\Rightarrow i(t) = \frac{V_m}{R} \cos(\omega t + 0)$$

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{V_m \angle 0}{\frac{V_m}{R} \angle 0} = R \angle 0 \rightarrow \text{No phase shift.}$$

$Z_R(j\omega) = R$ ohm.

* Same phase shift between voltage & current.



no phase shift

بدون هیچ تغییر فاز

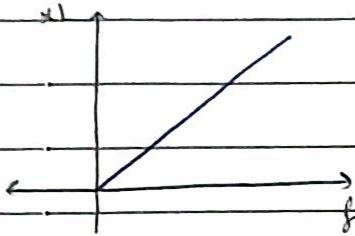
فاز

Notes:-

$$① Z_L(j\omega) = j\omega L = \omega L / 90^\circ$$

$$X_L = \omega L \Rightarrow \text{reactance}$$

$$\omega = 2\pi f \Rightarrow X_L = (2\pi f)L \Rightarrow X_L \propto f$$



① at $f = 0 \Rightarrow DC$ Inductors act as

$$X_L = 0 \Omega \Rightarrow \text{short circuit}$$

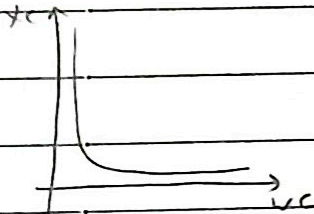
in DC-circuit

② at $f = \infty \Rightarrow X_L = \infty \Omega \Rightarrow \text{open circuit}$.

$$② Z_C(j\omega) = \frac{-j}{\omega C} = \frac{1}{\omega C} / -90^\circ$$

$$X_C = \frac{1}{\omega C} \Rightarrow \text{Reactance}$$

$$X_C \propto \frac{1}{\omega C}$$

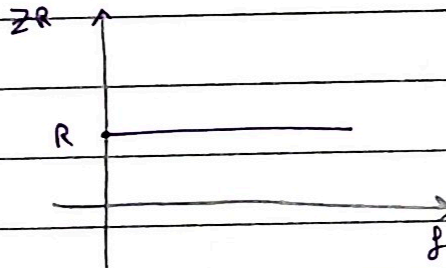


① at $f = 0 (DC) \rightarrow X_C = \infty$

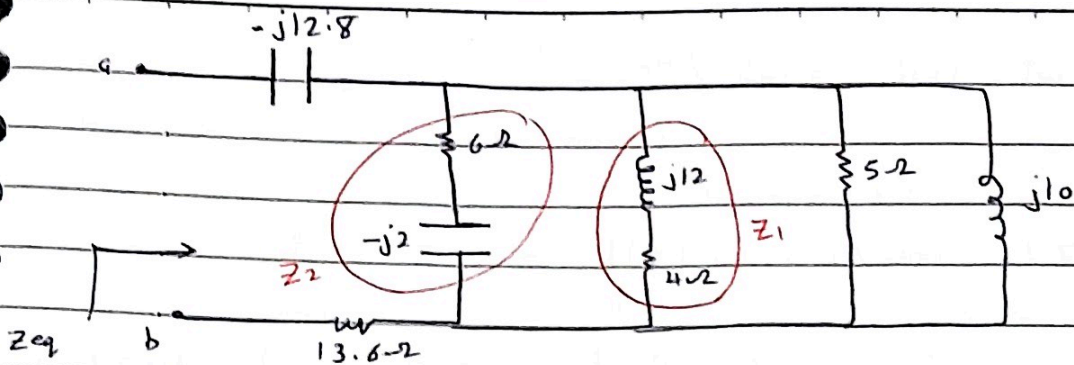
capacitor act as open circuit.

② at $f = \infty \rightarrow X_C = 0 \rightarrow \text{short circuit}$.

③ $Z_R(j\omega) = R \Rightarrow \text{independent of freq}$



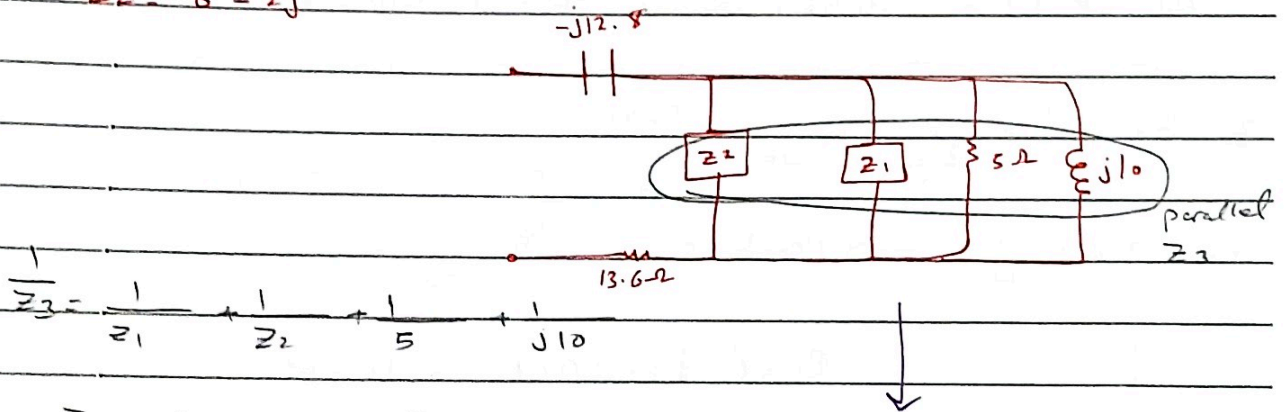
Ex:-



Find Z_{eq} between a & b?

$$Z_1 = 4 + j12$$

$$Z_2 = 6 - j2$$

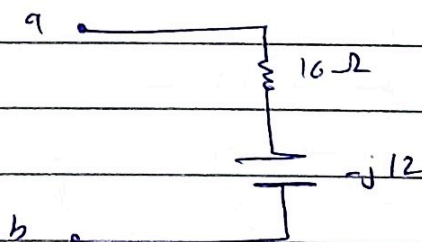


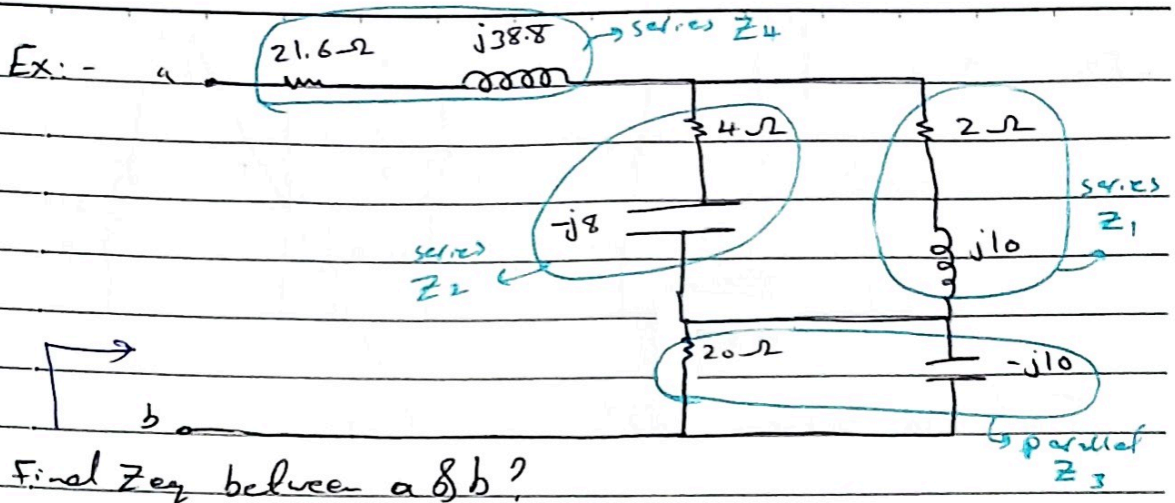
$$\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{5} + \frac{1}{j10}$$

$$Z_3 = 2.4 + j0.8 \Omega$$

$$Z_{eq} = -j12.8 + 13.6 + 2.4 + j0.8$$

$$Z_{eq} = 16 - j12$$

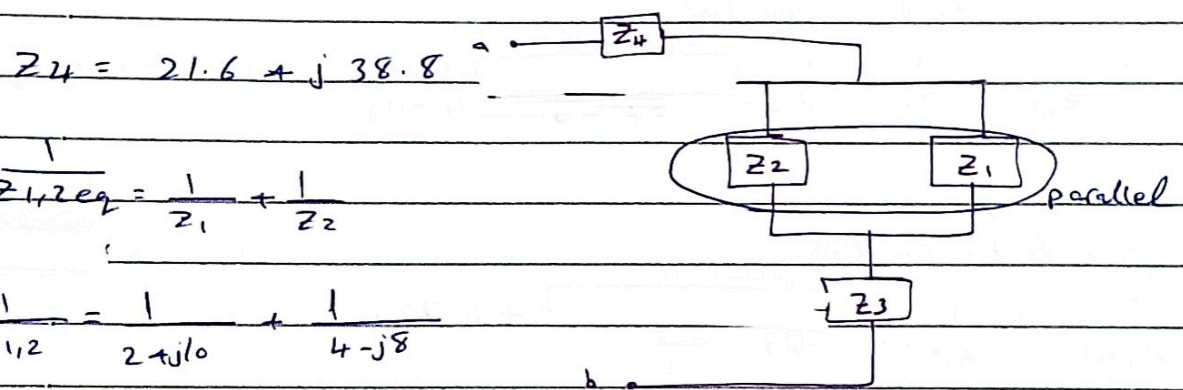




$$Z_1 = 2 + j10$$

$$Z_2 = 4 - j8$$

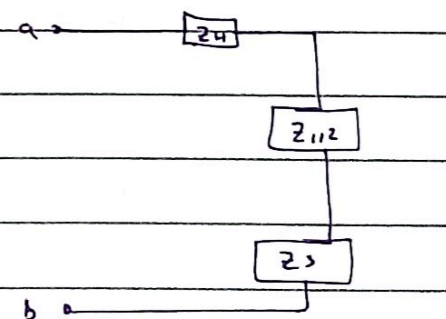
$$\frac{1}{Z_3} = \frac{1}{20} + \frac{1}{-j10} \Rightarrow Z_3 = 4 - j8$$



$$\frac{1}{Z_{1,2}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z_{1,2}} = \frac{1}{2 + j10} + \frac{1}{4 - j8}$$

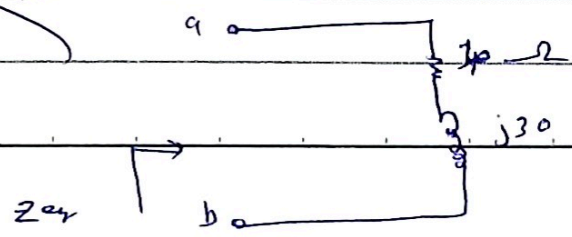
$$Z_{1,2} = \frac{72}{5} - \frac{4}{5}j$$



$$Z_{eq} = Z_4 + Z_{1,2} + Z_3$$

$$= 21.6 + j38.8 + \frac{72}{5} - \frac{4}{5}j + 4 - j8$$

$$Z_{eq} = 40 + 30j$$



* The average value:-

measuring the mean of the voltage or current over a period time.

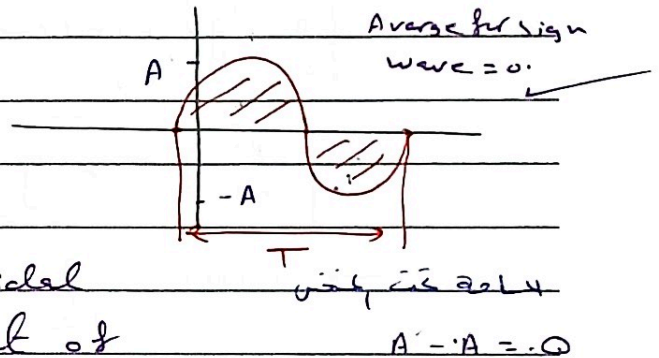
$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(t) dt \rightarrow \text{integral of the signal over period of time.}$$

Ex:- $x(t) = A \cos(\omega t + \theta)$ Find the average $\langle x(t) \rangle$

$$\Rightarrow \langle x(t) \rangle = \frac{1}{T} \int_0^T A \cos(\omega t + \theta) dt \rightarrow \text{Area under curve.}$$

$$= \frac{A}{T} \left[\frac{\sin(\omega t + \theta)}{\omega} \right]_0^T = 0$$

$$= \frac{A}{\omega T} [0] = 0$$



Note! The average of the sinusoidal signal is (zero) independent of Amplitude or freq.

Ex:- Find $\langle x(t) \rangle$

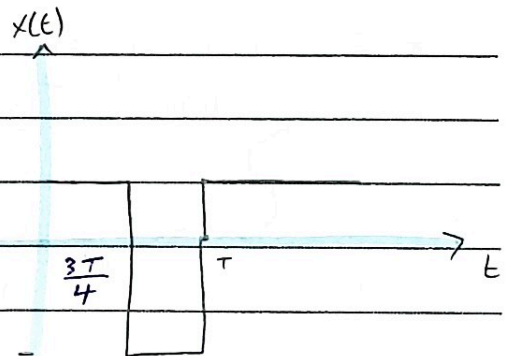
$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(t) dt$$

$$= \frac{1}{T} \left[\int_0^{\frac{3T}{4}} 10 dt + \int_{\frac{3T}{4}}^T -20 dt \right]$$

$$= \frac{1}{T} \left[10t \Big|_0^{\frac{3T}{4}} + (-20t) \Big|_{\frac{3T}{4}}^T \right] = \frac{1}{T} \left(\frac{30}{4} T - \frac{20}{4} T \right)$$

$$\text{So, } \langle x(t) \rangle = 2.5$$

$$= \frac{1}{T} \left(\frac{10}{4} T \right)$$



* if $Z = R + jX_L \Rightarrow$ behave like inductive.

$$0 < \theta < 90$$

* if $Z = R - jX_C \Rightarrow$ behave like capacitive

$$0 > \theta > -90$$

⊛ Admittance :-

$$Z = R + jX \rightarrow \text{Reactance (} X_L \text{ or } X_C \text{)}$$

↓ ↓
Resistance

Impedance

$$Y = G + jB \rightarrow \text{Susceptance}$$

↓ ↓
conductance

$$Y = \frac{1}{Z} \quad \text{which mean } G \neq 1/R$$

$$B \neq \frac{1}{X}$$



Ex :- $Z = 100 \angle 53$ Find Y :-

$$Y = \frac{1}{Z} = \frac{1}{100 \angle 53} = \underbrace{6.01 \times 10^{-3}}_G - j \underbrace{8 \times 10^{-3}}_{B_L}$$

$\frac{X_L}{L}$

④ $Z = R \pm jX$

resistance $\leftarrow$ $\rightarrow$ Reactances

$\rightarrow$ inductor
 $\rightarrow$ capacitor

a) if $Z = R$

assume $V = V_m \angle 0^\circ$

So $\Rightarrow I = \frac{V_m \angle 0^\circ}{R} = \frac{V_m}{R} \angle 0^\circ$

So current and voltage are in phase

phase shift between

phase shift between V & $I = 0$

current and voltage across the resistor.

b) if $Z = -jX_c \Rightarrow$ pure capacitive

assume $V = V_m \angle 0^\circ$

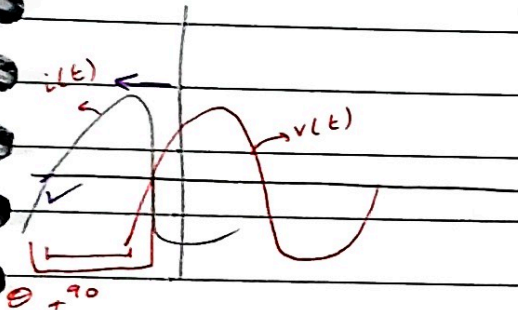
So, $I = \frac{V_m \angle 0^\circ}{X_c \angle -90^\circ} = \frac{V_m}{X_c} \angle 0^\circ + 90^\circ$

shift left

So, current I leads voltage

(voltage) by 90°

or $\Rightarrow$ voltage V lags the current by 90°



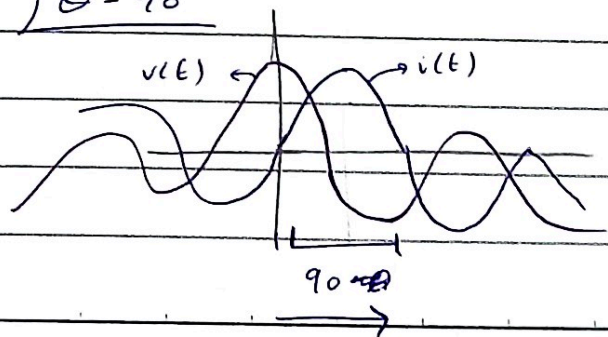
c) $Z = jX_L \Rightarrow$ pure inductive

assume $V = V_m \angle 0^\circ$

So, $I = \frac{V_m \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V_m}{X_L} \angle 0^\circ - 90^\circ$

shift from right

$\Rightarrow$ voltage leads the current
or current lags the voltage.



Note: $v(t) = v_1 \cos(\omega t) + v_2 \sin(\omega t) + \dots + v_n$
 DC value.

$$V_{RMS} = \sqrt{(V_{DC})^2 + \sum_{i=1}^N \frac{v_i^2}{2}}$$

$$\text{or } RMS(\text{Total}) = \sqrt{(RMS_1)^2 + (RMS_2)^2 + (RMS_3)^2 + \dots + (RMS_n)^2}$$

مثال 1

Ex:- $x(t) = \overset{(1)}{3} + \overset{(2)}{2} \cos(\omega t)$ Sign wave
 Find $x_{RMS} = ?$ $\rightarrow \frac{\text{Peak}}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$$RMS = \sqrt{(3)^2 + \left(\frac{2}{\sqrt{2}}\right)^2}$$

$$RMS = \sqrt{9+2} = \sqrt{11}$$

Ex:- $v(t) = \overset{(1)}{4} \cos(100t) + \overset{(2)}{12} \sin(100t) - \overset{(3)}{6} \cos(300t) + \overset{(4)}{5}$
 Find $V_{RMS} = ?$

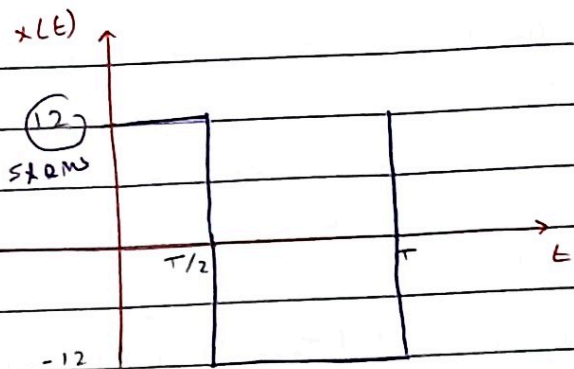
$$V_{RMS} = \sqrt{\left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + (5)^2}$$

$$= \sqrt{123} = 11.09$$



Ex:-

$$x(t) = \begin{cases} 12 & 0 \leq t \leq \frac{T}{2} \\ -12 & \frac{T}{2} \leq t \leq T \end{cases}$$



Find x_{RMS} ?

$$x_{RMS} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

$$= \sqrt{\frac{1}{T} \left[\int_0^{T/2} (12)^2 dt + \int_{T/2}^T (-12)^2 dt \right]}$$

$$= \sqrt{\frac{1}{T} \left(144 \times \frac{T}{2} + \frac{144 T}{2} \right)} = \sqrt{144} = 12$$

Note! ① for square wave signal

$$x_{RMS} = A \Rightarrow \text{max value.}$$

② for triangle signal

$$x_{RMS} = \frac{A}{\sqrt{3}}$$

③ for DC-signal

$$x_{RMS} = \text{DC value.}$$

Ex:- $V_{RMS} = 220$ volt. Find V_{peak} ? $V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$

$$V_{peak} = (V_{RMS}) \times \sqrt{2}$$

$$= 220 \times \sqrt{2} = 311 \text{ volt}$$



* Root mean Square (RMS):-

RMS = effective value of varying voltage or current.

$$X_{RMS} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} \Rightarrow X_{\text{effective}}$$

Ex:- $x(t) = A \cos(\omega t)$ Find X_{RMS}

$$\cos^2 x = \frac{1}{2} (1 + \cos(2x))$$

$$X_{RMS} = \sqrt{\frac{1}{T} \int_0^T A^2 \cos^2(\omega t) dt}$$

$$= \sqrt{\frac{A^2}{2T} \int_0^T (1 + \cos(2\omega t)) dt}$$

$$= \sqrt{\frac{A^2}{2T} [T + 0]} = \sqrt{\frac{A^2}{2}}$$

$$\Rightarrow X_{RMS} = \frac{A}{\sqrt{2}}$$

Note! * For sinusoidal signal

$$X_{RMS} = \frac{\text{Peak Value}}{\sqrt{2}}$$



-10j

At Node ③ $2I_{in} = 4I_{out}$

$$V_2 = 14.28 \angle -54.8^\circ \text{ Volt}$$

$$\frac{V_2 - V_3}{2 + j6} + 4 \angle 0^\circ = \frac{V_3 - 0}{-j25} + \frac{V_3 - 0}{10}$$

$$V_3 = 17.33 \angle 1.99^\circ \text{ Volt}$$

$$V_1(t) = 14.28 \cos(10^4 t - 54.8^\circ) \text{ V}$$

$$V_2(t) = 17.33 \cos(10^4 t + 1.99^\circ) \text{ V}$$

$$\textcircled{1} \rightarrow 0 = \frac{V_2 - V_1}{5} + \frac{V_2 - 0}{20} + \frac{V_2}{-50j} + \frac{V_2 - V_3}{2 + j6} \quad \dots \textcircled{1}$$

$$\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{-50j} + \frac{1}{2 + j6} \right) V_2 - \left(\frac{1}{2 + j6} \right) V_3 = \frac{V_1}{5}$$

$\downarrow$ Z_{22} $\downarrow$ Z_{23}

$$Z_{22} = 0.3 - 0.13j$$

$$Z_{23} = 0.05 - 0.13j$$

$$\frac{V_1}{5} = \frac{10 \angle -90^\circ}{5} = -2j$$

$$\text{So, } Z_{22} V_2 - Z_{23} V_3 = -2j \quad \dots \textcircled{1}$$



$$\frac{V_2 - V_3}{2 + j6} + 4 \angle 0^\circ = \frac{V_3 - 0}{-j25} + \frac{V_3 - 0}{10}$$

$$\left(\frac{1}{2 + j6} \right) V_2 - \left(\frac{1}{2 + j6} + \frac{1}{-j25} + \frac{1}{10} \right) V_3 = -4 \angle 0^\circ$$

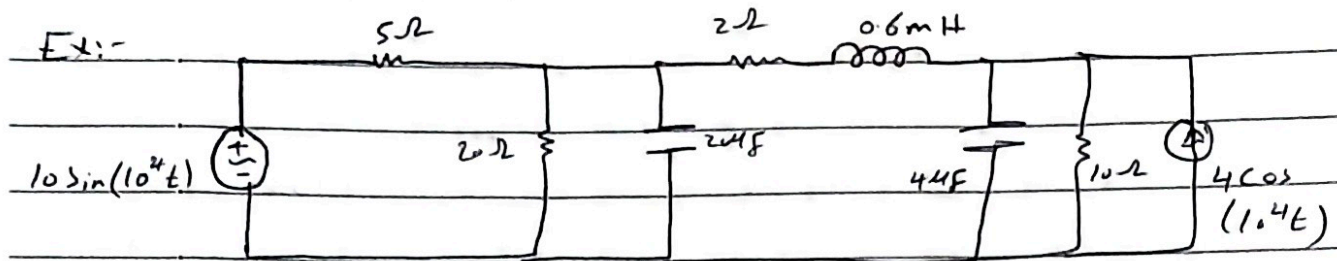
$\downarrow$ $Z_{33} = -0.11j$

$$Z_{23} V_2 - Z_{33} V_3 = -4 \angle 0^\circ \quad \dots \textcircled{2}$$

$$\hookrightarrow V_2 = \frac{-4 \angle 0^\circ + Z_{33} V_3}{Z_{23}}$$

$$\rightarrow Z_{22} \left(\frac{-4 \angle 0^\circ + Z_{33} V_3}{Z_{23}} \right) - Z_{23} V_3 = -2j$$

$$0.3 - 0.13j \left(\frac{-4 + -0.11j V_3}{0.05 - 0.15j} \right) - (0.05 - 0.15j) V_3 = -2j$$



Find the value of nodal voltage using nodal analysis
 $V_1(t)$ & $V_2(t)$

① $\omega = 10^4 \text{ rad/s}$

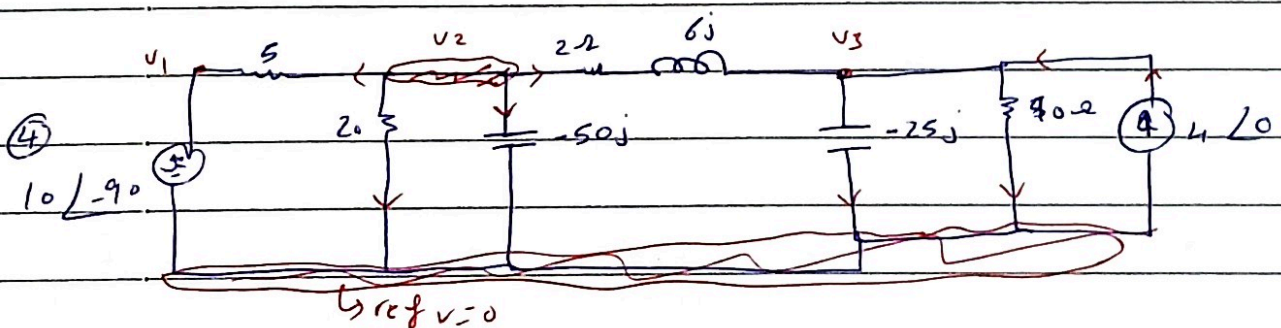
② $10 \sin(10^4 t) \Rightarrow 10 \cos(10^4 t - 90^\circ)$
 $= 10 \angle -90^\circ$

$4 \cos(10^4 t) \Rightarrow 4 \angle 0^\circ$

③ $2 \mu\text{F} \rightarrow Z_C = \frac{-j}{10^4(2\mu)} = -50j$

$0.6 \text{ mH} \rightarrow Z_L = j(10^4)(0.6 \text{ m}) = 6j$

$4 \mu\text{F} \rightarrow Z_C = \frac{-j}{10^4(4\mu)} = -25j$



$V_1 = 10 \angle -90^\circ \text{ volt}$

at Node ② $\Rightarrow \sum I_{in} = \sum I_{out}$

$$0 = \frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_2 - 0}{-50j} + \frac{V_2 - V_3}{2 + 6j}$$

AC - Analysis steps: -

- ① Note the frequency of the sinusoidal excitation.
- ② Convert all sources to phaser form (freq. domain)
- ③ Convert each circuit element to impedance form

$$Z_R(j\omega) = R$$

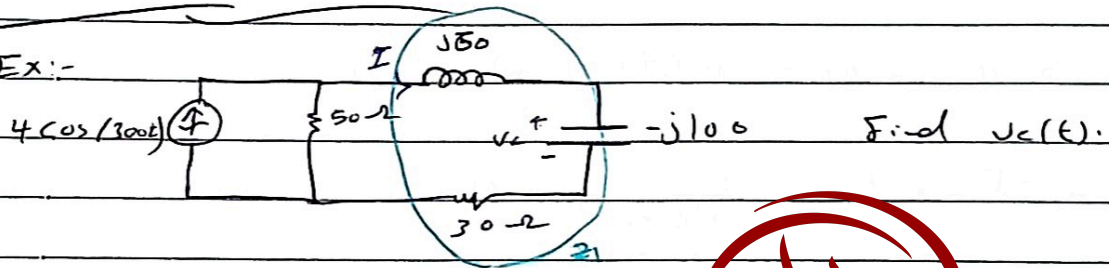
$$Z_L(j\omega) = j\omega L$$

$$Z_C(j\omega) = \frac{-j}{\omega C}$$

- ④ Solve the resulting phaser circuit using any method (mesh, Nodal, Super position, source transformation, Norton & Thevenin. - - -)

- ⑤ Convert to time domain.

