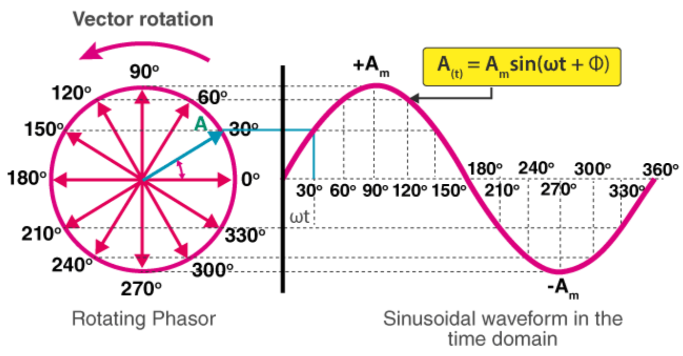
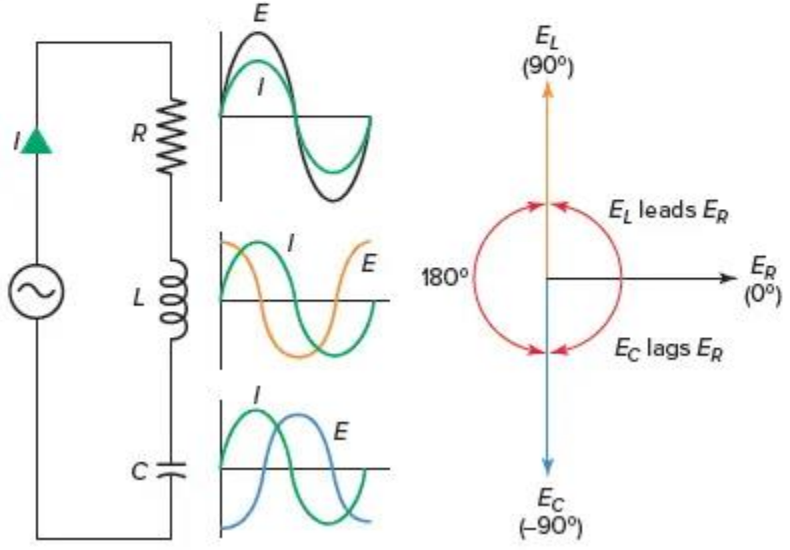
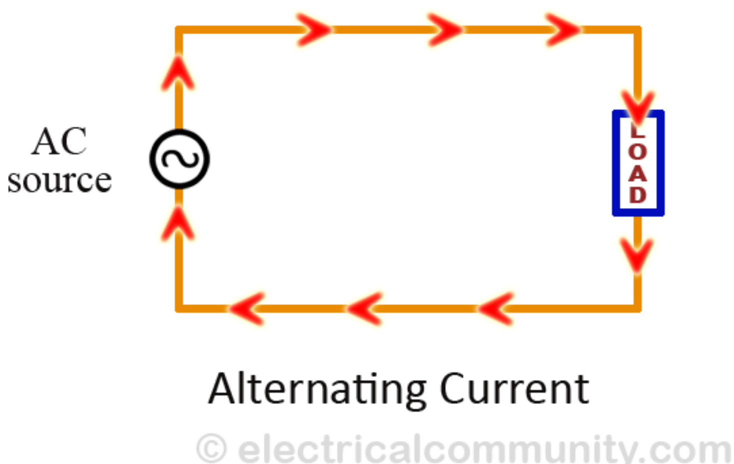
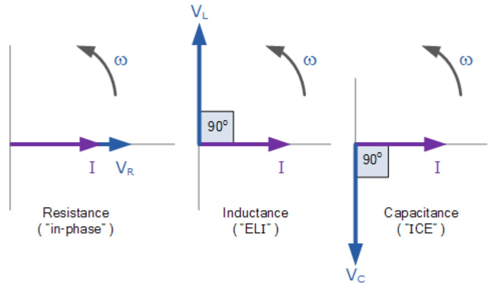
Sinusoidal Steady-State Analysis of Single-Phases Circuits







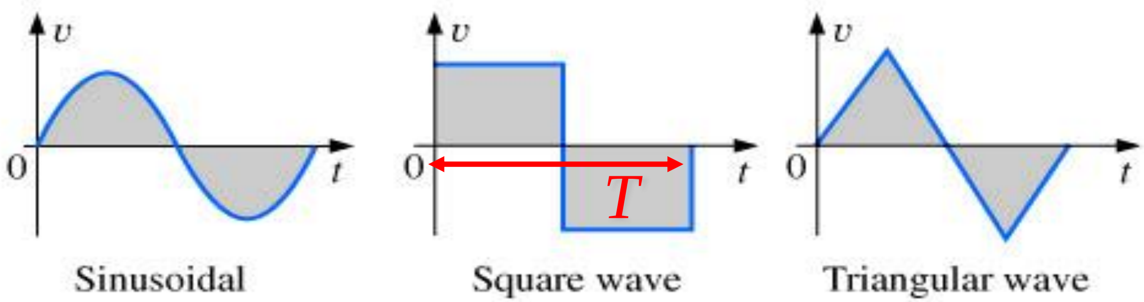


Sinusoidal Steady-State Analysis of Single-Phases Circuits

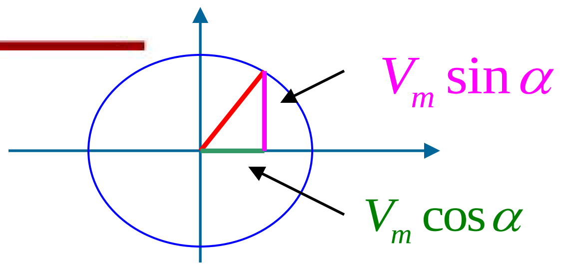
- Sinusoids' features
- Phasors
- Phasor relationships for circuit elements
- Impedance and admittance
- Kirchhoff's laws in the frequency domain
- Impedance combinations

Alternating (AC) Waveforms

- The term **alternating** indicates only that the waveform alternates between two prescribed levels in a set time sequence.
- **Instantaneous value:** The magnitude of a waveform at any instant of time; denoted by the lowercase letters (v_1, v_2).
- **Peak amplitude:** The maximum value of the waveform as measured from its average (or mean) value, denoted by the uppercase letters V_m.
- **Period (T):** The time interval between successive repetitions of a periodic waveform.
- **Cycle:** The portion of a waveform contained in one period of time.
- **Frequency:** (Hertz) the number of cycles that occur in 1 s $f = 1/T$
- The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.



Sinusoids



- The sinusoidal wave form can be derived from the length of the vertical projection of a radius vector rotating in a uniform circular motion about a fixed point.

- The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

- The angular velocity (ω) is: $\omega = \alpha / t$

Since ω is typically provided in radians per second, the angle α obtained using $\alpha = \omega t$ is usually in radians.

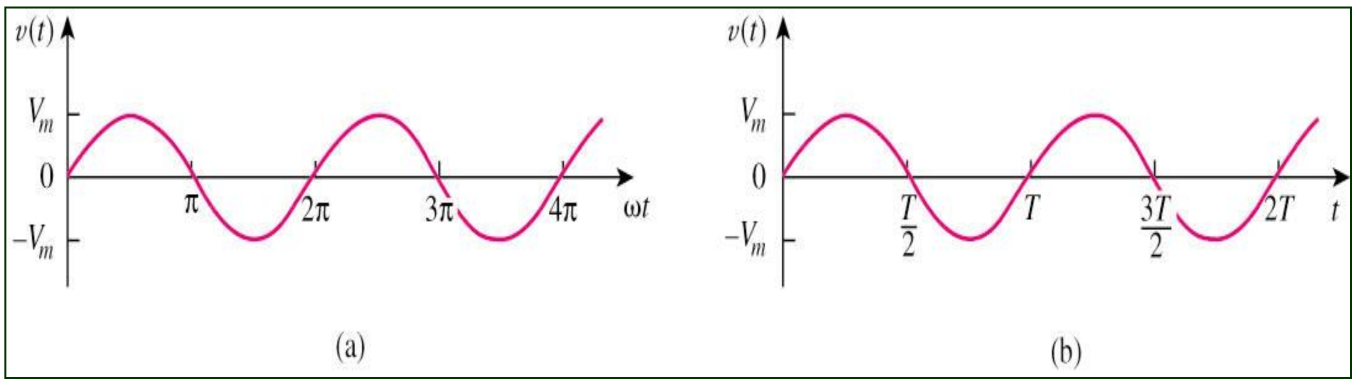
- The time required to complete one revolution is equal to the period (T) of the sinusoidal waveform. The radians subtended in this time interval are 2π.

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad \omega = 2\pi f$$

Sinusoids

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$

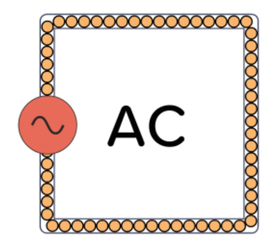


where

V_m = the **amplitude** of the sinusoid

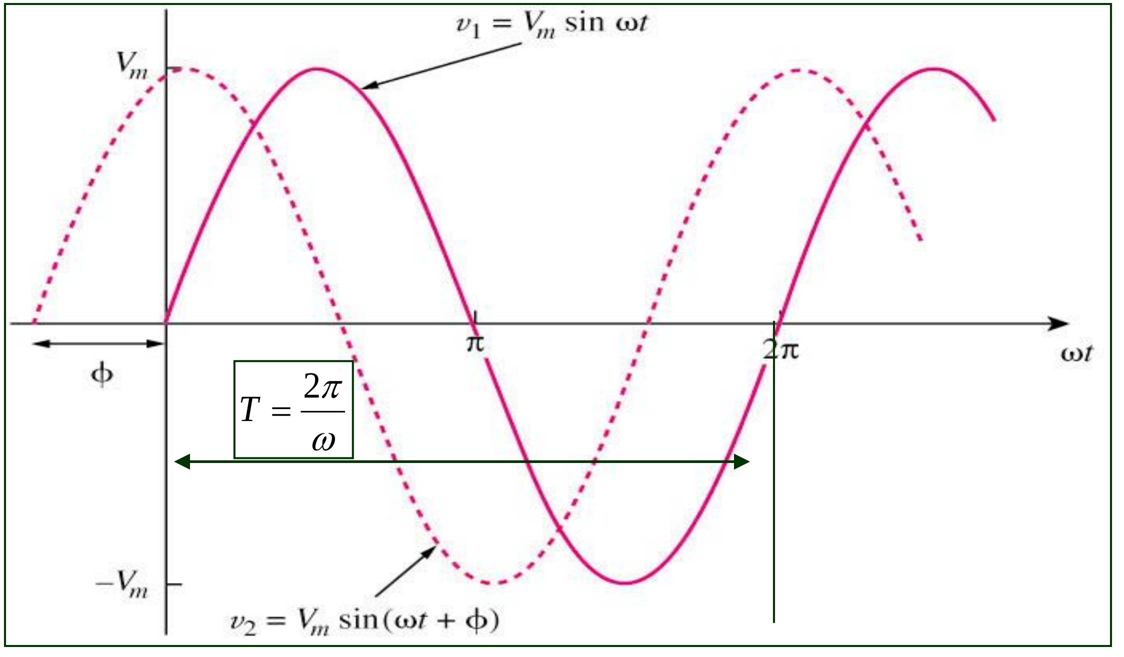
ω = the angular frequency in radians/s

Φ = the phase



Sinusoids

A <u>periodic function</u> is one that satisfies *v(t)* = *v(t + nT*), for all t and for all integers n.



$$f = \frac{1}{T} Hz$$

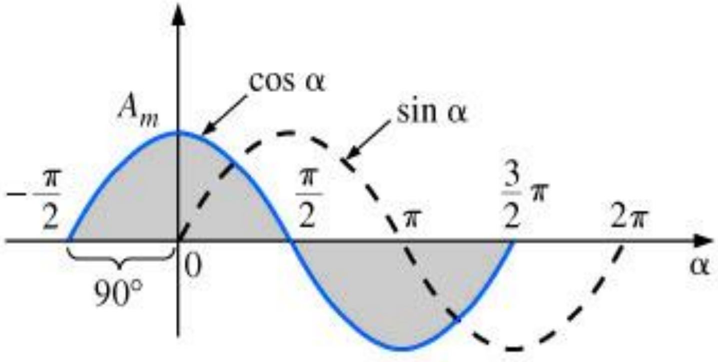
$$\omega = 2\pi f$$

- Only two sinusoidal values with the <u>same frequency</u> can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

Phase of Sinusoids

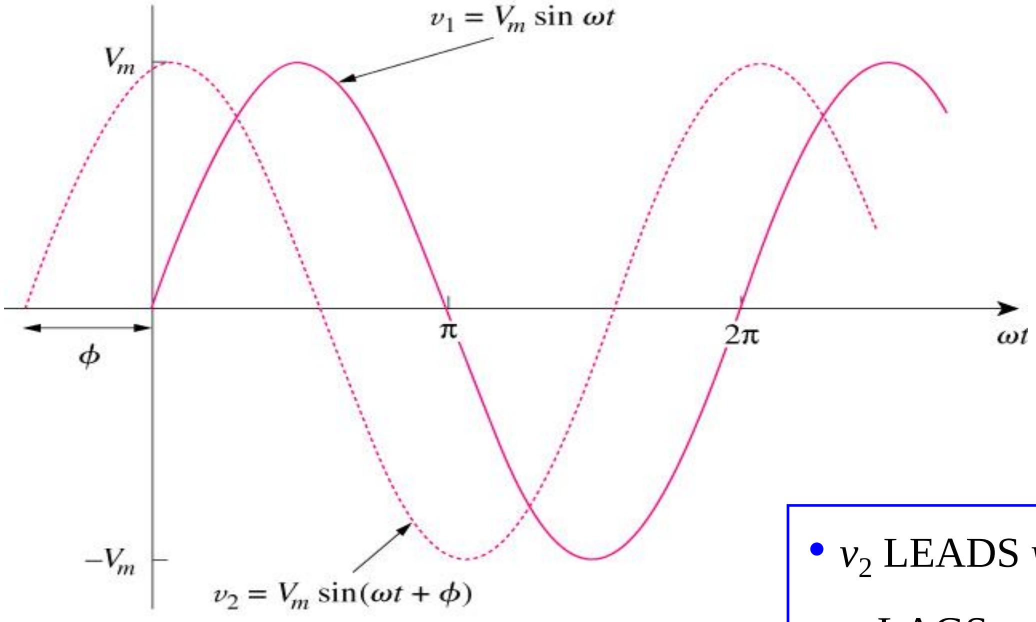
- The terms *lead* and *lag* are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes.
- The cosine curve is said to *lead* the sine curve by 90°.
- The sine curve is said to *lag* the cosine curve by 90°.
- 90 is referred to as the phase angle between the two waveforms.
- When determining the phase measurement we first note that each sinusoidal function has the same frequency, permitting the use of either waveform to determine the period.
- Since the full period represents a cycle of 360°, the following ratio can be formed:

$$\theta = \frac{\text{phase shift (no. of div.)}}{T\text{ (no. of div.)}} \times 360^\circ$$



Phase of Sinusoids

➢ Consider the sinusoidal voltage having phase φ, $v(t) = V_m \sin(\omega t + \phi)$



- v_2 LEADS v_1 by phase φ.
- v_1 LAGS v_2 by phase φ.
- v_1 and v_2 are out of phase.

Sinusoids

Example:

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Solution:

Amplitude = 5, phase = -60^o, angular frequency = 4π rad/s, Period = 0.5 s, frequency = 2 Hz.

Sinusoids

Example: Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$. Does i_1 lead or lag i_2?

Solution:

Since $\sin(\omega t + 90^o) = \cos \omega t$

$$i_2 = 5\sin(377t - 40^o + 90^o) = 5\sin(377t + 50^o)$$

$$i_1 = -4\sin(377t + 25^o) = 4\sin(377t + 180^o + 25^o) = 4\sin(377t + 205^o)$$

therefore, i_1 leads i_2 by 155^o.

Trigonometric Identities

- Sine and cosine form conversions.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

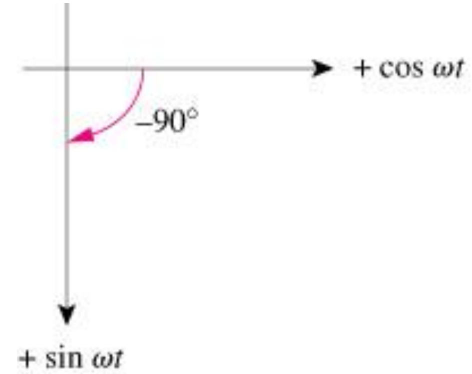
$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

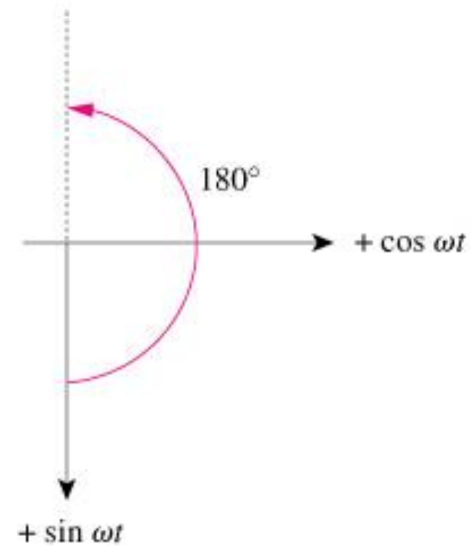
Where

$$C = \sqrt{A^2 + B^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{B}{A}$$

Graphically relating sine and cosine functions.



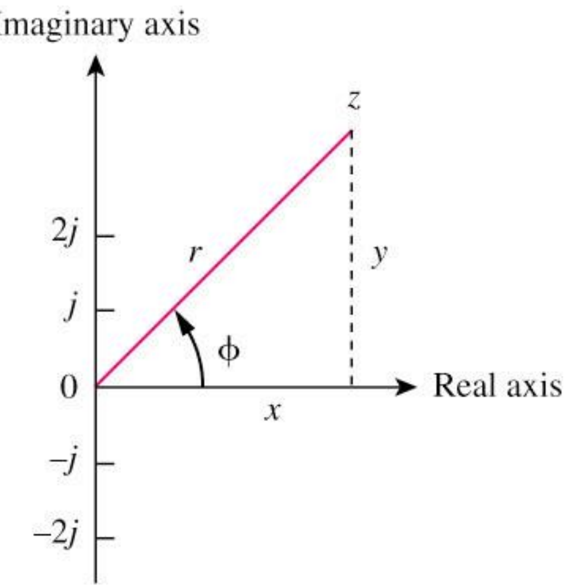
$$\cos(\omega t - 90°) = \sin \omega t$$



$$\sin(\omega t + 180°) = -\sin \omega t$$

Phasor

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.



- It can be represented in one of the following three forms:

a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where $r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\dfrac{y}{x}$

Example:

Evaluate $[(5 + j2)(-1 + j4) - 5\angle 60]^*$ and $\dfrac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ$

(a)

$$(5 + j2)(-1 + j4) = -5 + j20 - j2 - 8 = -13 + j18$$
$$5\angle 60^\circ = 2.5 + j4.33$$
$$(5 + j2)(-1 + j4) - 5\angle 60^\circ = -15.5 + j13.67$$
$$[(5 + j2)(-1 + j4) - 5\angle 60]^* = \underline{\mathbf{-15.5 - j13.67}} = \underline{\mathbf{20.67\angle 221.41^\circ}}$$

(b)

$$3\angle 40^\circ = 2.298 + j1.928$$
$$10 + j5 + 3\angle 40^\circ = 12.298 + j6.928 = 14.115\angle 29.39^\circ$$
$$-3 + j4 = 5\angle 126.87^\circ$$
$$\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} = \frac{14.115\angle 29.39^\circ}{5\angle 126.87^\circ} = 2.823\angle -97.48^\circ$$
$$2.823\angle -97.48^\circ = -0.3675 - j2.8$$
$$10\angle 30^\circ = 8.66 + j5$$
$$\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ = \underline{\mathbf{8.293 + j2.2}}$$

Phasor

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

Phasors

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.
- Phasor is the mathematical equivalent of a sinusoid with time variable dropped.
- Phasor representation is based on Euler's identity.

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi \quad \text{Euler's Identity}$$

$$\cos\phi = \mathrm{Re}\left\{e^{j\phi}\right\} \quad \text{Real part}$$

$$\sin\phi = \mathrm{Im}\left\{e^{j\phi}\right\} \quad \text{Imaginary part}$$

- Given a sinusoid $v(t)=V_m\cos(\omega t+\varphi)$.

$$v(t) = V_m\cos(\omega t+\phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)}) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t}) = \mathrm{Re}(\mathbf{V} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m\angle\phi = \text{PHASOR REP.}$$

$$v(t) = V_m\cos(\omega t+\phi) \Leftrightarrow \mathbf{V} = V_m\angle\phi$$

(Time Domain Re pr.) (Phasor Domain Re presentation)

$$v(t) = \mathrm{Re}\{\mathbf{V}e^{j\omega t}\} \quad \text{(Converting Phasor back to time)}$$

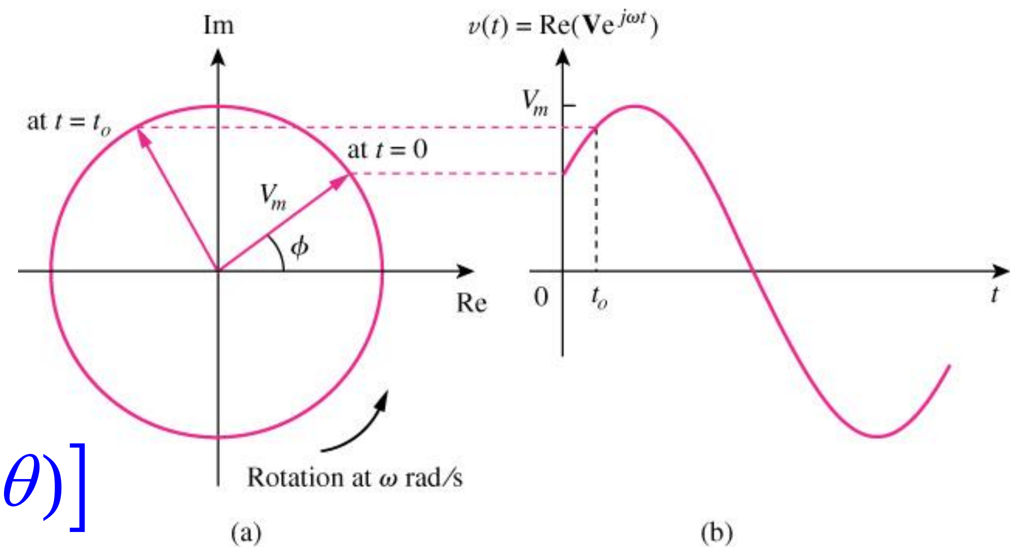
Phasor as Rotating Vectors

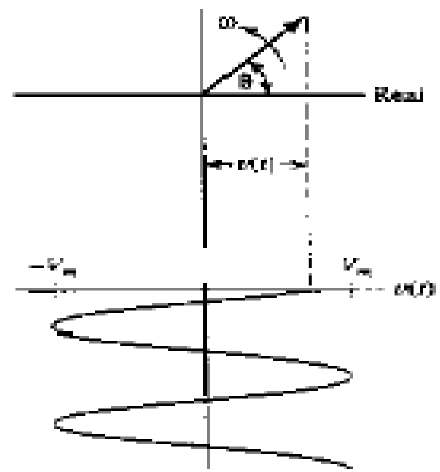
$$v(t) = V_m \cos(\omega t + \theta)$$

$$v(t) = \mathrm{Re}\left[V_m e^{(j\omega t + \theta)}\right]$$

$$v(t) = \mathrm{Re}\left[V_m \angle (j\omega t + \theta)\right]$$

Rotating Phasor





Phasor

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

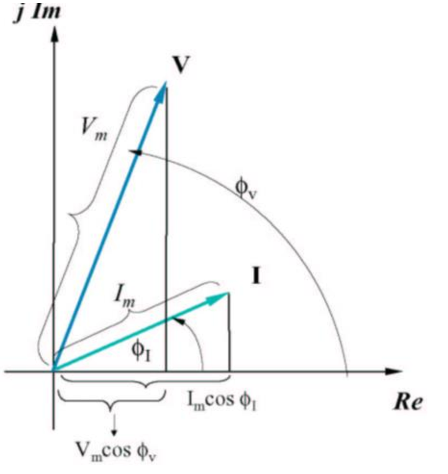
(time domain) (phasor domain)

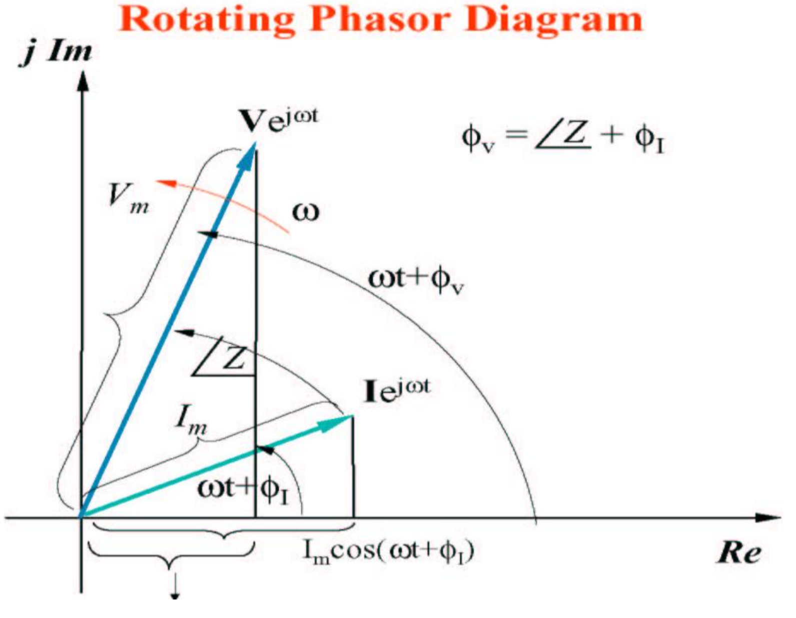
- Amplitude and phase difference are two principal concerns in the study of voltage and current sinusoids.
- Phasor will be defined from the cosine function in all our proceeding study. If a voltage or current expression is in the form of a sine, it will be changed to a cosine by subtracting from the phase.

Phasor Diagrams

- The SINOR $\mathbf{V}e^{j\omega t}$ Rotates on a circle of radius V_m at an angular velocity of ω in the counterclockwise direction.

Time Domain Representation	Phasor Domain Rep.
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$





Phasor

Example:

Transform the following sinusoids to phasors:

$i = 6\cos(50t - 40^o)$ A, $\quad v = -4\sin(30t + 50^o)$ V

Solution:

a. $I = 6\angle -40^\circ$ A

b. Since $-\sin(A) = \cos(A+90^o)$;

$v(t) = 4\cos(30t+50^o+90^o) = 4\cos(30t+140^o)$ V

Transform to phasor => $V = 4\angle 140^\circ$ V

Phasor

Example:

Transform the sinusoids corresponding to phasors

$$\mathbf{V} = -10\angle 30^\circ \text{ V}$$

$$\mathbf{I} = j(5 - j12) \text{ A}$$

Solution:

a) $v(t) = 10\cos(\omega t + 210^o)$ V

b) Since $\mathbf{I} = 12 + j5 = \sqrt{12^2 + 5^2}\angle \tan^{-1}(\frac{5}{12}) = 13\angle 22.62^\circ$

$i(t) = 13\cos(\omega t + 22.62^o)$ A

Phasor

The differences between v(t) and V:

- $v(t)$ is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- $v(t)$ is time dependent, V is not.
- $v(t)$ is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; when it is applied to two or more sinusoid signals only if they have the same frequency.

Differentiation and Integration in Phasor Domain

- Differentiating a sinusoid is equivalent to multiplying its corresponding phasor by $j\omega$.

$$v(t) = V_m \cos(\omega t + \theta) = \mathrm{Re}\left[\mathbf{V} e^{j\omega t}\right]$$

$$\frac{dv(t)}{dt} = -\omega V_m \sin(\omega t + \theta) = -\omega V_m \cos(\omega t + \theta + 90^\circ)$$

$$= \mathrm{Re}\left[j\omega \mathbf{V} e^{j\omega t}\right] \qquad \frac{dv}{dt} \Leftrightarrow J\omega\mathbf{V}$$

- Integrating a sinusoid is equivalent to dividing its corresponding phasor by $j\omega$.

(Time Domain)		(Phasor Domain)
$v(t) = V_m \cos(\omega t + \phi)$	$\Leftrightarrow$	$\mathbf{V} = V_m \angle \phi$
$v(t) = V_m \sin(\omega t + \phi)$	$\Leftrightarrow$	$\mathbf{V} = V_m \angle \phi - 90^\circ$
$\frac{dv}{dt}$	$\Leftrightarrow$	$J\omega\mathbf{V}$
$\int v\,dt$	$\Leftrightarrow$	$\frac{\mathrm{V}}{J\omega}$

Phasor

<u>Example:</u>

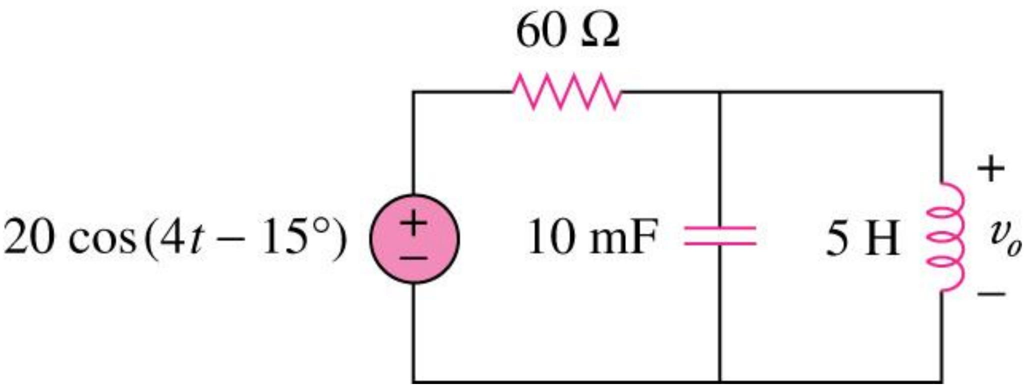
Use phasor approach, determine the current *i(t)* in a circuit described by the integro-differential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$$

<u>Answer</u>: ***i(t)* = 4.642*cos(*2*t* + 143.2º*) A***

Phasor

- We can derive the differential equations for the following circuit in order to solve for $v_o(t)$ in phase domain V_o.



$$\frac{d^2v_o}{dt^2}+\frac{5}{3}\frac{dv_0}{dt}+20v_0=-\frac{400}{3}\sin(4t-15^o)$$

- However, the derivation may sometimes be very tedious.

Is there any quicker and more systematic methods to do it?

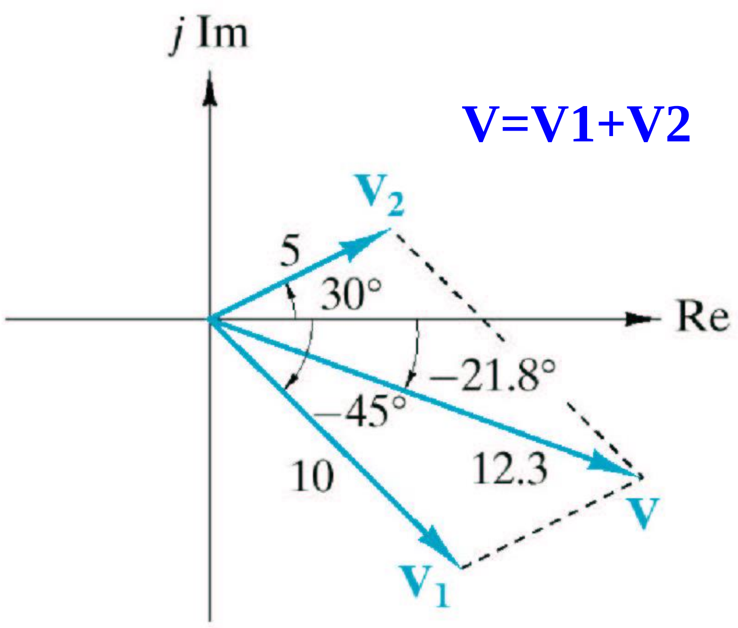
Phasor

The answer is YES!

Instead of first deriving the differential equation and then transforming it into phasor to solve for V_o, we can transform all the RLC components into phasor first, then apply the KCL laws and other theorems to set up a phasor equation involving V_o directly.

Adding Phasors Graphically

➢ Adding sinusoids of the same frequency is equivalent to adding their corresponding phasors.



