Solution of second order ODEs :

general form ODEs :

$y'' + P(x)y' + q(x)y = g(x)$

method (1) : I have constant coeff.

final answear : $y(x) = y_c(x) + y_p(x)$

$y'' + a_1 y' + a_0 y = g(x)$

$r_{1,2} = \frac{-a_1 \mp \sqrt{a_1^2 - 4a_0}}{2}$, $y_c(x)$

case (1) : $r_1 \neq r_2$ (real roots)

$y_c(x) = Ae^{r_1 x} + Be^{r_2 x}$

case (2) : $r_1 = r_2 = r$ (real roots)

$y_c(x) = (A + Bx)e^{rx}$

case (3) : $r_1 \neq r_2$ (complex)

$r_1 = \alpha + \beta i$, $r_2 = \alpha - \beta i$

$y_c(x) = (A\cos\beta x + B\sin\beta x)e^{\alpha x}$, $y_p(x)$

if $g(x) = c \rightarrow y_p(x) = c/a_0$

if $g(x) = cx \rightarrow y_p(x) = c/a_0 [x - a_1/a_0]$

if $g(x) = cx^2 \rightarrow y_p(x) = c/a_0 [x^2 - \frac{2a_1}{a_0} + \frac{2(a_1^2 - a_0)}{a_0^2}]$

method (2) : $y'' + P(x)y' = g(x)$, $y'' = f(x, y') \rightarrow y'' + 2xy' = x^2$

. assume $P = y' \rightarrow P' = dP/dx = y'' \rightarrow$ نعوض بالمعادلة

. 1^{st} ODEs solved by intigrating factor .

$\mu = e^{\int P(x)dx}$, $P = \frac{1}{\mu}\int \mu g(x)dx \rightarrow \int P \, dy = \int dx$

method (3) : $y'' = f(x, y, y')$, we assume x does not exist

. $y' = P$, $y'' = PdP/dy$ $y'' = f(y, y') \rightarrow 1 + yy'' + y'^2 = 0$, $y'' + 2y' - y = 0$

$dy \sqrt{(\frac{\alpha}{y})^2 - 1} = dx \rightarrow (x+b)^2 + y^2 = \alpha^2$ *

method (4) : $xy'' = f(y', y/x) \rightarrow xy'' + y'^2 - (y/x)^2 = 0$

. assume $v = y/x \rightarrow y = xv$, $y' = xv' + v$, $y'' = xv'' + 2v'$ نعوض بالمعادلة

. assume $x = e^t$, $t = \ln x$ to get v

. $v' = dv/dt \cdot \frac{1}{x}$, $v'' = [\frac{d^2v}{dt^2} - \frac{dv}{dt}] \cdot (\frac{1}{x})^2 \rightarrow$ نعوض

. assume $P = \frac{dv}{dt}$, $\frac{d^2v}{dt^2} = \frac{dP}{dv} \cdot P(1 - 2v + ce^{-v})$, $\mu = e^{\int P(v)dv}$

. intigrating factor 1^{st} ODE $\rightarrow$ by parts $P = \frac{1}{\mu}\int \mu Q(v)dv$

final answear take it : $\frac{dy}{dx} - \frac{y}{x} = 1 - 2(\frac{y}{x}) + c_2 e^{(-y/x)}$

Fourier series :

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$ $-\pi \leq x \leq \pi$

$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[a_n\cos nx + b_n\sin nx]$

$+ a_2$ $+ b_2$

$+ a_3$ $+ b_3$ ...

. when $f(x)$ is an even function the $b_n = 0$, if it was odd function $a_n = 0$, $a_0 = 0$

the even $a_{n,0} = 2\int_0^{\pi}$

odd $\rightarrow x, x^3, \sin x, \tan x$

even $\rightarrow x^2, x^4, \cos x$

ODD * ODD $\rightarrow$ even

odd . even $\rightarrow$ odd

even * even $\rightarrow$ even

Partial Diff equ. :

. $b^2 - 4ac = ?$

. if $b^2 - 4ac < 0$ elliptic PDE

. if $b^2 - 4ac > 0$ hyperbolic PDE

. if $b^2 - 4ac = 0$ Parabolic PDE

$\underbrace{\frac{\partial^2 u}{\partial x^2}}_{a} + \underbrace{\frac{\partial^2 u}{\partial x \partial y}}_{b} + \underbrace{\frac{\partial^2 u}{\partial y^2}}_{c} = x\frac{\partial u}{\partial x}$

. separation of variables :

$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial y^2} \rightarrow c = f(t) \cdot g(y)$

$\frac{dc}{dt} = f'g$ نشتق بالنسبة لـ t .