Ex:-



$$\textcircled{1} \omega = 300 \text{ rad/s}$$

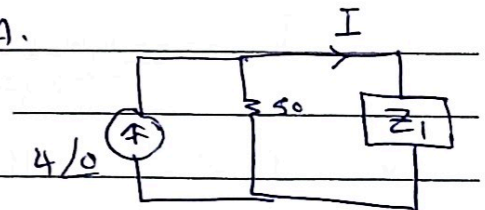
$$\textcircled{2} j60 \Omega$$

$$-j100 \Omega$$

$$\textcircled{3} 4 \cos(300t) \Rightarrow 4 \angle 0^\circ \text{ A.}$$

$$Z_1 = 30 + j60 - j100$$

$$Z_1 = 30 - j40 \Omega$$



$$-I = \frac{4 \angle 0 \left(\frac{1}{Z_1} \right)}{\frac{1}{Z_1} + \frac{1}{50}}$$

$$= 2 + j$$

Series ← Z1
3 ohm

3 ohm 3 ohm 3 ohm

$$\text{So, } V_C = I(-j100) \Rightarrow V_C(t) = 223.6 \cos(300t - 63.4^\circ) \text{ V}$$

$$= 223.6 \angle -63.4^\circ \text{ V}$$

$$v_o(t) = 38.65 \cos(25t + 165)$$

$$v_o(t) = 554.7 \cos(50t - 3.7)$$

$$v_{oR}(t) = v_o(t) + v_o''(t)$$

Ex:-

$$v(t) = 12 \sin(500t + 30)$$

$$i(t) = 0.2 \cos(500t)$$

$$v(t) = 12 \cos(500t - 60)$$

$$V = 12 \angle -60 \text{ volt}$$

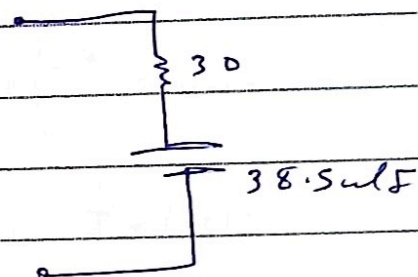
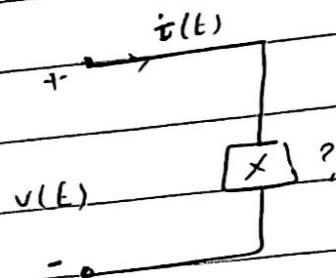
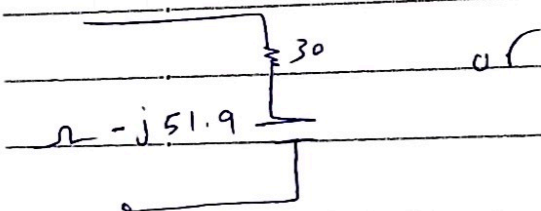
$$I = 0.2 \angle 0 \text{ A}$$

$$X = \frac{V}{I} = \frac{12 \angle -60}{0.2 \angle 0} = 60 \angle -60 \Omega$$

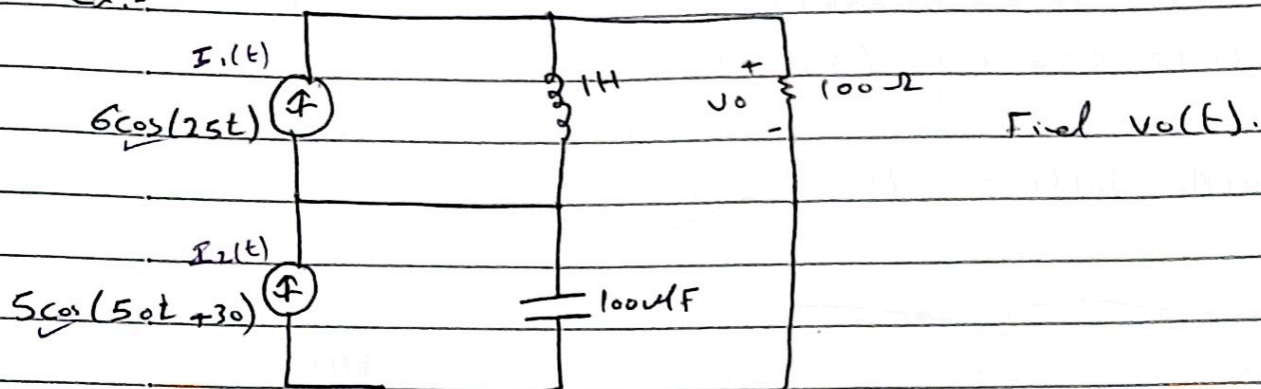
$$X = \underbrace{30}_{R} - j \underbrace{51.9}_{X_C} \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{500 \mu C} = 51.9$$

$$C = 38.5 \mu F$$

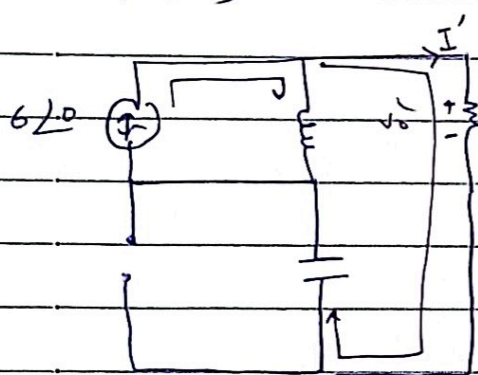


Ex:-



* ω not same value $\Rightarrow$ so I will solve using Superposition.

① Leave $I_1(t)$ & kill $I_2(t)$.



$$\omega = 25 \text{ rad/s}$$

$$1 \text{ H} \Rightarrow Z_L = j\omega L = j25 \Omega$$

$$100 \mu\text{F} \Rightarrow Z_C = \frac{-j}{25 + j400} = -j400 \Omega$$

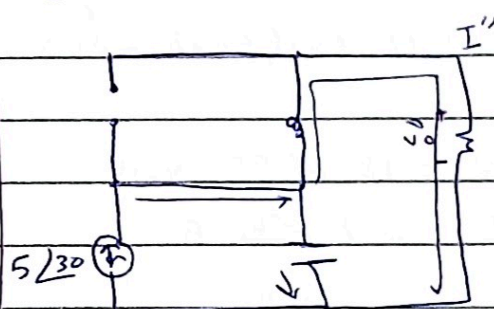
By current division

$$I' = \frac{6/20}{100 - j400}$$

$$\frac{1}{100 - j400} + \frac{1}{j25}$$

$$V_0 = (100) \times I' = 38.30 \angle 165^\circ$$

② leave $I_2(t)$ & kill $I_1(t)$



$$\omega = 50 \text{ rad/s}$$

$$1 \text{ H} \Rightarrow Z_L = j(50)(1) = j50 \Omega$$

$$100 \mu\text{F} \Rightarrow Z_C = \frac{-j}{50(100 \mu\text{F})} = -200j \Omega$$

By current division

$$I'' = \frac{(5/30) \left(\frac{1}{100 + j50} \right)}{\left(\frac{1}{100 + j50} \right) + \frac{1}{-j200}}$$

$$= -0.37 + 0.89j$$

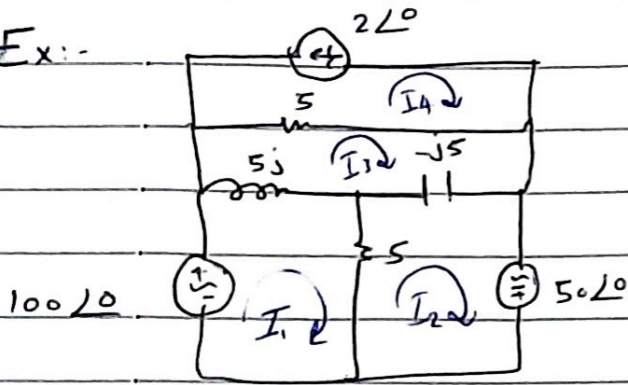
$$I'' = 4.55 \angle -3.7^\circ$$

$$V_0 = 100 \times I'' = 554.7 \angle -3.7^\circ$$

$V_0 \neq V' + V'' \Rightarrow$ Due to different freq.



Ex:-



write mesh equations:-

$$\Rightarrow I_4 = -2\angle 0 \text{ A.}$$

for mesh ① $\sum V = 0$

$$-100\angle 0 + 5jI_1 + 5I_1 - 5jI_3 - 5I_2 = 0$$

for mesh ② $\sum V = 0$

$$5I_2 - j5I_2 - 50\angle 0 - 5I_1 - (j5)I_3 = 0$$

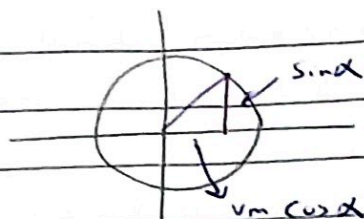
for mesh ③ $\sum V = 0$

$$5jI_3 + 5I_3 + (-j5I_2) - 5I_4 - 5jI_1 - (-5j)I_2 = 0$$

Sinusoids

The angular velocity $\omega = \alpha / t$

$$\omega = \frac{2\pi}{T} \text{ or } \omega = 2\pi f$$



$$v(t) = V_m \sin(\omega t + \phi)$$

amplitude

$$T = \frac{1}{f}$$

$$\theta = \frac{\text{phase shift (no. of div)}}{T (\text{no. of div})} \times 360^\circ$$

Ex: Given a sinusoid $5 \sin(4\pi t - 60^\circ)$, Calculate it's amplitude, phase, angular frequency, period, and frequency.

A $\rightarrow$ amplitude = 5

Phase $\rightarrow -60^\circ$

angular frequency = 4π rad/s

$$\text{period } T = \frac{1}{f} = \frac{1}{2} = 0.5 \text{ s}$$

$$\frac{\omega}{2\pi} = f = \frac{4\pi}{2\pi} = 2 \text{ Hz}$$

Ex: Find the phase angle between $i_1 = -4 \sin(377t + 25^\circ)$

and $i_2 = 5 \cos(377t - 40^\circ)$. Does i_1 lead i_2 or lag?

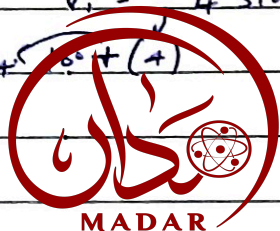
$$\sin(\omega t + 90^\circ) = \cos \omega t$$

$$i_2 = 5 \sin(377t - 40^\circ + 90^\circ) = 5 \sin(377t + 50^\circ)$$

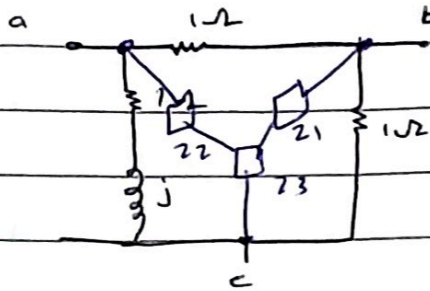
$$i_1 = -4 \sin(377t + 25^\circ) = 4 \sin(377t + 180^\circ + 25^\circ) = 4 \sin(377t + 205^\circ)$$

$$\theta = 205^\circ - 50^\circ = 155^\circ$$

i_1 leads i_2 by 155°



Ex:-



D

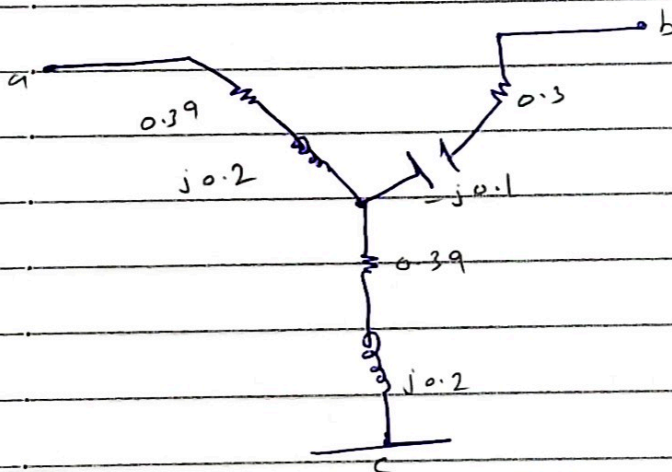


Convert from Δ to Y

$$Z_1 = \frac{(1)(1)}{1 + 1 + (1+j)} = \frac{1}{3+j} = 0.3 - j0.1 \Omega$$

$$Z_2 = \frac{(1)(1+j)}{3+j} = 0.39 + j0.2 \Omega$$

$$Z_3 = \frac{(1)(1+j)}{3+j} = 0.39 + j0.2 \Omega$$



* Max Power Transfer in AC Circuit: -

value of load to get max power transfer.

$$Z_L = Z_{th}^* \rightarrow \text{conjugate.}$$

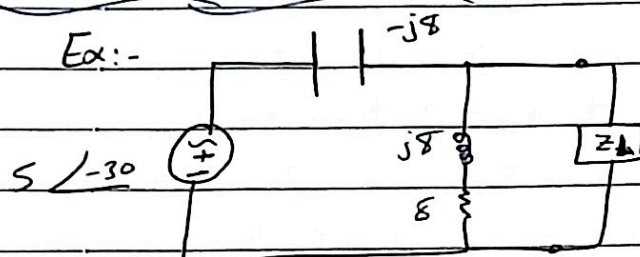
Ex:- $Z_1 = 3 + j2$

$Z_1^* = 3 - j2$

only change the sign of the Imaginary part

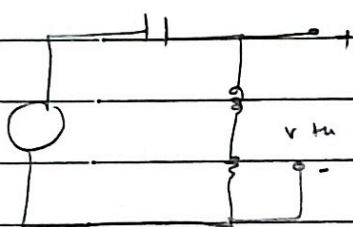
Ex: $Z_1 = 12 \angle -60^\circ$

$Z_1^* = 12 \angle 60^\circ \rightarrow$ change the sign.



Final V_{th} , I_N , Z_{th}
 Z_L for max power.

① Final V_{th}

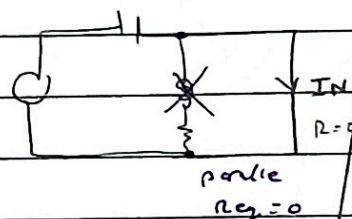


using voltage division

$$V_{th} = \frac{5 \angle -30^\circ \cdot (8 + j8)}{8 + j8 - j8}$$

$$V_{th} = 7.07 \angle 15^\circ$$

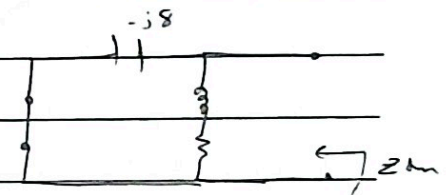
② Final I_N



$$I_N = \frac{5 \angle -30^\circ}{-j8}$$

$$I_N = 0.625 \angle 60^\circ \text{ A}$$

③ Final Z_{th}



$$Z_{th} = (-j8) \parallel (8 + j8)$$

$$Z_{th} = 8 - j8 \Omega$$

$$Z_L = Z_{th}^* \Rightarrow Z_L = 8 + j8 \Omega$$

Transfer a sinusoid to and from the time domain to the phasor domain:-

$$v(t) = V_m \cos(\omega t + \phi) \quad \longleftrightarrow \quad V = V_m \angle \phi$$

(time domain) (phasor domain)

Time domain

$$V_m \cos(\omega t + \phi)$$

$$V_m \sin(\omega t + \phi)$$

$$I_m \cos(\omega t + \phi)$$

$$I_m \sin(\omega t + \phi)$$

Phasor Domain

$$V_m \angle \phi$$

$$V_m \angle \phi - 90^\circ$$

$$I_m \angle \phi$$

$$I_m \angle \phi - 90^\circ$$



Ex:- Transform of the following sinusoids to phasors:-
 $i = 6 \cos(50t - 40^\circ) \text{ A}$, $v = -4 \sin(30t + 50^\circ) \text{ V}$

a) $I = 6 \angle -40^\circ \text{ A}$

b) $v = -4 \cos(30t + 50 + 90) = 4 \cos(30t + 140) \text{ V}$
 $V = 4 \angle 140^\circ \text{ V}$

Ex:- Transfer the sinusoids corresponding to phasors

$$V = -10 \angle 30^\circ \text{ V}, \quad I = j(5 - j12) \text{ A}$$

a) $v(t) = 10 \cos(\omega t + 180 + 30) = 10 \cos(\omega t + 210) \text{ V}$

b) $I = 12 + j5 = \sqrt{12^2 + 5^2} \angle \tan^{-1}\left(\frac{5}{12}\right) = 13 \angle 22.62^\circ$

$$i(t) = 13 \cos(\omega t + 22.62^\circ)$$

* Evaluate $[(5+j2)(-1+j4) - 5 \angle 60^\circ] \text{ and } \frac{10+j5+3 \angle 40^\circ}{-3+j4} + 10 \angle 70^\circ$

a) $(5+j2)(-1+j4) = -5 \angle 60^\circ$
 $= -15.5 - 13.67j = 20.67 \angle 138.59^\circ$

b) $\frac{10+j5+3 \angle 40^\circ}{-3+j4} + 10 \angle 70^\circ = 8.293 + 2.2j$
 $= 8.58 \angle 14.86^\circ$

Mathematic operation of complex number.

① Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

② subtraction $z_1 - z_2 =$

③ Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

④ Division $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$

⑤ Reciprocal $\frac{1}{z} = \frac{1}{r} \angle -\phi$



⑥ Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$

⑦ Complex conjugate $z^* = x - jy = r \angle -\phi = r e^{-j\phi}$

⑧ Euler's identity $e^{j\phi} = \cos \phi + j \sin \phi$

$\rightarrow \cos \phi = \text{Re}(e^{j\phi})$ real part

$\rightarrow \sin \phi = \text{Im}(e^{j\phi})$ imaginary part.

$v(t) = V_m \cos(\omega t + \phi) = \text{Re}(V_m e^{j(j\omega t + \phi)}) = \text{Re}(V_m e^{j\phi} e^{j\omega t}) = \text{Re}(V_m e^{j\omega t})$

Rotating phasor $\Rightarrow v(t) = V_m \cos(\omega t + \theta)$

$v(t) = \text{Re}[V_m e^{j(\omega t + \theta)}]$

$v(t) = \text{Re}[V_m \angle (j\omega t + \theta)]$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(\omega t \pm 180) = -\sin \omega t$$

$$\cos(\omega t \pm 180) = -\cos \omega t$$

$$\sin(\omega t \pm 90) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90) = \mp \sin \omega t$$



$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \Theta)$$

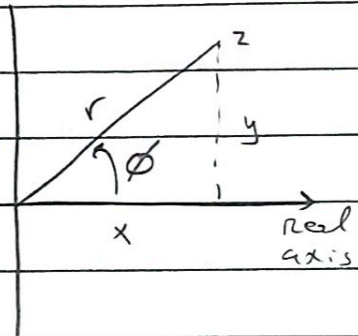
$$\text{where } C = \sqrt{A^2 + B^2} \text{ and } \Theta = \tan^{-1} \frac{B}{A}.$$

phasor

a. rectangular $Z = x + jy = r(\cos \phi + j \sin \phi)$

b. polar $Z = r \angle \phi$

c. Exponential $Z = r e^{j\phi}$



$$\text{where } r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1} \left(\frac{y}{x} \right)$$

$Q \quad p/x$

$$Z: \text{Impedance} = \frac{V}{I} = R + jX$$

$$Y: \text{reciprocal } Y = \frac{1}{Z} = \frac{1}{R + jX}$$



Impedances and admittances of passive elements

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

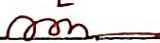
L  $Z = j\omega L$  short circuit at dc $\omega = 0, Z = 0$

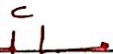
open circuit at high frequency $Z \rightarrow \infty, \omega \rightarrow \infty$

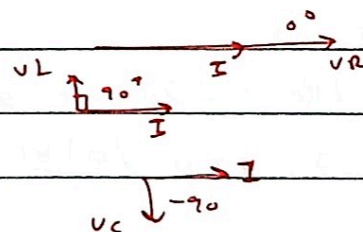
C  $Z = \frac{1}{j\omega C}$  open at dc $\omega = 0, Z \rightarrow \infty$

short at high frequency $\omega \rightarrow \infty, Z = 0$

R  $Z = R$

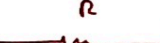
L  $Z = X_L$

C  $Z = X_C$



R  L  $Z = \sqrt{R^2 + X^2}$ positive between 0 & 90°

R  C  $Z = \sqrt{R^2 + X^2}$ negative between -90 & 0

R  L  C  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ negative if $X_C > X_L$
positive if $X_C < X_L$

Example:- Find the voltage $v(t)$ in a circuit described by the integrodifferential equation using the phasor approach.

$$2 \frac{dv}{dt} + 5v + 10 \int v dt = 20 \cos(5t - 30^\circ)$$

$$2j\omega V + 5V + \frac{10}{j\omega} V = 20 \angle -30^\circ$$

$$\omega = 5$$

$$V [j10 + 5 - j(10/5)] = V(5 + j8) = 20 \angle -30^\circ$$

$$V = \frac{20 \angle -30^\circ}{5 + j8} = 2.12 \angle -88^\circ$$

$$\text{So, } v(t) = 2.12 \cos(5t - 88^\circ) \text{ V}$$



The phasor relationships on the circuit element.

	Time domain	Frequency domain
• Resistor	$v = iR$	$V = IR$
• Inductor	$L \Rightarrow v = L \frac{di}{dt}$	$V = j\omega L I$
• Capacitor	$C \Rightarrow i = C \frac{dv}{dt}$	$I = j\omega C V \Rightarrow V = \frac{I}{j\omega C}$

Ex:- If voltage $v(t) = 6 \cos(100t - 30^\circ)$ is applied to a $50 \mu\text{F}$ capacitor calculate the current through the capacitor.