Example:

Find $v(t) = v_1(t) + v_2(t)$

$v_1(t) = -10\sin(\omega t + 30^o)$ **$v_2(t) = 20\cos(\omega t - 45^o)$**

Let $v = -10\sin(\omega t + 30°) + 20\cos(\omega t - 45°)$

Then, $v = 10\cos(\omega t + 30° + 90°) + 20\cos(\omega t - 45°)$

Taking the phasor of each term

$$\mathbf{V} = 10\angle 120° + 20\angle -45°$$

$$\mathbf{V} = -5 + j8.66 + 14.14 - j14.14$$

$$\mathbf{V} = 9.14 - j5.48 = 10.66\angle -30.95°$$

Converting **V** to the time domain

$$v(t) = \underline{\mathbf{10.66\cos(\omega t - 30.95°)V}}$$

Example:

Find The voltage *v*(*t*) in a circuit described by the integrodifferential equation using the phasor approach.

$$2\frac{dv}{dt}+5v+10\int v\,dt = 20\cos(5t-30°)$$

Given that

$$2\frac{dv}{dt}+5v+10\int v\,dt = 20\cos(5t-30°)$$

we take the phasor of each term to get

$$2j\omega\,V + 5\,V + \frac{10}{j\omega}V = 20\angle\text{-}30°, \qquad \omega = 5$$

$$V\,[j10+5-j(10/5)] = V\,(5+j8) = 20\angle\text{-}30°$$

$$V = \frac{20\angle\text{-}30°}{5+j8} = \frac{20\angle\text{-}30°}{9.434\angle 58°}$$

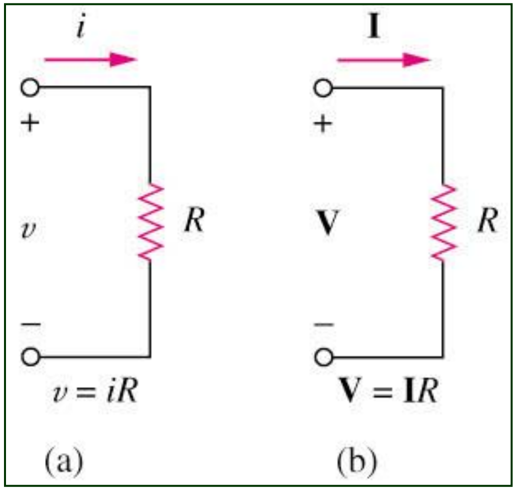
$$V = 2.12\angle\text{-}88°$$

Converting V to the time domain

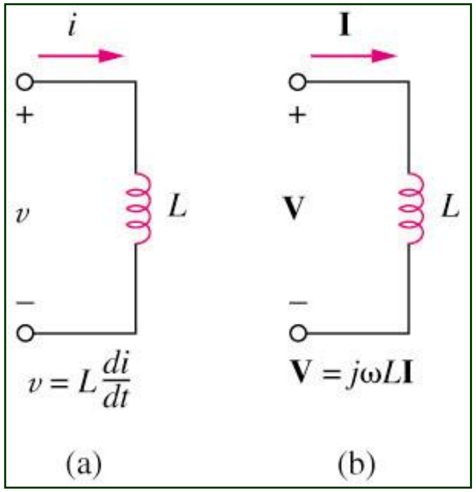
$$v(t) = \underline{\mathbf{2.12\cos(5t-88°)V}}$$

Phasor Relationships for Circuit Elements

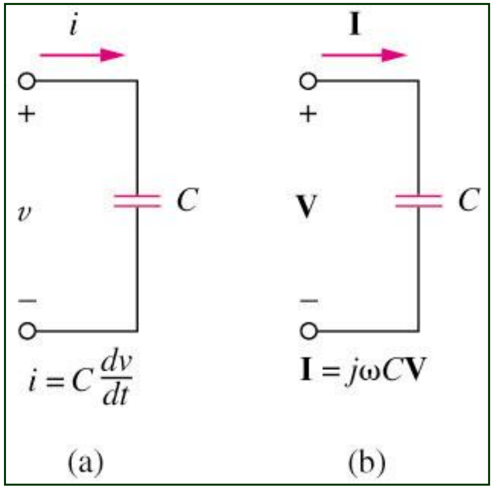
Resistor:

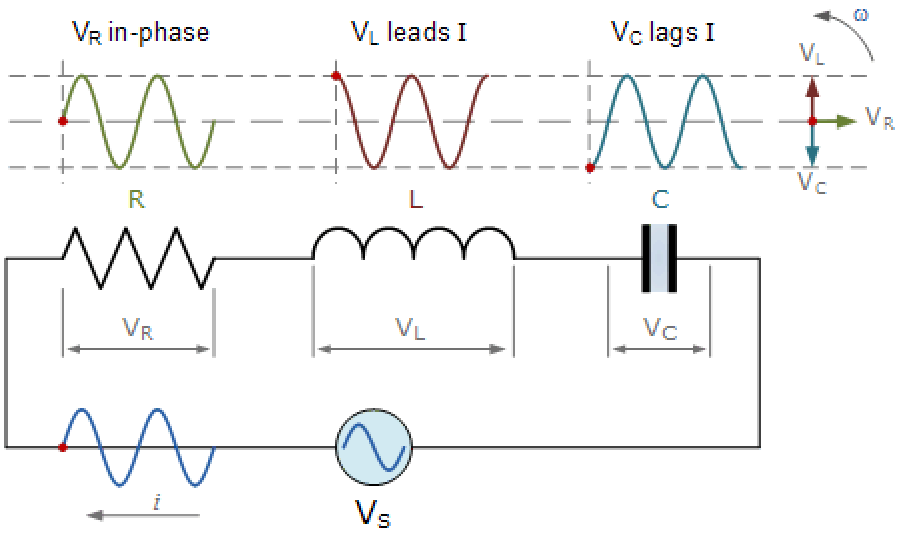


Inductor:



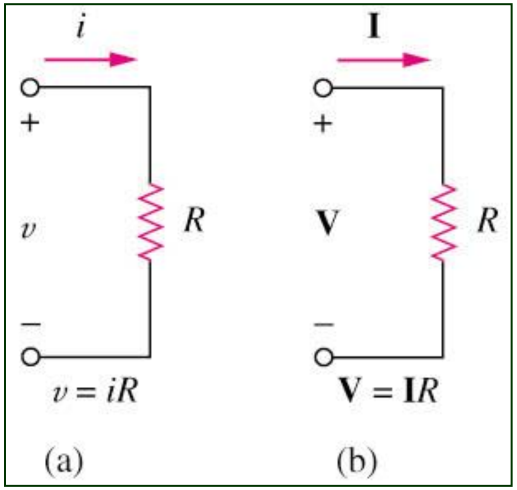
Capacitor:

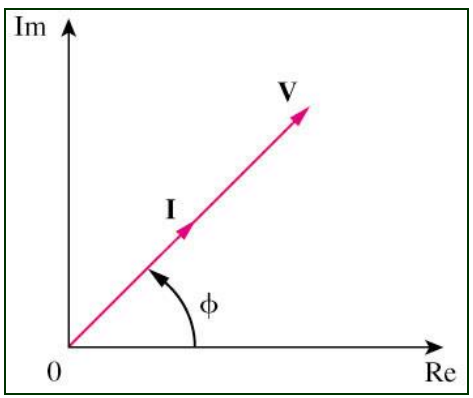




Phasor Relationships for Circuit Elements

Resistor:





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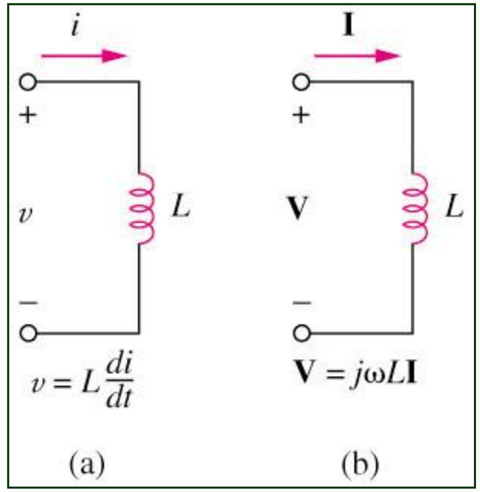
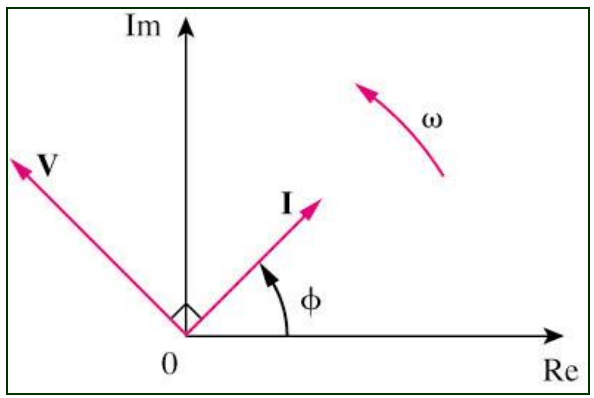
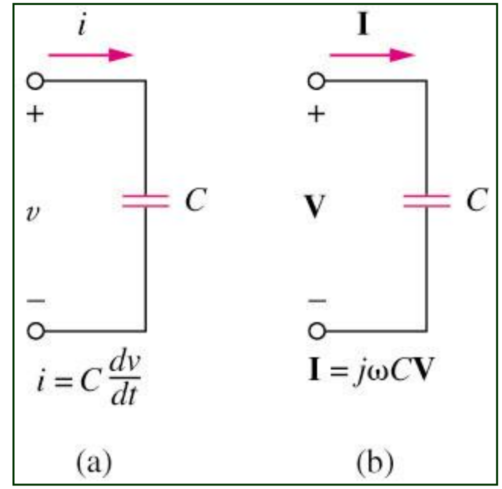
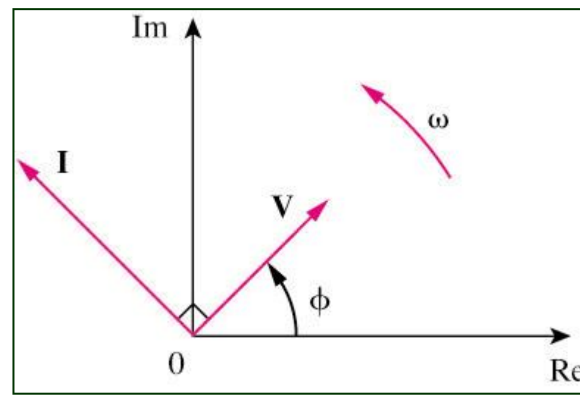




Capacitor:





Phasor Relationships for Circuit Elements

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

Phasor Relationships for Circuit Elements

Example:

If voltage $v(t) = 6\cos(100t - 30^o)$ is applied to a 50 μF capacitor, calculate the current, $i(t)$, through the capacitor.

Answer: $i(t)$ = **30 cos(100t + 60º) mA**

Impedance and Admittance

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

where R = Re(Z) is the resistance and X = Im(Z) is the reactance. Positive X is for L and negative X is for C.

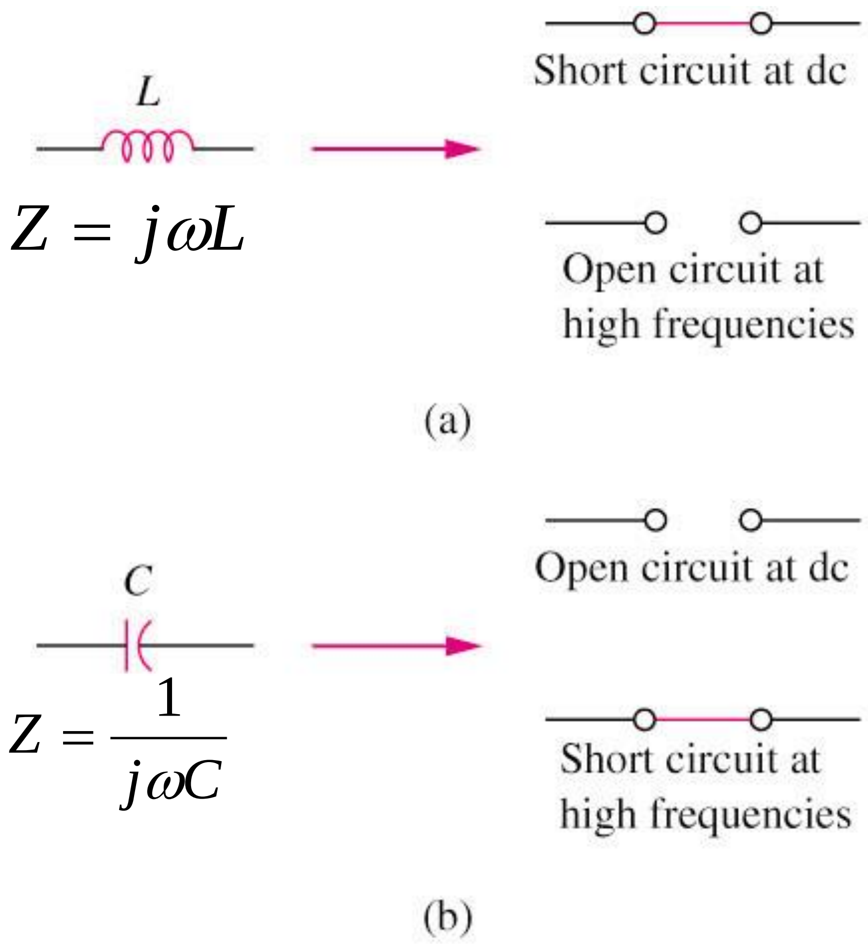
- The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

Impedance and Admittance

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Impedance and Admittance



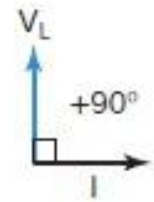
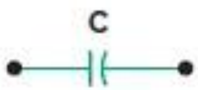
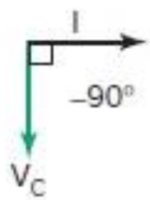
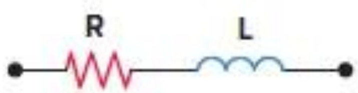
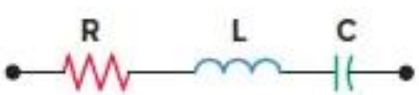
$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

Impedance and Phase Angle

Circuit Elements	Impedance Z	Phase Angle ϕ
	$Z = R$	
	$Z = X_L$	
	$Z = X_C$	
	$Z = \sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$Z = \sqrt{R^2 + X_C^2}$	Negative, between –90° and 0°
	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Impedance and Admittance

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

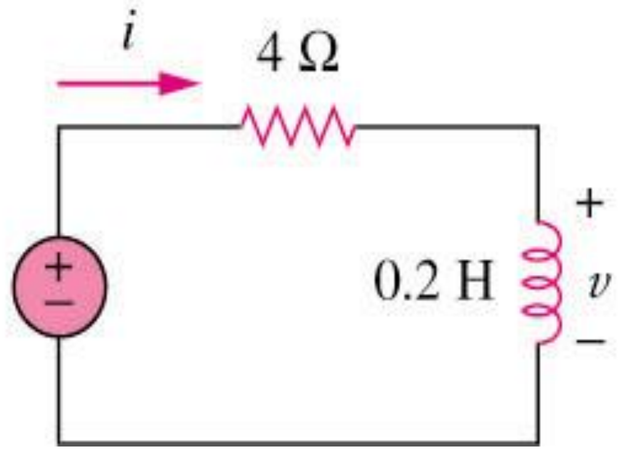
Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

Impedance and Admittance

Example:

Refer to Figure below, determine *v(t)* and *i(t)*.

$v_s = 5\cos(10t)$



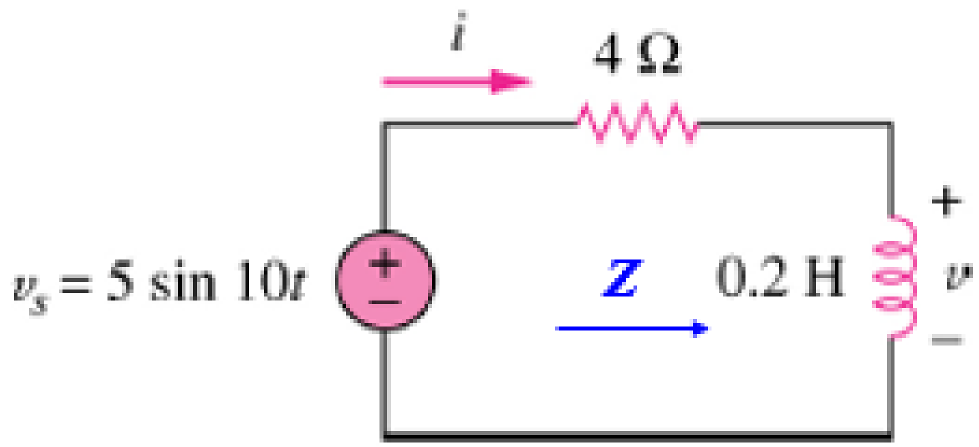
Answers:

$i(t) = 1.118\cos(10t - 26.56^{o})$ **A;**

$v(t) = 2.236\cos(10t + 63.43^{o})$ **V**

Example:

Calculate *v*(*t*) and *i*(*t*) in the circuit given.



$$\mathbf{V}_s = 5\angle 0°, \quad \omega = 10$$

$$\mathbf{Z} = 4 + j\omega L = 4 + j2$$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{5\angle 0°}{4 + j2} = \frac{5(4 - j2)}{16 + 4} = 1 - j0.5 = 1.118\angle\text{-}26.57°$$

$$\mathbf{V} = j\omega L\, \mathbf{I} = j2\, \mathbf{I} = (2\angle 90°)(1.118\angle\text{-}26.57°) = 2.236\angle 63.43°$$

Therefore,

$v(t) = \underline{2.236 \sin(10t + 63.43°)\ \text{V}}$

$i(t) = \underline{1.118 \sin(10t - 26.57°)\ \text{A}}$

Kirchhoff's Laws in the Frequency Domain

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

Impedance Combinations

- The following principles used for DC circuit analysis all apply to AC circuit.

- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

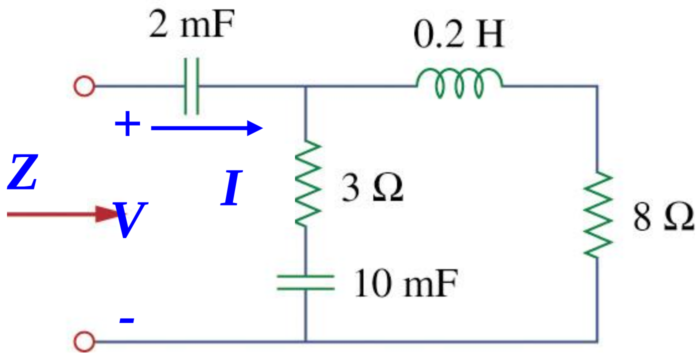
Impedance of Joint Elements

- The Impedance Z represents the opposition of the circuit to the flow of sinusoidal current.

$$Z = \frac{V}{I} = R + jX =$$

$$= \text{Resistance} + \text{j} \times \text{Reactance}$$

$$= |\text{Z}| \angle \theta$$



$$|Z| = \sqrt{R^2 + X^2} \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |Z|\cos\theta \qquad X = |Z|\sin\theta$$

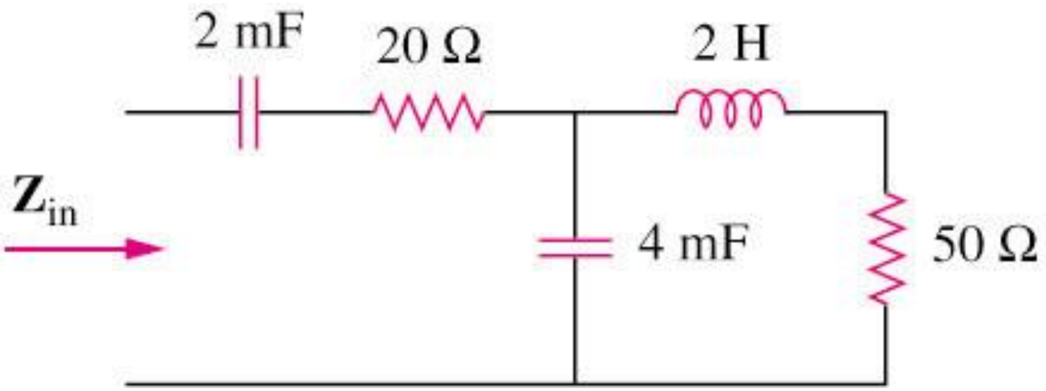
- The Reactance is Inductive if *X* is positive and it is Capacitive if *X* is negative.

Impedance Combinations

Example

Determine the input impedance of the circuit in figure below at

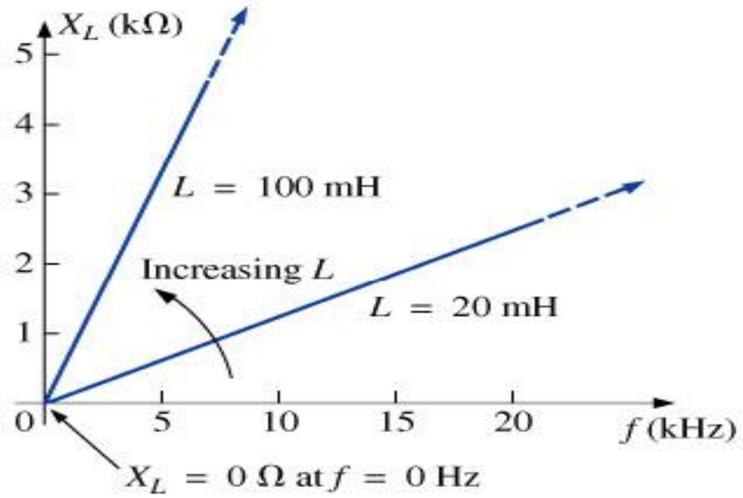
ω =10 rad/s.

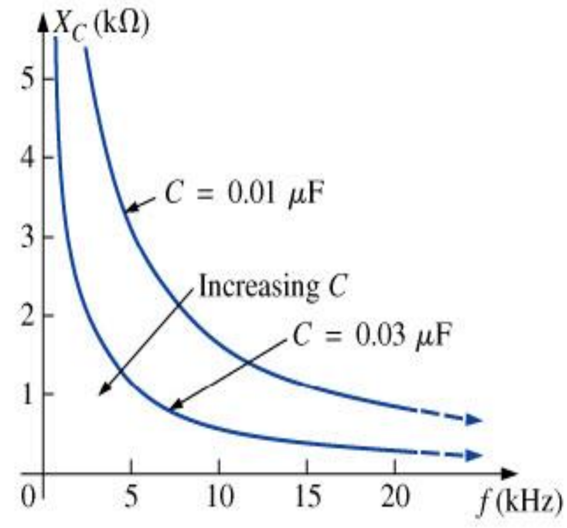


Answer: Z_{in} = **32.38 – j73.76 Ω**

Impedance as a Function of Frequency

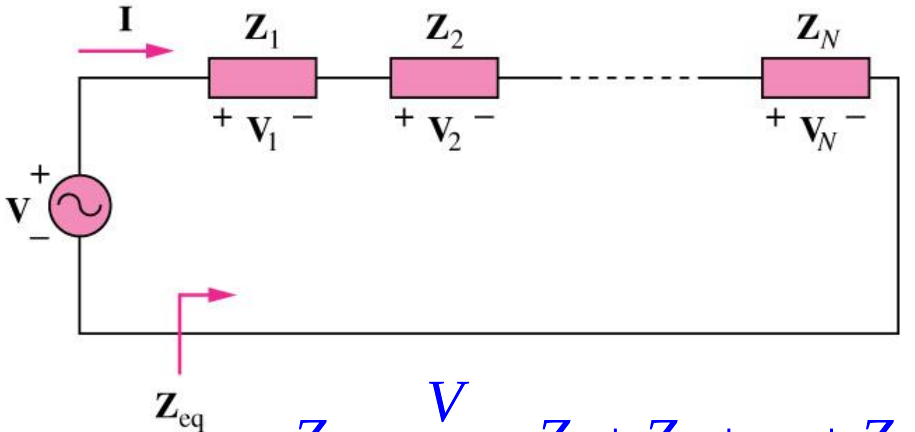
- As the applied frequency increases, the resistance of a resistor remains constant, the reactance of an inductor increases linearly, and the reactance of a capacitor decreases nonlinearly.





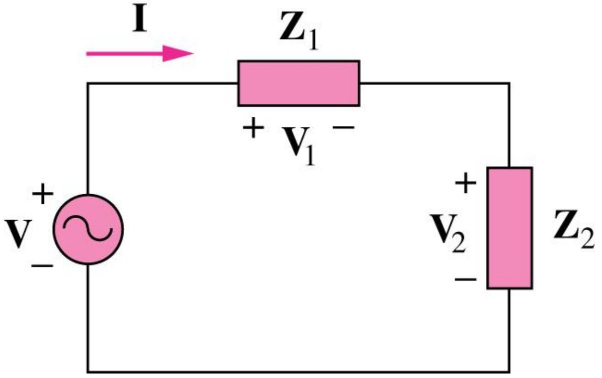
Application of KVL for Phasors

- The Kirchoff's Voltage Law (KVL) holds in the frequency domain. For series connected impedances:



$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \cdots + Z_N$$ (Equivalent Impedance)

- The Voltage Division for two elements in series is:



$$V_1 = \frac{Z_1}{Z_1 + Z_2} V$$

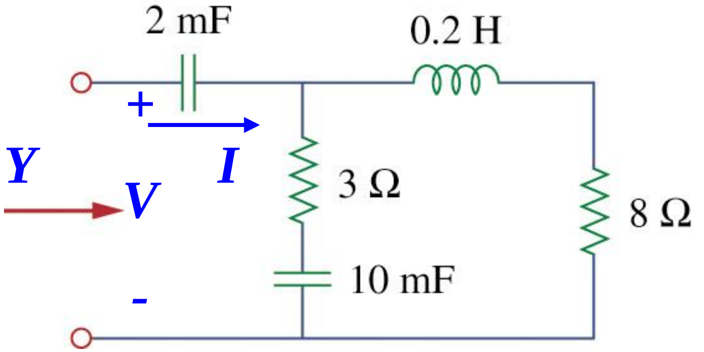
$$V_2 = \frac{Z_2}{Z_1 + Z_2} V$$

Admittance of Joint Elements

- The Admittance Y represents the admittance of the circuit to the flow of sinusoidal current. The admittance is measured in Siemens (s)

$$Y = \frac{1}{Z} = \frac{I}{V} = G + jB$$

$$= \text{Conductance} + \text{j} \times \text{Suseptance} = |\text{Y}| \angle \theta$$

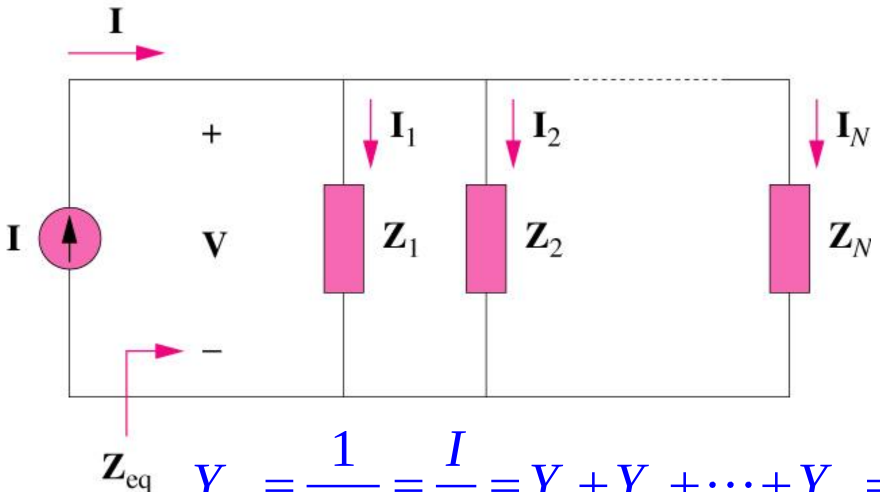


$$Y = G + jB = \frac{1}{R + jX} \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

$$G = \frac{R}{R^2 + X^2} \qquad B = -\frac{X}{R^2 + X^2}$$

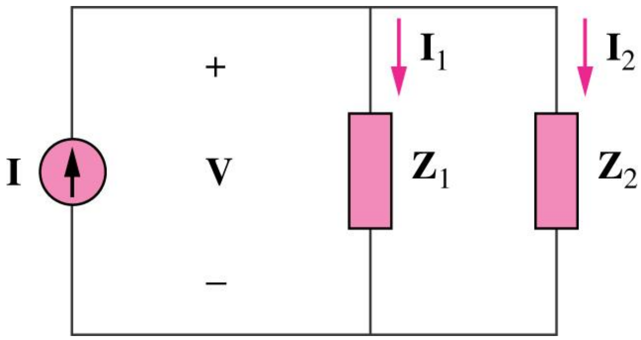
Parallel Combination for Phasors

- The Kirchoff''s Current Law (KCL) holds in the frequency domain. For series connected impedances:



$$Y_{eq} = \frac{1}{Z_{eq}} = \frac{I}{V} = Y_1 + Y_2 + \cdots + Y_N = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N} \quad \text{(Eqiv. Admitance)}$$

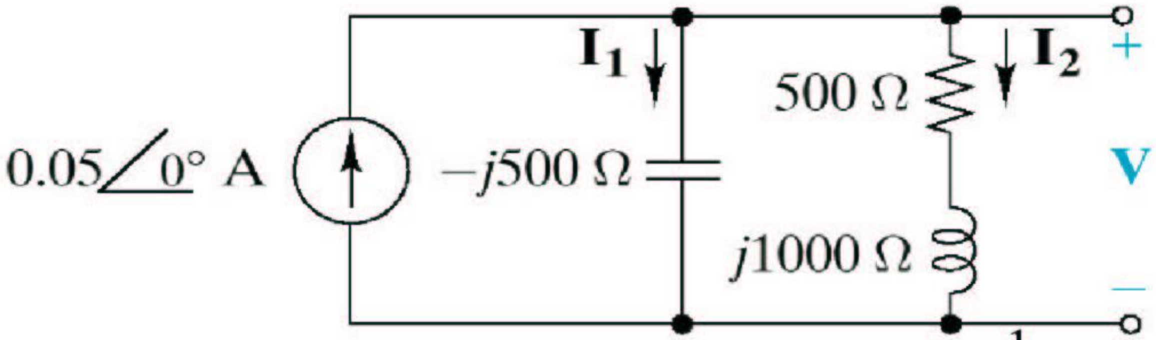
- The Current Division for two elements is:



$$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

Application of Current Division for Phasors



$$\hat{I}_1 = \frac{(R + j\omega L)}{R + j\omega L + \frac{1}{j\omega C}} \hat{I}$$

$$\hat{I}_1 = \frac{(500 + j1000)}{500 + j1000 - j500} 0.05\angle 0^\circ$$

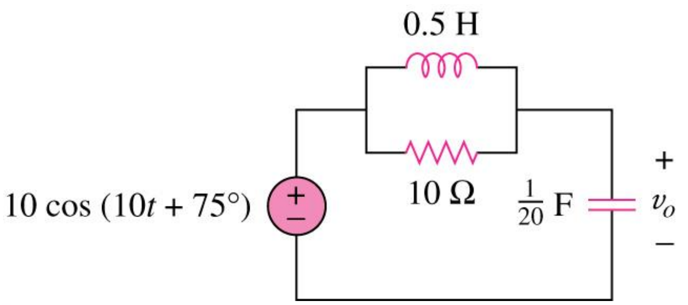
$$\hat{I}_1 = 0.079\angle 108.4^\circ \text{ A}$$

$$\hat{I}_2 = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \hat{I}$$

$$\hat{I}_2 = \frac{-j500}{500 + j1000 - j500} 0.05\angle 0^\circ$$

$$\hat{I}_2 = 0.03535\angle -45^\circ \text{ A}$$

Example: **Calculate $v_o(t)$ in the circuit given.**



In the frequency domain,
the voltage source is $\mathbf{V}_s = 10\angle 75°$
the 0.5-H inductor is $j\omega L = j(10)(0.5) = j5$
the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor
and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \,||\, j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\, \mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(10\angle 75°) = \frac{-j(10\angle 75°)}{1 + j} = \frac{10\angle(75° - 90°)}{\sqrt{2}\angle 45°} = 7.071\angle\text{-}60°$$

$v_o(t)$ = **7.071 cos(10t – 60°) V**

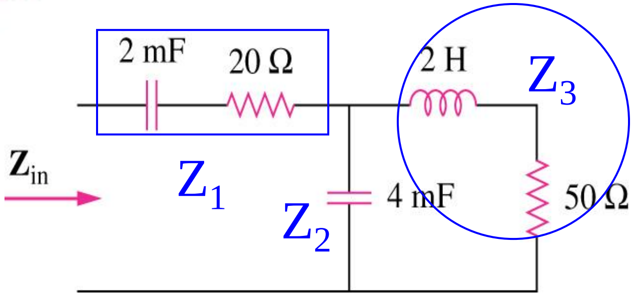
<u>Example:</u> Calculate Z_{in} of the circuit at ω = 10 rad/s