$\frac{d^2c}{dy^2} = fg''$ نشتق بالنسبة لـ y .

$f'g = fg''$ نساوي الطرفين .

نوزع ع f نجعل $\frac{g''}{g}$ ثابت λ^2 .

$\frac{1}{f} \cdot \frac{df}{dt} = \lambda^2 \rightarrow \int \frac{1}{f}df = \int \lambda^2 dt$.

$f = \alpha e^{\lambda^2 t}$.

$T(t, x) = T(t)\,X(x)$ *

$T(t, x) = [A'\cos\lambda x + B'\sin\lambda x]e^{-\lambda^2 t}$

$Bc_1 \checkmark \rightarrow T(t, x) \rightarrow Bc_2$ $B' \neq 0 \rightarrow T(t, x)$, 2

$Ic \rightarrow B' \checkmark, n \checkmark \rightarrow T(t, x)$ النهائي

Laplace *

$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$, $T(t=0) = 0$, $L[\frac{\partial T}{\partial t}] = sT(s) - T(0)$

$\frac{d^2T}{dx^2} - sT(s) = 0 \rightarrow T = e^{rx}$, $T' = re^{rx}$, $T'' = r^2 e^{rx}$

$r^2 e^{rx} - se^{rx} = 0$, $r^2 - s = 0$, $r = \mp\sqrt{s}$

$T = Ae^{\sqrt{s}x} + Be^{-\sqrt{s}x}$, $L^{-1}[e^{-\sqrt{s}}] = \frac{a}{2\sqrt{\pi t^3}}e^{\frac{-1}{4t}}$

$L^{-1}[e^{\sqrt{s}}] = \infty \rightarrow T = A(\infty) + B\frac{a}{2\sqrt{\pi t^3}}e^{\frac{-1}{4t}x}$

ex : $dy_1/dt - 2y_1 = 1$, $dy_2/dt + y_2 = y_1$, $y_1(0) = 0$, $y_2(0) = 1$

$sy_1(s) - \cancel{y_1(0)} - 2y_1(s) = \frac{1}{s}$

$y_1(s) = 1/s(s-2) = A_1/s + B_1/s-2$

$y_1(t) = -1/2[1 - e^{2t}]$

$[sy_2(s) - \cancel{y_2(0)}] + y_2(s) = y_1(s)$

$[s+1]y_2(s) = 1/s(s-2) + 1/1$

$1 + s(s-2)/s(s-2) \rightarrow = \frac{s^2 - 2s + 1}{s(s-2)}$

$y_2(s) = \frac{s^2 - 2s + 1}{s(s+1)(s-2)} = \frac{A_1}{s} + \frac{B_1}{s+1} + \frac{c_1}{s-2}$

$A_1 = \lim_{s\to 0}(s \cdot \frac{s^2 - 2s + 1}{s(s+1)(s-2)}) = -\frac{1}{2}$

$c_1 = \lim_{s\to 2}((s-2)\frac{s^2 - 2s + 1}{s(s+1)(s-2)}) = 1/6$

$B_1 = \lim_{s\to -1}((s+1)\frac{s^2 - 2s + 1}{s(s+1)(s-2)}) = \frac{4}{3}$

$A_1 = -\frac{1}{2}$, $B_1 = \frac{1}{2}$

$y_2(s) = -\frac{1}{2}(\frac{1}{s}) + \frac{4}{3} \cdot \frac{1}{s+1} + \frac{1}{6}(\frac{1}{s-2})$

$y_2(t) = -\frac{1}{2} + \frac{4}{3}e^{-t} + \frac{1}{6}e^{2t}$

ex : $f(t) = \sin\omega t \rightarrow F(s) = \frac{\omega}{s^2 + \omega^2}$

$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$

$L[f(t)] = L[\sin\omega t] = \frac{1}{2i}L[e^{i\omega t} - e^{-i\omega t}]$

$= \frac{1}{2i}\int_0^{\infty} e^{-st}[e^{i\omega t} - e^{-i\omega t}]dt$

$= \frac{1}{2i}\int_0^{\infty} e^{-(s-i\omega)t}dt - \int_0^{\infty} e^{-(s+i\omega)t}dt$

$= \frac{1}{2i}(\frac{(s+i\omega) - (s-i\omega)}{s^2 + \omega^2})$

$= \frac{\omega}{s^2 + \omega^2}$

$= \omega^2 + s^2$

$(s - i\omega)(s + i\omega) = s^2 - i^2\omega^2 = s^2 \sqrt{-1}^2 \omega^2$

Laplace Transformation

Change t-domain $\rightarrow$ s-domain .