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{100 \times 50 \times 10^{-6}} = -200j = 200 \angle -90^\circ$$

Initial value

$$V = 6 \angle -30^\circ \text{ volt.}$$

$$I = \frac{V}{Z_C} = \frac{6 \angle -30^\circ}{200 \angle -90^\circ} = 0.03 \angle 60^\circ$$

$$i(t) = 0.03 \cos(100t + 60^\circ) \text{ A}$$

$$= 30 \cos(100t + 60^\circ) \text{ mA}$$

$$\int v dt \leftrightarrow \frac{V}{j\omega}$$

Ex:- Use phasor approach, determine the current $i(t)$ in a circuit described by the integro-differential equation.

$$4i + 8 \int i dt - 3 \frac{di}{dt} = 50 \cos(2t + 75^\circ)$$

$$v(t) = 50 \cos(2t + 75^\circ)$$

$$\omega = 2 \text{ rad/s}, V = 50 \angle 75^\circ$$

$$\int Z_C$$

$$4i + 8 \int i dt - 3 \frac{di}{dt} = v(t)$$



$$4i \rightarrow Z_R = R \rightarrow 4 \Omega$$

$$8 \int i dt \rightarrow Z_C = \frac{8}{j\omega} = \frac{8}{j2} = -\frac{4}{j} = -4j \Omega$$

$$-3 \frac{di}{dt} \rightarrow Z_L = -3j\omega = -6j \Omega$$

$$Z_{eq} = 4 - j4 - 6j \Rightarrow Z_{eq} = 4 - 10j \Omega$$

$$I = \frac{V}{Z_{eq}} \Rightarrow \frac{50 \angle 75^\circ}{4 - 10j} = 4.64 \angle 143.2^\circ \text{ A}$$

$$\text{So, } i(t) = 4.64 \cos(2t + 143.2^\circ) \text{ A}$$

Ex:- Find $v(t) = v_1(t) + v_2(t)$

$$v_1(t) = -10 \sin(\omega t + 30^\circ)$$

$$v_2(t) = 20 \cos(\omega t - 45^\circ)$$

$$v(t) = 10 \cos(\omega t + 30 + 90) + 20 \cos(\omega t - 45)$$

$$= 10 \angle 120^\circ + 20 \angle -45^\circ = 9.14 - 5.4j$$

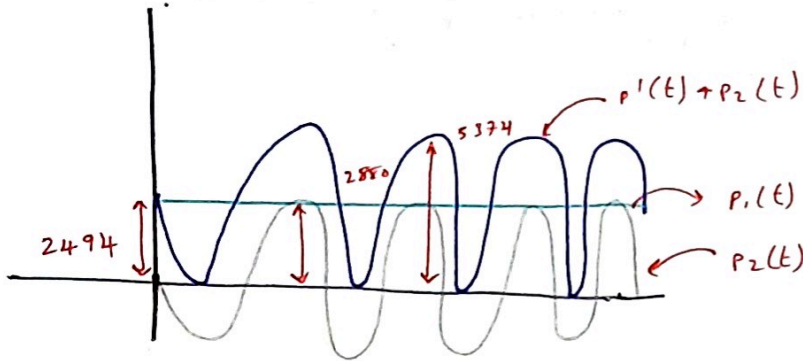
$$= 10.66 \angle -30.95^\circ \text{ So, } v(t) = 10.66 \cos(\omega t - 30.95^\circ) \text{ V}$$

$$\rightarrow p(t) = v(t)i(t) = \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{P_1(t)} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{P_2(t)}$$

Instantaneous power.

$$\rightarrow v(t) = 120\sqrt{2} \cos(377t + 60^\circ)$$

$$i(t) = 24\sqrt{2} \cos(377t + 30^\circ)$$



$$\phi_v = 60^\circ, \phi_i = 30^\circ, V_m = 120\sqrt{2}, I_m = 24\sqrt{2}$$

$$\text{So, } P_1(t) = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

$$P_1(t) = \frac{1}{2} (120\sqrt{2})(24\sqrt{2}) (\cos(60 - 30))$$

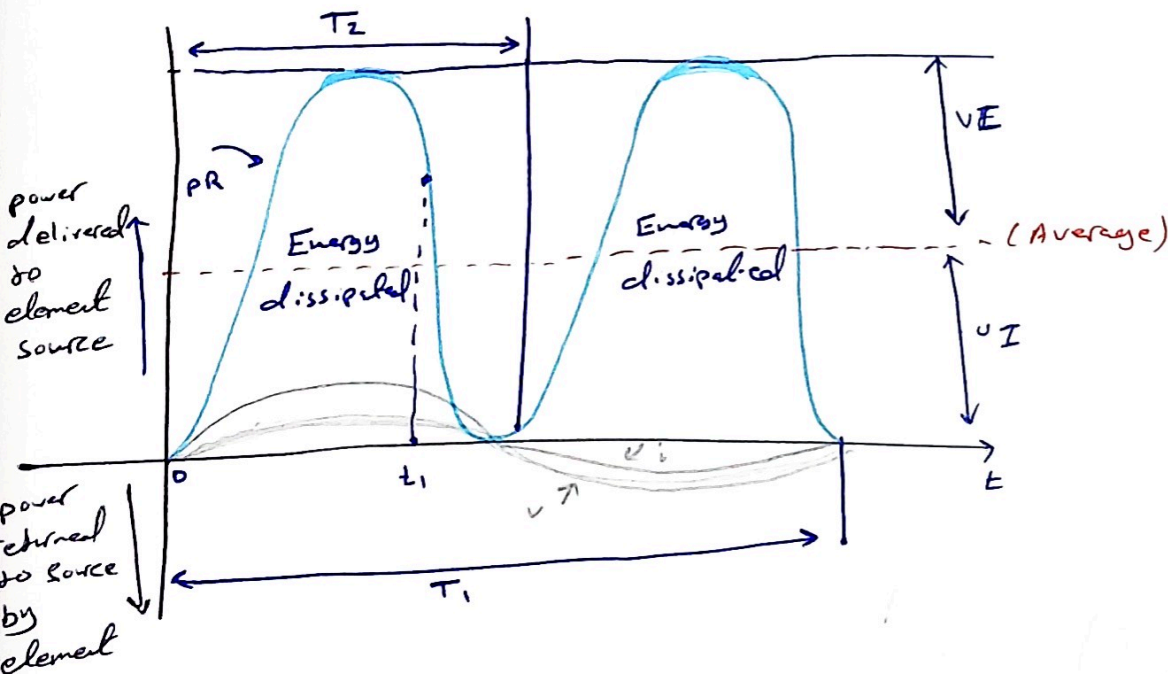
$$\rightarrow P_2(t) = \frac{1}{2} V_m I_m \cos(2\omega t + \phi_v + \phi_i)$$

$$= \frac{1}{2} (120\sqrt{2})(24\sqrt{2}) \cos(2\omega t + 60 + 30)$$



Resistive circuit and Real power.

الطاقة الحقيقية.

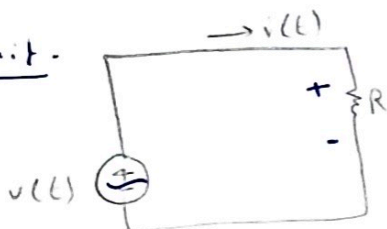


Effective or RMS value. (Root mean square value).

$$v(t) \rightarrow V_{rms} = v_{eff}$$

$$i(t) \rightarrow I_{rms} = i_{eff}$$

→ In AC circuit.



total power dissipated R is given by:

$$P = \frac{1}{T} \int_0^T i^2 R dt = \frac{R}{T} \int_0^T i^2 dt = I_{rms}^2 R.$$

$$P(t) = i^2(R) \text{ instantaneous power.}$$

$$P_{avg} = \frac{1}{T} \int_0^T i^2(R) dt.$$

AC

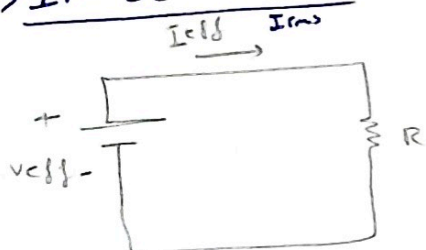
$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$[Watt]$
 $S [VA]$
 Apparent power
 power factor (pf)

$$\frac{V_m}{\sqrt{2}} = V_{rms} \quad \frac{I_m}{\sqrt{2}} = I_{rms}$$

$$\theta_v - \theta_i = \theta_2 \text{ (impedance)}$$

→ In DC circuit



$$P_{avg} = I_{eff}^2 R$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = I_{rms}$$



$$i(t) = I_{rms} \cos(\omega t)$$

$$I_{RMS} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2 \omega t dt} = \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_m I_m \cos(\theta_v - \theta_i)$$

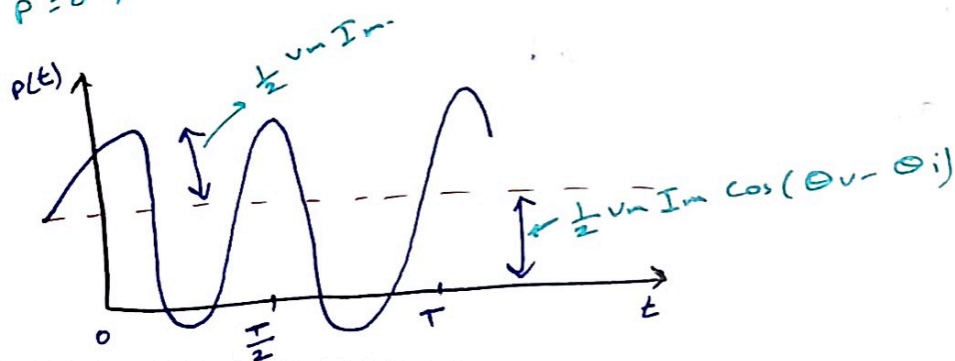
$$= \frac{1}{T} \int_0^T P(t) dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

1. $P \rightarrow$ is not time dependent.

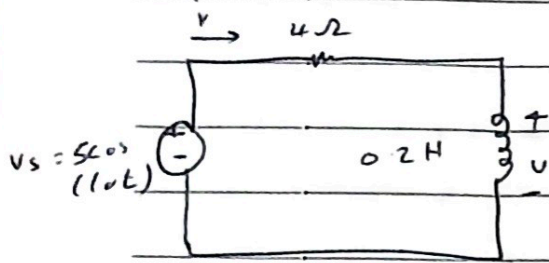
2. when $\theta_v = \theta_i$ it is a purely resistive load case.

3. when $\theta_v - \theta_i = \pm 90^\circ$ it is a purely reactive load case.

4. $P = 0$ mean that the circuit absorbs no average power.



Ex:- Refer to figure below, determine $v(t)$ and $i(t)$:-



$$v(t) = 5 \cos(10t)$$

$$R = 4 \Omega$$

$$L = 0.2 \text{ H}$$



$$A = 5 \text{ V}, \quad \omega = 10 \text{ rad/s}, \quad \theta = 0^\circ$$

$$V_s = 5 \angle 0^\circ \text{ V}$$

$$Z_R = R = 4, \quad Z_L = j\omega L = j(10)(0.2) = 2j \Omega$$

$$Z_{eq} = 4 + 2j \Omega \Rightarrow Z_{eq} = 4.47 \angle 26.56^\circ$$

$$I = \frac{V_s}{Z_{eq}} = \frac{5 \angle 0^\circ}{4.47 \angle 26.56^\circ}$$

$$I = 1.118 \angle -26.57^\circ \text{ A}$$

$$i(t) = 1.118 \cos(10t - 26.57^\circ) \text{ A}$$

$$V = I \cdot Z_L$$

$$V = 1.118 \angle -26.57^\circ \cdot j2$$

$$V = 2.236 \angle 63.43^\circ \text{ Volt}$$

$$v(t) = V_m \cos(\omega t + \phi)$$

$$= 2.236 \cos(10t + 63.43^\circ) \text{ V}$$

* Apparent power and power factor.

→ purely resistive load (R)

$$\theta_v - \theta_i = 0$$

$$pf = 1$$

$p/s = 1$, all power are consumed.

→ purely reactive load (L or C)
jωL
-j/ωC

$$\theta_v - \theta_i = \pm 90^\circ$$

$$pf = 0$$

$p = 0$, no real power consumption.

→ Resistive and reactive load (R and L/C)

$$\theta_v - \theta_i > 0 \quad (\theta +ve)$$

$$\theta_v - \theta_i < 0 \quad (\theta -ve)$$

• lagging - inductive load
current lags voltage

• leading - capacitive load.
voltage lags current.

$$pf = \cos(\theta_v - \theta_i) = \cos(\theta_z)$$

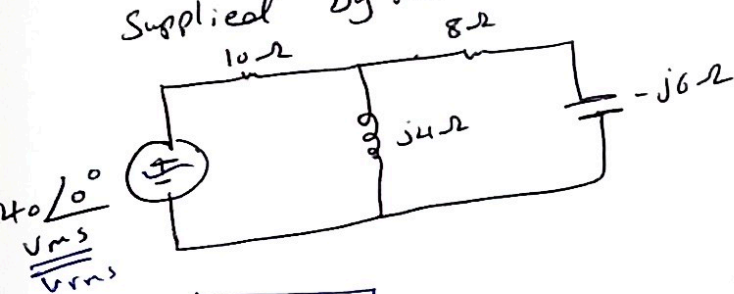
$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$$S = \frac{1}{2} V_m I_m = V_{rms} I_{rms} \quad [VA] \text{ apparent power.}$$



Ex:- Calculate the power factor seen by the source and the average power supplied by the source:-

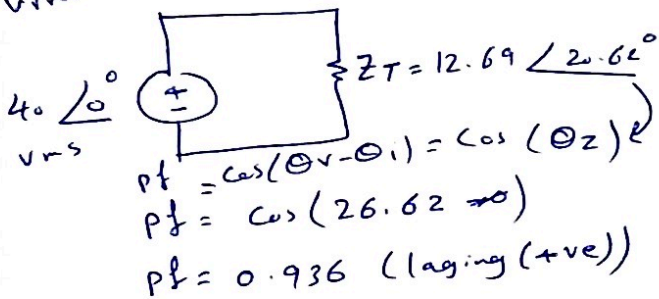


$$Z_T = 10 + j4 \parallel (8 - j6)$$

$$= 10 + \frac{j4 \parallel (8 - j6)}{1}$$

$$= 10 + \frac{1}{\frac{1}{j4} + \frac{1}{8 - j6}} = 12.69 \angle 20.62^\circ$$

$$Z_T = 12.69 \angle 20.62^\circ$$



$$pf = \cos(\theta_v - \theta_i) = \cos(\theta_z)$$

$$pf = \cos(20.62^\circ)$$

$$pf = 0.936 \text{ (lagging (+ve))}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40 \angle 0^\circ}{12.69 \angle 20.62^\circ} = 3.152 \angle -20.62^\circ$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{avg} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$= 40 \times (3.152) \times 0.936$$

$$= 118 \text{ watt.}$$

Ex:-

Calculate the instantaneous power and average power absorbed by a passive linear network if:-

$$v(t) = 80 \cos(10t + 20^\circ)$$

$$i(t) = 15 \sin(10t + 60^\circ) \Rightarrow i(t) = 15 \cos(10t - 30^\circ)$$

$$V_m = 80V, I_m = 15A, \theta_v = 20^\circ, \theta_i = -30^\circ$$

$$\theta_v - \theta_i = 20^\circ - (-30^\circ) = 50^\circ$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$2\omega t + \theta_v + \theta_i = 2(10t) + 20^\circ - 30^\circ$$

$$2\omega t + \theta_v + \theta_i = 20t - 30^\circ$$

$$\frac{1}{2} V_m I_m = \frac{1}{2} \times 15 \times 80 = 600$$

$$\cos(20^\circ - 30^\circ) = \cos(50^\circ) = 0.642$$

$$\text{So, } p(t) = 600 \times 0.642 + 600 \times \cos(2\omega t + \theta_v + \theta_i)$$

$$= 600 \times 0.642 + 600 \times \cos(20t - 30^\circ)$$

$$p(t) = 385.7 + 600 \cos(20t - 30^\circ) \text{ W} \quad \text{Ans}$$

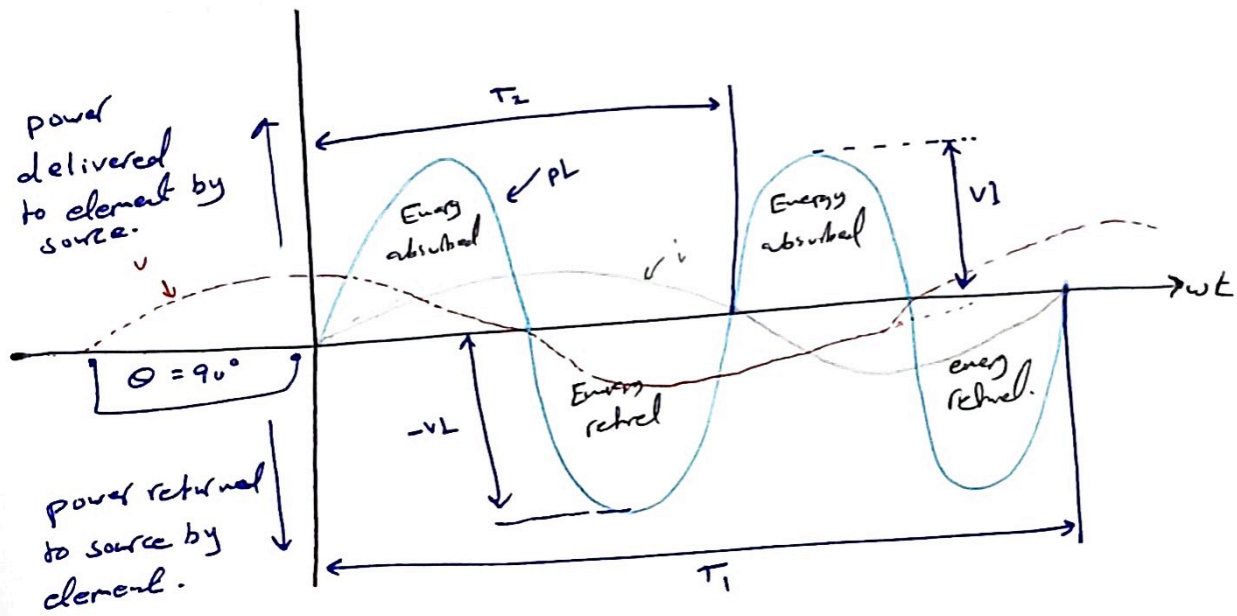
$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= 600 \cos(50^\circ)$$

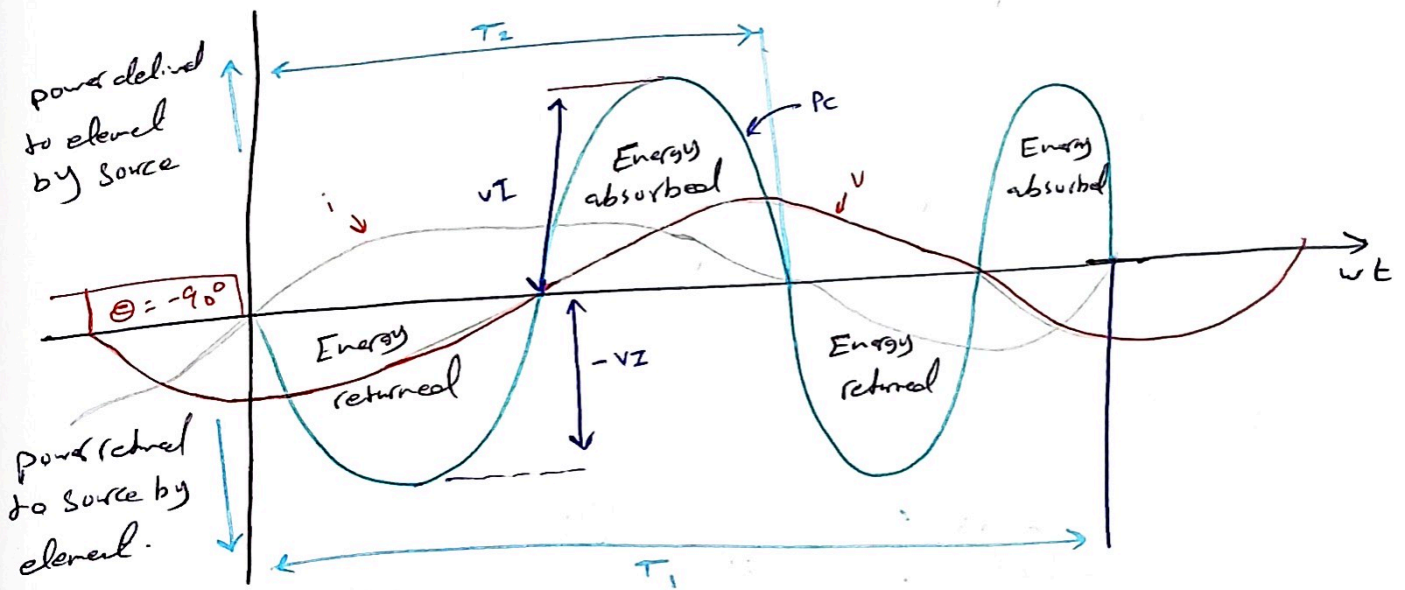
$$= 385.2 \text{ watt} \quad \text{Ans}$$

Real and Reactive Powers.

* Inductive circuit and Reactive power. ($\theta = +ve$)



* Capacitive circuit and Reactive power ($\theta = -ve$)

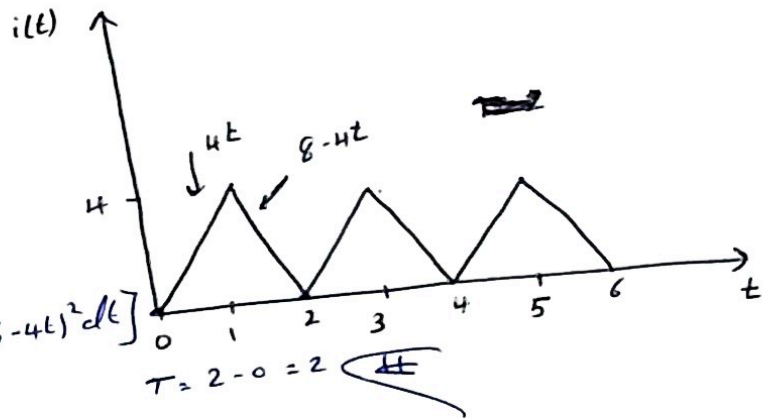


RMS values: -

Ex:- Find the RMS value of the current waveform. Calculate the average power if the current is applied to a 9 Ω resistor.

$$i(t) = \begin{cases} 4t & 0 \leq t < 1 \\ 8-4t & 1 \leq t < 2 \end{cases}$$

$$I_{rms}^2 = \frac{1}{T} \int_0^T i^2 dt = \frac{1}{2} \left[\int_0^1 (4t)^2 dt + \int_1^2 (8-4t)^2 dt \right]$$



$$I_{rms}^2 = \frac{1}{2} \left[\int_0^1 16t^2 dt + \int_1^2 (4-4t+t^2) dt \right]$$

$$\sqrt{I_{rms}^2} = \sqrt{\frac{16}{3}} \Rightarrow I_{rms} = 2.309 \text{ A}$$

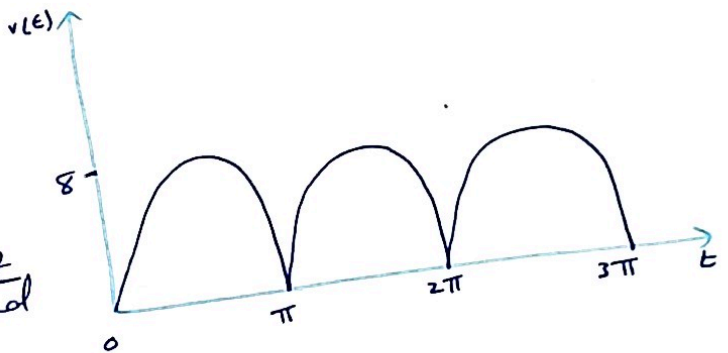
$$\text{So, } P = I_{rms}^2 R = \left(\frac{16}{3}\right)(9) = 48 \text{ watt}$$

Ex:- Find the RMS value of the full-wave rectified sine wave. Calculate the average power dissipated in 6 Ω resistor.

$$T = \pi \quad v(t) = 8 \sin(t) \quad 0 \leq t < \pi$$

$$V_{eff}^2 = \frac{1}{T} \int_0^T v^2 dt = \frac{1}{\pi} \int_0^{\pi} (8 \sin(t))^2 dt$$

$$V^2 = \frac{64}{\pi} \int_0^{\pi} \frac{1}{2} (1 - \cos 2t) dt = \frac{32}{\text{rad}}$$

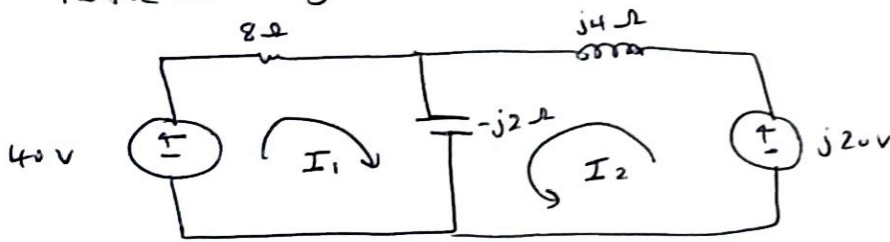


$$V_{eff} = \sqrt{32} = 5.657 \text{ V} \rightarrow \text{rad}$$

$$P = \frac{V_{eff}^2}{R} = \frac{32}{6} = 5.33 \text{ W} \rightarrow \text{rad}$$

Example:-

Calculate the average power absorbed by each of the five element in the circuit given.



mesh ①

$$-40 + 8I_1 + (-j2 \cdot I_1) + (-j2)I_2 = 0$$

$$-20 + 4I_1 - jI_1 - jI_2 = 0$$

$$20 = (4 - j)I_1 - jI_2 \quad \dots ①$$

mesh ②

$$-j20I_2 + I_2 4j + -j2I_2 + -j2I_1 = 0$$

$$-10j + 2jI_2 - jI_2 - jI_1 = 0$$

$$10j = -jI_1 + jI_2$$

$$\begin{bmatrix} 4-j & -j \\ -j & j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 10j \end{bmatrix}$$

$$I_1 = \frac{D_1}{D} = \frac{-10 + j20}{2 + j4} = 5 \angle 53.14^\circ$$

$$I_2 = \frac{D_2}{D} = \frac{10 + 60j}{2 + j4} = 13.6 \angle 17.11^\circ$$

for 40 volt source $V_s = 40 \angle 0^\circ$, $I_1 = 5 \angle 53.14^\circ$

$$P_s = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} 40 \times 5 \times \cos(0 - 53.14)$$

$$= -60 \text{ watt}$$

$$V_s = 20 \angle 90^\circ$$

for 20 volt source $\odot \checkmark$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \times 20 \times 13.6 \times \cos(90 - 17.11)$$

$$= -40 \text{ watt}$$

for resistor $I = |I_1| = 5$, $V = 8|I_1| = 40$

$$P = \frac{1}{2} (40)(5) = 100 \text{ watt}$$

for inductor and capacitor is equal zero watt.