Let Z_1 = impedance of the 2-mF capacitor in series with the 20-Ω resistor
Z_2 = impedance of the 4-mF capacitor
Z_3 = impedance of the 2-H inductor in series with the 50-Ω resistor

$$Z_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j(10)(2\times10^{-3})} = 20 - j50$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(4\times10^{-3})} = -j25$$

$$Z_3 = 50 + j\omega L = 50 + j(10)(2) = 50 + j20$$

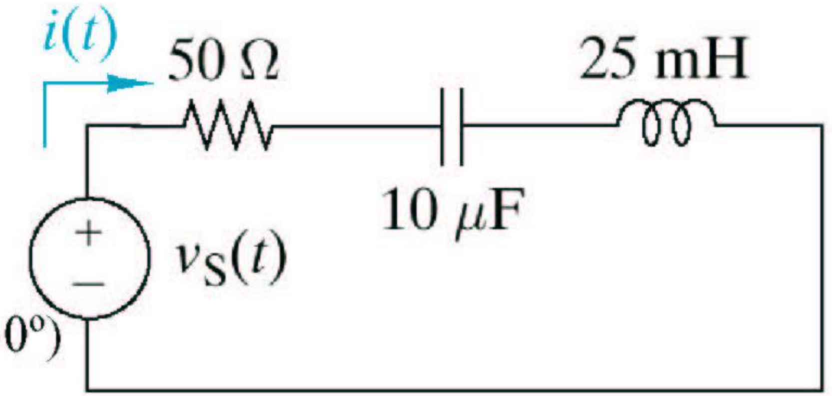


$$Z_{in} = Z_1 + Z_2 \,||\, Z_3 = Z_1 + Z_2 Z_3 / (Z_2 + Z_3)$$

$$Z_{in} = 20 - j50 + \frac{-j25x(50+j20)}{-j25+50+j20} = 20 - j50 + 12.38 - j23.76$$

$$Z_{in} = \underline{\mathbf{32.38 - j73.76\ \Omega}}$$

Example:



$v_s(t) = 35\cos(10^3 t + 30^o)$

Find

(a) Angular frequency in radians per sec

(b) Impedance of R in Ω

(c) Impedance of L in Ω

(d) Impedance of C in Ω

(e) Driving point impedance in Ω

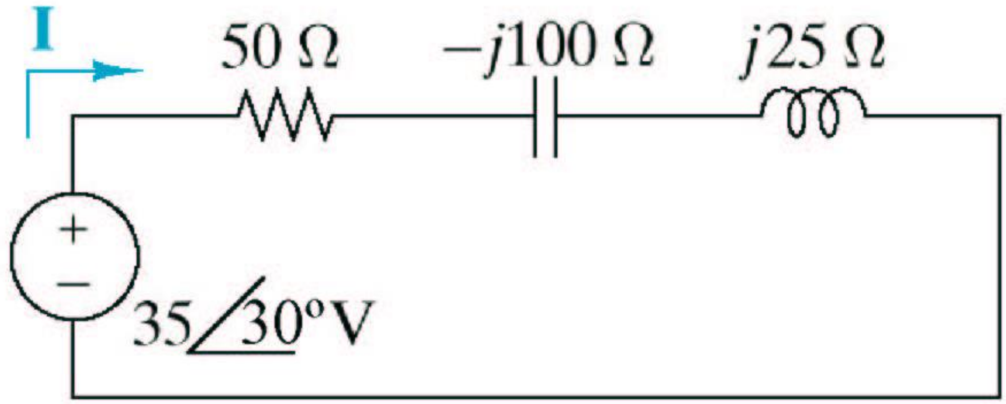
(f) Driving point admittance in S

(g) Phasor voltage and current

(h) Find particular response $i_p(t)$

Graph $i_p(t)$ as a function of time

Example: Cont'd



Driving Point Impedance $$Z = \frac{V_s}{I} = R + \frac{1}{j\omega C} + j\omega L$$

$$Z = 50 - j100 + j25 = 50 - j75\Omega = 90.14\angle -56.14^\circ\,\Omega$$

Driving Point Admittance

$$Y = \frac{1}{Z} = \frac{1}{90.14\angle -56.14^\circ} = 0.011\angle 56.14^\circ \ \text{ S}$$

Example: Cont'd

Phasor Voltage: $\hat{V}_s = 35\angle 30°$

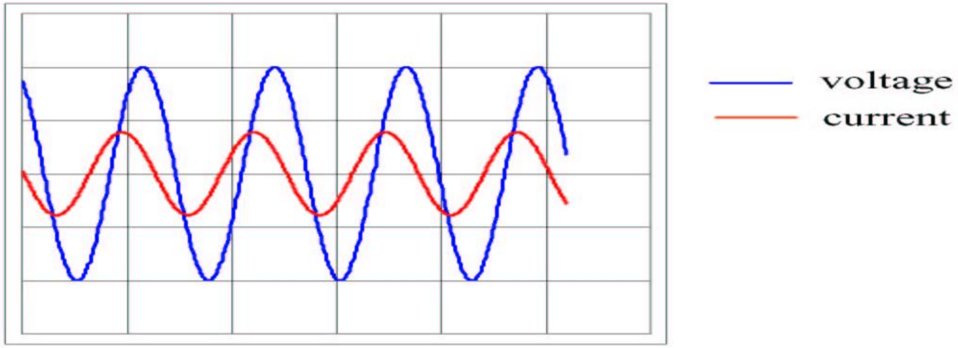
Phasor Current: $\hat{I} = \dfrac{\hat{V}_s}{Z} = \dfrac{35\angle 30°}{90.14\angle -56.14°}$

$$= 0.388\angle(30° + 56.14°) = 0.388\angle 86.14°$$

Particular response (called the steady-state response):

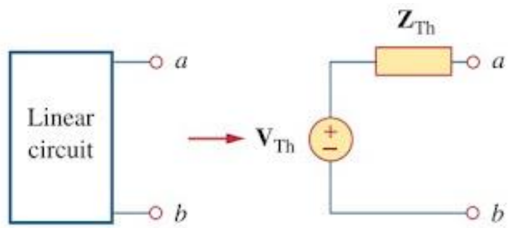
$$i_p(t) = 0.388\cos(10^3 t + 86.14^o)$$

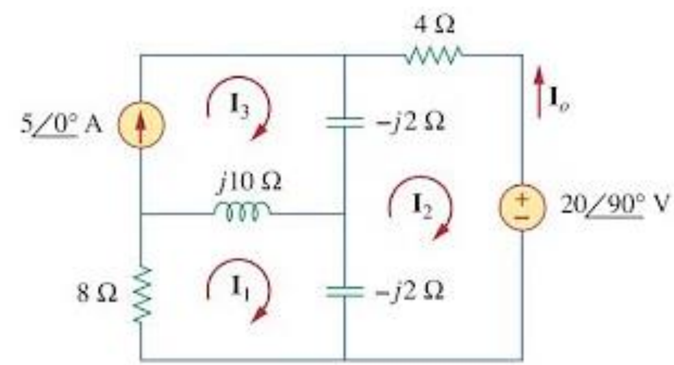
Note that current leads voltage by 56.14^o which is $\angle Z$

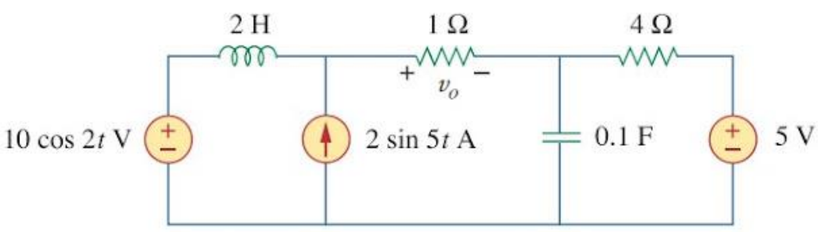


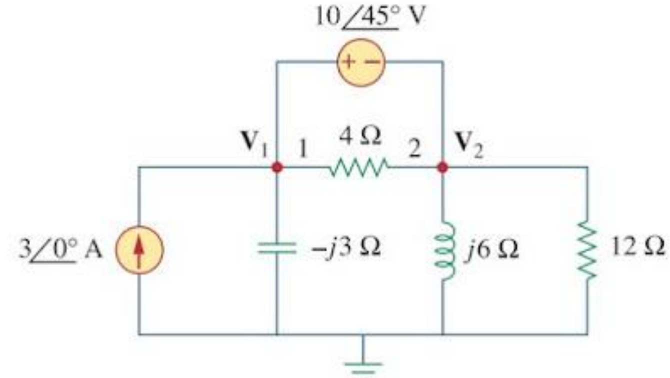
Sinusoidal Steady-State Analysis

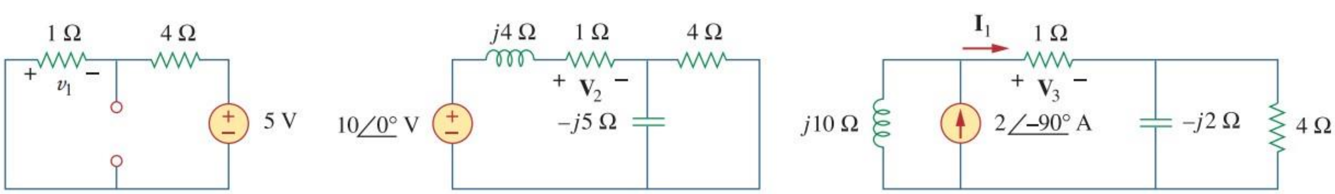
- Basic Approach
- Nodal Analysis
- Mesh Analysis
- Superposition Theorem
- Source Transformation
- Thevenin Equivalent Circuits









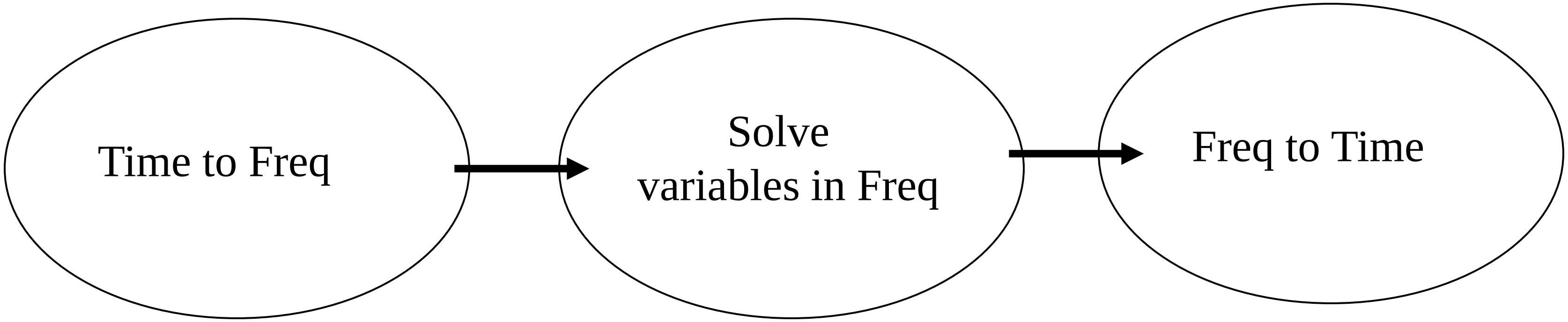


(a) (b) (c)

Basic Approach

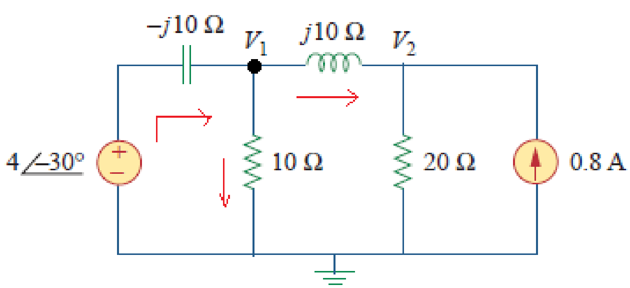
Steps to Analyze AC Circuits:

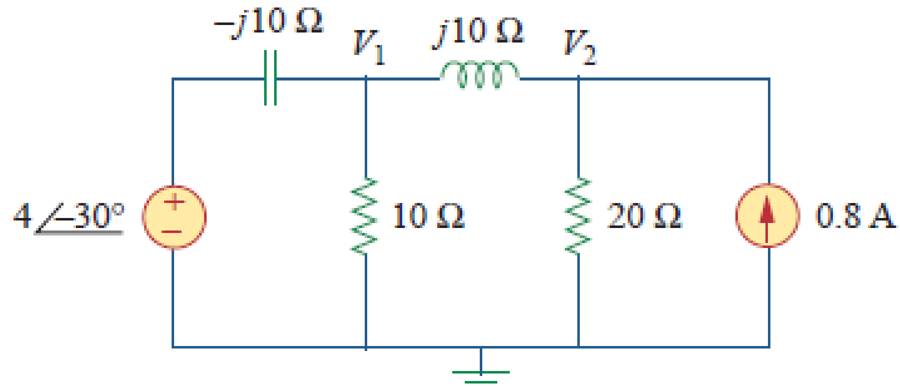
1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

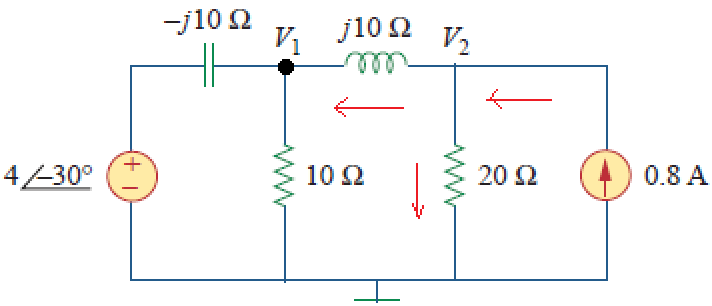


Nodal Analysis

Example: Using nodal analysis, find V_1 and V_2 in the circuit of figure below.







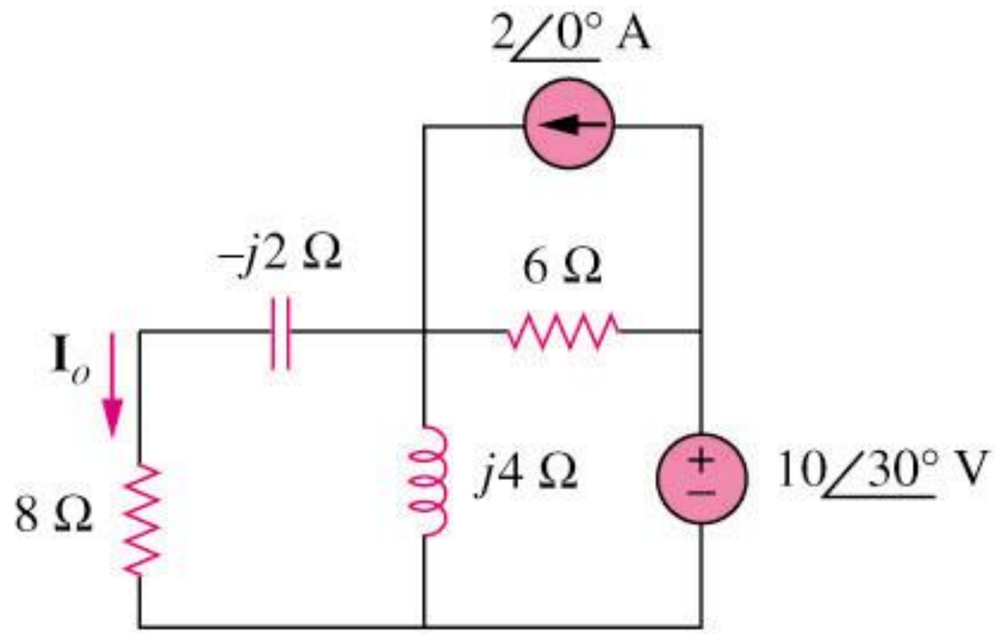
$$\frac{4\angle -30^\circ - V_1}{-j10} = \frac{V_1}{10} + \frac{V_1 - V_2}{j10} \longrightarrow 4\angle -30^\circ = 3.468 - j2$$

$$0.8 = \frac{V_2}{20} + \frac{V_2 - V_1}{j10} \longrightarrow j16 = -2V_1 + (2 + j)V_2$$

$$\begin{bmatrix} -j & 1 \\ -2 & (2+j) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.468 - j2 \\ j16 \end{bmatrix}$$

Mesh Analysis

Example: Find I_o in the following figure using mesh analysis.

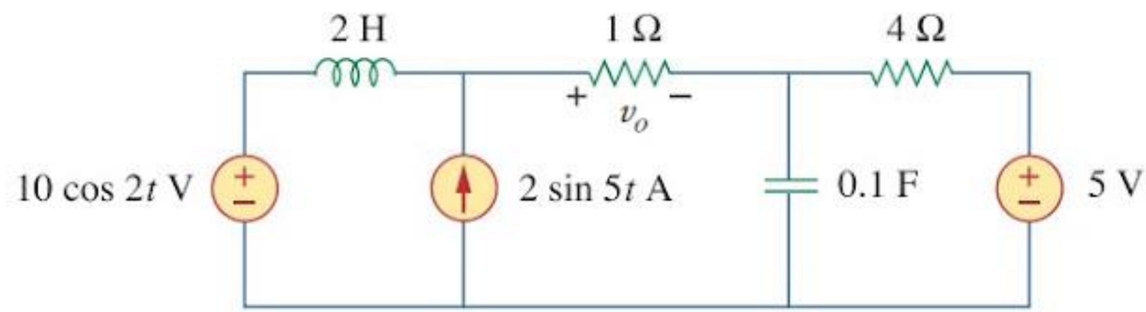


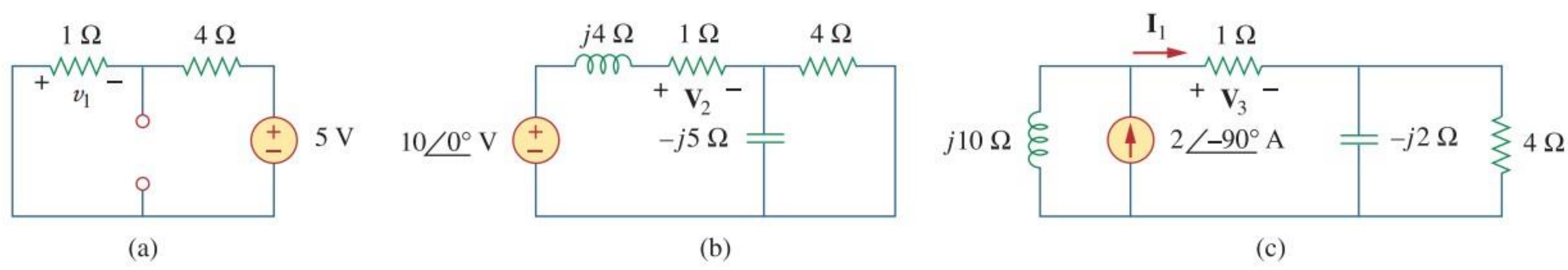
Answer: $I_o = 1.194\angle 65.44° \text{ A}$

Superposition Theorem

When a circuit has sources operating at different frequencies,

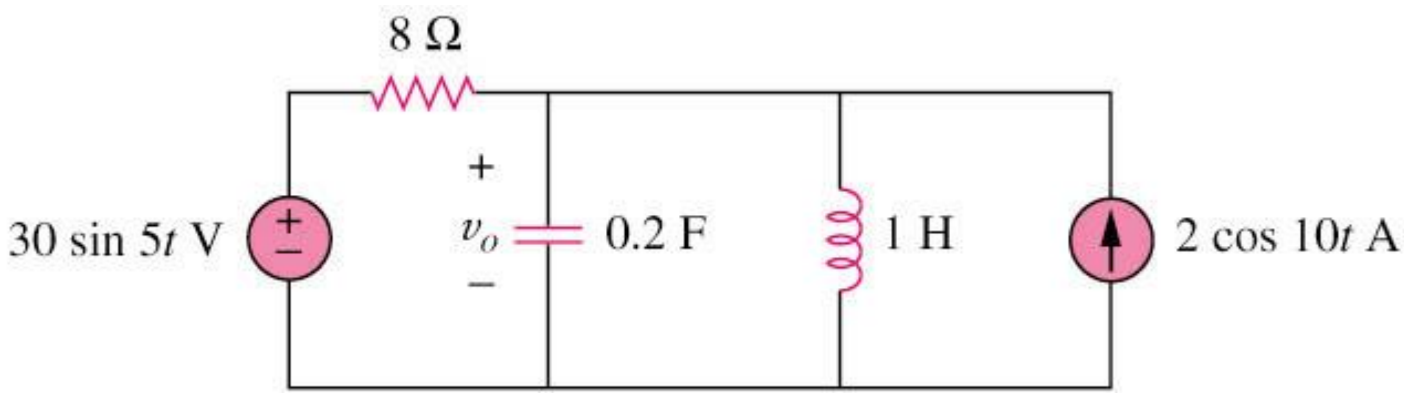
- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.





Superposition Theorem

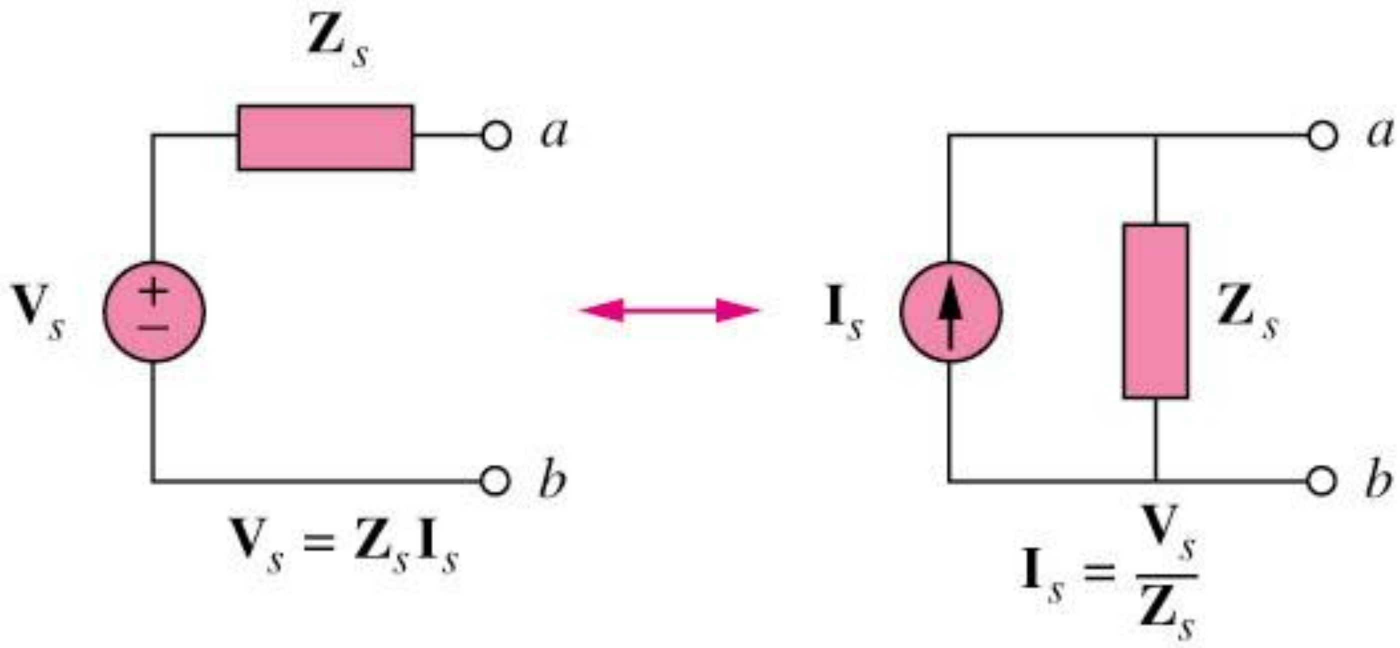
Example: Calculate v_o in the circuit using the superposition theorem.



Answer

v_o **= 4.631 sin(5t – 81.12°) + 1.051 cos(10t – 86.24°) V**

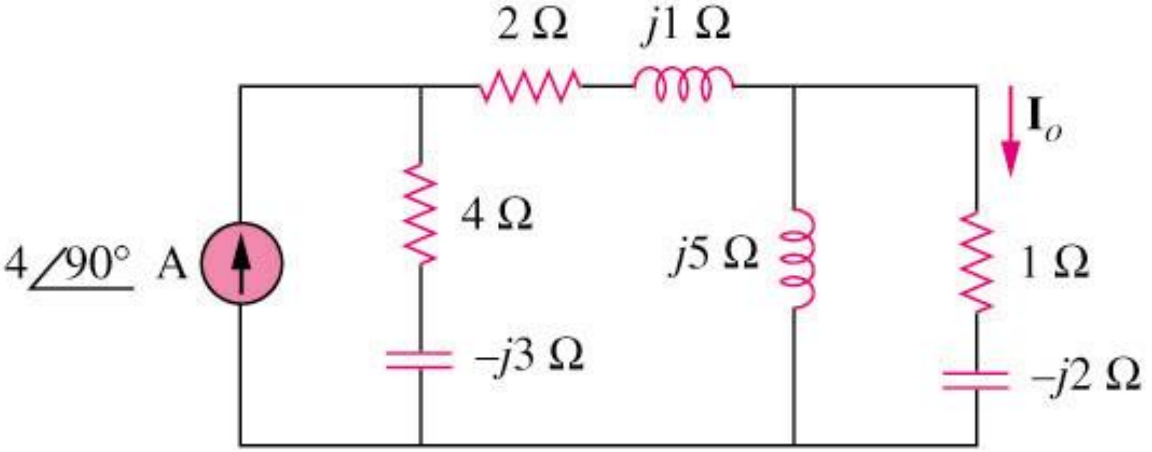
Source Transformation



Source Transformation

Example:

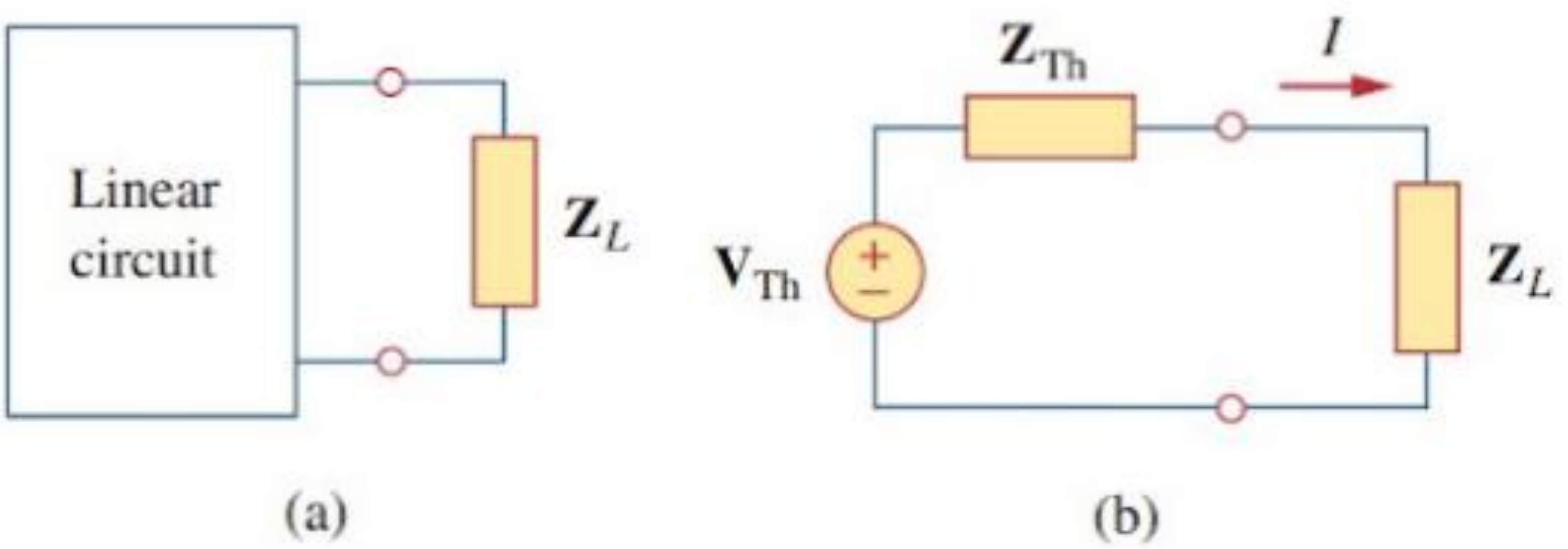
Find I_o in the circuit of figure below using the concept of source transformation.



I_o = 3.288∠99.46° A

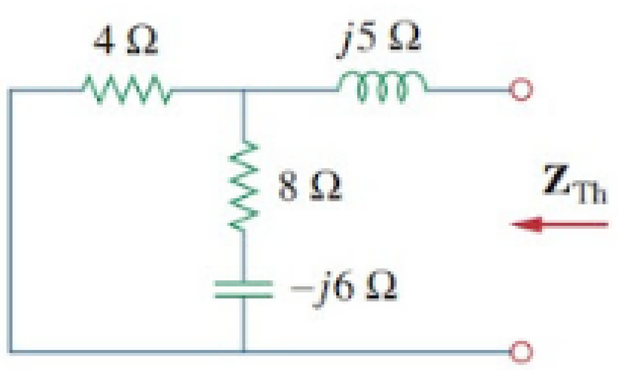
Thevenin Equivalent Circuit

Thevenin transform

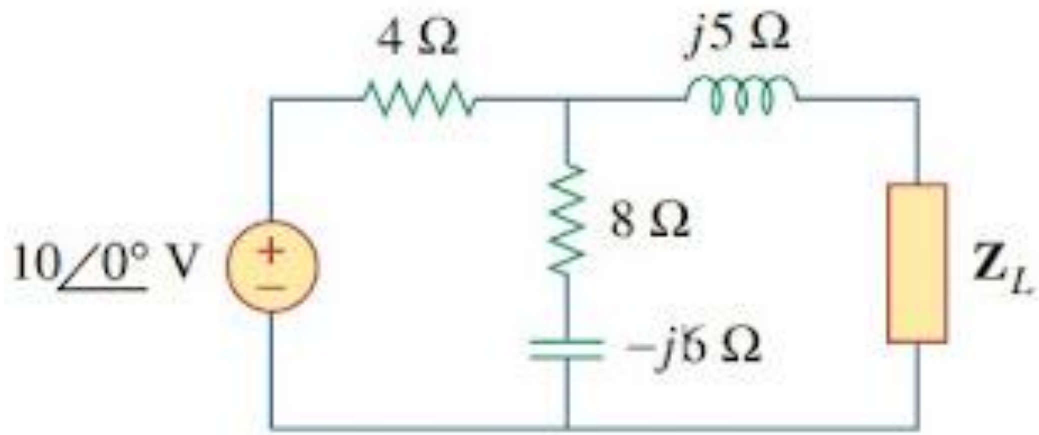


Thevenin Equivalent Circuit

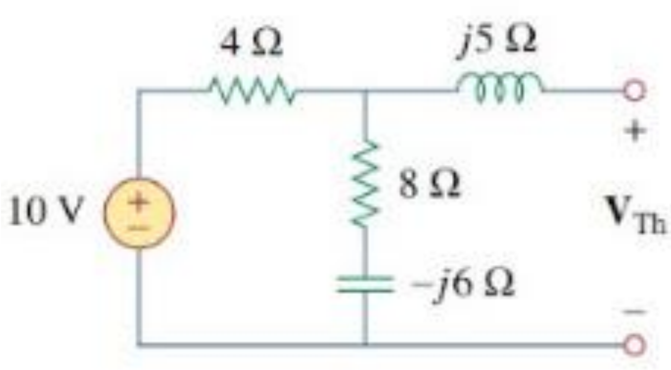
Example: Find the Thevenin equivalent as seen from the load side.



(a)



$$\mathbf{Z}_{Th} = j5 + 4 \| (8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467\ \Omega$$

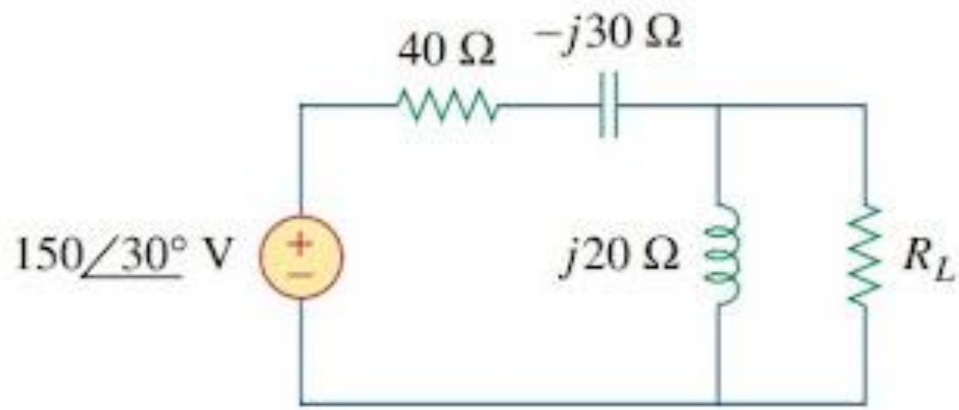


(b)

$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454\angle -10.3^\circ\ \text{V}$$

Thevenin Equivalent Circuit

Example: Find the Thevenin equivalent as seen from the load side.