$L[f(t)] = \int_0^{\infty} e^{-st}f(t)dt = F(s)$

$u = t$ $dv = e^t dt$

$du = dt$ $v = e^t$

$uv|_0^{\infty} - \int_0^{\infty} v\,du$ * by parts

$F(s) = \frac{1}{(s-a)(s-b)} = \frac{A_1}{s-a} + \frac{B_1}{s-b}$

$A_m = \lim_{s\to a_1}[\frac{d^{m-1}}{ds^{m-1}}(s - a_1)^{\text{الأس}} F(s)] \cdot \frac{1}{(m-1)!}$

B
⋮

المعادلات :

. $\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$

. $\int_{-\pi}^{\pi}\cos nx\cos mx\,dx = 0$ $m \neq n$

. $\int_{-\pi}^{\pi}\cos nx\cos nx\,dx = \int_{-\pi}^{\pi}\cos^2 nx\,dx$

. $\int_{-\pi}^{\pi}\cos nx\sin mx = 0$ any value of n & m

. $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

. $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

. $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

. $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$\int e^{ax}\cos(bx) = \frac{e^{ax}[a\cos(bx) + b\sin(bx)]}{a^2 + b^2}$

$\int e^{ax}\sin(bx)dx = \frac{e^{ax}[a\sin(bx) - b\cos(bx)]}{a^2 + b^2}$

integral functions : $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$

1) error functions : $\mathrm{erf}(0) = 0$, $\mathrm{erf}(\infty) = 1$

$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\beta^2}d\beta$

$\mathrm{erf}(x) = 1 - (\alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3)e^{-x^2}$, $t = 1/1 + Px$

$P = 0.47047$, $\alpha_1 = 0.34802$, $\alpha_2 = -0.09587$, $\alpha_3 = 0.74785$

2) Gamma functions :

$\Gamma(n) = (n-1)!$, $\Gamma(x) = \int_0^x t^{x-1}e^{-t}dt$

3) Beta function :

$\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, $\beta(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$

- exact diff. equ. of ODE's :

$M(x,y)\,dx + N(x,y)\,dy = 0$

$M_y = N_x \to$ exact , $M_y \neq N_x$ non-exact

Steps :

1) intigrate M w.r.t x $\int M\,dx + h(y)$ constant

2) differentiate (1) w.r.t y

3) equate (2) w.r.t N

4) $\int h'(y)\,dy \rightsquigarrow$ solu. in (1)

integrating factor :

$dy/dx + P(x)y = Q(x)$ Poly Poly

Steps :

$\mu = e^{\int P(x)dx}$, $y = \frac{1}{\mu}\int \mu Q(x)\,dx$

non-exact equ. :

case (1) : $\mu = \mu(x)$ only

$\ln\mu - \ln c = -\int \frac{N_x - M_y}{N}\,dx$

. we multiply $\mu *$ (function)

exact $\to$ solu.

case (2) : $\mu = \mu(y)$ only

$\ln\mu = c\int \frac{N_x - M_y}{M}\,dy$

Bernoulli equation :

$dy/dx + P(x)y = Q(x)y^n$

$M(x) = e^{\int P_1(x)dx}$

$v = \frac{1}{\mu}\int \mu Q_1(x)\,dx$

$y = v^{n-1}$, $P_1(x) = (1-n)P(x)$

$Q_1(x) = (1-n)Q(x)$

Riccati equ. :

$dy/dx = P(x)y^2 + Q(x)y + R(x)$

special case : $p(x) = -1$

Steps :

1) we assume $y = \frac{1}{u}\frac{du}{dx}$

2) $u'' - Q(x)u' - R(x)u = 0$

3) $u = e^{rx}$, $u' = re^{rx}$, $u'' = r^2e^{rx}$

4) $u = c_1e^{rx} + c_2e^{rx}$

5) $du/dx = \dots$, 6) $y = \frac{du/dx}{u}$

linear equ. coeff : non-exact

$(ax+by+c)dx + (\alpha x + \beta y + \gamma)dy = 0$

1) $\ln x = -\int \frac{\alpha + \beta u}{\beta u^2 + (b+\alpha)u + a}\,du + c$

$u = y/x \to y$ ✓

First order of 2nd type :

