Midterm

mathematical methods in chemical engineering first semester 2025

MADAR

MADAR TEAM

1- Solve the following differential equations

$$dy/dx = y / (1 + x) \quad \rightarrow \text{use separation}$$

$$dy/dx + x \cdot y - 1 = 0 \quad \rightarrow \text{integrating factor}$$

$$2yy' + y^2 + x + 1 = 0 \quad \rightarrow \text{exact method}$$

2) A perfectly mixed cylindrical tank (diameter = 2 m, height = 3 m, total volume = 9.42 m^3) initially contains 5 m^3 pure water.
A salty solution with concentration of 2 mol/m^3 is introduced into the tank at a fixed flow rate of 3 m^3/h. Simultaneously, the well-mixed liquid is withdrawn from the tank at a rate of 2 m^3/h.

a- Formulate a differential equation describing the variation of the tank volume
b- Determine the time required for the tank to become full.
c- determine the concentration of the outlet stream (the well-mixed liquid leaving the tank) after 5 minutes.

Midterm Solution: :: problem 1 ::

$\boxed{30/30}$

6 points 1) $dy/dx = y/1+x$ 1.5 $\rightarrow \int dy/y = \int dx/1+x$

$\rightsquigarrow \ln y = \ln(1+x) + c$ 1.5

$y = (1+x)e^{c_1}$

$y = c_2(1+x)$ 1.5

6 points 2) $dy/dx + xy - 1 = 0 \rightsquigarrow dy/dx + xy = 1$ 2

$\mu = e^{\int P(x)dx} = e^{\int x dx} = e^{\frac{x^2}{2}}$ 2

$y = \frac{1}{\mu}\int \mu Q(x) dx = \frac{1}{e^{\frac{x^2}{2}}}\int e^{\frac{x^2}{2}} dx = e^{-\frac{x^2}{2}}\int e^{\frac{x^2}{2}} dx$ 2

8 points 3) $2yy' + y^2 + x + 1 = 0 \rightarrow (2y\, dy/dx + y^2 + x + 1 = 0) * dx$

$\rightarrow \underbrace{2y}_{N} dy + (\underbrace{y^2 + x + 1}_{M}) dx = 0$

$\frac{\partial M}{\partial y} = 2y$, $\frac{\partial N}{\partial x} = 0 \rightsquigarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (non-exact)

to make it exact, $\mu = \mu(x) = c e^{\int \frac{M_y - N_x}{N} dx}$

$= c e^{\int \frac{2y - 0}{2y} dx}$

multiply by μ :: $\mu = ce^x$

$(\underbrace{y^2 + x + 1)e^x}_{M} dx + \underbrace{2ye^x}_{N} dy = 0$

$\frac{\partial M}{\partial y} = 2ye^x$, $\frac{\partial N}{\partial x} = 2ye^x \rightsquigarrow$ (exact)

* intigrate M w.r.t x ::

$F = \int (y^2 + x + 1)e^x dx$

$= e^x[y^2 + x] + h(y)$

* Diff. F w.r.t y

$\frac{\partial F}{\partial y} = 2ye^x + h'(y)$

* equate to N.

$2ye^x + h'(y) = 2ye^x$

$\rightarrow h'(y) = 0 \rightsquigarrow h(y) = c$

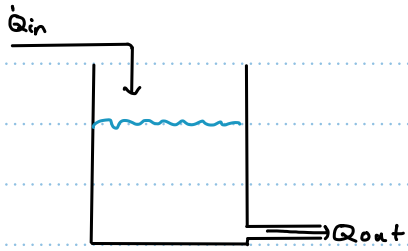
$\rightarrow$ Solu::

$F(x,y) = e^x[y^2 + x] = c$

$y = \pm\sqrt{ce^{-x} - x}$

10 points

problem 2 :



a)

$V_T = 9.42\,m^3$

$V_i = 5\,m^3$

$\dot{Q}_{in} = 3\,m^3/h \quad , \quad C_{in} = 2\,mol/m^3$

$\dot{Q}_{out} = 2\,m^3/h$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dV}{dt} \rightsquigarrow \frac{dV}{dt} = 3-2 \rightarrow \frac{dV}{dt} = 1$$

$$\int_{V_0}^{V} dV = \int_0^t dt \rightarrow \boxed{V = V_0 + t}$$

b) $\rightsquigarrow 9.42 = 5 + t \rightsquigarrow t = 4.42\,h$

c)

$$\frac{d(VC)}{dt} = \dot{Q}_{in} C_{in} - \dot{Q}_{out} C_{out}$$

$$V\frac{dC}{dt} + C\frac{dV}{dt} = (3)(2) - 2(C)$$

$$V\frac{dC}{dt} + C(1) = 6 - 2C \rightarrow V\frac{dC}{dt} = 6 - 3C$$

$$(V_0 + t)\frac{dC}{dt} = 6 - 3C \rightarrow \int \frac{dC}{6-3C} = \int \frac{dt}{V_0 + t}$$

5

$$-\frac{1}{3}\ln[6-3C] = \ln[5+t] + k \rightarrow C = 2 - k(5+t)^{-3}$$

at $t = 5\,min \rightarrow C = 0.0975\,mol/m^3$