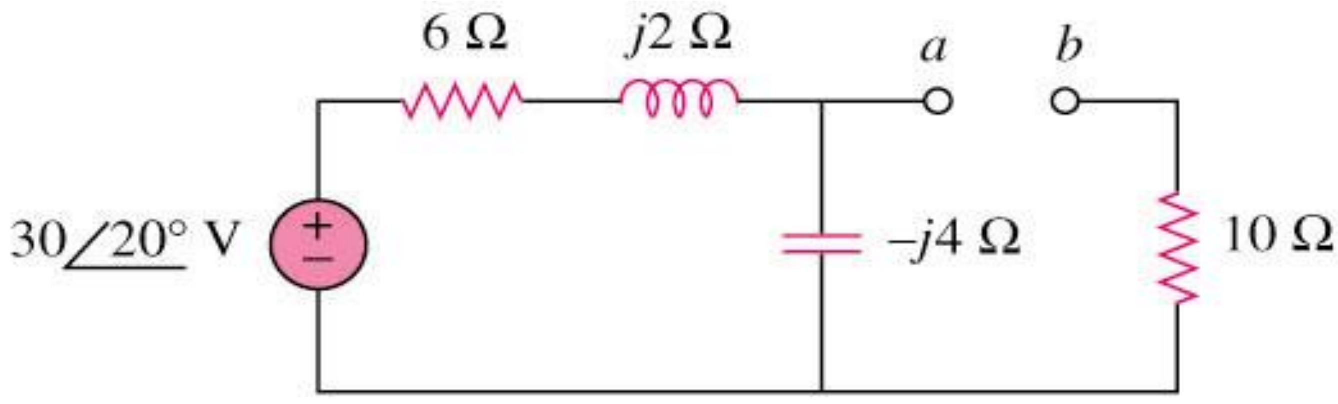


$$\mathbf{Z}_{Th} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

$$\mathbf{V}_{Th} = \frac{j20}{j20 + 40 - j30}(150\angle 30^\circ) = 72.76\angle 134^\circ\ \text{V}$$

Thevenin Equivalent Circuit

Example: Find the Thevenin equivalent at terminals *a*–*b* of the circuit below.

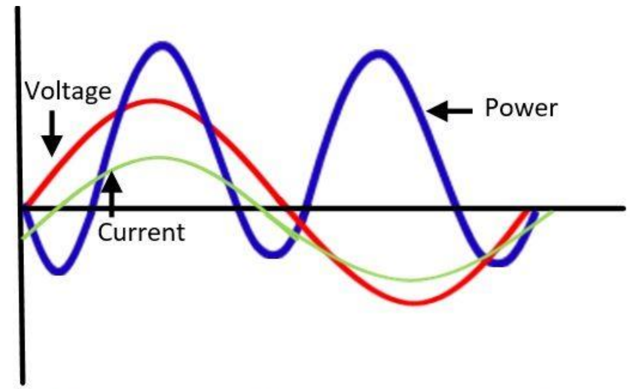


Z_{th} =12.4 – j3.2 Ω

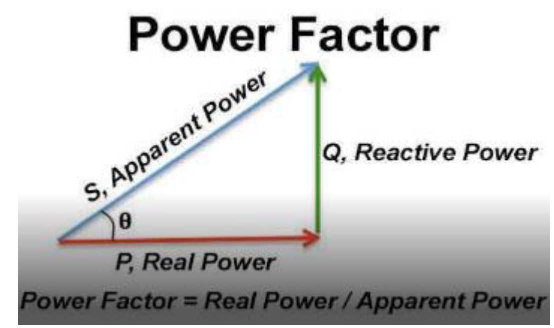
V_{TH} = 18.97∠-51.57° V

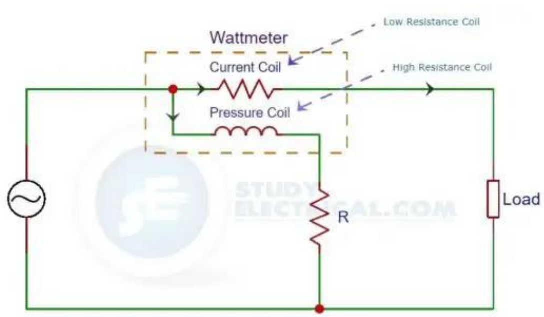
Topic 2 - AC Power Calculation

- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Conservation of AC Power
- Power Factor Correction
- Power Measurement



Plot of the Power of Instantaneous Voltage, Current & Power





Instantaneous and Average Power

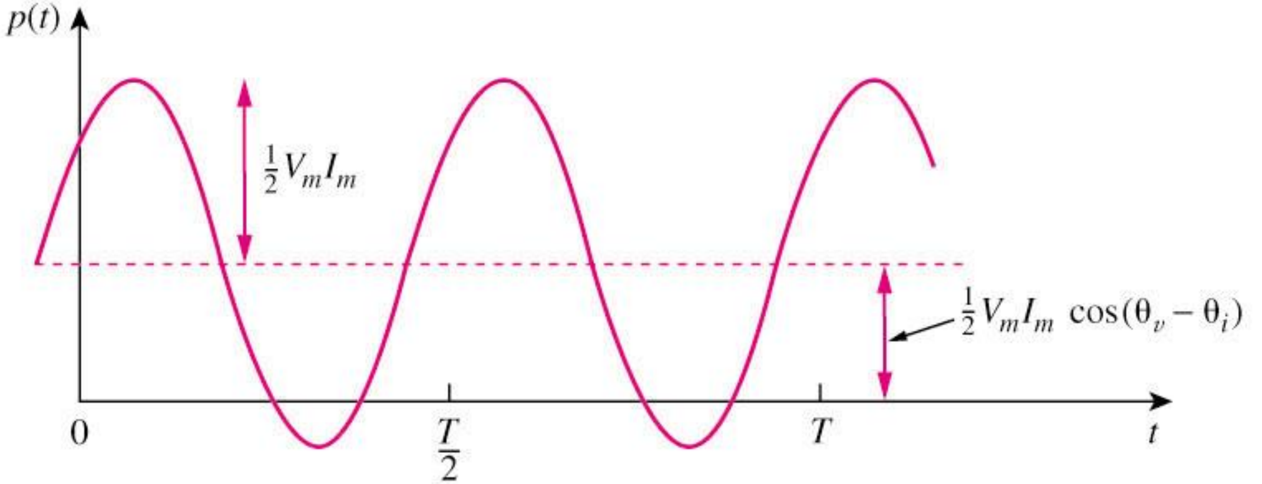
- The instantaneously power, $p(t)$

$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power **Sinusoidal power at 2ω**

The instantaneous power *p*(*t*) is composed of a constant part (DC) and a time dependent part having frequency 2ω.

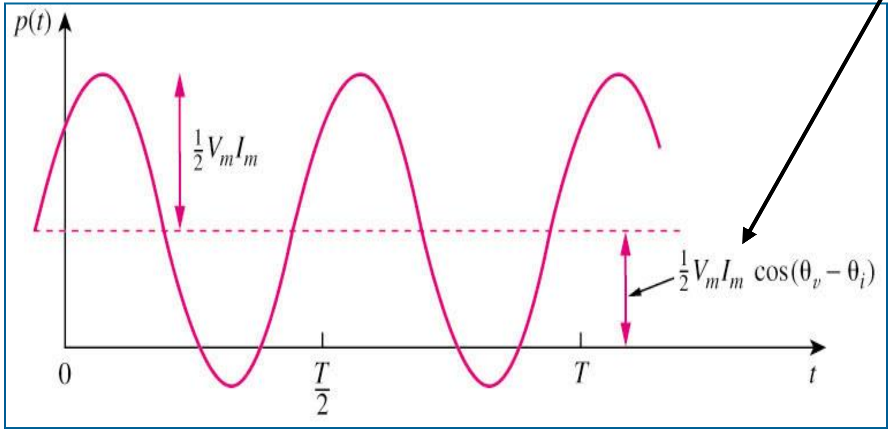


p(t) > 0: power is absorbed by the circuit; p(t) < 0: power is absorbed by the source.

Instantaneous and Average Power

- The average power, P, *is the average of the instantaneous power over one period.*

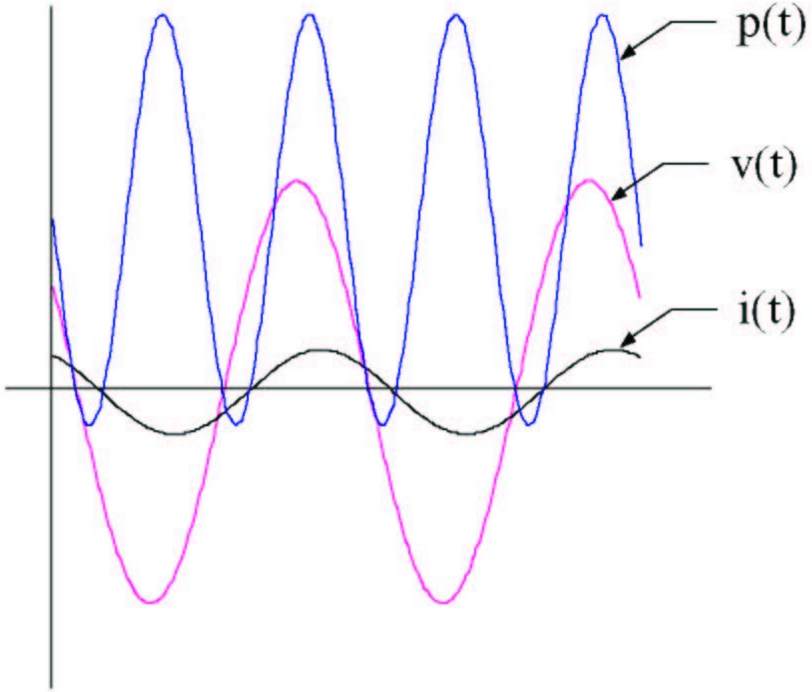
$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m\, I_m \cos(\theta_v - \theta_i)$$



1. **P is not <u>time dependent</u>.**
2. **When $\theta_v = \theta_i$, it is a <u>purely resistive</u> load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a <u>purely reactive</u> load case.**
4. **P = 0 means that the circuit absorbs <u>no average power</u>.**

Instantaneous Power

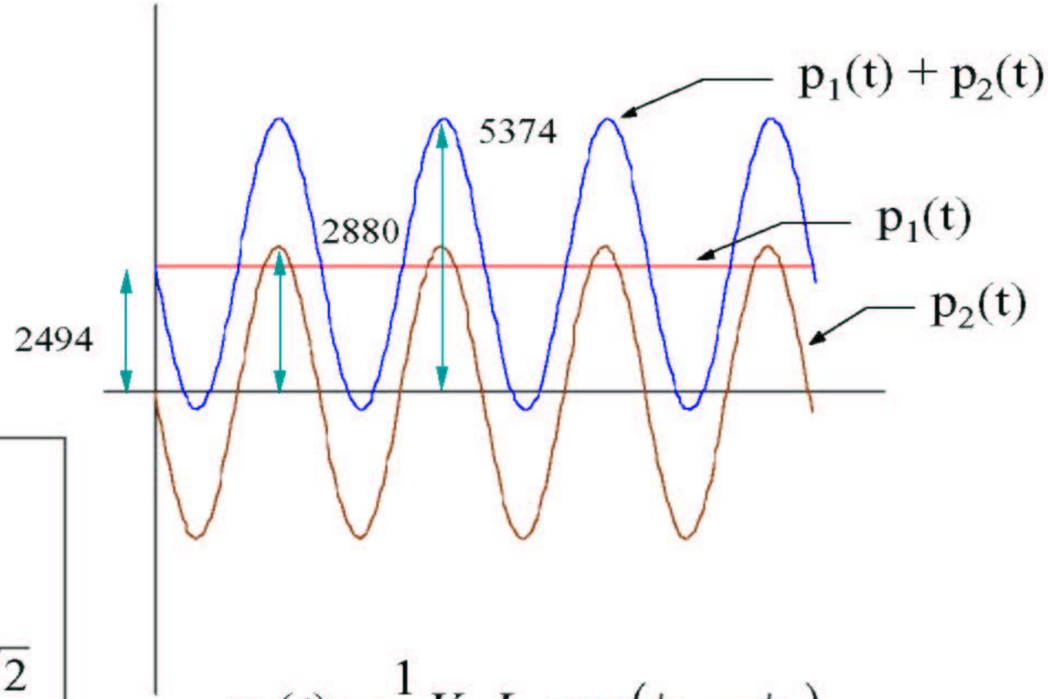
$$p(t) = v(t)i(t) = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \tfrac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



$v(t) = 120\sqrt{2}\cos(377t + 60^\circ)$ $\quad$ $i(t) = 24\sqrt{2}\cos(377t + 30^\circ)$

Instantaneous Power

$$p(t) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) = p_1(t) + p_2(t)$$



$\phi_V = 60°$

$\phi_I = 30°$

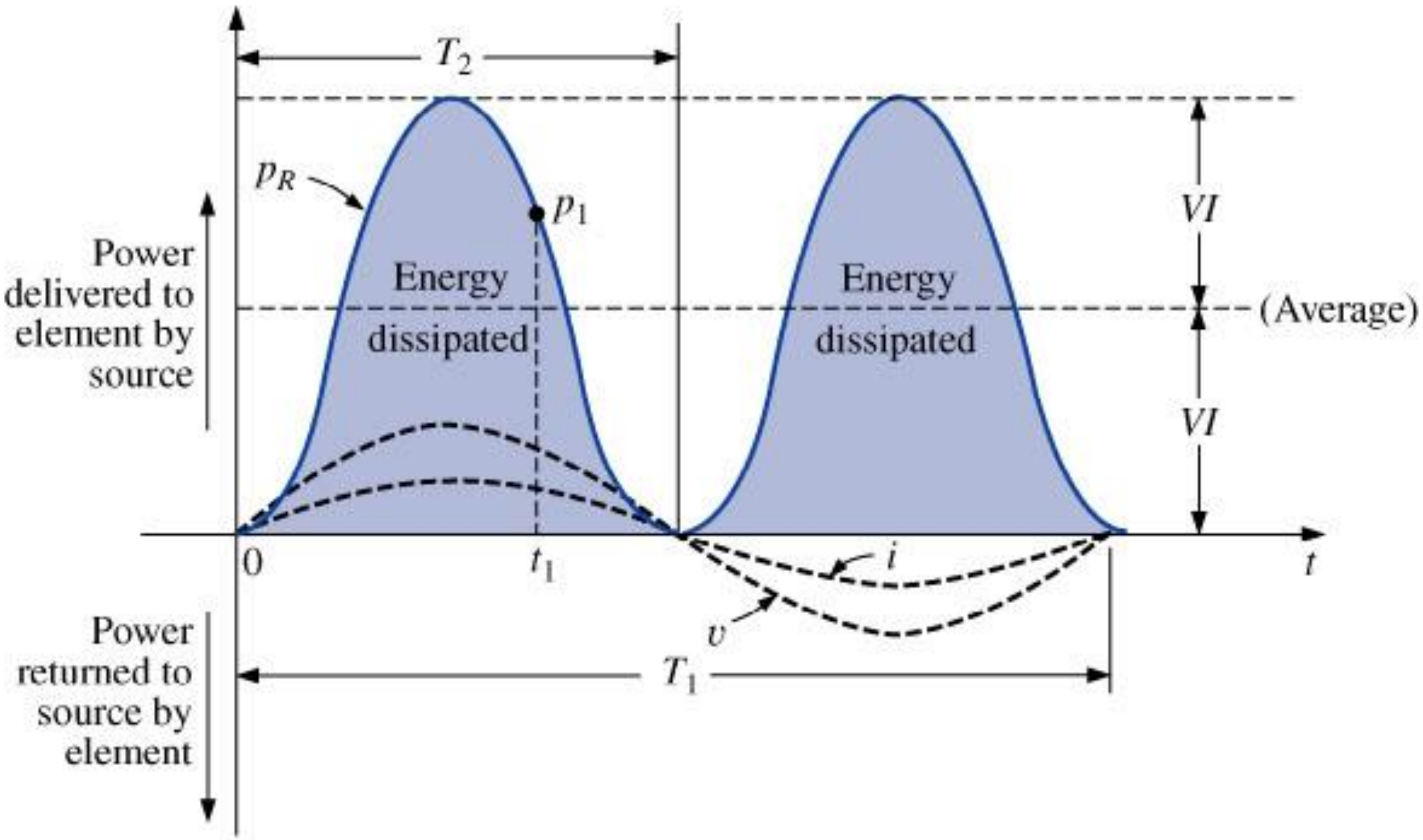
$V_m = 120\sqrt{2}$

$I_m = 24\sqrt{2}$

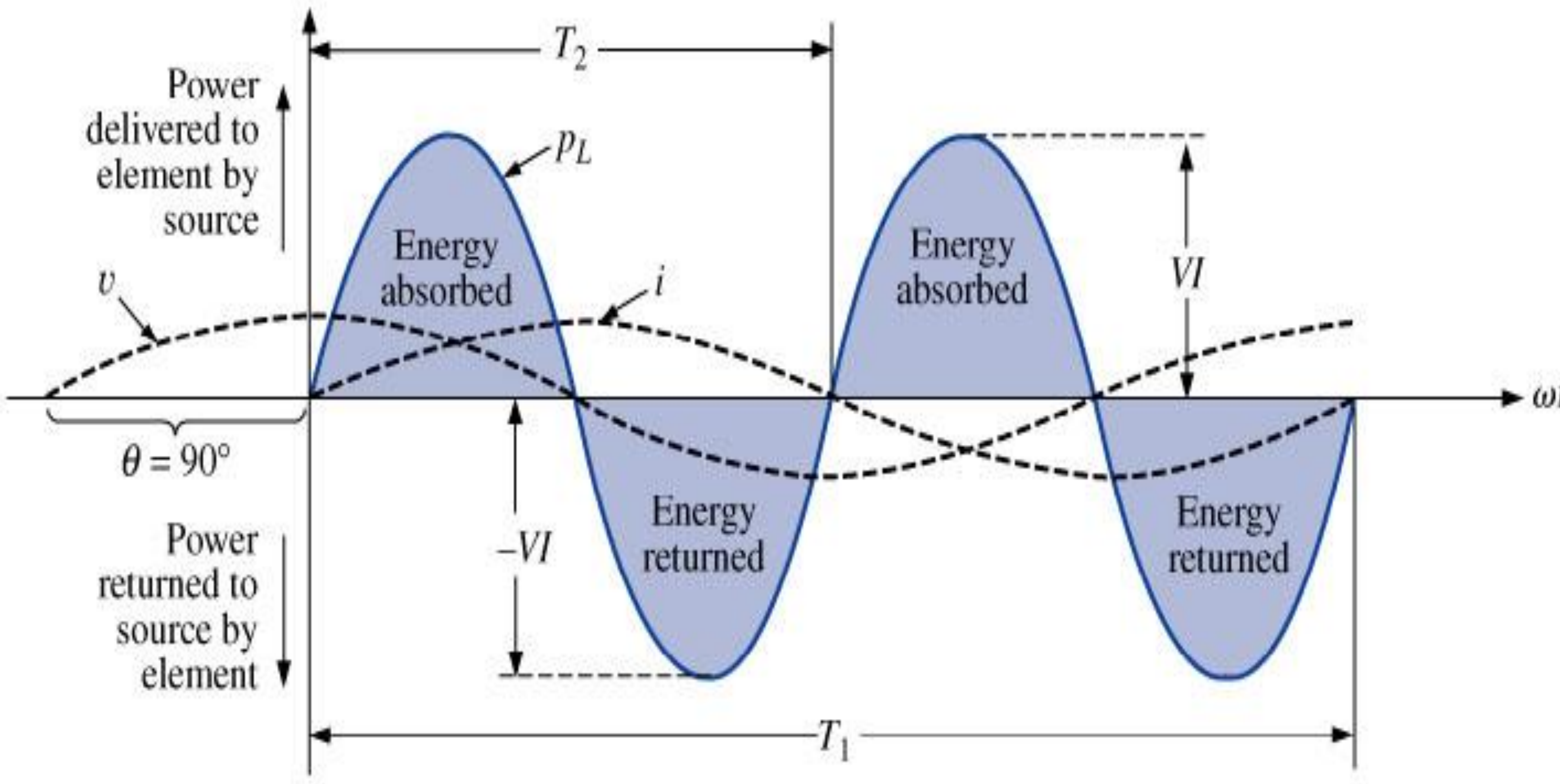
$$p_1(t) = \frac{1}{2} V_m I_m \cos(\phi_V - \phi_I)$$

$$p_2(t) = \frac{1}{2} V_m I_m \cos(2\omega t + \phi_V + \phi_I)$$

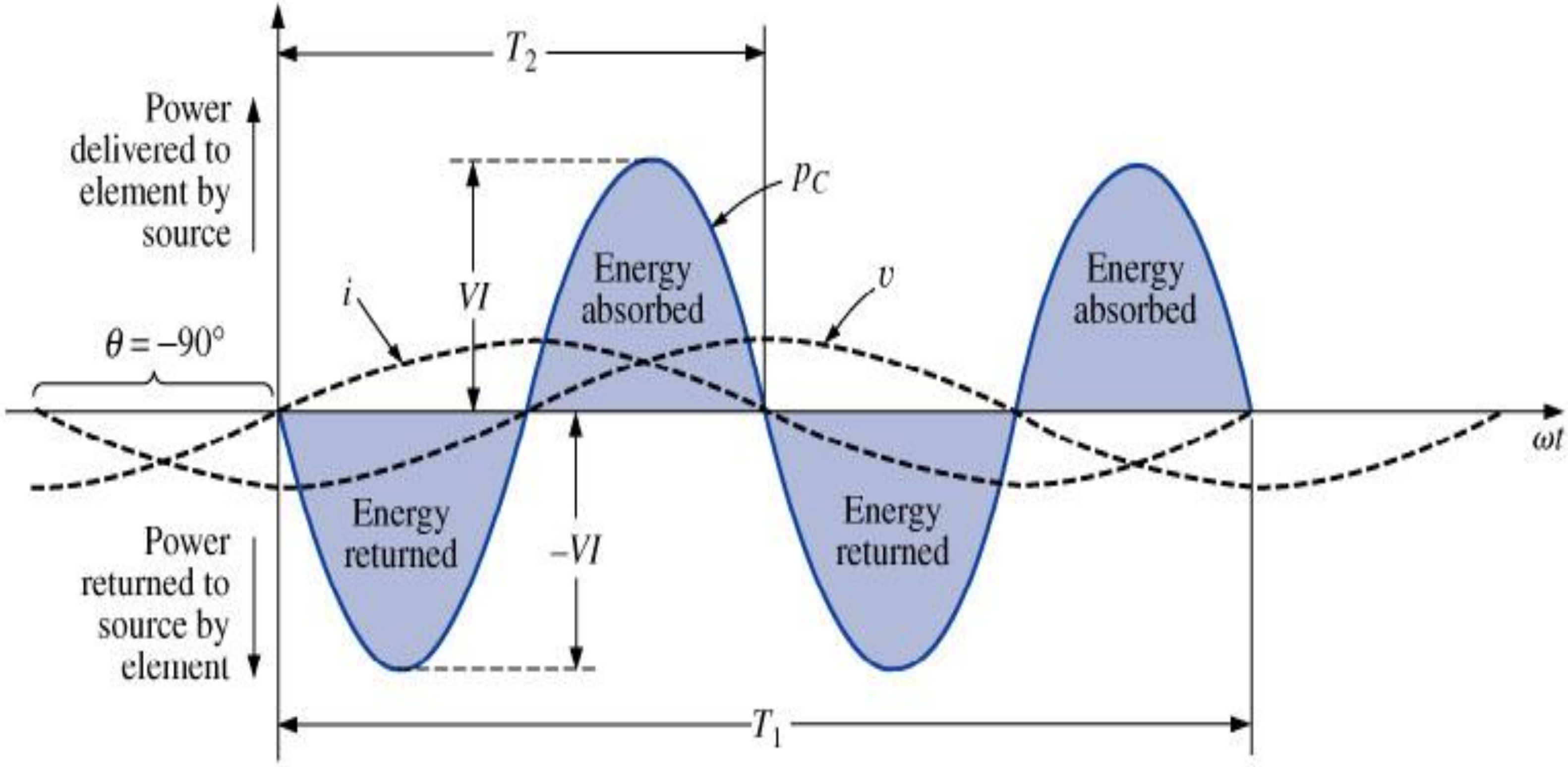
Resistive Circuit and Real Power



Inductive Circuit and Reactive Power



Capacitive Circuit and Reactive Power



Instantaneous and Average Power

Example:

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20^\circ)$$

$$i(t) = 15\sin(10t + 60^\circ)$$

Answer: $385.7 + 600\cos(20t - 10^\circ)\text{W},\ 387.5\text{W}$

Average Power

➢The average power P is the average of the instantaneous power over one period .

$$p(t) = v(t)i(t) \quad \text{Instantaneous Power}$$

$$P = \frac{1}{T}\int_0^T p(t)dt \quad \text{Average Power}$$

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)dt + \frac{1}{T}\int_0^T \tfrac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)dt$$

$$P = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \tfrac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)dt$$

$$= \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + 0 \qquad \text{(Integral of a Sinusoidal=0)}$$

$$P = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \tfrac{1}{2}\mathrm{Re}\left[\mathbf{VI}^*\right] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Average Power

Example:

A current $\mathbf{I} = 10\angle 30°$ flows through an impedance.

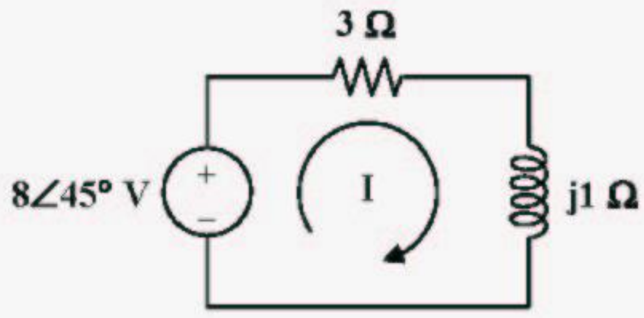
Find the average power delivered to the impedance.

$\mathbf{Z} = 20\angle -22°\Omega$

Answer: 927.2W

Average Power

Example: Find the average power absorbed by resistor and inductor. Find the average power supplied by the source



$$\mathbf{I} = \frac{8\angle 45^\circ}{3+j} = 2.53\angle 26.57^\circ$$

For the resistor, $\mathbf{I}_R = \mathbf{I} = 2.53\angle 26.57^\circ \quad \mathbf{V}_R = 3\mathbf{I} = 7.59\angle 26.57^\circ$

$$P_R = \frac{1}{2}V_m I_m = \frac{1}{2}(2.53)(7.59) = \underline{\mathbf{9.6\ W}}$$

For the inductor, $\mathbf{I}_L = 2.53\angle 26.57^\circ$, $\mathbf{V}_L = j\mathbf{I}_L = 2.53\angle(26.57^\circ + 90^\circ) = 2.53\angle 116.57^\circ$

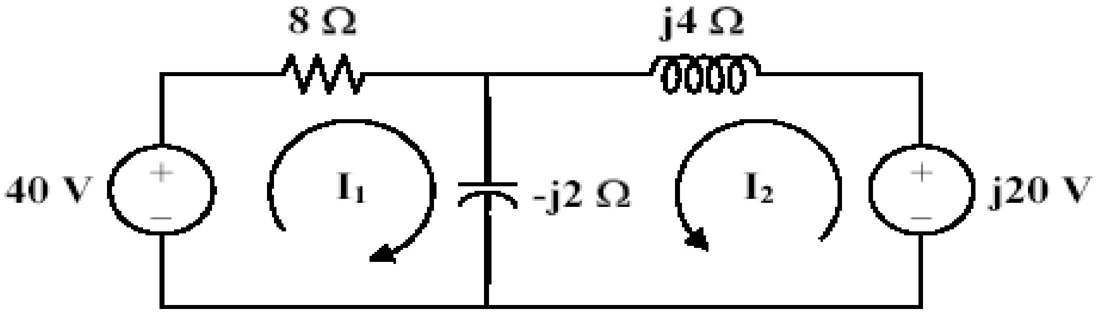
$$P_L = \frac{1}{2}(2.53)^2\cos(90^\circ) = \underline{\mathbf{0\ W}}$$

The average power supplied is

$$P = \frac{1}{2}(8)(2.53)\cos(45^\circ - 26.57^\circ) = \underline{\mathbf{9.6\ W}}$$

Average Power

Example: Calculate the average power absorbed by each of the five elements in the circuit given.



For mesh 1,

$$-40 + (8 - j2)\mathbf{I}_1 + (-j2)\mathbf{I}_2 = 0$$

$$(4 - j)\mathbf{I}_1 - j\mathbf{I}_2 = 20 \qquad (1)$$

For mesh 2,

$$-j20 + (j4 - j2)\mathbf{I}_2 + (-j2)\mathbf{I}_1 = 0$$

$$-j\mathbf{I}_1 + j\mathbf{I}_2 = j10 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 4-j & -j \\ -j & j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

Average Power

$$\Delta = 2 + j4\,, \qquad \Delta_1 = -10 + j20\,, \qquad \Delta_2 = 10 + j60$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 5\angle 53.14^\circ \quad \text{and} \quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = 13.6\angle 17.11^\circ$$

For the 40-V voltage source,

$$\mathbf{V}_s = 40\angle 0^\circ \qquad \mathbf{I}_1 = 5\angle 53.14^\circ$$

$$P_s = \frac{-1}{2}(40)(5)\cos(-53.14^\circ) = \underline{\mathbf{-\,60\ W}}$$

For the j20-V voltage source,

$$\mathbf{V}_s = 20\angle 90^\circ \qquad \mathbf{I}_2 = 13.6\angle 17.11^\circ$$

$$P_s = \frac{-1}{2}(20)(13.6)\cos(90^\circ - 17.11^\circ) = \underline{\mathbf{-\,40\ W}}$$

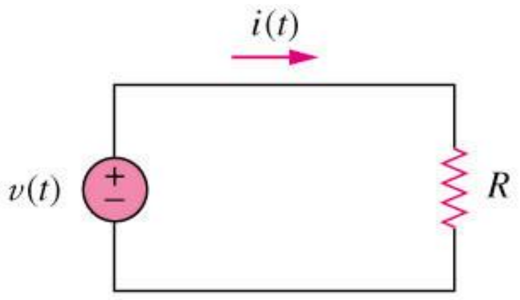
For the resistor,

$$I = |\mathbf{I}_1| = 5 \qquad V = 8|\mathbf{I}_1| = 40$$

$$P = \frac{1}{2}(40)(5) = \underline{\mathbf{100\ W}}$$

The average power absorbed by the inductor and capacitor is zero watts.

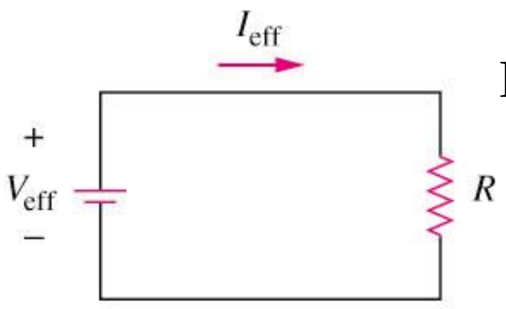
Effective or RMS Value



a) AC circuit

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$



b) DC circuit

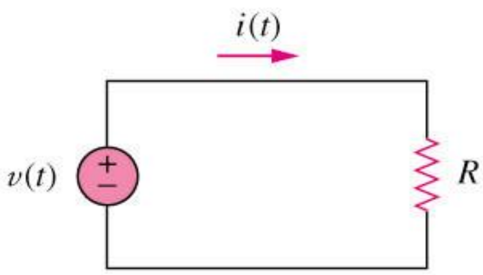
Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 \, dt} = I_{rms}$$

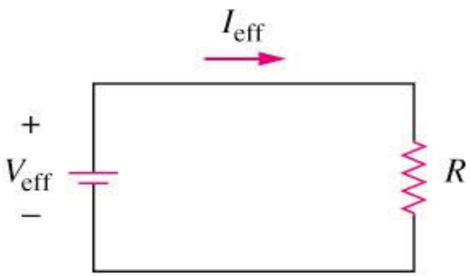
The rms value is a constant itself which depending on the shape of the function *i*(*t*).