$(dy/dx)^2 - \alpha\, dy/dx + \beta y = g(x)$

. assume $p = dy/dx$

. $p^2 - \alpha p + \beta y = g(x)$

. $p = \frac{\alpha \mp \sqrt{\alpha^2 - 4(\beta y - g(x))}}{2}$ dy/dx

. $u(x) =$ الجذر ما بداخل $\to \frac{du}{dx} = \square$

. $\int du = \int dx \rightsquigarrow + g - \{ \frac{y}{y}$

Solution of sys. of homogeneous linear ODE

. $AX = 0 \to \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix} = \begin{bmatrix} 0 \\ 0\end{bmatrix}$

. $Ax = \lambda I x$, $\lambda \to$ constant

. $(A - \lambda I)X = 0$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}$

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda\end{bmatrix} = \begin{bmatrix} 0 \\ 0\end{bmatrix}$

$\begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix} = \begin{bmatrix} 0 \\ 0\end{bmatrix}$

$(a_{11}-\lambda)(a_{22}-\lambda) - (a_{12})(a_{21}) = 0$ تربيعية معادلة

$\lambda_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} \rightsquigarrow$ eigenvalue

to get eigen vectors :

$\lambda_1 = \square \to (a)$ نعوض بـ

$x_1 + x_2 = 0$

$x_1 + x_2 = 0$ نحصل على معادلتين

$\begin{bmatrix} x_1 \\ x_2\end{bmatrix} = \alpha\begin{bmatrix} -1 \\ -1\end{bmatrix}$ باخذ العامل المشترك

$\lambda_2 = \square$ نعوض

$\begin{bmatrix} x_1 \\ x_2\end{bmatrix} = \beta\begin{bmatrix} \, \\ i\end{bmatrix}$

Solution of system of ODEs

case (1) : real + distinct eigen values

$\lambda_1, \lambda_2 \to \lambda_1 \neq \lambda_2$

$\begin{bmatrix} y_1 \\ y_2\end{bmatrix} = c_1\begin{bmatrix} A_1 \\ A_2\end{bmatrix}e^{\lambda_1 x} + c_2\begin{bmatrix} B_1 \\ B_2\end{bmatrix}e^{\lambda_2 x}$

first v second v

case (2) : Real + Repeated ($\lambda_1 = \lambda_2) = \lambda$

$\begin{bmatrix} y_1 \\ y_2\end{bmatrix} = c_1\begin{bmatrix} v_1 \\ v_2\end{bmatrix}e^{\lambda x} + c_2\left(x\begin{bmatrix} v_1 \\ v_2\end{bmatrix} + \begin{bmatrix} w_1 \\ w_2\end{bmatrix}\right)e^{\lambda x}$

case (3) : Complex roots [$\lambda_{1,2} = \alpha \mp \beta i$]

$\begin{bmatrix} y_1 \\ y_2\end{bmatrix} = \left\{c_1\left(\begin{bmatrix} P_1 \\ P_2\end{bmatrix}\cos\beta x - \begin{bmatrix} q_1 \\ q_2\end{bmatrix}\sin\beta x\right) + c_2\left(\begin{bmatrix} P_1 \\ P_2\end{bmatrix}\sin\beta x\right) + \begin{bmatrix} q_1 \\ q_2\end{bmatrix}\cos\beta x\right\}e^{\alpha x}$

case (1) : $dy_1/dx = 6y_1 - 3y_2$

$dy_2/dx = 2y_1 + y_2$

$A - \lambda I = \begin{bmatrix} 6-\lambda & -3 \\ 2 & 1-\lambda\end{bmatrix}$

$\begin{bmatrix} 6-\lambda & -3 \\ 2 & 1-\lambda\end{bmatrix}\begin{bmatrix} A \\ B\end{bmatrix} = \begin{bmatrix} 0 \\ 0\end{bmatrix}(a)$

λ_1, λ_2 ← نطلعهم

v_1 v_2 نعوض ← (a) بـ نعوضهم

$v_1 = \begin{bmatrix} A \\ B\end{bmatrix} = \alpha\begin{bmatrix} 3 \\ 2\end{bmatrix}$ ← خدنا أول