* power supplied by source (voltage source or current source):-
 $P_p = -\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

* power absorbed by a load (resistor, inductor capacitor)
 $P(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$



Ex:- A current $I = 10 \angle 30^\circ$ flows through an impedance.
Find the average power delivered to the impedance.

$$Z = 20 \angle -22^\circ \Omega$$

$$I_{rms} = 10 \text{ A.}$$

$$\theta_1 = 30^\circ$$

$$|Z| = 20 \Omega$$

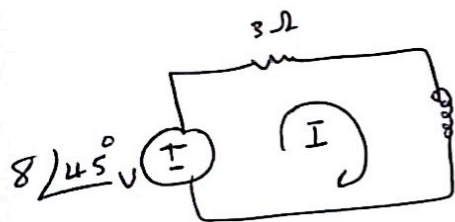
$$\theta_2 = -22^\circ$$

$$P_{avg} = \frac{|I|^2}{2} |Z| \cos(\theta_2)$$

$$= \frac{10^2}{2} \times 20 \times \cos(-22)$$

$$= 927.2 \text{ watt}$$

Ex:- Find the average power absorbed by resistor and inductor. Find the average power supplied by the source.



$$j1\Omega \Rightarrow Z_T = 3 + j1\Omega$$

$$\text{So, } I_r = \frac{V_r}{Z} = \frac{8 \angle 45^\circ}{3 + j1} \Rightarrow 2.53 \angle 26.57^\circ$$

$$\text{for } I_R \Rightarrow Z_R = I = 2.53 \angle 26.57^\circ$$

$$V_R = I \times R = 2.53 \angle 26.57^\circ \times 3 = 7.59 \angle 26.57^\circ$$

$$P_R = \frac{1}{2} V_m I_m = \frac{1}{2} (7.59 \angle 26.57^\circ) \times (2.53 \angle 26.57^\circ)$$

$$P_R = 9.6 \text{ watt}$$

$$\text{for } I_L = I = 2.53 \angle 26.57^\circ$$

$$V_L = jI = j \times 2.53 \angle 26.57^\circ = 2.53 \angle 116.57^\circ$$

$$P_L = \frac{1}{2} (V_m) (I_m) \cos(\theta_V - \theta_I)$$

$$= \frac{1}{2} (2.53)^2 \cos(90) = 0 \text{ watt.}$$

Power supplied is

$$P_{avg} = \frac{1}{2} \times V_m \times I_m \times \cos(\theta_V - \theta_I)$$

$$= \frac{1}{2} \times 8 \times 2.53 \times \cos(45 - 26.57)$$

$$= 9.6 \text{ watt}$$



Real and Reactive powers.

→ The Real power is the only useful power delivered to the load.

→ The REACTIVE power represents the energy exchange between the source and reactive part of the load. It is being transferred back and forth between the load and the sources.

→ The unit of Q is volt ampere reactive (VAR)

$$S = P + jQ = \text{Re}(S) + j\text{Im}(S)$$

= Real power + Reactive power.

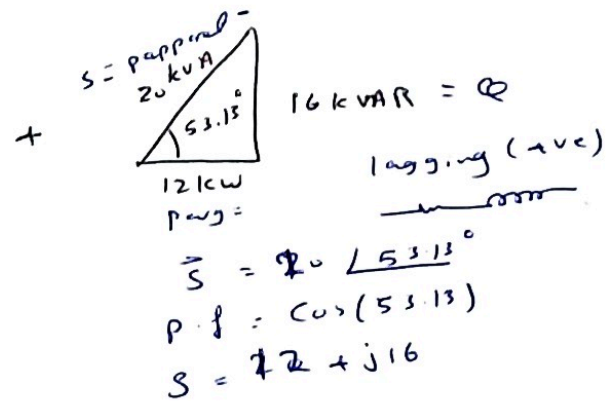
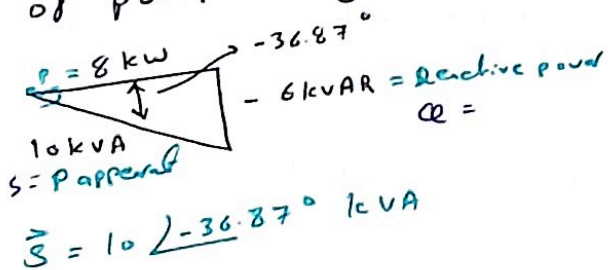
$$S = I_{\text{rms}}^2 Z = I_{\text{rms}}^2 (R + jX) = P + jQ.$$

$$\begin{bmatrix} P \text{ [W]} \\ S \text{ [VA]} \\ Q \text{ [VAR]} \end{bmatrix}$$

$$P = V_{\text{RMS}} I_{\text{RMS}} \cos(\theta_v - \theta_i) = \text{Re}(S) = I_{\text{RMS}}^2 R$$

$$Q = V_{\text{RMS}} I_{\text{RMS}} \sin(\theta_v - \theta_i) = \text{Im}(S) = I_{\text{RMS}}^2 X$$

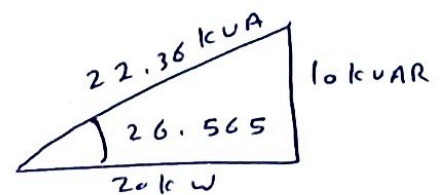
Use of power triangles



leading (-ve)° power factor
 $\text{P.f.} = \cos(-36.87)$

$$\begin{aligned} S &= P + jQ \\ &= 8 + j(-6) \\ &= 8 - j6 \end{aligned}$$

$$\begin{aligned} S_0, S_1 + S_2 &= (P_1 + P_2) + j(Q_1 + Q_2) \\ &= (8 + 12) + j(-6 + 16) \\ &= 20 + j10 \end{aligned}$$



$$S = \sqrt{P^2 + Q^2}$$

$$\theta_2 = \theta_v - \theta_i = \tan^{-1}\left(\frac{Q}{P}\right)$$

Apparent power: $- S = |\vec{S}| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$

Real power: $- P = S \cos(\theta_v - \theta_i)$

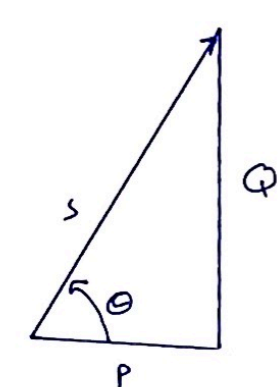
Reactive power: $- Q = \text{Im}(S) = S \sin(\theta_v - \theta_i)$

power factor: $- pf = P/S = \cos(\theta_v - \theta_i)$

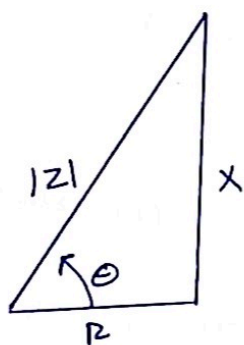
→ Real power is the actual power dissipated by the load.

→ Reactive power is a measure of the energy exchange between source and reactive part of the load.

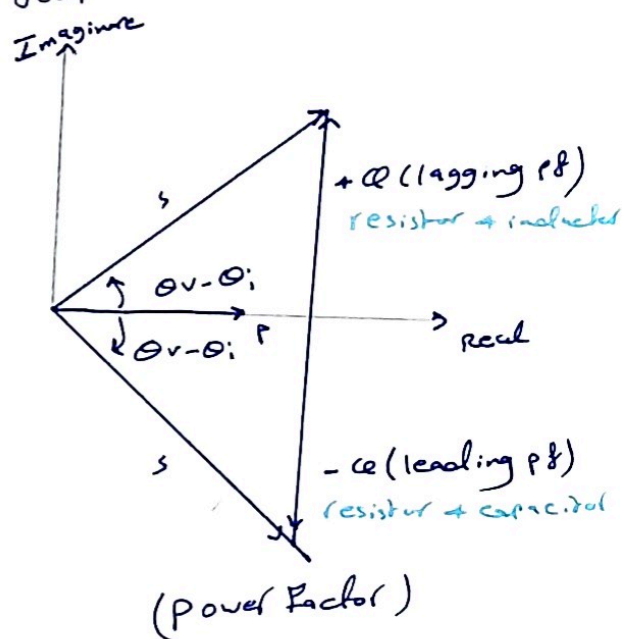
→ The complex power is represented by the power Triangle similar to Impedance triangle. power triangle has four items: P, Q, S and θ



power Triangle



Impedance Triangle



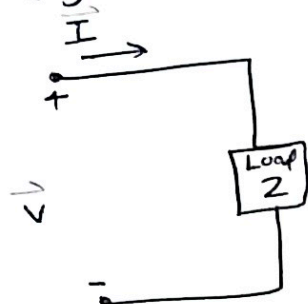
Real and Reactive Powers.

Complex power

$$\left\{ \begin{array}{l} \text{average power} = \frac{1}{2} R |I|^2 \\ L \rightarrow 0 \\ C \rightarrow 0 \end{array} \right\}$$

→ The complex power $\underline{S}$ contains all the information pertaining to the power absorbed by a given load.

→ Complex power $\underline{S}$ is the product of the voltage and the complex conjugate of the current: - $\underline{S}$: complex power



$$V = V_m \angle \theta_v \quad (I = I_m \angle \theta_i)$$

$$\underline{S} = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i = S \angle \theta_v - \theta_i$$

$$I^* = \text{conjugate} (* - 1)$$

$$I^* = I_m \angle -\theta_i$$

$$2 + j \xrightarrow{\text{conj}} = 2 - j$$

Ex: $\underline{S} = 2 \angle 30^\circ = \underbrace{2 \cos(30^\circ)}_{P(\text{avg power})} + j \underbrace{2 \sin(30^\circ)}_{Q(\text{reactive power})}$

$S = 2 \Rightarrow$ apparent power

P.S = power factor = $\cos(\theta_v - \theta_i) = \cos(30^\circ)$ $\Rightarrow$ 4ve S , lagging.

apparent average power = $P.S \times S = 2 \times \cos(30^\circ)$

$$S = \frac{1}{2} V I^* = \underbrace{V_{rms} I_{rms}}_S \angle \underbrace{\theta_v - \theta_i}_{\theta_2} \quad (\text{polar form})$$

$$S \Rightarrow V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j(V_{rms} I_{rms} \sin(\theta_v - \theta_i)) \quad (\text{rectangular})$$

$$S, S = P + jQ$$

$\rightarrow R$ —
P: is the average power in watts delivered to a load and it's the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- $Q = 0$ for resistive loads (unity pf)
- $Q < 0$ for capacitive loads (leading pf) (-ve)
- $Q > 0$ for inductive loads (lagging pf) (+ve)

[VAR $\Rightarrow$ volt Ampir reactive.]



Ex:- The 60Ω resistor absorbs 240 watt of average power. Calculate V and the complex power of each branch. what is the total complex power?

$$P = 240 \text{ W}, R = 60$$

$$P = I_2^2 R$$

$$I_2 = \sqrt{\frac{P}{R}} = \sqrt{\frac{240}{60}} = 2 = 2 \angle 0^\circ \text{ rms.}$$

$$V_0 = I_2 \cdot Z_2$$

$$= 2(60 + j20) = 120 + j40 \text{ rms.}$$

$$I_1 = \frac{V_0}{Z_1} = \frac{120 + j40}{30 - j10} = 3.4 + 2.4j = 4 \angle 36.87^\circ$$

$$I = I_1 + I_2 = 2 \angle 0^\circ + 4 \angle 36.87^\circ$$

$$I = 5.727 \angle 24.8^\circ \Rightarrow 5.2 + j2.4 \text{ rms.}$$

$$V_T = I \cdot 20 + V_0 = 20(5.2 + j2.4) + 120 + j40 = 224 + 88j$$

$$V = 240 \angle 21.45^\circ \text{ rms}$$

$$\text{For } R = 20\Omega$$

$$V = 20 \cdot I = 20(5.2 + j2.4) = 114 + 54j \angle 24.8^\circ$$

$$I = 5.2 + j2.4 = 5.727 \angle 24.8^\circ$$

$$S = V \cdot I^* = 114.54 \angle 24.8^\circ \cdot 5.727 \angle -24.8^\circ = 656 \text{ VA.}$$

$$\text{For } (30 - j10) \text{ impedance}$$

$$V = V_0 = 120 + j40 = 126.5 \angle 18.43^\circ$$

$$I_1 = 3.4 + 2.4j = 4 \angle 36.87^\circ$$

$$S_1 = V_0 \cdot I_1^* = 126.5 \angle 18.43^\circ \cdot 4 \angle -36.87^\circ = 506 \angle -18.44^\circ = 480 - j160 \text{ VA}$$

$$\text{For } (j20 + 60) \text{ impedance.}$$

$$V = V_0 = 126.5 \angle 18.43^\circ$$

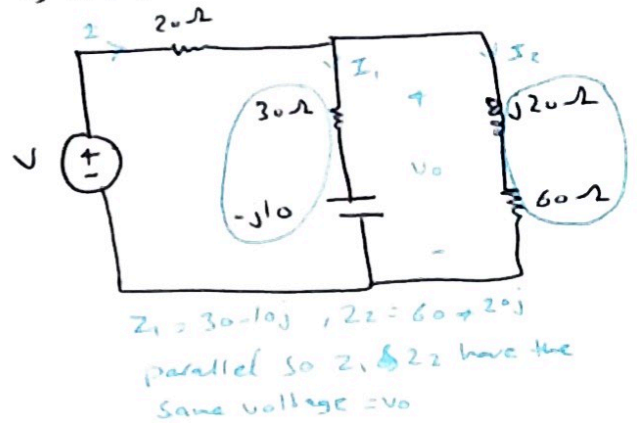
$$I_2 = 2 \angle 0^\circ$$

$$S_2 = V_0 \cdot I_2^*$$

$$S_2 = 126.5 \angle 18.43^\circ \cdot 2 \angle -0^\circ = 253 \angle 18.43^\circ$$

$$S_T = V \cdot I^* = 240 \angle 21.45^\circ \cdot 5.727 \angle -24.8^\circ$$

$$= 1378.3 \angle -3.35^\circ = 1376 - 80j \text{ VA}$$



Ex:- Two loads are connected in parallel. Load 1 has 2 kW, pf = 0.75 leading and load 2 has 4 kW, pf = 0.95 lagging. Calculate the pf of two loads and the complex power supplied by the source.

$$P_1 = 2 \text{ kW} = 2000 \text{ W}$$

$$\text{pf} = 0.75 \rightarrow \theta_1 = -ve$$

$$\cos(\theta_1) = 0.75 = -41.41^\circ$$

$$P_1 = S_1 \cos \theta_1 \Rightarrow S_1 = \frac{P_1}{\cos(\theta_1)} = \frac{2000}{\cos(-41.41)} = 2666.67$$

$$Q_1 = S_1 \sin \theta_1 = 2666.67 \times \sin(-41.41) = -1763.85$$

$$S_1 = P_1 + jQ_1$$

$$= 2000 - j1763.85 \text{ (leading) VAR}$$

$$P_2 = 4 \text{ kW} = 4000 \text{ W}$$

$$\text{pf} = 0.95 \Rightarrow \theta_2 = +ve$$

$$\cos(\theta_2) = 0.95 \Rightarrow \theta_2 = 18.19^\circ$$

$$P_2 = S_2 \cos \theta_2$$

$$S_2 = \frac{P_2}{\cos \theta_2} = \frac{4000}{\cos(18.19)} = 4210.41$$

$$Q_2 = S_2 \sin(\theta_2) = 4210.41 \times \sin(18.19) = 1314.4$$

$$S_2 = P_2 + jQ_2$$

$$= 4000 + j1314.4 \text{ (lagging)}$$

The Total power (Complex power) =

$$S = S_1 + S_2 = (P_1 + P_2) + j(Q_1 + Q_2)$$

$$= (2000 + 4000) + j(-1763.85 + 1314.4)$$

$$= 6000 - j449.45 \text{ VAR}$$

$$= 6000 - j0.44945 \text{ kVAR}$$

$$\text{pf} = \frac{P}{|S|} = \frac{6000}{\sqrt{P^2 + Q^2}} = \frac{6000}{\sqrt{6000^2 + (-449.45)^2}}$$

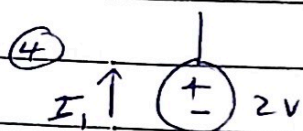
$$\text{pf} = 0.9972 \text{ (leading)}$$



$$\textcircled{3} \quad P_{abs} = (I_2)^2 \times R$$

$$= (-1.22)^2 \times 1 = 1.49 \text{ watt.}$$

(absorbed.) P_{abs} $\rightarrow$ absorbed.



$$P(2V) = -(2)(-0.33) = +0.66 \text{ Watt}$$

$\rightarrow$ abs.

$$P_{abs}(2V) = 0.66 \text{ watt}$$

$$\text{So, } P_{gen}(2V) = -P_{abs}(2V) = -0.66 \text{ watt.}$$

③ Super position

* Used for linear circuits & linear quantities (I & V) when there is multiple sources in the circuit.

* steps:-

① We will choose one supply & short circuit the other sources as follow:-

- 1) current source $\xrightarrow{\text{kill}}$ open circuit.
- 2) voltage source $\xrightarrow{\text{kill}}$ short circuit.

② Find the currents & voltages in the circuit.

③ repeat step (1 & 2) for other supplies.

④ The Total current or voltage is the sum of values found in previous steps.

Ex:-

6 Ω

9 Ω

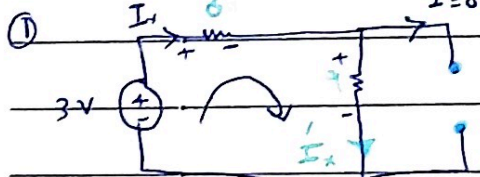
I_x

Final I_x using super position.

3V

2A

2 sources.



1st V.L:-

$$-3 + 6 I_x + 9 I_x = 0$$

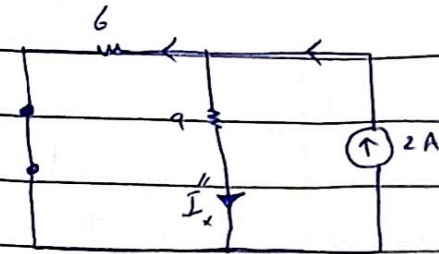
$$I_x = 0.2 \text{ A}$$

leave voltage source &

kill current source.



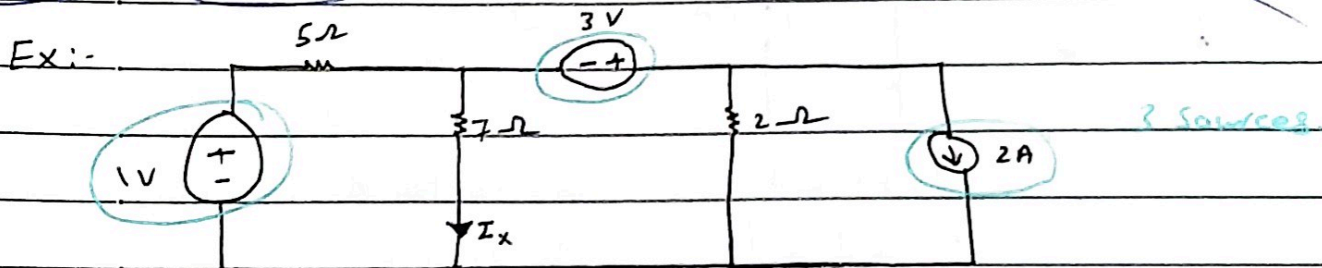
② leave current source & kill voltage.



use current ~~divider~~ division.

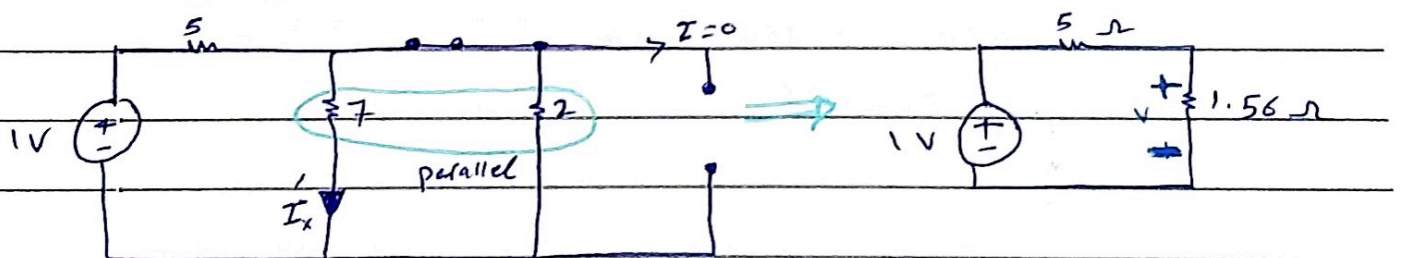
$$I_x'' = 2 \times \frac{\frac{1}{9}}{\frac{1}{9} + \frac{1}{6}} = \boxed{0.8 \text{ A}}$$

$$\text{So, } I_x = I_x' + I_x'' = 0.2 + 0.8 = \boxed{1 \text{ A}}$$



Find I_x using superposition:-

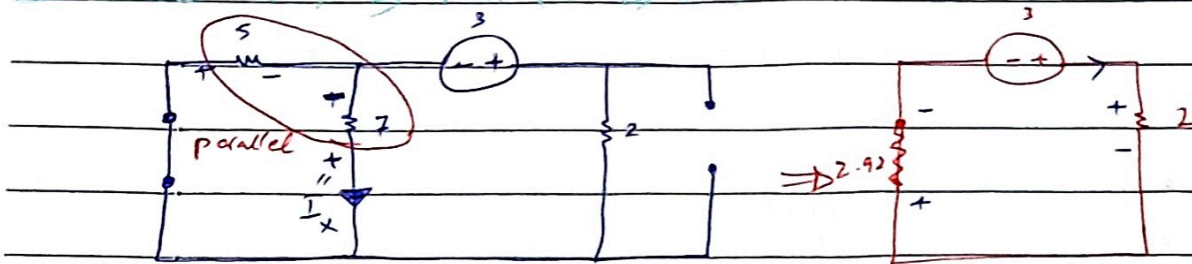
① leave 1V & kill 3V & 2A.



$$\Rightarrow V = \frac{1 \times 1.56}{1.56 + 5} = 0.24 \text{ V} \Rightarrow V = \text{voltage across } (7 \Omega \text{ \& } 2 \Omega) \text{ parallel.}$$

$$I_x' = \frac{0.24}{7} = 0.034 \text{ A.}$$

② leave (3V) & kill (1V & 2A).