➢ **The <u>EFFECTIVE</u> Value or the Root Mean Square (RMS) value of a periodic current is the DC value that delivers the same average power to a resistor as the periodic current.**

Effective or RMS Value



a) AC circuit



b) DC circuit

The rms value of a sinusoid $i(t) = I_m \cos(\omega t)$ is given by:

$$I_{Rms} = \sqrt{\frac{1}{T}\int_0^T I_m{}^2 \cos^2 \omega t dt} = \sqrt{\frac{I_m{}^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)dt} = \frac{I_m}{\sqrt{2}}$$

The average power can be written in terms of the rms values:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

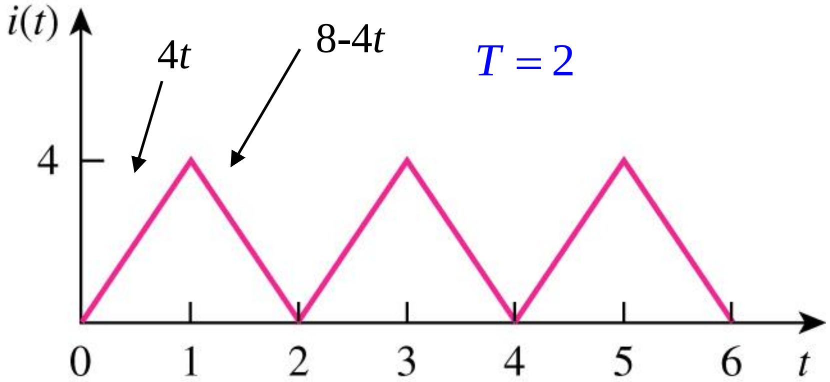
- The average power for resistive loads using the (RMS) value is:

$$P_R = I_{Rms}{}^2 R = \frac{V_{Rms}{}^2}{R}$$

RMS Value

➢ **Example:** **Find the RMS value of the current waveform. Calculate the average power if the current is applied to a 9 Ω resistor.**

$$i(t) = \begin{cases} 4t & 0 < t < 1 \\ 8 - 4t & 1 < t < 2 \end{cases}$$



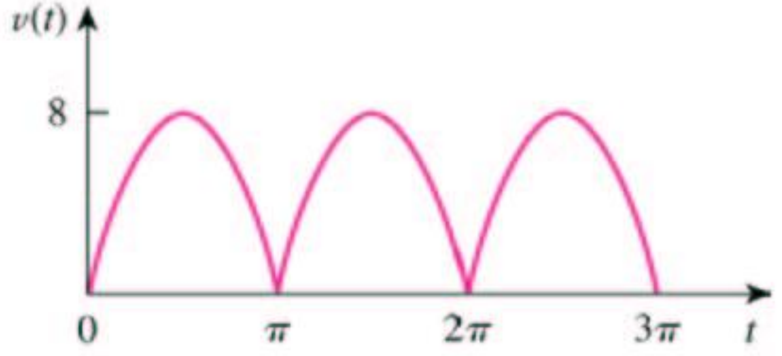
$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\, dt = \frac{1}{2}\left[\int_0^1 (4t)^2\, dt + \int_1^2 (8-4t)^2\, dt\right]$$

$$I_{rms}^2 = \frac{16}{2}\left[\int_0^1 t^2\, dt + \int_1^2 (4 - 4t + t^2)\, dt\right] \qquad I_{rms}^2 = 8\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{16}{3}$$

$$I_{rms} = \sqrt{\frac{16}{3}} = 2.309\text{A} \qquad P = I_{rms}^2 R = \left(\frac{16}{3}\right)(9) = 48\text{W}$$

RMS Value

Example: **Find the RMS value of the full-wave rectified sine wave. Calculate the average power dissipated in a 6 Ω resistor.**



$$T = \pi\ ,\ v(t) = 8\sin(t),\quad 0 < t < \pi$$

$$V_{emf}^2 = \frac{1}{T}\int_0^T v^2\ dt = \frac{1}{\pi}\int_0^\pi (8\sin(t))^2\ dt \qquad V_{emf}^2 = \frac{64}{\pi}\int_0^\pi \frac{1}{2}\left[1-\cos(2t)\right] dt = 32$$

$$V_{emf} = \underline{\mathbf{5.657\ V}} \qquad P = \frac{V_{emf}^2}{R} = \frac{32}{6} = \underline{\mathbf{5.333\ W}}$$

Apparent Power and Power Factor

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

Apparent Power, *S* **Power Factor, pf**

- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

Apparent Power and Power Factor

Purely resistive load (R)	$\theta_v - \theta_i = 0$, Pf = 1	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

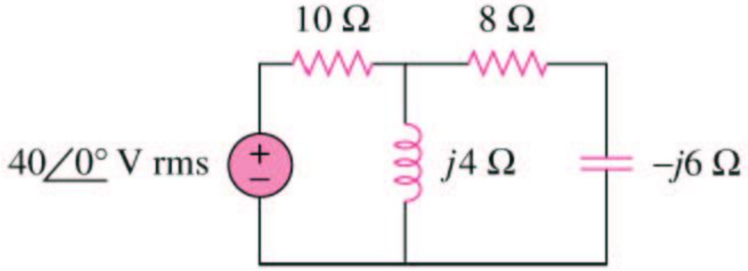
$$P = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{Rms} I_{Rms} \cos(\theta_v - \theta_i)$$

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$$S = \frac{1}{2} V_m I_m = V_{Rms} I_{Rms}$$

Power Factor

Example: **Calculate the power factor seen by the source and the average power supplied by the source**



The total impedance as seen by the source is

$$\mathbf{Z} = 10 + j4 \,||\, (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2} \qquad \mathbf{Z} = 12.69\angle 20.62°$$

The power factor is

$$pf = \cos(20.62°) = \underline{\mathbf{0.936 \quad (lagging)}}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40\angle 0°}{12.69\angle 20.62°} = 3.152\angle -20.62°$$

The average power supplied by the source is equal to the power absorbed by the load.

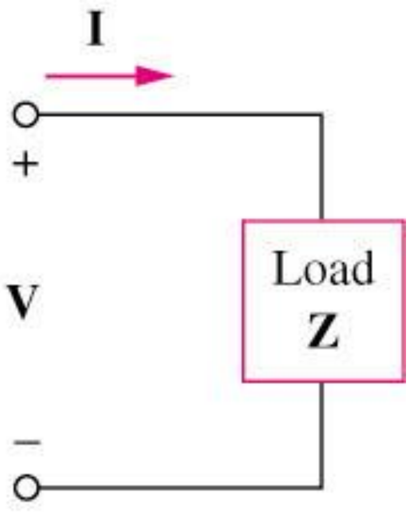
$$P = I_{rms}^2 R = (3.152)^2 (11.88) = 118 \text{ W}$$

or $$P = V_{rms} I_{rms} pf = (40)(3.152)(0.936) = \underline{\mathbf{118 \text{ W}}}$$

Complex Power

➢ The **<u>COMPLEX</u>** Power **S** contains all the information pertaining to the power absorbed by a given load.

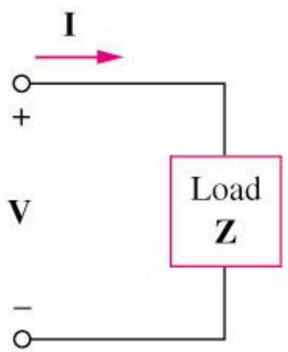
➢ Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathrm{V}\,\mathrm{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

Complex Power



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

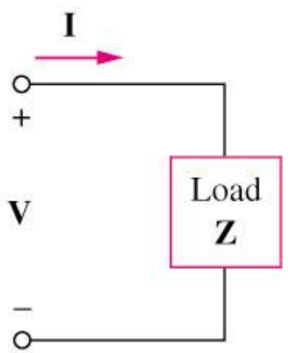
$$\boldsymbol{S} = \mathbf{P} + \mathbf{j}\,\mathbf{Q}$$

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

Complex Power



$$\Rightarrow S = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

$$S = P + j\, Q$$

Apparent Power, $S = |\mathbf{S}| = Vrms*Irms = \sqrt{P^2 + Q^2}$

Real power, $P = Re(\mathbf{S}) = S \cos(\theta_v - \theta_i)$

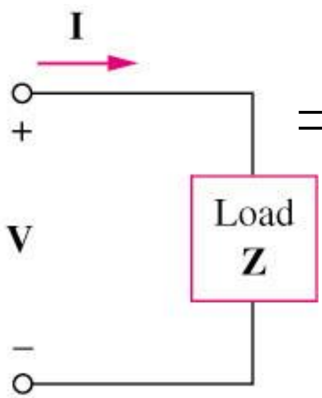
Reactive Power, $Q = Im(\mathbf{S}) = S \sin(\theta_v - \theta_i)$

Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

- Real Power is the actual power dissipated by the load.
- Reactive Power is a measure of the energy exchange between source and reactive part of the load.

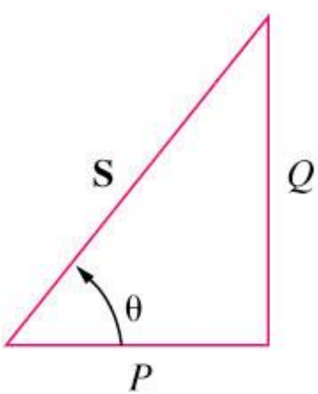
Complex Power

➢ The <u>COMPLEX Power</u> is represented by the <u>POWER TRIANGLE</u> similar to <u>IMPEDANCE TRIANGLE</u>. Power triangle has four items: P, Q, S and θ.

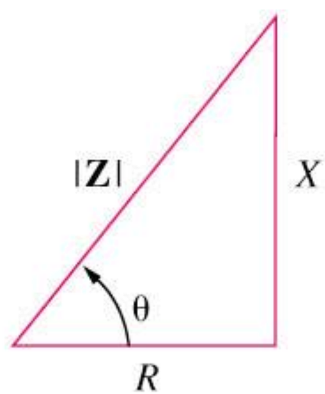


$$\Rightarrow S = \underbrace{V_{rms}\ I_{rms}\ \cos(\theta_v - \theta_i)}_{P} + j\,\underbrace{V_{rms}\ I_{rms}\ \sin(\theta_v - \theta_i)}_{Q}$$

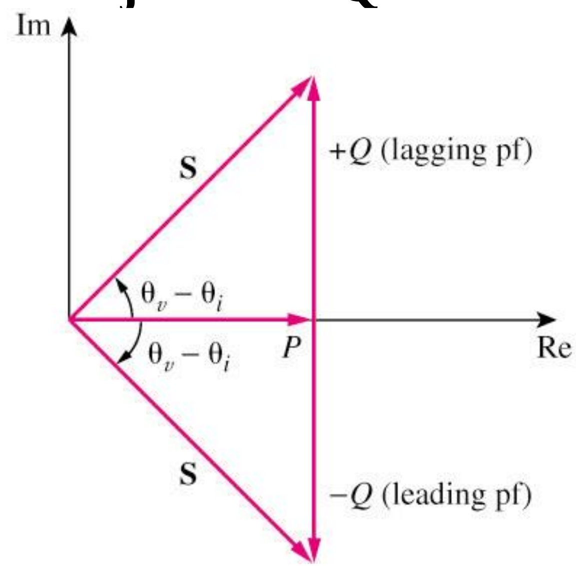
$$\mathbf{S} = \mathbf{P} + \mathbf{j}\ \mathbf{Q}$$



<u>Power Triangle</u>



<u>Impedance Triangle</u>



<u>Power Factor</u>

Real and Reactive Powers

- The REAL Power is the only useful power delivered to the load.
- The REACTIVE Power represents the energy exchange between the source and reactive part of the load. It is being transferred back and forth between the load and the source
- The unit of Q is volt-ampere reactive (VAR)

$$\mathbf{S} = P + jQ = \mathrm{Re}\{\mathbf{S}\} + j\,\mathrm{Im}\{\mathbf{S}\}$$

$$= \text{Real Power} + \text{Reactive Power}$$

$$\mathbf{S} = I^2_{Rms} Z = I^2_{Rms}(R + jX) = P + jQ$$

$$P = V_{Rms} I_{Rms} \cos(\theta_v - \theta_i) = \mathrm{Re}\{\mathbf{S}\} = I^2_{Rms} R$$

$$Q = V_{Rms} I_{Rms} \sin(\theta_v - \theta_i) = \mathrm{Im}\{\mathbf{S}\} = I^2_{Rms} X$$

Complex Power

Example: Two loads are connected in parallel. Load 1 has 2 kW, pf=0.75 leading and Load 2 has 4 kW, pf=0.95 lagging. Calculate the pf of two loads and the complex power supplied by the source.

For load 1,

$$P_1 = 2000, \qquad pf = 0.75 = \cos\theta_1 \longrightarrow \theta_1 = -41.41^\circ$$

$$P_1 = S_1 \cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 2666.67$$

$$Q_1 = S_1 \sin\theta_1 = -176.85$$

$$\mathbf{S}_1 = P_1 + jQ_1 = 2000 - j1763.85 \quad \text{(leading)}$$

For load 2,

$$P_2 = 4000, \qquad pf = 0.95 = \cos\theta_2 \longrightarrow \theta_2 = 18.19^\circ$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 4210.53$$

$$Q_2 = S_2 \sin\theta_2 = 1314.4$$

$$\mathbf{S}_2 = P_2 + jQ_2 = 4000 + j1314.4 \quad \text{(lagging)}$$

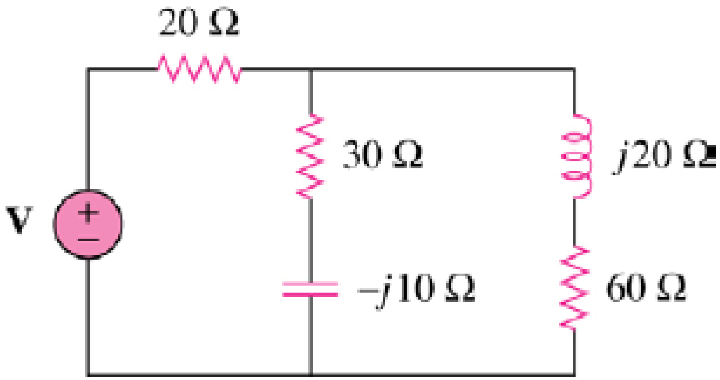
The total complex power is

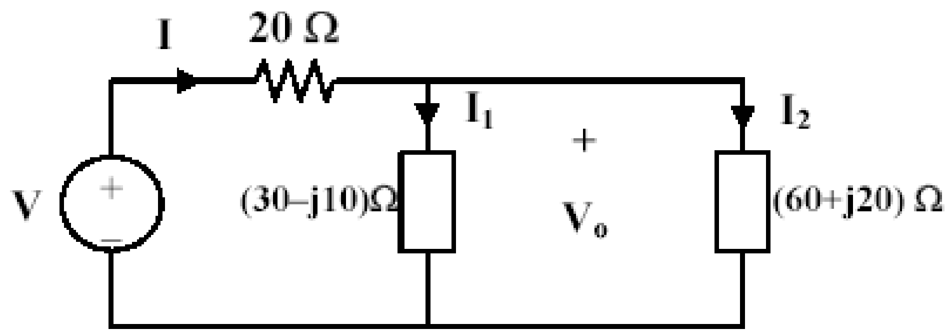
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = \underline{\mathbf{6 - j0.4495\ kVA}}$$

$$pf = \frac{P}{|\mathbf{S}|} = \frac{6000}{6016.18} = \underline{\mathbf{0.9972 \quad (leading)}}$$

Complex Power

Example: **The 60 Ω resistor absorbs 240 Watt of average power. Calculate V and the complex power of each branch. What is the total complex power?**





$$P = I_2^2 R \longrightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4 \quad \mathbf{I}_2 = 2 \quad \text{(rms)}$$

$$\mathbf{V}_o = \mathbf{I}_2(60 + j20) = 120 + j40$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{30 - j10} = 3.2 + j2.4 \quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 5.2 + j2.4$$

$$\mathbf{V} = 20\mathbf{I} + \mathbf{V}_o = (104 + j48) + (120 + j40)$$

$$\mathbf{V} = 224 + j88 = \underline{\mathbf{240.67\angle 21.45^\circ \quad (rms)}}$$ $$\mathbf{V} = 224 + j88 = \underline{\mathbf{240.67\angle 21.45^\circ \quad (rms)}}$$

Complex Power

For the 20-Ω resistor,

$$\mathbf{V} = 20\,\mathbf{I} = 204 + j48 = 114.54\angle 24.8^\circ$$

$$\mathbf{I} = 5.2 + j2.4 = 5.727\angle 24.8^\circ$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (114.54\angle 24.8^\circ)(5.727\angle -24.8^\circ) \qquad \mathbf{S} = \underline{\mathbf{656\ VA}}$$

For the (30 – j10)-Ω impedance,

$$\mathbf{V}_o = 120 + j40 = 126.5\angle 18.43^\circ$$

$$\mathbf{I}_1 = 3.2 + j2.4 = 4\angle 36.87^\circ$$

$$\mathbf{S}_1 = \mathbf{V}_o\,\mathbf{I}_1^* = (126.5\angle 18.43^\circ)(4\angle -36.87^\circ) \qquad \mathbf{S}_1 = 506\angle -18.44^\circ = \underline{\mathbf{480 - j160\ VA}}$$

For the (60 + j20)-Ω impedance, $\mathbf{I}_2 = 2\angle 0^\circ$

$$\mathbf{S}_2 = \mathbf{V}_o\,\mathbf{I}_2^* = (126.5\angle 18.43^\circ)(2\angle -0^\circ) \quad \mathbf{S}_2 = 253\angle 18.43^\circ = \underline{\mathbf{240 + j80\ VA}}$$

The overall complex power supplied by the source is

$$\mathbf{S}_T = \mathbf{V}\,\mathbf{I}^* = (240.67\angle 21.45^\circ)(5.727\angle -24.8^\circ)$$

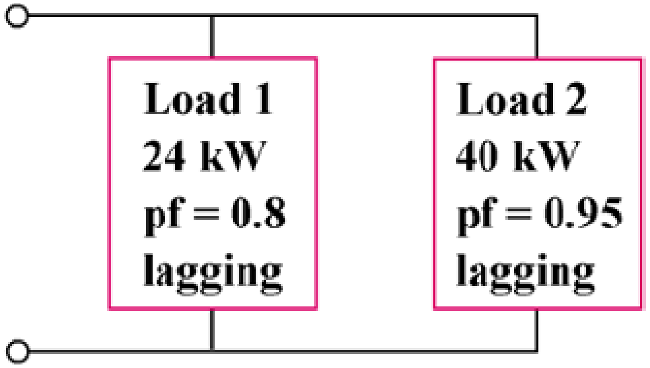
$$\mathbf{S}_T = 1378.3\angle -3.35^\circ = \underline{\mathbf{1376 - j80\ VA}}$$

Complex Power

Example: **A 120-V_{rms} 60-Hz source supplies two loads connected in parallel, as shown below.**

(a) Find the power factor of the parallel combination.

(b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.



Solution:

Chapter 11, Solution 74.

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$ $\quad S_1 = \dfrac{P_1}{\cos\theta_1} = \dfrac{24}{0.8} = 30\ \text{kVA}$

$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18\ \text{kVAR}$ $\quad \mathbf{S}_1 = 24 + j18\ \text{kVA}$

$\theta_2 = \cos^{-1}(0.95) = 18.19°$ $\quad S_2 = \dfrac{P_2}{\cos\theta_2} = \dfrac{40}{0.95} = 42.105\ \text{kVA}$

$Q_2 = S_2 \sin\theta_2 = 13.144\ \text{kVAR}$ $\quad \mathbf{S}_2 = 40 + j13.144\ \text{kVA}$

Complex Power

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$ $\quad S_1 = \dfrac{P_1}{\cos\theta_1} = \dfrac{24}{0.8} = 30\text{ kVA}$

$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18\text{ kVAR}$ $\quad \mathbf{S}_1 = 24 + j18\text{ kVA}$

$\theta_2 = \cos^{-1}(0.95) = 18.19°$ $\quad S_2 = \dfrac{P_2}{\cos\theta_2} = \dfrac{40}{0.95} = 42.105\text{ kVA}$

$Q_2 = S_2 \sin\theta_2 = 13.144\text{ kVAR}$ $\quad \mathbf{S}_2 = 40 + j13.144\text{ kVA}$

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144\text{ kVA}$

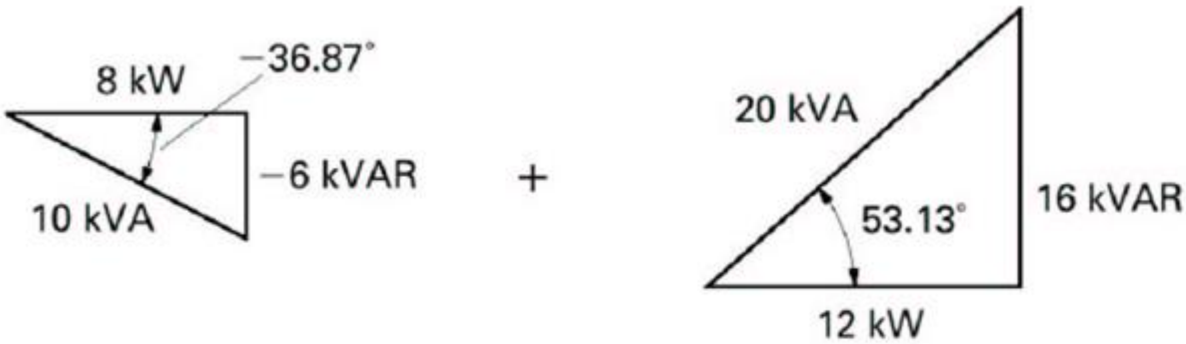
$\theta = \tan^{-1}\left(\dfrac{31.144}{64}\right) = 25.95°$ $\quad pf = \cos\theta =$ **<u>0.8992 (lagging)</u>**

(b) $\theta_2 = 25.95°, \quad \theta_1 = 0°$

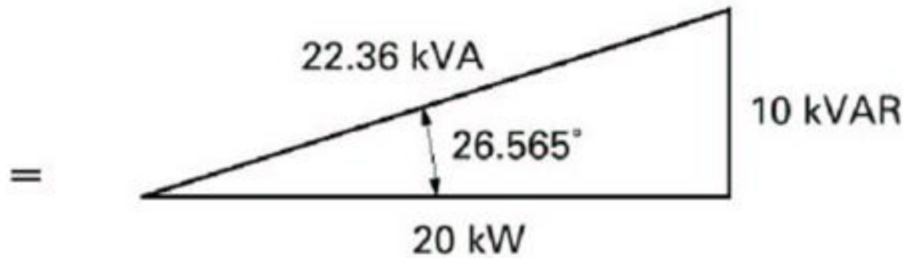
$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144\text{ kVAR}$

$C = \dfrac{Q_c}{\omega V_{rms}^2} = \dfrac{31{,}144}{(2\pi)(60)(120)^2} =$ **<u>5.74 mF</u>**

Use of Power Triangles

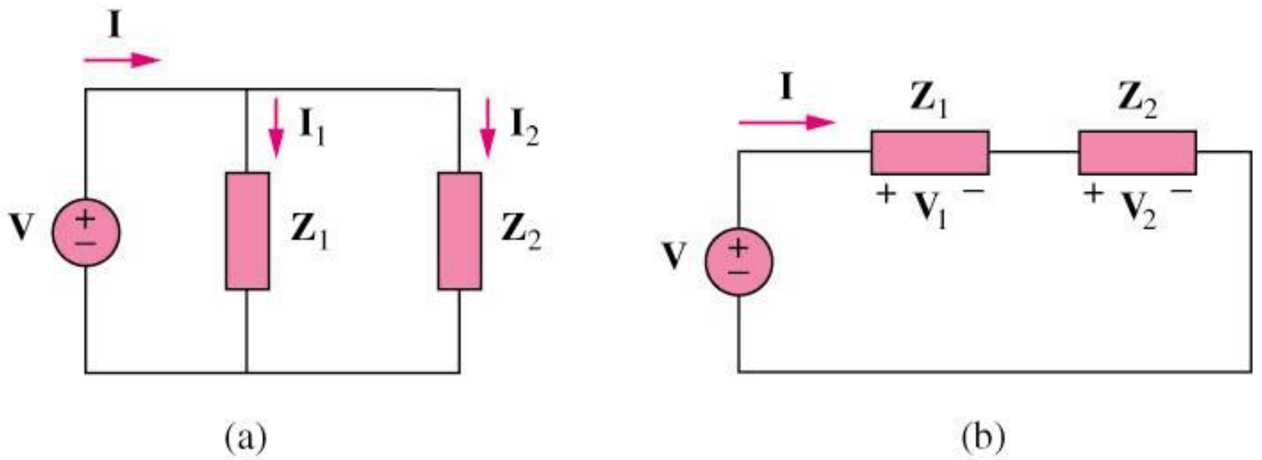


$$S = P + jQ = S_1 + S_2 = (P_1 + P_2) + j(Q_1 + Q_2)$$



Conservation of AC Power

➢The complex real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.

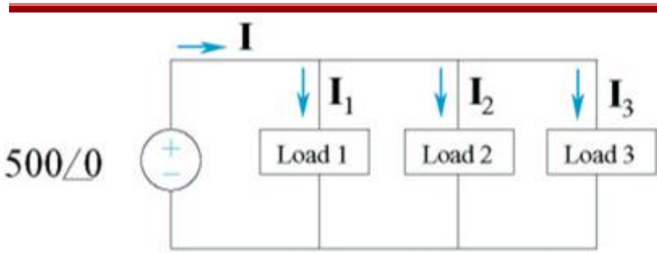


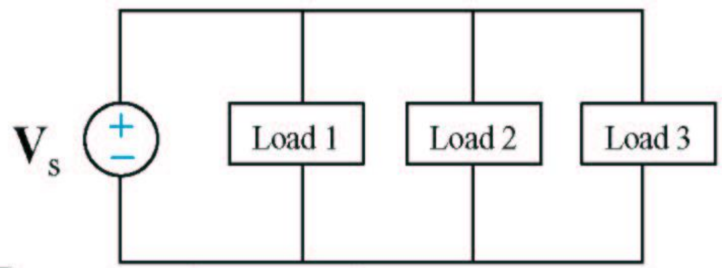
For parallel or series connection:

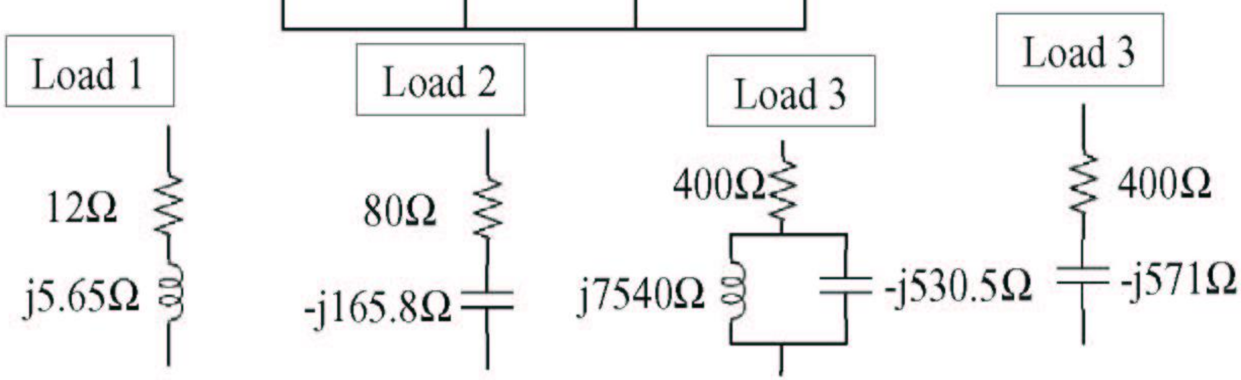
$$\mathbf{S} = \mathbf{V_1 I_1^*} + \mathbf{V_2 I_2^*} + \cdots + \mathbf{V_N I_N^*}$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \cdots + \mathbf{S_N}$$

Example of Complex Power Balance







$$Z_1 = 12 + j5.65 = 13.26\angle 25.21\ \Omega$$

$$Pf_1 = \cos(25.21) = 0.9 \text{ lag}$$

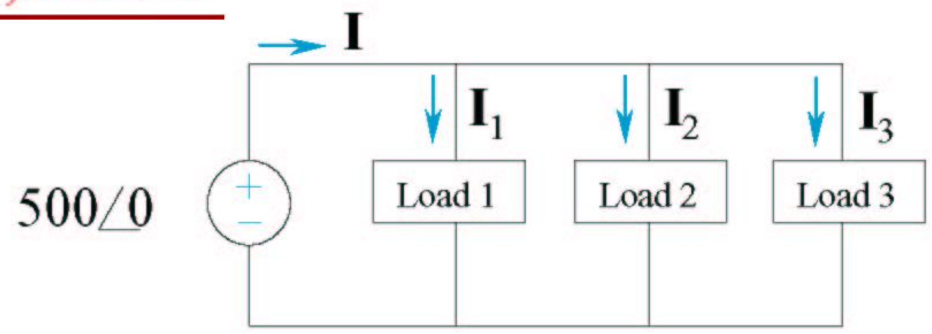
$$Z_2 = 80 - j165.8 = 184.1\angle -64.2\ \Omega$$

$$Pf_2 = \cos(-64.2) = 0.43 \text{ lead}$$

$$Z_3 = 400 + -j571 = 697\angle -55\ \Omega$$

$$Pf_1 = \cos(-55) = 0.57 \text{ lead}$$

Example, cont'd



$$\mathbf{I}_1 = \frac{500\angle 0^\circ}{13.26\angle 25.21^\circ} = 37.7\angle -25.21^\circ = 34.11 - j16.06$$

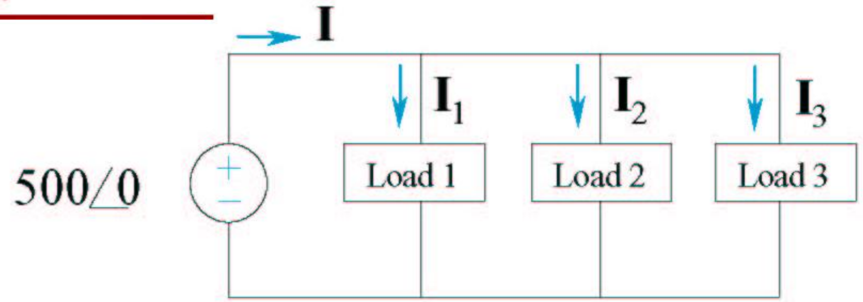
$$\mathbf{I}_2 = \frac{500\angle 0^\circ}{184.1\angle -64.2^\circ} = 2.72\angle 64.2^\circ = 1.18 + j2.45$$

$$\mathbf{I}_3 = \frac{500\angle 0^\circ}{697\angle -55^\circ} = 0.72\angle 55^\circ = 0.41 + j0.59$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 35.7 - j13.02 = 38\angle -20^\circ A$$

Combined Pf = cos(20) = 0.94 lag

Example, cont'd



$$\mathbf{S}_1 = \hat{V}\hat{I}_1^* = 500 \bullet 37.7\angle 25.21° = 18850\angle 25.21° = 17055 + j8029 \text{ VA}$$

$$\mathbf{S}_2 = \hat{V}\hat{I}_2^* = 500 \bullet 2.72\angle -64.2° = 1360\angle -64.2° = 592 - j1224 \text{ VA}$$

$$\mathbf{S}_3 = \hat{V}\hat{I}_3^* = 500 \bullet 0.72\angle -55° = 360\angle -55° = 207 - j295 \text{ VA}$$

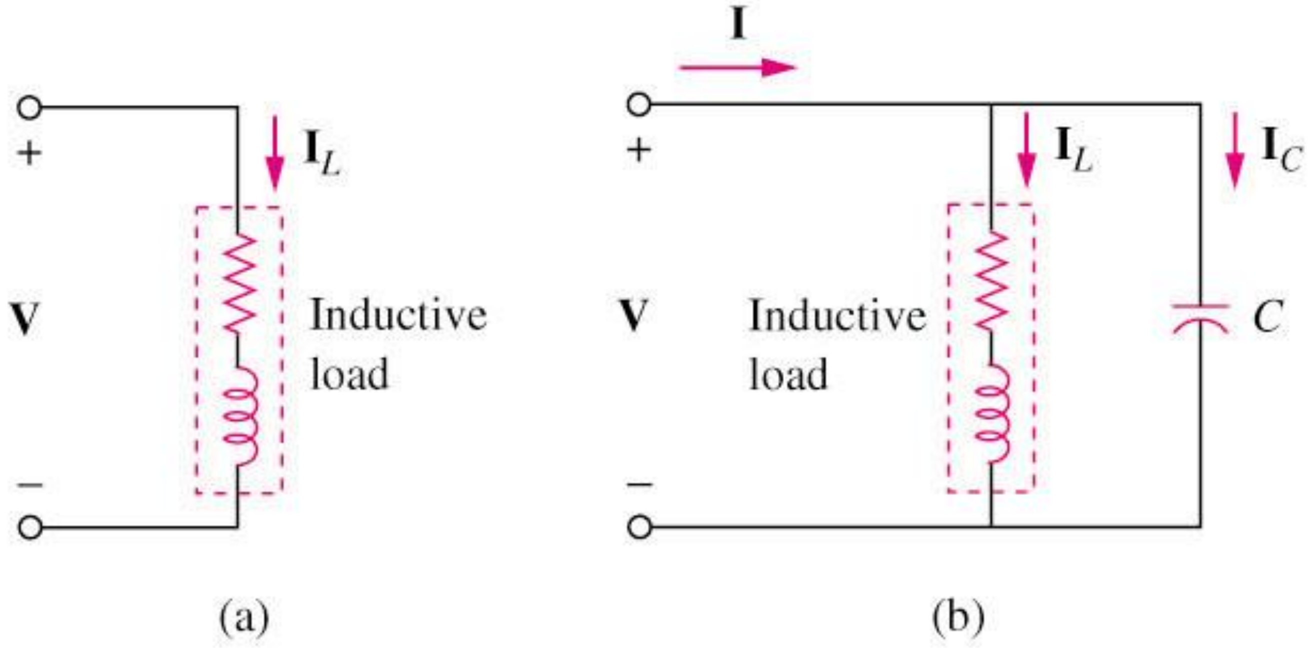
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 17854 - j6510 = 19000\angle -20° \text{ VA}$$

Check: $\mathbf{S} = \hat{V}\hat{I}^* = 500 \bullet 38\angle -20° = 19000\angle -20° \text{ VA}$

Complex power is Conserved

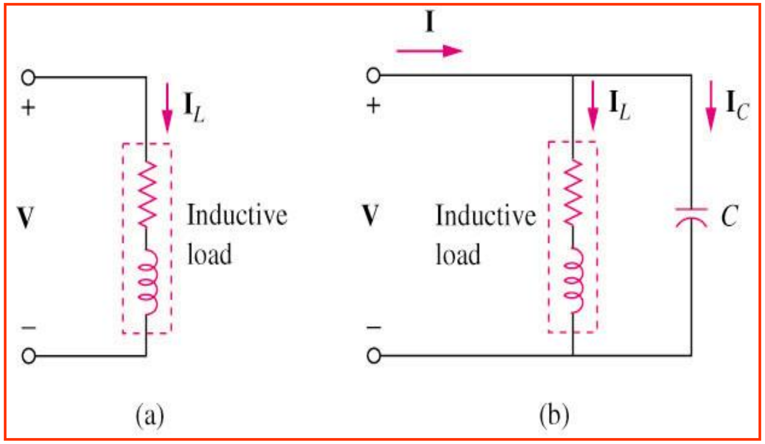
Power Factor Correction

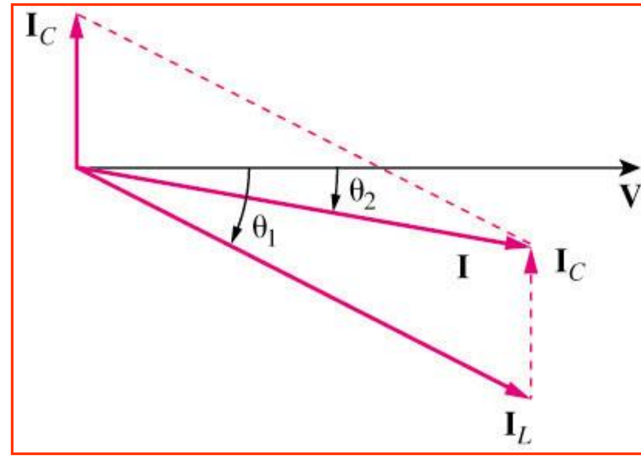
<u>Power factor correction</u> is the process of **<u>increasing</u>** the **power factor** <u>without altering</u> the voltage or current to the original load.