$v_2 = \begin{bmatrix} A \\ B\end{bmatrix} = \beta\begin{bmatrix} \, \\ \, \end{bmatrix}$ الثاني

we put them in case (1)

case (2) Same as case (1)

we get $\begin{bmatrix} v_1 \\ v_2\end{bmatrix}$ and $\begin{bmatrix} w_1 \\ w_2\end{bmatrix}$

$(A - \lambda I)w = v$ vector

$\begin{bmatrix} 0 & 13 \\ 0 & \, \end{bmatrix}\begin{bmatrix} w_1 \\ w_2\end{bmatrix} = \begin{bmatrix} v_1 \\ v_2\end{bmatrix} \to$ we get w_1, w_2

بعلاقة المعادلة

case (3) :

$\begin{bmatrix} P_1 \\ P_2\end{bmatrix} \to \begin{bmatrix} 1 \\ 0\end{bmatrix}$

$\begin{bmatrix} q_1 \\ q_2\end{bmatrix} \to \begin{bmatrix} 0 \\ 1/\sqrt{5}\end{bmatrix}i$

$v = \begin{bmatrix} 1 \\ i\end{bmatrix} = \begin{bmatrix} 1+0i \\ 0+i\end{bmatrix}$

$= \begin{bmatrix} 1 \\ 0\end{bmatrix} + \begin{bmatrix} 0 \\ 1\end{bmatrix}i$

Forbeneous method :

case (1) : $c_1 - c_2$ should be fraction

ex: $2xy'' + y' + y = 0 \leftrightarrow P_2y'' + P_1y' + P_0y = 0$

$P_2 = 2x$, $P_1 = 1$, $P_0 = 1$

$y = x^c$, $y' = cx^{c-1}$, $y'' = c(c-1)x^{c-2}$

$[2c(c-1) + c]x^{c-1} + x^c = 0$ g(c)

$f(c)x^{c-1} + g(c)x^c = 0$, $f(c) = 2c^2 - 2c + c = 0$

$c = 0$, $c = .5 \rightsquigarrow g(c) = 1$, $P = C - (c-1) = 1$

$y_1 = x^c + A_1x^{c+P} + A_2x^{c+2P} + \dots$

$= x^0 + A_1x^{0+1} + A_2x^{0+2(1)} + \dots$

$= 1 + A_1x + A_2x^2 + \dots$

$K = 0 \to A_1 = -A_0$, $K = 1 \to A_2 = A_0/6$

$K = 2 \to A_3 = -A_0/90$

$y_1 = A_0 - A_0x + \frac{A_0}{6}x^2 - \frac{A_0}{90}x^3$

$K = 0 \to B_1 = -B_0/3$

$K = 2 \to B_3 = -\frac{B_0}{630}$

$y_2 = B_0\, x^{\frac{1}{2}}\left[1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3\right]$

$y = y_1 + y_2$

case (2) $c_1 = c_2 = c$

ex: $xy'' + y' + xy = 0$, $P_2 = x$, $P_1 = 1$, $P_0 = x$

$y = x^c$, $y = cx^{c-1}$, $y'' = c(c-1)x^{c-2}$

$c^2x^{c-1} + (1)x^{c+1} = 0$, $c_1 = c_2 = 0$, $y_1 = A_0x^r + A_1x^{r+P}$

$+ A_2x^{r+2P} + A_3x^{r+3P}$

$K = 0 \to A_1 = -A_0/4$ $K = 1 \to A_2 = A_0/64$

$K = 2 \to A_3 = -A_0/2304$, $y_1 = A_0\left[1 - \frac{x^2}{4} + \frac{x^4}{64} + \dots\right]$

$y_2 = y_1\ln x + x^c\left[1 + \frac{2x^2}{(c+1)^2} - \frac{2x^4}{(c+2)^2(c+4)^2}\right]$

case (3) : $c_1 \neq c_2$ $c_1 - c_2$ integer

ex: $xy'' + y = 0$, $P_2 = x$, $P_1 = 0$, $P_0 = 1$

$y = x^c$, $y' = cx^{c-1}$, $y'' = c(c-1)x^{c-2}$

$c(c-1)x^{c-1} + x^c = 0$, $f(c) = c(c-1)$, $g(c) = 1$, $P = 1$

$y = A_0x^c + A_1x^{c+P} + A_2x^{c+2P}$

$K = 0 \to A_1 = -A_0/c(c+1)$

$K = 1 \to A_2 = A_0/c(c+1)^2(c+2)$

$y = A_0x^c - (A_0/c(c+1))\cdot x^{c+1} + \frac{A_0x^{c+2}}{c(c+1)^2(c+2)} + \dots$