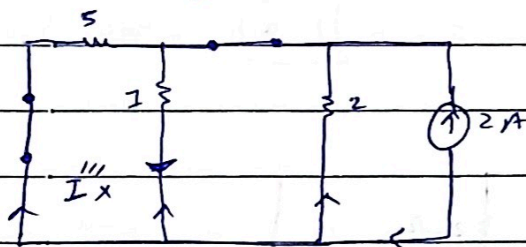


$$V = \frac{3 \times 2.92}{2.92 + 2} = 1.78 \text{ V.}$$

$$I_x'' = \frac{-1.78}{7} = -0.254 \text{ A.}$$



③ leave 2A & kill (3V, 1V)



By current division:-

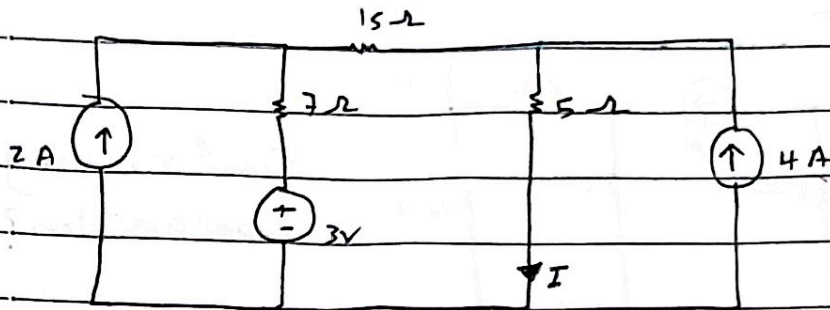
$$I_x''' = \frac{(-)(2) \times \frac{1}{7}}{\frac{1}{7} + \frac{1}{2} + \frac{1}{5}} = -0.34 \text{ A.}$$

$$\text{So, } I_x = I_x' + I_x'' + I_x'''$$

$$= 0.034 - 0.254 - 0.34$$

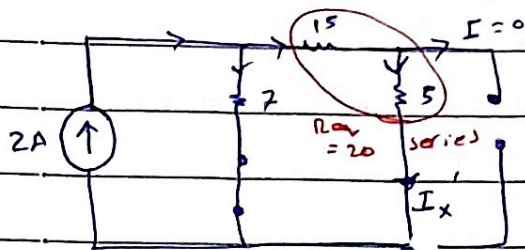
$$= -0.564 \text{ A}$$

Ex:-



Find I using superposition:-

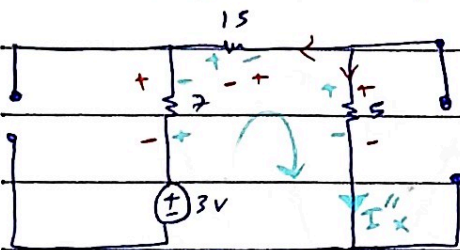
① leave 2A & kill (3V & 4A).



⇒ by current division

$$I'_x = \frac{2 * (\frac{1}{20})}{\frac{1}{20} + \frac{1}{7}} = 0.52 \text{ A}$$

② leave (3V) & kill (2A, 4A).

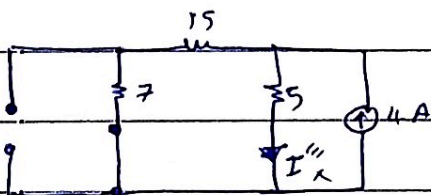


By KVL:

$$-3 + 7I''_x + 15I''_x + 5I''_x = 0$$

$$I''_x = 0.11 \text{ A}$$

③ leave (4A) & kill (3V, 2A)

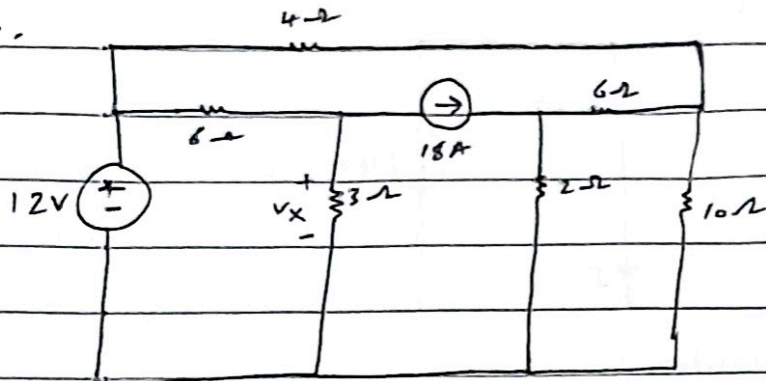


By current division:-

$$I'''_x = \frac{4(\frac{1}{5})}{\frac{1}{5} + \frac{1}{22}} = 0.326 \text{ A}$$

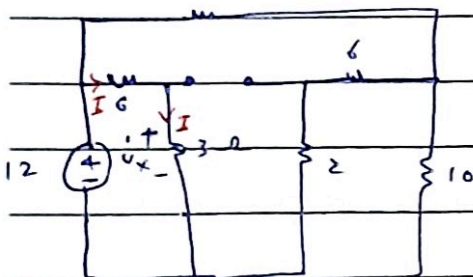
$$\text{So, } I = 0.52 + 0.11 + 0.326 = 3.89 \text{ A}$$

Ex:-



Final V_x using superposition?

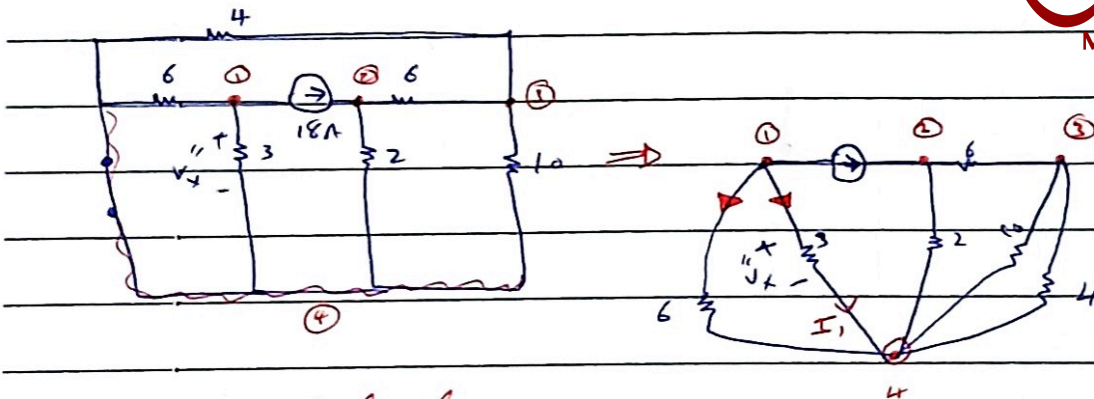
① leave (12V) & kill (18A).



by voltage divider:-

$$V_x = \frac{12 \times 3}{3 + 6} = 4 \text{ volt.}$$

② leave (18A) & kill (12V):-



By current divider

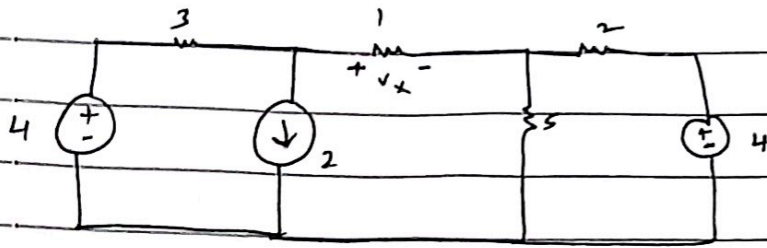
$$I_1 = \frac{(-18) \times \frac{1}{3}}{\frac{1}{3} + \frac{1}{16}} = -12 \text{ A}$$

$$V''_x = 3 \times -12 = -36 \text{ V}$$

$$\text{So, } V_x = 4 - 36 = -32 \text{ volt}$$

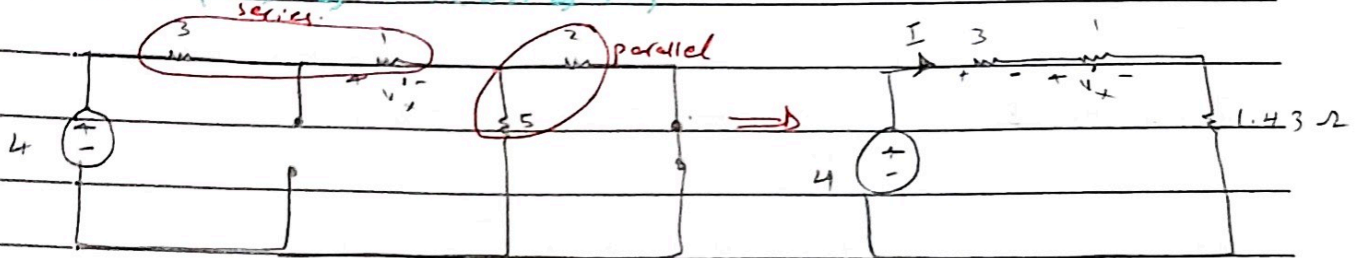


Ex:-



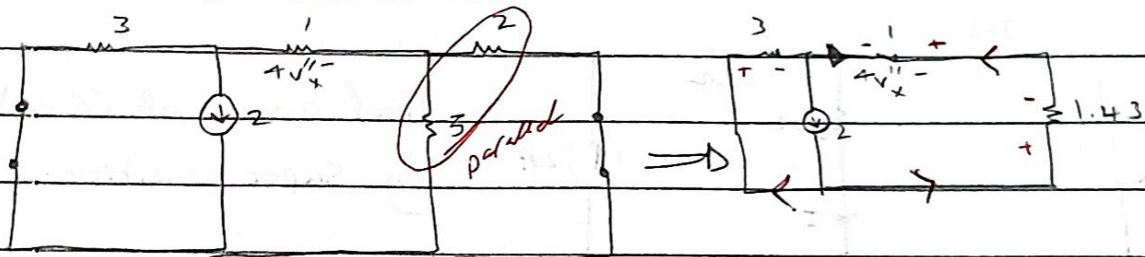
Find v_x using
super position ?

① leave (4V) & kill (2A & 4V).



$$v'_x = \frac{4 \times 1}{3 + 1 + 1.43} = 0.74 \text{ V.}$$

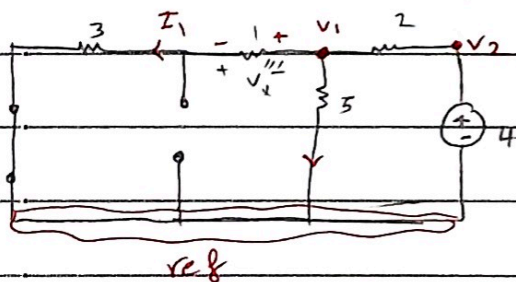
② leave (2A) & kill (4V & 4V)



$$I = 2 \times \frac{1}{\frac{1}{2.43} + \frac{1}{3}} = 1.1 \text{ A.}$$

$$v''_x = - (1.1) \times 1 = -1.1 \text{ Volt}$$

③ leave (4V) & kill (4V & 2A):-



By nodal



$$V_2 = 4$$

at node ①

$$0 = \frac{V_1 - 0}{4} + \frac{V_1}{5} + \frac{V_1 - 4}{2}$$

$$V_1 = 2.11 \text{ volt}$$

$$I_1 = \frac{V_1 - 0}{4} = \frac{2.11}{4} \Rightarrow I_1 = 0.53 \text{ A}$$

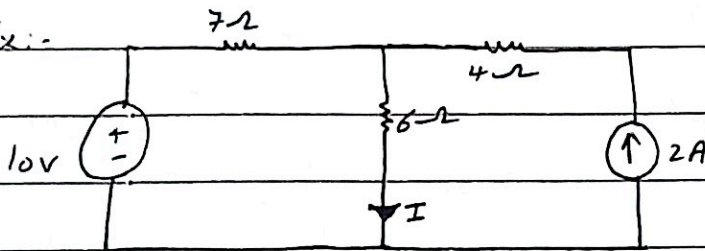
$$V_x''' = -0.53 \times 1 = -0.53 \text{ volt}$$

$$\text{So, } V_x = 0.74 + -1.1 + -0.53$$

$$V_x = -0.89 \text{ volt}$$

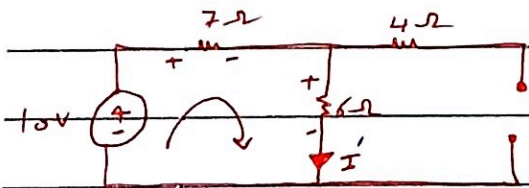


Ex:-



Find power of (6Ω)
by Super position.

① leave (10V) & kill (2A)

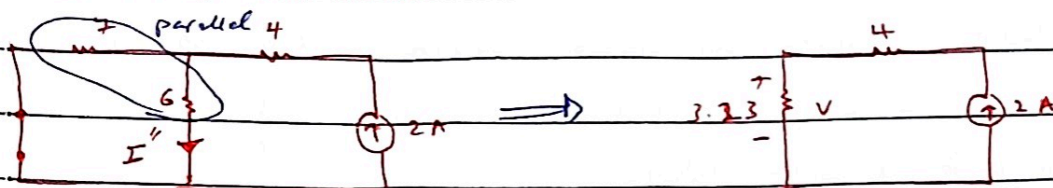


$$\text{By KVL: } -10 + 7I' + 6I' = 0$$

$$I' = 0.77 \text{ A}$$

$$P' = (I')^2 \times 6 = 3.56 \text{ W}$$

② leave (2A) & kill (10V):-



$$V = 2 \times 3.23 = 6.46 \text{ Volt}$$

$$I'' = \frac{6.46}{6} = 1.08 \text{ A.}$$

$$P''(6\Omega) = (1.08)^2 (6)$$

$$P'' = 6.99 \text{ W}$$



$$I = I' + I'' = 0.77 + 1.08$$

$$= 1.85 \text{ A}$$

$$P(6\Omega) = (1.85)^2 (6) = 20.5$$

$$= 20.52 \text{ watt.}$$

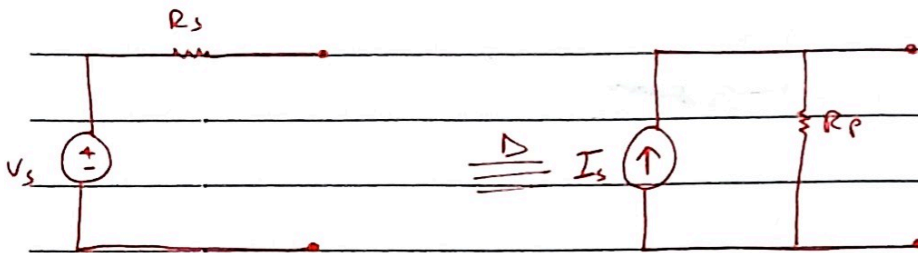
Note!! $I^2 R \neq (I')^2 R + (I'')^2 R$

* super position not applied on power.

④ Source Transformation.

In some circuits replacing current source with voltage source or ~~using~~ versa make the circuit
Visa

Simple & easy to solve.



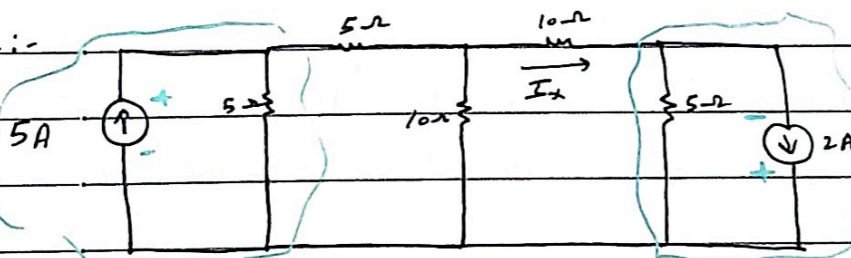
* اختيار بفتح من الباء الـ الحجب و لازم يكون الـ I_s و V_s نفس الاتجاه !!

$$R_s = R_p$$

$$V_s = I_s R_s$$

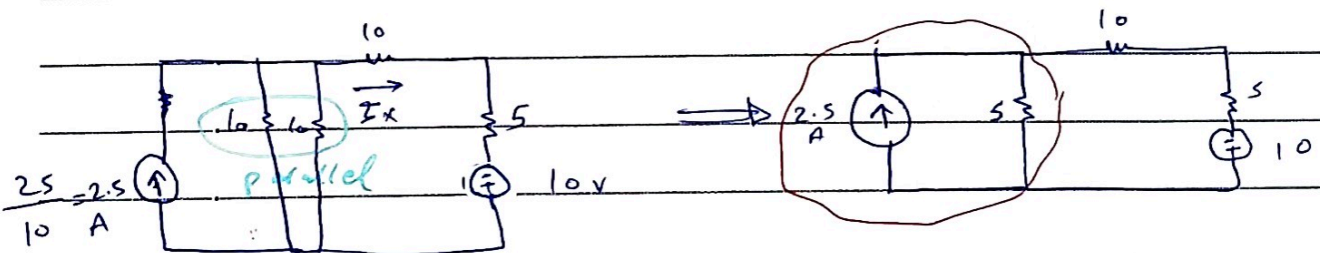
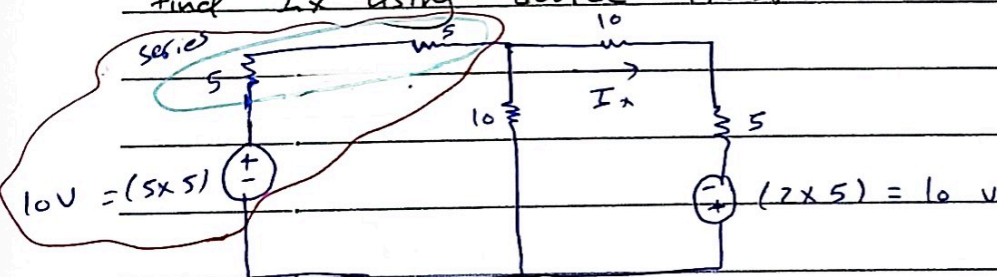
$$I_s = \frac{V_s}{R_s}$$

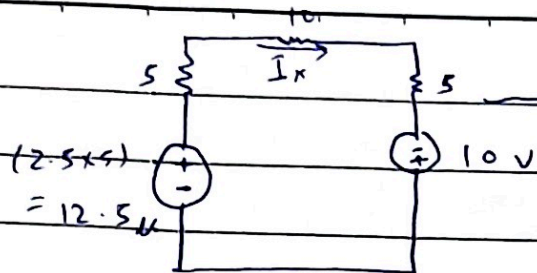
Ex:-



*) Branch لـ يكون فيه
الجهد و التيار في
نفس الاتجاه !!!

Find I_x using source Transformation.





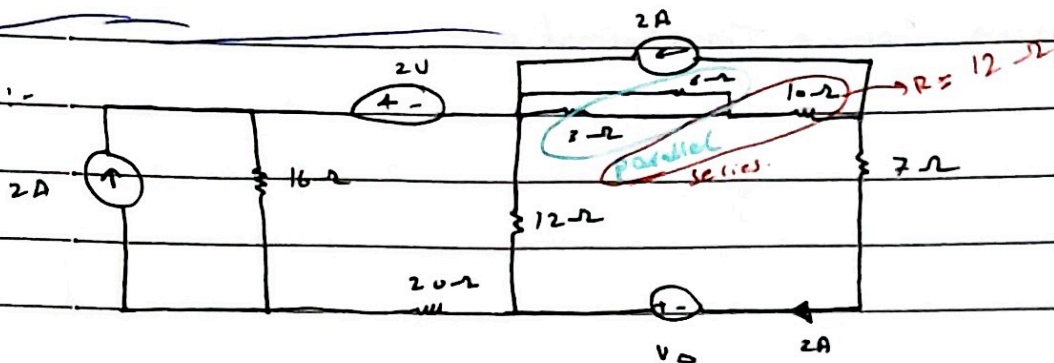
simple circuit (1 loop) ✓

By KVL :-

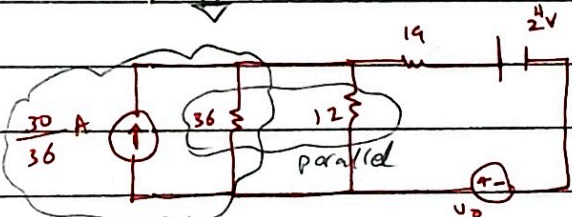
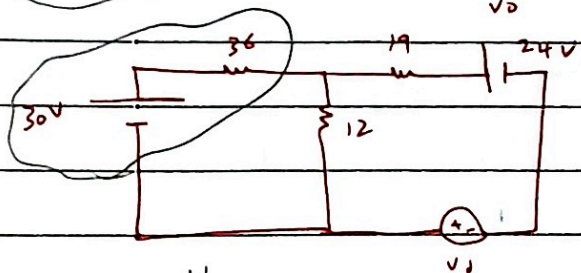
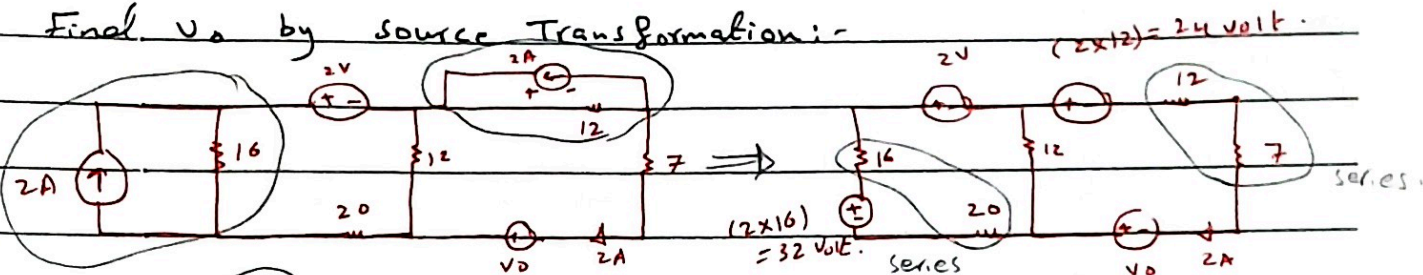
$$-12.5 + 5I_x + 10I_x + 5I_x - 10 = 0$$

$$I_x = 1.125 \text{ A}$$

Ex:-



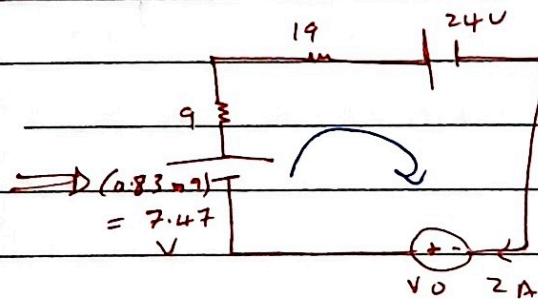
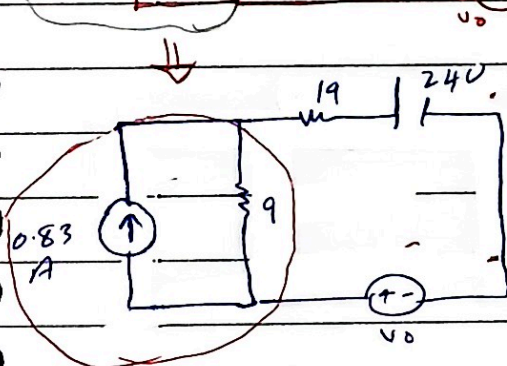
Final V_o by source Transformation:-



$$-V - 2 + 32 = 0$$

پیش از این

$$V = 30 \text{ V}$$

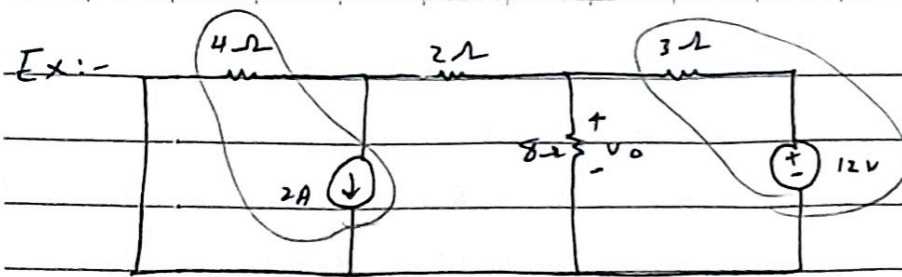


KVL:-

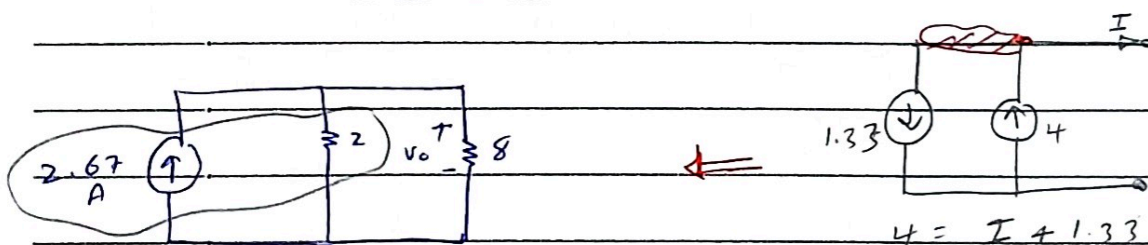
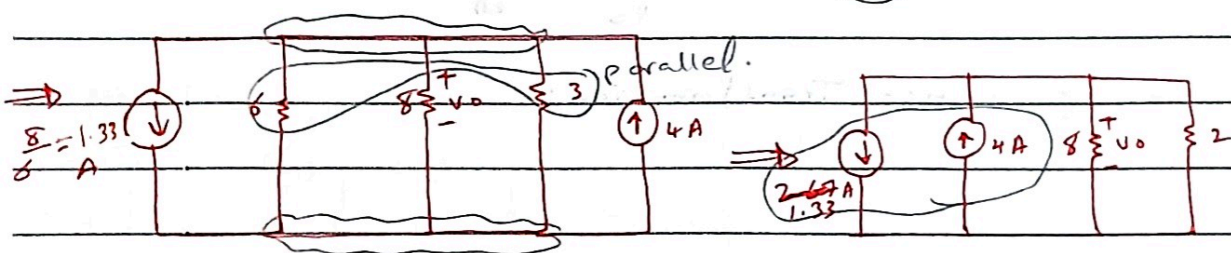
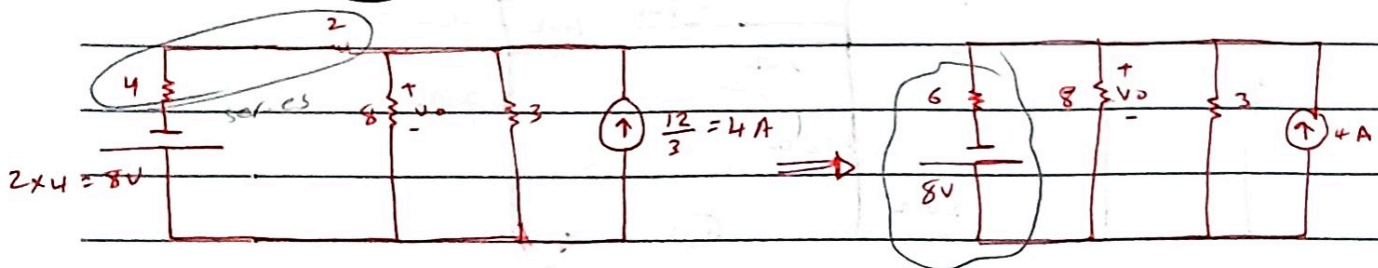
$$-7.47 + (19) \times 2$$

$$+ 24 - V_o = 0$$

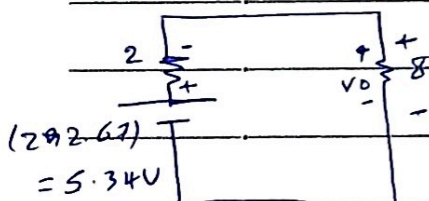
$$V_o = 72.5 \text{ volt}$$



Final v_o using Source Transformation:-



$\uparrow (4A) \text{ و } \uparrow (2.67A) \rightarrow I = 2.67A$

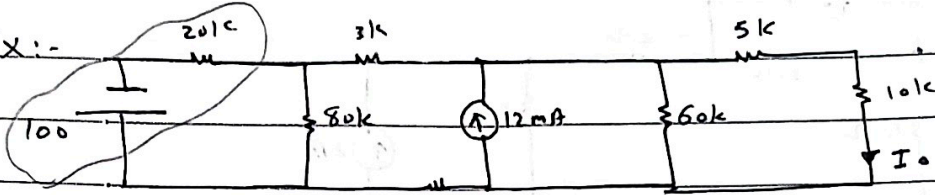


By voltage division.