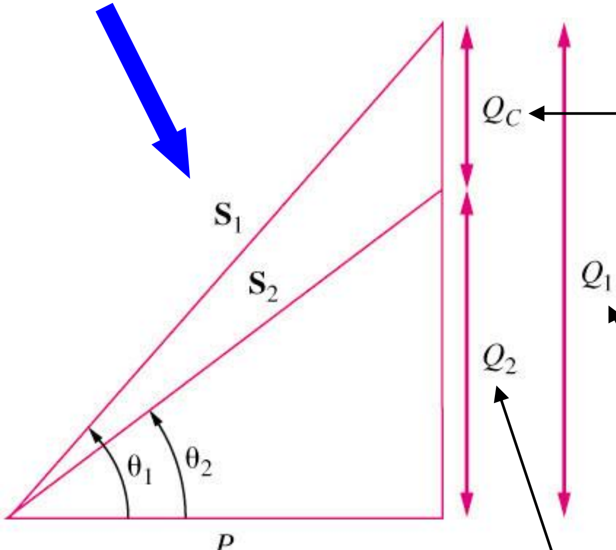


Power factor correction is necessary for <u>economic reason</u>.

Power Factor Correction







$$\mathbf{Q_c = Q_1 - Q_2}$$
$$\mathbf{= P\,(\tan\theta_1 - \tan\theta_2)}$$
$$\mathbf{= \omega C V^2_{rms}}$$

$$\mathbf{Q_1 = S_1 \sin\theta_1}$$
$$\mathbf{= P \tan\theta_1}$$

$$\mathbf{P = S_1 \cos\theta_1}$$

$$\mathbf{Q_2 = P \tan\theta_2}$$

$$C = \frac{Q_c}{\omega V^2_{rms}} = \frac{P\,(\tan\theta_1 - \tan\theta_2)}{\omega\, V^2_{rms}}$$

Power Factor Correction

➢ The process of increasing the power factor without altering the voltage or current to the original load is called power factor correction.

$$P_1 = P_2 = P \quad \text{Real power stays same}$$

$$P = S_1 \cos\theta_1 \quad Q_1 = S_1 \sin\theta_1 = P\tan\theta_1 \quad Q_2 = P\tan\theta_2$$

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2)$$

$$C = \frac{Q_C}{\omega V_{rms}^{\;2}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^{\;2}}$$

- The capacitance value needed to change the pf angle from θ_1 to θ_2 .
- Similarly the inductance value needed to change the pf angle from θ_1 to θ_2 for a capacitive load.

$$L = \frac{V_{rms}^{\;2}}{\omega Q_L}$$

Power Factor Correction

Example: Find the value of the capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. The load is supplied by a 110 Volt (rms), 60 Hz line.

$$pf = 0.85 = \cos\theta \longrightarrow \theta = 31.79°$$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{140}{\sin(31.79°)} = 265.8 \text{ kVA}$$

$$P = S\cos\theta = 225.93 \text{ kW}$$

For $pf = 1 = \cos\theta_1 \longrightarrow \theta_1 = 0°$

Since P remains the same,

$$P = P_1 = S_1\cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 225.93$$

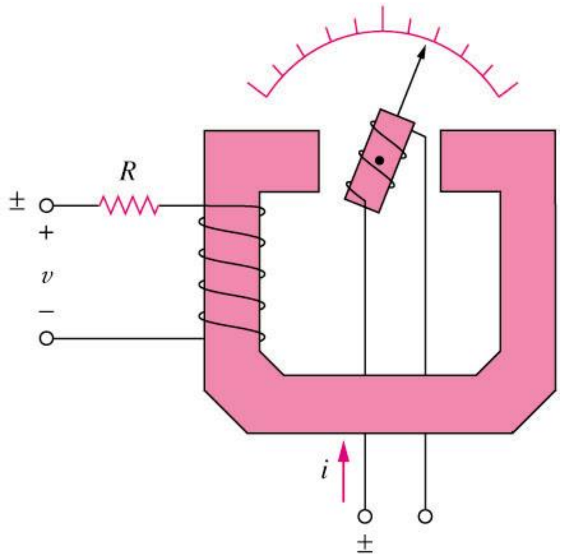
$$Q_1 = S_1\sin\theta_1 = 0$$

The difference between the new Q_1 and the old Q is Q_c.

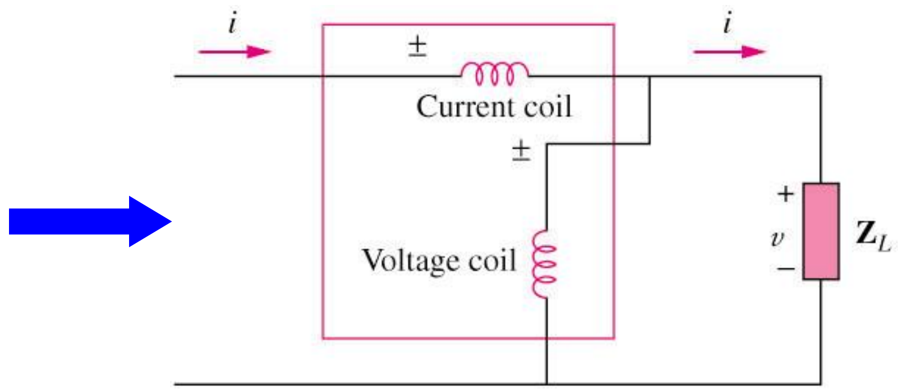
$$Q_c = 140 \text{ kVAR} = \omega C V_{rms}^2 \qquad C = \frac{140\times 10^3}{(2\pi)(60)(110)^2} = \underline{\mathbf{30.69\ mF}}$$

Power Measurement

The wattmeter is the instrument for measuring the average power. Two coils are used, the high impedance Voltage coil and the low impedance Current coil.



The basic structure



Equivalent Circuit with load

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

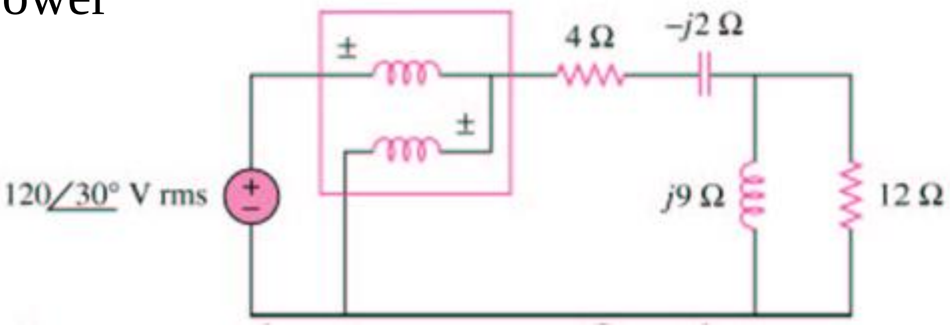
Wattmeter measures the average power given by:

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Power Measurement

<u>Example:</u> Find the wattmeter reading

The wattmeter measures the average power from the source



Let $\mathbf{Z}_1 = 4 - j2$

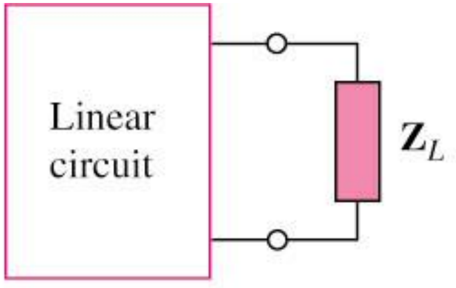
$$\mathbf{Z}_2 = 12 \,||\, j9 = \frac{(12)(j9)}{12 + j9} = 4.32 + j5.76$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = 8.32 + j3.76 = 9.13\angle 24.32^\circ$$

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{9.13\angle -24.32^\circ} = 1577.2\angle 24.32^\circ \text{ kVA}$$

$$P = |\mathbf{S}|\cos\theta = \underline{\mathbf{1437.2\ kW}}$$

Maximum Average Power Transfer



a) Circuit with a load

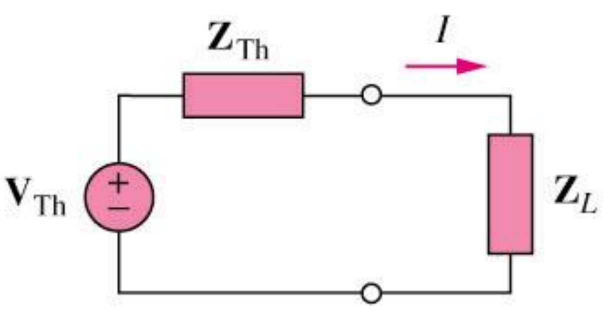
$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

The maximum average power can be transferred to the load if

$$\mathbf{X_L = -X_{TH} \text{ and } R_L = R_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$



b) Thevenin Equivalent circuit

If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

$$\boldsymbol{Z_L = R_{TH} - jX_{TH} = Z^*_{TH}}$$

Maximum Average Power Transfer

- Write the expression for average power associated with Z_L: $P(Z_L)$.

$$Z_L = R_L + jX_L$$

$$\text{Set } \frac{\partial P}{\partial R_L} = 0\text{: Solve for } R_L$$

$$\text{Set } \frac{\partial P}{\partial X_L} = 0\text{: Solve for } X_L$$

$$\mathbf{I} = \frac{\mathbf{V}_{th}}{Z_L + Z_{Th}} = I_m \angle \theta_i$$

$$I_m = \frac{|\mathbf{V}_{Th}|}{\sqrt{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]}}$$

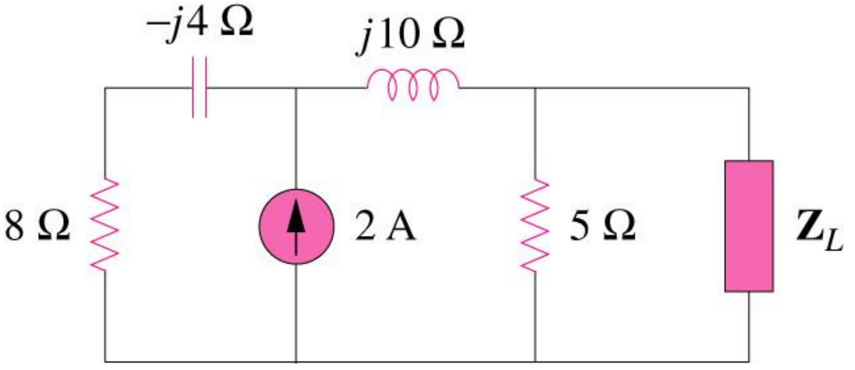
$$P = \frac{I_m^2 R_L}{2} = \frac{|V_{Th}|^2 R_L / 2}{(R_L + R_{Th})^2 + (X_L + X_{Th})^2}$$

$$\frac{\partial P}{\partial X_L} = 0 \rightarrow \boxed{X_L = -X_{Th}} \qquad \frac{\partial P}{\partial R_L} = 0 \rightarrow \boxed{R_L = R_{Th}}$$

Maximum Average Power Transfer

Example:

For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.



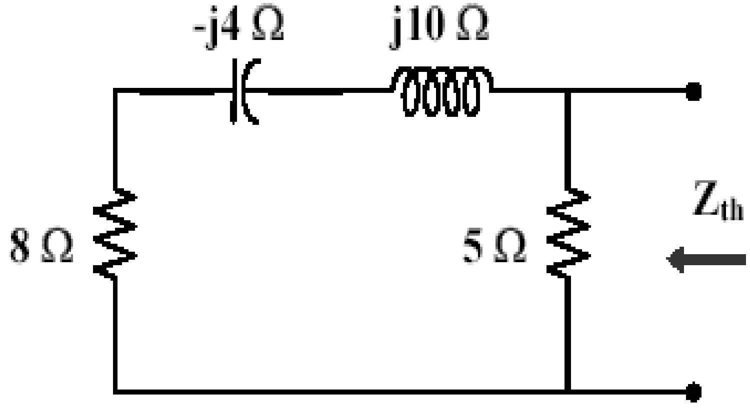
Answer:

$Z_L = 3.415 - j0.7317\ \Omega$

$P_{max} = 1.429W$

Maximum Average Power Transfer

Solution: We first obtain the Thevenin equivalent circuit across $\mathbf{Z}_L$. $\mathbf{Z}_{Th}$ is obtained from the circuit in Fig. (a).

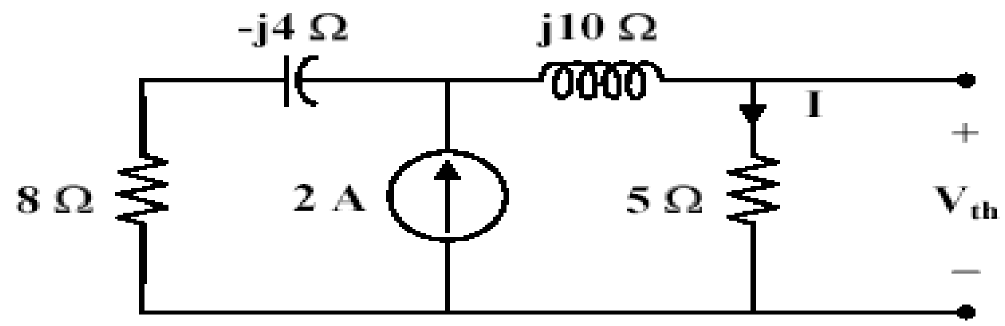


(a)

$$\mathbf{Z}_{Th} = 5 \,||\, (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

Maximum Average Power Transfer

V_{Th} is obtained from the circuit in Fig. (b).



(b)

By current division,

$$\mathbf{I} = \frac{8 - j4}{8 - j4 + j10 + 5}(2)$$

$$\mathbf{V}_{Th} = 5\mathbf{I} = \frac{(10)(8 - j4)}{13 + j6} = 6.25\angle -51.34°$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = \underline{\mathbf{3.415 - j0.7317\ \Omega}}$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{(6.25)^2}{(8)(3.415)} = \underline{\mathbf{1.429\ W}}$$

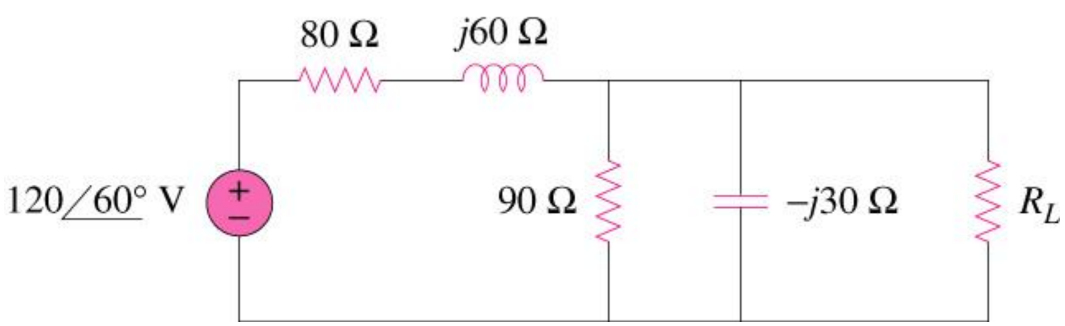
Maximum Average Power for Resistive Load

- When the load is <u>PURELY RESISTIVE</u>, the condition for maximum power transfer is:

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$$

- Now the maximum power can not be obtained from the P_{max} formula given before.

- **<u>Example:</u> Calculate the resistive load needed for maximum power transfer and the maximum average power.**



Solution: We first find $\mathbf{Z}_{Th}$ and $\mathbf{V}_{Th}$ across R_L.

Let $\mathbf{Z}_1 = 80 + j60$ and $\mathbf{Z}_2 = 90 \,||\, (-j30) = \dfrac{(90)(-j30)}{90 - j30} = 9(1 - j3)$

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 \,||\, \mathbf{Z}_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27} = 17.181 - j24.57\ \Omega$$

Maximum Average Power for Resistive Load

$$\mathbf{V}_{Th} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(120\angle 60°) = \frac{(9)(1 - j3)}{89 + j33}(120\angle 60°)$$

$$\mathbf{V}_{Th} = 35.98\angle -31.91°$$

$$R_L = |\mathbf{Z}_{Th}| = \underline{\mathbf{30\ \Omega}}$$

The current through the load is

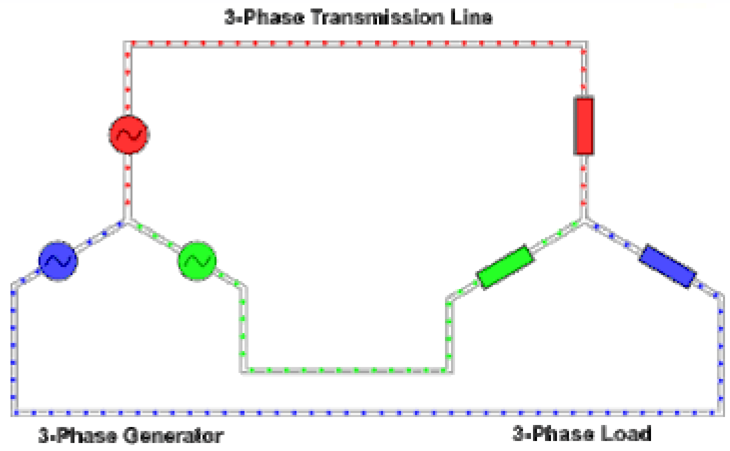
$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{35.98\angle -31.91°}{47.181 - j24.57} = 0.6764\angle -4.4°$$

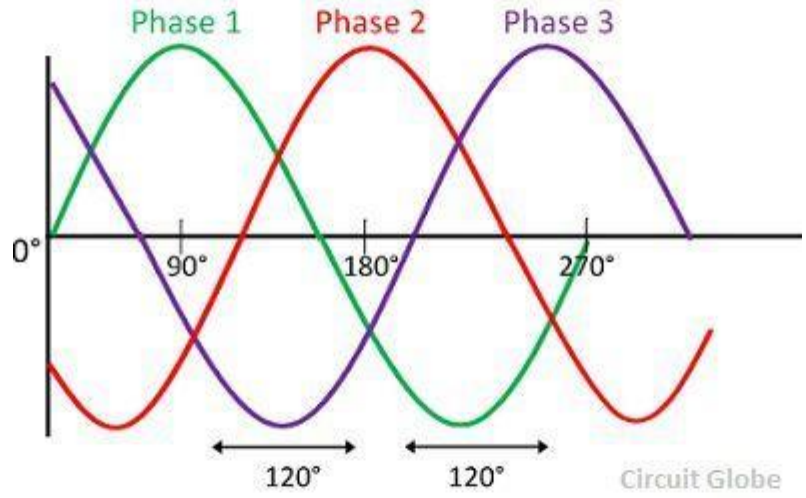
The maximum average power absorbed by R_L is

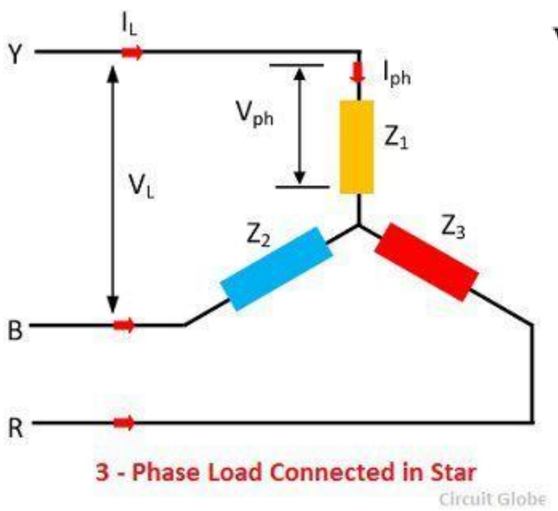
$$P_{max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(0.6764)^2(30) = \underline{\mathbf{6.863\ W}}$$

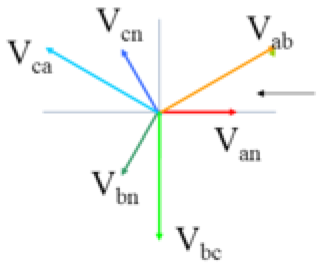
- Notice the way that the maximum power is calculated using the Thevenin Equivalent circuit.

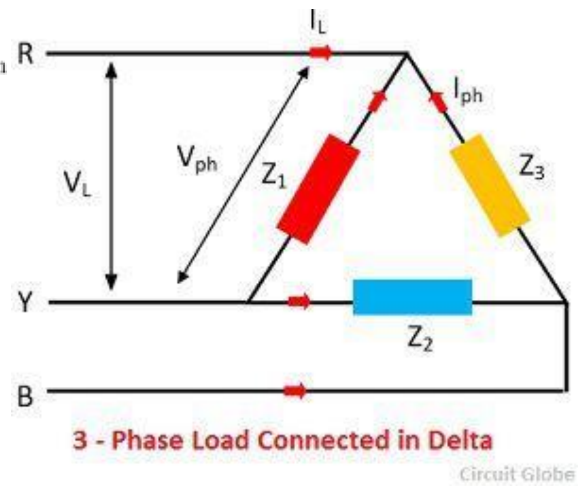
Topic 9: 3-Phase Circuit Analysis

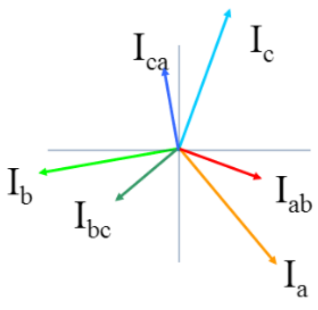










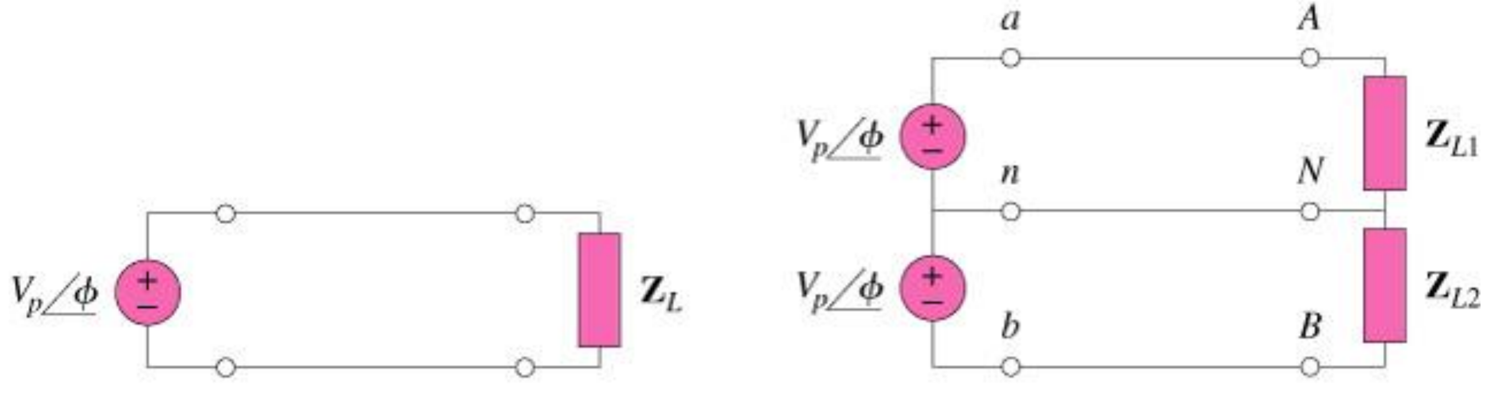


Three phase Circuits

- An AC generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase AC generator.**
- If the number of coils on the rotor is increased in a specified manner, the result is a **Polyphase AC generator**, which develops more than one AC phase voltage per rotation of the rotor
- In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons.

  1. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less).
  2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
  3. Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
  4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

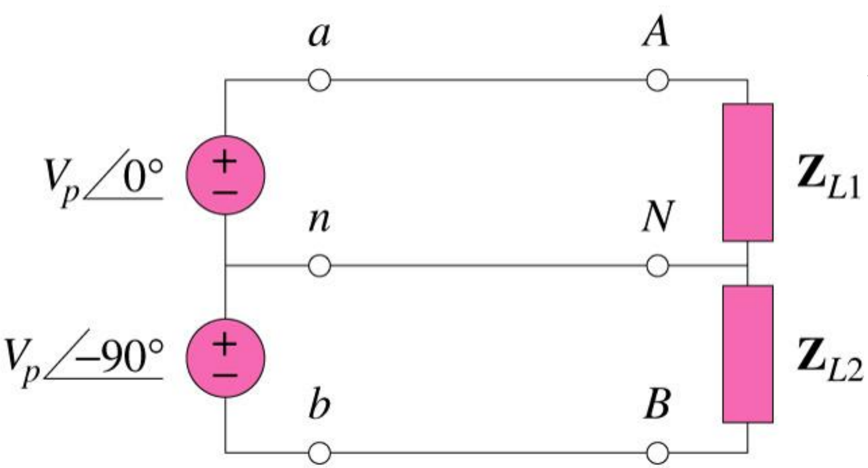
Single-Phase, Two-Phase, Three phase Circuits



(a) (b)

a) Single phase systems two-wire type

b) Single phase systems three-wire type. Allows connection to both 120 V and 240 V.

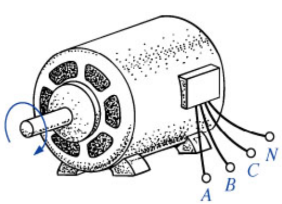


Two-phase three-wire system.

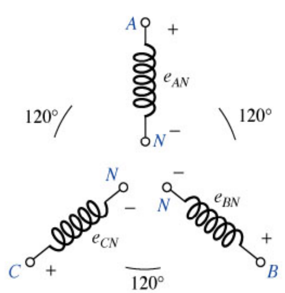
The AC sources operate at different phases.

Three-phase Generator

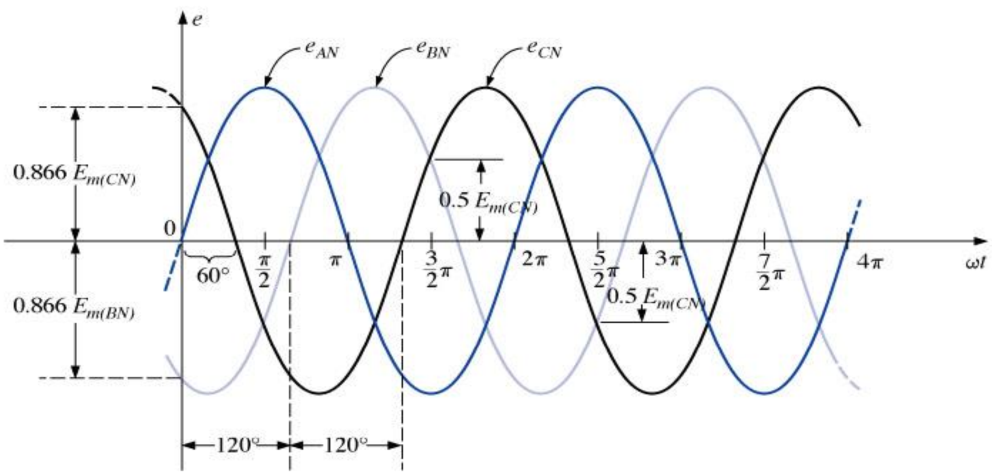
- The three-phase generator has three induction coils placed 120° apart on the stator.
- The three coils have an equal number of turns, the voltage induced across each coil will have the same peak value, shape and frequency.