$f(c) = 0 \to c_1 = 0$, $c_2 = 1$

$y_1 = A_0x[1 - x/2 + x^2/12]$, $y_2 = A_0x^c - \frac{A_0}{c(c+1)}x^{c+1} + \dots + (c - c_1)\Big[$ $c = 1$

$\frac{A_0x^{c+1}}{c(c+1)^2} + \frac{A_0x^{c+1}}{c^2(c+1)}\Big]$

a) $V_T = 9.42\,m^3$, $V_i = 5m^3$, $Q_{in} = 3m^3/h$, $C_{in} = 2mol/m^3$, $Q_{out} = 2m^3/h$

$Q_{in} - Q_{out} = dV/dt \rightsquigarrow dV/dt = 3 - 2 \to dV/dt = 1$, $\int_{V_0}^{V} dV = \int_0^t dt \to V = V_0 + t$

b) $9.42 = 5 + t \to t = 4.42h$

c) $\frac{d(VC)}{dt} = Q_{in}C_{in} - Q_{out}C_{out}$

$V\frac{dc}{dt} + C\frac{dV}{dt} = (3)(2) - 2(C) \to V\frac{dc}{dt} + C(1) = 6 - 2C \to \frac{dc}{dt}V = 6 - 3C$

$(V_0 + t)\frac{dc}{dt} = 6 - 3c \to \int dc/6 - 3c = \int \frac{dt}{5 + \cancel{V_0} + t}$

$-\frac{1}{3}\ln[6 - 3c] = \ln[5 + t] + K \to c = 2 - K(5 + t)^{-3}$

at $t = 5min \to c = .0975\ mol/m^3$

$3^x \rightsquigarrow L[3^x]$?

$= \int_0^\infty e^{-st}\cdot e^{(\ln 3)t}\,dt = \int_0^\infty e^{-(s-\ln 3)t}\,dt = \frac{1}{s - \ln 3}$

$\frac{e^x - e^{-x}}{2} = \sinh x$, $\frac{1}{2}[L[e^x] - L[e^{-x}]]$, $L[e^{ax}] = \frac{1}{s-a}$

$= \frac{1}{2}\left[\frac{1}{s-1} - \frac{1}{s+a}\right) = \frac{1}{s^2-1} \rightsquigarrow L[\sinh x] = \frac{1}{s^2-1}$

$L[y'] = sy(0) - y(0)$

$L[y''] = s^2y(s) - sy(0) - y'(0)$

$L[y'''] = s^3y(s) - s^2y(0) - sy'(0) - y''(0)$

$L[y''''] = s^4y(s) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0)$

$L[y'''''] = s^5y(s) - s^4y(0) - s^3y'(0) - s^2y''(0) - sy'''(0) - y^{IV}(0)$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

$y = y(s)$

$\mathcal{L}\left[\frac{dx}{dt}\right] = sX(s) - X(t=0)$

$\mathcal{L}\left[\frac{d^2x}{dt^2}\right] = s^2X(s) - sX(0) - X'(0)$

$\frac{1}{\alpha}\sin\alpha t \rightarrow \frac{1}{\alpha}\cdot\frac{\alpha}{s^2+\alpha^2}$, $? = \frac{1}{s(s^2+\alpha^2)}$

$k \rightarrow k/s$

$t \rightarrow 1/s^2$

$\frac{t^{n-1}}{(n-1)!} \rightarrow 1/s^n \quad n=1,2\ldots$

$\frac{1}{\sqrt{\pi t}} \rightarrow 1/\sqrt{s}$

$2\sqrt{\frac{t}{\pi}} \rightarrow s^{-3/2}$

$\frac{1}{\alpha}\sin\alpha t \rightarrow 1/s^2+\alpha^2$

$\cos\alpha t \rightarrow s/s^2+\alpha^2$

$\frac{1}{\alpha}\sinh(\alpha t) \rightarrow 1/s^2-\alpha^2$

PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$
$$\cos(\theta + 2\pi n) = \cos\theta$$
$$\tan(\theta + \pi n) = \tan\theta$$
$$\csc(\theta + 2\pi n) = \csc\theta$$
$$\sec(\theta + 2\pi n) = \sec\theta$$
$$\cot(\theta + \pi n) = \cot\theta$$

TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$
$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta$$
$$\sec(-\theta) = \sec\theta$$
$$\cot(-\theta) = -\cot\theta$$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$
$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$
$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$