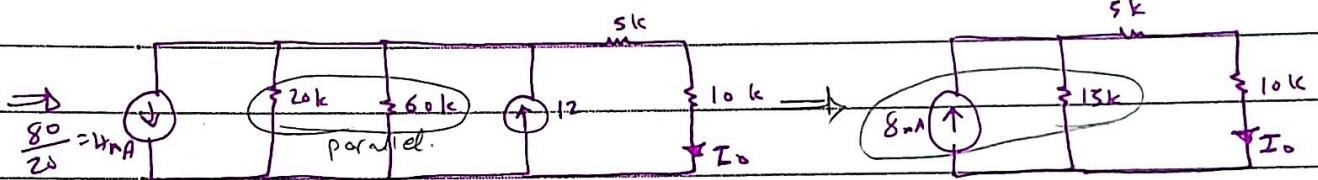
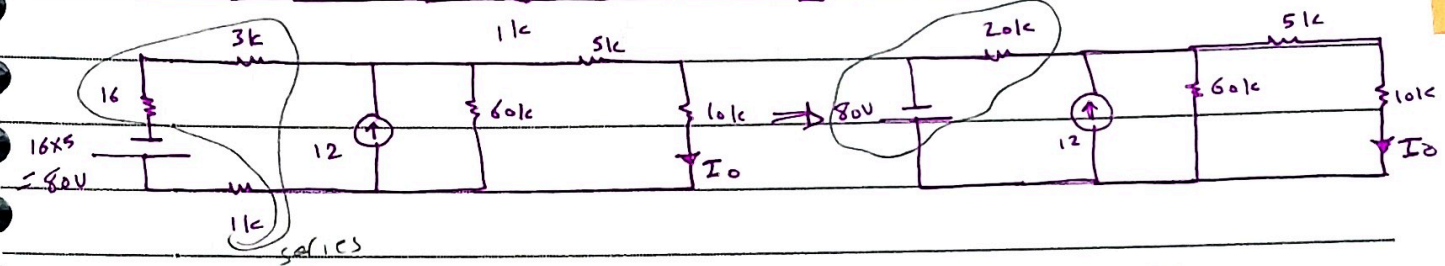
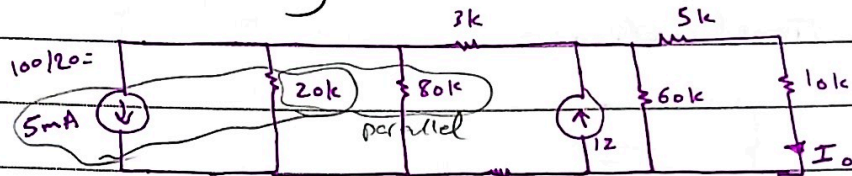
$$v_o = \frac{(5.34) * 8}{8 + 2} = 4.272 \text{ Volt}$$

v_o و 2, 12Vs check for 2Ω
(-) les ولتاژ (+) ولتاژ 2Ω

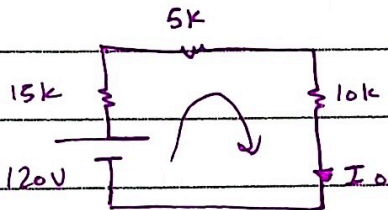
Ex:-



Find I_o using Source Transformation?



$$I = 12 - 4 = 8 \text{ mA} \uparrow$$

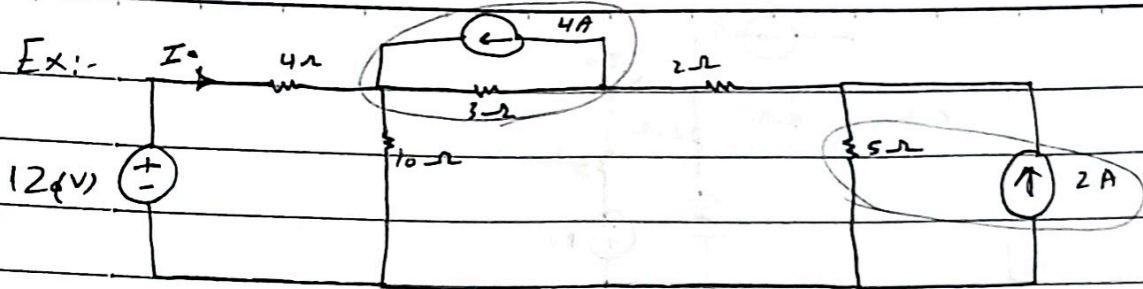


By KVL:-

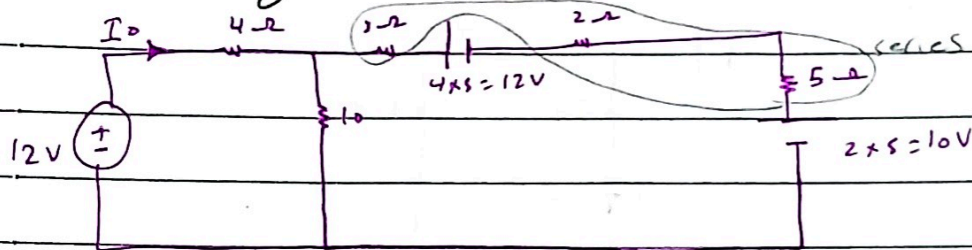
$$-120 + (30 \times I_o) = 0$$

$$I_o = 4 \text{ mA}$$

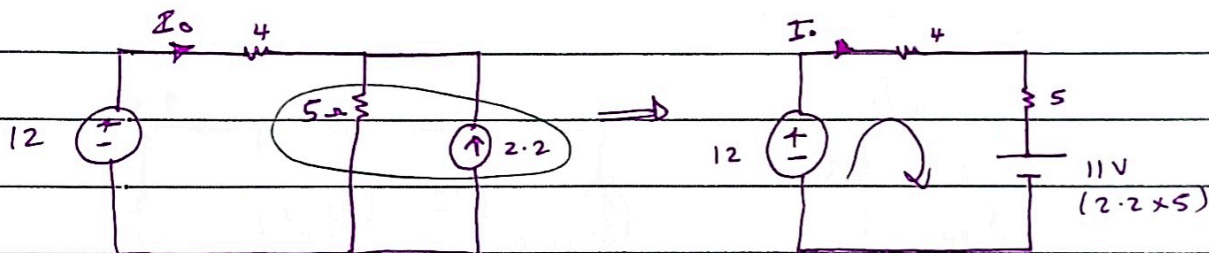
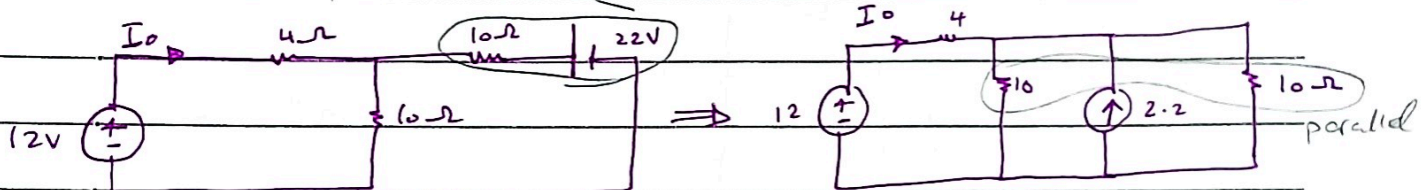




Find I_o using Source Transformation.



$$V = 12 + 10 = 22 \text{ V}$$

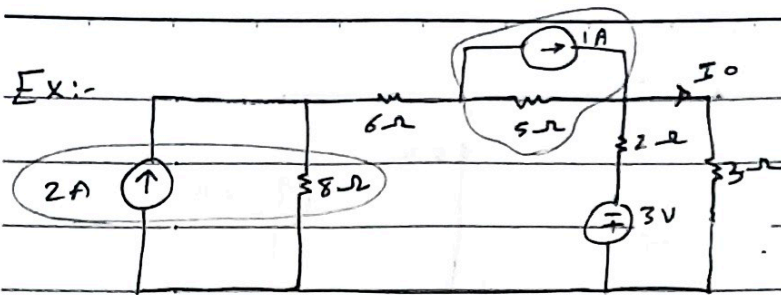


using KVL:- $-12 + (4 * I_o) + 11 = 0$

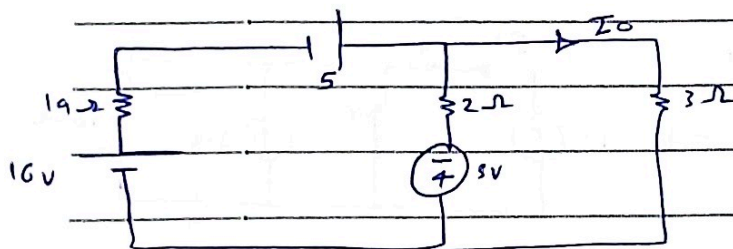
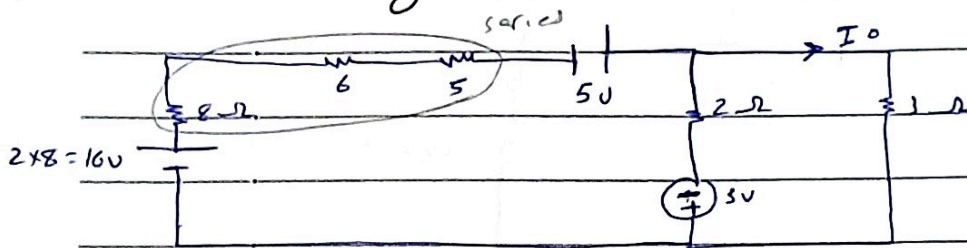
$$I_o = 0.111 \text{ A}$$



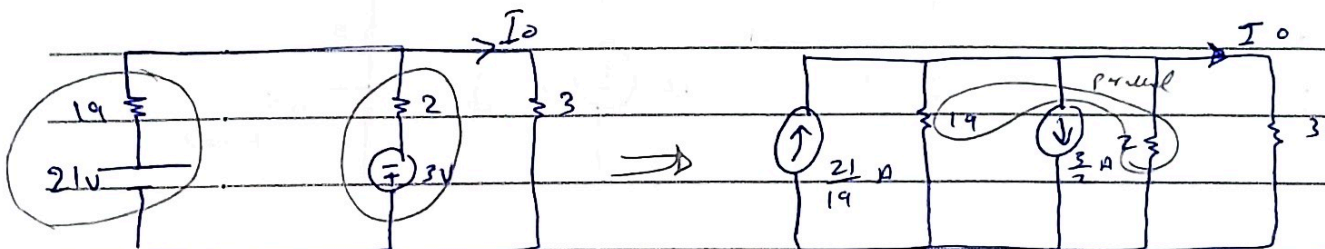
Ex:-



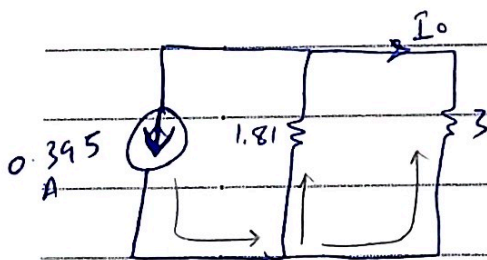
Find I_0 using Source Transformation:-



$$V = 16 + 5 = 21 \text{ Volt.}$$



$$I = \frac{3}{2} - \frac{21}{19} = 0.395 \text{ A.}$$



0.395 A

using current division ✓

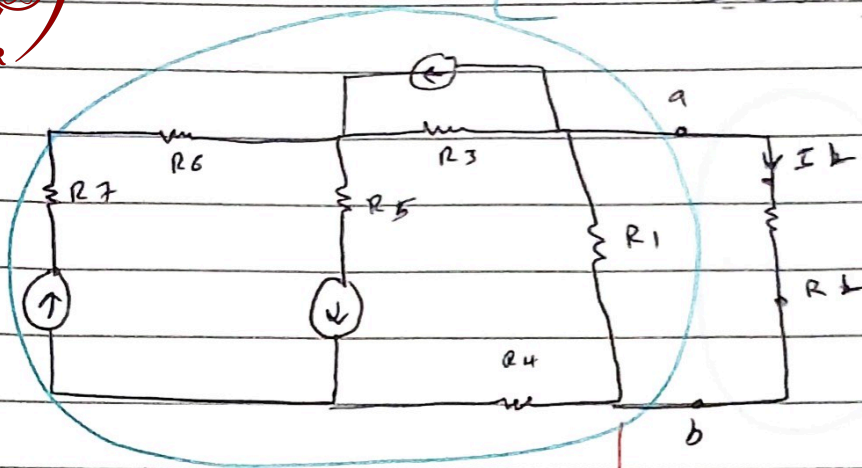
$$I_0 = \frac{(-0.395) \times \frac{1}{3}}{\frac{1}{3} + \frac{1}{1.81}}$$

$$= -0.15 \text{ A}$$

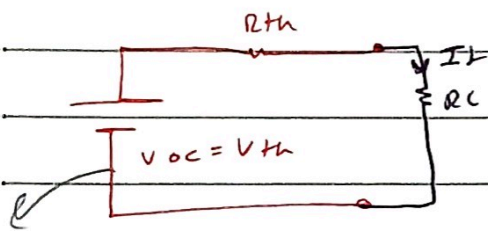
⑤ Thevenin and Norton method



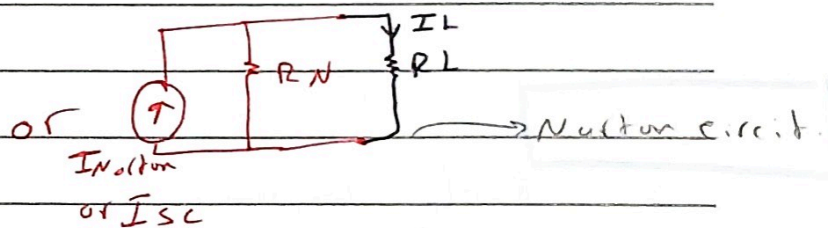
مخطط دارة ثيرنين
 فيها 7 sources و 20 resistors
 اكتب التيار في المقاومة



①
 Voltage source + المقاومة
 in series



Theremin circuit.



②
 like load

⇒ Steps to Find ($V_{th} = V_{open\ circuit}$) :-

- ① Remove (R_{Load})
- ② leave that place open circuit
- ③ Find $V_{OC} = V_{th}$ at that terminal

↳ using kvl, volt division, Nodal, Mesh...

⇒ Steps to Find ($I_{No} = I_{sc}$) :-

- ① Remove (R_{Load})
- ② put short circuit in the place of R_{Load}
- ③ Find current in that short circuit ($I_{sc} = I_N$)

↳ kcl, nodal, ...

⇒ steps to find $(R_{th} = R_N)$:-

① $R_{th} = \frac{V_{oc}}{I_{sc}}$

or ① Remove R_L ② kill all sources

→ voltage source → short

→ current source → open

③ find Req. seen at that terminals

↓ place of (R_L) ✓

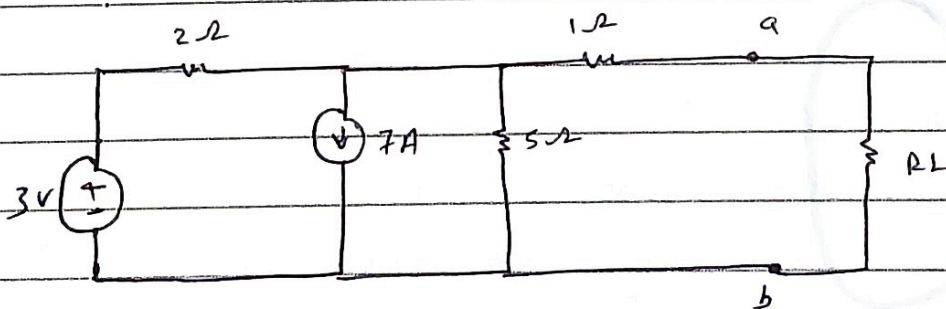
Note!

* $V_{th} = I_N \times R_{th}$

* $I_{sc} = \frac{V_{th}}{R_{th}}$

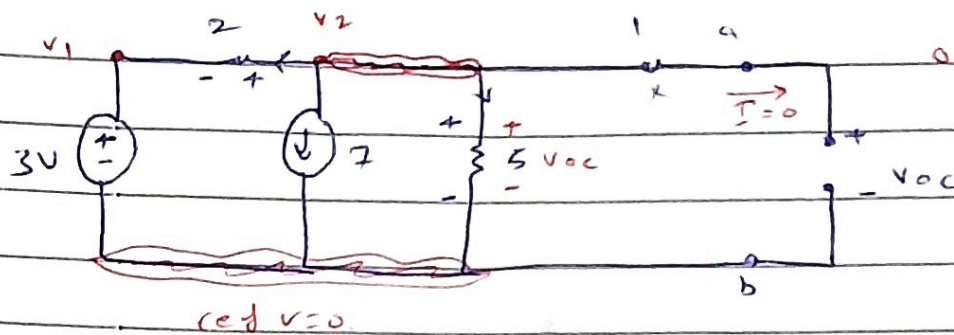


Ex:- find thevenin circuit &
find Norton circuit from a & b.



* $V_{th} = V_{oc}$





لأنها open circuit فيها $I=0$
 بفرضها منته صفر
 وسهلها لـ V_{OC} كجواب

* Using Nodal Method.

$$V_1 = 3V$$

at Node (2) $\Rightarrow$

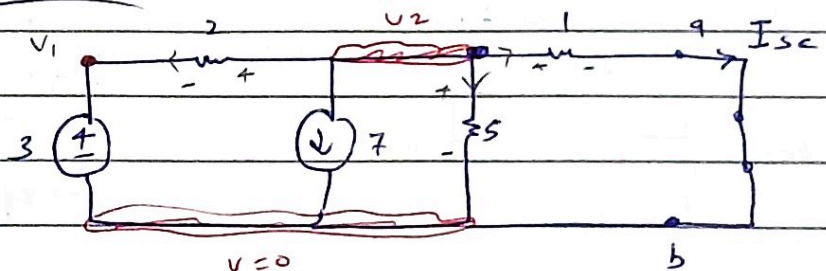
$$\sum I_{in} = \sum I_{out}$$

$$0 = 7 + \frac{V_2 - 3}{2} + \frac{V_2 - 0}{5}$$

$$V_2 = -7.85V \Rightarrow V_{OC} = V_2 = -7.85V$$

نفسه

* Final $I_{SC} = I_N$



* using Nodal.

$$V_1 = 3V$$

at Node (2) $\Rightarrow \sum I_{in} = \sum I_{out}$

$$0 = 7 + \frac{V_2 - 3}{2} + \frac{V_2 - 0}{5} + \frac{V_2 - 0}{1}$$

$$V_2 = -3.24V$$

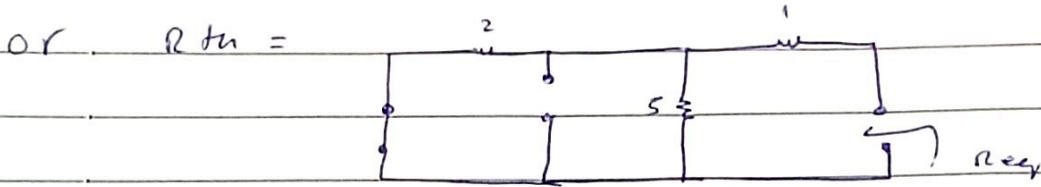
فرضها صفر

$$I_{SC} = \frac{V_2 - 0}{1} = V_2 \Rightarrow I_{SC} = -3.24A$$

HE

→ To Find R_{th} :-

$$\textcircled{1} R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-7.86}{-3.24} = 2.43 \Omega$$



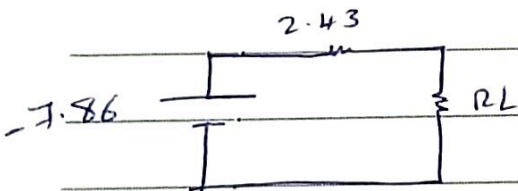
* جميع المقاومات بـ 1 أوم

$R_{eq} = (2 // 5) + 1$ in series.

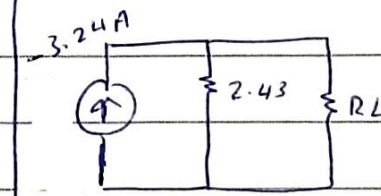
$$\text{So, } \frac{1}{2} + \frac{1}{5} \Rightarrow 1.43$$

$$R_{eq} = 1.43 + 1 = 2.43 \Omega$$

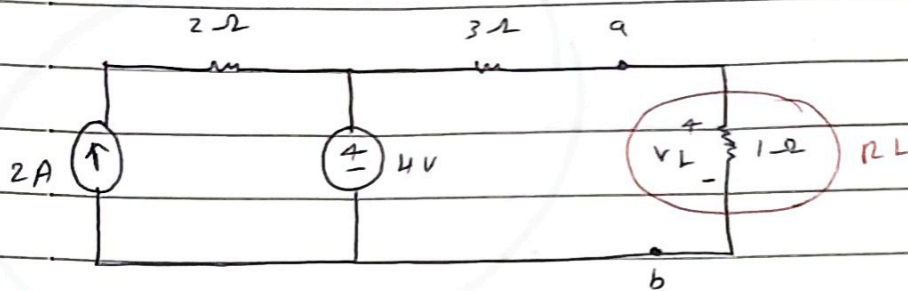
thevenin circuit =



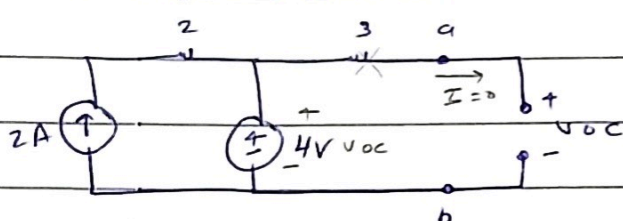
Norton circuit.



Ex:- Find V_L using thevenin & Norton Method:-

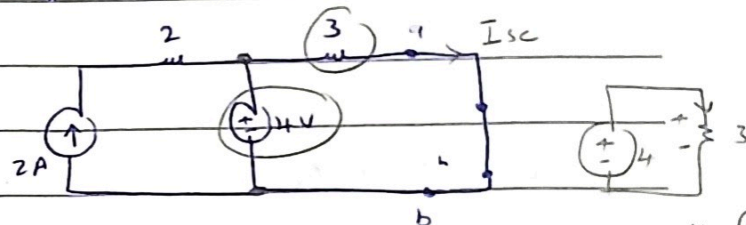


* Find $V_{th} = O.C$



So, $V_{OC} = 4 \text{ Volt}$.