(a)

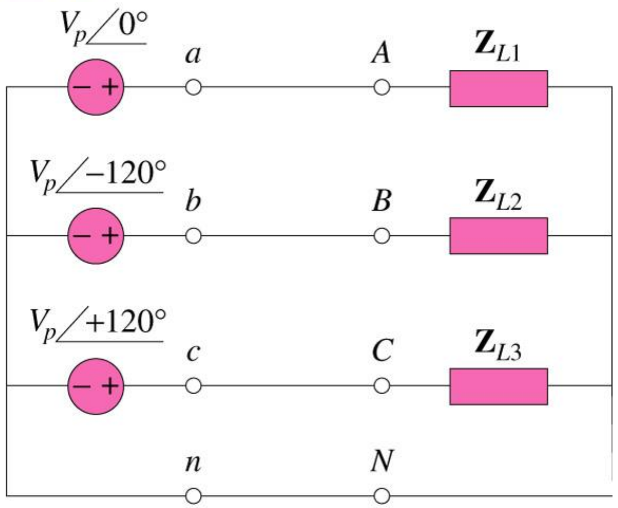


(b)



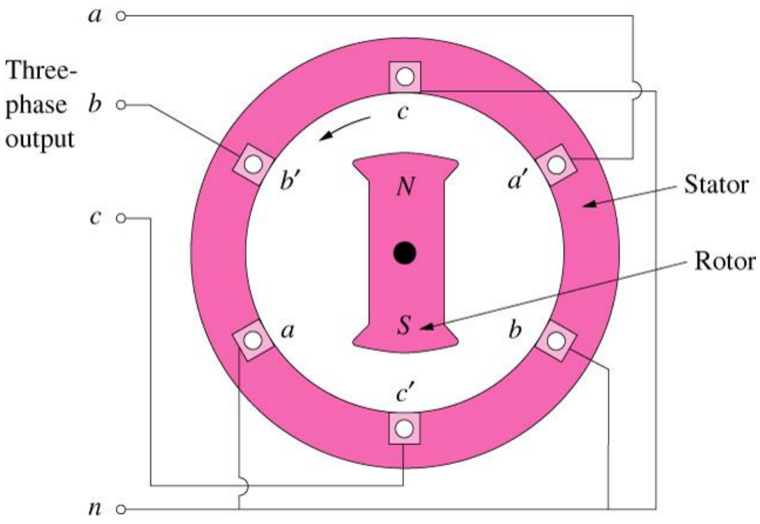
Balanced Three-phase Voltages

Three-phase four-wire system

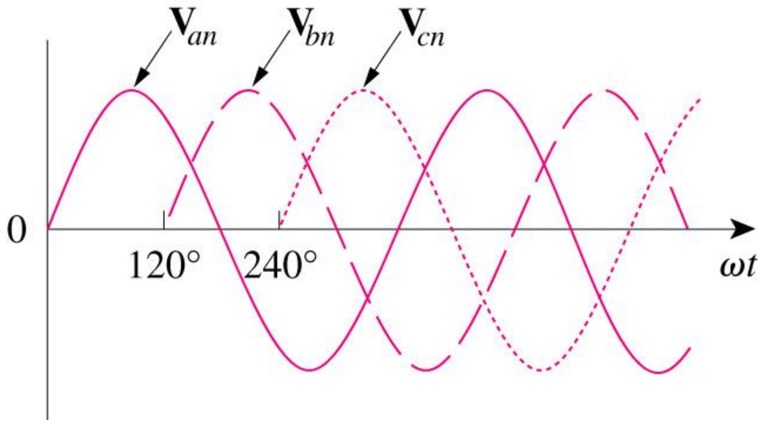


Neutral Wire

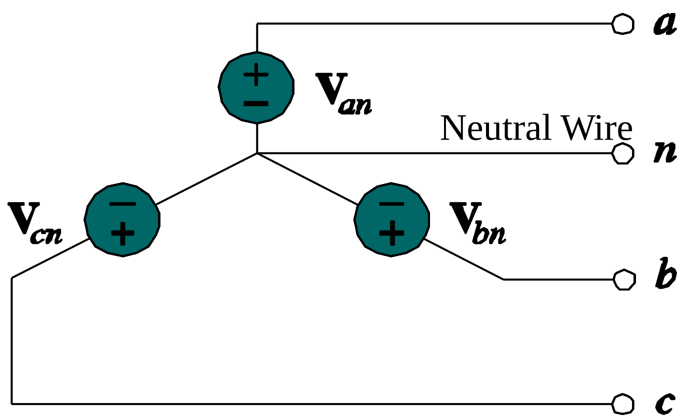
A Three-phase Generator



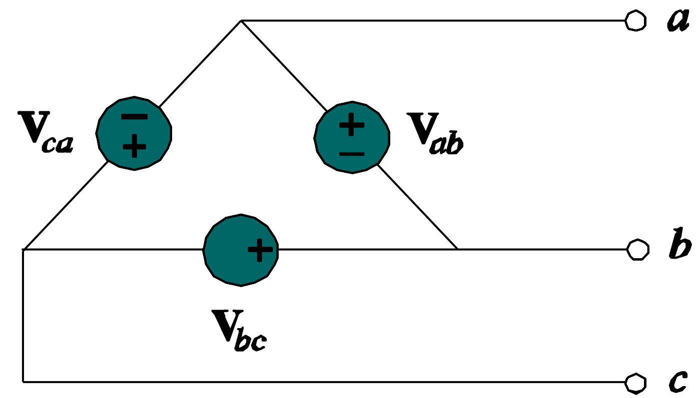
Voltages having 120° phase difference



Balanced Three phase Voltages

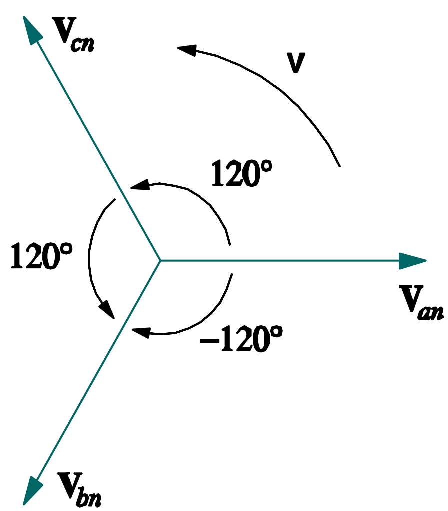


a) Wye Connected Source

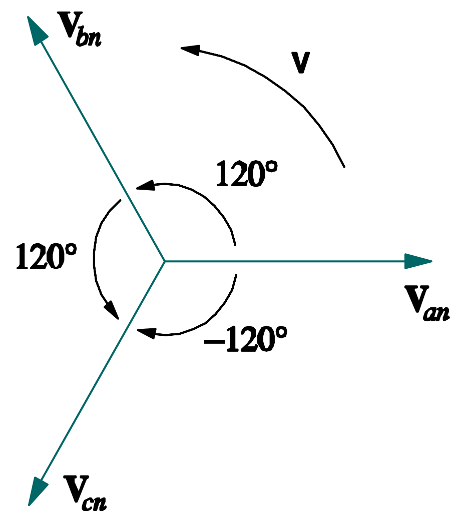


b) Delta Connected Source

$$V_{an} = V_p\angle 0^\circ$$
$$V_{bn} = V_p\angle -120^\circ$$
$$V_{cn} = V_p\angle -240^\circ$$



a) abc or positive sequence

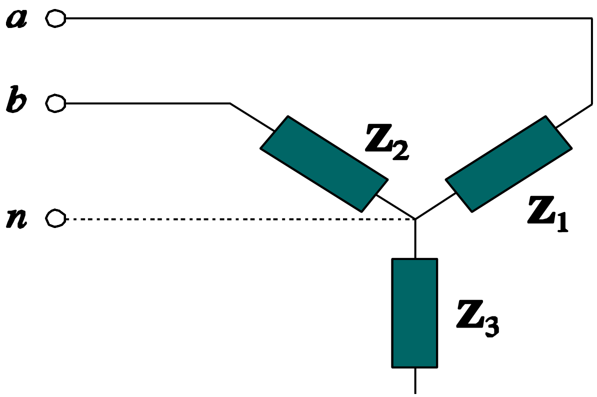


$$V_{an} = V_p\angle 0^\circ$$
$$V_{bn} = V_p\angle +120^\circ$$
$$V_{cn} = V_p\angle +240^\circ$$

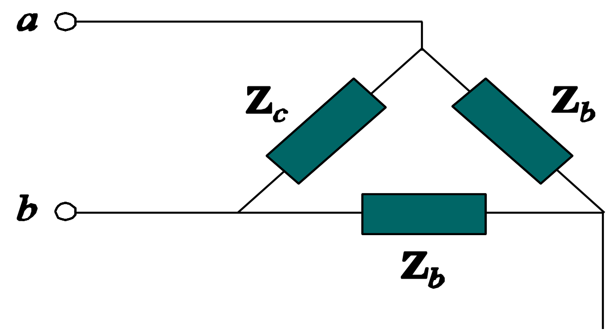
b) acb or negative sequence

Balanced Three phase Loads

- A Balanced load has equal impedances on all the phases



a) Wye-connected load



b) Delta-connected load

Balanced Impedance Conversion:

Conversion of Delta circuit to Wye or Wye to Delta.

$$Z_Y = Z_1 = Z_2 = Z_3$$

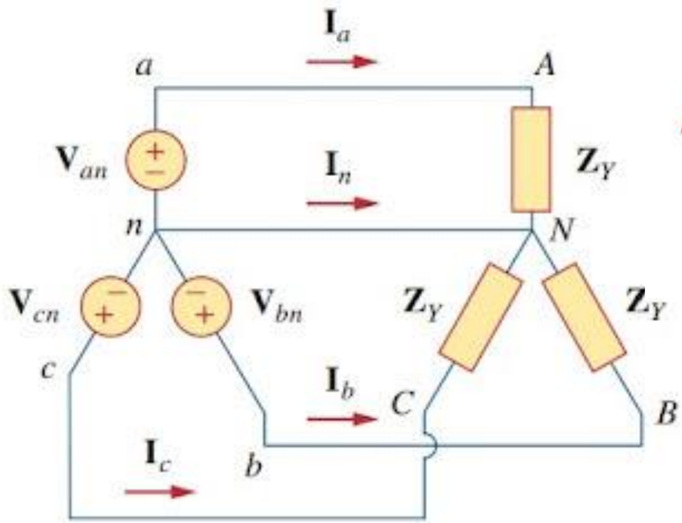
$$Z_\Delta = Z_a = Z_b = Z_c$$

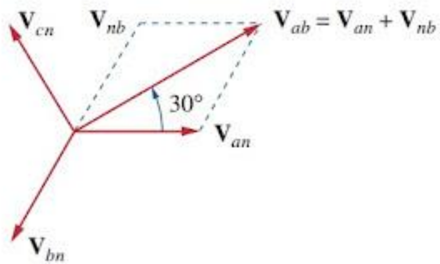
$$Z_\Delta = 3Z_Y \qquad Z_Y = \frac{1}{3} Z_\Delta$$

Three phase Connections Y-Y Connection

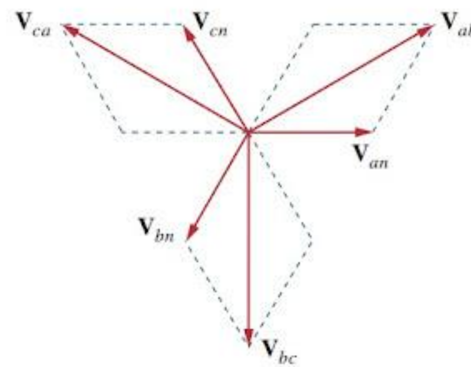
- Both the three phase source and the three phase load can be connected Wye Wye.

- Y-Y connection





(a)



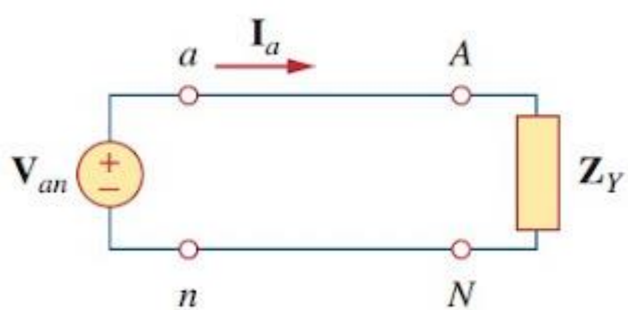
b)

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\underline{/-120^\circ}}{\mathbf{Z}_Y} = \mathbf{I}_a\underline{/-120^\circ}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\underline{/-240^\circ}}{\mathbf{Z}_Y} = \mathbf{I}_a\underline{/-240^\circ}$$

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$

$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}, \qquad \mathbf{V}_{cn} = V_p\underline{/+120^\circ}$$

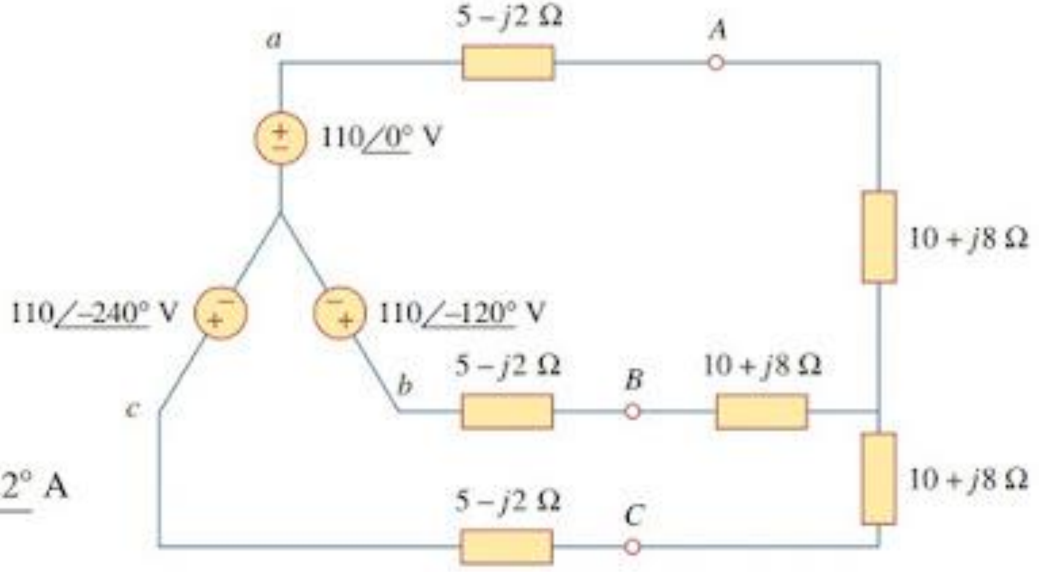


$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$V_L = \sqrt{3}V_p$$

Three phase Connections Y-Y Connection

➢ Example: Calculate the line currents in the three-wire Y-Y system shown below.



$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

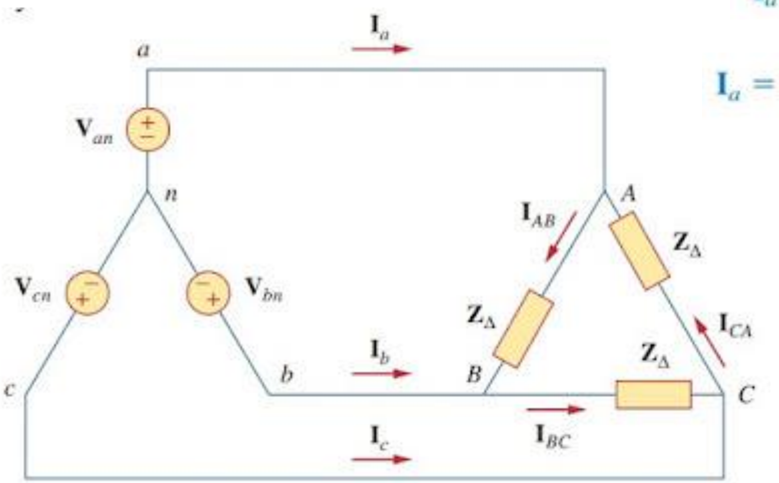
Three phase Connections Y-Δ Connection

- Both the three phase source and the three phase load can be connected Wye Wye.

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

- Y-Δ connection

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

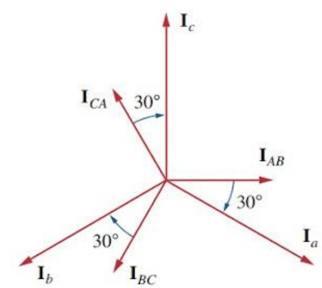


$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle{-240^\circ})$$
$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$

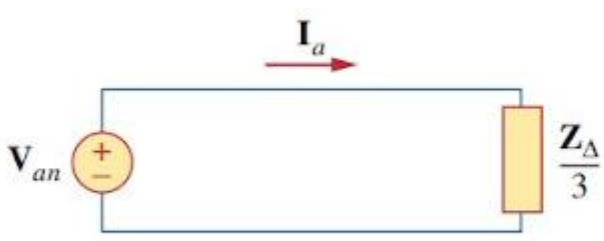
$$I_L = \sqrt{3} I_p$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$



$$\mathbf{V}_{an} = V_p\angle{0^\circ}$$
$$\mathbf{V}_{bn} = V_p\angle{-120^\circ}, \qquad \mathbf{V}_{cn} = V_p\angle{+120^\circ}$$



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle{30^\circ} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle{-90^\circ} = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle{-150^\circ} = \mathbf{V}_{CA}$$

Three phase Connections Y-Δ Connection

➢ Example: A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load (8 + j4) Ω per phase. Calculate the phase and line currents.

The load impedance is $\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB} \qquad \mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ} = 33.53\underline{/-16.57^\circ}\ \text{A}$$

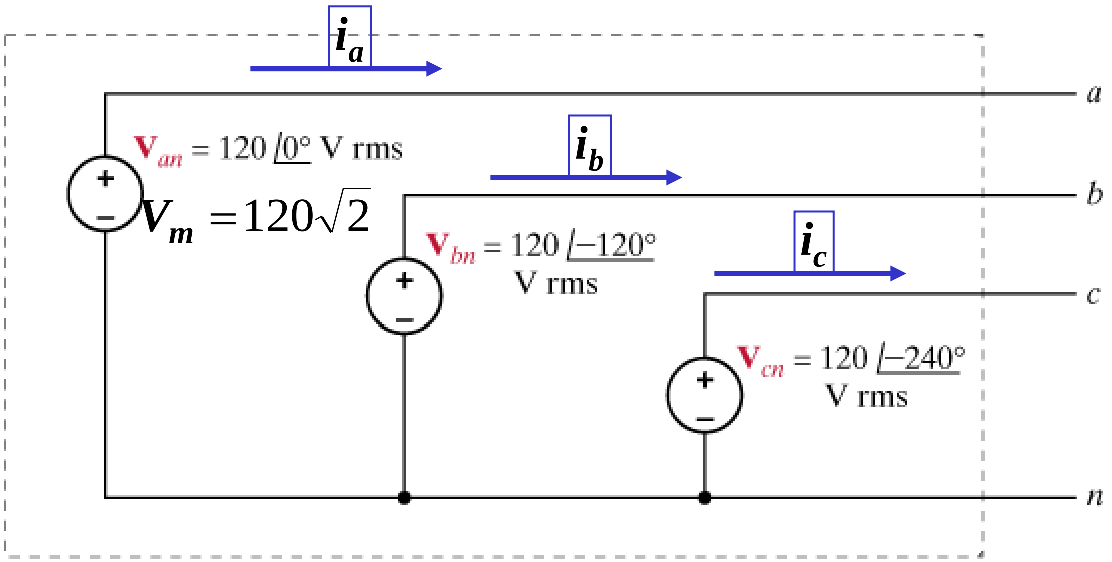
$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\ \text{A}$$

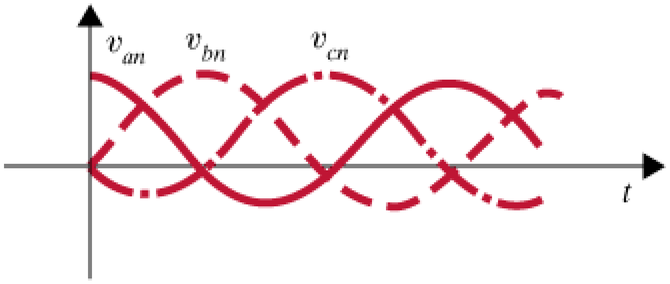
$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\ \text{A}$$

Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

THREE PHASE CIRCUITS





Instantaneous Phase Voltages

$$v_{an}(t) = V_m \cos(\omega t)(V)$$

$$v_{bn}(t) = V_m \cos(\omega t - 120°)(V)$$

$$v_c(t) = V_m \cos(\omega t - 240°)(V)$$

Balanced Phase Currents

$$i_a(t) = I_m \cos(\omega t - \theta)$$

$$i_b(t) = I_m \cos(\omega t - \theta - 120°)$$

$$i_c(t) = I_m \cos(\omega t - \theta - 240°)$$

Instantaneous power

$$p(t) = v_{an}(t)i_a(t) + v_{bn}(t)i_b(t) + v_{cn}(t)i_c(t)$$

Theorem

For a balanced three phase circuit the instantaneous power is constant

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta \quad W$$

Proof of Theorem

For a balanced three phase circuit the instantaneous power is constant

$$\boldsymbol{p}(\boldsymbol{t}) = 3\frac{\boldsymbol{V_m I_m}}{2}\cos\theta\,(\boldsymbol{W})$$

Instantaneous power

$$\boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{v_{an}}(\boldsymbol{t})\boldsymbol{i_a}(\boldsymbol{t}) + \boldsymbol{v_{bn}}(\boldsymbol{t})\boldsymbol{i_b}(\boldsymbol{t}) + \boldsymbol{v_{cn}}(\boldsymbol{t})\boldsymbol{i_c}(\boldsymbol{t})$$

$$\boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{V_m I_m}\begin{bmatrix} \cos\omega\boldsymbol{t}\cos(\omega\boldsymbol{t}-\theta) \\ +\cos(\omega\boldsymbol{t}-120)\cos(\omega\boldsymbol{t}-120-\theta) \\ +\cos(\omega\boldsymbol{t}-240)\cos(\omega\boldsymbol{t}-240-\theta) \end{bmatrix}$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{V_m I_m}\begin{bmatrix} 3\cos\theta + \cos(2\omega\boldsymbol{t}-\theta) \\ +\cos(2\omega\boldsymbol{t}-240-\theta) \\ +\cos(2\omega\boldsymbol{t}-480-\theta) \end{bmatrix}$$

$$\phi = \omega\boldsymbol{t}-\theta$$

$$\cos(\phi-240) = \cos(\phi+120)$$

$$\cos(\phi-480) = \cos(\phi-120)$$

$$\cos(120) = -0.5$$

Lemma

$$\cos\phi + \cos(\phi-120) + \cos(\phi+120) = 0$$

Proof

$$\cos\phi = \cos\phi$$

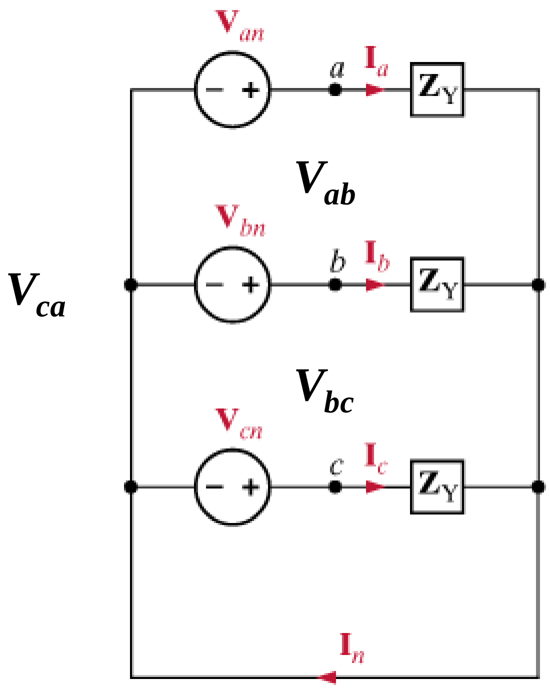
$$\cos(\phi-120) = \cos\phi\cos(120) + \sin\phi\sin(120)$$

$$\cos(\phi+120) = \cos\phi\cos(120) - \sin\phi\sin(120)$$

$$\boxed{\cos\phi + \cos(\phi-120) + \cos(\phi+120) = 0}$$

SOURCE/LOAD CONNECTIONS

BALANCED Y-Y CONNECTION

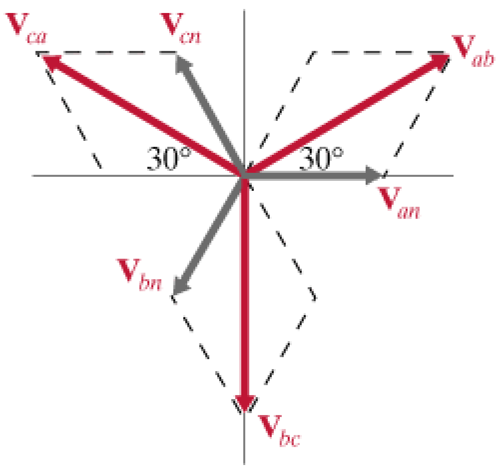


$$V_{an} = |V_p| \angle 0°$$

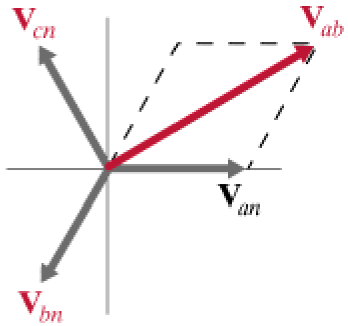
$$V_{bn} = |V_p| \angle -120°$$

$$V_{cn} = |V_p| \angle 120°$$

Positive sequence phase voltages



Line voltages



$$V_{ab} = V_{an} - V_{bn}$$

$$= |V_p| \angle 0° - |V_p| \angle -120°$$

$$= |V_p| \left(1 - (\cos 120 - j \sin 120)\right)$$

$$= |V|_p - |V_p| \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}\,|V_p| \angle 30°$$

$$V_{bc} = \sqrt{3}\,|V_p| \angle -90°$$

$$V_{ca} = \sqrt{3}\,|V_p| \angle -210°$$

$$V_L = \sqrt{3}\,|V_p| = \text{Line Voltage}$$

$$I_a = \frac{V_{an}}{Z_Y}; I_b = \frac{V_{bn}}{Z_Y}; I_c = \frac{V_{cn}}{Z_Y}$$

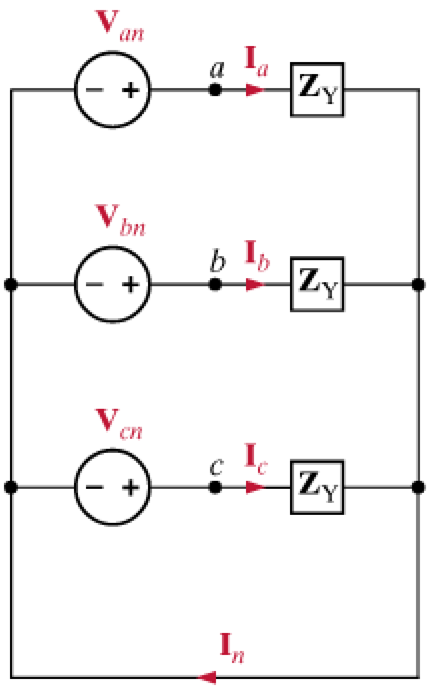
$$I_a = |I_L| \angle \theta°; I_b = |I_L| \angle \theta - 120°; I_c = |I_L| \angle \theta + 120°$$

$$I_a + I_b + I_c = I_n = 0$$

For this balanced circuit it is enough to analyze one phase

Example: **For an abc sequence, balanced Y-Y three phase circuit** $V_{ab} = 208\angle -30^\circ$

Determine the phase voltages.



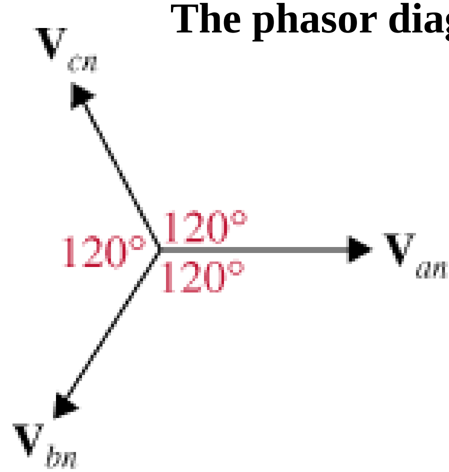
Balanced Y - Y

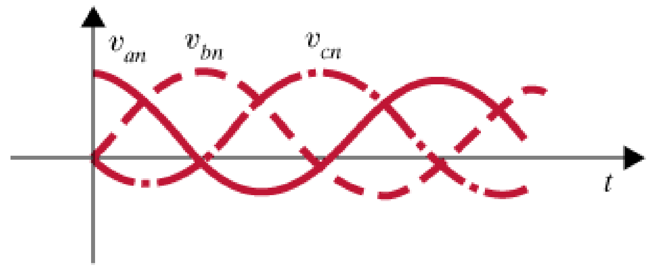
$V_{an} = 120\angle -60^\circ$

$V_{bn} = 120\angle -180^\circ$

$V_{an} = 120\angle 60^\circ$

The phasor diagram could be rotated by any angle





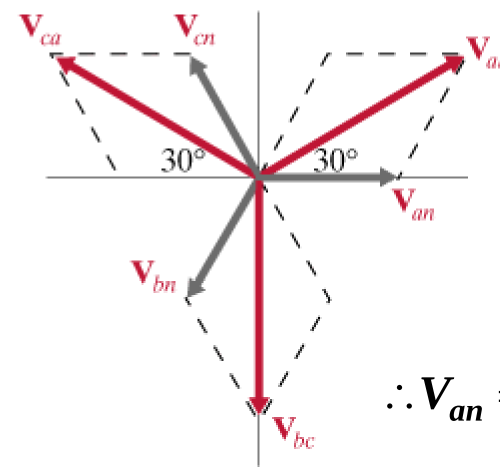
Positive sequence a-b-c

$V_{an} = |V_p|\angle 0^\circ$

$V_{bn} = |V_p|\angle -120^\circ$

$V_{cn} = |V_p|\angle 120^\circ$

Positive sequence phase voltages



$V_{ab} = \sqrt{3}|V_p|\angle 30^\circ$

V_{an} lags V_{ab} by 30°

$V_{ab} = 208\angle -30^\circ$

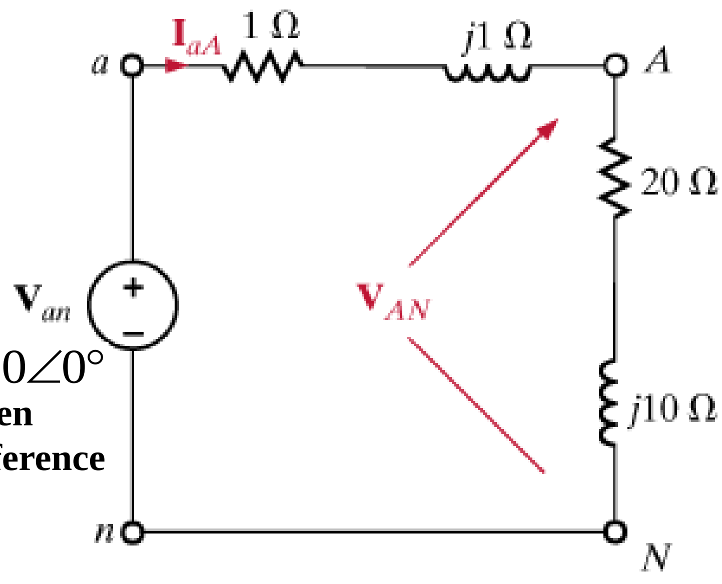
$\therefore V_{an} = \frac{|V_{ab}|}{\sqrt{3}}\angle(-30^\circ - 30^\circ)$

Relationship between phase and line voltages

Example: **For an abc sequence, balanced Y - Y three phase circuit**

$$\text{source } |V_{phase}| = 120(V)_{rms},\ Z_{line} = 1 + j1\Omega,\ Z_{phase} = 20 + j10\Omega$$

Determine line currents and load voltages.