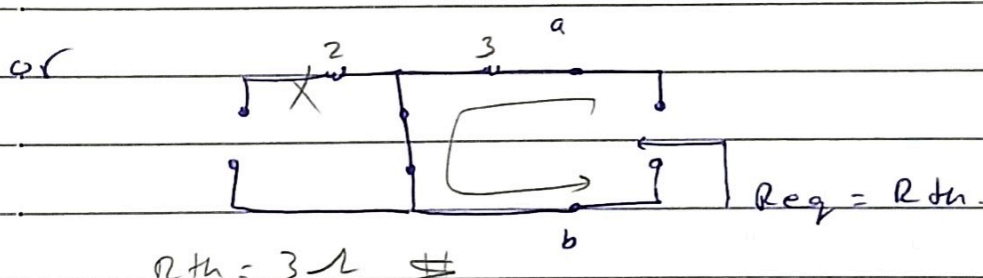
* Find $I_{SC} = I_N$



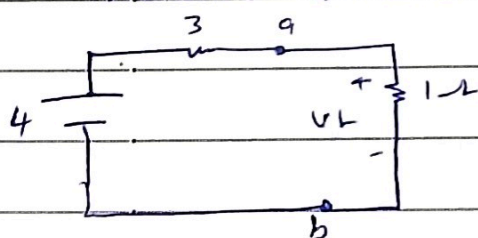
parallel
So, val 3Ω
= 4 volt

$$I_{SC} = \frac{4}{3} = 1.33 \text{ A}$$

$$R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{4}{1.33} = 3 \Omega$$



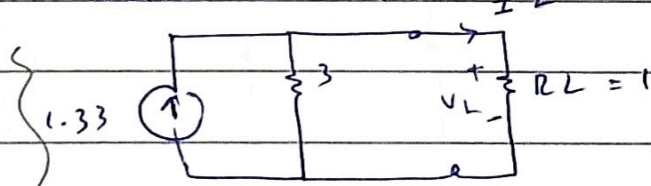
Thevenin Circuit



voltage division

$$V_L = \frac{(4)(1)}{1+3} = 1 \text{ Volt}$$

Norton circuit

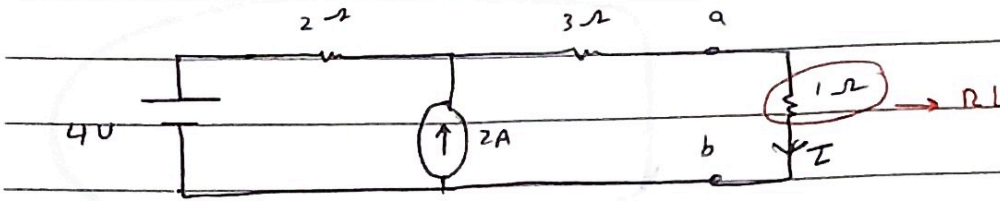


$$I_L = (1.33) \left(\frac{1}{1+3} \right) = 0.99 \approx 1$$

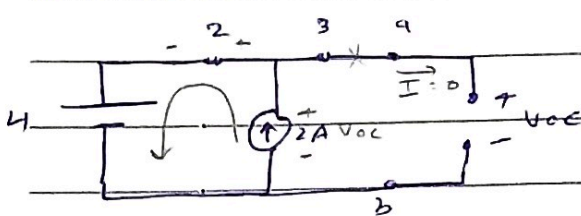
$$\text{So, } V_L = I_L \times 1 = 1 \times 1 = 1 \text{ Volt}$$

Ex:-

* Final Thevenin & Norton circuit seen from a & b:-



* Final V_{th}

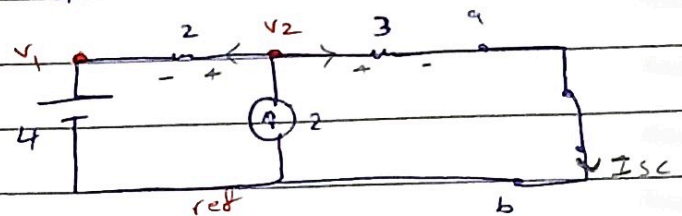


by KVL =

$$-V_{OC} + (2)(2) + 4 = 0$$

$$\boxed{V_{OC} = 8V}$$

* Final I_N



using Nodal:-

$$V_1 = 4V$$

at node ②:- $I_{in} = I_{out}$

$$2 = \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{3}$$

$$\boxed{V_2 = 4.8 \text{ volt}}$$

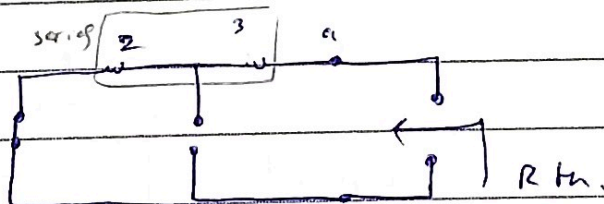
$$I_{SC} = \frac{4.8 - 0}{3} = \boxed{1.6 A}$$

* Final R_{th} :-

$$\textcircled{1} R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{8}{1.6} = 5 \Omega$$

or

②

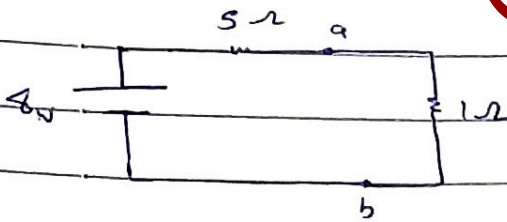


$$\text{So, } R_{th} = 3 + 2 = 5 \Omega$$

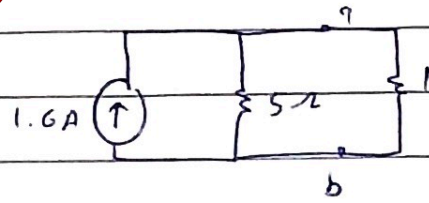




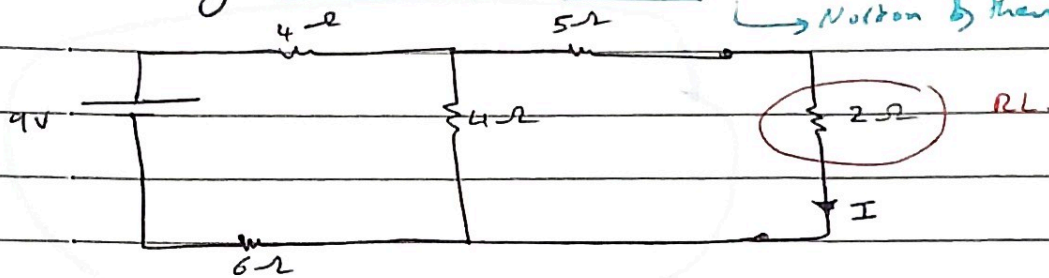
* Thevenin circuit:-



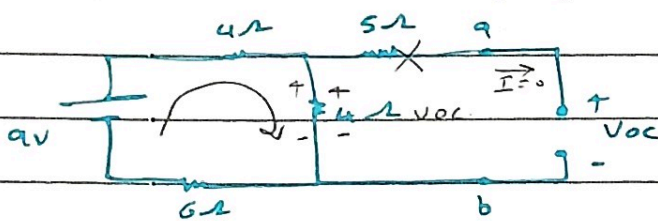
* Norton circuit



• Ex:- Using Thevenin & Norton method Find I:-



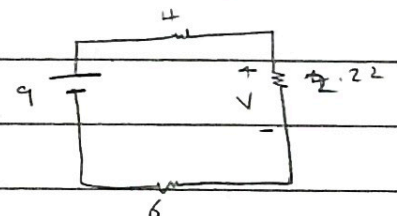
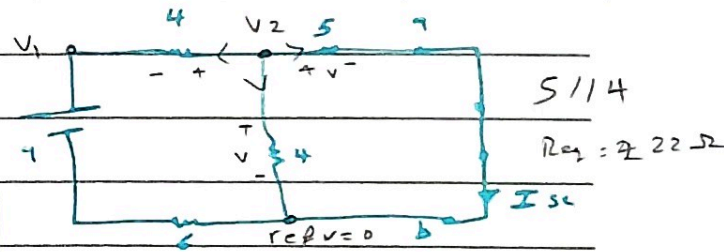
* Thevenin circuit $\rightarrow V_{OC}$



Using voltage division:-

$$V_{OC} = 9 \left(\frac{4}{4 + 4 + 6} \right) = 2.57 \text{ V}$$

* Norton circuit $\rightarrow I_{SC} = I_N$



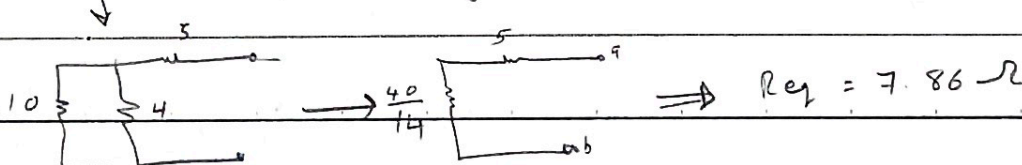
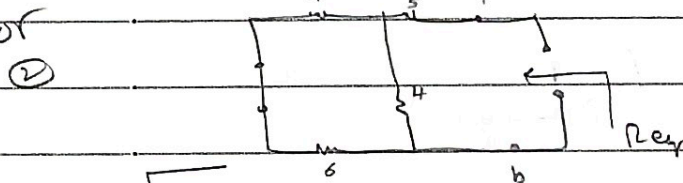
By voltage division

$$V = \frac{9(2.22)}{2.22 + 10} = 1.635 \text{ Volt.}$$

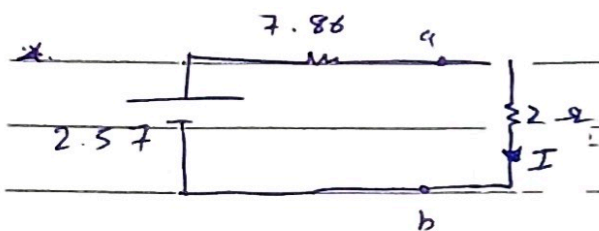
$$I_{SC} = \frac{V}{5} = \frac{1.635}{5} = 0.33 \text{ A}$$

To Find R_{th} :-

$$① R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{2.57}{0.33} = 7.86 \Omega$$



$$\Rightarrow R_{th} = 7.86 \Omega$$

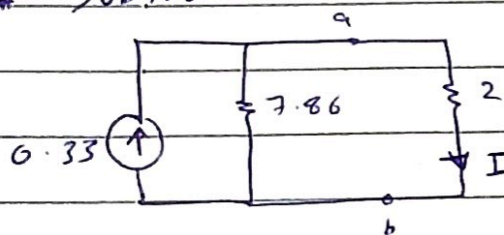


using Thevenine

$$I = \frac{2.57}{7.86} = 0.29 \text{ A}$$



Another

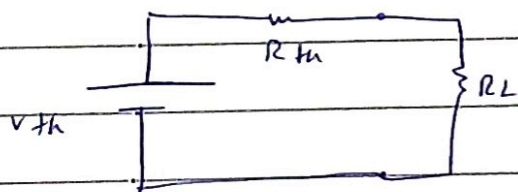


using current division

$$I = \frac{0.33 \times \frac{1}{2}}{\frac{1}{2} + \frac{1}{7.86}} = 0.29 \text{ A}$$

* Maximum power transfer:-

we concern about power consumption by load after simplification of the circuit.



→ thevenine circuit

نريد إيجاد قيمة RL التي تستهلك أقصى قدرة

Maximum Power

* what is the value of R_L to absorb max. power from source.

put ② in ①

$$P_L = I^2 R_L$$

$$I = \frac{V_{th}}{\Sigma R = R_{th} + R_L}$$

$$P_L = \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 R_L \quad \text{--- (3)}$$

for maximum (PL) $\Rightarrow \frac{dPL}{dRL} = 0$

Result $\Rightarrow RL = R_{th}$.

$\Rightarrow$ For any circuit to get maximum power at the load $\Rightarrow RL = R_{th}$. (4)

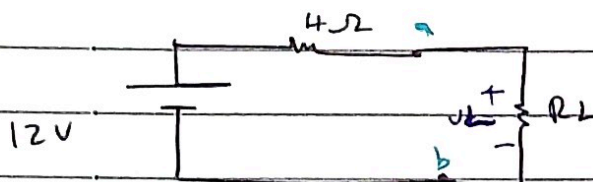
put (4) in (3)

$$PL(max) = \frac{V_{th}^2}{4 R_{th}} = \frac{I_N^2 R_L}{4}$$



• Ex: - Final power at load (PL) when: -

- ① $RL = 2\Omega$ ② $RL = 8\Omega$ ③ $RL = 4\Omega$



$$P = I^2 R = \frac{V^2}{R} = IV$$

① when $RL = 2\Omega$ (using volt division)

$$V_L = \frac{12 \times 2}{2 + 4} = 4 \text{ volt} \Rightarrow PL = \frac{16}{2} = 8 \text{ watt}$$

② when $RL = 8\Omega$ (using volt division)

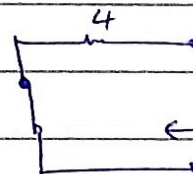
$$V_L = \frac{12 \times 8}{4 + 8} = 8 \text{ volt} \Rightarrow PL = \frac{64}{8} = 8 \text{ watt}$$

③ when $RL = 4$ (using volt division)

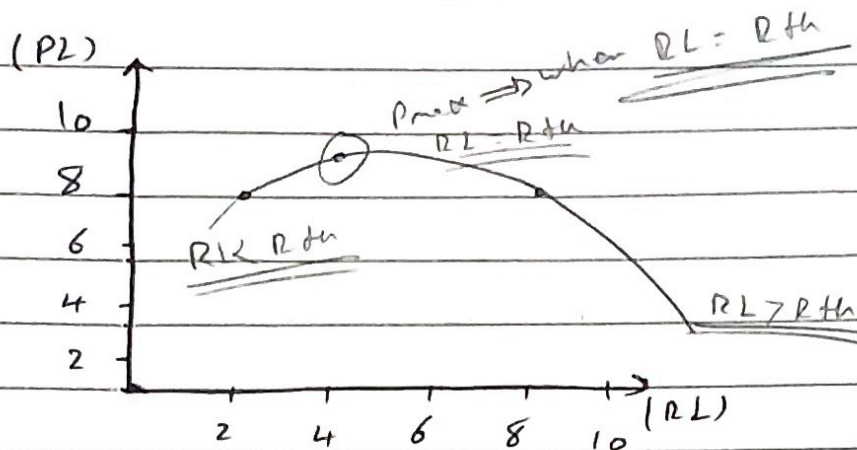
$$V_L = \frac{12 \times 4}{4 + 4} = 6 \text{ v} \Rightarrow PL = \frac{36}{4} = \boxed{9 \text{ watt}} \text{ max power.}$$

Refer to circuit $\Rightarrow$ find R_{th} .

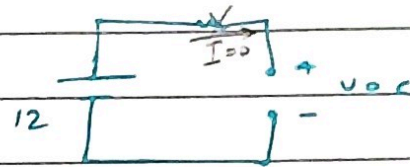
So, $R_{th} = RL = 4$



So, $R_{th} = 4\Omega$



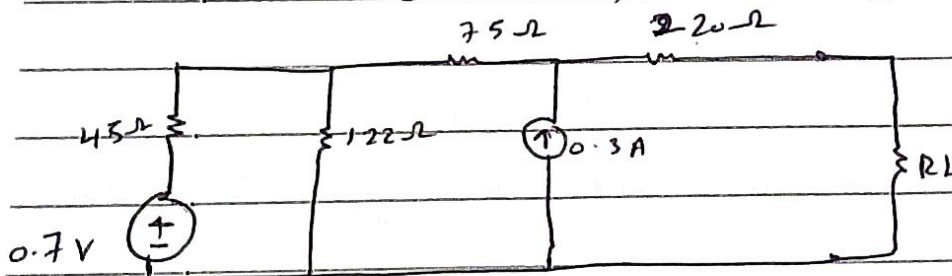
$$P_L(\max) = \frac{V_{th}^2}{R_{th}}$$



$$= \frac{(12)^2}{4 + 4} = 9 \text{ watt} \quad \text{Voc} = V_{th} = 12 \text{ volt.}$$

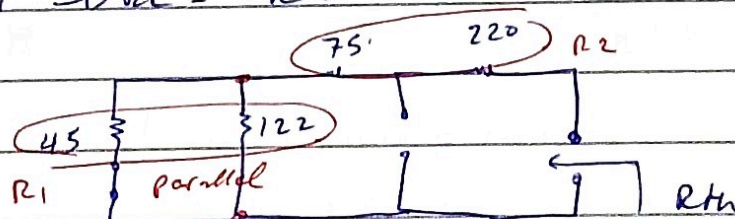
• Example:- Find ① R_L to get max power.

② $P_L(\max)$



⇒ for max power ⇒ $R_L = R_{th}$

to find R_{th}



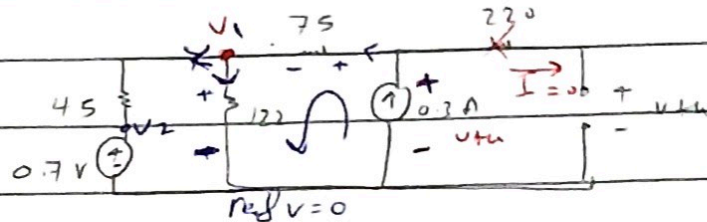
$$R_1 = \frac{1}{\frac{1}{45} + \frac{1}{122}} = 32.8 \Omega$$

$$R_1 \& R_2 \text{ Series} \Rightarrow 32.8 + 75 + 220 = 327.8 \Omega$$

① → for max power transfer $R_L = 327.9 \Omega$ ~~##~~

② $P_L(\max) = \frac{V_{th}^2}{4 R_{th}}$

To find V_{th} →



using Nodal analysis:-

$$\sum I_n = \sum I_{out}$$

$$V_2 = 0.7 \text{ volt.}$$

$$0.3 = \frac{V_1 - 0.7}{45} + \frac{V_1 - 0}{122} \Rightarrow V_1 = 10.45 \text{ volt.}$$

By KVL $\Rightarrow -10.45 - (75)(0.3) + V_{th} = 0$

$$V_{th} = 32.95 \text{ volt.}$$

$$P_L(\max) = \frac{(32.95)^2}{4 \times (327.9)} = 0.827 \text{ watt} \quad \text{##}$$



* The end of chapter, 4,

chapter "6" Capacitor theory. (c)

* AC- Network Analysis:-

we will take about energy storage element.

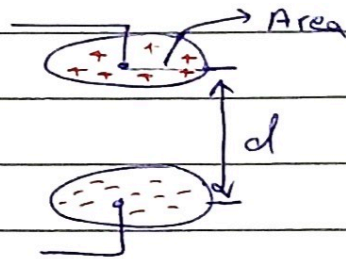
Capacitor circuit
Inductor circuit

① Capacitor :-

① device (element) store energy

② Consists of two parallel plates each plates has cross sectional area (A) & have distance (d) between them. the separation filled with dielectric material has certain permittivity (E)

$$C = \frac{\epsilon A}{d}$$



③ unit of capacitance $\Rightarrow$ Farad (F)

④ Symbol in electric circuit

⑤ Charge on plates of capacitor:-

$$Q = C \cdot V \Rightarrow q(t) = C v(t) \text{ (when I have variable voltage I have } q \text{ variable.)}$$

$$⑥ i(t) = \frac{dq(t)}{dt} \Rightarrow i_c(t) = C \frac{dv_c(t)}{dt} \text{ voltage on capacitor.}$$

$$⑦ v_c(t) = \frac{1}{C} \int_{t_0}^t i_c(\tau) d\tau + v_c(t_0) \text{ initial condition.}$$



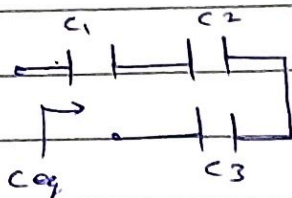
⑧ $i_c(t) = C \frac{dv_c(t)}{dt}$ & if $v_c(t) = \text{D.C. value} \Rightarrow \text{fixed}$

$\Rightarrow i_c(t) = 0$ | it's means that capacitor act like open circuit in D.C. circuit.

⑨ Energy stored in capacitor.

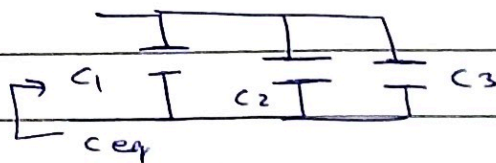
$$E = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C} \text{ Joule.}$$

⑩ Capacitor in series:-



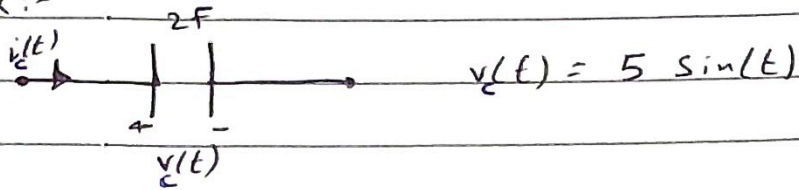
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

⑪ Capacitor in parallel:-



$$C_{eq} = C_1 + C_2 + C_3$$

Ex:-



Find ① energy stored

② Q

③ I

$$① E = \frac{1}{2} C V^2 \Rightarrow \frac{1}{2} \times 2 \times (5 \sin(t))^2$$

$$E = 25 \sin^2(t) \text{ Joule.}$$

$$② Q = C V = 2 \times 5 \sin(t)$$

$$Q = 10 \sin(t) \text{ Coulomb.}$$

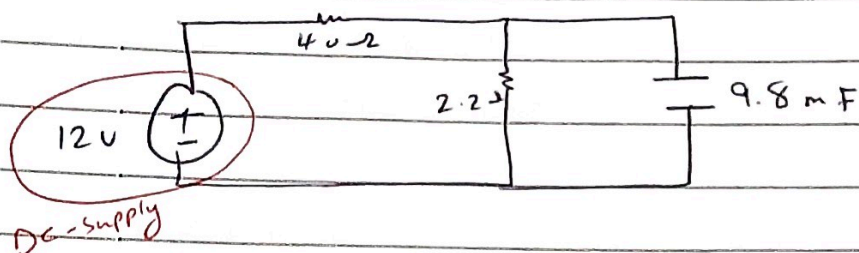
$$③ i_c(t) = C \frac{dv_c(t)}{dt}$$

$$= 2 (5 \cos(t))$$

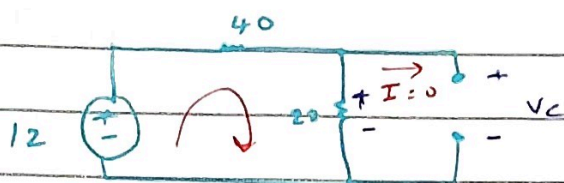
$$i_c(t) = 10 \cos(t) \text{ A.}$$



Ex :- Find :- ① Energy stored in the capacitor
② Q on capacitor.



* in DC - circuit
the capacitor
act like open
circuit $I_c = 0$



By voltage division :-

$$V_c = 12 \times \frac{2.2}{2.2 + 40} = 4.26 \text{ V}$$

$$\textcircled{1} E = \frac{1}{2} C V^2 = \frac{1}{2} \times 9.8 \times 10^{-3} (4.26)^2$$

$$E = 88.92 \text{ mJoule.} \quad \leftarrow$$

$$\textcircled{2} Q = CV$$

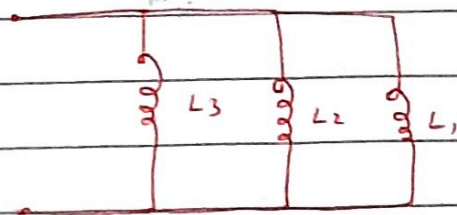
$$= 9.8 \times 10^{-3} (4.26)$$

$$= 41.75 \text{ mCoulomb}$$

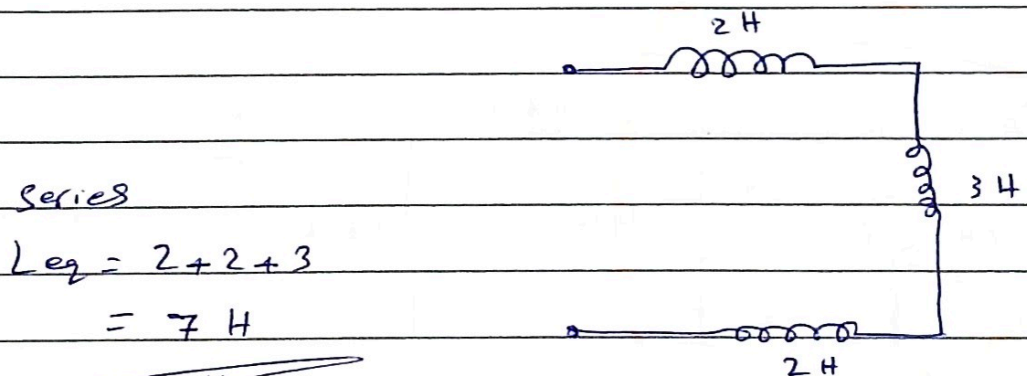
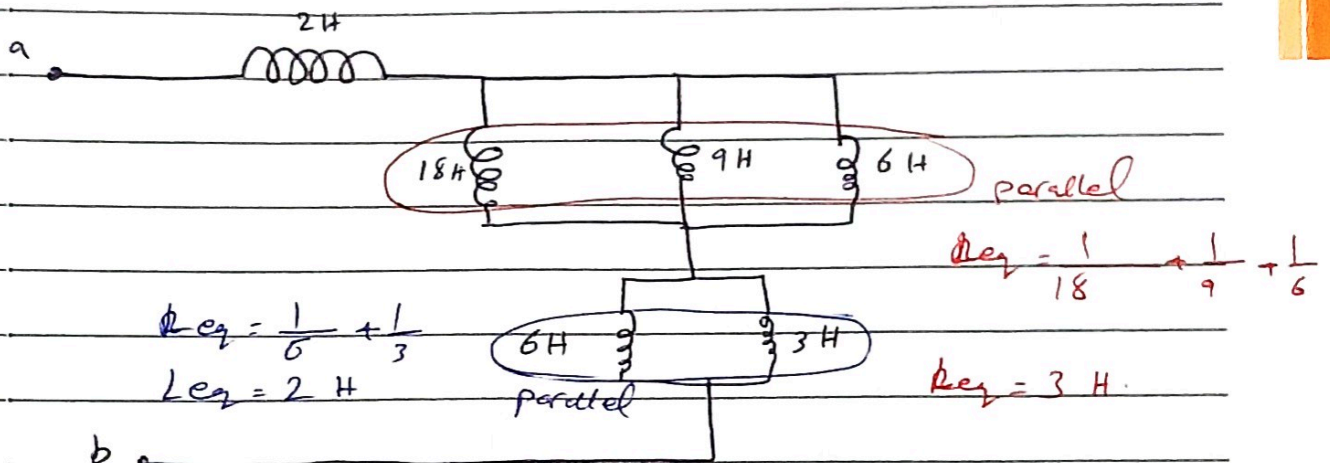


⑨ parallel connection:-

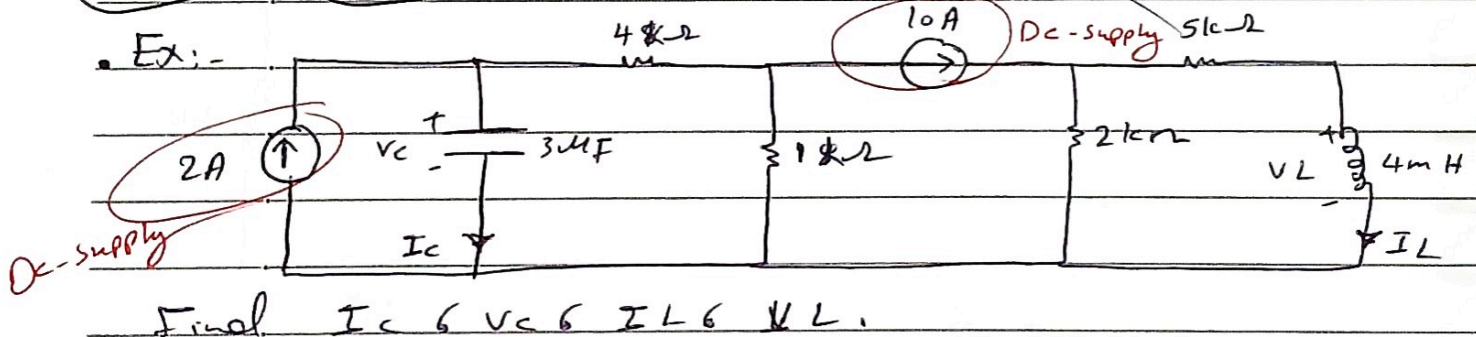
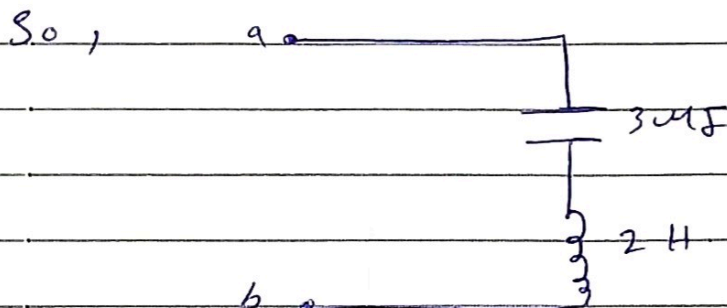
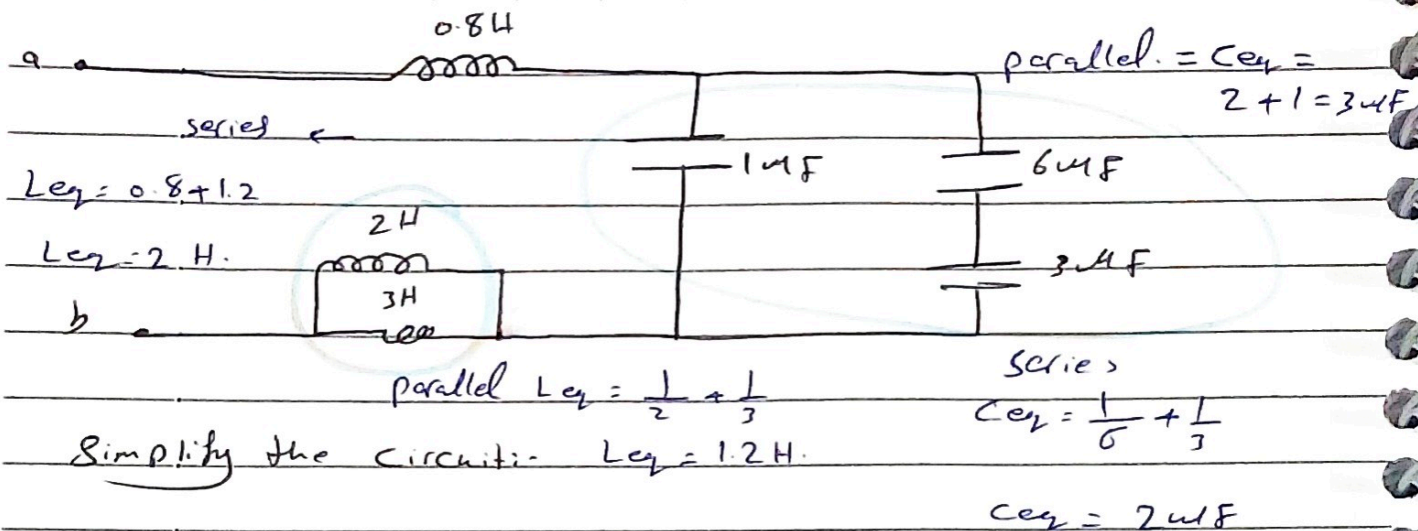
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



Ex:- Find L_{eq} from a & b.

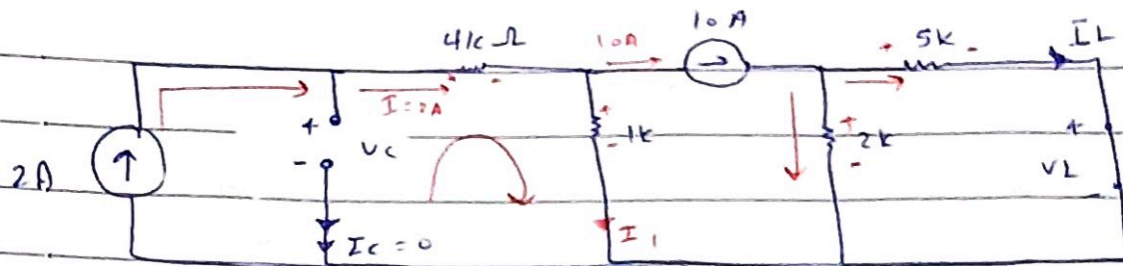


Ex



DC-supply so, I_c & $V_L = 0$ & Capacitor like open & Inductor short circuit.





By KCL $\Rightarrow 2 = I_1 + 10$
 $I_1 = -8 \text{ A}$



By KVL $\Rightarrow -V_c + 4 \times 2 + 1 \times (-8) = 0$

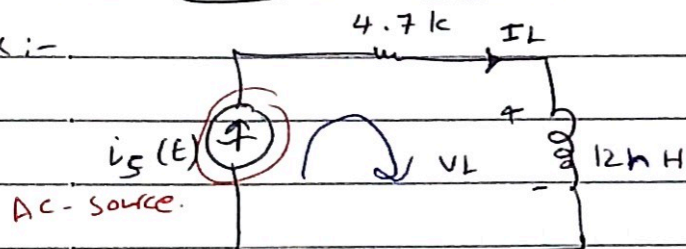
$V_c = 0$ volt
 $I_c = 0$ A $\rightarrow$ open circuit.

using current division (parallel)

$I_L = 10 \times \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{2}} = 2.86 \text{ A}$

$V_L = 0 \rightarrow$ short circuit

Ex:-



$i_s(t) = 4e^{-t}$

Find Energy stored in inductor? ② $V_L(t)$

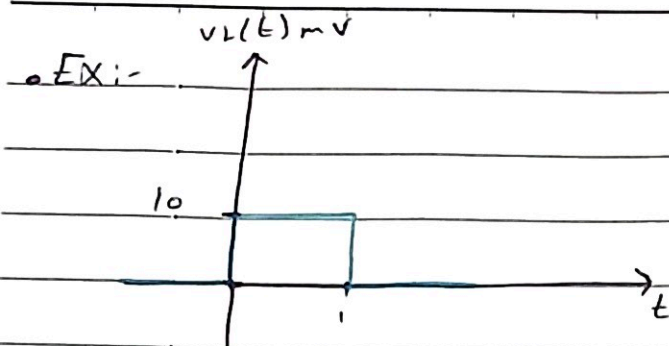
① $E = \frac{1}{2} L I_L^2$ $E = \frac{1}{2} (12 \times 10^{-3}) (4e^{-t})^2$

$I_L(t) = I_s(t)$ $E = 9.6 \times 10^{-8} e^{-2t}$ Joule

لأنه ما في ختم، ليس بهر
 بـ، بر ابرم.

② $V_L(t) = L \frac{di_L(t)}{dt} = (12 \times 10^{-3}) (4)(-1) e^{-t}$

$= -48 \times 10^{-9} e^{-t}$ volt



$$L = 10 \text{ mH}$$

$$i_L(0) = 0 \text{ A}$$

Find $i_L(t)$

$$i_L(t) = \left[\frac{1}{L} \int_{t_0}^t v_L(\tau) d\tau \right] + i_L(t_0)$$

for $t < 0 \rightarrow v_L(t) = 0 \rightarrow i_L(t) = 0 \text{ A}$

for $0 < t \leq 1 \rightarrow v_L(t) = 10 \text{ mV}$

for $i_L(t) = \left[\frac{1}{10 \times 10^{-3}} \int_0^t (10 \text{ m}) d\tau \right] + 0$

$$= \frac{1}{10 \times 10^{-3}} (10 \times 10^{-3}) t$$

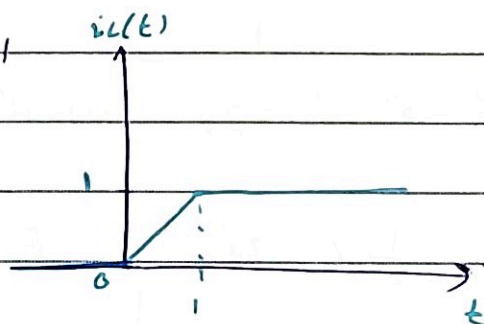
$$i_L(t) = t$$

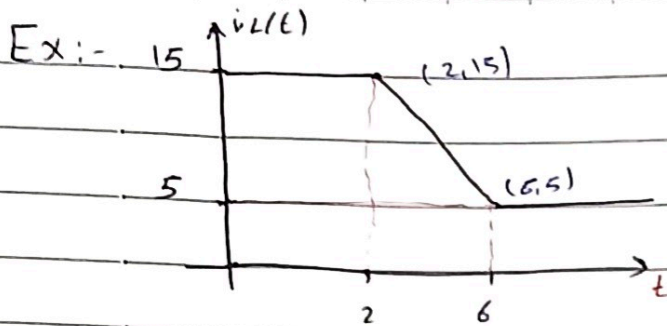
for $t \geq 1 \rightarrow v_L(t) = 0$

$$i_L(1) = 1 \text{ A}$$

$$i_L(t) = \left[\frac{1}{10 \times 10^{-3}} \int_1^t 0 d\tau \right] + 1$$

$$i_L(t) = 1 \text{ A}$$





$$L = 50 \text{ mH}$$

Find & plot $v_L(t)$

$$v_L(t) = L \frac{di_L(t)}{dt}$$

for $0 \leq t \leq 2 \Rightarrow i_L(t) = 15 \text{ A}$

So, $v_L(t) = 0$

for $2 \leq t \leq 6 \Rightarrow i_L(t) = -2.5t + 20$

$$m = \frac{dy}{dx} = \frac{15-5}{2-6} = -2.5$$

$$y - 5 = -2.5(x - 6)$$

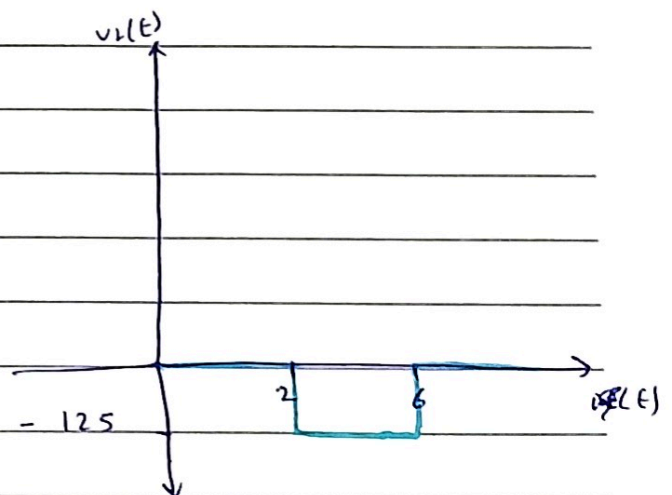
$$y = -2.5x + 20$$

$$v_L(t) = 50 \times 10^{-3} (-2.5)$$

$$v_L(t) = -125 \text{ mV} \quad \leftarrow \text{H}$$

for $t > 6 \Rightarrow i_L(t) = 5 \text{ A}$

$$v_L(t) = 0$$

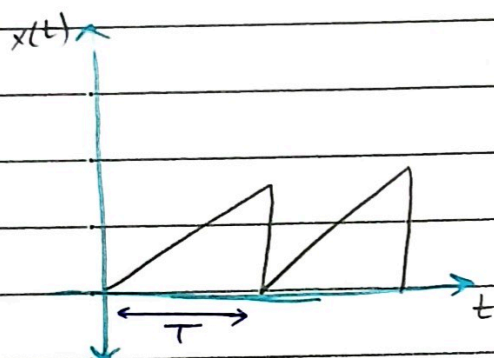


• chapter 11, 9, 11

① Time Dependent Signal sources:-

DC - sources $\Rightarrow$ Fixed value with time.
Fixed direction.

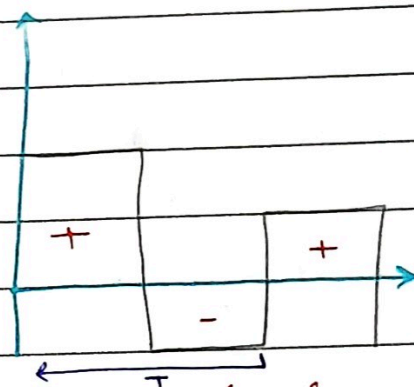
★ AC - sources $\Rightarrow$ Generate voltage or current varying with time.



Saw-tooth

Same direction

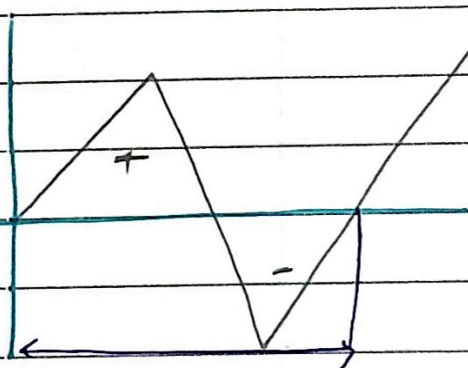
varying value



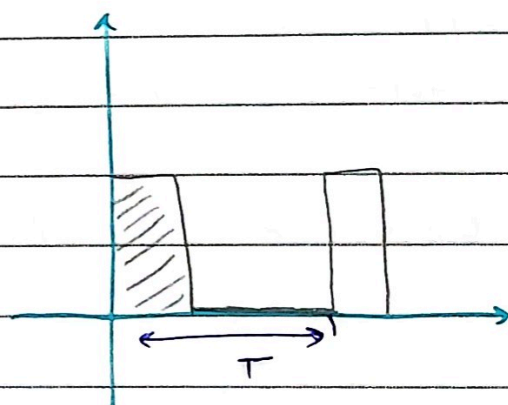
varying direction

and it's wave is

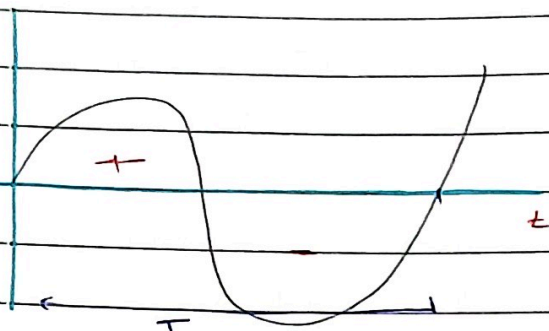
square wave.



triangle signal



pulse-train signal

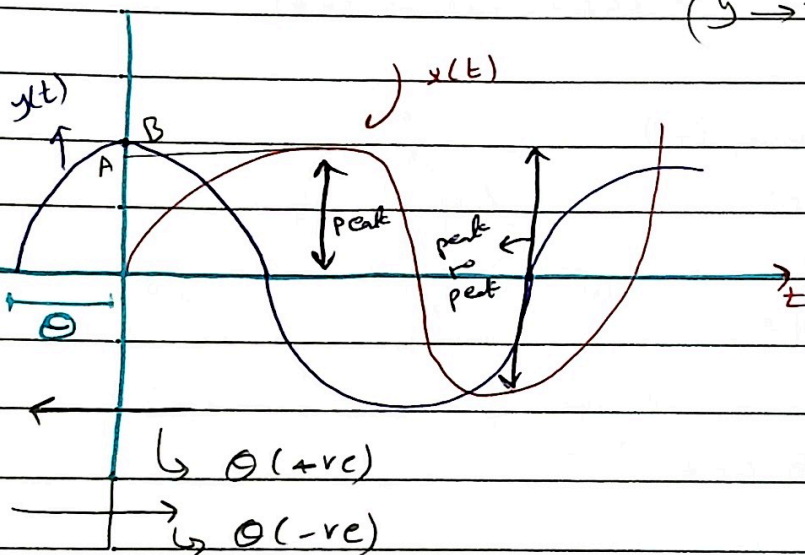


Sine-wave signal.

* Periodic signal :- It's a signal Repeat it self after one period (T)

④ Sinusoidal signal :-

y و x تفرقا بـ جينافيدار @ (y → x)



$$x(t) = A \cos(\omega t + \Theta)$$

$$y(t) = B \cos(\omega t)$$

الجمع دال (cos)

A :- amplitude of signal → peak value.

من 0 و ٩٢

$$\text{peak to peak} = 2 * \text{peak} = 2 * A$$

من ٩٢ و ٠

ω :- radian frequency ⇒ unit rad/s.

$$\omega = 2\pi f \rightarrow f \text{ :- Natural frequency} \Rightarrow \text{unit: Hz}$$

* Mathematical (1) Rectangular

$$x_1 = a + jb$$

(1) Addition:- $x_2 = c + jd$

$$\begin{aligned} x_1 + x_2 &= (a + jb) + (c + jd) \\ &= (a + c) + j(b + d) \end{aligned}$$

(2) Subtraction

$$\begin{aligned} x_1 - x_2 &= (a + jb) - (c + jd) \\ &= (a - c) + j(b - d) \end{aligned}$$

(3) Multiplication

$$\begin{aligned} x_1 \cdot x_2 &= (a + jb)(c + jd) \\ &= ac + ajd + jbc + \underbrace{jbjd}_{\rightarrow j \cdot j = -1} \\ &= (ac - bd) + j(ad + bc) \end{aligned}$$

(4) Division:-

$$\begin{aligned} \frac{x_1}{x_2} &= \frac{a + jb}{c + jd} \cdot \frac{c - jd}{c - jd} \\ &= \frac{ac + bd}{c^2 - d^2} + j \left(\frac{bc - ad}{c^2 - d^2} \right) \end{aligned}$$



(2) Polar

$$x_1 = r_1 \angle \theta_1$$

$$x_2 = r_2 \angle \theta_2$$

$$x_1 \cdot x_2 = (r_1 \angle \theta_1) \cdot (r_2 \angle \theta_2)$$

$$= r_1 \cdot r_2 \angle \theta_1 + \theta_2$$

$$x_1 \div x_2 = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

★ Complex Numbers:-

$$j \text{ or } i = \sqrt{-1}$$

$$j^2 = -1$$

Complex Number has two forms:-

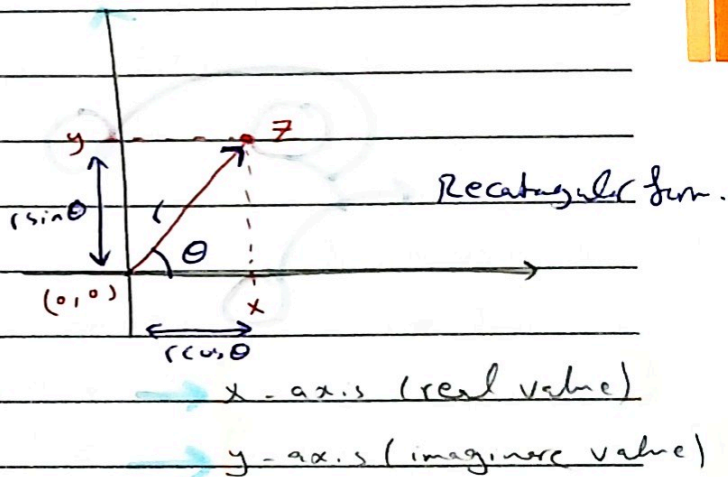
① Rectangular form $Z = x + jy$

② Polar form: $Z = r \angle \theta$

→ start from (0,0) to Z

ave is θ between x & y axis

$$x = r \cos \theta$$



★ Conversion from Rect to Polar

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow \begin{matrix} \text{angle} & \text{between} & \text{x-axis} & \text{and} & \text{y-axis} \\ (-, -) & & (+, +) \end{matrix}$$

Conversion from polar to Rect.

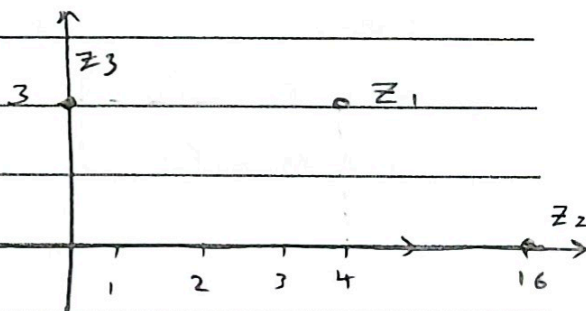
$$x = r \cos \theta$$

$$y = r \sin \theta$$

Ex:- ① $Z_1 = 4 + 3j$
 real imag

② $Z_2 = 16 \rightarrow \text{real}$

③ $Z_3 = 3j \rightarrow \text{imag}$



$$T (\text{period}) = \frac{1}{f} [\text{Sec}]$$

θ :- phase shift.

$$\theta = \frac{\text{distance between two signal in sec.}}{T} \times 360^\circ$$

$T \rightarrow \text{period}$



Pr * $\sin(\omega t) = \cos(\omega t - 90^\circ)$

Ex :- $i(t) = 10 \cos(377t + 30^\circ)$
 $\rightarrow i(t)$ moved to left.

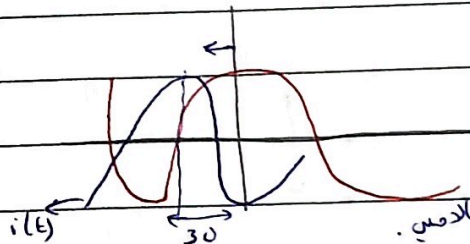
$$I_{\text{peak}} = 10 \text{ A}$$

$$I_{\text{peak to peak}} = 2 \times 10 = 20 \text{ A.}$$

$$\omega = 377 \text{ rad/s}$$

$$= 2\pi f$$

$$f = \frac{377}{2\pi} = 60 \text{ Hz}$$



$$T = \frac{1}{f} = \frac{1}{60} = 0.016 \text{ sec}$$

$$\theta = 30^\circ$$

$$i(0.1 \text{ sec}) = 10 \times \cos(377 \times 0.1 + 30^\circ)$$

$$= 7.8 \text{ A}$$