$120\angle 0°$

Chosen as reference

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

Because circuit is balanced data on any one phase are sufficient

$$I_{bB} = 5.06\angle -120 - 27.65°(A)rms$$

$$I_{cC} = 5.06\angle 120 - 27.65°(A)rms$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{an} = 120\angle 120°$$

Abc sequence

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

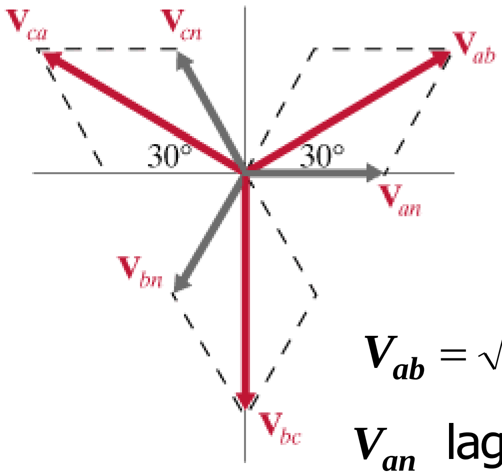
$$V_{BN} = 113.15\angle -121.08°(V)rms$$

$$V_{CN} = 113.15\angle 118.92°(V)rms$$

Example: **For an abc sequence, balanced Y - Y three phase circuit**

$\boldsymbol{V}_{an} = 120\angle 90°(\boldsymbol{V})\boldsymbol{rms}$. Find the line voltages

$\boldsymbol{V}_{ab}$ leads $\boldsymbol{V}_{an}$ by 30°



$$\boldsymbol{V}_{ab} = \sqrt{3}\times 120\angle 120°(\boldsymbol{V})\boldsymbol{rms}$$

$$\boldsymbol{V}_{bc} = \sqrt{3}\times 120\angle 0°(\boldsymbol{V})\boldsymbol{rms}$$

$$\boldsymbol{V}_{ca} = \sqrt{3}\times 120\angle 240°(\boldsymbol{V})\boldsymbol{rms}$$

$$\boldsymbol{V}_{ab} = \sqrt{3}\,|\boldsymbol{V}_p|\angle 30°$$

$\boldsymbol{V}_{an}$ lags $\boldsymbol{V}_{ab}$ by 30°

$\boldsymbol{V}_{ab} = 208\angle 0°(\boldsymbol{V})\boldsymbol{rms}$. Find the phase voltages

Relationship between phase and line voltages

$\boldsymbol{V}_{an}$ lags $\boldsymbol{V}_{ab}$ by 30°

$$\boldsymbol{V}_{an} = \frac{208}{\sqrt{3}}\angle -30°\ (\boldsymbol{V})\boldsymbol{rms}$$

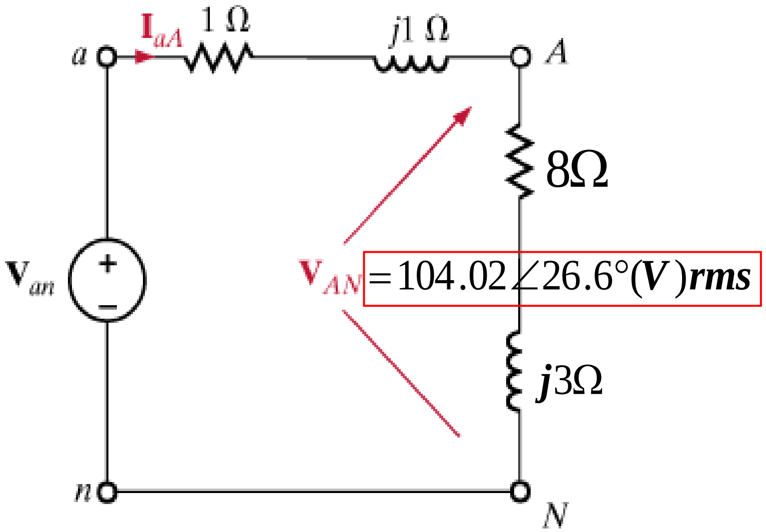
$$\boldsymbol{V}_{bn} = \frac{208}{\sqrt{3}}\angle -150°(\boldsymbol{V})\boldsymbol{rms}$$

$$\boldsymbol{V}_{cn} = \frac{208}{\sqrt{3}}\angle 90°(\boldsymbol{V})\boldsymbol{rms}$$

Example: **For an abc sequence, balanced Y - Y three phase circuit**

load, $|V_{phase}| = 104.02\angle 26.6°(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 8 + j3\Omega$

Determine source phase voltages



Currents are not required. Use inverse voltage divider

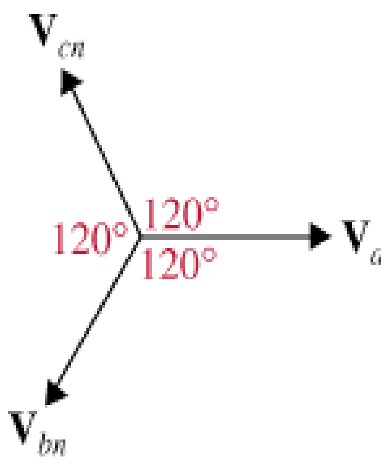
$$V_{an} = \frac{(8 + j3) + (1 + j1)}{8 + j3} V_{AN}$$

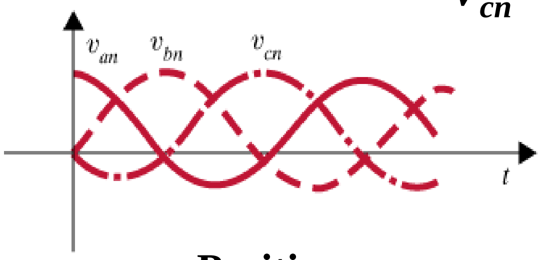
$$\frac{9 + j4}{8 + j3} \times \frac{8 - j3}{8 - j3} = \frac{84 + j5}{73} = 1.15\angle 3.41°$$

$$V_{an} = 120\angle 30°$$

$$V_{bn} = 120\angle -90°$$

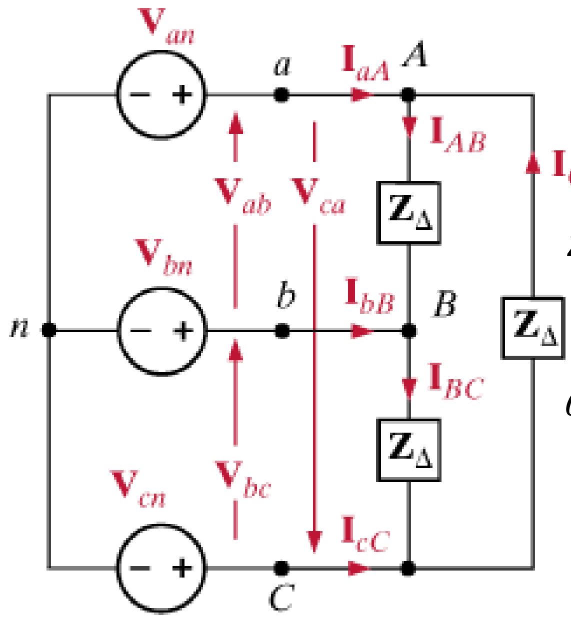
$$V_{cn} = 120\angle 150°$$





Positive sequence a-b-c

DELTA-CONNECTED LOAD



$$Z_\Delta = |Z_L| \angle \theta_Z$$

$$\theta_\Delta = 30° - \theta_Z$$

Load phase currents

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta - 120°$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta + 120°$$

Line currents

$$I_{aA} = I_{AB} - I_{CA}$$

$$I_{bB} = I_{BC} - I_{AB}$$

$$I_{cC} = I_{CA} - I_{BC}$$

Method 1: Solve directly

$$V_{an} = |V_p| \angle 0°$$

$$V_{bn} = |V_p| \angle -120°$$

$$V_{cn} = |V_p| \angle 120°$$

Positive sequence phase voltages

$$V_{ab} = \sqrt{3} |V_p| \angle 30°$$

$$V_{bc} = \sqrt{3} |V_p| \angle -90°$$

$$V_{ca} = \sqrt{3} |V_p| \angle -210°$$

$$|I_{line}| = \sqrt{3} |I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Method 2: We can also convert the delta connected load into a Y connected one. The same formulas derived for resistive circuits are applicable to impedances

Balanced case $Z_Y = \frac{Z_\Delta}{3}$

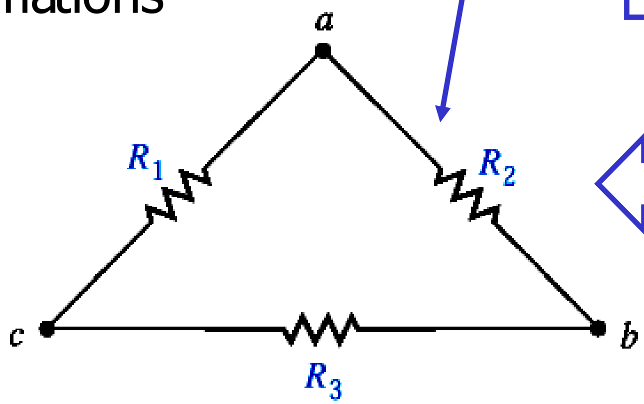
$$I_{aA} = \frac{V_{an}}{Z_Y} = |I_{aA}| \angle \theta_L \Rightarrow \begin{cases} |I_{aA}| = \dfrac{|V_{AB}|/\sqrt{3}}{|Z_\Delta|/3} \\ \theta_L = -\theta_Z \end{cases}$$

REVIEW OF

$\Delta \leftrightarrow$ Y

Transformations

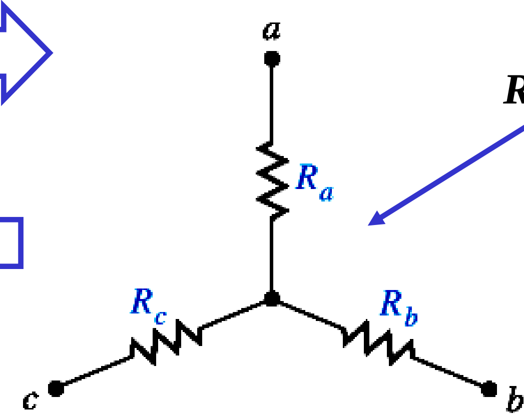
$R_{ab} = R_2 \,||\, (R_1 + R_3)$



Δ- Y

Y-Δ

$R_{ab} = R_a + R_b$



$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$\Delta \rightarrow Y$$

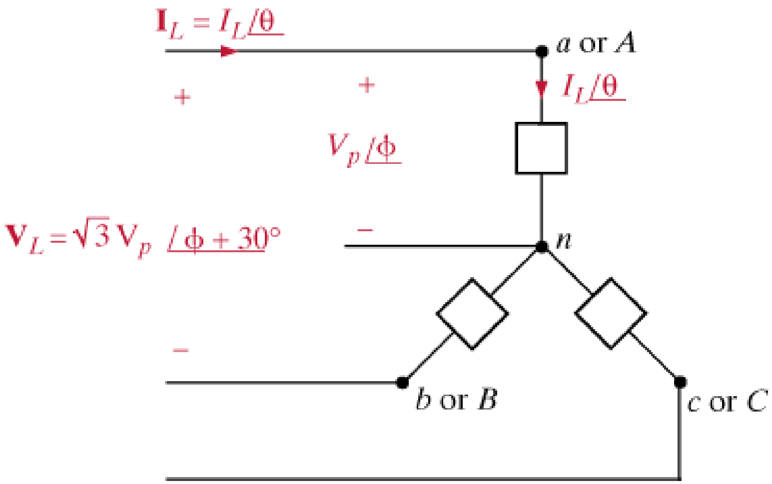
$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

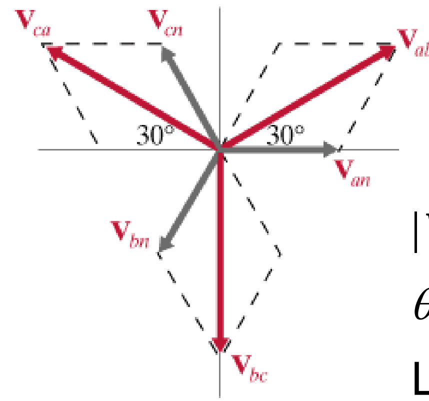
$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$Y - \Delta$$

$$R_\Delta = R_1 = R_2 = R_3 \Rightarrow R_Y = \frac{R_\Delta}{3}$$



(a)

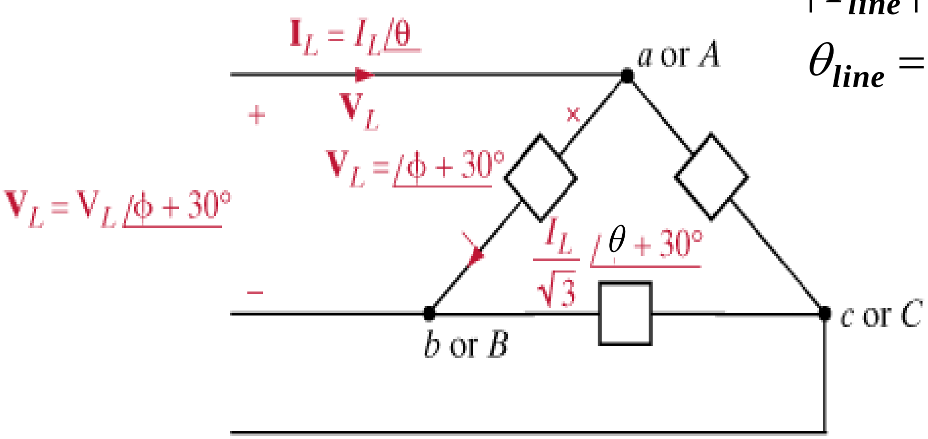


(b)

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line - phase voltage relationship



(b)

$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Example:

$I_{aA} = 12\angle 40°.$

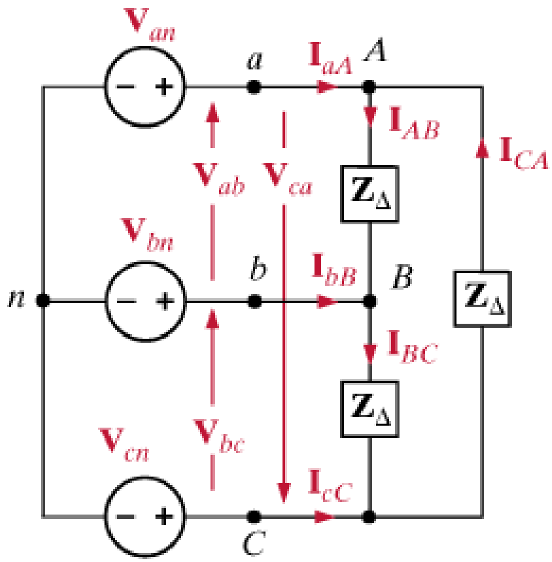
Find the phase currents

$I_{AB} = 6.93\angle 70°$

$I_{BC} = 6.93\angle -50°$

$I_{CA} = 6.93\angle 190°$

Example: **Delta-connected load consists of 10-Ohm resistance in series with 20-mH inductance. Source is Y-connected, abc sequence, 120-V rms, 60Hz. Determine all line and phase currents**



$$V_{an} = 120\angle 30°(V)rms$$

$$Z_{\text{inductance}} = 2\pi \times 60 \times 0.020 = 7.54\Omega$$

$$Z_\Delta = 10 + j7.54\Omega = 12.52\angle 37.02° \quad \Rightarrow Z_Y = 4.17\angle 37.02°$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{120\sqrt{3}\angle 60°}{10 + j7.54} = 16.60\angle 22.98°(A)rms$$

$$I_{BC} = 16.60\angle -97.02°(A)rms$$

$$I_{CA} = 16.60\angle 142.98°(A)rms$$

$$|V_\Delta| = \sqrt{3}|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$I_{aA} = 28.75\angle -7.02°(A)rms$$

$$I_{bB} = 28.75\angle -127.02°(A)rms$$

$$I_{cC} = 28.75\angle 112.98°(A)rms$$

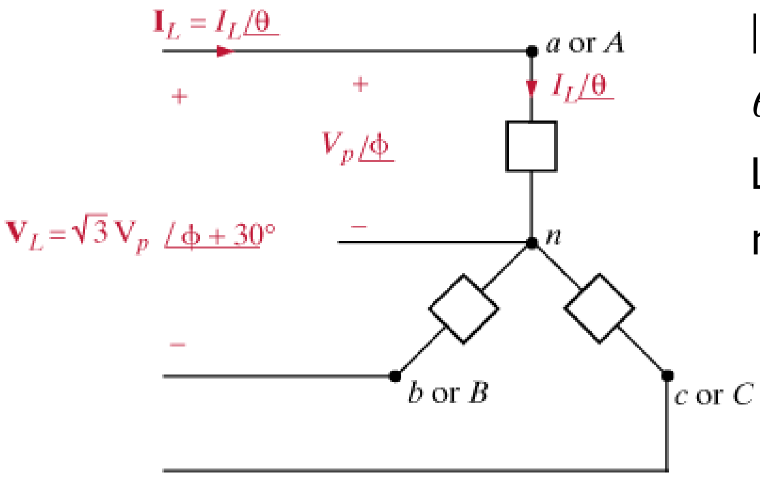
$$|I_{line}| = \sqrt{3}|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Alternatively, determine first the line currents and then the delta currents

POWER RELATIONSHIPS



(a)

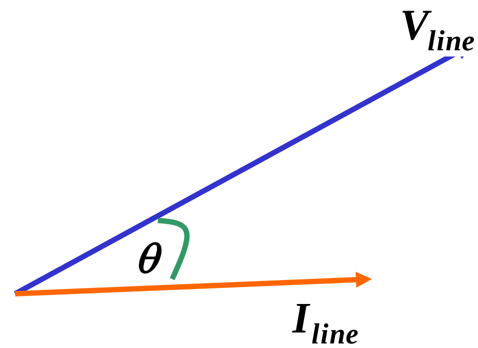
$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

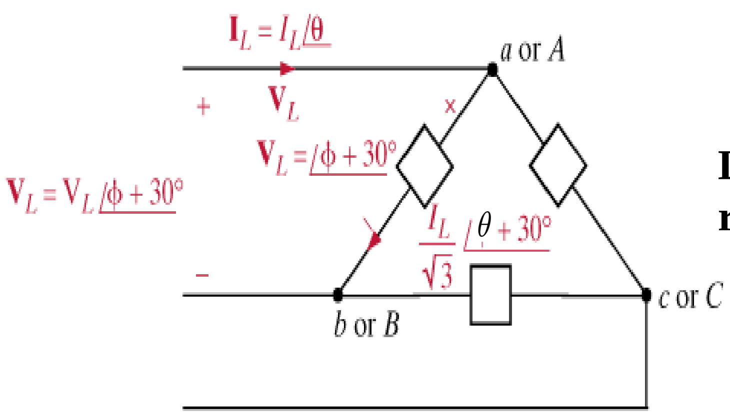
Line - phase voltage relationship

$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$



θ → **Impedance angle**, **Power factor angle**



(b)

$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

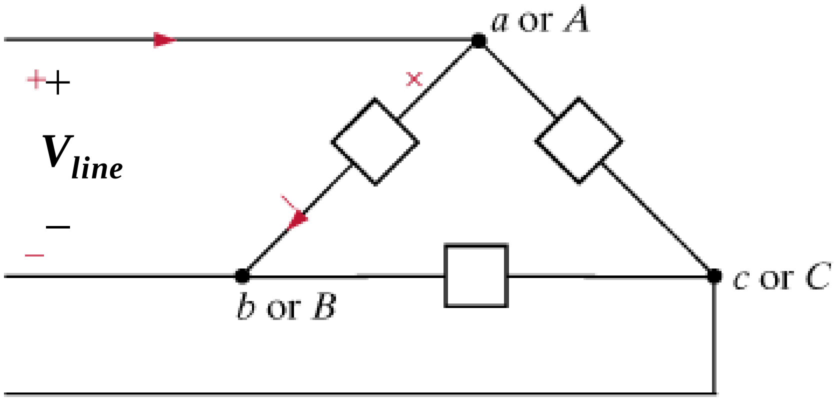
$$S_{total} = 3 V_{line} \times I^*_\Delta$$

$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$

$$P_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\sin\theta_f$$

<u>EXAMPLE:</u> Determine the magnitude of the line currents and the value of load impedance per phase in the delta



$$|V_{line}| = 208(V)rms$$

$$P_{total} = 1200W$$

$$\text{power factor angle} = 20° \text{ lagging}$$

$$|I_{line}| = \sqrt{3}\,|I_{\Delta}|$$

$$\theta_{line} = \theta_{\Delta} - 30°$$

Line-phase current relationship

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

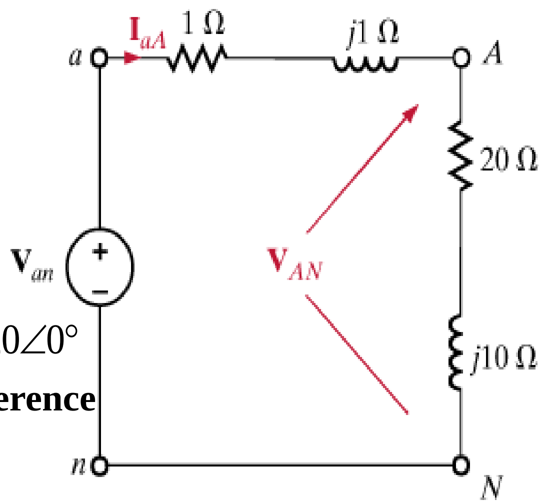
$$\frac{P_{total}}{3} = \frac{|V_{line}||I_{line}|}{\sqrt{3}}\cos\theta_f \quad \Rightarrow |I_{line}| = 3.54(A)rms$$

$$\Rightarrow |I_{\Delta}| = 2.05(A)rms \quad \Rightarrow |Z_{\Delta}| = \frac{|V_{line}|}{|I_{\Delta}|} = 101.46\Omega$$

Example: **For an abc sequence, balanced Y - Y three phase circuit**

$$\text{source } |V_{phase}| = 120(V)_{rms},\ Z_{line} = 1 + j1\Omega,\ Z_{phase} = 20 + j10\Omega$$

Determine real and reactive power per phase at the load and total real, reactive and complex power at the source



$120\angle 0°$

Reference

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$S_{phase} = V_{AN} I_{aA}^{*} = 113.15\angle -1.08° \times 5.06\angle 27.65°$$

$$S_{phase} = 572.54\angle 26.57° = 512 + j256.09(VA)rms$$

$$S_{\text{source phase}} = V_{an} \times I_{aA}^{*} = 120\angle 0° \times 5.06\angle 27.65°$$

$$S_{\text{source phase}} = 607.2\angle 27.65°$$

$$= 537.86 + j281.78VA$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{an} = 120\angle 120°$$

Because circuit is balanced data on any one phase are sufficient

Abc sequence

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

$$P_{\text{total source}} = 3 \times 537.86(W)$$

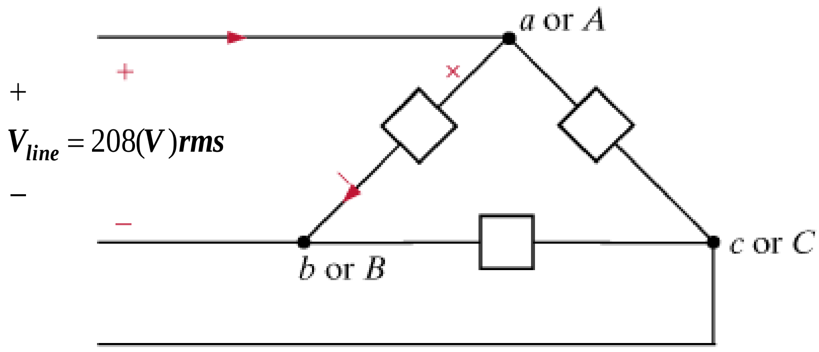
$$Q_{\text{total source}} = 3 \times 281.78(VA)$$

$$S_{\text{total source}} = P_{\text{total source}} + Q_{\text{total source}}$$

$$= 1613.6 + j845.2(VA)$$

$$|S_{\text{total source}}| = 1821.6(VA)$$

Example: Determine the line currents and the combined power factor



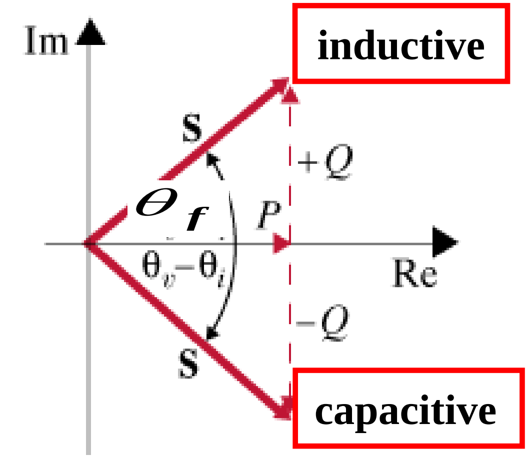
Circuit is balanced

Load 1 : 24kW at pf = 0.6 lagging

Load 2 : 10kW at pf = 1

Load 3 : 12kVA at pf = 0.8 leading

$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$
$$S_{total} = S_1 + S_2 + S_3$$



$$\left.\begin{matrix} P_1 = 24kW \\ pf = 0.6\,lagging \end{matrix}\right\} \Rightarrow |S_1| = 40kVA$$

$$|Q_1| = \sqrt{|S_1|^2 - |P_1|^2} = 32kVA$$

$$lagging \Rightarrow inductive \;\therefore S_1 = 24 + j32\,kVA$$

Load 2

$$\left.\begin{matrix} P_2 = 10kW \\ pf = 1 \end{matrix}\right\} \Rightarrow S_2 = 10 + j0\,kVA$$

Load 3

$$\left.\begin{matrix} |S_3| = 12kVA \\ pf = 0.8 \end{matrix}\right\} \Rightarrow \begin{cases} P_3 = 9.6kW \\ |Q_3| = 7.2kVA \end{cases}$$

$$leading\;\; pf \Rightarrow capacitive \;\therefore S_3 = 9.6 - j7.2kVA$$

$$S_{TOTAL} = S_1 + S_2 + S_3 = 43.6 + j24.8kVA = 50.160\angle 29.63^\circ kVA$$

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$
$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

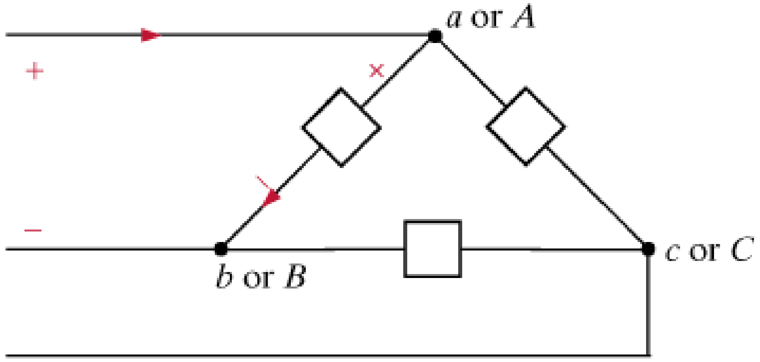
$$\Rightarrow \begin{cases} |S_{total}| = \sqrt{3}|V_{line}| \times |I_{line}| \\ \theta_f = 29.63^\circ \end{cases}$$

$$pf = 0.869\; lagging$$
$$|I_{line}| = 139.23(A)rms$$

Continued ...

EXAMPLE
continued ....

If the line impedances are $Z_{line} = 0.05 + j0.02\Omega$ determine line voltages and power factor at the source



$$|I_{line}| = 139.23(A) rms$$

$$S_{line} = 3 \times (Z_{line} I_{line}) I_{line}^{*} = 3 \times Z_{line} |I_{line}|^{2}$$

$$S_{line} = 2908 + j1163 (VA)$$

$$S_{\text{load total}} = 43.6 + j24.8 kVA = 50.160\angle 29.63° kVA$$

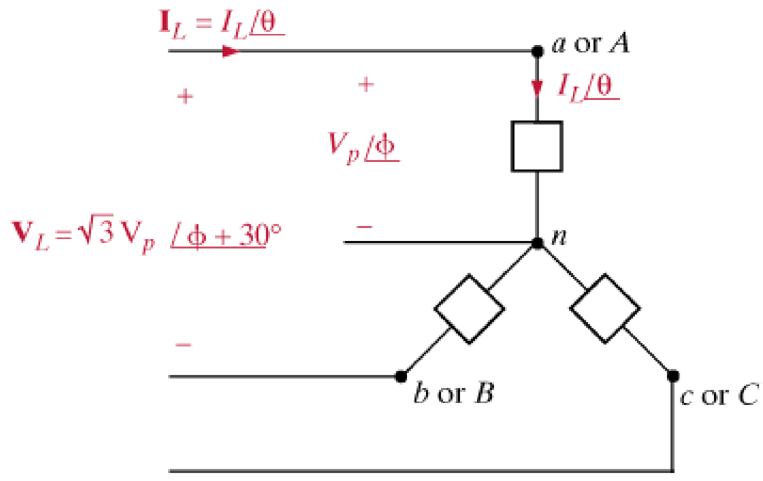
$$S_{\text{source total}} = 46.508 + j25.963 = 53.264\angle 29.17° kVA$$

$$\begin{cases} |S_{total}| = \sqrt{3}\,|V_{line}| \times |I_{line}| \\ \theta_f = 29.17° \end{cases}$$

$$V_{line} = \frac{53{,}264}{\sqrt{3} \times 139.13} = 220.87(V) rms$$

$$pf = \cos\theta_f = \cos(29.17°) = 0.873 \, lagging$$

<u>Example:</u> A Y -Y balanced three-phase circuit has a line voltage of 208-Vrms. The total real power absorbed by the load is 12kW at pf=0.8 lagging. Determine the per-phase impedance of the load



(a)

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

$$S = P + jQ$$

$$P = |S| \cos\theta_f$$

$$Q = |S| \sin\theta_f$$

$$pf = \cos\theta_f$$

$$\Rightarrow |V_{phase}| = \frac{208}{\sqrt{3}} = 120(V)rms$$

$$S_{total} = 3V_{phase} \times \left(\frac{V_{phase}}{Z_{phase}}\right)^* = 3 \times \frac{|V_{phase}|^2}{Z^*_{phase}}$$

$$pf = 0.8 = \cos\theta_f \Rightarrow \theta_f = 36.87°$$

$$|S_{total}| = \frac{P_{total}}{pf} = 15kVA$$

$$|Z_{phase}| = \frac{3 \times |V_{phase}|^2}{|S_{total}|} = 2.88\Omega$$

$$Z_{pahse} = 2.88\angle 36.87°\Omega$$

<u>Example:</u> A 480-V rms line feeds two balanced 3-phase loads. The loads are rated Load 1: 5kVA at 0.8 pf lagging Load 2: 10kVA at 0.9 pf lagging.

Determine the magnitude of the line current from the 408-V rms source

$$|S_1| = 5kVA = \frac{P}{0.8} \Rightarrow P_1 = 4kW$$

$$Q_1 = \sqrt{|S_1|^2 - P_1^2} = 3.0kVA$$

$$pf \ \ lagging \Rightarrow S_1 = 4 + j3kVA$$

$$|S_2| = 10kVA = \frac{P}{0.9} \Rightarrow P = 9kW$$

$$Q_2 = \sqrt{|S_2|^2 - P_2^2} = 4.36kVA$$

$$S_2 = 9 + j4.36kVA$$

$$S_{total} = 13 + j7.36kVA$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$

$$S_{total} = S_1 + S_2$$

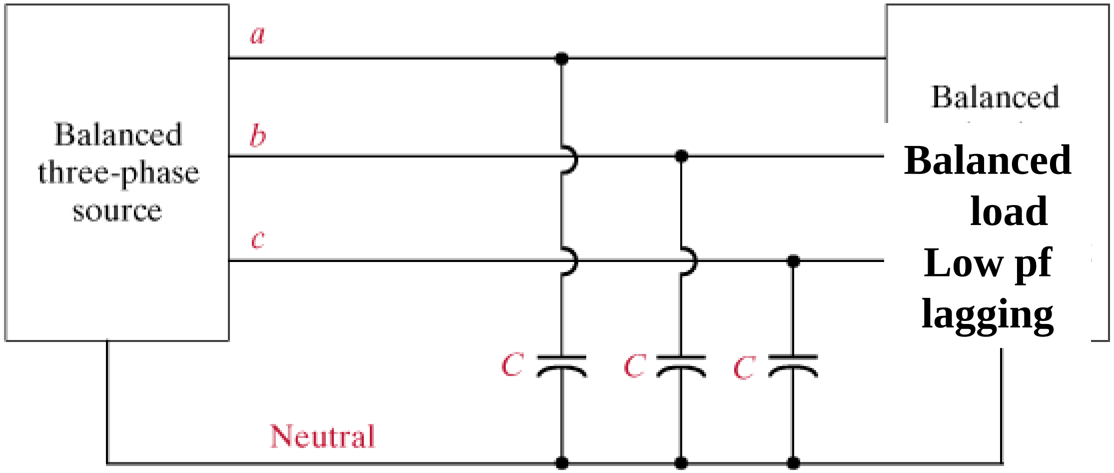
$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$|S_{total}| = \sqrt{3}|V_{line}||I_{line}|$$

$$|I_{lineq}| = \frac{|S_{total}|}{\sqrt{3} \times |V_{line}|} = \frac{14{,}939}{706.68} = 21.14(A)rms$$

POWER FACTOR CORRECTION



Similar to single phase case. Use capacitors to increase the power factor

Keep clear about total/phase power, line/phase voltages

$$\left.\begin{matrix} S_{old} \\ pf_{old} \end{matrix}\right\} \rightarrow Q_{old}$$

$$\Delta Q = Q_{new} - Q_{old}$$

Reactive Power to be added

$$Q_{Cphase} = \omega C V_{ph}^2$$

$$\left.\begin{matrix} P_{old} \\ pf_{new} \end{matrix}\right\} \rightarrow Q_{new}$$

$$S = P + jQ$$

$$P = |S| \cos\theta_f$$

$$Q = |S| \sin\theta_f$$

$$pf = \cos\theta_f$$

$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1 - pf^2}$$

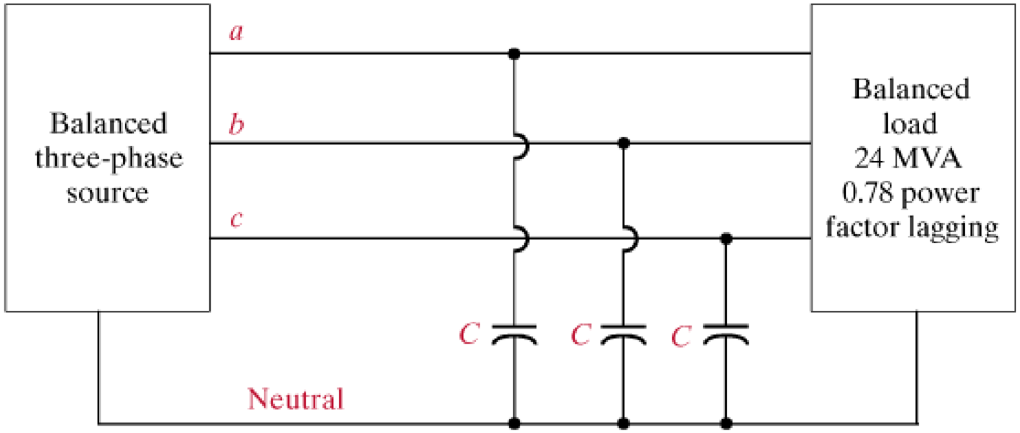
$$\tan\theta_f = \frac{pf}{\sqrt{1 - pf^2}} \qquad Q = P\tan\theta_f$$

$$Q_{\text{per capacitor}} = -\omega C V^2$$

The voltage depends on how the capacitors are connected

$$lagging \Rightarrow Q > 0$$

EXAMPLE:



***f*=60 Hz, V_{line}=34.5 kV$_{rms}$.**

Required *pf*=0.94 lagging.

$$P_{old} = 18.72\,MW$$
$$pf_{new} = 0.90\,lagging$$
$$\Rightarrow Q_{new} = -9.067\,MVA$$

$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$

$$Q = P\tan\theta_f$$

$$\tan\theta_f = \frac{pf}{\sqrt{1-pf^2}}$$

$$lagging \Rightarrow Q_{old} > 0$$

$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1-pf^2} = 0.626$$

$$|Q_{old}| = 15.02\,MVA$$
$$P_{old} = 18.72\,MW$$

$$\Delta Q = 9.067 - 15.02 = -5.953\,MVA$$

$$Q_{\text{per capacitor}} = -1.984\,MVA$$

$$Y - \text{connection} \Rightarrow V_{\text{capacitor}} = \frac{34.5}{\sqrt{3}}\,kV\ rms$$

$$Q_{Cphase} = \omega C V_{ph}^2$$

$$-1.984\times10^6 = -2\pi\times60\times C\times\left(\frac{34.5\times10^3}{\sqrt{3}}\right)^2$$

$$C = 13.26\,\mu F$$